MATHS

BOOKS - ML KHANNA

MAXIMA AND MINIMA

Problem Set 1 Multiple Choice Questions

1. If $f(x) = x^5 - 5x^4 + 5x^3 - 10$ has local max. and min. at x = p and x = q resp. , then (p,q) =

A. (0,1)

B. (1,3)

C. (1,0)

D. None of these

Answer: B

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2. The function $f(x) = \frac{2}{x} + \frac{x}{2}$ has a local minimum at x =

A. -2

B. 0

C. 1

D. 2

Answer: D

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3. If $a_1^s \in R$ then the expression $\Sigma_{i=1}^n (x - a_1)^2$ assumes its least. Value at x =

A. Σa_1

B. $2\Sigma a_1$

C. $n\Sigma a_1$

D. $\frac{1}{n}\Sigma a_1$

Answer: D

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4. The real number x when added to its inverse gives the minimum value of the sum at x equal to

A. -2

B. -1

C. 1

D. 2

Answer: C

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5. The max. and min. value of $\frac{x^2+x+1}{x^2-x+1}$ are

A. (2,1)

B. $\left(3, \frac{1}{3}\right)$

C. $(1, 0)$

D. $(3, 1)$

Answer: B

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6. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is(A) 0 (B) 1 (C) 3 (D) $\frac{1}{3}$

A. 0

B. $1/3$

C. 1

D. 3

Answer: B

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7. A function f(x) $= \dfrac{x^2 - 3x + 2}{x^2 + 2x - 3}$ is

A. min. at x= -3,max. at x= 1

B. max . atx=-3

C. Increasing in its domain

D. Decreasing in its ·domain

Answer: C

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8. The maximum value of $\left(\dfrac{\log x}{x}\right)$ is

A. 1

B. $2/e$

C. e

D. $1/e$

Answer: D

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9. x^x has a stationary point at

A. x = e

B. x = 1/e

C. x = 1

D. $x = \sqrt{e}$

Answer: B

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10. When x is positive, the minimum value of x^x is

A. e^{-1}

B. $e^{-1/e}$

C. $e^{1/e}$

D. e^e

Answer:

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11. Greatest value of $(1/x)^x$ is

A. e

B. $e^{1/e}$

C. $(1/e)^e$

D. None of these

Answer: B

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12. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

A. 1

B. e

C. $e^{1/e}$

D. None

Answer: C

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13. The minimum value of $ax+by$ when $xy=r^2$ is

A. $2r\sqrt{ab}$

B. -2ab$\sqrt{r}$

C. $-2r\sqrt{ab}$

D. none

Answer: A

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14. Given that $f(x) = x^{1/x}, x > 0$ has the maximum value at x = e, then

A. $e^{\pi} > \pi^{e}$

B. $e^{\pi} < \pi^{e}$

C. $e^{\pi} = \pi^{e}$

D. $e^{\pi} \leq \pi^{e}$

Answer: A

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15. In the interval $[0, 1]$, the function $x^{25}(1-x)^{75}$ takes its maximum value at the point 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. 0

Answer: C

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16. In a submarine telegraph cable the speed of signaling varies as $x^2 \log\left(\frac{1}{x}\right)$ where x is the ratio of the radius of the cable to that of covering. Then the greatest speed is attained when this ratio is

A. $1 : e$

B. $1 : \sqrt{e}$

C. $1 : e\sqrt{e}$

D. none

Answer: B

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17. If $y = a \log x + bx^2 + x$ has its exteme values at x=-1 and x=2, then find a and b.

A. $a = 2, b = -1$

B. $a = 2, b = -1/2$

C. $a = -2, b = 1/2$

D. None of these

Answer: B

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18. The function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0,2] at x = 1. Then the value of a is

A. 120

B. -120

C. 52

D. none of these

Answer: A

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19. The fuction $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1 = 0$ has a local maximum at $x = \alpha$ and a local minimum at $x = \beta$ such that $\beta = \alpha^2$ then a is equal to :

A. 0

B. $1/4$

C. 2

D. either 0 or 2

Answer: C

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20. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of f (x). As b varies, the range of m (b) is

A. [0,1]

B. $\left[0, \frac{1}{2}\right]$

C. $\left[\frac{1}{2}, 1\right]$

D. $[0, 1]$

Answer: D

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21. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$ on $[-2, 4]$. The relative maximum occurs at $x = -2$ (b) -1 (c) 2 (d) 4

A. -2

B. -1

C. 2

D. 4

Answer: D

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22. If minimum value of $f(x) = \left(x^2 + 2bx + 2c^2\right)$ is greater than the maximum value of $g(x) = -x^2 - 2cx + b^2$, then $(x \in R)$

A. $|c| > |b|\sqrt{2}$

B. $abcc\sqrt{2} > b$

C. $0 < c < \sqrt{2}b$

D. no real value of a

Answer: A

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23. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$ attains its maximum and minimum at $x = p$ and $x = q$ respectively, such that $p^2 = q$, then the value of a is-

A. $1/2$

B. 1

C. 2

D. 3

Answer: C

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24. For the curve $y = xe^{x}$, the point

A. x = -1 is a point of minima

B. x = 0 is a point of minima

C. x= -1 is a point of maxima

D. x=0 is a point of maxima

Answer: A

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25. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are

A. (10,10)

B. (12,8)

C. (15,5)

D. None

Answer: C

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26. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are

A. 44, 20

B. 16, 48

C. 05,15

D. 50, 14

Answer: C

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27. A particle is moving in a straight line such that its distance at any time t is given by

$x = \frac{t^4}{4} - 2t^3 + 4t^2 + 7$. Then

A. velocity is max. at $t = (6 - 2\sqrt{3})/3$

B. acceleration is min. at t = 2

C. min. distance is al t= 0, 4

D. None of these

Answer: A::B::C

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28. Find the points on the curve $5x^2 - 8xy + 5y^2 = 4$ whose distance from the origin is maximum or minimum.

A. $(\sqrt{2}, \sqrt{2})$

B. $(-\sqrt{2}, -\sqrt{2})$

C. $\left(\frac{\sqrt{2}}{3}, -\frac{\sqrt{2}}{3}\right)$

D. $\left(-\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)$

Answer: A::B::C::D

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29. The maximum distance of the point (a, 0) from the curve $2x^2 + y^2 - 2x = 0$ is -

A. $\sqrt{1 + 2a + 2a^2}$

B. $\sqrt{1 + 2a - a^2}$

C. $\sqrt{1 - 2a + 2a^2}$

D. $\sqrt{1 - 2a + a^2}$

Answer: C

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30. Find the coordinates of the point on the curve $y = \dfrac{x}{1 + x^2}$ where the tangent to the curve has the greatest slope.

A. (0,0)

B. $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

C. $\left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right)$

D. none

Answer: A

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31. Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

A. (-2, 8)

B. $(-2, -8)$

C. $(2, -8)$

D. none

Answer: B

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32. The point (0,5) is closest to the curve $x^2 = 2y$ at

A. $(2\sqrt{2}, 0)$

B. (0,0)

C. $(2, 2)$

D. None of these

Answer: D

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33. Find the point on the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$ which is nearest to the line $3x + 2y + 1 = 0$ and compute the distance between the point and the line.

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34. The maximum distance of the point (a, 0) from the curve $2x^2 + y^2 - 2x = 0$ is -

A. $\sqrt{1 + 2k - k^2}$

B. $\sqrt{1 - 2k + 2k^2}$

C. $\sqrt{1 + 2k + 2k^2}$

D. $\sqrt{1 - 2k + k^2}$

Answer: B

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35. The co-ordinates of the points on the barabola $y^2 = 8x$, which is at minium distance from the circle $x^2 + (y + 6)^2 = 1$ are

A. (2,4)

B. (2, -4)

C. (18,-12)

D. (8,8)

Answer: B

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36. Two towns A and Bare 60 km. apart. A school is to tbe built to serve 150 students is to be as small as possible, the the school should be built at

A. twon B

B. 45 km. from town A

C. townA

D. 45 km. from town B

Answer: C

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37. The fuel charges for running a train are proportional to the square of the speed generated in km/h, and the cost is Rs. 48 at 16 km/h. If the fixed charges amount to Rs. 300/h, the most economical speed is 60 km/h (b) 40 km/h 48 km/h (d) 36 km/h

A. 10

B. 20

C. 30

D. 40

Answer: D

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38. Assuming the petrol burnt (per hour) in driving a motor boat veries as the cube of its velocity, show that the most economical speed when going against the current of c miles per hour is $\left(\frac{3c}{2}\right)$ miles per hour.

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39. If a function $f(x)$ has $f'(a) = 0 and f^a = 0,$ then $x = a$ is a maximum for $f(x)$ $x = a$ is a minimum for $f(x)$ it is difficult to say $(a) and (b)$ $f(x)$ is necessarily a constant function.

A. x= a is a maximum for f (x)

B. x= a is a minimum for f (x)

C. It is difficult to say (a) and (b)

D. f (x) is necessarily a constant function

Answer: C

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40. If $f''(x) < 0, x \in (a, b), f(c)$ attains maximum value at $(c, f(c)),$ where a < c < b, then f'(c) is equal to :

A. $\frac{f(b) - f(a)}{2}$

B. $f'(c) = 0$

C. $f'(c) = \frac{f(a) - f(b)}{2}$

D. $f'(c) = 2(f(b) - f(a))$

Answer: B

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41. If $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ is a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. Then, the function $P(x)$ has

A. Neither a max. nor a min.

B. Only one maximum

C. Only one minimum

D. None of these

Answer: C

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42. The largest value of $2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 4$ occurs at x =

A. -2

B. -1

C. 2

D. 4

Answer: D

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43. The difference between the greatest and least values of the function f(x) = $\sin 2x - x$ on $[-\pi/2, \pi/2]$ is

A. π

B. $\frac{\sqrt{3}}{2}+\frac{\pi}{3}$

C. $\frac{\sqrt{3}+\sqrt{2}}{2}+\frac{\pi}{6}$

D. $\frac{1}{2}\left(\sqrt{3}+sqr2\right)$

Answer: A

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44. The difference between the greatest and the least values of the function $f(x)=\int_0^x \left(at^2+1+\cos t\right)dt,\ a>0$ for $x\in[2,3]$ is

A. $\frac{19}{3}a+1+\sin 3-\sin 2$

B. $\frac{18}{3}a+1+2\sin 3$

C. $\frac{18}{3}a-1+2\sin 3$

D. none

Answer: A

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45. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of p+q is

A. 2

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

Answer: D

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46. The function

$f(x) = \int_1^x \left\{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\right\}dt$ attains its maximum at x =

A. 1

B. 2

C. 3

D. 4

Answer: A

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47. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$ has a local minimum at $x =$ 0 (b) 1 (c) 2 (d) 3

A. 0

B. 1

C. 2

D. 3

Answer: B::D

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48. The poins of extremum of $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt$ are

A. x = 0

B. x = 1

C. x = 1/2

D. x = -1

Answer: B::D

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49. $f(x) = \int_0^x t(t-1)(t-2)dt$ takes on its minimum values of x =

A. 0

B. 1

C. 2

D. none

Answer: A::C

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50. If $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - e^{x-1}, & 1 < x \leq 2 \\ x - e, & 2 < x \leq 3 \end{cases}$ and $g'(x) = f(x), x \in [1, 3]$, then`

A. $g(x)$ has local minima at x = e and local maxima at x = 1 + ln 2

B. g(x) has local maxima at x = 1 and local minima at x = 2

C. g(x) does not have local maxima

D. g(x) does not have local minima

Answer: A:B

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51. A cubic function $f(x)$ vanishes at $x = -2$ and has relative minimum/maximum at $x = -1 and x = \frac{1}{3}$ if $\int_{-1}^{1} f(x)dx = \frac{14}{3}$. Find the cubic function $f(x)$.

A. $x^3 - x^2 - x$

B. $x^3 + x^2 - x + 1$

C. $x^3 + x^2 - x + 2$

D. $x^3 + x^2 - x - 2$

Answer: C

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52. Let three degree polynomial function f(x) has local maximum at x = -1 and f(-1) = 2, f(3) = 18, f'(x) has a minima at x = 0, then :

A. the distance between (-1,2) and (a,f(a)) where a denotes point where function has local max/min is $2\sqrt{5}$

B. the function decreases from 1 to $2\sqrt{5}$

C. the function increases from 1 to $2\sqrt{5}$

D. the function decreases from -1 to 1

Answer: A

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53. The minimum value of $ax + by$ when $xy = r^2$ is

A. $2c\sqrt{ab}$

B. $2ab\sqrt{c}$

C. $-2c\sqrt{a}b$

D. None of these

Answer: A

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54. If $a^2x^4 + b^2y^4 = c^6$, then maximum value of xy is

A. $\frac{c^3}{\sqrt{ab}}$

B. $\frac{c^3}{\sqrt{2ab}}$

C. $\frac{c^3}{ab}$

D. $\frac{c^3}{2ab}$

Answer: B

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55. The minimum value of

$f(x) = |3 - x| + |2 + x| + |5 - x|$ is

A. 7

B. 10

C. 8

D. 0

Answer: A

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56. The interval into which the function $y = \frac{x-1}{x^2 - 3x + 3}$ transforms the entire real line is

A. $[1/3, 2]$

B. $[-1/3, 1]$

C. $[-1/3, 2]$

D. none of these

Answer: B

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57. The set of values of λ for which the function $f(x) = (4\lambda - 3)(x + \log 5) + 2(\lambda - 7). \cot \frac{x}{2} \sin^2 \frac{x}{2}$ does not possess critical point is :

A. $[1, \infty]$

B. $(2, \infty)$

C. $(-\infty, -4/3)$

D. $(-\infty, -1)$

Answer: B::C

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58. Let f(x) = $\begin{cases} |x| & \text{for} \quad 0 < |x| \leq 2 \\ 1 & \text{for} \quad x = 0 \end{cases}$ Then at x = 0, f(x) has

A. a local maximum

B. no local maximum

C. a local minimum

D. no extermum

Answer: D

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59. Let (h, k) be a fixed point, where $h > 0, k > 0.$ A straight line passing through this point cuts the positive direction of the coordinate axes at the point $PandQ$. Find the minimum area of triangle OPQ, O being the origin.

A. hk

B. 2hk

C. $\frac{1}{2}hk$

D. None of these

Answer: B

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60. The minimum value of 2^{x^2-3} ^ $(3+27)$ is 2^{27} (b) 2 (c) 1 (d) none of these

A. 2^{27}

B. 2

C. 1

D. None of these

Answer: C

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61. If $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-2$ where a > 0, has + ive point of maximum then a varies over an interval of length

A. $\frac{8}{7}$

B. $\frac{6}{7}$

C. $\frac{4}{7}$

D. $\frac{3}{7}$

Answer: A

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62. If $f(x) = (x^2 - 1)^{n+1}(x^2 + x + 1)n \in N$ and f(x) has a local extremum at x = 1, then n =

A. 2

B. 3

C. 4

D. 5

Answer: A:B::C:D

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Problem Set 1 True And False

1. v24

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2. If $f'(x) = (x-a)^{2n}(x-b)^{2m+1}$ then

f at x = a is a point of neither max. nor min.

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3. If $f'(x) = (x-a)^{2n}(x-b)^{2m+1}$ then

x = b is a point of maxima.

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Problem Set 2 Multiple Choice Questions

1. the maximum value of function $\sin x(1+\cos x)$ is

A. 3

B. $3\sqrt{3}/4$

C. 4

D. $3\sqrt{3}$

Answer: B

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2. $f(x) = \sin x + \cos 2x (x > 0)$ has minma for x =

A. $n\frac{\pi}{2}$

B. $\frac{3}{2}(n+1)\pi$

C. $\frac{1}{2}(2n+1)\pi$

D. None of these

Answer: C

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3. At x = $\frac{5\pi}{6}$, f(x)= 2sin 3x+3 cos 3x is

A. maximum

B. minimum

C. zero

D. none of these

Answer: D

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4. The maximum and minimum value of f(x) $= ab\sin x + b\sqrt{1-a^2}\cos x + c$ lie in the interval (assuming $|a| < 1, b > 0)$

A. $\begin{bmatrix} b-c \\ b+c \end{bmatrix}$

B. $[b-c, b+c]$

C. $[c-b, b+c]$

D. None of these

Answer: C

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5. $f(x) = \sin^p x \cos^q x (p, q > 0, 0 < x < \pi/2)$ has point of maxima at

A. $x = \tan^{-1}\sqrt{(p/q)}$

B. $x = \tan^{-1}\sqrt{(q/p)}$

C. no such point exists

D. None of these

Answer: A

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6. The minimum value of $\frac{a^2}{\cos^2 x} + \frac{b^2}{\sin^2 x}$

A. $(a-b)^2$

B. $a^2 + b^2$

C. $(a+b)^2$

D. $a^2 - b^2$

Answer: C

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7. A tangent is drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at the point $(3\sqrt{3}\cos\theta \sin\theta)$ where $0 < \theta < \frac{\pi}{2}$. The sum of intecepts of the tangents with the coordinates axes is least when θ equals

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{4}$

Answer: A

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8. The min. intercept made by the axes on the lengent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

A. a^2+b^2

B. $a+b$

C. $a-b$

D. none

Answer: B

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9. Find the minimum length of radius vector of the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$

A. $a-b$

B. $a+b$

C. $2a+b$

D. None of these

Answer: B

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10. $f(x)=1+2\sin x+3\cos^2 x, \left(0\leq x<\frac{2\pi}{3}\right)$ is

A. Min. at $x=90^\circ$

B. Max. at $x=\frac{\sin^{-1}1}{\sqrt{3}}$

C. Min. at $x=30^\circ$

D. Max. at $x=\sin^{-1}\left(\frac{1}{3}\right)$

Answer: A::D

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11. Let $f(x) = 1 + 2\sin x + 2\cos^2 x, 0 \leq x \leq \pi/2$. Then

A. f (x) is greatest at $\pi/6$

B. least at $0, \pi/2$

C. increasing in $[0, \pi/6]$ and decreasing in $(\pi/6, \pi/2)$

D. f(x) is continuous in $[0, \pi/2]$

Answer: A:C:D

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12. The difference between the greatest and the least value of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$

A. $\frac{3}{8}$

B. $\frac{8}{7}$

C. $\frac{9}{4}$

D. $\frac{2}{3}$

Answer: C

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13. The greatest value of the function $f(x) = \frac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)}$ on the interval $\left[0, \frac{\pi}{2}\right]$ is

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. 1

D. $-\sqrt{2}$

Answer: C

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14. The minimum value of $\left(1+\frac{1}{\sin^{n}\alpha}\right)\left(1+\frac{1}{\cos^{n}\alpha}\right)$ is

A. 1

B. 2

C. $\left(1+2^{n/2}\right)^{2}$

D. none of these

Answer: C

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15. The least value of a for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ has at least one solution on the interval $\left(0, \frac{\pi}{2}\right)$ is

A. 9

B. 4

C. 8

D. 1

Answer: A

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16. The set of all values of k for which the function $f(x)=(k^2-3k+2)\left(\cos^2\frac{x}{4}-\sin^2\frac{x}{4}\right)+(k-1)x+\sin 1$ does not possess critical points is

A. $[1,\infty]$

B. $(0,1)\cup(1,4)$

C. $(-2,4)$

D. $(1,3)\cup(3,5)$

Answer: B

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17. If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$ then find the maximum value of $\tan A \cdot \tan B$.

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{3}$

C. 3

D. $\sqrt{3}$

Answer: B

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Problem Set 2 True And False

1. $\frac{x}{1 + x \tan x}$ is max. when $x = \cos x, 0 \leq x \leq \frac{\pi}{2}$

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Problem Set 3 Multiple Choice Questions

1. A wire of length a is cut into two parts which are bent, respectively, in the form of a square and a circle. The least value of the sum of the areas so formed is $\frac{a^2}{\pi+4}$ (b) $\frac{a}{\pi+4}$ $\frac{a}{4(\pi+4)}$ (d) $\frac{a^2}{4(\pi+4)}$

A. $\frac{a^2}{\pi+4}$

B. $\frac{a}{\pi+4}$

C. $\frac{a}{4(\pi+4)}$

D. $\frac{a^2}{4(\pi+4)}$

Answer: D

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2. A box, constructed from a rectangular metal sheet, is 21 cm by 16cm by cutting equal squares of sides x from the corners of the sheet and then turning up the projected portions. The value of x os that volume of the box is maximum is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: C

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3. If the sum of the lengths of the hypotenuse and another side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between these sides is $\frac{\pi}{3}$.

A. $\frac{\pi}{12}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: C

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4. The sides of the rectangle o.f greatest area which can be inscribed in the· ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ are given by

A. $(4, 2\sqrt{2})$

B. $(2\sqrt{2}, 2)$

C. $(2, \sqrt{2})$

D. $(4\sqrt{2}, 4)$

Answer: A

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5. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

A. $\sqrt{ab}$

B. a/b

C. $2ab$

D. ab

Answer: C

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6. The muinimum area of the triangle formed by the tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the coordinate axes is

A. $\frac{a^2+b^2}{2}$

B. $\frac{(a+b)^2}{2}$

C. ab

D. $\frac{(a-b)^2}{2}$

Answer: B

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7. A point P is given on the circumference of a circle of radius r. Chords QR are parallel to the tangent at P. Determine the maximum possible area of triangle PQR.

A. $3\frac{\sqrt{2}}{4}r^2$

B. $\frac{3\sqrt{3}}{4}r^2$

C. $\frac{3}{8}r$

D. None

Answer: B

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8. Rectangle of maximum area that can be inscribed in an equilateral triangle of side awill have area=

A. $\frac{a^2\sqrt{3}}{2}$

B. $\frac{a^2\sqrt{3}}{4}$

C. $\frac{a^2\sqrt{3}}{8}$

D. None

Answer: C

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9. A running track of 440 ft is to be laid out enclosing a football field, the shape of which is a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, then find the length of its sides.

A. 110,35

B. 120,60

C. 130,50

D. None

Answer: A

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10. Find the dimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible when revolved about one of its sides.

A. 10,8

B. 11,7

C. 12,6

D. None

Answer: C

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11. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

A. $\sin^{-1}\frac{1}{3}$

B. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. 45°

D. 30°

Answer: A

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12. A cylindrical gas container is closed at the top and open at the bottom. If the iron plate of the top is 5/4 times as thick as the plate forming the cylindrical sides, the ratio of the radius to the height of the cylinder using minimum material for the same capacity is 3:4 (b) 5:6 (c) 4:5 (d) none of these

A. $2/3$

B. $1/2$

C. $4/5$

D. $1/3$

Answer: C

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13. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

A. $2/3$

B. $3/4$

C. $1/3$

D. $1/4$

Answer: A

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14. The height of the cylinder of max. volume that can be inscribed in a sphere of radius a is

A. $\frac{2a}{\sqrt{3}}$

B. $\frac{a}{\sqrt{2}}$

C. $\frac{5a}{4}$

D. None

Answer: A

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15. The altitude of a right circular cone of minimum volume circumscribed about a sphere of radius r is

A. 2r

B. 3r

C. 5r

D. none of these

Answer: D

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Problem Set 3 True And False

1. A figure consists of a semi-circle with a rectangle on its diameter. Given that the parameter of the figure is 20 feei. In order that its area may be maximum, its length and breadth are equal.

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2. An open tank is to be constructed with square base and vertical sides so as to contain a given quantity of water. Show that the expenses of lining with lead will be least, if depth is made half of width.

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3. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

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4. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

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5. Show that a conical tent of given capacity will require the least amount of canvas if its height is equal to its base radius.

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6. The right circular cylinder of given surface and max. volume is such that its height is equal to radius of the base.

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7. The given quantity of metal is to be cost into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi : (\pi + 2)$.

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8. The curved surface of the cone inscribed in a given sphere is maximum if $h = \frac{4}{3}a$.

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Problem Set 3 Fill In The Blanks

1. The total area of a page is 150 square inches. The combined width of the margin at the top, bottom is 'S' and the side is Z. The dimensions of the page in order that the area of the printed matter may be maximum.

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2. A window of perimeter P (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

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3. A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

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4. The three sides of a trapezium are equal. If each of them is $6cm$ long, Find the area of the trapezium when it is maximum.

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5. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ having its vertex coincident with one extremity of the major axis.

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6. Find the point on the curve $4x^2 + a^2y^2 = 4a^2, 4 < a^2 < 8$ that is farthest from the point (0, –2).

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7. A manufacturer plans to construct a cylindrical can to hold one cu m of liquid. If the cost of constructing the top and bottom of the can is twice the cost of constructing the sides, what are the dimensions of the most economical can?

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8. From a variable point of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

normal is drawn to the ellipse. The maximum distance of the normal from the centre of the ellipse is ....

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Self Assessment Test

1. v24

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2. From a fixed point A on the circumference of a circle of radius r, the perpendicular AY falls on the tangent at P. Find the maximum area of triangle APY.

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3. At x = $\frac{5\pi}{6}$, f(x)= 2sin 3x+3 cos 3x is

A. maximum

B. minimum

C. zero

D. none of these

Answer: D

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4. $f(x) = (3 - x)e^{2x} - 4xe^{x} - x$ has

A. a maximum at x = 0

B. a minimum at x = 0

C. neither of two at x = 0

D. f (x) is not derivable at x = 0

Answer: C

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5. For what values of x, the function $f(x) = x^{5} - 5x^{4} + 5x(3) - 1$ is maximum or minimum? Prove that at x = 0, the function is neither maximum nor minimum.

A. one minimum and two maxima

B. two minima and one maximum,

C. two minima and two maxima,

D. one minimum and one maximum.

Answer: D

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6. $f(x) = \sin x + \cos 2x (x > 0)$ has minma for x =

A. $\frac{n\pi}{2}$

B. $\frac{3(n+1)\pi}{2}$

C. $\frac{(2n+1)\pi}{2}$

D. None of these

Answer: C

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7. If a function $f(x)$ has $f'(a) = 0 and f^{a} = 0,$ then $x = a$ is a maximum for $f(x)$ $x = a$ is a minimum for $f(x)$ it is difficult to say $(a) and (b)$ $f(x)$ is necessarily a constant function.

A. x = a is a maximum for f (x)

B. x = a is a minimum for f(x)

C. It is difficult to say (a) and (b)

D. f(x) is necessarily a constant function

Answer: C

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8. If $y = a \log x + bx^2 + x$ has its exteme values at x=-1 and x=2, then find a and b.

A. `a = 2, b = -1

B. a = 2, b = -1/2

C. a = -2, b = 1/2

D. None of these

Answer: B

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9. If $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ is a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. Then, the function $P(x)$ has

A. Neither a max. nor a min.

B. Only one maximum

C. only one minimum

D. None of these

Answer: C

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10. N Characters of information are held on magnetic tape, in batches of x characters each, the batch processing time is $\alpha + \beta x^2$seconds, α and β are constants. The optical value of x for fast processing is

A. α / β

B. β / α

C. $\sqrt{\alpha / \beta}$

D. $\sqrt{\beta / \alpha}$

Answer: C

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11. The maximum value of $\frac{\ln x}{x}$ is:

A. 1

B. 2/e

C. e

D. 1/e

Answer: D

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12. For the curve $y = xe^x$, the point

A. x = -1 is a point of minimum

B. x = 0 is a minimum

C. x = -1 is a maximum

D. x = 0 is a maximum

Answer: A

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13. The point (0,5) is closest to the curve $x^2 = 2y$ at

A. $(2\sqrt{2}, 0)$

B. (0,0)

C. (2,2)

D. none of these

Answer: D

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14. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

A. $2/3$

B. $3/4$

C. $1/3$

D. $1/4$

Answer: A

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15. The function $f(x) = |px - q| + r|x|, x \in (-\infty, \infty)$ where $p > 0, q > 0, r > 0$, assumes its minimum value only on one point if

A. $p \neq q$

B. $r \neq q$

C. $r \neq p$

D. $p = q = r$

Answer: D

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16. (a) The minimum value of $5^{(x^2 - 2)^{3+8}}$ is ....

(b) Max. value of $(x+1)^2(x+3)$ is ....

(c) If the of the triangle formed by the tangent at any point P to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and axes fo ellipse be minimum then point is $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$.

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17. The total number of local maxima and local minima of the function f(x) = $\begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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18. For $x\varepsilon\left(0, \frac{5\pi}{2}\right)$, definite $f(x) = \int_0^x \sqrt{t}\sin t dt$. Then f has

A. local maximum at π and 2π

B. local minimum at π and 2π

C. local minimum at π and local maximum at 2π

D. local maximum at π and local minimum at 2π

Answer: D

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19. Given P(x) $= x^4 + ax^3 + bx^2 + cx + d$ such that x=0 is the only real root of P'(x) =0 . If P(-1) lt P(1),$then \in the \int erval$[-1,1]`

A. P (-1) is minimum and P(1) is maximum of P

B. P (-1) is not minimum but P(1) is the maximum of P

C. P (-1) is the minimum but P (1) is n ot the maximum of P

D. neither P(-1) is the minimum nor P(1) is the maximum of P.

Answer: B

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20. Let $f,\ g\,and\,h$ be real-valued functions defined on the interval $[0,\ 1]$ by

$f(x) = e^{x}\ \hat{}\ 2 + e^{-x}\ \hat{}\ 2,\ g(x) = xe^{x}\ \hat{}\ 2 + e^{-x}\ \hat{}\ 2\,and\,h(x) = x^2e^{x}\ \hat{}\ 2$

. If $a, b,\ and\,c$ denote respectively, the absolute maximum of $f,\ g\,and\,h$ on $[0,\ 1]$, then $a = b\,and\,c \ne b$ (b) $a = c\,and\,a \ne b\ \ a \ne b\,and\,c \ne b$

(d) $a = b = c$

A. $a = b$ and $c \ne b$

B. $a = c$ and $a \ne b$

C. $a \ne b$ and $c \ne d$

D. $a = b = c$

Answer: D

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21. The shortest distance between the lines $y - x = 1$ and the curve $x = y^2$ is

A. $\frac{3\sqrt{2}}{8}$

B. $\frac{2\sqrt{3}}{8}$

C. $\frac{3\sqrt{2}}{5}$

D. $\frac{\sqrt{3}}{4}$

Answer: A

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22. If $f(x) = \int_0^x t^2(t-2)(t-3)dt$for $x \in [0, \infty]$ then

A. f has a local maximum at x= 2

B. f is decreasing on [2,3]

C. $\exists$ some $c \in]0, \infty$ such that $f'(c) = 0$

D. f has a local minimum at x = 3

Answer: A::B::C::D

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23. Let $IR\overrightarrow{I}R$ be defined as $f(x) = |x| + + x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is______

A. 5

B. 4

C. 3

D. 2

Answer: A

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24. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6 and p(3) = 2$, then $p'(0)$ is____

A. 9

B. 8

C. 7

D. 6

Answer: A

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25. Let f be a function defined on R such that

$f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4 \forall x \in R$

If g is a function defined on R with values in the interval $]0, \infty[$ such that

$f(x) = \log(g(x)) \forall x \in R$ then the number of point in R at which g has a local maximum is

A. 1

B. 2

C. 3

D. 4

Answer: A

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26. Let `f(theta)=sin(tan^(-1)((sintheta)/(sqrt(cos2theta)))),w h e r e-pi/4

A. 1

B. 2

C. 3

D. 4

Answer: A

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27. The minimum value of the sum of real number $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8, and a^{10} with a > 0$ is

A. 8

B. 10

C. 4

D. 1

Answer: A

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Miscelaneous Exercise Matching Entries

1.

List-A		List-B
(a)	The max. value of $\left(\frac{1}{x}\right)^{2x^2}$ is	1. ab
(b)	P is a point on the parabola $y^2 = 8x$ such that its distance from the circle $x^2 + (y+6)^2 = 1$ is minimum is	2. $e^{1/e}$
(c)	Minimum area of triangle formed by any tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the axes of co-ordinates is	3. $\tan^{-1}\sqrt{(p/q)}$
(d)	$\sin^p x \cos^q x$ where p, q are +ive and x acute is maximum at $x =$	4. $(2, -4)$

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Miscelaneous Exercise Comprehension

1. If $f : R \in R$ is defined by

$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}$, $0 < a < 2$, the which of the following is true:

A. $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$

B. $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$

C. $f'(1) f'(-1) = (2-a)^2$

D. $f'(1) f'(-1) = -(2+a)^2)$

Answer:

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2. Consider the function $f:(-\infty, \infty) \rightarrow (-\infty, \infty)$ defined by $f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}; 0 < a < 2$. Which of the following is true?

A. f(x) is decreasing on [-1,1] and has a local minimum at x = 1

B. f(x) is increasing on [-1,1] but has a local maximum at x = 1

C. f(x) is increasing on [-1,1] but has neither a local maximum nor a local minimum at x =1

D. f (x) is decreasing on [-1,1] but has neither a local maximum nor a local minimum at x= 1

Answer:

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3. Consider the function f : $(-\infty, \infty) \to (-\infty, \infty)$ defined by f(x) = $\frac{x^2 - ax + 1}{x^2 + ax + 1}, 0 < a < 2$

Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$

Which of the following is true ?

A. $g'(x)$ is positive on $[-\infty, 0]$ and negative on $[0, \infty]$

B. $g'(x)$is negative on $[-\infty, 0]$ and positive on $[0, \infty]$

C. $g(x)'$ changes sign on both $[-\infty, 0]$ and $[0, \infty]$

D. $g(x)$ does not change sign on $[-\infty, \infty]$

Answer:

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