MATHS

BOOKS - ARIHANT MATHS

COMPLEX NUMBERS

Examples

1. Is the following computation correct ? If not give the correct compuatation:

$$\sqrt{(-2)}\sqrt{(-3)} = \sqrt{(-2)(-3)} = 6$$

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2. A student writes the formula $\sqrt{ab} = \sqrt{a}\sqrt{b}$. Then he substitutes $a = -1 \; and \; b = -1 \; and$ finds $1 = -1$. Explain where is he wrong?

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3. Explain the fallacy

$$-1 = i \times i = \sqrt{-1} \times \sqrt{-1} = \sqrt{(-1) \times (-1)} = \sqrt{1} = 1.$$

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4. Evaluate.

(i) i^{1998}

(ii)i^{-9999}

(iii) $\left(-\sqrt{-1}\right)^{4n-3}$,n $\neq$ N

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5. Find the value of $1 + i^2 + i^4 + i^6 + \ldots + i^{2n}$,

where$i = \sqrt{-1}$ and n in N.

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6. If $a = \frac{1+i}{\sqrt{2}}$, where $i = \sqrt{-1}$, then find the value of a^{1929}.

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7. The value of sum $\sum_{n=1}^{13} \left(i^n + i^{n+1}\right)$,where $i = \sqrt{-1}$ equals

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8. The value of $\sum_{n=0}^{100} i^{n!}$ equals (where $i = \sqrt{-1}$)

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9. Find he value of $\sum_{r=1}^{4n+7} i^r$ where, $i = \sqrt{-1}$.

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10. Show that the polynomial $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$ is divisible by $x^3 + x^2 + x + 1$, $wherep, q, r, s \in n$.

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11. What is the digit in the unit's place of

$(5172)^{11327}$?

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12. What is the digit in the unit's place of

$(143)^{86}$?

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13. What is the digit in unit's place of

$(1354)^{22222}$?

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14. What is the digit in the unit's place of

$(13057)^{941120579}$?

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15. Evaluate $\int \frac{1}{\cos^2 x} dx$

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16. What is the digit in the unit's place of $(2419)^{111213}$?

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17. Evaluate $\int \frac{1}{\sin^2 x} dx$

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18. If $(x+iy)^5 = p+iq$, then prove that $(y+ix)^5 = q+ip$.

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19. Find the least positive integral value of

n, for which $\left(\frac{1-i}{1+i}\right)^n$, where $i=\sqrt{-1}$, is purely

imaginary with positive imaginary part.

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20. If the multicative inverse of a comlex number is $(\sqrt{3}+4i)\backslash 19$, where

$i=\sqrt{-}1$, find the complex number.

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21. Evaluate $\int(\tan^2 x+1)dx$

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22. Find real value of x and y for which the complex numbers $-3+ix^2y$ and x^2+y+4i are conjugate of each other.

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23. If $x = -5+2\sqrt{-4}$, find the value of $x^4+9x^3+35x^2-x+4$.

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24. Evaluate $\int (\cot^2 x + 1) dx$

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25. Find the argument s of $z_1 = 5+5i,\ z_2 = -4+4i,\ z_3 = -3-3i$ and $z_4 = 2-2i$, where $i = \sqrt{-1}$.

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26. Evaluate $\int \left(\sin^{-1} x + \cos^{-1} x\right) dx$

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27. Evaluate $\int \frac{1}{\sin^{-1} x + \cos^{-1} x} dx$

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28. If $|z - 2 + i| \leq 2, wherei = \sqrt{-1},$ then find the greatest and least value of $|z|$.

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29. Evaluate $\int \left(\tan^{-1} x + \cot^{-1} x\right) dx$

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30. If $|z_1| = 1, |z_2| = 2, |z_3| = 3,$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12,$ then find the value of $|z_1 + z_2 + z_3|$.

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31. Evaluate $\int_0^1 \frac{1}{x+5} dx$

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32. Evaluate $\int_0^1 \frac{1}{2x+8} dx$

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33. Find the principal value of $\cos ec^{-1}(2)$

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34. If z_1 and z_2 are conjugate to each other , find the principal argument of $(-z_1 z_2)$.

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35. Write the value of $arg(z) + arg(zconjugate)$.

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36. Write the polar form of $-\frac{1}{2} - \frac{i\sqrt{3}}{2}$ (Where, $i = \sqrt{-1}$).

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37. Given that $|z-1| = 1$, where z is a point on the argand planne , show that $\frac{z-2}{z} = i\tan(argz)$, where $i = \sqrt{-1}$.

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38. Let z be a non-real complex number lying on $|z| = 1$, prove that $z = \frac{1 + i\tan\left(\frac{arg(z)}{2}\right)}{1 - i\tan\left(\frac{arg(z)}{2}\right)}$ (where $i = \sqrt{-1}$.)

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39. Prove that $tan\left(i(\log)_e\left(\frac{a - ib}{a + ib}\right)\right) = \frac{2ab}{a^2 - b^2}(wherea, b \in R^+)$

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40. If m and x are two real numbers where $m \in I$, then $e^{2mi\cot^{-1}x}\left(\frac{x \cdot i + 1}{x \cdot i - 1}\right)^m$

(A) $\cos x + i\sin x$ (B) $\frac{m}{2}$ (C) 1 (D) $\frac{m + 1}{2}$

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41. Express $(1+i)^{-1}$,where, i=$\sqrt{-1}$ in the

form A+iB.

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42. If $\sin\left(\log_e i^i\right) = a + ib$, where $i = \sqrt{-1}$,

find a and b, hence and find $\cos\left(\log_e i^i\right)$.

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43. Find the general value of $\log_2(5i)$,

where $i = \sqrt{-1}$.

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44. If $|z_1| = |z_2|$ and arg $(z_1 / z_2) = \pi$, then

z 1 +z 2 is equal to

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45. Let z and w are two non zero complex number such that $|z| = |w|$, and $Arg(z) + Arg(w) = \pi$ then (a) $z = w$ (b) $z = \overline{w}$ (c) $\bar{z} = \overline{w}$ (d) $\bar{z} = -\overline{w}$

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46. Find the square root of

$X + \sqrt{(-X^4 - X^2 - 1)}$.

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47. Solve that equation $z^2 + |z| = 0$, where z is a complex number.

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48. Number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in C$, is

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49. Find the all complex numbers satisying the equation $2|z|^2 + z^2 - 5 + i\sqrt{3} = 0, where i = \sqrt{-1}.$

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50. If $z_r = \cos\left(\frac{\pi}{3_r}\right) + i\sin\left(\frac{\pi}{3_r}\right), r = 1, 2, 3, ,$ prove that $z_1 z_2 z_3 z_\infty = i \cdot$

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51. Evaluate $\int_0^1 \frac{1}{e^x} dx$

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52. Find all roots of $X^5 - 1 = 0.$

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53. Find all roots of the equation

$X^6 - X^5 + X^4 - X^3 + X^2 - X + 1 = 0.$

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54. if α, β, γ are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ then $\frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$

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55. If $z = \frac{\sqrt{3 + i}}{2}$ (where $i = \sqrt{-1}$) then $\left(z^{101} + i^{103}\right)^{105}$ is equal to

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56. If $\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy)$, where x and y are reals, then the ordered pair (x,y) is given by

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57. If the polynomial $7x^3+ax+b$ is divisible by x^2-x+1, find the value of $2a+b$.

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58. If $1, \omega, \omega^2, \ldots \omega^{n-1}$ are n, nth roots of unity, find the value of $(9-\omega)\left(9-\omega^2\right)\ldots\left(9-\omega^{n-1}\right)$.

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59. If $a=\cos(2\pi/7)+i\sin(2\pi/7)$, then find the quadratic equation whose roots are

$\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$.

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60. Find the value of

$$\sum_{k=1}^{10}\left[\sin\left(\frac{2\pi k}{11}\right) - i\cos\left(\frac{2\pi k}{11}\right)\right], wherei = \sqrt{-1}.$$

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61. Complex numbers z_1, z_2 and z_3 are the vertices A,B,C respectivelt of an isosceles right angled triangle with right angle at C. show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

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62. Complex numbers z_1, z_2, z_3 are the vertices of A,B,C respectively of an equilteral triangle. Show that $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$.

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63. If z_1, z_2 and z_3 are the vertices of an equilasteral triangle with z_0 as its circumcentre , then changing origin to z^0 ,show that $z_1^2 + z_2^2 + z_3^2 = 0,$ where$z_1, z_2, z_3,$ are new complex numbers of the vertices.

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64. Show that inverse of a point a with respect to the circle $|z - c| = R(a$ and c are complex numbers and center respectively and R is the radius) is the point $c + \dfrac{R^2}{\bar{a} - \bar{c}}$,

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65. Find the perpendicular bisector of $3 + 4i$ and $-5 + 6i, where i = \sqrt{-1}.$

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66. If z_1, z_2 and z_3 are the vertices of an equilasteral triangle with z_0 as its circumcentre , then changing origin to z^0 ,show that $z_1^2 + z_2^2 + z_3^2 = 0$, where$z_1, z_2, z_3$, are new complex numbers of the vertices.

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67. Let $z_1 z_2$ and z_3 be three complex numbers and $a, b, c \in R$, such that $a + b + c = 0$ and $az_1 + bz_2 + cz_3 = 0$ then show that $z_1 z_2$ and z_3 are collinear.

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68. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, zi and $z + zi$ is $= \frac{1}{2} |z|^2$

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69. Show that the point a' is the reflection of the point a in the line $z\bar{b} + \bar{z}b + c = 0$, If $a'\bar{b} + \bar{a}b + c = 0.$

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70. Find the center and radius of the circle $2z\bar{z} + (3 - i)z + (3 + i)z - 7 = 0, \quad \text{where} \quad i = \sqrt{-1}.$

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71. Find all circles which are orthogonal to $|z| = 1$ and $|z - 1| = 4.$

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72. If z is any complex number, prove that : $|z|^2 = |z^2|.$

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73. 1. If $|z-2+i| \leq 2$ then find the greatest and least value of $|z|$

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74. In the argand plane, the vector $z = 4 - 3i, where i = \sqrt{-1},$ is turned in the clockwise sense by 180°. Find the complex number represented by the new vector .

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75. In a parallelogram ABCD, diagonals AC and BD intersect at O and AC = 6.8 cm and BD = 5.6 cm. Find the measure of OC and OD.

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76. Find the maximum and minimum values of $|z|$ satisfying $\left|z+\frac{1}{z}\right|=2$

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77. If $\left|z+\frac{4}{z}\right|=2$, find the maximum and minimum values of $|z|$.

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78. If $|z| \geq 3$, then determine the least value of $\left|z+\frac{1}{z}\right|$.

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79. If $|z|=1$ and $w=\frac{z-1}{z+1}$ (where $z \neq -1$), then $Re(w)$ is

A. 0

B. $\frac{-1}{|z+1|^2}$

C. $\left|\frac{z}{z=1}\right| \cdot \frac{1}{|z+1|^2}$

D. $\frac{\sqrt{2}}{|z+1|^2}$

Answer: a

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80. if a, b, c, a_1, b_1 and c_1 are non-zero complexnumbers satisfying $\frac{a}{a_1}+\frac{b}{b_1}+\frac{c}{c_1}=1+i$ and $\frac{a_1}{a}+\frac{b_1}{b}+\frac{c_1}{c}=0$, where $i=\sqrt{-1}$, the value of $\frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}$ is

$(a)2i(b)2+2i(c)2$ (d)None of these

A. 2i

B. 2+2i

C. 2

D. None of these

Answer: a

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81. Let z and ω be complex numbers. If $Re(z) = |z-2|, Re(\omega) = |\omega - z|$ and $arg(z-\omega) = \frac{\pi}{3}$, then the value of $Im(z+w)$, is

A. $\frac{1}{\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\sqrt{3}$

D. $\frac{4}{\sqrt{3}}$

Answer: d

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82. The mirror image of the curve $arg\left(\frac{z-3}{z-i}\right) = \frac{\pi}{6}, i = \sqrt{-1}$ in the real axis

A. $arg\left(\frac{z+3}{z+i}\right) = \frac{\pi}{6}$

B. $arg\left(\frac{z-3}{z+i}\right)=\frac{\pi}{6}$

C. $arg\left(\frac{z+i}{z+3}\right)=\frac{\pi}{6}$

D. $arg\left(\frac{z+i}{z-3}\right)=\frac{\pi}{6}$

Answer: d

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83. Expand $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix}$

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84. If $z+\frac{1}{z}=1$ and $a=z^{2017}+\frac{1}{z^{2017}}$ and b is the lastdigit of the number $2^{2^n}-1$,, when the integer $n>1$, the value of a^2+b^2 is

A. 23

B. 24

C. 26

D. 27

Answer: c

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85. if$\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)]+[1/(b+\omega)]+[1/(c+\omega)]=2\omega^2$ and $\left[1/(a+\omega)^2\right]+\left[1/(b+\omega)^2\right]+\left[1/(c+\omega)^2\right]=2\omega$, then find the value of $[1/(a+1)]+[1/(b+1)]+[1/(c+1)]\cdot$

A. -2

B. -1

C. 1

D. 2

Answer: d

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86. If a,b,c are distinct integers and $\omega(\neq 1)$ is a cube root of unity, then the minimum value of $\left|a+b\omega+c\omega^2\right|+\left|a+b\omega^2+c\omega\right|$ is

A. (a)$\sqrt{3}$

B. (b)3

C. (c)$6\sqrt{2}$

D. (d)2

Answer: a

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87. If $|z-2i| \leq \sqrt{2}$, where $i=\sqrt{-1}$, then the maximum value of $|3-i(z-1)|$, is

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $2+\sqrt{2}$

D. $3+2\sqrt{2}$

Answer: C

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88. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $Re(z_1\bar{z}_2) = 0$, then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies

a. $|\omega_1| = 1$

b. $|\omega_2| = 1$

c. $Re(\omega_1\overline{\omega}_2) = 0$

d. None of these

A. $|\omega_1| = 1$

B. $|\omega_2| = 1$

C. $Re(\omega_1\overline{\omega}_2) = 0$

D. None of these

Answer: a,b,c

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89. The complex numbers z_1, z_2, z_3 stisfying $(z_2 - z_3) = (1 + i)(z_1 - z_3). where i = \sqrt{-1},$ are vertices of a triangle which is

A. equilateral

B. isosceles

C. right angled

D. scalene

Answer: b,c

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90. Express in a complex number if $z = (2 - i)(5 + i)$

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91. The equation $z^2 - i|z-1|^2 = 0$, where $i = \sqrt{-1}$, has.

A. no real root

B. no purely imaginary root

C. all roots inside $|z| = 1$

D. atleast two roots

Answer: a,b,c

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92. If two complex numbers z_1, z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$?

A. $\max . |2z_1 + z_2| = 4$

B. $\min . |z_1 + z_2| = 1$

C. $\left|z_2 + \frac{1}{z_1}\right| \leq 3$

D. $\left|z_1 + \frac{2}{z_2}\right| \leq 2$

Answer: a,b,c,d

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93. Consider a quadratic equation $az^2 + bz + c = 0,$ where a,b and c are complex numbers.

The condition that the equation has one purely real root, is

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94. Consider a quadratic equation $az^2 + bz + c = 0,$ where a,b and c are complex numbers.

The condition that the equation has one purely real root, is

A. $(a\bar{b} + \bar{a}b)(b\bar{c} - \bar{b}c) = (c\bar{a} + \bar{c}a)^2$

B. $\left(a\bar{b} - \bar{a}b\right)\left(b\bar{c} + \bar{b}c\right) = (c\bar{a} + \bar{c}a)^2$

C. $\left(a\bar{b} - \bar{a}b\right)\left(\bar{b}c - \bar{b}c\right) = (c\bar{a} - \bar{c}a)^2$

D. $\left(a\bar{b} - \bar{a}b\right)\left(b\bar{c} - \bar{b}c\right) = (c\bar{a} + \bar{c}a)^2$

Answer: c

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95. Consider the quadratic equation $az^2 + bz + c = 0$ where a, b, c are non-zero complex numbers. Now answer the following.

The condition that the equation has both roots purely imaginary is

A. $\frac{a}{a} = \frac{b}{b} = \frac{c}{c}$

B. $\frac{a}{a} = \frac{b}{b} = \frac{c}{c}$

C. $\frac{a}{a} = \frac{b}{b} = -\frac{c}{c}$

D. $\frac{a}{a} = -\frac{b}{b} = \frac{c}{c}$

Answer: d

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96. Let Papoint denoting a comples number z on the complex plane.

$i.\ e. \qquad z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

$\text{if} \qquad Re(z) = x \ \text{ and } \ Im(z) = y, thenz = x + iy$

Number of integral solutions satisfying the eniquality

$|Re(z)| + \ | Im(z) < 21, . is$

A. a parallelogram which is not arhombus

B. a rhombus which is not a square

C. a rectangle which is not a square

D. a square

Answer: d

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97. Let Papoint denoting a comples number z on the complex plane.

$i.e. \qquad z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

if $\qquad Re(z) = x$ and $Im(z) = y, thenz = x + iy$

Number of integral solutions satisfying the eniquality

$|Re(z)| + \; | Im(z) < 21, .\, is$

A. $50\pi \quad \text{sq units}$

B. $100\pi \quad \text{sq units}$

C. $55 \quad \text{sq units}$

D. $110 \quad \text{sq units}$

Answer: a

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98. If z is a comlex number in the argand plane, the equation $|z-2|+|z+2|=8$ represents

A. 841

B. 839

C. 840

D. 842

Answer: c

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99. If $z_1, z_2 \in C, z_1^2 + z_2^2 \in R, z_1(z_1^2 - 3z_2^2) = 2$ and $z_2(3z_1^2 - z_2^2) = 11,$ the value of $z_1^2 + z_2^2$ is

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100. Consider four complex numbers $z_1 = 2 + 2i, ,$

$z_2 = 2 - 2i, z_3 = -2 - 2i \text{ and } z_4 = -2 + 2i), wherei = \sqrt{-1},$

Statement -1 $z_1, z_2, z_3 \text{ and } z_4$

constitute the vertices of a

square on the complex plane because

Statement -2 The non-zero complex numbers $z, \bar{z}, \ -z, \ -\bar{z}$

always constitute the vertices of a square.

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101. Consider $z_1 \text{ and } z_2$ are two complex numbers

such that $|z_1 + z_2| = |z_1| + |z_2|$

Statement -1 $amp(z_1) - amp(z_2) = 0$

Statement -2 The complex numbers z_1 and z_2 are collinear.

Check for the above statements.

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102. If $|z - iRe(z)| = |z - Im(z)|,$ then prove that z

lies on the bisectors of the quadrants, $\text{ where } i = \sqrt{-1}.$

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103. Find the gratest and the least values of $|z_1 + z_2|$,

if $z_1 = 24 + 7i \text{ and } |z_2| = 6, \quad \text{where } i = \sqrt{-1}$

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104. Given that $|z - 1| = 1$, where z is a point

on the argand planne , show that $\frac{z-2}{z} = i \tan(argz)$,

where $i = \sqrt{-1}$.

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105. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, where $i = \sqrt{-1}$,then lies on

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106. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, where

$i = \sqrt{-1}$, then the maximum value of $|2z - 6 + 5i|$, is

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107. Prove that the complex numbers z_1 and z_2 and the origin form an isosceles triangle with vertical angle $\frac{2\pi}{3}$, if $z_1^2+z_2^2+z_1z_2=0.$

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108. If $\alpha=e^{i2\pi/7} and f(x)=a_0+\sum_{k=0}^{20}a_kx^k$, then prove that the value of $f(x)+f(\alpha x)+....+f(\alpha^6x)$ is independent of α.

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109. Show that all the roots of the equation $a_1z^3+a_2z^2+a_3z+a_4=3$, $(where|a_i|\leq 1, i=1, 2, 3, 4,)$ lie outside the circle with centre at origin and radius $2/3$.

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110. Complex numbers z_1, z_2 and z_3 are the vertices A,B,C respectivelt of an isosceles right angled triangle with right angle at C. show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

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111. If z_1 and z_2 are two complex number such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$, Prove that $i\frac{z_1}{z_2} = k$ where k is a real number Find the angle between the lines from the origin to the points $z_1 + z_2$ and $z_1 - z_2$ in terms of k

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112. If $z = x + iy$ is a complex number with $x, y \in Q and |z| = 1,$ then show that $|z^{2n} - 1|$ is a rational number for every $n \in N$.

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113. If a is a complex number such that $|a| = 1$, then find the value of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root.

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114. If $n \in N > 1$, then the sum of real part of roots of $z^n = (z+1)^n$ is equal to

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115. Convert the complex numbers given below in the polar form: i

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116. Two different non-parallel lines cut the circle $|z| = r$ at points a, b, c and d, respectively. Prove that these lines meet at the point z given by $\dfrac{a^{-1} + b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}$

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117. Find $\frac{dy}{dx}$ if $x-y=\cos x$

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118. Show that the triangle whose vertices are $z_1z_2z_3$ and $z_1'z_2'z_3'$ are directly similar , if $\begin{vmatrix} z_1 & z'_1 & 1 \\ z_2 & z'_2 & 1 \\ z_3 & z'_3 & 1 \end{vmatrix}=0$

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119. if ω is the nth root of unity and Z_1, Z_2 are any two complex numbers , then prove that .

$\Sigma_{k=0}^{n-1}\left|z_1+\omega^k z_2\right|^2=n\left\{\left|z_1\right|^2+\left|z_2\right|^2\right\}$ where $n \in N$

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120. If $z_1+z_2+z_3+z_4=0$ where $b_i \in R$ such that the sum of no two values being zero and $b_1z_1+b_2z_2+b_3z_3+b_4z_4=0$ where z_1, z_2, z_3, z_4 are arbitrary complex numbers such that no three of them are collinear, prove that the four complex numbers would be concyclic if $|b_1b_2||z_1-z_2|^2=|b_3b_4||z_3-z_4|^2$.

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Example

1. Evaluate $\int \frac{1}{\sin^2 x+\cos^2 x}dx$

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2. If

$\theta_i \in [0, \pi/6], i=1, 2, 3, 4, 5,$ and $\sin\theta_1 z^4+\sin\theta_2 z^3+\sin\theta_3 z^2+\sin\theta_4 z$

show that $\frac{3}{4}<|Z|<1.$

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3. Let z and w be two non-zero complex numbers such that |z|=|w| and arg(z)+arg(w)=π, then z equals

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4. Show that the following four conditions are equivalent : (i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$

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5. Evaluate $\int_0^1 \frac{1}{e^{2x}} dx$

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6. If $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ are the n, nth

roots of the unity , then find the value of $\sum_{i=0}^{n-1} \frac{1}{2-a_i}$.

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7. The roots z_1, z_2 and z_3 of the equation $x^3 + 3ax^2 + 3bx + c = 0$ in which a,b and c are complex numbers, correspond to the points A,B,C on the Gaussian plane. Find the centroid of the $\triangle ABC$ and show that it will be equilateral, ifa^(2)=b`.

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8. Find the multiplicative inverse of $z = 4 - 3i$

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9. If $z_1 = 2 + 5i, z_2 = 3 - i, where \quad i = \sqrt{-1},$ find

(i)$Z_1 \cdot Z_2$

(ii) $Z_1 \times Z_2$

(iii) $Z_2 \cdot Z_1$

(iv) $Z_2 \times Z_1$

(v) acute angle between $Z_1 \text{ and } Z_2$.

(vi) projection of $Z_1 on Z_2$.

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10. Express in a complex number if $z = (2 - i)(5 + i)$

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Example Single Integer Answer Type Questions

1. Number of solutions of the equation $\sqrt{x^2} - \sqrt{(x-1)^2} + \sqrt{(x-2)^2} = \sqrt{5}$ is

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Example Matching Type Questions

1. Express in the complex form if $z = i^{19}$

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2. Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{\sin^2 x + \cos^2 x} dx$

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Subjective Type Examples

1. Express in the form of complex number $z = (5 - 3i)(2 + i)$

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2. Express in the complex number if $3(7+7i)+i(7+7i)$

Watch Video Solution

3. Find the multiplicative inverse of $z=6-3i$

Watch Video Solution

4. Find $\frac{dy}{dx}$ if $ax-by=\sin x$

Watch Video Solution

5. if α and β the roots of $z+\frac{1}{z}=2(\cos\theta+I\sin\theta)$ where $0<\theta<\pi$ and $i=\sqrt{-1}$ show that $|\alpha-i|=|\beta-i|$

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Exercise For Session 1

1. If $(1+i)^{2n}+(1-i)^{2n}=-2^{n+1}\left(where, i=\sqrt{-1}\right.$ for all those n, which are

A. even

B. odd

C. multiple of 3

D. None of these

Answer:

Watch Video Solution

2. If $i=\sqrt{-1},$ the number of values of i^{-n} for a different $n \in I$ is

A. 1

B. 2

C. 3

D. 4

Answer:

Watch Video Solution

3. If $a > 0$ and $b < 0$, $then \sqrt{a}\sqrt{b}$ is equal to (where, $i = \sqrt{-1}$)

A. $-\sqrt{a \cdot |b|}$

B. $\sqrt{a \cdot |b|i}$

C. $\sqrt{a \cdot |b|}$

D. none of these

Answer:

Watch Video Solution

4. The value of $\sum_{r=-3}^{1003} i^r \left(where i = \sqrt{-1}\right)$ is

A. 1

B. -1

C. i

D. $-i$

Answer:

Watch Video Solution

5. Evaluate $\int_0^{\frac{\pi}{4}} \left(\cot^2 x - \cos ec^2 x\right) dx$

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6. Evaluate $\int_0^1 8x^3 dx$

Watch Video Solution

Exercise For Session 4

1. Find $\frac{dy}{dx}$ if $x^2 + xy = \tan x$

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Exercise For Session 5

1. Express in the complex number $(-5i)\left(\frac{i}{8}\right)$

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Exercise For Session 2

1. If $\frac{1-ix}{1+ix} = a - ib$ and $a^2 + b^2 = 1$, $where\ a, b \in R$ and $i = \sqrt{-1}$, then x is equal to

A. $\dfrac{2a}{(1+a)^2+b^2}$

B. $\dfrac{2b}{(1+a)^2+b^2}$

C. $\dfrac{2a}{(1+b)^2+a^2}$

D. $\dfrac{2b}{(1+b)^2+a^2}$

Answer:

Watch Video Solution

2. The least positive integer n for which $\left(\dfrac{1+i}{1-i}\right)^n = \dfrac{2}{\pi}\left(\sec^{-1}\dfrac{1}{x}+\sin^{-1}x\right)$ (where, $X \neq 0, \ -1 \leq X \leq 1$ and $i=\sqrt{-1}$, is

A. 2

B. 4

C. 6

D. 8

Answer:

Watch Video Solution

3. If $z = (3 + 4i)^6 + (3 - 4i)^6$, where $i = \sqrt{-1}$, then Find the value of Im(z) .

A. -6

B. 0

C. 6

D. none of these

Answer:

Watch Video Solution

4. If $(x + iy)^{1/3} = a + ib$, where $i = \sqrt{-1}$, then $\left(\frac{x}{a} + \frac{y}{b}\right)$ is equal to

A. $4a^2b^2$

B. $4(a^2 - b^2)$

C. $4a^2 - b^2$

D. $a^2 + b^2$

Answer:

Watch Video Solution

5. If

$$\frac{3}{2 + \cos\theta + i\sin\theta} = a + ib \text{ where } i = \sqrt{-1} \text{ and } a^2 + b^2 = \lambda a - 3, \text{ th}$$

A. 3

B. 4

C. 5

D. 6

Answer:

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6. Evaluate $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x) dx$

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7. The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other, for

A. $x = n\pi, n \in I$

B. $x = 0$

C. $x = \left(n + \frac{1}{2}\right), n \in I$

D. 2

Answer:

Watch Video Solution

8. ਜੇਕਰ α ਅਤੇ β ਅਲੱਗ-ਅਲੱਗ ਮਿਸ਼ਰਿਤ ਸੰਖਿਆਵਾਂ ਹਨ ਅਤੇ $|\beta| = 1$ ਹੈ, ਤਾਂ $\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$ ਪਤਾ ਕਰੋ।

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

Answer:

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9. If $x = 3 + 4i$ find the value of $x^4 - 12x^3 - 70x^2 - 204x + 225$

A. -45

B. 0

C. 35

D. 15

Answer:

Watch Video Solution

10. If $|z_1 - 1| \leq , |z_2 - 2| \leq 2, |z_3 - 3| \leq 3,$ then find the greatest value of $|z_1 + z_2 + z_3|$

A. 6

B. 12

C. 17

D. 23

Answer:

Watch Video Solution

11. The principal value of $arg(z)$, where $z = 1 + \cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)$ (where, $i = \sqrt{-1}$) is given by

A. $-\frac{\pi}{5}$

B. $-\frac{4\pi}{5}$

C. $\frac{\pi}{5}$

D. $\frac{4\pi}{5}$

Answer:

Watch Video Solution

12. If

$|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|z_1 + z_2 + z_3| = 5.$ then$|4z_2z_3 + 9z_3z_1 + 1$

is

A. 24

B. 60

C. 120

D. 240

Answer:

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13. If $|z-i| \leq 5$ and $z_1 = 5+3i \left(\text{where, } i = \sqrt{-1}, \right.$ the greatest and least values of $|iz+z_1|$ are

A. 7 and 3

B. 9 and 1

C. 10 and 0

D. none of these

Answer:

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14. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, the find the value of $arg\left(\frac{z_1}{z_4}\right) + arg(z_2/z_3)$.

A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

Answer:

Watch Video Solution

Exercise For Session 3

1. Find the real part of $(1-i)^{-i}$.

A. $e^{-\pi/4}\cos\left(\frac{1}{2}\log_e 2\right)$

B. $-e^{-\pi/4}\sin\left(\frac{1}{2}\log_e 2\right)$

C. $e^{-\pi/4}\cos\left(\frac{1}{2}\log_e 2\right)$

D. $e^{-\pi/4}\sin\left(\frac{1}{2}\log_e 2\right)$

Answer:

Watch Video Solution

2. The amplitude of $e^{e^{-(i\theta)}}$, where $\theta \in R$ and $i = \sqrt{-1}$, is

A. $\sin\theta$

B. $-\sin\theta$

C. $e^{\cos\theta}$

D. $e^{\sin\theta}$

Answer:

Watch Video Solution

3. If $z = i\log_e\left(2-\sqrt{3}\right)$, where$i = \sqrt{-1}$ then the cos z is equal to

A. i

B. 2i

C. 1

D. 2

Answer:

Watch Video Solution

4. If $z = (i)^{(i)^{i}}$ where $i = \sqrt{-1}$, then z is equal to

A. 1

B. -1

C. i

D. $-i$

Answer:

Watch Video Solution

5. $\sqrt{(-8-6i)}$ is equal to (where, $i=\sqrt{-1}$

A. (a)$1 \pm 3i$

B. (b)$\pm(1-3i)$

C. (c)$\pm(1+3i)$

D. (d)$\pm(3-i)$

Answer:

Watch Video Solution

6. Simplify: $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$

A. $-\frac{3}{2}i$

B. $\frac{3}{4}i$

C. $-\frac{3}{4}i$

D. $-\frac{3}{2}$

Answer:

Watch Video Solution

7. If $0 < amp(z) < \pi, 'then' amp(z) - amp(-z)$` is equal to

A. 0

B. $2amp(z)$

C. π

D. $-\pi$

Answer:

Watch Video Solution

8. If $|z_1| = |z_2|$ and $amp(z_1) + amp(z_2) = 0,$ then

A. $z_1 = z_2$

B. $\bar{z}_1 = z_2$

C. $z_1 + z_2 = -0$

D. $\bar{z}_1 = \bar{z}_2$

Answer: B

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9. Solve the equation $|z| = z + 1 + 2i$.

A. $2 - \frac{3}{2}i$

B. $\frac{3}{2} + 2i$

C. $\frac{3}{2} - 2i$

D. $-2 + \frac{3}{2}i$

Answer: C

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10. The number of solutions of the equation $z^2 + \bar{z} = 0$ is .

A. 1

B. 2

C. 3

D. 4

Answer: D

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11. If $z_r = \frac{\cos(r\alpha)}{n^2} + i\frac{\sin(r\alpha)}{n^2}$, where r=1,2,3,....n, then $\lim_{n \to \infty} (z_1 . z_2 .... . z_n)$ is equal to

A. $e^{i\alpha}$

B. $e^{-i\alpha/2}$

C. $e^{i\alpha/2}$

D. $\sqrt[3]{e^{i\alpha}}$

Answer:

Watch Video Solution

12. Evaluate $\int_0^{\frac{\pi}{4}}\left(\tan^2 x+1\right)dx$

A. `

B.

C.

D.

Answer:

Watch Video Solution

13. If $iz^4+1=0$, then prove that z can take the value $\cos\pi/8+is\in\pi/8.$

A. $\dfrac{1+i}{\sqrt{2}}$

B. $\cos\left(\dfrac{\pi}{8}\right)+i\sin\left(\dfrac{\pi}{8}\right)$

C. $\dfrac{1}{4i}$

D. i

Answer:

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14. If $\omega(\neq 1)$ is a cube root of unity, then $\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)\left(1-\omega^4+\omega^8\right)$...upto $2n$ is factors, is

A. 2^n

B. 2^{2n}

C. 0

D. 1

Answer:

Watch Video Solution

15. If α, β, γ are the cube roots of p, then for any x,y,z $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ =

A. $\frac{1}{2}(-1 - i\sqrt{3}), i = \sqrt{-1}$

B. $\frac{1}{2}(1 + i\sqrt{3}), i = \sqrt{-1}$

C. $\frac{1}{2}(1 - i\sqrt{3}), i = \sqrt{-1}$

D. none of these

Answer:

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Exercise For Session 4

1. If z_1, z_2, z_3 and z_4 are the roots of the equation $z^4 = 1$, the value of $\sum_{i=1}^{4} z_i^3$ is

A. 0

B. 1

C. $i, i = \sqrt{-1}$

D. $1 + i, i = \sqrt{-1}$

Answer: A

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2. If $z_1, z_2, z_3, \ldots\ldots\ldots\ldots, z_n$ are n nth roots of unity, then for $k = 1, 2, , \ldots\ldots\ldots, n$

A. (a)$|z_k| = k \mid z_{k+1}|$

B. (b)$|z_{k+1}| = k \mid z_{k1}|$

C. (c)$|z_{k+1}| = |zk| + |z_{k-1}|$

D. (d)$|z_k| = |z_{k+1}|$

Answer: D

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3. If $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are n, nth roots of unity, then $(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)\ldots(1-\alpha_{n-1})$ equals to

A. 0

B. 1

C. n

D. n^2

Answer: C

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4. Evaluate $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \left(\cot^2 x + 1\right) dx$

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5. If α is the nth root of unity then prove that $1 + 2\alpha + 3\alpha^2 + \ldots.$ upto n terms

A. $\frac{2n}{1-\alpha}$

B. $-\frac{2n}{1-\alpha}$

C. $\frac{n}{1-\alpha}$

D. $-\frac{n}{1-\alpha}$

Answer:

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6. a and b are real numbers between 0 and 1 such that the points $Z_1 = a + i, Z_2 = 1 + bi, Z_3 = 0$ form an equilateral triangle, then a and b are equal to

A. $a = b = 2 + \sqrt{3}$

B. $a = b = 2 - \sqrt{3}$

C. $a = b = -2 - \sqrt{3}$

D. none of these

Answer: B

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7. If $|z| = 2,$ the points representing the complex numbers $-1 + 5z$ will lie on

A. a circle

B. a straight line

C. a parabola

D. an ellipse

Answer:

Watch Video Solution

8. If $|(z-2)/(z-3)|=2$ represents a circle, then find its radius.

A. 1

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{2}{3}$

Answer:

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9. Evaluate $\int_0^1 \frac{1}{x^2+4} dx$

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10. If z is a comlex number in the argand plane, the equation $|z-2|+|z+2|=8$ represents

A. a parabola

B. an ellipse

C. a hyperbola

D. a circle

Answer: D

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11. Evaluate $\int_0^1 \frac{1}{x^2+16} dx$

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12. locus of the point z satisfying the equation $|iz-1|+|z-i|=2$ is

A. a straight line

B. a circle

C. an ellipse

D. a pair of straight lines

Answer:

Watch Video Solution

13. If z, iz and $z + iz$ are the vertices of a triangle whose area is 2units, the value of $|z|$is

A. 1

B. 2

C. 4

D. 8

Answer:

Watch Video Solution

14. If $\left|z-\frac{4}{z}\right|=2$ then the greatest value of $|z|$ is:

A. (A) $\sqrt{5}-1$

B. (B) $\sqrt{5}+1$

C. (C) $\sqrt{5}$

D. (D) 2

Answer:

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Exercise Single Option Correct Type Questions

1. if cos (1-i) = a+ib, where a , b $\in$ R and $i=\sqrt{-1}$, then

a. $a=\frac{1}{2}\left(e-\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e+\frac{1}{e}\right)\sin 1$

b. $a=\frac{1}{2}\left(e+\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e-\frac{1}{e}\right)\sin 1$

c. $a=\frac{1}{2}\left(e+\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e+\frac{1}{e}\right)\sin 1$

d. $a=\frac{1}{2}\left(e-\frac{1}{e}\right)\cos 1, b=\frac{1}{2}\left(e-\frac{1}{e}\right)\sin 1$

A. $a = \frac{1}{2}\left(e - \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e + \frac{1}{e}\right)\sin 1$

B. $a = \frac{1}{2}\left(e + \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e - \frac{1}{e}\right)\sin 1$

C. $a = \frac{1}{2}\left(e + \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e + \frac{1}{e}\right)\sin 1$

D. $a = \frac{1}{2}\left(e - \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e - \frac{1}{e}\right)\sin 1$

Answer: B

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2. Number of roots of the equation $z^{10} - z^5 - 992 = 0$ with negative real part is

A. 3

B. 4

C. 5

D. 6

Answer: C

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3. If z and $\bar{z}$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)} = \sqrt{2} - 1$, then n is equal to:

A. (A) 8

B. (B) 16

C. (C) 24

D. (D) 32

Answer: D

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4. If $\prod_{p=1}^{r} e^{ip\theta} = 1,$ where $\prod$ denotes the continued product and $i = \sqrt{-1}$, the most general value of θ is (where, n is an integer)

A. (a) $\frac{2n\pi}{r(r-1)}, n \in I$

B. (b) $\frac{2n\pi}{r(r+1)}, n \in I$

C. (c) $\frac{4n\pi}{r(r-1)}, n \in I$

D. (d) $\frac{4n\pi}{r(r+1)}, n \in I$

Answer: D

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5. If $(3+i)(z+\bar{z})-(2+i)(z-\bar{z})+14i=0$, where $i=\sqrt{-1}$, then z $\bar{z}$ is equal to

A. (a)10

B. (b)8

C. (c)-9

D. (d)-10

Answer: A

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6. The centre of a square ABCD is at z=0, A is z_1. Then, the centroid of $\triangle ABC$ is (where, $i=\sqrt{-1}$)

A. (a)$z_1(\cos\pi \pm i\sin\pi)$

B. (b)$\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$

C. (c)$z_1\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

D. (d)$\frac{z_1}{3}\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

Answer: D

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7. Evaluate $\int_0^{\frac{\pi}{4}}\left(\tan^2 x - \sec^2 x\right)dx$

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8. Let α and β be two fixed non-zero complex numbers and 'z' a variable complex number. If the lines $\alpha\bar{z} + \bar{a}z + 1 = 0$ and $\beta\bar{z} + \bar{\beta}z - 1 = 0$ are mutually perpendicular, then

A. $ab + \bar{a}\bar{b} = 0$

B. $ab - \bar{a}\bar{b} = 0$

C. $\bar{a}b - a\bar{b} = 0$

D. $a\bar{b} + \bar{a}b = 0$

Answer: D

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9. Evaluate $\int_0^1 \frac{1}{x^2 + 1} dx$

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10. Evaluate $\int \frac{1}{x^2 + 25} dx$

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11. If $f(x) = g(x^3) + xh(x^3)$ is divisiblel by $x^2 + x + 1$, then

A. g(x) is divisible by (x-1) but not h(x) but not h(x)

B. h(x) is divisible by (x-1) but not g(x)

C. both g(x) and h(x) are divisible by (x-1)

D. None of above

Answer: C

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12. If the points represented by complex numbers $z_1 = a + ib, z_2 = c + id$ and $z_1 - z_2$ are collinear, where $i = \sqrt{-1}$, then

A. ad+bc=0

B. ad-bc=0

C. ab+cd=0

D. ab-cd=0

Answer: B

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13. Let C and R denote the set of all complex numbers and all real numbers respectively. Then show that $f: C \to R$ given by $f(z) = |z|$ for all $z \in C$ is neither one-one nor onto.

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14. Let α and β be two distinct complex numbers, such that $|\alpha| = |\beta|$. If real part of α is positive and imaginary part of β is negative, then the complex number $(\alpha + \beta) / (\alpha - \beta)$ may be

A. zero

B. real and negative

C. real and positive

D. purely imaginary

Answer: D

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15. The complex number z satisfies thc condition $\left|z-\frac{25}{z}\right|=24.$ The maximum distance from the origin of co-ordinates to the points z is

A. 25

B. 30

C. 32

D. None of these

Answer: A

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16. The points A,B and C represent the complex numbers $z_1, z_2, (1-i)z_1 + iz_2$ respectively, on the complex plane (where, $i = \sqrt{-1}$). The $\triangle ABC$, is

a. isosceles but not right angled

b. right angled but not isosceles

c. isosceles and right angled

d. None of the above

A. isosceles but not right angled

B. right angled but not isosceles

C. isosceles and right angled

D. None of the above

Answer: C

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17. Find the $6th$ term of A.P if $a = 1, d = 2$

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18. Find the $3rd$ term of A.P if $a = 1, d = 2$

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19. The centre of circle represented by $|z+1| = 2|z-1|$ in the complex plane is

A. 0

B. $\frac{5}{3}$

C. $\frac{1}{3}$

D. None of these

Answer: B

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20. If $x = 9^{\frac{1}{3}} 9^{\frac{1}{9}} 9^{\frac{1}{27}} \ldots\ldots$ and if $y = 4^{\frac{1}{3}} 4^{-\frac{1}{9}} 4^{\frac{1}{27}} \ldots\ldots$ and if $z = \sum_{r=1}^{\infty} (1+i)^{-r}$ then , the argument of the complex number $w = x + yz$ is

A. 0

B. $-\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

C. $-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

D. $\pi - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

Answer: B

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21. Find the $5th$ term of A.P if $a = 1, d = 2$

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22. Let $|Z_r - r| \le r, Aar = 1, 2, 3. \ldots, n.$ Then $\left|\sum_{r=1}^{n} z_r\right|$ is less than

A. n

B. 2n

C. n(n+1)

D. $\frac{n(n+1)}{2}$

Answer: C

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23. If arg $\left(\frac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \frac{\pi}{2}$ and $\left|\frac{z}{|z|} - z_1\right| = 3$, then $|z_1|$ equals to a. $\sqrt{3}$ b. $2\sqrt{2}$ c. $\sqrt{10}$ d. $\sqrt{26}$

A. $\sqrt{3}$

B. $2\sqrt{2}$

C. $\sqrt{10}$

D. $\sqrt{26}$

Answer: C

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24. Find the $7th$ term of A.P if $a = 1, d = 2$

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25. Find the $10th$ term of A.P if $a = 1, d = 2$

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26. If $z = (3 + 7i)(\lambda + i\mu), \quad \text{when} \quad \lambda, \mu \in I - \{0\} \quad \text{and} \quad i = \sqrt{-1}$, is purely imaginary then minimum value of $|z|^2$ is

a. 0

b. 58

c. 3364/3

d. 3364

A. 0

B. 58

C. $\frac{3364}{3}$

D. 3364

Answer: D

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27. Given $z = f(x) + ig(x)$ where $f, g: (0, 1) \to (0, 1)$ are real valued functions. Then which of the following does not hold good?

A. $z = \frac{1}{1 - ix} + i\left(\frac{1}{1 + ix}\right)$

B. $z = \frac{1}{1 + ix} + i\left(\frac{1}{1 - ix}\right)$

C. $z = \frac{1}{1 + ix} + i\left(\frac{1}{1 + ix}\right)$

D. $z = \frac{1}{1 - ix} + i\left(\frac{1}{1 - ix}\right)$

Answer: B

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28. If $z^3 + (3 + 2i)z + (- 1 + ia) = 0$ has one real roots, then the value of a lies in the interval $(a \in R)$ `

A. (-2,-1)

B. (-1,0)

C. (0,1)

D. (1,2)

Answer: B

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29. If m and n are the smallest positive integers satisfying the relation $\left(2CiS\frac{\pi}{6}\right)^m = \left(4CiS\frac{\pi}{4}\right)^n$, where $i = \sqrt{-1}$, $(m+n)$ equals to

A. (a)60

B. (b)72

C. (c)96

D. (d)36

Answer: B

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30. Number of imaginergy complex numbers satisfying the equation, $z^2 = \bar{z} \cdot 2^{1-|z|}$ is

s. 0

b. 1

c. 2

d. 3

A. 0

B. 1

C. 2

D. 3

Answer: C

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Exercise More Than One Correct Option Type Questions

1. If $\frac{z+1}{z+i}$ is a purely imaginary number (where$\left(i=\sqrt{-1}\right)$, then z lies on a

A. straight line

B. circle

C. circle with radius = $\frac{1}{\sqrt{2}}$

D. circle passing through the origin

Answer: B::C::D

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2. Find the multiplicative inverse of $z = 2 - 3i$

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3. If the complex numbers is $(1 + ri)^3 = \lambda(1 + i)$, when $i = \sqrt{-1}$, for some real λ, the value of r can be

A. $\cos\frac{\pi}{5}$

B. $\cos ec\frac{3\pi}{2}$

C. $\cot\frac{\pi}{12}$

D. $\tan\frac{\pi}{12}$

Answer: B::C::D

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4. If z $\in$ C, which of the following relation(s) represents a circle on an Argand diagram? (where,$i = \sqrt{-1}$)

A. $|z-1| + |z+1| = 3$

B. $|z-3| = 2$

C. $|z-2+i| = \frac{7}{3}$

D. $(z-3+i)(\bar{z}-3-i) = 5$

Answer: B::C::D

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5. Find the modules of $4+3\iota$

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6. If z is a complex number which simultaneously satisfies the equations $3|z-12|=5|z-8i|$ and $|z-4|=|z-8|$, where $i=\sqrt{-1}$, then Im(z) can be

A. 8

B. 17

C. 7

D. 15

Answer: A::B

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7. If $P(z_1), Q(z_2), R(z_3)$ and $S(z_4)$ are four complex numbers representing the vertices of a rhombus taken in order on the complex plane, which one of the following is held good?

A. $\frac{z_1 - z_4}{z_2 - z_3}$ is purely real

B. $\frac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary

C. $|z_1 - z_3| \neq |z_2 - z_4|$

D. $amp\left(\frac{z_1 - z_4}{z_2 - z_4}\right) \neq amp\left(\frac{z_2 - z_4}{z_3 - z_4}\right)$

Answer: A::B::C

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8. If $a|z - 3| = \min\{|z1, |z - 5|\}, thenRe(z)$ equals to 2 b. $\frac{5}{2}$ c. $\frac{7}{2}$ d. 4

A. 2

B. 2.5

C. 3.5

D. 4

Answer: A::D

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9. Find the $8th$ term of A.P if $a = 1, d = 2$

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10. If z=x+iy, where $i = \sqrt{-1}$, then the equation $\left|\left(\frac{2z - i}{z + 1}\right)\right| = m$ represents a circle, then m can be

A. $\frac{1}{2}$

B. 1

C. 2

D. $\in (3, 2\sqrt{3})$

Answer: A::B::D

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11. Equation of tangent drawn to circle $|z| = r$ at the point $A(z_0)$, is

A. $Re\left(\frac{z}{z_0}\right) = 1$

B. $Im\left(\frac{z}{z_0}\right) = 1$

C. $Im\left(\frac{z_0}{z}\right) = 1$

D. $z\overline{z_0} + z_0\bar{z} = 2r^2$

Answer: A::D

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12. z_1 and z_2 are the roots of the equation $z^2 - az + b = 0$ where $|z_1| = |z_2| = 1$ and a,b are nonzero complex numbers, then

A. (a)$|a| \leq 1$

B. (b)$|a| \leq 2$

C. (c)$arg(a) = arg(b^2)$

D. (d)$arg(a^2) = arg(b)$

Answer: B::D

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13. If α is a complex constant such that $\alpha z^2 + z + \overline{\alpha} = 0$ has a real root, then

A. $\alpha + \overline{\alpha} = 1$

B. $\alpha + \overline{\alpha} = 0$

C. $\alpha + \overline{\alpha} = -1$

D. the absolute value of real root is 1

Answer: A::C::D

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14. If the equation $z^3 + (3+i)z^2 - 3z - (m+i) = 0, \quad \text{where } i = \sqrt{-1} \text{ and } m \in R,$ has atleast one real root, value of m is

A. 1

B. 2

C. 3

D. 5

Answer: A::D

Watch Video Solution

15. If $z^3 + (3+2i)z + (-1+ia) = 0$ has one real roots, then the value of a lies in the interval $(a \in R)$ `

A. (-2,1)

B. (-1,0)

C. (0,1)

D. (-2,3)

Answer: A::B::D

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Exercise Passage Based Questions

1.

$$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if arg (z)} & < 0 \\ -\pi, & \text{if arg (z)} & > 0 \end{cases}, \text{where} -\pi < arg(z) \le \pi$$

.

If $arg(z) < 0$, then arg (-z)-arg(z) is equal to

A. $-\pi$

B. $-\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. π

Answer: A

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2. Find $\frac{dy}{dx}$ if $x + y^2 = \tan x + y$

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3.

$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if arg (z)} < 0 \\ -\pi, & \text{if arg (z)} > 0 \end{cases}, \text{where} - \pi < arg(z) \leq \pi$

.

If $arg(z) < 0$, then arg (-z)-arg(z) is equal to

A. 1

B. 1.25

C. 1.5

D. 2.5

Answer: B

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4. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in I.$

If $\sum_{n=1}^{25} i^{n!} = a + ib, \quad \text{where} \quad i = \sqrt{-1}$, then a-b, is

A. prime number

B. even number

C. composite number

D. perfect number

Answer: A

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5. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in I.$

If $\sum_{r=-2}^{95} i^r + \sum_{r=0}^{50} i^{r!} = a + ib, \quad \text{where} \quad i = \sqrt{-1},$ the unit digit of $a^{2011} + b^{2012}$, is

A. (a)2

B. (b)3

C. (c)5

D. (d)6

Answer: C

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6. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in I.$

If $\sum_{r=4}^{100} i^{r!} + \prod_{r=1}^{101} i^r = a + ib, \quad \text{where} \quad i = \sqrt{-1}$, then a+75b, is

A. 11

B. 22

C. 33

D. 44

Answer: B

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7. For any two complex numbers $z_1 \text{and} z_2$,

$$|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$$

and equality holds iff origin $z_1 \text{ and } z_2$ are collinear and z_1, z_2 lie on the same side of the origin .

If $\left|z - \frac{1}{z}\right| = 2$ and sum of greatest and least values of $|z|$ is λ, then λ^2, is

A. 2

B. 4

C. 6

D. 8

Answer: D

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8. For any two complex numbers $z_1 \text{and} z_2$, $|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$ and equality holds iff origin z_1 and z_2 are collinear and z_1, z_2 lie on the same side of the origin . If $\left|z - \frac{2}{z}\right| = 4$ and sum of greatest and least values of $|z|$ is λ, then λ^2, is

A. 12

B. 18

C. 24

D. 30

Answer: C

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9. For any two complex numbers $z_1 \text{and} z_2$,

$$|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$$

and equality holds iff origin z_1 and z_2 are collinear and z_1, z_2 lie on the same side of the origin .

If $\left|z-\frac{3}{z}\right|=6$ and sum of greatest and least values of $|z|$ is 2λ, then λ^2, is

A. 12

B. 18

C. 24

D. 30

Answer: A

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10. Express in the complex form $z=(7-i)(2+i)$

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11. Express in the complax form if $z=(4-3i)(2+i)$

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Exercise Single Integer Answer Type Questions

1. The number of values of z (real or complex) e simultaneously satisfying the system of equations $1 + z + z^2 + z^3 + \dots z^{17} = 0$ and $1 + z + z^2 + z^3 + .. + z^{13} = 0$ is

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2. Number of complex numbers satisfying $z^3 = \bar{z}$ is

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3. Let z=9+ai, where $i = \sqrt{-1}$ and a be non-zero real.

If $Im(z^2) = Im(z^3)$, sum of the digits of a^2 is

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4. Numbers of complex numbers z, such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is

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5. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 1i$, $where$i= $\sqrt{-1}$, $then(a + b)$ equal to ______.

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6. If $z = \frac{\pi}{4}(1 + i)^4\left(\frac{1 - (\pi)i}{(\pi) + i} + \frac{(\pi) - i}{1 + (\pi)i}\right)$, $then\left(\frac{|z|}{amp(z)}\right)$ equal

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7. Suppose A is a complex number and $n \in N$, such that $A^n = (A + 1)^n = 1,$ then the least value of n is

a. 3

b. 6

c. 9

d. 12

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8. Let $z_r, r = 1, 2, 3, \ldots, 50$ be the roots of the equation $\sum_{r=0}^{50} (z)^r = 0$. If $\sum_{r=1}^{50} \frac{1}{z_r - 1} = -5\lambda$, then λ equals to

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9. Evaluate $p = \sum_{p=1}^{32} (3p+2)\left(\sum_{q=1}^{10}\left(\sin\frac{2q\pi}{11} - i\cos\frac{2q\pi}{11}\right)\right)^p$, where $i = \sqrt{-1}$

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10. Find the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n$ = 1

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Complex Number Exercise 5

1. Find the Sum of $5th$ term of A.P if $a = 1, d = 2$

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2. Find the $4th$ term of A.P if $a = 1, d = 2$

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3. Find the value of 2^3

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4. Find the Sum of $4th$ term of A.P if $a = 1, d = 2$

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Exercise Statement I And Ii Type Questions

1. **Statement-1** $3+7i > 2+4i, \quad \text{where} \quad i=\sqrt{-1}.$

Statement-2 $3 > 2 \quad \text{and} \quad 7 > 4$

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2. Which statement is correct.?

statement-1 $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta, i = \sqrt{-1}$

statement-2 $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^2 = i$

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3. **statement-1** Locus of z satisfying the equation $|z-1|+|z-8|=5$ is an ellipse.

statement-2 Sum of focal distances of any point on ellipse is constant for an ellipse.

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4. Let $z_1, z_2 \text{ and } z_3$ be three complex numbers in AP.

Statement-1 Points representing $z_1, z_2 \text{ and } z_3$ are collinear

Statement-2 Three numbers a,b and c are in AP, if b-a=c-b

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5. **Statement-1** If the principal argument of a complex numbere z is θ ., the principal argument of z^2 is 2θ.

Statement-2 $arg(z^2) = 2arg(z)$

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Complex Number Exercise 6

1. Complex numbers z_1, z_2, z_3 are the vertices of A,B,C respectively of an equilteral triangle. Show that $z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$.

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2. **Statement-1** If the principal argument of a complex numbere z is θ ., the principal argument of z^2 is 2θ.

Statement-2 $arg(z^2)=2arg(z)$

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Exercise Subjective Type Questions

1. If z_1, z_2, z_3 are any three complex numbers on Argand plane, then $z_1(Im(\bar{z}_2z_3))+z_2(Im\bar{z}_3z_1))+z_3(Im\bar{z}_1z_2))$ is equal to

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2. The roots z_1, z_2, z_3 of the equation $x^3 + 3ax^2 + 3bx + c = 0$ in which a, b, c are complex numbers correspond to points A, B, C. Show triangle will be an equilateral triangle if $a^2 = b$.

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3. If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots $x^5 - 1 = 0$, then value of $\frac{\omega - \alpha_1}{\omega^2 - \alpha_1} \cdot \frac{\omega - \alpha_2}{\omega^2 - \alpha_2} \cdot \frac{\omega - \alpha_3}{\omega^2 - \alpha_3} \cdot \frac{\omega - \alpha_4}{\omega^2 - \alpha_4}$ is (where ω is imaginary cube root of unity)

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4. If $z_1 and z_2$ both satisfy $z + \bar{z}r = 2|z - 1|$ and $arg(z_1 - z_2) = \frac{\pi}{4}$, then find I m(z_1+z_2) .

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5. Find the all complex numbers satisying the equation $2|z|^2 + z^2 - 5 + i\sqrt{3} = 0, where i = \sqrt{-1}.$

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6. Express in the complex form if $(5i)\left(\frac{-3i}{5}\right)$

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7. Find the point of intersection of the curves $arg(z - 3i) = \frac{3\pi}{4} and arg(2z + 1 - 2i) = \pi / 4.$

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8. Show that if a and b are real, the principal value of arg a is 0 or π according as a is positive or negative and that of bi is $\frac{\pi}{2}$ or $-\frac{\pi}{2}$ according as b is positive or negative.

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9. Let z and ω be complex numbers. If $Re(z) = |z-2|, Re(\omega) = |\omega - z|$ and $arg(z-\omega) = \frac{\pi}{3}$, then the value of $Im(z+w)$, is

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10. If z_1 and z_2 are two complex numbers and $c > 0$, then prove that $|z_1 + z_2|^2 \leq (1+c)|z_1|^2 + (1+c^{-1})|z_2|^2$.

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11. Find the circumstance of the triangle whose vertices are given by the complex numbers z_1, z_2 and z_3.

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12. Find the circumstance of the triangle whose vertices are given by the complex numbers z_1, z_2 and z_3.

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Complex Number Exercise 7

1. Find $\frac{dy}{dx}$ if $y = \cos(\sin x)$

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2. Express in the form of complax number $z = (2 - i)(3 + i)$

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3. Two different non-parallel lines meet the circle $|z| = r$. One of them at points a and b and the other which is tangent to the circle at c. Show that the point of intersection of two lines is $\frac{2c^{-1} - a^{-1} - b^{-1}}{c^{-2} - a^{-1}b^{-1}}$.

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4. A,B and C are the points respectively the complex numbers z_1, z_2 and z_3 respectivley, on the complex plane and the circumcentre of $\triangle ABC$ lies at the origin. If the altitude of the triangle through the vertex. A meets the circumcircle again at P, prove that P represents the complex number $\left(-\frac{z_2 z_3}{z_1}\right)$.

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5. Let z, z_0 be two complex numbers. It is given that $|z| = 1$ and the numbers $z, z_0, \overline{z_0}, 1$ and 0 are represented in an Argand diagram by the points P, P_0, Q, A and the origin, respectively. Show that $\triangle POP_0$ and $\triangle AOQ$ are congruent. Hence, or otherwise, prove that

$|z - z_0| = |z\overline{z_0} - 1|$.

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6. Express in a complex form if $z = i^7$

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7. Let a, b and c be any three nonzero complex number. If $|z| = 1$ and $'z'$ satisfies the equation $az^2 + bz + c = 0$, prove that $a.\bar{a}$ = $c.\bar{c}$ and |a||b|=$\sqrt{ac(\bar{b})^2}$

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8. Let z_1, z_2 and z_3 be three non-zero complex numbers and $z_1 \neq z_2$. If $\begin{vmatrix} |z_1| & |z_2| & |z_3| \\ |z_2| & |z_3| & |z_1| \\ |z_3| & |z_1| & |z_2| \end{vmatrix} = 0$, prove that

(i) z_1, z_2, z_3 lie on a circle with the centre at origin.

(ii)$arg\left(\frac{z_3}{z_2}\right) = arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$.

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9. The roots of the equation

$8x^3 - 4x^2 - 4x + 1 = 0$ are $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}$ and $\cos\frac{5\pi}{7}$.

Evaluate $\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7}$

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10. What is 8% Equals to`

A. 0.08

B. 0.8

C. 0.008

D. 0.0008

Answer:

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11. Find $\frac{dy}{dx}$ if $x - 5y = \tan y$

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Exercise Questions Asked In Previous 13 Years Exam

1. If ω is a cube root of unity but not equal to 1, then minimum value of $\left|a + b\omega + c\omega^2\right|$, (where a,b and c are integers but not all equal), is

A. 0

B. $\frac{\sqrt{3}}{2}$

C. 1

D. 2

Answer: C

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2. If one of the vertices of the square circumscribing the circle $|z-1|=\sqrt{2}$ is $2+\sqrt{3}\iota$. Find the other vertices of square

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3. If $z_1 and z_2$ are two nonzero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then $arg z_1 - arg z_2$ is equal to

A. $-\pi$

B. $-\pi/2$

C. $\pi/2$

D. 0

Answer: D

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4. If the cube roots of unity are $1, \omega, \omega^2,$ then the roots of the equation $(x-1)^3 + 8 = 0$ are

A. $-1, 1+2\omega, 1+2\omega^2$

B. $-1, 1-2\omega, 1-2\omega^2$

C. $-1-1-1$

D. None of these

Answer: B

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5. If $\omega = z/[z-(1/3)i]$ and $|\omega| = 1$, then find the locus of z.

A. a straight line

B. a parabola

C. an ellipse

D. a circle

Answer: A

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6. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \overline{w}z}{1 - z}\right)$ is a purely real, then the set of values of z is $|z| = 1, z \neq 2$ (b) $|z| = 1 and z \neq 1$ (c)$z = \bar{z}$ (d) None of these

A. $\{z : |z| = 1\}$

B. $\{z : z = \bar{z}\}$

C. $\{z : z \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

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7. Find the value of $\sum_{k=1}^{10}\left[\sin\left(\frac{2\pi k}{11}\right) - i\cos\left(\frac{2\pi k}{11}\right)\right], where i = \sqrt{-1}.$

A. i

B. 1

C. -1

D. $-i$

Answer: D

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8. If $z^2+z+1=0$ where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\dots.+\left(z^6+\frac{1}{z^6}\right)^2$ is

A. 18

B. 54

C. 6

D. 12

Answer: D

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9. A man walks a distance of 3 units from the origin towards the North-East $\left(N45^{0}E\right)$ direction.From there, he walks a distance of 4 units towards the North-West $\left(N45^{0}W\right)$ direction to reach a point P. Then, the position of P in the Argand plane is

A. $3e^{i\pi/4}+4i$

B. $(3-4i)e^{i\pi/4}$

C. $(4+3i)e^{i\pi/4}$

D. $(3+4i)e^{i\pi/4}$

Answer: D

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10. If $|z|=1$ and $z\neq 1$, then all the values of $\dfrac{z}{1-z^{2}}$ lie on

A. a line not passing through the origin

B. $|z| = \sqrt{2}$

C. the X-axis

D. the Y-axis

Answer: D

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11. If $|z+4| \leq 3$, the maximum value of $|z+1|$ is

A. 4

B. 10

C. 6

D. 0

Answer: C

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12. Let A, B, C be three sets of complex number as defined below:

$A = \{z : Im \geq 1\}, B = \{z : |z - 2 - i| = 3\}, C : \{z : Re((1 - i)z) = \sqrt{2}\}$

The number of elements in the set $A \cap B \cap C$ is

A. 0

B. 1

C. 2

D. ∞

Answer: B

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13. What is 5% Equals to

A. 0.05

B. 0.5

C. 0.005

D. 0.0005

Answer: C

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14. Express in the form of complex number $i^9 + i^{19}$

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15. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

A. 6+7i

B. $-7+6i$

C. 7+6i

D. $-6+7i$

Answer: D

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16. If the conjugate of a complex numbers is $\frac{1}{i-1}$, where $i=\sqrt{-1}$. Then, the complex number is

A. $\frac{-1}{i-1}$

B. $\frac{1}{i+1}$

C. $\frac{-1}{i+1}$

D. $\frac{1}{i-1}$

Answer: C

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17. Let $z = x + iy$ be a complex number where x and y are integers. Then ther area of the rectangle whose vertices are the roots of the equaiton $\bar{z}z^3 + z\bar{z}^3 = 350.$

A. 48

B. 32

C. 40

D. 80

Answer: A

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18. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m \to 1-15} Img\left(z^{2m-1}\right)$ at $\theta = 2^\circ$ is:

A. $\dfrac{1}{\sin 2^\circ}$

B. $\frac{1}{3\sin 2^\circ}$

C. $\frac{1}{2\sin 2^\circ}$

D. $\frac{1}{4\sin 2^\circ}$

Answer: D

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19. If $\left|z - \frac{4}{z}\right| = 2$ then the greatest value of $|z|$ is:

A. $2 + \sqrt{2}$

B. $\sqrt{3} + 1$

C. $\sqrt{5} + 1$

D. 2

Answer: C

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20. Let z_1 and z_2 be two distinct complex numbers and $z = (1-t)z_1 + tz_2$, for some real number t with $0 < t < 1$ and $i = \sqrt{-1}$. If arg(w) denotes the principal argument of a non-zero complex number w, then

a. $|z - z_1| + |z - z_2| = |z_1 - z_2|$

b. $arg(z - z_1) = arg(z - z_2)$

c. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

d. $arg(z - z_1) = arg(z_2 - z_1)$

A. $|z - z_1| + |z - z_2| = |z_1 - z_2|$

B. $arg(z - z_1) = arg(z - z_2)$

C. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D. $arg(z - z_1) = arg(z_2 - z_1)$

Answer: A:B:C::D

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21. Find the value of 2^4

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22. If α and β are the roots of the equation x^2-$x+1=0$, then $\alpha^{2009}+\beta^{2009}=$

A. -1

B. 1

C. 2

D. -2

Answer: B

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23. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ is

A. 1

B. 2

C. ∞

D. 0

Answer: A

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24. If z is any complex number satisfying $|z-3-2i| \leq 2$, where $i = \sqrt{-1}$, then the maximum value of $|2z-6+5i|$, is

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25. The set $\left\{Re\left(\frac{2iz}{1-z^2}\right): z \text{ is a complex number}, |z| = 1, z = \pm 1\right\}$ is_______.

A. $(-\infty, -1] \cap [1, \infty)$

B. $(-\infty, 0) \cup (0, \infty)$

C. $(-\infty, -1] \cup [1, \infty)$

D. $[2, \infty)$

Answer: A

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26. The maximum value of $\left|arg\left(\frac{1}{1-z}\right)\right| f \text{ or } |z| = 1, z \neq 1$ is given by

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer: C

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27. Find the value of 3^4

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28. Let α and β be real numbers and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with Re(z)=1, then it is necessary that

A. $\beta \in (-1, 0)$

B. $|\beta| = 1$

C. $\beta \in (1, \infty)$

D. $\beta \in (0, 1)$

Answer: C

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29. If ω is a cube root of unity and $(1+\omega)^7 = A + B\omega$ then find the values of A and B`

A. (1,1)

B. (1,0)

C. (-1,1)

D. (0,1)

Answer: A

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30. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + z + 1$ is real. Then a cannot take the value.

A. -1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: D

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31. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

A. on a circle with centre at the origin

B. either on the real axis or on a circle not passing through the origin

C. on the imaginary axis

D. either on the real axis or on a circle passing through the origin

Answer: D

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32. Let complex numbers α and $\frac{1}{\alpha}$ lies on circle $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$ respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$ then $|\alpha|$ is equal to

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{7}}$

D. $\frac{1}{3}$

Answer: C

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33. What is 3% Equals to

A. 0.03

B. 0.3

C. 0.003

D. 0.0003

Answer: C

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34. Express in the form of complex number if $z = i^{-39}$

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35. Express in the form of complex number $(1-i)^4$

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36. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

A. is strictly greater than $\frac{5}{2}$

B. is equal to $\frac{5}{2}$

C. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

D. lies in the interval (1,2)

Answer: D

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37. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 \bar{z_2}}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a

A. circle of radius z

B. circle of radius $\sqrt{2}$

C. straight line parallel to X-axis

D. straight line parallel to y-axis

Answer: A

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38. Let $\omega \neq 1$ be a complex cube root of unity. If $\left(3 - 3\omega + 2\omega^2\right)^{4n+3} + \left(2 + 3\omega - 3\omega^2\right)^{4n+3} + \left(-3 + 2\omega + 3\omega^2\right)^{4n+3} = 0$, then the set of possible value(s) of n is are

A. 1

B. 2

C. 3

D. 4

Answer: A::B::D

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39. For any integer k, let $\alpha_k = \frac{\cos(k\pi)}{7} + i\frac{\sin(k\pi)}{7}, where i = \sqrt{-1}$.

Value of the expression $\frac{\sum k = 112|\alpha_{k+1} - \alpha_k|}{\sum k = 13|\alpha_{4k-1} - \alpha_{4k-2}|}$ is

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40. What is 4% Equals to

A. 0.04

B. 0.4

C. 0.004

D. 0.0004

Answer: C

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41. If the equation $z^3 + (3+i)z^2 - 3z - (m+i) = 0$, where $i = \sqrt{-1}$ and $m \in R$, has atleast one real root, value of m is

A. the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$

B. the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

C. the X-axis for $a \neq 0, b = 0$

D. the Y-axis for $a = 0, b \neq 0$

Answer: A::C::D

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42. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{bmatrix} = 3k$, then k is equal to

A. 1

B. $-z$

C. z

D. -1

Answer: B

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Complex Number Exercise 8

1. Find the modules of $2+3\iota$

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2. Find the value of 4^3

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3. Find the value of 5^3

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