MATHS

BOOKS - ARIHANT MATHS

CONTINUITY AND DIFFERENTIABILITY

Examples

1. If $f(x) = \frac{|X|}{X}$. Discuss the continuity at $x \to 0$

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2. If $f(x) = \begin{cases} 2x+3, & \text{when} \quad x < 0 \\ 0, & \text{when} \quad x = 0 \\ x^2+3, & \text{when} \quad x > 0 \end{cases}$ Discuss the continuity.

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3. If $f(x)=\dfrac{x^2-1}{x-1}$ Discuss the continuity at $x\to 1$

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4. Show that the function $f(x)=\begin{cases}2x+3, & -3\le x<-2\\ x+1, & -2\le x<0\\ x+2, & 0\le x\le 1\end{cases}$ is discontinuous at x = 0 and continuous at every point in interval $[-3, 1]$

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5. Examination the function $f(x)$ given by $f(x)=\begin{cases}\dfrac{\cos x}{\frac{\pi}{2}-x} & x\ne\frac{\pi}{2}\\ 1 & x=\frac{\pi}{2}\end{cases}$; for continuity at $x=\dfrac{\pi}{2}$

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6. Discuss the continuity of $f(x)=\tan^{-1}x$

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7. Let y = f(x) be defined parametrically as $y = t^2 + t|t|, x = 2t - |t|, t \in R$. Then, at x = 0,find f(x) and discuss continuity.

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8. Let $f(x) = \frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continous function at $x = 0$. The value of $f(0)$ equals:

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

Answer: C

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9. If $f(x) = \sqrt{\frac{1}{\tan^{-1}(x^2 - 4x + 3)}}$, then f(x) is continuous for

a. (1, 3)

b. $(-\infty, 0)$

c. $(-\infty, 1) \cup (3, \infty)$

d. None of these

A. (1, 3)

B. $(-\infty, 0)$

C. $(-\infty, 1) \cup (3, \infty)$

D. None of these

Answer: C

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10. If $f(x) = [x]$, where $[\cdot]$ denotes greatest integral function. Then, check the continuity on $(1, 2]$

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11. Examine the function, $f(x) = \begin{cases} x-1, & x<0 \\ 1/4, & x=0 \\ x^2-1, & x>0 \end{cases}$ Discuss the continuity and if discontinuous remove the discontinuity.

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12. The function $f(x) = \left\{ \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, x \neq 0, at x = 0, f(x) = 0 \right.$

a. is continuous at $x = 0$

b. is not continuous at $x = 0$

c. is not continuous at $x = 0$, but can be made continuous at $x = 0$

(d) none of these

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13. Show $f(x) = \dfrac{1}{|x|}$ has discontinuity of second kind at x = 0.

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14. $f(x) = \begin{cases} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ for what value of k, f(x) is continuous at x = 0 ?

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15. A function f(x) is defined by, $f(x) = \begin{cases} \frac{[x^2] - 1}{x^2 - 1}, & \text{for} \quad x^2 \neq 1 \\ 0, & \text{for} \quad x^2 = 1 \end{cases}$ Discuss the continuity of f(x) at x = 1.

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16. Discuss the continuity of the function $f(x) = \lim_{x \to \infty} \frac{\log(2 + x) - x^{2n} \sin x}{1 + x^{2n}}$ at x=1.

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17. Discuss the continuity of $f(x) \in [0, 2], where f(x) = (\lim)_{n \to \infty} \left(\sin\left(\pi \frac{x}{2}\right)\right)^{2n}$

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18. Let $f(x) = \begin{cases} \{1 + |\sin x|\}^{a/|\sin x|}, & -\pi/6 < x < 0 \\ b, & x = 0 \\ e^{\tan 2x/\tan 3x}, & 0 < x < \pi/6 \end{cases}$ Determine a and b such that f(x) is continuous at x = 0

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19. Fill in the blanks so that the resulting statement is correct. Let $f(x) = [x + 2]\sin\left(\frac{\pi}{[x + 1]}\right)$, where $[\cdot]$ denotes greatest integral function. The domain of f is ....and the points of discontinuity of f in the domain are

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20. Let $f(x+y) = f(x) + f(y)$ for all $x and y$. If the function $f(x)$ is continuous at $x = 0$, show that $f(x)$ is continuous for all x.

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21. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$ then the value of $f(1.5)$ is :

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22. Discuss the continuity for $f(x) = \dfrac{1-u^2}{2+u^2}$, where u = tan x.

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23. Find the points of discontinuity of $y = \dfrac{1}{u^2+u-2}$, where $u = \dfrac{1}{x-1}$

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24. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$.

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25. Suppose that f(x) is continuous in [0, 1] and $f(0) = 0, f(1) = 0$. Prove $f(c) = 1 - 2c^2$ for some $c \in (0, 1)$

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26. The left hand derivative of $f(x) = [x]\sin(\pi x)$ at $x = k, k \in Z$, is

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1}k\pi$

Answer: A

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27. Which of the following functions is differentiable at $x = 0$?

A. $\cos(|x|) + |x|$

B. $\cos(|x|) - |x|$

C. $\sin(|x|) + |x|$

D. $\sin(|x|) - |x|$

Answer: D

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28. Show that $f(x) = \begin{cases} \text{x} \sin\frac{1}{x}, & \text{when} \quad x \neq 0 \\ 0, & \text{when} \quad x = 0 \end{cases}$ is continuous but not differentiable at x = 0

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29. Let $f(x) = xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}; x \neq 0, f(0) = 0,$ test the continuity & differentiability at $x = 0$

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30. Let $f(x) = |x-1| + |x+1|$ Discuss the continuity and differentiability of the function.

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31. Discuss the continuity and differentiability for $f(x) = [\sin x]$ when $x \in [0, 2\pi]$, where $[\cdot]$ denotes the greatest integer function x.

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32. If $f(x) = \{|x| - |x-1|\}^2$, draw the graph of f(x) and discuss its continuity and differentiability of f(x)

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33. If $f(x) = \begin{cases} x - 3, & x < 0 \\ x^2 - 3x + 2, & x \geq 0 \end{cases}$ and let $g(x) = f(|x|) + |f(x)|$.

Discuss the differentiability of g(x).

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34. Let f(x) = [n + p sin x], $x \in (0, \pi), n \in Z$, p is a prime number and [x] = the greatest integer less than or equal to x. The number of points at which f(x) is not not differentiable is :

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35. Differentiate $2x^2 + 4\sin x$ w.r.t x

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36. Differentiate $4x^4 + 4\cos x$ w.r.t x

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37. Let $f(x)=\begin{cases}\int_0^x \{5+|1-t|\}dt, & \text{if} \quad x>2\\ 5x+1, & \text{if} \quad x\leq 2\end{cases}$

Test $f(x)$ for continuity and differentiability for all real x.

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38. Draw the graph of the function and discuss the continuity and differentiability at x = 1 for, $f(x)=\begin{cases}3^x, & \text{when} \quad -1\leq x\leq 1\\ 4-x, & \text{when} \quad 1<x<4\end{cases}$

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39. Expand $\begin{vmatrix}7x & 6\\ 2x & 1\end{vmatrix}$

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40. The set of points where ,$f(x)=x|x|$ is twice differentiable is

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41. The function $f(x)=\left(x^{2}-1\right)\left|x^{2}-3x+2\right|+\cos(|x|)$ is differentiable not differentiable at (a)-1 (b)0 (c)1 (d)2

A. -1

B. 0

C. 1

D. 2

Answer: D

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42. If $f(x)=\sum_{r=1}^{n} a_r|x|^r$, where a_i s are real constants, then f(x) is

a. continuous at x = 0, for all a_i

b. differentiable at x = 0, for all $a_i \in R$

c. differentiable at x = 0, for all $a_{2k+1} = 0$

d. None of the above

A. continuous at x = 0, for all a_i

B. differentiable at x = 0, for all $a_i \in R$

C. differentiable at x = 0, for all $a_{2k+1} = 0$

D. None of the above

Answer: A::C

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43. Let f and g be differentiable functions satisfying $g(a) = b, g'(a) = 2$ and fog =I (identity function). then f' (b) is equal to

A. 2

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. None of these

Answer: C

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44. If $f(x)=\frac{x}{1+(\log x)(\log x)....\infty}, \forall x \in [1,3]$ is non-differentiable at x = k. Then, the value of $[k^2]$, is (where $[\cdot]$ denotes greatest integer function).

A. 5

B. 6

C. 7

D. 8

Answer: C

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45. If f(x) = |1-X|, then the points where $\sin^{-1}(f(|x|)$ is non-differentiable are

A. {0, 1}

B. {0, -1}

C. {0, 1, -1}

D. None of these

Answer: C

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46. Discuss the differentiability of $f'(x) = \frac{\sin^{-1}(2x)}{1+x^2}$

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47. Let [] donots the greatest integer function and $f(x) = [\tan^2 x]$, then

A. $\lim_{x \to 0} f(x)$ doesn't exist

B. f(x) is continuous at x = 0

C. f(x) is not differentiable at x = 0

D. $f'(0) = 1$

Answer: B

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48. Let $h(x) = \min \{x, x^2\}$ for every real number of x. Then, which one of the following is true?

A. h is not continuous for all x

B. h is differentiable for all x

C. $h'(x) = 1$ for all x

D. h is not differentiable at two values of x

Answer: D

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49. let $f: R \rightarrow R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of values where f(x) is differentiable is:

A. {-1, 1}

B. {-1, 0}

C. {0, 1}

D. {-1, 0, 1}

Answer: D

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50. Let f(x) be a continuous function such that f(0) = 1 and f(x)-f$\left(\frac{x}{7}\right)$=$\frac{x}{7}$ $\forall x \in R$, then f(42) is

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51. The total number of points of non-differentiability of $f(x) = \max\left\{\sin^2 x, \cos^2 x, \frac{3}{4}\right\}$ in $[0, 10\pi]$, is

A. 40

B. 30

C. 20

D. 10

Answer: C

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52. Differentiate $7x^3 + e^{4x}$ w.r.t x

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53. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2)$, $[.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

A. $a \in [8, 64]$

B. $a \in (0, 8]$

C. $a \in [64, \infty)$

D. None of these

Answer: C

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54. If $f(x) = x^2 - 2x$ then find the derivative of this function.

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55. Let $f(x) = \phi(x) + \Psi(x) where, \phi'(x)$ and $\Psi'(x)$ are finite and definite. Then,

a. f(x) is continuous at x = a

b. f(x) is differentiable at x = a

c. f'(x) is continuous at x = a

d. f'(x) is differentiable at x = a

A. f(x) is continuous at x = a

B. f(x) is differentiable at x = a

C. f'(x) is continuous at x = a

D. f'(x) is differentiable at x = a

Answer: A::B

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56. If $f(x) = x + \tan x$ and $g(x) is the$inverseoff(x), $then$differentiation of g(x)$is(a)$1/(1+[g(x)-x]^2)(b) 1/(2-[g(x)+x]^2)(c)1/(2+[g(x)-x]^2)(d) none of these`

A. $\dfrac{1}{1+(g(x)-x)^2}$

B. $\dfrac{1}{2+(g(x)+x)^2}$

C. $\dfrac{1}{2+(g(x)-x)^2}$

D. None of these

Answer: C

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57. If $f(x)=\int_0^x (f(t))^2 dt, f: R \to R$ be differentiable function and $f(g(x))$ is differentiable at $x=a$, then

A. (a)g(x) must be differentiable at x = a

B. (b)g(x) is discontinuous, then f(a) = 0

C. (c)$f(a) \neq 0$, then g(x) must be differentiable

D. (d)None of these

Answer: B::C

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58. If $f(x) = [x^{-2}[x^2]]$, (where $[\cdot]$ denotes the greatest integer function) $x \neq 0$, then incorrect statement

a. f(x) is continuous everywhere

b. f(x) is discontinuous at $x = \sqrt{2}$

c. f(x) is non-differentiable at x = 1

d. f(x) is discontinuous at infinitely many points

A. f(x) is continuous everywhere

B. f(x) is discontinuous at $x = \sqrt{2}$

C. f(x) is non-differentiable at x = 1

D. f(x) is discontinuous at infinitely many points

Answer: A

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59. If

$f(x)=\{'x^2(sgn[x])+\{x\}, 0 \leq x \leq 2''\sin x+|x-3|, 2<x<4,$

(where[.] & {.} greatest integer function & fractional part functiopn respectively), then -

A. f(x) is differentiable at x = 1

B. f(x) is continuous but non-differentiable at x

C. f(x) is non-differentiable at x = 2

D. f(x) is discontinuous at x = 2

Answer: C::D

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60. Expand $\begin{vmatrix} 2 & 0 \\ 5 & 7 \end{vmatrix}$

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61. The values of a and b so that the function $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x < \pi/4 \\ 2x\cot x + b, & \pi/4 \le x \le \pi/2 \\ a\cos 2x - b\sin x, & \pi/2 < x \le \pi \end{cases}$ is continuous for $x \in [0, \pi]$, are

A. $a = \frac{\pi}{6}, b = -\frac{\pi}{6}$

B. $a = -\frac{\pi}{6}, b = \frac{\pi}{12}$

C. $a = \frac{\pi}{6}, b = -\frac{\pi}{12}$

D. None of these

Answer: C

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62. Let f be an even function and f'(x) exists, then f'(0) is

A. 1

B. 0

C. -1

D. -2

Answer: B

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63. Find the set of profit where $f(x) = x^2|x|$is thrice differentiable .

A. R

B. R - {0, 1}

C. $[0, \infty)$

D. R - {0}

Answer: D

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64. The function $f(x)=\dfrac{|x+2|}{\tan^{-1}(x+2)}$, is continuous for x∈R x∈R−{0} x∈R−{−2} None of these

A. $x \in R$

B. $x \in R-\{0\}$

C. $x \in R-\{-2\}$

D. None of these

Answer: C

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65. If $f(x)=\begin{cases}\dfrac{\sin[x^2]\pi}{x^2-3x+8}+ax^3+b & 0\le x\le 1\\ 2\cos\pi x+\tan^{-1}x & 1<x\le 2\end{cases}$ is differentiable in $[0,2]$ then: ([.] denotes greatest integer function)

A. (A)$a=\dfrac{1}{6}, b=\dfrac{\pi}{4}-\dfrac{13}{6}$

B. (B)$a=-\dfrac{1}{6}, b=\dfrac{\pi}{4}$

C. (C)$a = -\frac{1}{6}, b = \frac{\pi}{4} - \frac{13}{6}$

D. (D)None of these

Answer: A

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66. Expand $\begin{vmatrix} 9 & 1 \\ 2 & 0 \end{vmatrix}$

A. 0

B. 1

C. -2

D. 3

Answer: B

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67. Let $g(x)=\ln f(x)$ where f(x) is a twice differentiable positive function on $(0,\infty)$ such that $f(x+1)=xf(x)$. Then for N = 1,2,3

$$g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=$$

A. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N+1)^2}\right\}$

D. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N+1)^2}\right\}$

Answer: A

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68. Let y=f(x) be a differentiable function $\forall x \in R$ and satisfies:

$$f(x)=x+\int_0^1 x^2 z f(z)dz+\int_0^1 xz^2 f(z)dz.$$

A. $f(x)=\frac{20x}{119}(2+9x)$

B. $f(x) = \frac{20x}{119}(4 + 9x)$

C. $f(x) = \frac{10x}{119}(4 + 9x)$

D. $f(x) = \frac{5x}{119}(4 + 9x)$

Answer: B

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69. A function $f: R \to R$ satisfies the equation $f(x + y) = f(x).\ f(y)$ for all, $f(x) \neq 0$. Suppose that the function is differentiable at x = 0 and f'(0) = 2. Then,

A. f'(x) = 2f(x)

B. f'(x) = f(x)

C. f'(x) = f(x) + 2

D. f'(x) = 2f(x) + x

Answer: A

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70. Let f be a function such that $f(x+f(y))=f(x)+y, \forall x, y \in R$, then find f(0). If it is given that there exists a positive real δ such that f(h) = h for $0<h<\delta$, then find f'(x)

A. 0, 1

B. $-1, 0$

C. 2, 1

D. $-2, 0$

Answer: A

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71. If the function of $f(x)=\left[\frac{(x-5)^3}{A}\right]\sin(x-5)+a\cos(x-2)$, where$[\cdot]$ denotes the greatest integer function, is continuous and differentiable in (7, 9), then find the value of A

A. $A \in [8, 64]$

B. $A \in [0, 8)$

C. $A \in [64, \infty)$

D. $A \in [8, 16]$

Answer: C

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72. If $f(x) = [2 + 5|n|\sin x]$, where $n \in I$ has exactly 9 points of non-derivability in $(0, \pi)$, then possible values of n are (where [x] dentoes greatest integer function)

A. ± 3

B. ± 2

C. ± 1

D. None of these

Answer: C

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73. The number of points of discontinuity of $f(x) = [2x]^2 - \{2x\}^2$ (where [] denotes the greatest integer function and {} is fractional part of x) in the interval $(-2, 2)$, is 1 b. 6 c. 2 d. 4

A. 6

B. 8

C. 4

D. 3

Answer: A

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74. Find $\frac{dy}{dx}$ if $f(x) = \frac{2}{1-x}$

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75. Let $f: R \rightarrow R$ be a differentiable function at x = 0 satisfying f(0) = 0 and f'(0) = 1, then the value of $\lim_{x \rightarrow 0} \frac{1}{x} \cdot \sum_{n=1}^{\infty} (-1)^n \cdot f\left(\frac{x}{n}\right)$, is

a. 0

b. $-\log 2$

c. 1

d. e

A. (a)0

B. (b)$-\log 2$

C. (c)1

D. (d)e

Answer: B

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76. Let f(x) is a function continuous for all $x \in R$ except at x = 0 such that $f'(x) < 0, \forall x \in (-\infty, 0)$ and $f'(x) > 0, \forall x \in (0, \infty)$. If $\lim_{x \to 0^+} f(x) = 3, \lim_{x \to 0^-} f(x) = 4$ and $f(0) = 5$, then the image of the point (0, 1) about the line, $y. \lim_{x \to 0} f(\cos^3 x - \cos^2 x) = x. \lim_{x \to 0} f(\sin^2 x - \sin^3 x)$, is

a. $\left(\frac{12}{25}, \frac{-9}{25}\right)$

b. $\left(\frac{12}{25}, \frac{9}{25}\right)$

c. $\left(\frac{16}{25}, \frac{-8}{25}\right)$

d. $\left(\frac{24}{25}, \frac{-7}{25}\right)$

A. $\left(\frac{12}{25}, \frac{-9}{25}\right)$

B. $\left(\frac{12}{25}, \frac{9}{25}\right)$

C. $\left(\frac{16}{25}, \frac{-8}{25}\right)$

D. $\left(\frac{24}{25}, \frac{-7}{25}\right)$

Answer: D

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77. If f(x) be such that $f(x) = \max\left(|3-x|, 3-x^3\right)$, then

(a) f(x) is continuous $\forall x \in R$

(b) f(x) is differentiable $\forall x \in R$

(c) f(x) is non-differentiable at three points only

(d) f(x) is non-differentiable at four points only

A. (a) f(x) is continuous $\forall x \in R$

B. (b) f(x) is differentiable $\forall x \in R$

C. (c) f(x) is non-differentiable at three points only

D. (d) f(x) is non-differentiable at four points only

Answer: A::D

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78. Let $f(x) = |x-1|([x]-[-x])$, then which of the following statement(s) is/are correct. (where [.] denotes greatest integer function.)

a.f(x) is continuous at x = 1

b. f(x) is derivable at x = 1

c. f(X) is non-derivable at x = 1

d. f(x) is discontinuous at x = 1

A. f(x) is continuous at x = 1

B. f(x) is derivable at x = 1

C. f(X) is non-derivable at x = 1

D. f(x) is discontinuous at x = 1

Answer: A::C

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79. If y = f(x) defined parametrically by $x = 2t - |t - 1|$ and $y = 2t^2 + t|t|$, then

(a)f(x) is continuous for all $x \in R$

(b)f(x) is continuous for all $x \in R - \{2\}$

(c)f(x) is differentiable for all $x \in R$

(d)f(x) is differentiable for all $x \in R - \{2\}$

A. (a)f(x) is continuous for all $x \in R$

B. (b)f(x) is continuous for all $x \in R - \{2\}$

C. (c)f(x) is differentiable for all $x \in R$

D. (d)f(x) is differentiable for all $x \in R - \{2\}$

Answer: A::D

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80. $f(x) = \sin^{-1}[e^x] + \sin^{-1}[e^{-x}]$ where [.] greatest integer function then

a. domain of $f(x) = (-\text{In } 2, \text{In } 2)$

b. range of f(x) = $\{\pi\}$

c. f(x) has removable discontinuity at x = 0

d. $f(x) = \cos^{-1} x$ has only solution

A. domain of $f(x) = (-\text{In } 2, \text{In } 2)$

B. range of f(x) = $\{\pi\}$

C. f(x) has removable discontinuity at x = 0

D. $f(x) = \cos^{-1} x$ has only solution

Answer: A::C

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81. $f: R \to R$ is one-one, onto and differentiable and graph of y = f (x) is symmetrical about the point (4, 0), then

a. $f^{-1}(2010) + f^{-1}(-2010) = 8$

b. $\int_{-2010}^{2018} f(x)dx = 0$

c. if $f'(-100) > 0$, then roots of $x^2 - f'(10)x - f'(10) = 0$ may be non-real

d. if $f'(10) = 20$, then f'(-2) = 20

A. $f^{-1}(2010) + f^{-1}(-2010) = 8$

B. $\int_{-2010}^{2018} f(x)dx = 0$

C. if $f'(-100) > 0$, then roots of $x^2 - f'(10)x - f'(10) = 0$ may be non-real

D. if $f'(10) = 20$, then f'(-2) = 20

Answer: A::B::D

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82. Let f be a real valued function defined on the interval $(0, \infty)$ by $f(x) = In x + \int_0^x \sqrt{1 + \sin t}dt$. Then which of the following statement (s) is (are) true?

A. f''(x) exists for all $x \in (0, \infty)$

B. f'(x) exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$ but not differentiable on $(0, \infty)$

C. There exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (0, \infty)$

D. There exists $\beta > 0$ such that $|f(x)| + |f'(x)| \leq \beta$ from all $x \in (0, \infty)$

Answer: B::C

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83. $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all$x, y \in R.$ $(xy \neq 1)$,and $\lim_{x \to 0} \frac{f(x)}{x} = 2$.Find $f\left(\frac{1}{\sqrt{3}}\right) and f'(1).$

A. $f\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$

B. $f\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{3}$

C. $f'(1) = 1$

D. $f'(1) = -1$

Answer: A::C

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84. Let $f:R\overrightarrow{R}$ be a function satisfying condition $f(x+y^3)=f(x)+[f(y)]^3f$ or $allx, y \in R$. If $f'(0)\geq 0,$ find $f(10)$.

A. f(x) = 0 only

B. f(x) = x only

C. f(x) = 0 or x only

D. f(10) = 10

Answer: C::D

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85. Let $f(x)=x^3-x^2+x+1$ and $g(x)=\begin{cases}\max\ f(t), & 0\leq t\leq x \quad \text{for} \quad 0\leq x\leq 1\\ 3-x, & 1<x\leq 2\end{cases}$ Then, g(x) in [0, 2] is

a. continuous for $x\in[0,2]-\{1\}$

b. continuous for $x\in[0,2]$

c. differentiable for all $x\in[0,2]$

d. differentiable for all $x\in[0,2]-\{1\}$

A. continuous for $x \in [0, 2] - \{1\}$

B. continuous for $x \in [0, 2]$

C. differentiable for all $x \in [0, 2]$

D. differentiable for all $x \in [0, 2] - \{1\}$

Answer: B::D

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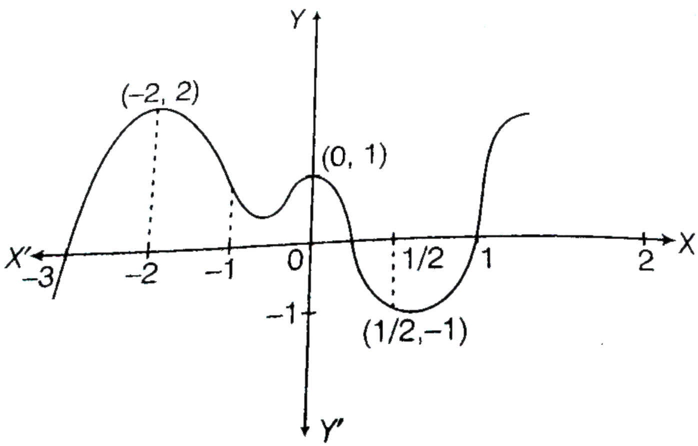
86. If $p''(x)$ has real roots α, β, γ. Then , $[\alpha] + [\beta] + [\gamma]$ is



A. -2

B. -3

C. -1

D. 0

Answer: B

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87. If $f(x) = \frac{1}{1-x}$, then the set of points discontinuity of the function $f(f(f(x)))is$ {1} (b) {0,1} (c) {-1,1} (d) none of these

A. x = 0, -1

B. x = 1 only

C. x = 0 only

D. x = 0, 1

Answer: D

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88. If α, β (where $\alpha < \beta$) are the points of discontinuity of the function g(x) = f(f(f(x))), where $f(x) = \frac{1}{1-x}$, and $P(a, a^2)$ is any point on XY - plane. Then,

The domain of f(g(x)), is

A. $x \in R$

B. $x \in R - \{1\}$

C. $x \in R - \{0, 1\}$

D. $x \in R - \{0, 1, -1\}$

Answer: C

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89. Find $\frac{dy}{dx}$ if $y = \frac{x}{\sin x}$

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90. If [x] dnote the greatest integer less than or equal to x then the equation $\sin x = [1 + \sin x] + [1 - \cos x][$ has no solution in

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91. Differentiate $x^3 + \sin 4x + e^{3x}$ w.r.t x

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92. Suppose a function f(x) satisfies the following conditions

$$f(x+y) = \frac{f(x)+f(y)}{1+f(x)f(y)}, \forall x \in R, y \text{ and } f'(0) = 1. \text{ Also}, -1 < f(x) <$$

,

The value of the limit $lt_{x \to \infty}(f(x))^x$ is:

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93. Given that $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{if } x > 0 \end{cases}$

If f(x) is continuous at x=0 find the value of a.

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94. $f(x) =$ maximum $\{4, 1+x^2, x^2-1)\ \forall x \in R$. Total number of points, where $f(x)$ is non-differentiable,is equal to

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95. Let $f(x) = x^n$,n being a non negative integer. The value of n for which the equality$f'(a+b) = f'(a) + f'(b)$ is valid for all $a.\ b > 0$ is

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96. The number of points where $f(x) = [\sin x + \cos x]$ (where [.] denotes the greatest integer function) $x \in (0, 2\pi)$ is not continuous is

a. 3

b. 4

c. 5

d. 6

A. (A) 3

B. (B) 4

C. (C) 5

D. (D) 6

Answer: 5

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97. Find $\frac{dy}{dx}$ if $2x - 3y = \log y$

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98. If $f\left(\frac{xy}{2}\right) = \frac{f(x).\ f(y)}{2}, x, y \in R, f(1) = f'(1).$ Then, $\frac{f(3)}{f'(3)}$ is.........

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99. Let f be a differentiable function satisfying the relation f(xy)=xf(y)+yf(x) −2xy .(where x, y >0) and f ' (1)=3, then

find f(x)

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100. Let f(x) is a polynomial function and $f(\alpha))^2 + f'(\alpha))^2 = 0$, then find $\lim_{x \to \alpha} \frac{f(x)}{f'(x)}\left[\frac{f'(x)}{f(x)}\right]$, where [.] denotes greatest integer function, is........

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101. Let $f: R \to R$ be a function satisfying $f(2-x) = f(2+x)$ and $f(20-x) = f(x)\,\forall x \in R$. For this function f, answer the following.

If $f(2) \neq f(6)$, then the

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102. Find $\frac{dy}{dx}$ if $2x - 10y = \log x$

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103. Discuss the differentiability of $f(x) = max\{2\sin x, 1-\cos x\}\,\forall x \in (0, \pi)$.

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104. Discuss the continuity of the function g(x) = [x] + [-x] at integral values of x.

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105. If $f(x) = \dfrac{\sin 2x + A\sin x + B\cos x}{x^3}$ is continuous at x = 0. Find the values of A and B. Also, find f(0)

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106. Let f: $R \rightarrow R$ satisfies $|f(x)| \leq x^2 \forall x \in R.$ then show thata f(x) is differentiable at x=0

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107. Show that the function defined by $f(x) = \begin{cases} x^2 \sin 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable for every value of x, but the derivative is not continuous for x=0

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108. Find $\frac{dy}{dx}$ if $x - 3y = x^2$

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109. Prove that $f(x) = [\tan x] + \sqrt{\tan x - [\tan x]}$. (where [.] denotes greatest integer function) is continuous in $\left[0, \frac{\pi}{2}\right)$.

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110. Determine the values of x for which the following functions fails to be continuous or differentiable $f(x) = \begin{cases} (1-x), & x < 1 \\ (1-x)(2-x), & 1 \leq x \leq 2 \\ (3-x), & x > 2 \end{cases}$

justify your answer.

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111. If g(x) is continuous function in $[0, \infty)$ satisfying $g(1) = 1. If \int_0^x 2x.\, g^2(t)dt = \left(\int_0^x 2g(x-t)dt\right)^2$, find g(x).

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112. Differentiate $x^5 + e^x$ w.r.t x

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113. If a function $f: [-2a, 2a] \rightarrow R$ is an odd function such that, $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left-hand derivative at $x = a$ is 0, then find the left-hand derivative at $x = -a$.

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114. Discuss the continuity of f(x) in [0, 2], where $f(x)=\begin{cases}[\cos \pi x], & x\leq 1\\ |2x-3|[x-2], & x>1\end{cases}$ where [.] denotes the greatest integral function.

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115. Let $f: R \rightarrow R$ be a differentiable function such that $f(x)=x^2+\int_0^x e^{-t} f(x-t) dt. f(x)$ increases for

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116. Let $f: R^+ \rightarrow R$ satisfies the functional equation $f(xy)=e^{xy-x-y}\{e^y f(x)+e^x f(y)\}, \forall x, y \in R^+$. If f'(1) = e, determine f(x).

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117. Let f be a differentiable function such that $f'(x) = f(x) + \int_0^2 f(x)dx$ and $f(0) = \frac{4 - e^2}{3}$. Find $f(x)$.

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118. A function $f(x)$ satisfies the following property: $f(x + y) = f(x)f(y)$. Show that the function is continuous for all values of x if its is continuous at $x = 1$.

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119. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x) + f(y)}{2}$ for all real x and y. If $f'(0)$ exits and equals -1 and $f(0) = 1$, then find $f(2)$.

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120. Let $f(x) = 1 + 4x - x^2, \forall x \in R$

$g(x) = \{ \max \{f(t), x \leq t \leq (x+1)0 \leq x < 3$

$\min \{(x+3)3 \leq x \leq 5\}$ Verify continuity of g(x), for all $x \in [0, 5]$

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121. Let $f(x) = x^4 - 8x^3 + 22x^2 - 24x - 55$ and

$g(x) = \begin{cases} \min f(x), & x \leq t \leq x+1 : -1 \leq x \leq 1 \\ x - 10, & x \geq 1 \end{cases}$ Discuss the continuity and differentiability of g(x) in $[-1, \infty)$

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122. Solve the differential equation

$(1+y^2)dx - (\tan^{-1} y - x)dy = 0$

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123. Let f be a one-one function such that $f(x).\, f(y)+2=f(x)+f(y)+f(xy), \forall x, y \in R-\{0\}$ and $f(0)=1, f'$

. Prove that $3\left(\int f(x)dx\right)-x(f(x)+2)$ is constant.

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124. Find $f'(x)$. if $f(x)=e^{x}-\log x-\sin x$

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125. Let f be a function such that $f(xy)=f(x).\, f(y), \forall y \in R$ and $R(1+x)=1+x(1+g(x))$. where $\lim_{x \to 0} g(x)=0$. Find the value of $\int_{1}^{2} \frac{f(x)}{f'(x)} \cdot \frac{1}{1+x^{2}} dx$

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126. If $f(x) = ax^2 + bx + c$ is such that $|f(0)| \leq 1, |f(1)| \leq 1$ and $|f(-1)| \leq 1,$ prove that $|f(x)| \leq 5/4, \forall x \in [-1, 1]$

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127. Let $\alpha + \beta = 1, 2\alpha^2 + 2\beta^2 = 1$ and $f(x)$ be a continuous function such that $f(2 + x) + f(x) = 2$ for all $x \in [0, 2]$ and $p = \int_0^4 f(x)dx - 4, q = \frac{\alpha}{\beta}$. Then, find the least positive integral value of 'a' for which the equation $ax^2 - bx + c = 0$ has both roots lying between p and q, where $a, b, c \in N$.

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128. Prove that the function $f(x) = a\sqrt{x-1} + b\sqrt{2x-1} - \sqrt{2x^2 - 3x + 1}$, where a + 2b = 2 and $a, b \in R$ always has a root in $(1, 5) \forall b \in R$

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129. Let $\alpha \in R$. Prove that a function $f: R \rightarrow R$ is differentiable at α if and only if there is a function $g: R \rightarrow R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha), \forall x \in R$.

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Exercise For Session 1

1. If function $f(x) = \dfrac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ is continuous function at x = 0, then f(0) is equal to

A. 2

B. $\dfrac{1}{4}$

C. $\dfrac{1}{6}$

D. $\dfrac{1}{3}$

Answer: C

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2. If $f(x) = \begin{cases} \frac{1}{e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

A. $\lim_{x \to 0^-} f(x) = 0$

B. $\lim_{x \to 0^+} f(x) = 1$

C. f(x) is discontinuous at x = 0

D. f(x) is continuous at x = 0

Answer: C

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3. If $f(x) = \begin{cases} \frac{x^2 - (a+2)x + 2a}{x-2}, & x \neq 2 \\ 2, & x = 2 \end{cases}$ is continuous at x = 2, then a is equal to

A. 0

B. 1

C. -1

D. 2

Answer: A

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4. If $f(x) = \begin{cases} \frac{\log(1+2ax) - \log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then k is equal to

A. 2a + b

B. 2a - b

C. b - 2a

D. a + b

Answer: A

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5. If $f(x)=\begin{cases}[x]+[-x], & x\neq 2\\ \lambda, & x=2\end{cases}$ and f is continuous at x = 2, where $[\cdot]$ denotes greatest integer function, then λ is

A. -1

B. 0

C. 1

D. 2

Answer: A

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Exercise For Session 2

1. Let $f(x) = \begin{cases} -2\sin x & \text{for} \quad -\pi \leq x \leq -\frac{\pi}{2} \\ a\sin x + b & \text{for} \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & \text{for} \quad \frac{\pi}{2} \leq x \leq \pi \end{cases}$.

If f is continuous on $[-\pi, \pi)$, then find the values of a and b.

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2. Draw the graph of the function $f(x) = x - \left|x - x^2\right|, \ -1 \leq x \leq 1$ and discuss the continuity or discontinuity of f in the interval $-1 \leq x \leq 1$

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3. Discuss the continuity of 'f' in [0, 2], where $f(x) = \begin{cases} |4x - 5|[x] & \text{for} \quad x > 1 \\ [\cos \pi x] & \text{for} \quad x \leq 1 \end{cases}$, where [x] is greastest integer not greater than x.

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4. Let $f(x) = \begin{cases} Ax - B, & x \le -1 \\ 2x^2 + 3Ax + B, & -1 < x \le 1 \\ 4, & x > 1 \end{cases}$

Statement I f(x) is continuous at all x, if $A = \frac{3}{4}$.

Statement II Polynomial function is always continuous.

A. Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect

Answer: D

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Exercise For Session 3

1. which of the following function(s) not defined at $x = 0$ has/have removable discontinuity at $x = 0$.

A. $f(x) = \dfrac{1}{1 + 2^{\cot x}}$

B. $f(x) = \cos\left(\dfrac{(|\sin x|)}{x}\right)$

C. $f(x) = \text{x} \sin\dfrac{\pi}{x}$

D. $f(x) = \dfrac{1}{\text{In}|x|}$

Answer: B::C::D

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2. Function whose jump (non-negative difference of LHL and RHL) of discontinuity is greater than or equal to one. is/are

A. $f(x) = \begin{cases} \dfrac{(e^{1/x} + 1)}{e^{1/x} - 1}, & x < 0 \\ \dfrac{(1 - \cos x)}{x}, & x > 0 \end{cases}$

B. $g(x) = \begin{cases} \dfrac{(x^{1/3} - 1)}{x^{1/2} - 1}, & x > 0 \\ \dfrac{\text{In x}}{(x - 1)}, & \frac{1}{2} < x < 1 \end{cases}$

C. $u(x) = \begin{cases} \dfrac{\sin^{-1} 2x}{\tan^{-1} 3x}, & x \in \left(0, \frac{1}{2}\right] \\ \dfrac{|\sin x|}{x}, & x < 0 \end{cases}$

D. $v(x) = \begin{cases} \log_3(x+2), & x > 2 \\ \log_{1/2}(x^2+5), & x < 2 \end{cases}$

Answer: A::C::D

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3. Consider the piecewise defined function $f(x) = \begin{cases} \sqrt{-x} & \text{if} \quad x < 0 \\ 0 & \text{if} \quad 0 \le x \le 4 \\ x - 4 & \text{if} \quad x > 4 \end{cases}$ describe the continuity of this function.

A. the function is unbounded and therefore cannot be continuous

B. the function is right continuous at x = 0

C. the function has a removable discontinuity at 0 and 4, but is continuous on the rest of the real line.

D. the function is continuous on the entire real line

Answer: D

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4. If $f(x) = sgn(\cos 2x - 2\sin x + 3)$, where sgn () is the signum function, then f(x)

A. is continuous over its domain

B. has a missing point discontinuity

C. has isolated point discontinuity

D. has irremovable discontinuity

Answer: C

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5. If $f(x) = \frac{2\cos x - \sin 2x}{(\pi - 2x)^2}$,$g(x) = \frac{e^{-\cos x} - 1}{8x - 4\pi}$

$h(x) = f(x)$ for $x < \frac{\pi}{2}$

$h(x) = g(x)$ for $x > \frac{\pi}{2}$ then which of the following holds?

A. h is not differentiable at $x = \pi/2$

B. h has an irremovable discontinuity at $x = \pi/2$

C. h has a removable discontinuity at $x = \pi/2$

D. $f\left(\frac{\pi^+}{2}\right) = g\left(\frac{\pi^-}{2}\right)$

Answer: A::C::D

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Exercise For Session 4

1. If $f(x) = \frac{1}{x^2 - 17x + 66}$, then $f\left(\frac{2}{x-2}\right)$ is discontinuous at x =

A. 2

B. $\frac{7}{3}$

C. $\frac{24}{11}$

D. 6, 11

Answer: A::B::C

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2. Let f be a continuous function on R such that $f\left(\frac{1}{4n}\right) = \frac{\sin e^n}{e^{n^2}} + \frac{n^2}{n^2+1}$ Then the value of $f(0)$ is

A. not unique

B. 1

C. data sufficient to find f(0)

D. data insufficient to find f(0)

Answer: B::C

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3. $f(x)$ is continuous at $x = 0$ then which of the following are always true ?

A. A. $\lim_{x \to 0} f(x) = 0$

B. B. f(x) is non coninuous at x = 1

C. C. $g(x) = x^2 f(x)$ is continuous x = 0

D. D. $\lim_{x \to 0^+} (f(x) - f(0)) = 0$

Answer: C::D

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4. If $f(x) = \cos\left[\frac{\pi}{x}\right]\cos\left(\frac{\pi}{2}(x-1)\right)$; where [x] is the greatest integer function of x,then $f(x)$ is continuous at :

A. x = 0

B. x = 1

C. x = 2

D. None of these

Answer: B::C

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5. Let $f(x) = [x]$ and $g(x) = \{0, x \in Zx^2, x \in R - Z$ then (where [.]denotest greatest integer funtion)

A. $\lim_{x \to 1} g(x)$ exists, but g(x) is not continuous at x = 1

B. $\lim_{x \to 1} f(x)$ does not exist and f(x) is not continuous at x = 1

C. gof is continuous for all x.

D. fog is continuous for all x.

Answer: A::B::C

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6. Let $f(x) = \begin{cases} a \sin^{2n} x & \text{for } x \geq 0 \text{ and } n \to \infty \\ b \cos^{2m} x - 1 & \text{for } x < 0 \text{ and } m \to \infty \end{cases}$ then

A. $f(0^-) \neq f(0^+)$

B. $f(0^+) \neq f(0)$

C. $f(0^-) = f(0)$

D. f is continuous at x = 0

Answer: A

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7. $Let f(x) = \lim_{n \to \infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$ for $x > 0, x \ne 1$, and $f(1) = 0$

Discuss the continuity at x=1.

A. f is continuous at x = 1

B. f has a finite discontinuity at x = 1

C. f has an infinite or oscillatory discontinuity at x = 1

D. f has a removal type of discontinuity at x = 1

Answer: B

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Exercise For Session 5

1.

if $f(x) = \frac{x}{(1+x)} + \frac{x}{(1+x)(1+2x)} + \frac{x}{(1+2x)(1+3x)} + \dots$

infinite terms , Discuss continuity at x=0

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2. If $g : [a, b]$ on to [a, b] is continous, then show that there is some $c \in [a, b]$ such that g(c) = c

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3. Find $\frac{dy}{dx}$ if $y = \frac{x}{\cos x}$

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4. Let

$$y_n(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots\ldots \frac{x^2}{(1+x^2)^{n-1}} \text{ and } y(x) = \lim_{n \to \infty}$$

. Discuss the continuity of $y_n(x)(n = 1, 2, 3.... n)$ and $y(x)$at x $= 0$

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Exercise For Session 6

1. If a function f(x) is defined as $f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ x^2 - x + 1, & x > 1 \end{cases}$ then

a. f(x) is differentiable at x = 0 and x = 1

b. f(x) is differentiable at x = 0 but not at x = 1

c. f(x) is not differentiable at x = 1 but not at x = 0

d. f(x) is not differentiable at x = 0 and x = 1

A. f(x) is differentiable at x = 0 and x = 1

B. f(x) is differentiable at x = 0 but not at x = 1

C. f(x) is not differentiable at x = 1 but not at x = 0

D. f(x) is not differentiable at x = 0 and x = 1

Answer: D

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2. If $f(x) = x^3$sgn (x), then

A. f is differentiable at x = 0

B. f is continuous but not differentiable at x = 0

C. $f'(0^-) = 1$

D. None of these

A. f is differentiable at x = 0

B. f is continuous but not differentiable at x = 0

C. $f'(0^-) = 1$

D. None of these

Answer: A

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3. Which one of the following functions is continuous everywhere in its domain but has atleast one point where it is not differentiable ?

A. $f(x) = x^{1/3}$

B. $f(x) = \frac{|x|}{x}$

C. $f(x) = e^{-x}$

D. f(x) = tan x

Answer: A

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4. If $f(x) = \begin{cases} x + \{x\} + x\sin\{x\}, & \text{for} \quad x \neq 0 \\ 0, & \text{for} \quad x = 0 \end{cases}$, where {x} denotes the fractional part function, then

A. f is continuous and differentiable at x = 0

B. f is continuous but not differentiable at x = 0

C. f is continuous and differentiable at x = 2

D. None of these

Answer: D

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5. If $f(x) = \begin{cases} x\left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at x = 0 f(x) is

A. differentiable

B. not differentiable

C. $f'(0^+) = -1$

D. $f'(0^-) = 1$

Answer: B

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Exercise For Session 7

1. Number of points of non-differererentiable of $f(x) = \sin \pi(x - [x])\text{in } \ (-\pi/2, [\pi/2)$. Where [.] denotes the greatest integer function is

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2. Consider the function $f(x) = f(x) = \begin{cases} x-1, & -1 \leq x \leq 0 \\ x^2, & 0 \leq x \leq 1 \end{cases}$

and $g(x) = \sin x$.

If $h_1(x) = f(|g(x)|)$

$\text{and} \quad h_2(x) = |f(g(x))|$.

Which of the following is not true about $h_1(x)$?

A. h(x) is continuous for $x \in [-1, 1]$

B. h(x) is differentiable for $x \in [-1, 1]$

C. h(x) is differentiable for $x \in [-1, 1] - \{0\}$

D. h(x) is differentiable for $x \in (-1,)$-{0}`

Answer: C

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3. If $f(x) = \begin{cases} |1 - 4x^2|, & 0 \leq x < 1 \\ [x^2 - 2x], & 1 \leq x < 2 \end{cases}$, where [] denotes the greatest integer function, then

A. f(x) is continuous for all $x \in [0, 2)$

B. f(x) is differentiable for all $x \in [0, 2) - \{1\}$

C. f(X) is differentiable for all $x \in [0, 2) - \left\{\frac{1}{2}, 1\right\}$

D. None of these

Answer: C

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4. Let $f(x) = \int_0^1 |x - t| t dt$, then

A. f(x) is continuous but not differentiable for all $x \in R$

B. f(x) is continuous and differentiable for all $x \in R$

C. f(x) is continuous for $x \in R - \left\{\frac{1}{2}\right\}$ and $f(x)$ is differentiable for $x \in R - \left\{\frac{1}{4}, \frac{1}{2}\right\}$

D. None of these

A. f(x) is continuous but not differentiable for all $x \in R$

B. f(x) is continuous and differentiable for all $x \in R$

C. f(x) is continuous for $x \in R - \left\{\frac{1}{2}\right\}$ and $f(x)$ is differentiable for $x \in R - \left\{\frac{1}{4}, \frac{1}{2}\right\}$

D. None of these

Answer: B

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5. Let f be a function such that $f(x+y)=f(x)+f(y)$ for all $x and y and f(x)=(2x^2+3x)g(x)$ for all$x$, where $g(x)$ is continuous and $g(0)=3$. Then find $f'(x)$.

A. 6

B. 9

C. 8

D. None of these

Answer: B

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6. Find $\dfrac{dy}{dx}$ if $y=3x^3+e^{7x}+5$

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7. Let $f: R \rightarrow R$ be a function satisfying $f\left(\frac{xy}{2}\right) = \frac{f(x) \cdot f(y)}{2}, \forall x, y \in R \text{ and } f(1) = f'(1) = \neq 0.$ Then, $f(x) + f(1 - x)$ is (for all non-zero real values of x) a.) constant b.) can't be discussed c.) $xd.\Big)\frac{1}{x}$

A. constant

B. can't be discussed

C. x

D. $\frac{1}{x}$

Answer: A

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8. Let $f: R \rightarrow R$ satisfying $f\left(\frac{x+y}{k}\right) = \frac{f(x) + f(y)}{k} (k \neq 0, 2)$.Let $f(x)$ be differentiable on R and $f'(0) = a$, then determine $f(x)$.

A. A. even function

B. B. neither even nor odd function

C. C. either zero or odd function

D. D. either zero or even function

Answer: C

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9. $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all$x, y \in R$. $(xy \neq 1)$,and $\lim_{x \to 0} \frac{f(x)}{x} = 2$.Find $f\left(\frac{1}{\sqrt{3}}\right) and f'(1)$.

A. $2\tan^{-1}x$

B. $\frac{1}{2}\tan^{-1}x$

C. $\frac{\pi}{2}\tan^{-1}x$

D. $2\pi\tan^{-1}x$

Answer: A

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10. Let $f(x) = \sin x$ and $g(x) = \begin{cases} \max\{f(t), 0 \leq x \leq \pi\} & \text{for } 0 \leq x \leq \pi \\ \frac{1 - \cos x}{2}, & \text{for } x > \pi \end{cases}$

Then, g(x) is

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Exercise Single Option Correct Type Questions

1. If $f(x) = \begin{cases} \sin\frac{\pi x}{2}, & x < 1 \\ [x], & x \geq 1 \end{cases}$, where [x] denotes the greatest integer function, then

A. f(x) is continuous at x = 1

B. f(x) is discontinuous at x = 1

C. $f(1^{+}) = 0$

D. $f(1^{-}) = -1$

Answer: A

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2. Consider $f(x) = \begin{cases} \frac{8^x - 4^x - 2^x + 1}{x^2}, & x > 0 \\ e^x \sin x + \pi x + k \log 4, & x < 0 \end{cases}$, f(x) is continuous at x = 0, then k is

A. log 4

B. log 2

C. (log 4) (log 2)

D. None of these

Answer: C

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3. Let $f(x) = \begin{cases} \frac{a(1 - x\sin x) + b\cos x + 5}{x^2}, & x < 0 \\ 3, & x = 0 \\ \left[1 + \left(\frac{cx + dx^3}{x^2}\right)\right]^{1/x}, & x > 0 \end{cases}$ If f is continuous at x = 0,

then

$(a + b + c + d)$ is

A. (a)5

B. (b)-5

C. (c)log 3 - 5

D. (d)5 - log 3

Answer: C

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4. f(x)=$\begin{cases} \cos^{-1}(\cot x) & x < \frac{\pi}{2} \\ \pi[x] - 1 & x > \frac{\pi}{2} \end{cases}$ where $[.]$ represents the greatest function and $\{.\}$ represents the fractional part function. Find the jump of discontinuity.

A. 1

B. $\pi/2$

C. $\frac{\pi}{2}-1$

D. 2

Answer: C

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5. Let $f:[0,1]\to[0,1]$ be a continuous function. Then prove that $f(x)=x$ for at least one $0\leq x\leq 1.$

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6. If $f(x)=\frac{x+1}{x-1}$ and $g(x)=\frac{1}{x-2}$, then (fog)(x) is discontinuous at

A. (a) x = 3 only

B. (b) x = 2 only

C. (c) x = 2 and 3 only

D. (d) x = 1 only

Answer: C

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7. Let

$$y_n(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots\ldots \frac{x^2}{(1+x^2)^{n-1}} \text{ and } y(x) = \lim_{n \to \infty}$$

. Discuss the continuity of $y_n(x)(n = 1, 2, 3.... n)$ and $y(x)$at x $= 0$

A. continuous for $x \in R$

B. continuous for $x \in R - \{0\}$

C. continuous for $x \in R - \{1\}$

D. data unsufficient

Answer: B

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8. If $g(x) = \dfrac{1 - a^x + xa^x \log a}{x^2 \cdot a^x}, x < 0 \quad \dfrac{(2a)^x - x\log(2a) - 1}{x^2}, x > 0$

(where a > 0) then find a and $g(0)$ so that $g(x)$ is continuous at $x = 0$.

A. (a)$\dfrac{-1}{\sqrt{2}}$

B. (b)$\dfrac{1}{\sqrt{2}}$

C. (c)2

D. (d)-2

Answer: B

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9. Find $\dfrac{dy}{dx}$ if $y = \dfrac{\pi}{2} - \sin x$

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10. Find $\dfrac{dy}{dx}$ if $y = \sin 2x - x^3$

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11. Let $f(x) = \begin{cases} \frac{1}{|x|} & f \text{ or } |x| \geq 1 \dots . ax^2 + b & f \text{ or } |x| < 1 \end{cases}$. If $f(x)$ is continuous and differentiable at any point, then $a = \frac{1}{2}, \quad b = -\frac{3}{2}$ (b) $a = -\frac{1}{2}, \quad b = \frac{3}{2}$ (c) $a = 1, \quad b = -1$ (d) none of these

A. $\frac{-1}{2}, \frac{3}{2}$

B. $\frac{1}{2}, \frac{-3}{2}$

C. $\frac{1}{2}, \frac{3}{2}$

D. None of these

Answer: A

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12. If $f(x) = \begin{cases} A + Bx^2, & x < 1 \\ 3Ax - B + 2, & x \geq 1 \end{cases}$, then A and B, so that f(x) is differentiabl at x = 1, are

A. $-2, 3$

B. $2, \ -3$

C. 2, 3

D. $-2, \ -3$

Answer: C

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13. If $f(x) = \begin{cases} |x-1|([x]-x), & x \neq 1 \\ 0, & x = 1 \end{cases}$, then

A. A. $f'(1^+) = 0$

B. B. $f'(1^-) = 0$

C. C. $f'(1^-) = -1$

D. D. f(x) is differentiable at x = 1

Answer: A

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14. If $f(x) = \begin{cases} [\cos \pi x], & x \leq 1 \\ 2\{x\} - 1, & x > 1 \end{cases}$, where [.] and {.} denotes greatest integer and fractional part of x, then

a. $f'(1^-) = 2$

b. $f'(1^+) = 2$

c. $f'(1^-) = -2$

d. $f'(1^+) = 0$

A. $f'(1^-) = 2$

B. $f'(1^+) = 2$

C. $f'(1^-) = -2$

D. $f'(1^+) = 0$

Answer: B

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15. If $f(x) = \begin{cases} x-3, & x<0 \\ x^2-3x+2, & x \geq 0 \end{cases}$, then $g(x) = f(|x|)$ is

a. $g'(0^+) = -3$

b. $g'(0^-) = -3$

c. $g'(0^+) = g'(0^-)$

d. g(x) is not continuous at x = 0

A. $g'(0^+) = -3$

B. $g'(0^-) = -3$

C. $g'(0^+) = g'(0^-)$

D. g(x) is not continuous at x = 0

Answer: A

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16. Find $\frac{dy}{dx}$ if $y = x \sin x$

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17. Let f be differentiable function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y) \text{for all} x, y > 0$. If f'(1) = 1, then f(x) is

A. $2\log_e x$

B. $3\log_e x$

C. $\log_e x$

D. $\frac{1}{2}\log_e x$

Answer: C

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18. Let $f(x+y) = f(x) + f(y) - 2xy - 1$ for all x and y. If f'(0) exists and $f'(0) = -\sin\alpha$, then $f\{f'(0)\}$ is

A. -1

B. 0

C. 1

D. 2

Answer: C

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19. A derivable function $f: R^{+} \rightarrow R$ satisfies the condition $f(x) - f(y) \geq \log\left(\frac{x}{y}\right) + x - y, \forall x, y \in R^{+}$. If g denotes the derivative of f, then the value of the sum $\sum_{n=1}^{100} g\left(\frac{1}{n}\right)$ is

A. (a)5050

B. (b)5510

C. (c)5150

D. (d)1550

Answer: C

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20. If $\frac{d(f(x))}{dx} = e^{-x} f(x) + e^{x} f(-x)$, then f(x) is, (given f(0) = 0)

a. an even function

b. an odd function

c. neither even nor odd function

d. can't say

A. an even function

B. an odd function

C. neither even nor odd function

D. can't say

Answer: B

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21. Let $f:(0, \infty) \to R$ be a continuous function such that $f(x) = \int_0^x t f(t) dt$. If $f(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$, is equal to

A. 216

B. 219

C. 222

D. 225

Answer: B

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22. For let $h(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ and } 0 \end{cases}$ if x is irrational where $p \& q > 0$ are relatively prime integers 0 then which one does not hold good?

(a)h(x) is discontinuous for all x in $(0, \infty)$

(b)h(x) is continuous for each irrational in $(0, \infty)$

(c)h(x) is discontinuous for each rational in $(0, \infty)$

(d)h(x) is not derivable for all x in $(0, \infty)$

A. (a)h(x) is discontinuous for all x in $(0, \infty)$

B. (b)h(x) is continuous for each irrational in $(0, \infty)$

C. (c)h(x) is discontinuous for each rational in $(0, \infty)$

D. (d)h(x) is not derivable for all x in $(0, \infty)$

Answer: B

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23. Let $f(x) = \frac{g(x)}{h(x)}$, where g and h are continuous functions on the open interval (a, b). Which of the following statements is true for $a < x < b$?

A. (a)f is continuous at all x for which $x \neq 0$

B. (b)f is continuous at all x for which g(x) = 0

C. (c)f is continuous at all x for which $g(x) \neq 0$

D. (d)f is continuous at all x for which $h(x) \neq 0$

Answer: D

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24. Find $\frac{dy}{dx}$ if $y = 2x^7$

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25. if $f(x) = \frac{x - e^x + \cos 2x}{x^2}$, $x \neq 0$, is continuous at $x = 0$, then

A. $f(0) = \frac{5}{2}$

B. $[f(0)] = -2$

C. $\{f(0)\} = -0.5$

D. $[f(0)].\{f(0)\} = -1.5$

Answer: D

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26. Consider the function $f(x) = \begin{cases} x\{x\} + 1, & \text{if } 0 \le x < 1 \\ 2 - \{x\}, & \text{if } 1 \le x \le 2 \end{cases}$, where {x} denotes the fractional part function. Which one of the following statements is not correct ?

A. $\lim_{x \to 1} f(x)$ exists

B. $f(0) \ne f(2)$

C. f(x) is continuous in [0, 2]

D. Rolle's theorem is not applicable to f(x) in [0, 2]

Answer: C

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27. Let $f(x) = \begin{cases} \dfrac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}, & \text{if } x > 2 \\ \dfrac{x^2 - 4}{x - \sqrt{3x - 2}}, & \text{if } x < 2 \end{cases}$,then

A. (a)$f(2) = 8 \Rightarrow f$ is continuous at x = 2

B. (b)$f(2) = 16 \Rightarrow f$ is continuous at x = 2

C. (c)$f(2^{-}) \neq f(2^{+}) \Rightarrow f$ is discontinuous

D. (d)f has a removable discontinuity at x = 2

Answer: C

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28. Let [x] denote the integral part of $x \in R$ and $g(x) = x - [x]$. Let $f(x)$ be any continuous function with $f(0) = f(1)$ then the function $h(x) = f(g(x)$:

A. has finitely many discontinuities

B. is discontinuous at some x = c

C. is continuous on R

D. is a constant function

Answer: C

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29. Let f be a differentiable function on the open interval(a, b). Which of the following statements must be true?

(i) f is continuous on the closed interval [a,b],

(ii) f is bounded on the open interval (a,b)

(iii)If $a < a1 < b1 < b$, and $f(a1) < 0 < f(b1)$, then there is a number c such that $a1 < c < b$, and $f(c) = 0$

(a)Only I and II

(b)Only I and III

(c)Only II and III

(d)Only III

A. Only I and II

B. Only I and III

C. Only II and III

D. Only III

Answer: D

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30. Number of points where the function $f(x) = (x^2 - 1)|x^2 - x - 2| + \sin(|x|)$ is not differentiable, is:

A. 0

B. 1

C. 2

D. 3

Answer: C

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31. Consider function $f: R - \{-1, 1\} \to R.\ f(x) = \dfrac{x}{1 - |x|}$ Then the incorrect statement is

A. A. it is continuous at the origin

B. B. it is not derivable at the origin

C. C. the range of the function is R

D. D. f is continuous and derivable in its domain

Answer: B

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32. Find $\frac{dy}{dx}$ if $2y - e^x = 6$

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33. The total number of points of non-differentiability of $f(x) = \min\left[|\sin x|, |\cos x|, \frac{1}{4}\right] \text{in} (0, 2\pi)$ is

A. 8

B. 9

C. 10

D. 11

Answer: D

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34. The function $f(x) = [x]^2 - [x^2]$ is discontinuous at (where $[\gamma]$ is the greatest integer less than or equal to γ), is discontinuous at

a. all integers

b. all integers except 0 and 1

c. all integers except 0

d. all integers except 1

A. all integers

B. all integers except 0 and 1

C. all integers except 0

D. all integers except 1

Answer: D

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35. The function $f(x) = (x^2 - 1)\left|x^2 - 3x + 2\right| + \cos(|x|)$ is differentiable not differentiable at (a)-1 (b)0 (c)1 (d)2

A. -1

B. 0

C. 1

D. 2

Answer: D

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36. If $f(x) = \begin{cases} \frac{1}{e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

A. 0

B. 1

C. -1

D. desn't exist

Answer: A

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37. Given $f(x)=\frac{e^x-\cos 2x-x}{x^2}$, for x $\in R-\{0\}$

$$g(x)=\begin{cases} f(\{x\}), & \text{for } \ n<x<n+\frac{1}{2} \\ f(1-\{x\}), & \text{for } \ n+\frac{1}{2}\leq x<n+1, n\in I \\ & \begin{cases}\text{where } \{\text{x}\} \text{ denotes} \\ \text{fractional part function}\end{cases} \\ \frac{5}{2}\text{otherwise}, \end{cases}$$ then g(x) is

A. discontinuous at all integral values of x only

B. continuous everywhere except for x = 0

C. discontinuous at $x=n+\frac{1}{2}, n\in I$ and at some $x\in I$

D. continuous everywhere

Answer: D

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38. The function $g(x) = \begin{cases} x+b, & x<0 \\ \cos x, & x \geq 0 \end{cases}$ can be made differentiable at x = 0

A. (a) if b is equal to zero

B. (b) if b is not equal to zero

C. (c) if b takes any real value

D. (d) for no value of b

Answer: D

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39. The graph of function f contains the point $P(1, 2)$ and $Q(s, r)$. The equation of the secant line through P and Q is $y = \left(\frac{s^2+2s-3}{s-1}\right)x - 1 - s$. The value of $f'(1)$, is

A. (a)2

B. (b)3

C. (c)4

D. (d)non-existent

Answer: C

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40. Consider $f(x) = \left[\dfrac{2(\sin x - \sin^3 x) + |\sin x - \sin^3 x|}{2(\sin x - \sin^3 x) - |\sin x - \sin^3 x|}\right], x \neq \dfrac{\pi}{2}$ for $x \in (0, \pi), f\left(\dfrac{\pi}{2}\right) = 3$ where [ ] denotes the greatest integer function then,

A. f is continuous and differentiable at $x = \pi/2$

B. f is continuous but not differentiable at $x = \pi/2$

C. f is neither continuous nor differentiable at $x = \pi/2$

D. None of the above

Answer: A

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41. If $f(x+y)=f(x)+f(y)+|x|y+xy^2, \forall x, y \in R$ and $f'(0)=0$, then

A. f need not be differentiable at every non-zero x

B. f is differentiable for all $x \in R$

C. f is twice differentiable at x = 0

D. None of the above

Answer: B

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42. Let $f(x)=\max\left\{\left|x^2-2\right|x\right||, |x|\}$ and $g(x)=\min\left\{\left|x^2-2\right|x\right||, |x|\}$then

A. (a) both f(x) and g(x) are non-differentiable at 5 points

B. (b) $f(x)$ is not differentiable at 5 points whether $g(x)$ is non-differentiable at 7 points

C. (c) number of points of non-differentiability for $f(x)$ and $g(x)$ are 7 and 5 points, respectively

D. (d) both $f(x)$ and $g(x)$ are non-differentiable at 3 and 5 points, respectively

Answer: B

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43. If $x\,dy = y\,(dx + y\,dy)$, $y > 0$ and $y(1) = 1$,

then $y(-3)$ is equal to ________.

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44. Let f(x) be continuous and differentiable function for all reals and f(x + y) = f(x) - 3xy + ff(y). If $\lim_{h \to 0} \frac{f(h)}{h} = 7$, then the value of f'(x) is

A. $-3x$

B. 7

C. $-3x+7$

D. $2f(x)+7$

Answer: C

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45. Let [x] be the greatest integer function $f(x) = \left(\frac{\sin\left(\frac{1}{4}(\pi[x])\right)}{[x]}\right)$ is

(a)Not continuous at any point

(b)continuous at $x = \frac{3}{2}$

(c)discontinuous at x = 2

(d)differentiable at $x = \frac{4}{3}$

A. Not continuous at any point

B. Continuous at 3/2

C. Discontinuous at 2

D. Differentiable at 4/3

Answer: C

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46. If $f(x) = \begin{cases} b\left([x]^2 + [x]\right) + 1, & \text{for } x > -1 \\ \sin(\pi(x+a)), & \text{for } x < -1 \end{cases}$, where [x] denotes the integral part of x, then for what values of a, b, the function is continuous at x = - 1?

A. $a = 2n + (3/2), b \in R, n \in I$

B. $a = 4n + 2, b \in R, n \in I$

C. $a = 4n + (3/2), b \in R^+, n \in I$

D. $a = 4n + 1, b \in R^+, n \in I$

Answer: A

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47. If f(x) and g(x) are non-periodic functions, then h(x)=f(g(x)) is

A. is always differentiable at $x = x_0$

B. is never differentiable at $x = x_0$

C. is differentiable at $x = x_0$ when $f(x_0) \neq g(x_0)$

D. cannot be differentiable at $x = x_0$, if $f(x_0) = g(x_0)$

Answer: C

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48. Number of points of non-differentiability of the function $g(x) = [x^2]\{\cos^2 4x\} + \{x^2\}[\cos^2 4x] + x^2 \sin^2 4x + [x^2][\cos^2 4x] + \{x^2\}\{\cos^2 4x\}$ in $(-50, 50)$ where $[x]$ and $\{x\}$ denotes the greatest integer function and fractional part function of x respectively, is equal to :

a.98

b. 99

c. 100

d. 0

A. 98

B. 99

C. 100

D. 0

Answer: D

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49. Find $\frac{dy}{dx}$ if $y = x \tan x$

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50. If $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \forall x, y \in R, y \neq 0$ and $f'(x)$ exists for all x, $f(2) = 4$. Then, $f(5)$ is

A. 3

B. 5

C. 25

D. None of the above

Answer: C

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Exercise More Than One Correct Option Type Questions

1. Function whose jump (non-negative difference of LHL and RHL) of discontinuity is greater than or equal to one. Is/are

A. $f(x) = \begin{cases} \frac{e^{1/x}+1}{e^{1/x}-1}, & x < 0 \\ \frac{1-\cos x}{x}, & x > 0 \end{cases}$

B. $g(x) = \begin{cases} \dfrac{x^{1/3}-1}{x^{1/2}-1}, & x > 1 \\ \dfrac{\log x}{x-1}, & \frac{1}{2} < x < 1 \end{cases}$

C. $u(x) = \begin{cases} \dfrac{\sin^{-1} 2x}{\tan^{-1} 3x}, & x \in \left[0, \frac{1}{2}\right] \\ \dfrac{|\sin x|}{x}, & x < 0 \end{cases}$

D. $v(x) = \begin{cases} \log_3(x+2), & x > 2 \\ \log_{1/2}(x^2+5), & x < 2 \end{cases}$

Answer: A::C

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2. Indicate all correct alternatives: if $f(x) = \dfrac{x}{2} - 1$, then on the interval $[0, \pi]$:

A. (a) $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both continuous

B. (b) $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both discontinuous

C. (c) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous

D. (d) $\tan(f(x))$ is continuous but $\dfrac{1}{f(x)}$ is not continuous

Answer: C::D

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3. On the interval $I=[-2,2]$, the function

$$f(x)=\begin{cases}(x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x\neq 0\\ 0 & x=0\end{cases}$$

A. f(x) is continuous for all values of $x\in I$

B. f(x) is continuous for $x\in I-\{0\}$

C. f(x) assumes all intermediate values from f(-2) to f(2)

D. f(x) has a maximum value equal to 3/e

Answer: B::C::D

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4. Given $f(x)=\begin{cases}3-\left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right] & x>0\\ \{x^2\}\cos\left(e^{\frac{1}{x}}\right) & x<0\end{cases}$ (where {} and [] denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does not hold good?

A. $f\left(0^{0-}\right)=0$

B. $f\left(0^{+}\right)=0$

C. $f(0)=0 \Rightarrow$ Continuous at x = 0

D. Irremovable discontinuity at x = 0

Answer: A::B::C

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5. If $f(x)=\begin{cases} b\left([x]^2+[x]\right)+1, & \text{for} \quad x>-1 \\ \sin(\pi(x+a)), & \text{for} \quad x<-1 \end{cases}$, where [x] denotes the integral part of x, then for what values of a, b, the function is continuous at x = - 1?

A. $a=2n+\frac{3}{2}, b \in R, n \in I$

B. $a=4n+2, b \in R, n \in I$

C. $a=4n+\frac{3}{2}, b \in R^{+}, n \in I$

D. $a=4n+1, b \in R^{+}, n \in I$

Answer: A::C

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6. Find $\frac{dy}{dx}$ if $y = \frac{x}{\tan x}$

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7. If $f(x) = |x+1|(|x| + |x-1|)$, then at what point(s) is the function not differentiable over the interval $[-2, 2]$?

a. -1

b. 0

c. 1

d. 1/2

A. (a) -1

B. (b) 0

C. (c) 1

D. (d) $\frac{1}{2}$

Answer: A::B::C

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8. Let [x] be the greatest integer function $f(x) = \left(\frac{\sin\left(\frac{1}{4}(\pi[x])\right)}{[x]}\right)$ is

(a)Not continuous at any point

(b)continuous at $x = \frac{3}{2}$

(c)discontinuous at x = 2

(d)differentiable at $x = \frac{4}{3}$

A. (a)Not continuous at any point

B. (b)continuous at $x = \frac{3}{2}$

C. (c)discontinuous at x = 2

D. (d)differentiable at $x = \frac{4}{3}$

Answer: B::C::D

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9. If $f(x)=\begin{cases}(\sin^{-1}x)^2\cos\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$ then f(x) is

a. continuous nowhere in $-1\leq x\leq 1$

b. continuous everywhere in $-1\leq x\leq 1$

c. differentiable nowhere in $-1\leq x\leq 1$

d. differentiable everywhere in $-1\leq x\leq 1$

A. continuous nowhere in $-1\leq x\leq 1$

B. continuous everywhere in $-1\leq x\leq 1$

C. differentiable nowhere in $-1\leq x\leq 1$

D. differentiable everywhere in $-1\leq x\leq 1$

Answer: B::D

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10. $f(x) = \cos x$ and $H(x) = \begin{cases} \min\{f(t), 0 \le t < x\} & 0 \le x \le \frac{\pi}{2} \\ \frac{\pi}{2} - x & \frac{\pi}{2} < x \le 3 \end{cases}$

then

A. H(x) is continuous and derivable in [0, 3]

B. H(x) is continuous but not derivable at $x = \frac{\pi}{2}$

C. H(x) is neither continuous nor derivable at $x = \frac{\pi}{2}$

D. maximum value of H(x) in [0, 3] is 1

Answer: A::D

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11. If $f(x) = 3(2x+3)^{2/3} + 2x + 3$, then:

(a) $f(x)$ is continuous but not differentiable at $x = -\frac{3}{2}$

(b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is continuous at $x = 0$

(d) $f(x)$ is differentiable but not continuous at $x = -\frac{3}{2}$

A. (a) $f(x)$ is continuous but not differentiable at $x = -\frac{3}{2}$

B. (b) $f(x)$ is differentiable at $x = 0$

C. (c) $f(x)$ is continuous at $x = 0$

D. (d) $f(x)$ is differentiable but not continuous at $x = -\frac{3}{2}$

Answer: A::B::C

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12. If $f(x) = \{\left(-x = \frac{\pi}{2}, x \le -\frac{\pi}{2}\right), \left(-\cos x, -\frac{\pi}{2} < x, \le 0\right)$,(

x-1, 0 lt x le 1),("in"x, x gt1):}`

A. f(x) is continuous at $x = -\frac{\pi}{2}$

B. f(x) is not differentiable at x = 0

C. f(x) is differentiable at x = 1

D. All of the above

Answer: A::B::C::D

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13. if $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1 + x^2)} & x \neq 0 \\ 0 & x = 0 \end{cases}$

a. f is continuous at x = 0

b. f is continuous at x = 0 but not differentiable at x = 0

c. f is differentiable at x = 0

d. f is not continuous at x = 0

A. f is continuous at x = 0

B. f is continuous at x = 0 but not differentiable at x = 0

C. f is differentiable at x = 0

D. f is not continuous at x = 0

Answer: A::C

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14. Let [x] denote the greatest integer less that or equal to x. If $f(x) = [x \sin \pi x]$, then f(x) is

(a) Continuous at x = 0

(b) Continuous in (-1, 0)

(c) Differentiable at x = 1

(d) Differentiable in (-1, 1)

A. continuous at x = 0

B. continuous in (-1, 0)

C. differentiable at x = 1

D. differentiable in (-1, 1)

Answer: A::B::C

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15. The function $f(x) = x - [x]$, where [·] denotes the greatest integer function is

(a) continuous everywhere

(b) continuous at integer points only

(c) continuous at non-integer points only

(d) differentiable everywhere

A. is continuous for all positive integers

B. is discontinuous for all non-positive integers

C. has finite number of elements in its range

D. is such that its graph does not lie above the X-axis

Answer: A::B::C::D

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16. The function $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

a. has its domain $-1 \leq x \leq 1$

b. has finite one sided derivates at the point x = 0

c. is continuous and differentiable at x = 0

d. is continuous but not differentiable at x = 0

A. has its domain $-1 \leq x \leq 1$

B. has finite one sided derivates at the point x = 0

C. is continuous and differentiable at x = 0

D. is continuous but not differentiable at x = 0

Answer: A::B::D

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17. Consider the function $f(x) = |x^3 + 1|$. Then,

A. domain of f $x \in R$

B. range of f is R^+

C. f has no inverse

D. f is continuous and differentiable for every $x \in R$

Answer: A::B::C

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18. f is a continuous function in [a,b]; g is a continuous function in [b,c]. A function h(x) is defined as h(x)=f(x) for x∈[a,b) =g(x) for x∈(b,c]. If f(b)=g(b), then

A. h(x) has a removable discontinuity at x = b

B. h(x) may or may not be continuous in [a, c]

C. $h(b^-) = g(b^+) = f(b^-)$

D. $h(b^+) = g(b^+) = f(b^+)$

Answer: A::B

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19. Which of the following function(s) has/have the same range ?

A. A. $f(x) = \frac{1}{1+x}$

B. B. $f(x) = \frac{1}{1+x^2}$

C. C. $f(x) = \frac{1}{1+\sqrt{x}}$

D. D. $f(x) = \frac{1}{\sqrt{3-x}}$

Answer: B::C

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20. If f(x) = sec 2x + cosec 2x, then f(x) is discontinuous at all points in

A. A. $\{n\pi, n \in N\}$

B. B. $\left\{(2n \pm 1)\frac{\pi}{4}, n \in I\right\}$

C. C. $\left\{\frac{n\pi}{4}, n \in I\right\}$

D. D. $\left\{(2n \pm 1)\frac{\pi}{8}, n \in I\right\}$

Answer: A::B::C

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21. Show that $f(x) = \begin{cases} \text{x} \sin\frac{1}{x}, & \text{when} \quad x \neq 0 \\ 0, & \text{when} \quad x = 0 \end{cases}$ is continuous but not differentiable at x = 0

A. $\lim_{x \to 0} f(x)$ exists for every $n > 1$

B. f is continuous at x = 0 for $n > 1$

C. f is differentiable at x= 0 for every $n > 1$

D. None of the above

Answer: A::B::C

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22. A function is defined as $f(x) = \begin{cases} e^x, & x \leq 0 \\ |x-1|, & x > 0 \end{cases}$, then f(x) is

A. A. continuous at x = 0

B. B. continuous at x = 1

C. C. differentiable at x = 0

D. D. differentiable at x = 1

Answer: A::B

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23. Let $f(x) = \int_{-2}^{x} |t+1| dt$, then

a. f(x) is continuous in [-1, 1]

b. f(x) is differentiable in [-1, 1]

c. f'(x) is continuous in [-1, 1]

d. f'(x) is differentiable in [-1, 1]

A. f(x) is continuous in [-1, 1]

B. f(x) is differentiable in [-1, 1]

C. f'(x) is continuous in [-1, 1]

D. f'(x) is differentiable in [-1, 1]

Answer: A::B::C::D

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24. A function f(x) satisfies the relation $f(x+y) = f(x) + f(y) + xy(x+y), \forall x, y \in R$. If f'(0) = - 1, then

A. f(x) is a polynomial function

B. f(x) is an exponential function

C. f(x) is twice differentiable for all $x \in R$

D. f'(3) = 8

Answer: A::C::D

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25. Show that the function

$f(x) = \begin{cases} 3x^2 + 12x - 1 & -1 \le x \le 2 \\ 37 - x & 2 < x \le 3 \end{cases}$ is continuous at x = 2

A. f(x) is increasing on [-1, 2]

B. f(x) is continuous on [-1, 3]

C. f'(2) doesn't exist

D. f(x) has the maximum value at x = 2

Answer: A::B::D

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26. If f(x) = 0 for $x < 0$ and $f(x)$ is differentiable at x = 0, then for $x > 0, f(x)$ may be

A. A. x^2

B. B. x

C. C. $-x$

D. D. $-x^{3/2}$

Answer: A::D

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Exercise Statement I And Ii Type Questions

1. Statement I $f(x) = \sin x + [x]$ is discontinuous at x = 0.

Statement II If g(x) is continuous and f(x) is discontinuous, then g(x) + f(x) will necessarily be discontinuous at x = a.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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2. Consider $f(x) = \begin{cases} 2\sin\left(a\cos^{-1}x\right), & \text{if } \quad x \in (0, 1) \\ \sqrt{3}, & \text{if } \quad x = 0 \\ ax + b, & \text{if } \quad x < 0 \end{cases}$

Statement I If b = $\sqrt{3}$ and $a = \frac{2}{3}$, then f(x) is continuous in $(-\infty, 1)$.

Statement II If a function is defined on an interval I and limit exists at every point of interval I, then function is continuous in I.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: C

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3. Let $f(x) = \begin{cases} \frac{\cos x - e^{x^2/2}}{x^3}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

Statement I f(x) is continuous at x = 0.

Statement II $\lim_{x \to 0} \frac{\cos x - e^{x^2/2}}{x^3} = -\frac{1}{12}$

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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4. Statement I The equation $\frac{x^3}{4} - \sin \pi x + \frac{2}{3} = 0$ has atleast one solution in [-2, 2].

Statement II Let $f:[a, b] \rightarrow R$ be a function and c be a number such that $f(a) < c < f(b)$, then there is atleast one number $n \in (a, b)$ such that f(n) = c.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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5. Statement I Range of $f(x) = x\left(\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\right) + x^2 + x^4$ is not R.

Statement II Range of a continuous evern function cannot be R.

A. (a)Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. (b)Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. (c)Statement I is correct, Statement II is incorrect

D. (d)Statement I is incorrect, Statement II is correct.

Answer: A

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6. Let $f(x) = \begin{cases} Ax - B & x \leq 1 \\ 2x^2 + 3Ax + B & x \in (-1, 1] \\ 4 & x > 1 \end{cases}$

Statement I f(x) is continuous at all x if $A = \frac{3}{4}, B = -\frac{1}{4}$. Because

Statement II Polynomial function is always continuous.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: B

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7. If $y = 3x^4 + 5$ then $\frac{dy}{dx}$

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8. Statement I f(x) = |x| sin x is differentiable at x = 0.

Statement II If g(x) is not differentiable at x = a and h(x) is differentiable at x = a, then g(x).h(x) cannot be differentiable at x = a

A. A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. C. Statement I is correct, Statement II is incorrect

D. D. Statement I is incorrect, Statement II is correct.

Answer: C

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9. If $y = 2x^6 + \sin 3x$ then $\frac{dy}{dx}$

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10. Let f(x) = $x - x^2$ and $g(x) = \{x\}, \forall x \in R$ where denotes fractional part function.

Statement I f(g(x)) will be continuous, $\forall x \in R$.

Statement II $f(0) = f(1)$ and $g(x)$ is periodic with period 1.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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11. Find $\frac{dy}{dx}$ if $y = ax^2$

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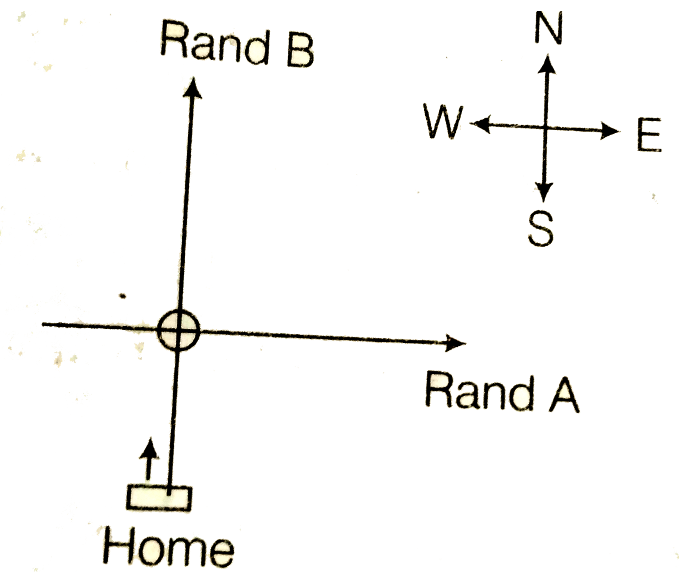
Exercise 4

1. A man leaves his home early in the morning to have a walk. He arrives at a junction of roads A and B as shown in figure. He takes the following steps in later journies :



(i) 1 km in North direction.

(ii) Changes direction and moves in North-East direction for $2\sqrt{2}$ km.

(iii) Changes direction and moves Southwards for distance of 2 km.

(iv) Finally he changes the direction and moves in South-East direction to reach road A again.

Now if roads A and B are taken as X-axis and y-axis ,then visible point representing the graph of y=f(x)

Then the value of x at which function f(x) is discontinous

A. (a) 2

B. (b) 3

C. (c) 19

D. (d) None of these

Answer: A

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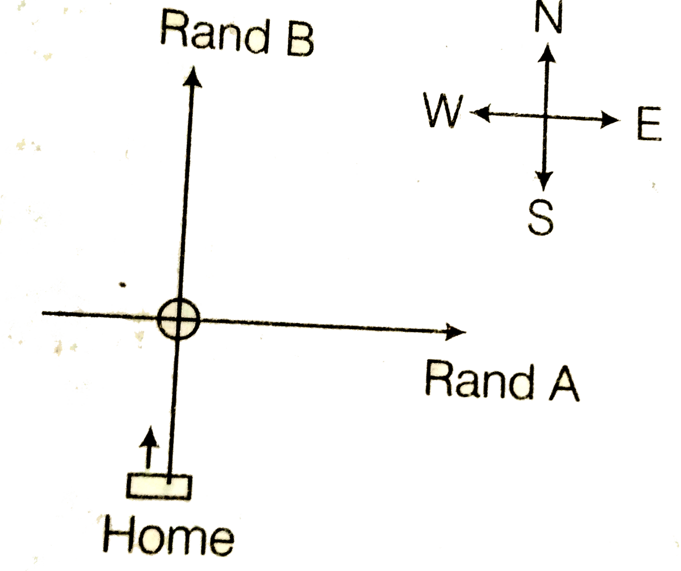
2. A man leaves his home early in the morning to have a walk. He arrives at a junction of roads A and B as shown in figure. He takes the following steps in later journies :



(i) 1 km in North direction.

(ii) Changes direction and moves in North-East direction for $2\sqrt{2}$ km.

(iii) Changes direction and moves Southwards for distance of 2 km.

(iv) Finally he changes the direction and moves in South-East direction to reach road A again.

Now if roads A and B are taken as X-asix and Y-axis, then visible point representing the graph of y = f(x).

Then the value of x at which function f(x) is discontionous

A. 0

B. 2

C. 3

D. 19

Answer: B::C

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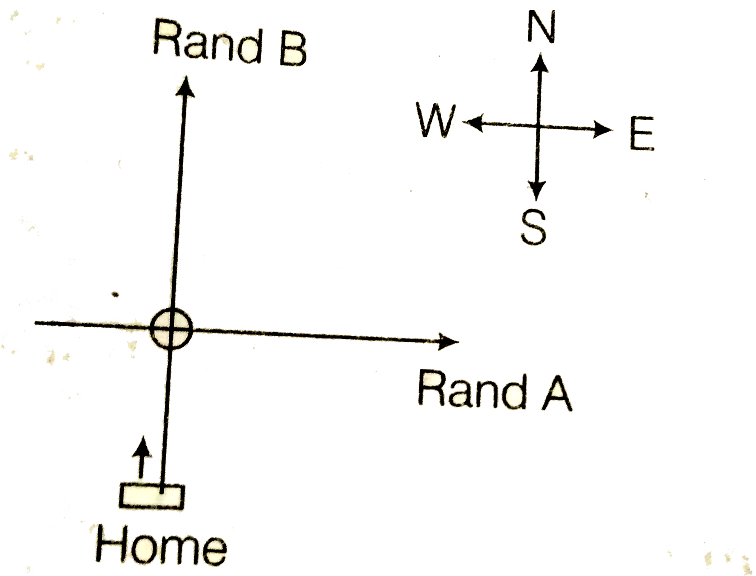
3. A man leaves his home early in the morning to have a walk. He arrives at a junction of roads A and B as shown in figure. He takes the following steps in later journies :



(i) 1 km in North direction.

(ii) Changes direction and moves in North-East direction for $2\sqrt{2}$ km.

(iii) Changes direction and moves Southwards for distance of 2 km.

(iv) Finally he changes the direction and moves in South-East direction to reach road A again.

Now if roads A and B are taken as X-asix and Y-axis, then visible point representing the graph of y = f(x).

Then the value of x at which function f(x) is discontionous

A. 1

B. 2

C. 19

D. None of these

Answer: A

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Exercise Passage Based Questions

1. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\, f(h)$

A standard result is $\frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

The range fo the function $\Delta = f(|x|)$ is

A. R

B. $R - \{0\}$

C. R^{+}

D. (0, e)

Answer: C

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2. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\,f(h)$

A standard result is $\frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

Range of f(x) is

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3. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\,f(h)$

A standard result is $\frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

The range fo the function $\Delta = f(|x|)$ is

A. odd

B. even

C. increasing

D. decreasing

Answer: D

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4. Find $\frac{dy}{dx}$ if $y = \frac{\cos x}{x}$

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5. Let y = f(x) be defined in [a, b], then

(i) Test of continuity at $x = c, a < c < b$

(ii) Test of continuity at x = a

(iii) Test of continuity at x = b

Case I Test of continuity at $x = c, a < c < b$

If y = f(x) be defined at x = c and its value f(c) be equal to limit of f(x) as $x \rightarrow c$ i.e. f(c) = $\lim_{x \rightarrow c} f(x)$

or $\lim_{x \rightarrow c^-} f(x) = f(c) = \lim_{x \rightarrow c^+} f(x)$

or LHL = f(c) = RHL

then, y = f(x) is continuous at x = c.

Case II Test of continuity at x = a

If RHL = f(a)

Then, f(x) is said to be continuous at the end point x = a

Case III Test of continuity at x = b, if LHL = f(b)

Then, f(x) is continuous at right end x = b.

Max ([x],|x|) is discontinuous at

A. $\frac{\pi}{2}, \frac{3\pi}{2}, 2\pi, 3\pi$

B. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 3\pi$

C. $\frac{\pi}{2}, 2\pi$

D. None of these

Answer: A

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6. Let y = f(x) be defined in [a, b], then

(i) Test of continuity at $x = c, a < c < b$

(ii) Test of continuity at x = a

(iii) Test of continuity at x = b

Case I Test of continuity at $x = c, a < c < b$

If y = f(x) be defined at x = c and its value f(c) be equal to limit of f(x) as

$x \rightarrow c$ i.e. f(c) = $\lim_{x \rightarrow c} f(x)$

or $\lim_{x \rightarrow c^-} f(x) = f(c) = \lim_{x \rightarrow c^+} f(x)$

or LHL = f(c) = RHL

then, y = f(x) is continuous at x = c.

Case II Test of continuity at x = a

If RHL = f(a)

Then, f(x) is said to be continuous at the end point x = a

Case III Test of continuity at x = b, if LHL = f(b)

Then, f(x) is continuous at right end x = b.

Number of points of discontinuity of $\left[2x^3 - 5\right]$ in [1, 2) is (where [.] denotes the greatest integral function.)

A. 14

B. 13

C. 10

D. None of these

Answer: B

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7. Let y = f(x) be defined in [a, b], then

(i) Test of continuity at $x = c, a < c < b$

(ii) Test of continuity at x = a

(iii) Test of continuity at x = b

Case I Test of continuity at $x = c, a < c < b$

If y = f(x) be defined at x = c and its value f(c) be equal to limit of f(x) as $x \to c$ i.e. f(c) = $\lim_{x \to c} f(x)$

or $\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x)$

or LHL = f(c) = RHL

then, y = f(x) is continuous at x = c.

Case II Test of continuity at x = a

If RHL = f(a)

Then, f(x) is said to be continuous at the end point x = a

Case III Test of continuity at x = b, if LHL = f(b)

Then, f(x) is continuous at right end x = b.

Max ([x],|x|) is discontinuous at

A. x = 0

B. ϕ

C. $x = n, n \in I$

D. None of these

Answer: B

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8. Find $\frac{dy}{dx}$ if $x = \cos y$

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9.

$\left(f(x) = \cos x \text{ and } H_1(x) = \min\{f(t), 0 \leq t < x\},\right), \left(0 \leq x \leq \frac{\pi}{2} = \frac{\pi}{2}\right.$

$\left(0 \leq x \leq \frac{\pi}{2} = \frac{\pi}{2} - x, \frac{\pi}{2} < x \leq \pi\right), \left(g(x) = \sin x \text{ and } H_3(x) = \min\{\right.$

$\left(g(x) = \sin x \text{ and } H_4(x) = \max\{g(t), 0 \leq t \leq x\},\right), \left(0 \leq x \leq \frac{\pi}{2} = \frac{\pi}{2}\right.$

Which of the following is true for $H_1(x)$?

A. Continuous and derivable in $[0, \pi]$

B. Continuous but not derivable at $x = \frac{\pi}{2}$

C. Neither continuous nor derivable at $x = \frac{\pi}{2}$

D. None of the above

Answer: B

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10.

$\left(f(x) = \cos x \text{ and } H_1(x) = \min\{f(t), 0 \le t < x\},\right), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

$\left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2} - x, \frac{\pi}{2} < x \le \pi\right), \left(g(x) = \sin x \text{ and } H_3(x) = \min\{\right.$

$\left(g(x) = \sin x \text{ and } H_4(x) = \max\{g(t), 0 \le t \le x\},\right), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

Which of the following is true for $H_1(x)$?

A. Continuous and derivable in $[0, \pi]$

B. Continuous but not derivable at $x = \frac{\pi}{2}$

C. Neither continuous nor derivable at $x = \frac{\pi}{2}$

D. None of the above

Answer: C

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11. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x + y^{2n+1}\right) = f(x) + (f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f(1), is

A. (a)0

B. (b)1

C. (c)2

D. (d)Not defined

Answer: B

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12. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x+y^{2n+1}\right)=f(x)+(f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f(x), is

A. $2x$

B. $x^2 + x + 1$

C. x

D. None of these

Answer: C

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13. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x + y^{2n+1}\right) = f(x) + \left(f(y)\right)^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f'(10), is

A. 10

B. 0

C. 2n + 1

D. 1

Answer:

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14. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x+y^{2n+1}\right)=f(x)+(f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f(x), is

A. odd

B. even

C. neither even nor odd

D. both even as well as odd

Answer: A

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15. Find $\frac{dy}{dx}$ if $x = y\sin x$

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16. Find $\frac{dy}{dx}$ if $y = x^4 - x^7$

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Exercise 5

1. Find $\frac{dy}{dx}$ if $y = 2x - 3$

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2. Find $\frac{dy}{dx}$ if $x - ay = bx^2$

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3. Find $\frac{dy}{dx}$ if $10x - 4y = \sin y$

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Exercise Single Integer Answer Type Questions

1. Number of points of discontinuity of $f(x) = \tan^2 x - \sec^2 x$ in $(0, 2\pi)$ is

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2. Number of points of discontinuity of the function $f(x) = \left[x^{\frac{1}{x}}\right], x > 0,$ where [.] represents GIF is

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3. Let $f(x) = x + \cos x + 2$ and $g(x)$ be the inverse function of f(x), then g'(3) equals to ....

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4. Let $f(x) = x \tan^{-1}(x^2)$ then find the $f'(x)$

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5. Let $f_1(x)$ and $f_2(x)$ be twice differentiable functions where $F(x) = f_1(x) + f_2(x)$ and $G(x) = f_1(x) - f_2(x)$, $\forall x \in R$, $f_1(0) = 2$ an

then the number of solutions of the equation $(F(x))^2 = \frac{9x^4}{G(x)}$ is...... .

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6. Suppose, the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative 7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at $x = 1$, has the value $10 + \lambda$, then the value of λ is equal to........

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7. If $y = \sin 7x + \cos 5x + e^x$ then $\frac{dy}{dx}$

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8. Let $f(x) = x^3 - x^2 + x + 1$ and

$g(x) = \begin{cases} \max f(t), & 0 \leq t \leq x \quad \text{for} \quad 0 \leq x \leq 1 \\ 3 - x, & 1 < x \leq 2 \end{cases}$ Then, g(x) in [0, 2] is

a. continuous for $x \in [0, 2] - \{1\}$

b. continuous for $x \in [0, 2]$

c. differentiable for all $x \in [0, 2]$

d. differentiable for all $x \in [0, 2] - \{1\}$

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9. If $f(x) = \begin{cases} \dfrac{\frac{\pi}{2} - \sin^{-1}\left(1 - \{x\}^2\right)\sin^{-1}(1 - \{x\})}{\sqrt{2}\left(\{x\} - \{x\}^3\right)}, & x > 0 \\ k, & x = 0 \\ \dfrac{A\sin^{-1}(1 - \{x\})\cos^{-1}(1 - \{x\})}{\sqrt{2\{x\}}(1 - \{x\})}, & x < 0 \end{cases}$ is continuous at x = 0, then the value of $\sin^2 k + \cos^2\left(\dfrac{A\pi}{\sqrt{2}}\right)$, is..... (where {.} denotes fractional part of x).

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Exercise 6

1. In a ΔABC, angles A, B, C are in AP. If $f(x) = \lim_{A \to c} \dfrac{\sqrt{3 - 4\sin A \sin C}}{|A - C|}$, then f(x) is equal to ....

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2. The number of points at which the function $f(x) = (x - |x|)^2 (1 - x + |x|)^2$ is not differentiable in the interval $(-3, 4)$ is ___

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Exercise Subjective Type Questions

1. Check continuity and differentiabilty of $f(x) = [x] + |1 - x|$ where [ ] denotes the greatest integer function

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2. If $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1)[x] & 2 \leq x < 3 \end{cases}$ where [.] denotes the greatest integer function, then continutity and diffrentiability of f(x)

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3. Let f be a twice differentiable function such that $f''(x) = -f(x), and f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2.$ Find $h(10)$ if $h(5) = 11$

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4. A function $f: R \to R$ satisfies the equation $f(x+y) = f(x).f(y)$ for all $xy \in R, f(x) \neq 0.$ Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$, then prove that $f' = 2f(x).$

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5. A function $f: R \to R$ satisfies the relation $f\left(\frac{x+y}{3}\right) = \frac{1}{3}|f(x) + f(y) + f(0)|$ for all $x, y \in R.$ If $f'(0)$ exists, prove that $f'(x)$ exists for all $x, \in R.$

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6. Let $f(x+y) = f(x) + f(y) + 2xy - 1$ for all real $x and y$ and $f(x)$ be a differentiable function. If $f'(0) = \cos\alpha$, the prove that $f(x) > 0 \forall x \in R$.

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Exercise 7

1. Examine the continuity or discontinuity of the following :

(i) $f(x) = [x] + [-x]$

(ii) $g(x) = \lim_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$

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Exercise Questions Asked In Previous 13 Years Exam

1. For every pair of continuous function $f, g : [0, 1] \to R$ such that max $\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$. The correct statement(s) is (are)

A. $[f(c)]^2 + 3f(c) = [g(c)]^2 + 3g(c)$for some c $\in [0, 1]1$

B. $[f(c)]^2 + f(c) = [g(c)]^2 + 3g(c)$for some c $\in [0, 1]$

C. $[f(c)]^2 + 3f(c) = [g(c)]^2 + g(c)$for some c $\in [0, 1]$

D. $[f(c)]^2 = [g(c)]^2$for some c $\in [0, 1]$

Answer: A::D

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2. Let f:$R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1)$. Define $h: R \to R$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \le 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

then number of point at which h(x) is not differentiable is

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3. Let $f(x) = \left\{x^2 \left|(\cos)\frac{\pi}{x}\right|, x \neq 0 \text{ and } 0, x = 0, x \in \mathbb{R},\right.$ then f is

a. differentiable both at x = 0 and at x = 2

b. differentiable at x = 0 but not differentiable at x = 2

c. not differentiable at x = 0 but differentiable at x = 2

d. differentiable neither at x = 0 nor at x = 2

A. differentiable both at x = 0 and at x = 2

B. differentiable at x = 0 but not differentiable at x = 2

C. not differentiable at x = 0 but differentiable at x = 2

D. differentiable neither at x = 0 nor at x = 2

Answer: B

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4. Q. For every integer n, let a_n and b_n be real numbers. Let function $f: R \to R$ be given by a $f(x) = \begin{cases} a_n + \sin \pi x & f \text{ or } x \in [2n, 2n+1] \\ b_n + \cos \pi x & f \text{ or } x \in (2n+1, 2n) \end{cases}$

for all integers n.

A. $a_{n-1} - b_{n-1} = 0$

B. $a_n - b_n = 1$

C. $a_n - b_{n+1} = 1$

D. $a_{n-1} - b_n = -1$

Answer: D

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5. Let $f: R \rightarrow R$ be a function such that $f(x+y) = f(x) + f(y), \forall x, y \in R.$

a. f(x) is differentiable only in a finite interval containing zero

b. f(x) is continuous for all $x \in R$

c. f'(x) is constant for all $x \in R$

d. f(x) is differentiable except at finitely many points

A. f(x) is differentiable only in a finite interval containing zero

B. f(x) is continuous for all $x \in R$

C. f'(x) is constant for all $x \in R$

D. f(x) is differentiable except at finitely many points

Answer: B::C

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6. If $f(x) = \left\{\left(- x = \frac{\pi}{2}, x \leq -\frac{\pi}{2}\right), \left(- \cos x, -\frac{\pi}{2} < x, \leq 0\right)\right.$,(x-1, 0 lt x le 1),("in"x, x gt1):}`

A. f(x) is continuous at $x = -\frac{\pi}{2}$

B. f(x) is not differentiable at x = 0

C. f(x) is differentiable x = 1

D. f(x) is differentiable at $x = -\frac{3}{2}$

Answer: D

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7. For the function $f(x) = x\cos\frac{1}{x}, x \geq 1$which one of the following is incorrect ?

A. (a)for atleast one x in the interval $[1, \infty), f(x+2) - f(x) < 2$

B. (b) $\lim_{x \to \infty} f'(x) = 1$

C. (c)for all x in the interval $[1, \infty), f(x+2) - f(x) > 2$

D. (d)f'(x) is strictly decreasing in the interval $[1, \infty)$

Answer: C

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8. If $\lim_{x \to \infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$, then

A. n = 1, m = 1

B. n = 1, m = - 1

C. n = 2, m = 2

D. $n > 2, m = n$

Answer: C

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9. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is constinous, $g(0) = 0$, $g'(0) = 0, g''(0) = 0$ and $f(x) = g(x)\sin x$.

Statement I $\lim_{x \to 0} (g(x)\cot x - g(0)\cos ecx) = f''(0)$

Statement II $f'(0) = g'(0)$

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: B

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10. If f(x)=x-[x], where [x] is the greatest integer less than or equal to x, then $f\left(+\frac{1}{2}\right)$ is:

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11. Check the differentiability if $f(x) = \min . \{1, x^2, x^3\}$.

A. f(x) is continuous everywhere

B. f(x) is continuous and differentiable everywhere

C. f(x) is not differentiable at two points

D. f(x) is not differentiable at one point

Answer: A::D

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12. Fill in the blank, in the statement given below.

Let $f(x) = x \mid x$ The set of points where f(x) is twice differentiable is

....

A. $0, \ \pm 1$

B. ± 1

C. 0

D. 1

Answer: A

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13. If is a differentiable function satisfying $f\left(\frac{1}{n}\right) = 0, \ \forall n \geq 1, n \in I,$

then

A. (a)$f(x) = 0, x \in (0, 1]$

B. (b)$f'(0) = 0 = f(0)$

C. (c)f(0) = 0 but f'(0) not necessarily zero

D. (d)$|f(x)| \leq 1, x \in (0, 1]$

Answer: B

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14. The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x & \text{if } |x| \leq 1 \\ \frac{1}{2}(|x| - 1) & \text{if } |x| > 1 \end{cases}$

A. (a)$R - \{0\}$

B. (b)$R - \{1\}$

C. (c)$R - \{-1\}$

D. (d)$R - \{-1, 1\}$

Answer: D

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15. The left hand derivative of $f(x) = [x]\sin(\pi x)$at $\mathrm{x} = k$ is an integer, is

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1}k\pi$

Answer: A

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16. Which of the following functions is differentiable at x = 0 ?

A. (a) $\cos(|x|) + |x|$

B. (b) $\cos(|x|) - |x|$

C. (c) $\sin(|x|) + |x|$

D. (d) $\sin(|x|) - |x|$

Answer: D

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17. For $x \in R$, $f(x) = |\log_e 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at x = 0

B. g'(0) = cos (log 2)

C. g'(0) = - cos (log 2)

D. g is differentiable at x = 0 and g'(0) = - sin(log 2)

Answer: B

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18. If the function $g(X) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx + 2 & 3 < x \leq 5 \end{cases}$ is differentiable , then the value of K+ m is

A. 2

B. $\frac{16}{5}$

C. $\frac{10}{3}$

D. 4

Answer: A

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19. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in [0, 1]$

(1) $2f'(c) = g'(c)$

(2) $2f'(c) = 3g'(c)$

(3) $f'(c) = g'(c)$

(4) $f'(c) = 2g'(c)$

A. $2f'(c) = g'(c)$

B. $2f'(c) = 3g'(c)$

C. $f'(c) = g'(c)$

D. $f'(c) = 2g'(c)$

Answer: D

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20. The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$ where [ ] denotes the greatest integer function, is discontinuous

A. continuous for every real x

B. discontinuous only at x = 0

C. discontinuous only at non-zero integral values of x

D. continuous only at x = 0

Answer: D

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