MATHS

BOOKS - ML KHANNA

LIMITS, CONTINUITY AND DIFFERENTIABILITY

PROBLEM SET (1) (MULTIPLE CHOICE QUESTIONS)

1. $\lim_{x \to 0} \frac{\sin x}{x} =$

A. 0

B. 1

C. 2

D. none

Answer: B

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2. Deduction : $\lim_{x \to 0} \frac{\sin x^0}{x}$ =

A. 1

B. π

C. $\pi / 180$

D. none

Answer: C

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3. $\lim_{x \to \infty} \frac{\sin x}{x}$

A. 1

B. 0

C. ∞

D. $-\infty$

Answer: B

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4. If $f(x)=\sqrt{\frac{x+\sin x}{x+\cos^2 x}}$. Then $\lim_{x\to\infty} f(x)$ is:

A. 0

B. ∞

C. 1

D. none of these

Answer: C

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5. $\lim_{x\to\infty}\sqrt{\frac{x+\sin x}{x-\cos x}}=$

A. 0

B. 1

C. -1

D. none of these

Answer: B

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6. If $f(x) = x \sin(1/x), x \neq 0$, then $\lim_{x \to 0} f(x)$=

A. 1

B. 0

C. -1

D. not exist

Answer: B

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7. Evaluate: $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

A. 1

B. $\sqrt{2}$

C. 2

D. $-\sqrt{2}$

Answer: B

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8. $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} \times f(t) dt}{x^2 - \frac{\pi^2}{16}}$ equals

A. $\frac{8}{\pi} f(2)$

B. $\frac{2}{\pi} f(2)$

C. $\frac{2}{\pi} f\left(\frac{1}{2}\right)$

D. $4f(2)$

Answer: A

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9. $\lim_{x \to 0} \frac{2 \sin^2 3x}{x^2}$ =

A. 9

B. 2

C. 18

D. 3

Answer: C

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10. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

A. 0

B. $1/4$

C. $1/2$

D. 1

Answer: C

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11. $\lim_{x \to \pi} \frac{\sqrt{2+\cos x}-1}{(\pi - x)^2}$

A. $\frac{2}{\pi}$

B. π

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: C

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12. $Lt_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{x} =$

A. 0

B. 1

C. $\sqrt{2}$

D. doest not exist

Answer: D

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13. $\lim_{x \to 0} \frac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x} =$

A. $10/3$

B. $3/10$

C. $6/5$

D. $5/6$

Answer: A

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14. The function $f: R/\{0\} \rightarrow R$ is given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continous at x=0 by defining f(0) as:

A. 2

B. -1

C. 0

D. 1

Answer: D

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15. $Lt_{x \to 0} \frac{f(1 - \cos 3x)}{x^2}$ where $f(x)$ is a continous function satisfying the condition $f\left(\frac{9}{2}\right) = \frac{2}{9}$ is equal to:

A. 0

B. $9/2$

C. $2/9$

D. none

Answer: C

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16. Let α and β be the roots of $ax^2 + bx + c = 0$, then $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is euqal to:

A. 0

B. $\frac{1}{2}(\alpha - \beta)^2$

C. $\frac{\alpha^2}{2}(\alpha-\beta)^2$

D. $-\frac{\alpha^2}{2}(\alpha-\beta)^2$

Answer: C

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17. If α, β are the roots of $x^2-ax+b=0$, then $\lim_{x\to\alpha}\frac{e^{x^2-ax+b}}{x-\alpha}=$

A. $\alpha-\beta$

B. $\beta-\alpha$

C. 1

D. none

Answer: D

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18. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

A. $-\pi$

B. π

C. $\frac{\pi}{2}$

D. 1

Answer: B

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19. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} =$

A. 1

B. 2

C. $\frac{1}{2}$

D. -1

Answer: C

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20. $Lt_{x \to 0} \dfrac{\tan^{-1} x - \sin^{-1} x}{x^3}$ =

A. -1

B. $-1/2$

C. $1/2$

D. 1

Answer: B

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21. The value of $\lim_{x \to 1} \left\{ \dfrac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right\}^{\frac{1 - \cos(x-1)}{(x-1)^2}}$ is equal to:

A. e

B. $e^{1/2}$

C. 1

D. $\sqrt{5/6}$

Answer: D

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22. $\lim_{x \to \pi/4} \dfrac{\sqrt{2} - \cos\theta - \sin\theta}{(4\theta - \pi)^2} =$

A. $\dfrac{1}{16}$

B. $\dfrac{1}{16\sqrt{2}}$

C. $\dfrac{1}{8}$

D. $\dfrac{1}{8\sqrt{2}}$

Answer: B

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23. $\lim_{x \to 0} \frac{1 - \cos 2x}{x}$

A. 0

B. 1

C. 2

D. 4

Answer: A

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24. $\lim_{x \to \infty} x \cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$ equal to ?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. π

D. 0

Answer: A

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25. $Lt_{x \to 0} \dfrac{\log(3+x) - \log(3-x)}{x} = k$ then the value of :

A. 0

B. $-\dfrac{1}{3}$

C. $\dfrac{2}{3}$

D. $-\dfrac{2}{3}$

Answer: C

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26. $\lim_{x \to \infty} a^x \sin\left(\dfrac{b}{a^x}\right)$, where $a > 1$ =

A. b log a

B. a log b

C. b

D. a

Answer: C

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27. $Lt_{x \to 1}(1 + \cos \pi x)\cot^2 \pi x$=

A. -1

B. $1/2$

C. 1

D. none

Answer: B

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28. $Lt_{x \to 0} \frac{\sin \sqrt{\theta}}{\sqrt{\sin \theta}}$ =

A. 1

B. -1

C. 0

D. none

Answer: A

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29. $\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$ =

A. 1

B. 2

C. 0

D. none

Answer: A

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30. $\lim_{x \to 0} \frac{\sqrt[3]{1+\sin x} - \sqrt[3]{1-\sin x}}{x}$ =

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. 0

Answer: B

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31. $\lim_{h \to 0} \left[\frac{1}{h\sqrt[3]{8+h}} - \frac{1}{2h}\right]$ =

A. $\frac{1}{12}$

B. $-\frac{4}{3}$

C. $-\frac{16}{3}$

D. $-\frac{1}{48}$

Answer: D

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32. Evaluate $\lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$.

A. $a^2 \cos a + a \sin a$

B. $a^2 \cos a + 2a \sin a$

C. $2a^2 \cos a + a \sin a$

D. none

Answer: B

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33. $\lim_{x \to 0} \left[\frac{\sin(x+a) + \sin(a-x) - 2\sin a}{x \sin x}\right]$ =

A. sin a

B. cos a

C. $-\sin a$

D. $\frac{1}{2}\cos a$

Answer: C

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34. $\lim_{x \to 0} \frac{\sin nx[(a-n)nx - \tan x]}{x^2} = 0$, where n is non-zero positive integer, then a is equal to:

A. $(n+1)/n$

B. n^2

C. $\frac{1}{n}$

D. $n+\frac{1}{n}$

Answer: D

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35. Evaluate: $(\lim)_{h \vec{0}}\frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}$

A. $2/3$

B. $4/3$

C. $-2/\sqrt{3}$

D. $-4/3$

Answer: B

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36. The value of: $\lim_{n \to \infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\ldots\ldots\cos\left(\frac{x}{2^n}\right)$ is:

A. 1

B. $\frac{\sin x}{x}$

C. $\frac{x}{\sin x}$

D. none of these

Answer: B

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37. $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ is:

A. $\frac{1}{\sqrt{x}}$

B. $\sqrt{x}$

C. $2\sqrt{x}$

D. $\frac{1}{2\sqrt{x}}$

Answer: D

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38. $\lim_{x \to 2a} \frac{\sqrt{x-2a}+\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}} =$

A. $\frac{1}{2\sqrt{a}}$

B. $2\sqrt{a}$

C. $\sqrt{a}$

D. $\frac{1}{\sqrt{a}}$

Answer: A

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39. $\text{lt}_{n \to \infty} \frac{1-2+3-4+5-6+\ldots. -2n}{\sqrt{n^2+1}+\sqrt{4n^2+1}}$=

A. $-1/3$

B. $-1/5$

C. $1/3$

D. none

Answer: A

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40. $\lim_{x \to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}, a \neq 0 =$

A. $\frac{1}{3}$

B. $\frac{1}{3\sqrt{3}}$

C. $\frac{2}{3\sqrt{3}}$

D. 0

Answer: B

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41. $\lim_{x \to 3} \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$ =

A. 2

B. 3

C. 1

D. 0

Answer: C

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42. $\lim_{x \to \infty} \left[x - \sqrt{x^2 + x}\right]$ =

A. $1/2$

B. 1

C. $-1/2$

D. 0

Answer: C

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43. $\operatorname{lt}_{x \to \infty} \sqrt{x}\left(\sqrt{x+c}-\sqrt{x}\right)$ =

A. $c/2$

B. $c/3$

C. $c/4$

D. none

Answer: A

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44. $\lim_{x \to \infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$ is equal to

A. 0

B. $1/2$

C. log 2

D. e^4

Answer: B

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45. The value of $\lim_{x \to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$ is equal to:

A. 3

B. 4

C. 1

D. 2

Answer: A

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46. If $f(9) = 9, f'(9) = 4$ then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ =_____.

A. 3

B. 4

C. 1

D. 2

Answer: B

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47. If $f(a) = a^2, \phi(a) = b^2$ and $f'(a) = k\phi'(a\}$, then $\text{lt}_{x \to a} \frac{\sqrt{f(x) - a}}{\sqrt{\phi(x) - b}}$ is:

A. $k\frac{a}{b}$

B. $k\frac{b}{a}$

C. $\frac{a}{b}$

D. $\frac{b}{a}$

Answer: B

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48. $\lim_{x \to \infty} \frac{x^3 - 3x + 2}{2x^3 + x - 3}$ =

A. 2

B. $1/2$

C. 0

D. not exist

Answer: B

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49. $\lim_{x \to \infty} \frac{\sqrt{x^2 - 1}}{2x + 1}$=

A. 1

B. 0

C. -1

D. $1/2$

Answer: D

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50. $\lim_{x \to \infty} \frac{(2+x)^{40}(4+x)^5}{(2-x)^{45}}$ =

A. -1

B. `1

C. 16

D. 32

Answer: A

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51. Evaluate : $(\lim)_{x \to \frac{\pi}{4}} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$

A. $\frac{3}{4}$

B. $\frac{11}{4}$

C. $\frac{1}{2}$

D. none of these

Answer: A

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52. If $f(x) = \frac{2}{x-3}, g(x) = \frac{x-3}{x+4}, and h(x) = -\frac{2(2x+1)}{x^2+x-12}, then (\lim)_{x \to 3}[$ is -2 (b) -1 (c) $-\frac{2}{7}$ (d) 0

A. -2

B. -1

C. $-2/7$

D. 0

Answer: C

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53. The value of $\lim_{x \to \infty} \sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2}$ is:

A. $1/2$

B. 1

C. 2

D. none of these

Answer: A

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54. $\lim_{x \to \infty} \frac{\sqrt[4]{x^5+2} - \sqrt[3]{x^2+1}}{\sqrt[5]{x^4+2} - \sqrt[2]{x^3+1}} =$

A. -1

B. 0

C. 1

D. ∞

Answer: A

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55. $\lim_{x \to -\infty} \sqrt{x^2-x+1} - ax + b = 0$, then (a,b) is:

A. $\left(-1, \frac{1}{2}\right)$

B. $\left(1, -\frac{1}{2}\right)$

C. $(-1, 2)$

D. none of these

Answer: B

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56. The values of constants a and b so that

$$\lim_{x \to \infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=0,$$ are

A. a=0, b=0

B. $a=1, b=-1$

C. $a=-1, b=1$

D. $a=2, b=-1$

Answer: B

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57. The value of $\lim_{x \to \infty}\left(\frac{x^2\sin(1/x)-x}{1-|x|}\right)$ is:

A. 0

B. 1

C. -1

D. none of these

Answer: A

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58. $\lim_{x \to 0} \frac{x \cos x - \sin x}{x^2 \sin x} =$

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

Answer: C

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59. $\lim_{x \to 0} \dfrac{x \cos x - \sin x}{x^2 \sin x}$ equal to ?

A. $\dfrac{1}{2}$

B. $-\dfrac{1}{2}$

C. $\dfrac{1}{3}$

D. $-\dfrac{1}{3}$

Answer: D

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60. If $\lim_{x \to 0} \dfrac{x(1 + a \cos x) - b \sin x}{x^3} = 1$, then a,b are

A. $\dfrac{1}{2}, -\dfrac{3}{2}$

B. $\dfrac{5}{2}, \dfrac{3}{2}$

C. $-\dfrac{5}{2}, -\dfrac{3}{2}$

D. none

Answer: C

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61. Let $L = \lim_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$. If L is finite, then

A. a=2

B. a=1

C. $L = \dfrac{1}{64}$

D. $L = \dfrac{1}{32}$

Answer: A::C

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62. If $Lt_{x \to \infty}\left(\dfrac{x^2 - 1}{x + 1} - ax - b\right) = 2$, then (a,b) =

A. (1,3)

B. $(\,-1, 3)$

C. $(1,\ -3)$

D. none

Answer: C

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63. If $\mathrm{lt}_{x \to \infty}\left[\frac{x^2+1}{x+1} - ax - b\right] = \infty$, then (a,b) =

A. [1,2]

B. (2,3}

C. (3,4)

D. none

Answer: D

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64. If $\operatorname{lt}_{x \to \infty}\left(\frac{x^3+1}{x^2+1} - ax - b\right) = 2$, then (a,b) =

A. (1,-2)

B. (1,2)

C. (1,1)

D. none

Answer: A

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65. $\lim_{x \to \infty} \frac{x \cos x - \log(1+x)}{x^2}$ is equal to:

A. $\frac{1}{2}$

B. 0

C. 1

D. none of these

Answer: A

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66. $\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{x^2}\log(1+x)\right]$ =

A. 1

B. $1/2$

C. -1

D. none

Answer: B

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67. The value of $\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to:

A. $\frac{1}{5}$

B. $\frac{1}{6}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: B

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68. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$, is

A. 2

B. -2

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: C

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69. If $\sum_{r=1}^{k} \cos^{-1} \beta_r = \frac{k\pi}{2}$ for any $k \geq 1$ and $A = \sum_{r=1}^{k} (\beta_r)^r$ then $\lim_{x \to A} \frac{(1+x)^{\frac{1}{3}} - (1-2x)^{\frac{1}{4}}}{x + x^2}$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{7}$

Answer: A

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70. $\lim_{x \to \pi/2} \frac{2x - \pi}{\cos x}$ =

A. 1

B. 2

C. -2

D. `0

Answer: C

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71. $\lim_{\theta \to \pi/2} (\sec\theta - \tan\theta) =$

A. 0

B. 1

C. -1

D. 2

Answer: A

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72. $\lim_{x \to 0} x \log \sin x =$

A. 1

B. -1

C. 0

D. none

Answer: C

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73. $\lim_{x \to \pi/2} \tan x \log_e \sin x =$

A. 1

B. -1

C. 0

D. none

Answer: C

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74. $\lim_{x \to \pi/2}\left[x\tan x - \left(\frac{\pi}{2}\right)\sec x\right]$ =

A. 1

B. -1

C. 0

D. none

Answer: B

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75. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2,$ then the value of $(\lim)_{x \vec{a}} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is -5 (b) $\frac{1}{5}$ (c) 5 (d) none of these

A. -5

B. $\frac{1}{5}$

C. 5

D. none of these

Answer: C

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76. If $f(2) = 4$ and $f'(2) = 4$, then $\text{lt}_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2}$ =

A. -4

B. -2

C. 2

D. 3

Answer: A

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77. Let $f(a) = g(a) = k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if

$$\mathrm{lt}_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4,$$ then the value of k =

A. 0

B. 1

C. 2

D. 4

Answer: D

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78. $f(x)$ is a differentiable functions given $f'(1) = 4, f'(2) = 6$, where $f'(c)$ means the derivatives of function at x = c, then

$$\lim_{h \to 0} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$$

A. not exist

B. -3

C. 3

D. $3/2$

Answer: C

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79. f(x) is differentiable, increasing functions, then $\lim_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is equal to:

A. 2

B. 1

C. -1

D. 0

Answer: C

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80. Suppose $f: R \to R$ is a differential function and f(1) =4. Then the value of :

$\lim_{x \to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is:

A. $8f'(1)$

B. $4f'(1)$

C. $2f'(1)$

D. $f'(1)$

Answer: A

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81. If $G(x) = -\sqrt{25-x^2}$, then $\lim_{x \to 1} \frac{G(x)-G(1)}{x-1}$ has the value of:

A. $1/24$

B. $1/5$

C. $-\sqrt{24}$

D. none of these

Answer: D

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82. Let $f(x) = \dfrac{1}{\sqrt{18 - x^2}}$. The value of: $\lim_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$ is:

A. 0

B. $-1/9$

C. $-1/3$

D. $1/9$

Answer: D

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83. Let $f(x) = \cot^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ and $g(x) = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\lim_{x \to t} \frac{f(x) - f(t)}{g(x) - g(t)}$ is equal to :

A. $\frac{3}{2(1 + t^2)}$

B. $-\frac{3}{2}$

C. $\frac{3}{2}$

D. $-\frac{3}{2(1 + t^2)}$

Answer: C

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84. $\lim_{x \to 0} \frac{\log \cos x}{x}$ is equal to:

A. 0

B. a=0

C. a=e

D. none of these

Answer: A

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85. $\lim_{x \to 0} \frac{\log \cos x}{x}$ is equal to:

A. 1

B. 0

C. e

D. none of these

Answer: B

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86. $\text{lt}_{x \to \infty} \frac{3^{x+1} - 5^{x+1}}{3^x - 5^x} =$

A. -5

B. $1/5$

C. 5

D. none

Answer: C

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87. $\text{lt}_{x \to 2} \dfrac{2^x - x^2}{x^x - 2^x}$ =

A. $\dfrac{\log 2 - 1}{\log 2 + 1}$

B. $\dfrac{\log 2 + 1}{\log 2 - 1}$

C. 1

D. -1

Answer: A

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88. $\lim_{x \to \pi/4} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ equals

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{2\sqrt{2}}$

D. 1

Answer: B

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89. $\lim_{x \to \pi/4} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$ equals:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{2\sqrt{2}}$

D. 1

Answer: D

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90. $\lim_{x \to a} \frac{x^n - a^n}{x - a}$=

A. na^n

B. na^{n-1}

C. 0

D. does not exist

Answer: B

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91. $\lim_{x \to a} \left(\frac{\cos x - \cos a}{\cot x - \cot a}\right)$=

A. $\frac{1}{2}\sin^3 a$

B. $\frac{1}{2}\operatorname{cosec}^3 a$

C. $\sin^3 a$

D. $\operatorname{cosec}^3 a$

Answer: C

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92. $\lim (t \to 0) \frac{\sqrt{\cos t} - \sqrt[3]{\cos t}}{\sin^2 t} =$

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $-\frac{1}{12}$

D. $\frac{2}{3}$

Answer: C

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93. If $f(x)=\frac{\sin(e^{x-2}-1)}{\log(x-1)}$ then $\lim_{x\to 2}(f(x))$ is given

A. -2

B. -1

C. 0

D. 1

Answer: D

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94. $\lim_{x\to -1}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is given by:

A. $\frac{1}{\sqrt{\pi}}$

B. $\frac{1}{\sqrt{2\pi}}$

C. 1

D. 0

Answer: B

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95. $Lt_{x \to a}\sqrt{(a^2 - x^2)}\cot\left\{\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right)\right\} =$

A. $-\frac{4a}{\pi}$

B. $-\frac{2a}{\pi}$

C. $\frac{2a}{\pi}$

D. $\frac{4a}{\pi}$

Answer: D

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96. $\mathrm{lt}_{x \to \infty}\left[\sqrt{x+\sqrt{x}} - \sqrt{x}\right]$ =

A. $1/2$

B. 1

C. 0

D. none

Answer: A

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97. If $\lim_{x \to 1} \sec^{-1}\left[\frac{\lambda^2}{\log x} - \frac{\lambda^2}{x-1}\right]$ exists then $\lambda \in$

A. $[-\infty, -\sqrt{2}]$

B. $[\sqrt{2}, \infty]$

C. $[-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty]$

D. none

Answer: A::B::C

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98. The value of $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$ is:

A. 1

B. -1

C. 0

D. none of these

Answer: A

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99. $\lim_{x \to 0} \left(\frac{\log_{\sec(x/2)} \cos x}{\log_{\sec x} \cos\left(\frac{x}{2}\right)} \right) =$

A. 2

B. 4

C. 8

D. 16

Answer: D

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100. The value of $(\lim)_{x \overrightarrow{\infty}}\left(\frac{\cos x}{m}\right)^{m}$ is 1 (b) e (c) e^{-1} (d) none of these

A. 1

B. e

C. e^{-1}

D. none of these

Answer: A

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101. $\text{lt}_{n \to \infty}\left[\tan\left\{\frac{\pi - 4}{4} + \left(1 + \frac{1}{n}\right)^{a}\right\}\right]^{n}$ =

A. e^{a}

B. e^{2a}

C. e^{0}

D. does not exist

Answer: B

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102. The value of $\lim_{x \to 1} \frac{x + x^2 + \dots . + x^n - n}{x - 1}$ is

A. n

B. $\frac{n+1}{2}$

C. $\frac{n(n+1)}{2}$

D. $\frac{n(n-1)}{2}$

Answer: C

Watch Video Solution

103. $\lim_{n \to \infty} \log_{n-1}(n)\log_{n}(n+1)\ldots\log_{n^k-1}\left(n^k\right)$ is equal to:

A. k

B. 2k

C. 3k

D. 4k

Answer: A

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104. $\lim_{x \to 0} \frac{1}{x}\left[\int_{y}^{0} e^{\sin^2 t}dt - \int_{x+y}^{0} e^{\sin^2 t}dt\right]$ is equal to:

A. $e^{\sin^2 y}$

B. $\sin 2ye^{\sin^2}y$

C. 0

D. none of these

Answer: A

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105. $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{2/x}$, $a, b, c > 0$ is equal to:

A. abc

B. $a^2b^2c^2$

C. $(abc)^{2/3}$

D. $(abc)^2$

Answer: C

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106. $\lim_{x \to 9} [\log_2 \log_3 x]^{\frac{1}{x-9}}$ =

A. 1

B. 0

C. ∞

D. $-\infty$

Answer: C

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107. If the normal of $f(x)=0$ at x=0 is given by $3x-y+3=0$, then $\lim_{x\to 0}\frac{x^2}{f(x^2)-5f[4x^2)+4f(7x^2)}$ is equal to:

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. 0

Answer: A

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108. Let $f''(x)$ be continuous at x = 0 and $f''(0) = 4$, Then value of $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is:

A. 1

B. 2

C. 3

D. 0

Answer: C

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109. $\lim_{x \to \pi/2} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \log \sin x}$ =

A. 1

B. 2

C. 3

D. 0

Answer: B

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110. $\lim_{x \to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1} =$

A. log 2

B. 2 log 2

C. $\frac{1}{2}\log 2$

D. 0

Answer: B

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111. $\text{lt}_{x \to 0} \dfrac{1 - \cos x}{x(2^x - 1)}$

A. $\dfrac{1}{2}\log_e 2$

B. $\dfrac{1}{2}\log_2 e$

C. 1

D. none

Answer: B

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112. $\lim_{x \to 0} \dfrac{a^x - b^x}{x} =$

A. log (ab)

B. $\dfrac{\log a}{b}$

C. $\dfrac{\log b}{a}$

D. none

Answer: B

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113. $\mathrm{lt}_{x \to 0} \dfrac{1 - 3^x - 4^x + 12^x}{\sqrt{2 \cos x + 7} - 3}$ is:

A. 2 log 4. log 3

B. $-6 \log 4. \log 3$

C. 3 log 4. log 3

D. none

Answer: B

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114. $\mathrm{lt}_{x \to 0} \dfrac{8^x - 4^x - 2^x + 1^x}{x^2}$ =

A. 2 log 2

B. $2(\log 2)(2)$

C. log 2

D. none

Answer: B

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115. $\operatorname{lt}_{x \to 0} \dfrac{64^x - 32^x - 16^x + 4^x + 2^x - 1}{\left[\sqrt{15 + \cos x} - 4\right] \sin x} =$

A. $-96(\log 2)^2$

B. $48(\log 2)^2$

C. $(2 \log 2)^3$

D. none

Answer: D

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116. $\mathrm{lt}_{x \to 0} \dfrac{8^x - 4^x - 2^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} =$

A. $4\sqrt{2}\log 2$

B. $8\sqrt{2}\log 2$

C. $8\sqrt{2}(\log 2)^2$

D. none

Answer: C

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117. $\lim_{x \to \infty} [1 + 2/x)]^x$ equals:

A. e

B. ∞

C. e^2

D. $1/e$

Answer: C

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118. The value of $\mathrm{lt}_{x \to 1}[\log_5 5x]^{\log_x 5}$ is:

A. 1

B. -1

C. e

D. e^{-1}

Answer: C

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119. The value of $\mathrm{lt}_{x \to 1}(\log_2 2x)^{\log_x 5}$ is:

A. $\log_2 5$

B. $\frac{5}{2}$

C. $e^{\log_2 5}$

D. none

Answer: C

Watch Video Solution

120. $\lim_{x \to 0} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x} =$

A. e

B. e^2

C. $\sqrt{e}$

D. $1/\sqrt{e}$

Answer: B

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121. If a $\lim_{x \to 1} x^{1/(1-x)} + b = e^{-1}$ where $a \geq 1$ and $b \geq 0$ then (a,b) is equal to:

A. $(1, e^{-1})$

B. $(2, e^{-1})$

C. (1,0)

D. none of these

Answer: C

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122. If $\mathrm{lt}_{x \to \infty} \left[1 + \frac{a}{x} + \frac{b}{x^2}\right]^{2x} = e^2$, then (a,b) is:

A. (2,1)

B. (1,2)

C. [1, any real constant]

D. (1,1)

Answer: C

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123. If $\mathrm{lt}_{x \to \infty}\left\{\sqrt{x^2 - x + 1} - ax - b\right\} = 0$, then (a,b)=

A. $\{-1, 1/2\}$

B. $\{1, 1/2\}$

C. $[1, -1/2]$

D. none

Answer: A

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124. If $\lim_{x \to 0} (1+ax)^{b/x} = e^2$, where $a, b \in N$ such that $a+b=3$, then the value of (a,b) is equal to:

A. (16,8)

B. (8,4)

C. (2,1)

D. (1,2)

Answer: C::D

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125. If a,b,c,d are positive, then $\lim_{x \to \infty} \left(1+\frac{1}{a+bx}\right)^{c+dx}$

A. $e^{d/b}$

B. $e^{c/a}$

C. $e^{(c+d)/(a+b)}$

D. e

Answer: A

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126. The value of $\lim_{x \to 0} [\cos x + a \sin bx]^{1/x}$ is:

A. 1

B. ab

C. e^{ab}

D. $e^{b/a}$

Answer: C

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127. If $l \lim_{x \to 0} (\cos x + a \sin bx)^{1/x} = e^2$, then (a,b) is equal to:

A. (1,2)

B. $\left(2, \frac{1}{2}\right)$

C. $\left(2\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

D. (4,2)

Answer: A::C

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128. $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ =

A. a^2

B. e

C. $\sqrt{e}$

D. $e^{1/3}$

Answer: D

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129. $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ =

A. e

B. e^2

C. e^3

D. none

Answer: B

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130. $(\text{limit})_{x\to a}\frac{x^m-a^m}{x^n-a^n}$ is:

A. $\frac{m}{n}a^{m-n}$

B. $\frac{n}{m}a^{n-m}$

C. mna^{mn}

D. none

Answer: A

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131. $\text{lt}_{x\to1}\frac{x^4-1}{x-1}=\text{lt}_{x\to k}\frac{x^3-k^3}{x^2-k^2}$ then k=

A. $\frac{2}{3}$

B. $\frac{4}{3}$

C. $\frac{8}{3}$

D. none

Answer: C

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132. $\text{lt}_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$ =

A. e

B. e^3

C. e^5

D. e^7

Answer: C

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133. $\lim_{x \to \infty} \left(\frac{x+2}{x+1}\right)^{x+3}$ =

A. e

B. e^2

C. e^4

D. e^6

Answer: A

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134. $\lim_{x \to \infty} \left(\frac{x+3}{x-1}\right)^{x+3}$ =

A. e^2

B. e^4

C. e^5

D. e^6

Answer: B

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135. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\lim_{x \to \alpha} \left(ax^2 + bx + c + 1\right)^{1/x-\alpha}$ is equal to

A. $2a(\alpha - \beta)$

B. $2\log[a(\alpha - \beta)]$

C. $e^{2a(\alpha - \beta)}$

D. $e^{a^2|\alpha - \beta|}$

Answer: C

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136. $\lim_{n \to \infty} (4^n + 5^n)^{1/n}$ is equal to:

A. 4

B. 5

C. e

D. none of these

Answer: B

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137. $\mathrm{lt}_{n \to \infty} \dfrac{x^n + y^n}{x^n - y^n}$, where $x > y > 1$ is equal to:

A. 0

B. 1

C. -1

D. none

Answer: B

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138. If $f(x) = \left(\dfrac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$, then $\lim_{x \to \infty} f(x)$ is:

A. e^4

B. e^3

C. e^2

D. 2^4

Answer: A

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139. The value of $\lim_{x \to \infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)$ is

A. e^2

B. e^{-2}

C. e^6

D. none of these

Answer: A

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140. $\text{lt}_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$ is:

A. e^{-1}

B. e

C. 1

D. none

Answer: C

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141. $\mathrm{lt}_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$ is:

A. e^{-1}

B. e

C. 1

D. none

Answer: A

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142. For $x \in R$, $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$ is equal to

A. e

B. e^{-1}

C. e^{-5}

D. e^5

Answer: C

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143. If $f(x) = x(-1)^{\left[\frac{1}{x}\right]}$, $x \leq 0$ where [x] denotes the greatest integer less than or equal to x, then the value of $\lim_{x \to 0} f(x)$ is equal to:

A. -1

B. 0

C. 2

D. none of these

Answer: B

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144. The integer $'n'$ for which $\left(\lim\right)_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number, is 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: C

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145. If $\operatorname{lt}_{x \to 0} \dfrac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^2}{2}}{x^n}$ is a finite non-zero number, then the integer n is:

A. 2

B. 3

C. 4

D. 5

Answer: C

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146. Let $f: R \to R$ be such that f (1) = 3 and f'(1) = 6. Then, $\lim_{x \to 0} \left[\dfrac{f(1+x)}{f(1)}\right]^{1/x}$ equals

A. 1

B. $e^{1/2}$

C. e^2

D. e^3

Answer: C

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147. The value of $\lim_{x \to b} \frac{\sqrt{x-a} - \sqrt{b-a}}{x^2 - b^2}$, for$b > a$,is

A. $\frac{1}{4b\sqrt{a-b}}$

B. $\frac{1}{4b\sqrt{b-a}}$

C. $\frac{1}{4a\sqrt{a-b}}$

D. $\frac{1}{b\sqrt{b-a}}$

Answer: B

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148. For $x>0$, $\lim_{x\to 0}\left((\sin x)^{1/x}+\left(\frac{1}{x}\right)^{\sin x}\right)$ is equal to:

A. -1

B. 0

C. 2

D. 1

Answer: D

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149. If $\lim_{t\to a}\frac{\int_a^t f(t)dt-\frac{t-a}{2}(f(t)-f(a))}{(t-a)^3}=0$, then maximum degree of $f(x)$ is:

A. 4

B. 3

C. 2

D. 1

Answer: D

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150. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x} =$

A. 4

B. $\frac{1}{2}$

C. 3

D. 2

Answer: D

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PROBLEM SET (1) (TRUE AND FALSE)

1. $\lim_{x \to 0} \frac{x}{\tan^{-1}(2x)}$ is equal to $\frac{1}{2}$

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2. $\lim_{\theta \to \pi/2} \frac{1 - \sin\theta}{(\pi - 2\theta)^2} = \frac{1}{8}$

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3. $\lim_{\theta \to \pi/2} \left(\frac{1 - \sin^3\theta}{\cos^2\theta}\right) = \frac{1}{2}$

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4. $\lim_{x \to \infty} \left[\sqrt{x^2 + ax + a^2} - \sqrt{x^2 + a^2}\right]$ is equal to $\frac{a}{2}$.

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PROBLEM SET (1) (FILL IN THE BLANKS)

1. $f(x)$ is the integral of $\frac{2\sin x - \sin 2x}{x^3}, x \neq 0, \lim_{x \to 0} f'(x)$=..........

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2. $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$= finite, then a=............ And limit =.............

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3. $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$ = ...

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4. The value of the limit $\lim_{x \to 0} \frac{e^{\sqrt{x}} - e^{1/\sqrt{x}}}{e^{\sqrt{x}} + e^{1/\sqrt{x}}}$ is..........

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5. $\lim_{h \to 0} \frac{\ln(1+2h) - 2\ln(1+h)}{h^2}$ = ...

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6. Find the constant a, b and c such that $\lim_{x \to 0} \frac{axe^x - b\log(1+x) + cxe^{-x}}{x^2 \sin x} = 2$

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7. $\lim_{n \to \infty} \left[1 - \log\left(1 + \frac{1}{n}\right)^{n-1}\right]$ is............

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PROBLEM SET (2) (MULTIPLE CHOICE QUESTIONS)

1. $\lim_{x \to 0} \frac{\sqrt{\left(\frac{1}{2}\right)(1 - \cos 2x)}}{x} =$

A. 1

B. -1

C. 0

D. none

Answer: D

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2. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

A. exists and it equals $\sqrt{2}$

B. exists and it equals $-\sqrt{2}$

C. does not exist because $x - 1 \to 0$

D. does not exist because L.H.L is not equal to R.H.L.

Answer: D

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3. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & \text{for } [x]\neq 0\\ 0, & \text{for } [x]=0\end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. Then find $\lim_{x\to 0} f(x)$.

A. 1

B. 0

C. -1

D. none of these

Answer: D

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4. $\text{lt}_{x\to\infty}\frac{\log x}{[x]}$, where [•] has the usual meaning is

A. 1

B. -1

C. 0

D. none

Answer: C

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5. The left hand limit of $f(x)=\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]^3\right\},(a>0)$

where [x] denotes the greatest integer less than or equal to x is

A. a^2

B. a^2-1

C. a^2-3

D. none of these

Answer: A

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6. $\lim_{x \to 3} \frac{|x-3|}{x-3} =$

A. 0

B. 1

C. -1

D. Not exist

Answer: D

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7. Let $f(x) = \begin{cases} \int_0^x \{5 + |1-t| dt\} & \text{if } x > 2 \\ 5x + 1 & \text{if } x \le 2 \end{cases}$, then

A. f(x) is not continuous at $x = 2$

B. f is differentiable everywhere

C. right hand limit at x = 2 does not exist

D. f is continuous but riot differentiable at x=2

Answer: D

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8. The value of $\lim_{x \to 1^+} \frac{\int_1^x |t-1|dt}{\sin(x-1)}$ is

A. 0

B. 1

C. -1

D. none

Answer: A

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9. $\lim_{x \to a} \frac{x}{x-a} \int_a^x f(x)dx$ equals

A. f(a)

B. a f(a)

C. 0

D. none

Answer: B

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10. If $f(x) = \begin{cases} x & x < 0 \\ 1 & x = 0 \\ x^2 & x > a \end{cases}$, then $\lim_{x \to 0} f(x) =$

A. 0

B. 1

C. 2

D. not exist

Answer: A

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11. If [x] denotes the greatest integer less than or equal to x,then the value of $\lim_{x \to 1} (1 - x + [x - 1] + [1 - x])$ is

A. 0

B. 1

C. -1

D. none of these

Answer: C

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12. Let $f(x) = \begin{cases} x^2 & x \notin Z \\ \frac{k(x^2 - 4)}{2 - x} & x \notin Z \end{cases}$

Then, $\lim_{x \to 2} f(x)$

A. exists only when k = 1

B. exists for every real k

C. exists for every real k except k = 1

D. does not exist

Answer: B

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13. If $f(x) = \{(x, x \leq 1), (x^2 + bx + c, x > 1$ and f'(x)) exists finitely for all $x \in R$, then

A. p=1, q=-1

B. $p \in R, q = 1$

C. $p = -1, q \in R$

D. $p = -1, q = 1$

Answer: D

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14. If f(x) is an odd function of x and $\mathrm{lt}_{x \to 0} f(x)$ exists then the limit must be zero. True or False.

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15. $Lt_{x \to 0} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}$ is equal to:

A. 1

B. -1

C. 0

D. does not exist

Answer: D

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16. $\lim_{x \to 0} (1 + \sin x)^{1/x^2}$ is equal to:

A. 0

B. ∞

C. $e^{1/2}$

D. does not exist

Answer: B

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17. $\lim_{x \to 0} \frac{\sin[\cos x]}{1 + [\cos x]}$, ([.] denotes the greatest integer function)

A. is equal to 1

B. is equal to zero

C. does not exist

D. none of these

Answer: B

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18. The number of points at which the function $f(x)=\frac{1}{\log|x|}$ is discontinuous is

A. 1

B. 2

C. 3

D. 4

Answer: C

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19. The function $f(x)=\frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0, is

A. a-b

B. 1+b

C. log a + log b

D. none of these

Answer: D

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20. The value of f(0) so that the function $f(x) = \frac{\log(1 + x^2 \tan x)}{\sin x^3}, (x \neq 0)$ ` is continuous at x = 0 is:

A. 1

B. 2

C. 3

D. none

Answer: A

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21. If the function:

$f(x) = \begin{cases} \frac{x^2-(A+2)x+A}{x-2} & \text{for} \quad x \neq 2 \\ 2 & \text{for} \quad x = 2 \end{cases}$ is continous at x=2, then A is:

A. 0

B. 1

C. -1

D. none of these

Answer: A

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22. If $f(x) = \frac{\cos^2 \pi x}{e^{2x} - 2ex}, x \neq \frac{1}{2}$, the value of $f\left(\frac{1}{2}\right)$, so that f (x) is continuous at $x = \frac{1}{2}$ is:

A. $\frac{\pi}{2e^2}$

B. $\frac{\pi}{2e}$

C. $\frac{\pi^2}{2e^2}$

D. $\frac{\pi^2}{2e}$

Answer: D

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23. The value of b for which the function $f(x) = \begin{cases} 5x - 4 & 0 < x \leq 1 \\ 4x^2 + 3bx & 1 < x < 2 \end{cases}$ is continuous at every point of its domain, is

A. -1

B. 0

C. 1

D. $13/3$

Answer: A

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24. if $f(x) = \begin{cases} x+\lambda & -1 < x < 3 \\ 4 & x = 3 \\ 3x-5 & x > 3 \end{cases}$, is continous at 3 then λ =

A. 1

B. -1

C. 0

D. none

Answer: A

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25. If the function $f(x) = \begin{cases} (\cos x)^{1/x} & x \neq 0 \\ = k & x = 0 \end{cases}$, is continous at x=0, then the value of k is:

A. 1

B. -1

C. 0

D. e

Answer: A

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26. Let $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} & If x \neq 2 \\ = k & If x = 2 \end{cases}$, If $f(x)$ is continous for all x, then k=

A. 3

B. 5

C. 7

D. none

Answer: C

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27. Let $f(x)=\dfrac{1-\tan x}{4x-\pi}, x \neq \dfrac{\pi}{4}, x \in \left[0, \dfrac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$=

A. 1

B. $1/2$

C. $-1/2$

D. -1

Answer: C

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28. The value of f(0), so that the function

$f(x)=\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x,

is given by:

A. $a\sqrt{a}$

B. $\sqrt{a}$

C. $-\sqrt{a}$

D. $-a\sqrt{a}$

Answer: C

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29. The value of $f(0)$, so that the function $f(x)=\frac{(27-2x)^2-3}{9-3(243+5x)^{1/5}-2}(x\neq 0)$ is continuous, is given $\frac{2}{3}$ (b) 6 (c) 2 (d) 4

A. $2/3$

B. 6

C. 2

D. 4

Answer: C

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30. $f(x) = \begin{cases} \frac{\sqrt{1+px}-\sqrt{1-px}}{x} & -1 \leq x < 0 \\ \frac{2x+1}{x-2} & 0 \leq x \leq 1 \end{cases}$ is continuous in the interval [-1,1], then p is equal to:

A. -1

B. $-1/2$

C. $1/2$

D. 1

Answer: B

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31. $f(x) = (x-1)^{\frac{1}{2-x}}$ is not defined at x = 2. If f(x) is continuous, then f(2) is equal to:

A. e

B. e^{-1}

C. e^{-2}

D. 1

Answer: B

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32. The function $f(x) = \begin{cases} x^2/a & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ (2b^2 - 4b)/x^2 & \sqrt{2} \leq x < \infty \end{cases}$ is continuous for $0 \leq x < \infty$, then the most suitable values of a and b are

A. a=1, b=-1

B. $a = -1, b = 1 + \sqrt{2}$

C. $a = -1, b = 1$

D. none of these

Answer: C

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33. If $f(x) = x^a \log x$ and $f(0) = 0$ then the value of α for which Rolle's theorem can be applied in [0,1] is

A. -2

B. -1

C. 0

D. $\frac{1}{2}$

Answer: D

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34. The value of a for which the function $f(x) = f(x) = \begin{cases} \frac{(4^x - 1)\hat{3}}{\sin(xa)\log\{(1 + x^2 3)\}}, x \neq 012(\log 4)^3, x = 0 \end{cases}$ may be continuous at $x = 0$ is 1 (b) 2 (c) 3 (d) none of these

A. 1

B. 2

C. 3

D. none of these

Answer: D

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35. If $f(x)=\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{\lambda^2}\right)\log\left(1+\frac{x^2}{2}\right)}, x\neq 0$ and f (0) = 8 be a continuous function then λ =

A. 2

B. 1

C. -1

D. -2

Answer: A::D

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36. Let $f(x)=\frac{x(1+a\cos x)-b\sin x}{x^3}, x\neq 0$ f(0) = 1. If f(x) is continuous at x = 0, a and b are given by

A. $5/2, 3/2$

B. $-5, \ -3$

C. $-5/2, \ -3/2$

D. none of these

Answer: C

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37. $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & x<0\\ =a & x=0\\ =\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x>0\end{cases}$ If the function be continuous at x = 0, then a =

A. 4

B. 6

C. 8

D. 10

Answer: C

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38. The function $f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \pi/4 \\ 2x\cot x + 6 & \pi/4 \le x \le \pi/2 \\ a\cos 2x - b\sin x & \pi/2 < x \le \pi \end{cases}$ is continuous for $0 \le x \le \pi$ then a, b are

A. $\frac{\pi}{6}, \frac{\pi}{12}$

B. $\frac{\pi}{3}, \frac{\pi}{6}$

C. $\frac{\pi}{6}, -\frac{\pi}{12}$

D. none of these

Answer: C

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39. In order that the function $f(x)=(x+1)^{\cot x}$ is continuous at $x=0, f(0)$ must be defined as

A. f(0)=0

B. f(0)=e

C. f(0)=1/e

D. none of these

Answer: B

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40. Function $f(x)=(\sin 2x)^{\tan^2 2x}$ is not defined at $x=\frac{\pi}{4}$. If f(x) is continuous at $x=\frac{\pi}{4}$, then $f\left(\frac{\pi}{4}\right)$ is equal to:

A. 1

B. 2

C. $\sqrt{e}$

D. none of these

Answer: C

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41. Let $f(x) = \dfrac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}, x \neq \dfrac{\pi}{4}$. The value which should be assigned to $f(x)$ at $x = \dfrac{\pi}{4}$, so that it is continuous everywhere is 1 (b) $\dfrac{1}{2}$ (c) 2 (d) none of these

A. $1/2$

B. 1

C. 2

D. none of these s

Answer: A

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42. If $f(x)=\frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}$, $(x\neq 2)$, then for f to be continuous everywhere, $f(0)$ is equal to:

A. -1

B. 1

C. 2^6

D. $7/64$

Answer: D

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43. The value of λ that makes the function $f(x) = \begin{cases} (\cos x)^{1/\sin x} & x \neq 0 \\ \lambda & x = 0 \end{cases}$ continuous at x = 0 is:

A. 0

B. 1

C. $1/2$

D. none of these

Answer: B

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44. Let $f''(x)$ be continuous at x = 0 and $f''(0) = 4$, Then value of $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is:

A. 11

B. 2

C. 12

D. none of these

Answer: C

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45. If the function $f(x)=\begin{cases}\left(1+|\sin x|^{\frac{a}{\sin x}}\right) & -\pi/6 < x < 0 \\ b & x=0 \\ e^{\frac{\tan 2x}{\tan 3x}} & 0 < x < \pi/6\end{cases}$, is continous at x=0, then

A. $a=\log_e b, a=2/3$

B. $b=\log_e a, a=2/3$

C. $a=\log_e b, b=2$

D. none of these

Answer: A

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46. Let $f(x) = \begin{cases} -2\sin x & x \leq -\pi/2 \\ a\sin x + b & -\pi/2 < x < \pi/2 \\ \cos x & x \geq \pi/2 \end{cases}$ If f(x) is continuous everywhere then (a,b) =

A. (0,1)

B. (1,1)

C. (-1,1)

D. (-1,0)

Answer: C

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47. The value of $f(0)$ so that the function $f(x) = \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point on its domain is:

A. 2

B. $1/3$

C. $2/3$

D. $-1/3$

Answer: B

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48. If $f(x) = \begin{cases} \dfrac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} & x \neq 0 \\ k & x = 0 \end{cases}$, is continuous at x = 0, then k equals

A. 16log2 log 3

B. $16\sqrt{2}$ log 6

C. $16\sqrt{2}$ log 2 log 3

D. none of these

Answer: C

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49. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & x < 4 \\ \frac{x-4}{|x-40|} + b & x > 4 \end{cases}$, Then $f(x)$ is continuous at x = 4, when

A. a=b=0

B. a=b=1

C. a=-1, b=1

D. a=1, b=1

Answer: D

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50. If $f(x) = \begin{cases} \frac{1}{(\pi - 2x)^2} \frac{\log \sin x}{(\log(1 + \pi^2 - 4\pi x + 4x^2)), x \neq \frac{\pi}{2} k, x = \frac{\pi}{2}} \end{cases}$ is continuous at $x = \frac{\pi}{2}, then k = -\frac{1}{16}$ (b) $-\frac{1}{32}$ (c) $-\frac{1}{64}$ (d) $-\frac{1}{28}$

A. $-\frac{1}{16}$

B. $-\frac{1}{32}$

C. $-\frac{1}{64}$

D. $-\frac{1}{26}$

Answer: C

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51. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & x < 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx\sqrt{x}} & x < 0 \end{cases}$, is continous at x=0, then

A. $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$

B. $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$

C. $a = -\frac{3}{2}, b \in R, c = \frac{1}{2}$

D. none of these

Answer: B

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52. let $f(x) = \dfrac{ae^{|\sin x|} - b\cos x - |x|}{x^2}$ if $f(x)$ is continuous at $x = 0$ then find the values of a and b

A. f(x) is always discontinuous at x = 0

B. f(x) is continuous at,x = 0 if a=b=c=1

C. $f'(0) = 1$

D. f(x) is continuous but not differentiable at x=0

Answer: C

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53. Let $f(x) = \begin{cases} x^p \frac{\sin 1}{x} & x \geq 0 \\ 0 & x = 0 \end{cases}$ Then $f(x)$ is continuous but not differentiable at x =0 if

A. $p \in [0, 1]$

B. $p \in [1, \infty)$

C. $p \in (-\infty, 0)$

D. $p = 0$

Answer: A

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54. The value of k which makes $f(x) = \begin{cases} \sin(1/x) & x \neq 0 \\ k & x = 0 \end{cases}$, continous at x =0 is:

A. 8

B. 1

C. -1

D. none of these

Answer: D

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55. $f(x) = \begin{cases} -1 & x < -1 \\ -x & -1 \le x \le 1 \\ 1 & x > 1 \end{cases}$ is continous

A. at x=1 but not at x=-1

B. at x=-1 but not at x=1

C. at both x=1 and x=-1

D. at none of x=1 and -1

Answer: D

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56. If f(x)= $\int_{-1}^{x} |t| dt, x \ge -1$ then

A. f and f' are continuous for $x + 1 > 0$

B. f is continuous but f' is not continuous for $x + 1 > 0$

C. f and f' are not continuous at x = 0

D. f is continuous at x = 0 but f' is not so

Answer: A

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57. The following functions are continuous on $(0, \pi)$

A. tan x

B. $\int_0^x \frac{\sin 1}{t} dt$

C. $\begin{cases} 1 & 0 < x \leq \frac{3\pi}{4} \\ 2\sin\left(\frac{x}{3}\right) & \frac{3\pi}{4} < x < \pi \end{cases}$

D. $\begin{cases} x\sin x & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}\sin(2\pi + x) & \frac{\pi}{2} < x < \pi \end{cases}$

Answer: B::C

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58. Given the function $f(x) = \frac{1}{1-x}$. The points of discontinuity of the composite function, $y = f(f[f(x))]$ are at x=

A. 0

B. 1

C. 2

D. -1

Answer: A::B

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59. If $f(x)$ is defined by: $f(x)=\begin{cases}\frac{|x^2-x|}{x^2-x} & (x\neq 0,1)\\ 1 & x=0\\ -1 & x=1\end{cases}$ then f(x) is continuous for all

A. x

B. x except at x=0

C. x except at x=1

D. x except at x=0 and x=1

Answer: D

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60. Let $f(x) = |x| + |x - 1|$, then

A. f(x) is continuous both at x = 0 and 1

B. f(x) is continuous at x = 0 but not at x = 1

C. f(x) is continuous at x = 1 but not at x = 0

D. none of these.

Answer: A

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61. The function $f(x) = |x| + |x - 1|, is$

A. continuous at x =1, but not differentiable

B. both continuous and differentiable at x = l

C. not continuous at x = 1

D. none of these

Answer: A

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62. Let $f(x)=\begin{cases}\frac{x^4-5x^2+4}{|(x-1)(x-2)|} & (x\ne 1, 2)\\ 6 & x=1\\ 12 & x=2\end{cases}$. Then f(x) is continuous on the set

A. R

B. R-{1}

C. R-{2}

D. R-{1,2}

Answer: D

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63. Let $f(x) = x - |x - x^2|, x \in [-1, 1]$.Then the number of points at which f(x) is discontinuous is:

A. 0

B. 1

C. 2

D. none

Answer: A

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64. The function $f(x) = [x]^2 - [x^2]$ (where [y] is thegreatest integer less than or equal to y), is discontinuous at:

A. all integers

B. all integrers except 0 and 1

C. all integers except 0

D. all integers except 1

Answer: D

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65. On the interval [-2,2] the function:

$$f(x) = \begin{cases} (x+1)e^{-\left\{\frac{1}{|x|}+\frac{1}{x}\right\}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

A. is continuous for all $x \in Z$

B. is continuous for all $x \in Z - \{0\}$

C. assumes all intermediate values from f(-2) to f(2)

D. has a maximum value equal to $\frac{3}{e}$.

Answer: B::C::D

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66. Let $f(x) = \begin{cases} \int_0^x \{5 + |1 - t|dt\} & \text{if} \quad x > 2 \\ 5x + 1 & \text{if} \quad x \leq 2 \end{cases}$, then

A. f(x) is not continuous at x = 2

B. f(x) is continuous but not differentiable at x = 2

C. f(x) is differentiable everywhere

D. the right derivative of $f(x)$ at x = 2 does not exist.

Answer: B

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67. The function $f(x) = [x]\cos\{(2x - 1)/2\}\pi$ denotes the greatest integer function, is discontinuous at:

A. all x

B. all integer points

C. no x

D. x whixh is not an integer

Answer: B

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68. The number of points where $f(x) = [\sin x + \cos x]$ (where [.] denotes the greatest integer.function] $x \in (0, 2\pi)$ is discontinuous is:

A. 3

B. 4

C. 5

D. 6

Answer: C

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69. Let $f: R \rightarrow R$ be any function. Define $g: R \rightarrow R$ by $g(x) = |f(x)|, \forall x$. Then, g is

A. onto if f is onto

B. one-one if f is one-one

C. continuous if f is continuous

D. differentiable if f is differentiable

Answer: C

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70. If $f(x) = \begin{cases} x(e)^{-\left[\frac{1}{|x|} + \frac{1}{x}\right]} & x \neq 0 \\ 0 & x = 0 \end{cases}$, then f(x) is:

A. continuous and differentiable $\forall$ x

B. continuous for $\forall$ x but not differentiable at x = 0

C. neither differentiable nor continuous at x = 0

D. discontinuous everywhere

Answer: B

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71. The function f defined as - $f(x) = \left(\sin x^2\right)/x$ for $x \neq 0$ and f(0) = 0 is:

A. continuous, and derivable at x = 0

B. neither continuous nor derivable at x = 0

C. continuous but not derivable at x =.0

D. none of these

Answer: A

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72. If $f(x) = \begin{cases} 1 & x < 0 \\ 1 + \sin x & 0 \leq x < \pi/2 \end{cases}$ Then at x=0, the derivative $f'(x)$ is:

A. 1

B. 0

C. infinite

D. does not exist

Answer: D

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73. For a real number y, let $[y]$ denotes the greatest integer less than or equal to y. Then, the function $f(x) = \frac{\tan \pi[(x - \pi)]}{1 + [x]^2}$ is

A. discontinuous at some x

B. continuous at all x, but the derivative f' (x) does not exist for some x

C. $f'(x)$ exists for all x but second derivative $f''(x)$ does not exist.

D. $f'(x)$ exists for all x.

Answer: D

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74. If $f(x) = x\left[\sqrt{x} - \sqrt{x+1}\right]$, then

A. f(x) is continuous but not differentiable at x = 0

B. f(x) is continuous and differentiable at x=0

C. f(x) is not differentiable at x = 0

D. none of these

Answer: C

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75. The function $f(x) = \begin{cases} |x-3| & x \geq 1 \\ x^2/4 - 3x/2 + 13/4 & x < 1 \end{cases}$ is

A. continous at x=1

B. continous at x=3

C. differentiable at x=1

D. differentiable at x=3

Answer: A::B::C

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76. The value of the derivative of $|x-1|+|x-3|$ at x=2 is:

A. -2

B. 0

C. 2

D. not defined

Answer: B

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77. Let [ ] denote the greatest integer function and $f(x) = [\tan^2 x]$ Then

A. $\lim_{x \to 0} f(x)$ does not exist

B. f(x) is continuous at x = 0

C. f(x) is not differentiable at x = 0

D. $f'(0) = 1$

Answer: B

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78. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)} & x \neq -2 \\ 2 & x = -2 \end{cases}$,

A. continuous at x = - 2

B. not continuous at x = - 2

C. differentiable at x = - 2

D. continuous but not differentiable at x = - 2.

Answer: B

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79. If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$, then

A. f(x) is increasing on $[-1, 2]$

B. f (x) is continuous in $[-1, 3]$

C. f'(2) does not exist

D. f(x) has the maximum value at x = 2

Answer: A::B::C

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80. The set of all points, where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable, is

A. $(-\infty, \infty)$

B. $(-0, \infty)$

C. $(-\infty, 0) \cup (0, \infty)$

D. none of these

Answer: A

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81. The set of points where the function $f(x) = x|x|$ is differentiable is

A. $(-\infty, \infty)$

B. $(-\infty, 0) \cup [0, \infty)$

C. $(0, \infty)$

D. $[0, \infty)$

Answer: A

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82. Prove that the function If $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is not differentiable

A. $(0, \infty)$

B. $(-\infty, \infty)$~$\{0\}$

C. $(-\infty, 0)$

D. $(-\infty, \infty)$

Answer: B

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83. The set of allpoints of differentiability of the function $f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is

A. $(-\infty, 0)$

B. $(-\infty, \infty)$~$\{0\}$

C. $[0, \infty)$

D. $(-\infty, \infty)$

Answer: D

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84. At the point x = 1, the function:

$f(x) = \begin{cases} x^3 - 1 & 1 < x < \infty \\ x - 1 & -\infty < x \leq 1 \end{cases}$ is:

A. continuous and differentiable

B. continuous and not differentiable

C. discontinuous and differentiable

D. none of these

Answer: B

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85. Let $f(x) = \min\{1, x^2, x^3\}$, then:

A. f(x) is continous $\forall x \in R$

B. $f'(x) > 0, \ \forall x > 1$

C. continous but not differentiable $\forall x \in R$

D. differentiable everywhere

Answer: B::C

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86. The function f(x) is defined as:

$f(x) = \frac{1}{3} - x, x, \ < \frac{1}{3}$

$= \left(\frac{1}{3} - x\right)^2, x \geq \frac{1}{3}$

then in.the itnerval (0,1), the mean value, theorem is not true because

A. f(x) is not continuous

B. f(x) is not differentiable

C. $f(0) \neq f(1)$

D. none

Answer: B

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87. If $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then

A. f and f' are continous

B. f is differentiable at x=0

C. f is diff. at x=0 and f' is not continous at x=0

D. none of these

Answer: B::C

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88. Let $g(x) = x f(x)$, where $f(x) = \begin{cases} x^2 \sin. \frac{1}{x} & : \quad x \neq 0 \\ 0 & : \quad x = 0 \end{cases}$. At $x = 0$,

A. g is differentiable but g' is not continuous

B. g is differentiable while f is not differentiable

C. both f and g are differentiable

D. g is differentiable but g' is continuous

Answer: A::B

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89. Let $f(x) = \begin{cases} 0 & x < 0 \\ x^2 & x \geq 0 \end{cases}$, then for all values of x

A. f' is-differentiable

B. f is differentiable

C. f' is continuous

D. f is continuous

Answer: B::C::D

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90. Let [x] denotes the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then $f(x)$ is

A. continuous at x = 0

B. continuous in $(-1, 0)$

C. differentiable at x = 1

D. differentiable in $(-1, 1)$

Answer: A::B::D

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91. The function f (x) = 1+ $|\sin x|$ is

A. continuous nowhere

B. continuous everywhere

C. differentiable nowhere

D. not differentiable at x=0

Answer: B::D

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92. If f(x)= $\begin{cases} \frac{x \log \cos x}{\log(1+x^2)} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then

A. f(x) is not continous at x=0

B. f(x) is continuous at x=0

C. f(x) is continuous at x = 0 but not differentiable at x=0

D. $f(x)$ is differentiable at x=0.

Answer: B::D

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93. If $x+4|y|=6y$ then y as a function of x is

A. defined for all real x

B. continous at x=0

C. derivable at x=0

D. $\frac{dy}{dx}=\frac{1}{2}$ for $x>0$.

Answer: B

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94. If $f'(x_0)$ exists, then $\lim_{h\to 0}\left(\frac{f(x_0+h)-f(x_0-h)}{2h}\right)$ is equal to:

A. $\frac{1}{2}f'(x_0)$

B. $f'(x_0)$

C. $2f'(x_0)$

D. none of these

Answer: B::C

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95. The function $f(x) = \begin{cases} |2x-3|[x] & x \geq 1 \\ \sin\left(\frac{\pi x}{2}\right) & x < 1 \end{cases}$

A. is continuous at x = 2

B. is differentiable at x = 1

C. is continuous but not differentiable at x = 1

D. none of these

Answer: C

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96. The function f (x) is defined as under :

$$f(x) = \begin{cases} 3^x & -1 \le x \le 1 \\ 4 - x & 1 < x < 4 \end{cases}$$

The above function is:

A. continuous at x = 1

B. differentiable at x = 1

C. continuous but not differentiable at x = 1

D. none of these

Answer: A::C

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97. A function is defined as follows :

$$f(x) = \begin{cases} x^3 & x^2 < 1 \\ x & x^2 \ge 1 \end{cases}$$

The function is:

A. continuous at x = 1

B. differentiable at x = 1

C. continuous but not differentiable at x = 1

D. none of these

Answer: A::C

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98. The left-hand derivative of $f(x) = [x]\sin(\pi x)$ at k an interger, is:

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1} k\pi$

Answer: A

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99. If the derivative of the function

$$f(x) = \begin{cases} ax^2 + b & x < -1 \\ bx^2 + ax + 4 & x \geq -1 \end{cases}$$

is every where continuous, then what are the values of a and b?

A. a=2,b=3

B. a=3, b=2

C. a=-2, b=-3

D. a=-3, b=-2

Answer: A

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100. If $f(x) = \begin{cases} ax^2 + b & b \neq 0 \quad x \leq 1 \\ bx^2 + ax + c & x > 1 \end{cases}$

Then f[x] is continuous and differentiable at x = 1 if:

A. c=0, a=2b

B. $a = b, c \in R$

C. $a = b, c = 0$

D. $a = b, c \neq 0$

Answer: A

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101. Let $f(x) = a|x|^2 + b|x| + c$ where a,b,c are real constants. Then f'(x) exists at x=0 if:

A. a=0

B. b=0

C. c=0

D. a=b

Answer: B

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102. Let $h(x) = \min\{x, x^2\}$, for every real number of x. Then

A. h continuous for all x

B. h is differentiable for all x

C. $h'(x) = 1$ for all $x > 1$

D. h is not differentiable at two values of x

Answer: A::C::D

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103. Let $f: R \to R$ be a function defined by $f(x) = \max .\{x, x^3\}$. The set of all points where $f(x)$ is NOT differenctiable is

(a) $\{-1, 1\}$

(b) $\{-1, 0\}$

(c) $\{0, 1\}$

(d) $\{-1, 0, 1\}$

A. $\{-1, 1\}$

B. $\{-1, 0\}$

C. {0,1}

D. $\{-1, 0, 1\}$

Answer: D

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104. The derivative of $f(x) = |x|$ at x = 0 is:

A. 1

B. 0

C. -1

D. does not exist

Answer: D

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105. For a differentiable function f, the value of $\lim_{h \to 0} \frac{[f(x+h)]^2 - [f(x)]^2}{2h}$ is equal to:

A. $[f'(x)]^2$

B. $f(x)f'(x)$

C. $\frac{1}{2}[f'(x)]^2$

D. $\frac{1}{2}\left[[f'(x)]^2 - [f(x)]^2\right]$

Answer: B

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106. If for a continuous function $f, f(0) = f(1) = 0, f'(1) = 2 and y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to a. 1 b. 2 c. 0 d. none of these

A. 1

B. 2

C. 0

D. none of these

Answer: B

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107. The function $f(x) = e^{|x|}$ is

A. continuous everywhere but not differentiable at x = 0

B. continuous and differentiable everywhere

C. not continuous at x=0

D. none of these

Answer: A

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108. Let f(x) be defined as

$f(x)=\begin{cases}\sin 2x & 0<x<\frac{\pi}{6}\\ px+q & \frac{\pi}{6}<x<1\end{cases}$, If f and f' are continuous then [p,q) is equal to

A. $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

B. $\left(1, \frac{1}{\sqrt{2}}+\frac{\pi}{6}\right)$

C. $\left(1, \frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)$

D. none of these

Answer: C

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109. Let $f(x)=\begin{cases}-\frac{1}{|x|} & \text{for } |x|\geq 1\\ ax^2-b & \text{for } |x|<1\end{cases}$, If f(x) is continuous and differentiable at any point, then:

A. $a=\frac{1}{2}, b=-\frac{3}{2}$

B. $a = \frac{1}{2}, b = \frac{3}{2}$

C. $a = 1, b = -1$

D. none of these

Answer: B

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110. The derivative of $f(x) = |x|^3$ at x=0 is:

A. -1

B. not defined

C. 0

D. $1/2$

Answer: C

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111. If $y = \left|\tan\left(\frac{\pi}{4} - x\right)\right|$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is

A. -1

B. 1

C. does not exist

D. none

Answer: C

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112. Which of the following functions is differentiable at x = 0 ?

A. $\cos(|x|) + |x|$

B. $\cos(|x|) - |x|$

C. $\sin(|x|) - |x|$

D. $\sin(|x|) + |x|$

Answer: C

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113. $f(x) = ||x| - 1|$ is not differentiable at

A. 0

B. $\pm 1, 0$

C. 1

D. ± 1

Answer: B

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114. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0,2) is:

A. 1

B. 2

C. 3

D. 4

Answer: C

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115. Consider, $f(x)=\begin{cases}\frac{x^2}{|x|} & x\neq 0\\ 0 & x=0\end{cases}$

A. f(x) is discontinuous everywhere

B. f(x) is continuous everywhere

C. f'(x) exists in (-1,1}

D. f'(x) exists in (- 2,2)

Answer: B

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116. The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is not differentiable at

A. -1

B. 0

C. 1

D. 2

Answer: D

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117. Consider the following statements S and R:

S: Both sin x and cos x are decreasing functions in the interval $(\pi/2, \pi)$

R: If a differentiable function decreases in an interval [a,b), then its derivative also decreases in [a,b). Which of the following is true ?

A. both S and R are wrong.

B. both S and R are correct, but R is not the correct explanation for S.

C. S is correct and R is the correct explanation for S.

D. S is correct and R is wrong.

Answer: D

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118. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{\left(1+x^2\right)^2} + \ldots\ldots + \frac{x^2}{\left(1+x^2\right)^n}$ + ...

Then at x =0

A. f (x) has no limit

B. f(x) is discontinuous

C. f(x) is continuous but not differentiable

D. f(x) is differentiable

Answer: B

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119. Let f(x) be a function satisfying $f(x+y)=f(x)+f(y)$ and $f(x)=xg(x)\text{For all }\ x, y \in R$, where g(x) is continuous. Then,

A. $f'(x)=g'(x)$

B. $f'(x)=g(x)$

C. $f'(x)=g\{0\}$

D. none of these

Answer: C

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120. Let f(x + y)=f(x)+f (y) and $f(x)=x^2g(x)$ for all $x, y \in R$, where g(x) is continuous function. Then f'(x) is equal to

A. g'(x)

B. g(0)

C. g(0) + g'(0)

D. 0

Answer: D

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121. A differentiable function f (x) satisfies the condition $f(x+y)=f(x)+f(y)+xy$ and $\lim_{h\to 0}\frac{1}{h}f(h)=3$ then f is:

A. linear

B. $f(x)=3x+\frac{x^2}{2}$

C. $f(x)=3x+x^2$

D. none of these

Answer: B

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122. Let f(x + y) = f(x) f (y) for all x and y. Suppose that f(3) = 3 and f' (0) = 11 then f' (3) is given by:

A. 22

B. 44

C. 28

D. 33

Answer: D

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123. Let $f(x+y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2x)g(x)$ where g(x) is continuous. Then, f'(x) equals

A. f(x) g(0)

B. 2f(x) g(0)

C. 2g(0)

D. none of these

Answer: B

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124. Suppose the function f satisfies the conditions :

(i) $f(x+y)=f(x)f(y)$ for all x and y.

(ii) $f(x)=1+xg(x)$, where $\lim_{x\to0}g(x)=1$

Then $f'(x)$ is:

A. $f(x)$

B. $f(x)g(0)$

C. $2f(x)g(0)$

D. none

Answer: A

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125. A function $f : R \to R$ satisfies the equation $f(x+y) = f(x)f(y)$ for all values of x and y and for any $x \in R, f(x) \neq 0$. Suppose the function is differentiable at x=0 and f' (0) = 2, then for all $x \in R, f(x) =$

A. e^x

B. e^{2x}

C. e^{-x}

D. none

Answer: B

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126. If f is twice differentiable function such that

$f''(x) = -f(x)$, and $f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2$ amd h(5)=11, then h(10)=

A. 22

B. 11

C. 15

D. none

Answer: B

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127. Let $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2 . F(5) = 5$ and $f''(x) = -f(x), g(x) = f'(x)$, then $F(10)$ is equal to:

A. 5

B. 10

C. 0

D. 3

Answer: A

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128. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$ then $f^2(16) + g^2(16)$ is:

A. 16

B. 32

C. 64

D. none

Answer: B

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129. Let $f(x+y) = f(x)f(y)$ for all x and y. Suppose f(5)=2 and f' (0) = 3, then f' (5) =

A. 0

B. 2

C. 6

D. none

Answer: C

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130. Let f be a continuous function on [1,3] which takes rational values for all x. If f(2) =10 then f(2.5) is equal to:

A. 25

B. 20

C. $\frac{f(1)+f(3)}{2}$

D. 10

Answer: D

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131. Let f(x) be differentiable $\forall x$. If f(1)=-2 and $f'(x) \geq 2 \forall x \in x[1, 6]$, then:

A. $f[6] < 5$

B. $f[6] = 5$

C. $f[6] \geq 8$

D. $f(6) < 8$

Answer: C

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132. If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2, x, y \in R$ and f(0)=0, then f(1) equals :

A. 2

B. 1

C. -1

D. 0

Answer: C

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133. Suppose f(x) is differentiable at $x = 1$ and $\mathrm{lt}_{h \to 0} \frac{1}{h} f(1+h) = 5$ then f'(1) equals :

A. 5

B. 6

C. 3

D. 4

Answer: A

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PROBLEM SET (2) (TRUE AND FALSE)

1. Let R be the set of real numbers and $f : R \to R$ suchthat for all x and y in R.

$|f(x) - f(y)| \leq |x - y|^3$, then $f(x)$ is a constant.

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2. The function f defined by:

$f(x) = x\left\{1 + \frac{1}{3}\sin\left(\log x^2\right)\right\}, x \neq 0$ and $f(0) = 0$, is everywhere continuous but has no differential coefficient at the origin.

A. True

B. False

C.

D.

Answer: t

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3. If f(x) be a continuous function and g(x) be discontinuous function then f{x)+g (x) is a discontinuous function.

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4. The derivative of an odd function is always an even function.

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PROBLEM SET (2) (FILL IN THE BLANKS)

1. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & \text{for } x < 0 \\ = c & \text{for } x = 0 \\ \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{1/2}} & \text{for } x > 0 \end{cases}$ is continous at x=0, then a=..............., b=.......... , c=...........

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2. A function f is defined as follows :

$f(x) = x^p \cos(1/x), x \neq 0, f(0) = 0$

What conditions whould be imposed on p so that

(i) f may be continuous at x=0

(ii) f may have a differential coefficient at x=0

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PROBLEM SET (3)

1. Discuss the applicability of Rolle's theorem in the interval [-1,1] to the function f(x) = | x|.

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2. Verify Rolle's theorem the function $f(x) = x^3 - 4x$ on [-2,2]

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3. Discuss the applicability of Rolle's theorem to $f(x) = \log\left[\frac{x^2 + ab}{(a+b)x}\right]$, in the interval [a,b].

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4. Verify Rolle's theorem for the following function $f(x) = 2x^3 + x^2 - 4x - 2$

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5. Verify Rolles theorem for the function: $f(x) = x(x+3)e^{-x/2}$ on $[-3, 0]$.

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6. Show that there is no real number p for which the equation $x^2 - 3x + p = 0$ has two distinct roots in [0,1]

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7. Verify Rolle's theorem for the function f(x) = x^2 in the interval $[-1, 1]$.

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8. Verify the Rolle's theorem for the function $f(x) = x^2 - 3x + 2$ on the interval [1,2].

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9. If a+b + c = 0, then show that the quadratic equation $3ax^2 + 2bx + c = 0$ has at least one root in [0,1].

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10. Discuss the applicability of Rolles theorem for the function $f(x) = x^{2/3}$ on $[-1, 1]$

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11. Discuss the applicability of Rolle's theorem on the function given by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } 0 \leq x \leq 1 \\ 3 - x & \text{if } 1 \leq x \leq 2 \end{cases}$$

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12. Discuss the application of Rolle's theorem to the function $f(x) = \log\left[\frac{x^2 + ab}{(ax + b)x}\right]$ in (a,b).

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13. Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ in interval [1,3]

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14. Verify Rolle's Theorem for the function $f(x) = 10x - x^2$

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15. Find 'c' of the mean value theorem, if:

$f(x) = x(x-1)(x-2), a = 0, b = 1/2$

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16. If f(x) =log x find all numbers strictly between e^2 and e^3 such that

$$f'(x) = \frac{f(e^3) - f(e^2)}{e^3 - e^2}$$

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17. Separate the intervals in which the polynomial $2x^3 - 15x^2 + 36x + 1$ increasing or decreasing.

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18. Use the function $f(x) = x^{\frac{1}{x}}, x > 0,$ to determine the bigger of the two numbers $e^{\pi} and \pi^{e}$.

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19. Show that Lagrange's mean value theorem does not holdior the function $f(x) = [x]$ in the interval [-1,1].

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20. Verify Lagrange's mean yalue'theorem for the function $f(x) = \sin x$ in $\left[0, \frac{\pi}{2}\right]$

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21. If f'[x) exists for all points in [a,b] and $\frac{f(c) - f(a)}{c - a} = \frac{f(b) - f(c)}{b - c}$ where $a < c < b$, then there is a number l such that $a < l < b$ and $f' '(l) = 0$

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22. If $f(x) = (x - 1)(x - 2)$ and $a = 0, b = 4$, find 'c' using Lagrange's mean value theorem.

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23. Examine if Mean Value Theorem applies to $f(x) = x^3 + 3x^2 - 5x$ in the interval [1,2]. If it does, then fintl tHe intermediate point whose existence is asserted by theorem

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PROBLEM SET (4)

1. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = 2 + (x - 1)^{3/2}$ in the interval [0,2]

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2. Discuss the applicability the Rolle's theorem for the function f(x) = x^2 in the interval [2,3].

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3. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = \tan x$ in $0 \leq x \leq \pi$

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4. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = x^4 - 3x^2 + 4$ in the interval $[-4, 4]$

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5. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = \frac{1}{x^2 + 1}$ in the interval $[-3, 3]$

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6. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = e^x \sin x$ in the interval $[0, \pi]$

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7. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = [x]$ in the interval $[-1, 1]$

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8. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = (x-2)\sqrt{m}$ in the interval [0,2]

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9. Discuss the applicability of Rolle's theorem of the following functions:

$f(x) = (x-a)^m (x-b)^n, m, n \in Z^+$, in the interval [a,b]

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10. Show that between any two roots of $e^{x}\cos x = 1$, there exists at least one root of $e^{x}\sin x - 1 = 0$

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11. Let $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots . + \frac{a_n - 1}{2} + a_n = 0$. Show that there exists at least one real x between 0 and 1 such that $a_0 x^n + a_1 x^{n-1} + \ldots . + a_n = 0$

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12. Verify the Rolle's theorem for the following functions:

(a) $f(x) = x^4 - 1$ on the interval $[-1, 1]$

(b) $f(x) = e^{x}(\sin x - \cos x)$ in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

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13. $f(x) = \begin{vmatrix} \sin x & \sin\alpha & \sin\beta \\ \cos x & \cos\alpha & \cos\beta \\ \tan x & \tan\alpha & \tan\beta \end{vmatrix}$, where $0 < x < \beta < \frac{\pi}{2}$ Show that $f'(l) = 0$, where $\alpha < l < \beta$

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14. A function f(x) is, continuous in the closed interval [0,1] and differentiable in the open interval [0,1] prove that $f'(x_1) = f(1) - f(0), 0 < x_1 < 1$

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15. Show that the set of all x for which $\log(1 + x) \leq x$ is equal to $[0, \infty]$

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16. Compute the value of θ the first mean value theorem $f(x + h) = f(x) + hf'(x + \theta h)$

if $f(x) = ax^2 + bx + c$

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17. Show that $x^n - a = 0$ has at most one real positive root if n is a positive integer.

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18. Show that the function f' if it exists in an interval, can not have an ordinary or removable discontinuity in that interval.

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19. Verify the Lagrange's theorem for the following functions:

(a) $f(x) = x^3$ in $[-1, 1]$

(b) $f(x) = \sin x$ in $\left[0, \frac{\pi}{2}\right]$

(c) $f(x) = x^n$ in $[-1, 1], n \in Z^+$

(d) $f(x) = 2x^2 - 7x + 10, x \in [2, 5]$

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20. Find the value of c, of mean value theorem.when

(a) $f(x) = \sqrt{x^2 - 4}$, in the interval [2,4]

(b) $f(x) = 2x^2 + 3x + 4$ in the interval [1,2]

(c) $f(x) = x(x-1)$ in the interval [1,2].

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PROBLEM SET (5) (MULTIPLE CHOICE QUESTIONS)

1. A value of c for which the conclusion of mean value theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is:

A. $2 \log_3 e$

B. $\frac{1}{2}\log_e 3$

C. $\log_e e$

D. $\log_e 3$

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2. In the Mean value theorem

$f(b) - f(a) = (b - a)f'(c)$, if a=4, b=9

and f(x)=$\sqrt{x}$, then the value of c is

A. 8

B. 5.25

C. 4

D. 6.25

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3. Rolle's theorem is not applicable to the function $f(x) = |x|$ defined on [-1,1] because

A. f is not continuous on $[-1, 1]$ because

B. f is not differentiate on (-1,1)

C. $f(-1) + f(1)$

D. $f(-1) = f(1) \neq 0$

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4. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval [1,3] and $f'\left[\frac{2\sqrt{3}+1}{\sqrt{3}}\right] = 0$, then a=

A. -11

B. -6

C. 6

D. 11

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5. For the function $f(x) = x + \frac{1}{x}, x \in [1, 3]$, the value of c for mean value therorem is

A. 1

B. $\sqrt{3}$

C. 2

D. None of these

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6. A function f is defined by $f(x) = x^x \sin x$ in $[0, \pi]$. Which of the following is not correct?

A. f is continous in $[0, \pi]$

B. f is differentiable in $[0, \pi]$

C. $f(0) = f(\pi)$

D. Rolle's theorem is not true is $[0, \pi]$

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7. The mean value theorem is $f(b) - f(a) = (b - a)f'(c)$.

Then for the function $x^2 - 2x + 3$, in $\left[1, \frac{3}{2}\right]$, the value of c:

A. $\frac{6}{5}$

B. $\frac{5}{4}$

C. $\frac{4}{3}$

D. $\frac{7}{6}$

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8. For the function $f(x) = e^{\cos x}$, Rolle's theorem is applicable when

A. $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

B. $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

C. $0 \leq x \leq 2\pi$

D. $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

Answer: A:B:C:D

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9. In which of the following functions, Rolle's theorem is applicable?

A. $f(x) = |x| \in -1 \leq x \leq 1$

B. $f(x) = \tan x$ in $0 \leq x \leq \pi$

C. $f(x) = 1 - (x-2)^{2/3}$ in $1 \leq x \leq$

D. $f(x) = x(x-2)^2$ in $0 \leq x \leq 2$

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10. The value of c in (0,2) satisfying the Mean Value theorem for the function $f(x) = x(x-1)^2, x\varepsilon[0,2]$ is equal to

A. $\frac{5}{4}$

B. $\frac{4}{5}$

C. $\frac{5}{3}$

D. $\frac{4}{3}$

Answer: D

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11. If f(x) satisfying the conditions of Rolle's theorem in [1,2] and f(x) is continuous in [1,2] then $\int_1^2 f'(x)dx$=

A. 4

B. 0

C. 1

D. 2

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12. If $f(x) = \sqrt{x-1} + \sqrt{x + 24 - 10\sqrt{x-1}}, 1 < x < 26$ be real valued function, then 'f' (x) for $1 < x < 26$ is:

A. 0

B. $\sqrt{x-1}$

C. $2\sqrt{x-1}$

D. $\sqrt{x-1} + 5$

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SELF ASSESSMENT TEST (MULTIPLE CHOICE QUESTIONS)

1. $\lim_{x \to \pi/4} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}$ equals

A. $\dfrac{1}{\sqrt{2}}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{2\sqrt{2}}$

D. 1

Answer: B

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2. $\lim_{x \to \pi/4} \dfrac{1 - \tan x}{1 - \sqrt{2}\sin x}$ equals:

A. $\dfrac{1}{\sqrt{2}}$

B. $\dfrac{1}{2}$

C. $\frac{1}{2\sqrt{2}}$

D. 1

Answer: D

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3. $\lim_{x \to 0} \frac{\sin 2x}{x}$ is equal to:

A. 0

B. 1

C. $1/2$

D. 2

Answer: D

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4. The function f defined as $f(x) = (\sin x^2)/x$ for $x \neq 0$ and $f(0) = 0$ is:

A. continuous and derivable at x = 0,

B. neither continuous nor derivable at x = 0,

C. continuous but not derivable at x = 0

D. none of these.

Answer: A

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5. For a real number y, let [y] denotes the greatest integer less than or equal to y. Then, the function $f(x) = \frac{\tan \pi[(x - \pi)]}{1 + [x]^2}$ is

A. discontinuous at some x,

B. continuous at all x, but the derivative f'(x)does not exist for some x.

C. f'(x) exists for all x but second derivative f" (x) does not exist.

D. f'(x) exists for all x.

Answer: D

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6. There exists a function f(x) satisfying

$f\{0\} = 1, f'\{0\} = -1, f(x) > 0$ for all x and

A. $f'(x) < 0$ for all x.

B. $-1 < f''(x) < 0$ for all x.

C. $-2 \leq f''(x) \leq -1$ for all x.

D. $f''(x) < -2$ for all x.

Answer: A

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7. The function f (x) = 1+ $|\sin x|$ is

A. continuous nowhere,

B. continuous everywhere,

C. differentiate nowhere,

D. not differentiable at x = 0,

Answer: B::D

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8. Let [x] denotes the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then $f(x)$ is

A. continuous at x=0,

B. continuous in (-1,0),

C. differentiable at x = 1

D. differentiable in (-1,1),

Answer: A::B::D

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9. The following functions are continuous on $(0, \pi)$

A. $\tan x$

B. $\int_0^x \frac{\sin 1}{t} dt$

C. $\begin{cases} 1 & 0 < x \leq \frac{3\pi}{4} \\ 2\sin\left(\frac{x}{3}\right) & \frac{3\pi}{4} < x < \pi \end{cases}$

D. $\begin{cases} x \sin x & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}\sin(2\pi + x) & \frac{\pi}{2} < x < \pi \end{cases}$

Answer: B::C

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10. If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$

A. tan [f (x)] and 1/f (x) are both continuous

B. tan [f (x)] and 1/f (x) are both discontinuous

C. tan [f (x)] and $f^{-1}(x)$ are both continuous

D. tan [f (x]] is discontinuous but 1/f (x) is not.

Answer: B

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11. If $f(x) = 3x - 5$, then $f^{-1}(x)$

A. is given by $\frac{1}{3x - 5}$

B. is given by $\frac{x + 5}{3}$

C. does not exist because f is not one-one

D. does not exist because f is not onto.

Answer: B

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12. If $\lim_{x \to 0} \left[1 + x \log\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b \sin^2 \theta, b > 0 \text{ and } \theta \in (- \pi, \pi]$,

then the value of θ is

A. $\pm \frac{\pi}{4}$

B. $\pm \frac{\pi}{3}$

C. $\pm \frac{\pi}{6}$

D. $\pm \frac{\pi}{2}$

Answer: D

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13. Show that the $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x - 2)\}}}{x - 2}\right)$ doesnot exist.

A. does not exist

B. equals $\sqrt{2}$

C. equals $-\sqrt{2}$

D. equals $\frac{1}{\sqrt{2}}$

Answer: A

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14. Let $f : R \to R$ be a positive increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ =

A. 1

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

Answer: A

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15. If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$ then

A. a=1, b=4

B. a=1, b=-4

C. a=2, b=-3

D. a=2, b=3

Answer: C

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16. The value of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{1/2}} & , \quad x > 0 \end{cases}$$

is continuous for all x in R, are

A. $p = \frac{1}{2}, q = -\frac{3}{2}$

B. $p = \frac{5}{2}, q = \frac{1}{2}$

C. $p = -\frac{3}{2}, q = \frac{1}{2}$

D. $p = \frac{1}{2}, q = \frac{3}{2}$

Answer: C

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17. Let f : (-1,1) $\rightarrow$ R be a differentiabale function with f(0) = -1 and f'(0) = 1. Let

g(x) = $\left[f(2f(x)+2)\right]^2$, then g'(0)=

A. 4

B. -4

C. 0

D. -2

Answer: B

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18. If $f: R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$ where [x] denotes the greatest integer function, then f is:

A. continous for every real R

B. discontinous only at x=0

C. discontinuous only at non-zero integral values of k

D. continuous only at x = 0

Answer: A

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19. Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$. If L is finite, then

A. a=2

B. $a = 1$

C. $L = \frac{1}{64}$

D. $L = \frac{1}{32}$

Answer: A::C

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20. Let $f: R \to R$ be a function such that $f(x+y) = f(x) + f(y) \forall x, y \in R$. If If f(x) is differentiable at x = 0, then

A. f(x) is differentiable only in a finite interval containing 0

B. f(x) is continuous $\forall x \in R$

C. $f'(x)$ is constant $\forall x \in R$

D. f[x) is differentiable except at finitely many points

Answer: B::C

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21. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$, then

A. f(x) is continous at $x = -\frac{\pi}{2}$

B. $f(x)$ is not differentiable at x=0

C. $f(x)$ is differentiable at x=1

D. f(x) is differentiable at $x = -\frac{3}{2}$

Answer: A::B::C::D

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22. Let $f:(0,1) \to R$ be defined by $f(x) = \frac{b-x}{1-bx}$ where b is a constant such that $0 < b < 1$ Then:

A. f is not invertible on $[0, 1]$

B. $f \ne f^{-1}$ on [0,1] and f'(b) = $\frac{1}{f'(0)}$

C. $f = f^{-1}$ on [0,1] and $f'(b) = \frac{1}{f'(0)}$

D. f^{-1} is differentiable on [0,1]

Answer: A::B

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23. Let f be a real valued function defined on the interval $[0, \infty]$ by $f(x) = \log x + \int_0^x \sqrt{1 + \sin t}.\, dt$ then which is (are) true:

A. $f''(x)$ exists $\forall x \in [0, \infty]$

B. $f'(x)$ exists $\forall x \in [0, \pi]$ and f' is continous on $[0, \infty]$ but not differentiable on $[0, \infty]$

C. $\alpha > 1$ such that $|f'(x)| < |f(x)| \forall x \in [\alpha, \infty|$

D. $3\beta > 0$ such that $|f(x)| + |f'(x)| \leq \beta \forall x \in [0, \infty]$

Answer: B::C

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24. Q. For every integer n, let an and bn be real numbers. Let function $f: R \rightarrow R$ be given by a $f(x) = \{a_n + \sin \pi x, f \text{ or } x \in [2n, 2n+1],$

$\_\ n + \cos \pi x, f \text{ or } x \in (2n+1, 2n)$ for all integers n.

A. $a_{n-1} - b_{n-1} = 0$

B. $a_n - b_n = 1$

C. $a_n - b_{n+1} = 1$

D. $a_{n-1} - b_n = -1$

Answer: B::D

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25. Let $f: (-1, 1) \rightarrow R$ be such that $f(\cos 4\theta) = \frac{2}{2 - \sec^2 \theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is/are

A. $1 - \sqrt{\frac{3}{2}}$

B. $1+\sqrt{\frac{3}{2}}$

C. $1-\sqrt{\frac{2}{3}}$

D. $1+\sqrt{\frac{2}{3}}$

Answer: A::B

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26. Let $f(x)=\left\{x^2\left|\frac{\cos\pi}{x}\right|, , x\neq 0, x\in R, x=0\right.$ then f is

A. diff. at x = 0 and x = 2

B. diff. at x = 0 but not at x = 2

C. not diff at x = 0,but diff. at x = 2

D. differentiable neither at x = 0 nor x = 2

Answer: A

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SELF ASSESSMENT TEST (TRUE AND FALSE TYPE QUESTIONS)

1. If f(x) and g(x) have no derivative at a point, then f(x) g(x) has no derivative at that point. True or False ?

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SELF ASSESSMENT TEST (FILL IN THE BLANKS)

1. A discontinuous function y = f(x) satisfying $x^2 + y^2 = 4$ is given by f(x) = ...

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2. Let $f(x) = \{x - 1)^2 \sin\left(\frac{1}{x - 1} - |x|\right.$,if $x \neq 1$ and -1, if $x = 1$ 1valued function. Then, the set of pointsf, where $f(x)$ is not differentiable, is .... .

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3. Let A be a set of n distinct elements. Then the total number of distinct function from $A \rightarrow A$ is _____ and out of these, ____ are onto functions.

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4. If $f(x) = \sin x, x \neq n\pi, n = 0, \pm 1, \pm 2, \pm 3 = 2,$ otherwise and $g(x) = x^2 + 1, x \neq 0, 2, g(0) = 4, g(2) = 5,$ then $\lim_{x \to 0} g[f(x)]$ is-

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MISCELLANEOUS EXERCISE (MATCHING ENTRIES)

1. Functions

List-A		List-B	
(a)	If $f(x) = \frac{a^x + a^{-x}}{2}$ $(a > 2)$ then $f(x+y) + f(x-y) =$	1.	$\frac{1}{2}\log\left(\frac{x-1}{3-x}\right)$
(b)	If $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$, then $f^{-1}(x) =$	2.	odd
(c)	The period of function $\left\lvert \sin \frac{x}{2} \right\rvert + \lvert \cos x \rvert$ is	3.	$2f(x)f(y)$
(d)	If $f(x) = \sin[\log(x + \sqrt{x^2+1})]$, then $f(x)$ is odd or even	4.	x
(e)	If $f(x) = \frac{1}{1-x}$, then $(f \circ f \circ f)x =$	5.	2π

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2. Range and domain of a functions

List-A		List-B	
(a)	Domain of $y = \sin^{-1}\left[\log_2\left(\frac{x^2}{2}\right)\right]$ is	1.	$1 \le x < 4$
(b)	Domain of $y = \sin^{-1}\frac{x-3}{2} - \log(4-x) =$	2.	$\left]-\frac{3\pi}{2}, \frac{\pi}{2}\right[$
(c)	Range of $y = \frac{1}{2 - \sin 3x}$ is	3.	$[-2, -1] \cup [1, 2]$
(d)	Range of $y = \frac{x^2+x+2}{x^2+x+1}, x \in R$ is	4.	$\left[\frac{1}{3}, 1\right]$
(e)	Range of $y = \sin x - \cos x$ is	5.	$\left[1, \frac{7}{3}\right]$
(f)	Range of $\tan^{-1} x - \cot^{-1} x$ is	6.	$[-\sqrt{2}, \sqrt{2}]$

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3. Increasing and decreasing functions

List-A		List-B	
(a)	$y = x^2 e^{-x}$ is an increasing function in the interval ...	1.	$\pi / 3$
(b)	$y = \frac{x}{\log x}$ is an increasing function in the interval ...	2.	$(1, 2)$
(c)	The length of the longest interval in which $y = 3\sin x - 4\sin^3 x$ is increasing is	3.	(e, ∞)
(d)	If $f(x) = \int e^x (x-1)(x-2)\,dx$ then f decreases in the interval	4.	$(0, 2)$

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4. Limits

List-A	List-B
(a) $\underset{x \to \infty}{\text{Lt}}\ x \cos \frac{\pi}{4x} \sin \frac{\pi}{4x}$ is	1. -1
(b) $\underset{x \to 0}{\text{Lt}}\ \frac{\tan[-\pi^2]x^2 - [\pi^2]x^2}{\sin^2 x}$	2. 4
(c) $\underset{x \to \infty}{\text{Lt}}\ \sqrt{\left\{\frac{2x - \sin x \cos x}{x + \cos^2 x + \sin^2 x}\right\}}$	3. $\frac{\pi}{4}$
(d) If $f(a) = 9, f'(a) = 4$, then $\underset{x \to 9}{\text{Lt}}\ \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$	4. $\sqrt{2}$
(e) If $f(x)$ is differentiable increasing function, then $\underset{x \to 0}{\text{Lt}}\ \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is	5. 0

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5.

List-A	List-B
(a) $\underset{x \to 0}{\text{Lt}}\ \frac{\log_{\sec(x/2)} \cos x}{\log_{\sec x} \cos(x/2)}$	1. $e^{(n-1)/2}$
(b) $\underset{x \to 0}{\text{Lt}}\ \frac{8^x - 4^x - 2^x - 1^x}{x^2} =$	2. 16
(c) $\underset{x \to 0}{\text{Lt}}\ \left[\frac{1 + 5x^2}{1 + 3x^2}\right]^{1/x^2} =$	3. $2(\log 2)^2$
(d) $\underset{x \to \infty}{\text{Lt}}\ \left[\frac{x^2 + 5x + 3}{x^2 + x + 2}\right]^x =$	4. e^2
(e) $\underset{x \to 1}{\text{Lt}}\ \left[\frac{x^n - 1}{n(x - 1)}\right]^{1/(x-1)} =$	5. e^4

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6.

List-A		List-B
(a)	$\underset{x\to 3}{\text{Lt}} \dfrac{x-3}{\lvert x-3\rvert} =$	1. $16\sqrt{2}\log 2 . \log 3$
(b)	If $f(x) = \dfrac{\log(1+ax) - \log(1+bx)}{x}$ be continuous at $x = 0$, then $f(0) =$	2. $\sqrt{e}$
(c)	If $f(x) = (\sin 2x)^{\tan^2 2x}$ be continuous at $x = \pi/4$, then $f\left(\dfrac{\pi}{4}\right) =$	3. $a+b$
(d)	If $f(x) = \begin{cases} \dfrac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$ be continuous at $x = 0$, then $k =$	4. Does not exist

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7.

List-A		List-B
(a)	If $y = \lvert x-1\rvert + \lvert x-3\rvert$, then at $x = 2, \dfrac{dy}{dx} = \ldots$	1. 32
(b)	If $f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then the set of all points of differentiability $= \ldots$	2. 0
(c)	If $f(x) = a\lvert x\rvert^2 + b\lvert x\rvert + c = 0$ and if $f'(x)$ exists at $x = 0$, then the value of b is	3. $(-\infty, \infty)$
(d)	If $f'(x) = g(x)$ and $g'(x) = -f(x)\ \forall\ x$ and $f(2) = 4 = f'(2)$, then $f^2(16) + g^2(16) = \ldots.$	4. 0

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8.

Column I		Column II	
(a)	$f(x) = (x+1)^{2/3} - (x-1)^{2/3}$	(p)	1
(b)	$f(x) = x^{3/2}\,[\sqrt{x^3+1} - \sqrt{x^3-1}]$	(q)	$\dfrac{3}{2}$
(c)	$f(x) = \sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x + 3}$	(r)	0
(d)	$f(x) = \sqrt{(x+1)(x+2)} - x$	(s)	$\dfrac{5}{2}$

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9.

Column I	Column II
(a) $f(x)=\frac{\log(1+4x)}{x}$	(p) $\frac{1}{4}$
(b) $f(x)=\frac{\log(4+x)-\log 4}{x}$	(q) 0
(c) $f(x)=\frac{1}{\sin x}-\frac{1}{\cos x}$	(r) 4
(d) $f(x)=\frac{1-\cos^3 x}{x\sin 2x}$	(s) $\frac{3}{4}$

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MISCELLANEOUS EXERCISE (ASSERTION/ REASONS)

1. Let $f(x)=(x+1)^2-1, x \geq -1$,IF $f(x):[-1,\infty] \rightarrow [-1,\infty]$

Statement 1: The set $\left\{x : f(x)=f^{-1}(x)\right\}=\{0, -1\}$

Statement-2: f is a bijection.

A. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

B. Statement 1 is ture, statement 2 is true, statement 2 is not a correct explanation for statement 1.

C. Statement 1 is ture, statement 2 is false.

D. Statement 1 is false, statement 2 is true.

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2. Let $f(x) = x|x|$ and $g(x) = \sin x$

Statement-1: gof is differentiable at x=0 and derivative is continous at that point.

Statement-2: gof is twice differentiable at x=0

A. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

B. Statement 1 is ture, statement 2 is true, statement 2 is not a correct explanation for statement 1.

C. Statement 1 is ture, statement 2 is false.

D. Statement 1 is false, statement 2 is true.

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3. Statement-1: $\text{lt}_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots. + (x+100)^{10}}{x^{10} + 9^{10}} = 100$

Statement-2: If $f(x)$ and $\phi(x)$ are polynomials of same degree, then $\text{lt}_{x \to \infty} \frac{f(x)'}{\phi(x)} = \frac{\text{Leading coeff. Of f(x)}}{\text{Leading coeff. of} \phi(x)}$

A. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

B. Statement 1 is ture, statement 2 is true, statement 2 is not a correct explanation for statement 1.

C. Statement 1 is ture, statement 2 is false.

D. Statement 1 is false, statement 2 is true.

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