MATHS

BOOKS - ARIHANT MATHS

DEFINITE INTEGRAL

Example

1. Evaluate $\displaystyle\int_0^1 \frac{1}{3+4x}dx$

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2. Find the value of $\displaystyle\int_{-1}^1 \frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right)dx$

A. $\pi/2$

B. $\pi/4$

C. $-\pi/2$

D. None of these

Answer: C

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3. If $I_n = \int (\text{In } x)^n dx,$ then $I_n + nI_{n-1} =$

A. (A) $x(\log x)^{n-1}$

B. (B) $x(\log x)^n$

C. (C) $nx(\log x)^n$

D. (D) None of these

Answer: B

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4. All the values of a for which $\int_1^2 \left[a^2 + (4 - 4a)x + 4x^3\right]dx \leq 12$ are

given by (A) $a = 3$ (B) $a \leq 4$ (C) $0 \leq a < 3$ (D) none of these

A. $a = 3$

B. $a \leq 4$

C. $0 \leq a < 3$

D. None of these

Answer: A

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5. Evaluate $\int_0^3 |(x - 1)(x - 2)|dx.$

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6. Evaluate: $\int_0^{\pi/2} \dfrac{1}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2}dx$

7. Evaluate $\displaystyle\int_{-2}^{2} \frac{dx}{4 + x^2}$ directly as well as by

8. Evaluate: $\displaystyle\int_{0}^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$

9. For any $n > 1$, evaluate the integral

$$\int_{0}^{\infty} \frac{1}{\left(x + \sqrt{x^2 + 1}\right)^n} dx$$

10. Evaluate: $\displaystyle\int_{0}^{e-1} \frac{x^2 + 2x - 1}{2 \cdot x + 1} dx + \int_{1}^{e} x \log x e^{\frac{x^2 - 2}{2}} dx$

A. $\left(\sqrt{e}\right)^{\left(e^2+1\right)}$

B. $\left(\sqrt{e}\right)^{e^2-1}$

C. 0

D. $\left(\sqrt{e}\right)^{e^2-2}$

Answer: D

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11. Let $f(x) = \int_2^x \dfrac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then, the value of $g'(0)$ is

A. 1

B. 17

C. $\sqrt{17}$

D. None of these

Answer: C

12. Let $a_n = \int_0^{\pi/2} (1 - \sin t)^n \sin 2t,$

then $\lim\limits_{n \to \infty} \sum\limits_{n=1}^{n} \dfrac{a_n}{n}$ is equal to

A. $\dfrac{1}{2}$

B. 1

C. $\dfrac{4}{3}$

D. $\dfrac{3}{2}$

Answer: A

13. The value of $x > 1$ satisfying the equation

$$\int_1^x t \ln t \, dt = \dfrac{1}{4} \text{ is}$$

A. $\sqrt{e}$

B. e

C. e^2

D. $e - 1$

Answer: A

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14. Prove that the function f(x) = 6x - 9 is continuous at x = 3 .

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15. If the value of definite integral $\int_1^a x \cdot a^{-[\log_a x]}\,dx$, where $a > 1$ and

[x] denotes the greatest integer, is, $\dfrac{e - 1}{2}$ then the value of a equal to

A. $\sqrt{e}$

B. e

C. $\sqrt{e + 1}$

D. $e - 1$

Answer: A

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16. Show that (i) $\displaystyle\int_0^{\pi/2} f(\sin x)dx = \int_0^{\pi/2} f(\cos x)dx$

(ii) $\displaystyle\int_0^{\pi/2} f(\tan x)dx = \int_0^{\pi/2} f(\cot x)dx$

(iii) $\displaystyle\int_0^{\pi/2} f(\sin 2x)\sin x\, dx = \int_o^{\pi/2} f(\sin 2x).\cos x\, dx$

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17. If f and g are continuous functions on $[0, a]$ satisfying $f(x) = (a - x)$ and $g(x) + g(a - x) = 2,$ then show that $\displaystyle\int_0^a f(x)g(x)dx = \int_0^a f(x)dx.$

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18. Evaluate

(i) $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$

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19. The value of $\displaystyle\int_0^a \log(\cot a + \tan x)dx$,

where $a \in (0, \pi/2)$ is equal to

A. $a\log(\sin a)$

B. $-a\log(\sin a)$

C. $-a\log(\cos a)$

D. None of these

Answer: B

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20. Evaluate $\displaystyle\int_{\pi/6}^{\pi/3} \left(\frac{1}{1 + \sqrt{\tan x}} \right) dx$

21. Prove that: $\displaystyle\int_{a}^{b} \frac{f(x)}{f(x) + f(a + b - x)} dx = \frac{b - a}{2}$

22. Solve $\displaystyle l = \int_{\cos^4 t}^{-\sin^4 t} \frac{\sqrt{f(z)}dz}{\sqrt{f(\cos 2t - z)} + \sqrt{f(z)}}$

23. Let the function f satisfies f(x).f ' (−x)=f(−x).f ' (x) for all x and f(0)=3 The value of $f(x). f(-x)$ for all x is

A. 4

B. 9

C. 12

D. 16

Answer: B

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24. If f(x) f '(-x) = f(-x) f'(x) and f(0)=9 Then

$$\int_{-51}^{51} \frac{dx}{3 + f(x)}$$ has the value equal to

A. (a) 17

B. (b) 34

C. (c) 102

D. (d) 0

Answer: A

25. Let $f(x) = |x|$ Then number of solutions of $f(x) = 0$ in$[-2, 2]$ is

A. 0

B. 1

C. 2

D. 4

Answer: A

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26. Given fuction, $\begin{cases} x^2 & \text{for} 0 \le x < 1 \\ \sqrt{x} & \text{for} 1 \le x \le 2 \end{cases}$

Evaluate $\displaystyle\int_0^2 f(x)dx$.

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27. Evaluate the integral $l = \int_0^2 |1 - x| dx$.

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28. Evaluate (i) $\int_0^\pi |\cos x| dx$ (ii) $\int_0^2 |x^2 + 2x - 3| dx$

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29. Evaluate $\int_{-1}^1 (x - [x]) dx$, where $[.]$

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30. Evaluate $\int_0^2 \{x\} dx$, where $\{x\}$ denotes the fractional part of x.

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31. $\int_0^9 \{\sqrt{x}\}dx$, where $\{x\}$ denotes the fractional part of x, is 5 (b) 6 (c) 4 (d) 3

32. If for a real number y, $[y]$ is the greatest integral function less, then or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x]dx$ is $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

33. The value of $\int_0^{100} [\tan^{-1}x]dx$ is equal to (where $[.]$ denotes the greatest integer function)

A. $\tan 1 - 100$

B. $\pi/2 - \tan 1$

C. $100 - \tan 1$

D. None of these

Answer: C

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34. Expand $\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}$

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35. The value of $\int_1^2 \left(x^{[x^2]} + [x^2]^x \right) dx$ is equal to where $[.]$ denotes the greatest integer function

A. $\frac{5}{4}\sqrt{3} + \left(2^{\sqrt{3}} - 2^{\sqrt{2}} \right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}} \right)$

B. $\frac{5}{4} + \sqrt{3} + \frac{\sqrt{2}}{3} + \frac{1}{\log 2}\left(2^{\sqrt{3}} - 2^{\sqrt{2}} \right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}} \right)$

C. $\frac{5}{4} + \frac{\sqrt{2}}{2} + \frac{1}{\log 2}\left(2^{\sqrt{3}} - 2^{\sqrt{2}} \right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}} \right)$

D. None of the above

Answer: B

36. $\int (\sin x + \cos x) dx$ is equal to :

A. $\dfrac{\pi}{2}$

B. π

C. 0

D. 2π

Answer: D

37. The value of $\int_0^{\pi/2} \sin|2x - \alpha| dx$, where $a\varepsilon[0, \pi]$ is

A. 1

B. $\cos \alpha$

C. $\dfrac{1 + \cos \alpha}{2}$

D. $\dfrac{1 - \cos \alpha}{2}$

Answer: A

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38. Let f be a continuous function satisfying $f'(lnx) = [1$ for $0 < x \le 1$, x for $x > 1$ and $f(0) = 0$ then $f(x)$ can be defined as

(a)$f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 1 - e^x & \text{if } x > 0 \end{cases}$ (b)$f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

(c)$f(x) = \begin{cases} x & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases}$ (d)$f(x) = \begin{cases} x & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

A. $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 1 - e^x & \text{if } x > 0 \end{cases}$

B. $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

C. $f(x) = \begin{cases} x & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases}$

D. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

Answer: D

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39. The integral $\int_{\frac{\pi}{4}}^{5\frac{\pi}{4}} (|\cos t|\sin t + |\sin t|\cos t)$ has the value equal to

A. 0

B. $1/2$

C. $1/\sqrt{2}$

D. 1

Answer: A

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40. If $f(x) = \begin{cases} 0 & \text{,where} x = \frac{n}{n+1} n = 1,2,3...... \\ 1 & \text{,else where} \end{cases}$, , then the value of

$\int_0^2 f(x)dx.$

A. (a) 1

B. (b) 2

C. (c) 3

D. (d) None of thses

Answer: B

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41. Evaluate $\displaystyle\int_{-1}^{1} \left(x^3 + 5x + \sin x\right) dx$

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42. Evaluate: $\displaystyle\int_{-(\pi/3)}^{\pi/3} x^3 \sin^4 x \, dx$

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43. Evaluate : $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

44. The value of $\displaystyle\int_{-2}^{2} \log\left(\frac{2-x}{2+x}\right) dx$ is equal to

A. $\dfrac{1}{2}$

B. 1

C. -1

D. 0

Answer: D

45. Evaluate: $\displaystyle\int_{0}^{\pi} x \sin 2x \, \frac{\sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi} dx$

A. $\dfrac{8}{\pi}$

B. $\dfrac{\pi}{8}$

C. $\dfrac{8}{\pi^2}$

D. $\dfrac{\pi^2}{8}$

Answer: C

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46. Let $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\frac{\cos^2 x}{2} \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$ then the value of

$\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^2 + 1)(f(x) + f''(x))dx$

A. (a)1

B. (b)−1

C. (c)2

D. (d)None of these

Answer: D

47. The value of $\int_{-1}^{1} \dfrac{x}{\sqrt{1-x^2}} \cdot \sin^{-1}\left(2x\sqrt{1-x^2}\right) dx$ is equal to

A. $4\sqrt{2}$

B. $4\left(\sqrt{2}-1\right)$

C. $4\left(\sqrt{2}+1\right)$

D. `None of these

Answer: B

48. Suppose the function $g_n(x) = x^{2n+1} + a_n x + b_n (N \in N)$ satisfes

the equation $\int_{-1}^{1} (px + q)g_n(x)dx = 0$ for all linear functions $(px + q)$

then

A. (a) $a_n = b_n = 0$

B. (b) $b_n = 0$, $a_n = -\dfrac{3}{2n+3}$

C. (c) $a_n = 0$, $b_n = -\dfrac{3}{2n+3}$

D. (d) $a_n = \dfrac{3}{2n+3}$, $b_n = -\dfrac{3}{2n+3}$

Answer: B

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49. Evaluate $\displaystyle\int_0^\pi \dfrac{x}{1+\cos^2 x}\,dx.$

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50. Prove that $\displaystyle\int_0^{\pi/2} \log(\sin x)\,dx = \int_0^{\pi/2} \log(\cos x)\,dx = -\dfrac{\pi}{2}\log 2.$

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51. If $\displaystyle\int_0^\pi \left(\dfrac{x}{1+\sin x}\right)^2 dx = A,$ then the value for

$\displaystyle\int_0^\pi \dfrac{2x^2 \cdot \cos^2 x/2}{(1+\sin x^2)} dx$ is equal to

A. $A + 2\pi - \pi^2$

B. $A - 2\pi + \pi^2$

C. $2\pi - A - \pi^2$

D. None of these

Answer: A

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52. What is the general solution of the equation cosx=1?

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53. Evaluate: $\displaystyle\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{\frac{1}{3}}^{\frac{2}{3}} e^{9\left(x-\frac{2}{3}\right)^2} dx$

54. If $f(x) = \int_0^x \log\left(\dfrac{1-t}{1+t}\right) dt$, then discuss whether even or odd?

55. Evaluate $\displaystyle\int_0^{4\pi} (\cos x)\, dx.$

56. Prove that $\displaystyle\int_0^{25} e^{x-[x]}\, dx = 25(e-1).$

57. The value of $\displaystyle\int_0^{2\pi} [\sin x + \cos x]\, dx$ is equal to

A. $-n\pi$

B. $n\pi$

C. $-2n\pi$

D. None of these

58. The value of $\int_{-5}^{5} f(x)dx$, where $f(x) = \text{minimum}(\{x+1\}, \{x-1\}), \forall x \in R$ and $\{.\}$ denotes fractional part of x, is equal to

A. 3

B. 4

C. 5

D. 6

Answer: C

59. Show that $\int_{0}^{n\pi+v} |\sin x|\,dx = 2n + 1 - \cos v$, where n is a positive integer and $0 \le v < \pi$.

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60. The value of $\int_{-2\pi}^{5\pi} \cot^{-1}(\tan x)\,dx$ is equal to

A. $\dfrac{7\pi}{2}$

B. $-\left(7\pi^2\right)$

C. $\dfrac{3\pi}{2}$

D. $21\dfrac{\pi^2}{4}$

Answer: B

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61. Let g(x) be a continuous and differentiable function such that

$$\int_0^2 \left\{ \int_{\sqrt{2}}^{\frac{\sqrt{5}}{2}} \left[2x^2 - 3 \right] dx \right\} . g(x) dx = 0, \text{ then g(x) = 0 when } x \in (0, 2)$$

has (where [*] denote greatest integer function)

A. exactly one real root

B. atleat one real root

C. no real root

D. None of these

Answer: B

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62. The value of x satisfying $\int_0^{2[x+14]} \left\{ \frac{x}{2} \right\} dx = \int_0^{\{x\}} [x + 14] dx$ is

equal to (where, $[.\,]$ and $\{.\,\}$ denotes the greates integer and fractional

part of x)

A. $[-14, \ -13)$

B. $(0, 1)$

C. $(-15, -14]$

D. None of these

Answer: A

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63. Find the derivative of the following with respect to x.

(i) $\displaystyle\int_0^x \cos t\, dt$

(ii) $\displaystyle\int_0^{x^2} \cos^2 dt$

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64. Evaluate $\dfrac{d}{dx}\left(\displaystyle\int_{1/x}^{\sqrt{x}} \cos t^2\, dt\right)$

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65. If $\dfrac{d}{dx} \left(\displaystyle\int_0^y e^{-t^2} dt + \int_0^{x^2} \sin^2 t\, dt \right) = 0$, find $\dfrac{dy}{dx}$.

66. The points of extremum of $\displaystyle\int_0^{x^2} \dfrac{t^2 - 5t + 4}{2 + e^t} dt$ are

67. If $y = \displaystyle\int_{x^2}^{x^3} \dfrac{1}{\log t} dt \, (x > 0)$, then find $\dfrac{dy}{dx}$

68. If $y = \displaystyle\int_0^x f(t)\sin\{k(x - t)\}dt$, then prove that $\dfrac{d^2 y}{dx^2} + k^2 y = kf(x)$.

69. If $\int_{\frac{\pi}{3}}^{x} \sqrt{(3 - \sin^2 t)} \, dt + \int_{0}^{y} \cos t \, dt = 0$, then evaluate $\dfrac{dy}{dx}$

70. Let $\dfrac{d}{dx} F(x) = \dfrac{e^{\sin x}}{x}$. $x < 0$. If $\int_{1}^{4} \dfrac{2e^{\sin x^2}}{x} \, dx = F(k) - F(1)$ then find the possible value of k.

A. 10

B. 14

C. 16

D. 18

Answer: C

71. The function $f(x) = \int_0^x \log_{|\sin t|}\left(\sin t + \dfrac{1}{2}\right)dt$, where $x \in (0, 2\pi)$,

then $f(x)$ strictly increases in the interval

A. $\left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$

B. $\left(\dfrac{5\pi}{6}, 2\pi\right)$

C. $\left(\dfrac{\pi}{6}, \dfrac{7\pi}{6}\right)$

D. $\left(\dfrac{5\pi}{6}, \dfrac{7\pi}{6}\right)$

Answer: D

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72. $f:(0, \infty) \to R$ and $F(x) = \int_0^x tf(t)dt$

If $F(x^2) = x^4 + x^5$, then $\Sigma_{r=1}^{12} f(r^2)$ is equal to S

A. (a) 216

B. (b) 219

C. (c) 221

D. (d) 223

Answer: B

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73. A function f(x) satisfies

$$f(x) = \sin x + \int_0^x f'(t)\left(2\sin t - \sin^2 t\right)dt \text{ Then } f(x) =$$

(a) $\dfrac{x}{1 - \sin x}$ (b) $\dfrac{\sin x}{1 - \sin x}$

(c) $\dfrac{1 - \cos x}{\cos x}$ (d) $\dfrac{\tan x}{1 - \sin x}$

A. $\dfrac{x}{1 - \sin x}$

B. $\dfrac{\sin x}{1 - \sin x}$

C. $\dfrac{1 - \cos x}{\cos x}$

D. $\dfrac{\tan x}{1 - \sin x}$

Answer: B

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74. If $F(x) = \displaystyle\int_1^x f(t)dt$, where $f(t) = \displaystyle\int_1^{t^2} \frac{\sqrt{1+u^4}}{u}du$, then the value of F''(2) equals to

A. $\dfrac{7}{4\sqrt{17}}$

B. $\dfrac{15}{\sqrt{17}}$

C. $\sqrt{257}$

D. $\dfrac{15\sqrt{17}}{68}$

Answer: C

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75. Evaluate $I(b) = \displaystyle\int_0^1 (x^b)dx = \int_0^1 \frac{x^b - 1}{\ln x}dx, b \geq 0.$

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76. Prove that $\displaystyle\int_0^{\pi/2} \frac{dx}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2} = \frac{\pi\left(a^2 + b^2\right)}{4a^3 b^3}.$

77. The value of

$$\int_0^{\pi/2} \frac{\log(1 + x\sin^2\theta)}{\sin^2\theta}\,d\theta, \; x \geq 0 \text{ is equal to}$$

A. $\pi\left(\sqrt{1+x} - 1\right)$

B. $\pi\left(\sqrt{1+x} - 2\right)$

C. $\sqrt{\pi}\left(\sqrt{1+x} - 1\right)$

D. None of these

Answer: A

78. Let $f(x)$ be a continuous function for all x, which is not identically zero such that $\{f(x)\}^2 = \int_0^x f(t)\,\frac{2\sec^2 t}{4 + \tan t}\,dt$ and $f(0) = \ln 4,$ then

A. $f\left(\dfrac{\pi}{4}\right) = \log(5)$

B. $f\left(\dfrac{\pi}{4}\right) = \dfrac{3}{4}$

C. $f\left(\dfrac{\pi}{2}\right) = 2$

D. None of these

Answer: A

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79. Evaluate the following

(i) $\displaystyle\lim_{n \to \infty} \left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2}.... + \dfrac{n-1}{n^2}\right)$

(ii) $\displaystyle\lim_{n \to \infty} \left(\dfrac{1}{n+1} + \dfrac{1}{n+2} + ... + \dfrac{1}{2n}\right)$

(iii) $\displaystyle\lim_{n \to \infty} \left(\dfrac{n}{n^2+1^2} + \dfrac{n}{n^2+2^2} + ... + \dfrac{n}{2n^2}\right)$

(iv) $\displaystyle\lim_{n \to \infty} \dfrac{\left(1^p + 2^p + ... + n^p\right)}{n^{p+1}}, p > 0$

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80. Differentiate the following function w.r.t x $\sin(ax+b)$

81. Evaluate $\lim\limits_{n \to \infty} \left(\dfrac{1}{2n+1} + \dfrac{1}{2n+2} + \ldots\ldots + \dfrac{1}{6n} \right).$

82. Evaluate $\displaystyle\int_{1}^{4} \left(ax^2 + bx + c \right) dx$.

83. The value of $\lim\limits_{n \to \infty} \left(\sin\dfrac{\pi}{2n} \cdot \sin\dfrac{2\pi}{2n} \cdot \sin\dfrac{3\pi}{2n} \ldots \sin\dfrac{(n-1)\pi}{2n} \right)^{\frac{1}{n}}$ is equal to

A. $\dfrac{1}{2}$

B. $\dfrac{1}{3}$

C. $\dfrac{1}{4}$

D. None of these

Answer: C

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84. The interval $[0, 4]$ isdivided into n equal sub-intervals by the points $x_0, x_1, x_2, \ldots, x_{n-1}, x_n$ where $0 = x_0 < x_1 < x_2 < x_3 < \ldots < x_n = 4$

If $\delta x = x_i - x_{i-1}$ for $i = 1, 2, 3, \ldots, n$, then $\lim\limits_{\delta x \to 0} \sum\limits_{i-1}^{n} x_i \delta x$ is equal to

A. (a)4

B. (b)8

C. (c)$\dfrac{32}{3}$

D. (d)16

Answer: B

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85. The value of $\lim\limits_{n \to \infty} \dfrac{1}{n} \cdot \sum\limits_{r=1}^{2n} \dfrac{r}{\sqrt{n^2 + r^2}}$ is equal to

A. $1 + \sqrt{5}$

B. $-1 + \sqrt{5}$

C. $-1 + \sqrt{2}$

D. $1 + \sqrt{2}$

Answer: B

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86. Differentiate the following function w.r.t x

$\sin(x^2 + 5)$

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87. The minimum odd value of 'a' $(a > 1)$ for which

$\int_{10}^{19} \dfrac{\sin x}{1 + x^a} dx < \dfrac{1}{9}$, is equal to

A. 1

B. 3

C. 5

D. 9

Answer: B

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88. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}\,dx$ cannot exceed $\sqrt{15/8}$.

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89. If $f(x)$ is a continous function such that $f(x) \mid 0, \forall x \in [2, 10]$ and $\int_4^8 f(x)\,dx = 0$, then find f (6)

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90. Prove that $\dfrac{\pi}{6} < \displaystyle\int_0^1 \dfrac{dx}{\sqrt{4 - x^2 - x^3}} < \dfrac{\pi}{4\sqrt{2}}$

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91. Prove that $4 \le \displaystyle\int_1^3 \sqrt{3 + x^3}\,dx \le 2\sqrt{30}.$

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92. Prove that $1 \le \displaystyle\int_0^1 e^{x^2}\,dx \le e$

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93. Evaluate

(i) ⌐1

(ii) ⌐2

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94. Evaluate $\int_0^\infty e^{-x^2} x^3 dx$.

95. Evaluate $\int_0^1 \left(\left(\log\left(\frac{1}{x} \right) \right)^{n-1} dx \right.$.

96. Evaluate $\int_0^2 \left(x^2 + 4 \right) dx$ as ' limit of a sum'.

97. Evaluate $\int_0^{\pi/2} \sin^4 x . \cos^6 x dx$.

98. The value of $\displaystyle\int_0^\infty e^{-(a^2x)}\,dx$ is equal to

99. What is the general solution of tanx=1?

100. The true set values of 'a' for which the inequality $\displaystyle\int_a^0 \left(3^{-2x} - 2.3^{-x}\right)dx \geq 0$ is true, is

A. $[0, 1]$

B. $(-\infty, -1]$

C. $[0, \infty)$

D. $(-\infty, -1] \cup [0, \infty)$

Answer: D

101. The value of the definite integral $\int_0^{2n\pi} \max\left(\sin x, \sin^{-1}(\sin x)\right) dx$

equals to (where, n in I)

A. $\dfrac{n(\pi^2 - 4)}{2}$

B. $\dfrac{n(\pi^2 - 4)}{4}$

C. $\dfrac{n(\pi^2 - 8)}{4}$

D. $\dfrac{n(\pi^2 - 2)}{4}$

Answer: C

102. $\int \dfrac{\sqrt{a^2 - x^2}}{x^2} dx$

103. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ is equal to

A. $\dfrac{8}{\pi}f(2)$

B. $\dfrac{2}{\pi}f(2)$

C. $\dfrac{2}{\pi}f\left(\dfrac{1}{2}\right)$

D. $4f(2)$

Answer: A

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104. Let f be a non-negative function defined on the interval $[0, 1]$. If
$$\int_0^x \sqrt{1 - (f'(t))^2}\, dt = \int_0^x f(t)dt, \, 0 \le x \le 1, \, and \, f(0) = 0, \text{then}$$

a. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

b. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

c. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

d. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

A. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

B. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

C. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

D. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

Answer: C

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105. The value of $\displaystyle\lim_{x \to 0} \dfrac{1}{x^3} \int_0^x \dfrac{t\ln(1+t)}{t^4 + 4} dt$

A. 0

B. $\dfrac{1}{12}$

C. $\dfrac{1}{24}$

D. $\dfrac{1}{64}$

Answer: B

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106. The value of $\int_0^1 \dfrac{x^4(1-x)^4}{1+x^2} dx$ is (are)

A. $\dfrac{22}{7} - \pi$

B. $\dfrac{2}{105}$

C. 0

D. $\dfrac{71}{15} - \dfrac{3\pi}{2}$

Answer: A

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107. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx$ (where $\theta \neq \dfrac{n\pi}{2}$, $n \in I$) is equal to

A. $-\cos\theta \int_1^{\tan\theta} f(x\sin\theta)dx$

B. $-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x)dx$

C. $-\tan\theta \int_1^{\sin\theta} f(x\cos\theta)dx$

D. $\dfrac{1}{\tan\theta}\displaystyle\int_{\sin\theta}^{\sin\theta\,\tan\theta} f(x)\,dx$

Answer: A

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108. Evaluate: $\displaystyle\int \dfrac{1}{x\sqrt{1+x^3}}\,dx$

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109. Find $\dfrac{dy}{dx}$ if $x^4 - y^4 = \sin x$

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110. Statement I $If f(x) = \displaystyle\int_0^1 (x f(t) + 1)\,dt,$ then $\displaystyle\int_0^3 f(x)\,dx = 12$

Statement II $f(x) = 3x + 1$

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: c

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111. Statement I The function $f(x) = \int_0^x \sqrt{1 + t^2} dt$ is an odd function and STATEMENT 2 :$g(x) = f'(x)$ is an even function , then $f(x)$ is an odd function.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the

correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: A

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112. Given, $f(x) = \sin^3 x$ and $P(x)$ is a quadratic polynomial with coefficient unity.

Statement I $\int_0^{2\pi} P(x). f''(x)dx$ vanishes.

Statement II $\int_0^{2\pi} f(x)dx$ vanishes.

A. Statement I is true, Statement II is also true, Statement II is the

correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the

correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: a

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113. Find the second order derivative if $y = x^3$

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114. If $f(a + b - x) = f(x),$ then $\int_a^b x f(x) dx$ is equal to:

A. 1

B. 2

C. 3

D. 4

Answer: A

---

115. Find the general solution of the equation tanx=0.

---

116. Find the value of $f(\alpha, \beta) = \begin{vmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) & \cos 2\beta \\ \sin \alpha & \cos \alpha & \sin \beta \\ -\cos \alpha & \sin \alpha & \cos \beta \end{vmatrix}$

---

117. Let $f(\alpha, \beta) = = \begin{vmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) & \cos 2\beta \\ \sin \alpha & \cos \alpha & \sin \beta \\ -\cos \alpha & \sin \alpha & \cos \beta \end{vmatrix}$

if $i = \int_{-\pi/2}^{\pi/2} \cos^2 \beta \left(f(0, \beta) + f\left(0, \frac{\pi}{2} - \beta\right) \right) d\beta$ then i is

A. $e^{\pi/2}$

B. 3

C. $2\left(2e^{\pi/2} - 1\right)$

D. `None of these

Answer: B

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118. Match the following

Column I		Column II	
(A)	$\int_{-1}^{1} \dfrac{dx}{1+x^2}$	(p)	$\dfrac{1}{2}\log\left(\dfrac{2}{3}\right)$
(B)	$\int_{0}^{1} \dfrac{dx}{\sqrt{1-x^2}}$	(q)	$\dfrac{1}{2}\log\left(\dfrac{3}{2}\right)$
(C)	$\int_{2}^{3} \dfrac{dx}{1-x^2}$	(r)	$\dfrac{\pi}{3}$
(D)	$\int_{1}^{2} \dfrac{dx}{x\sqrt{x^2-1}}$	(s)	$\dfrac{\pi}{2}$

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119. Let $f : R \to R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt.$ Then the value of $f(1n5)$ is_____

120. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad$ if $[x]$ is odd, $1 + [x] - x, \quad$ if $[x]$ is even Then the value of $\dfrac{\pi^2}{10} \int_{-10}^{10} f(x)\cos \pi x \, dx$ is____

121. Let f(x) be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x - t)dt$ then $\int_0^1 f(x)dx = $

122. Find the error in steps to evaluate the following integral

$$\int_0^\pi \frac{dx}{1 + 2\sin^2 x} = \int_0^\pi \frac{\sec^2 x\, dx}{\sec^2 x + 2\tan^2 x} = \int_0^\pi \frac{\sec^2 x\, dx}{1 + 3\tan^2 x}$$

$$= \frac{1}{\sqrt{3}} \left[\tan^{-1}(\sqrt{3}\tan x)\right]_0^\pi = 0$$

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123. Find $\displaystyle\int_0^a \sin x\, dx$.

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124. Evaluate $\displaystyle\int_{\cos(\cos^{-1}\alpha)}^{\sin(\sin^{-1}\beta)} \left| \left(\frac{\cos(\cos^{-1} x)}{\sin(\sin^{-1} x)} \right) \right| dx$

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125. $\displaystyle\int_\alpha^\beta 2x\ dx$ is equal to

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126. Evaluate: $\displaystyle\int_{\sqrt{\frac{3a^2+b^2}{2}}}^{\sqrt{\frac{a^2+b^2}{2}}} \frac{x \cdot dx}{\sqrt{(x^2-a^2)(b^2-x^2)}}$

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127. Evaluate $\displaystyle\int_0^{\pi/4} \frac{e^{\sec x}\left[\sin\left(x+\frac{\pi}{4}\right)\right]}{\cos x(1-\sin x)}dx$

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128. $\displaystyle\int_0^{\pi/4} \frac{x^2(\sin 2x-\cos 2x)}{(1+\sin 2x)\cos^2 x}dx$

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129. Evaluate $\displaystyle\int_0^{\pi} x^2\left\{(1+\sin x)^{-2}\cos x\right\}dx$

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130. Compute the following integrals.

(i) $\displaystyle\int_0^\infty f(x^n + x^{-n}) \ln x \,\frac{dx}{x} = 0$

(ii) $\displaystyle\int_0^\infty f(x^n + x^{-n}) \ln x \,\frac{dx}{1+x^2} = 0$

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131. Show that

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$

(b) $\displaystyle\int_0^\pi \cos x \, dx = 0$

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132. Find a function $g: R \to R$, continuous in $[0, \infty)$ and positive in $(0, \infty)$ satisfying $g(1) = 1$ and $\displaystyle\frac{1}{2}\int_0^x g^2(t) dt = \frac{1}{x}\left(\int_0^x g(t) dt\right)^2$

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133. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$, $(n > 1$ and is an integer), then

A. (a) $I_n + I_{n-2} = \dfrac{1}{n+1}$

B. (b) $I_n + I_{n-2} = \dfrac{1}{n-1}$

C. (c) $I_2 + I_4$, $I_4 + I_6$, $I_6 + I_8$ are in HP

D. (d) $\dfrac{1}{2(n+1)} < I_n < \dfrac{1}{2(n-1)}$

Answer: $\dfrac{1}{2(n+1)} < I_n < \dfrac{1}{2(n-1)}$

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134. If $U_n = \int_0^{\pi} \dfrac{1 - \cos nx}{1 - \cos x}\, dx$ where n is positive integer of zero, then

The value of $\int_0^{\pi/2} \dfrac{\sin^2 n\theta}{\sin^2 \theta}\, d\theta$ is

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135. Prove that for any positive integer k, $\dfrac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + + \cos(2k-1)x].$

Hence, prove that $\displaystyle\int_0^{\frac{\pi}{2}} \sin 2xk \cot x\, dx = \dfrac{\pi}{2}.$

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136. Evaluate $\displaystyle\int_0^{\sqrt{3}} \left(\dfrac{1}{1+x^2}\right) \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) dx.$

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137. Prove that $\displaystyle\int_0^x e^{xt} e^{-t^2}\, dt = e^{\frac{x^2}{4}} \int_0^x e^{-\frac{t^2}{4}}\, dt.$

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138. A function f(x) which satisfies the relation $f(x) = e^x + \displaystyle\int_0^1 (e^x + te^{-x}) f(t)\, dt,$ find $f(x)$.

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139. If $|a| < 1,$ show that $\displaystyle\int_0^\pi \frac{\log(1 + a\cos x)}{\cos x}\,dx = \pi\sin^{-1}a$

140. Evaluate $\displaystyle\int_0^{\pi/2} \cos\theta\tan^{-1}(\sin\theta)\,d\theta.$

141. Evaluate $\displaystyle\int_0^{\pi/2} \cos\theta\tan^{-1}(\sin\theta)\,d\theta.$

142. Let f be a continuous function on $[a, b].$ Prove that there exists a number $x \in [a, b]$ such that $\displaystyle\int_a^x f(t)\,dt = \int_x^b f(t)\,dt.$

143. If $f(x) = x + \int_0^1 (xy^2 + x^2 y)(f(y))dy$, find $f(x)$ if x and y are independent. Find $f(x)$.

144. $\int_0^x \left\{ \int_0^u f(t)dt \right\} du$ is equal to

(a) $\int_0^x (x - u)f(u)du$

(b) $\int_0^x uf(x - u)du$

(c) $x \int_0^x f(u)du$

(d) $x \int_0^x uf(u - x)du$

145. Evaluate: $\int_0^{\frac{3\pi}{2}} (\log|\sin x|)\cos(2nx)dx, n \in N$

146. Evaluate $\int_0^\infty e^{-x} \sin^n x \, dx$, if n is an even integer.

147. Evaluate $\int_0^1 (tx + 1 - x)^n \, dx$, where n is a positive integer and t is a parameter independent of x . Hence , show that $\int_0^1 x^k (1 - x)^{n-k} \, dx = \dfrac{P}{[.^n C_k(n+1)]}$ for $k = 0, 1, \ldots\ldots n$, then P=

148. Given a real valued fuction f(x) which is monotonic and differentiable , prove that for any real number a and b, $\int_a^b \{f^2(x) - f^2(a)\} \, dx = \int_{f(a)}^{f(b)} 2x\{b - f^{-1}(x)\} \, dx$

149. $\int_0^1 \dfrac{\sin \theta \left(\cos^2 \theta - \cos^2 \pi/5\right)\left(\cos^2 \theta - \cos^2 2\pi/5\right)}{\sin 5\theta} \, d\theta$

150. Show that $\displaystyle\lim_{n \to \infty} \sum_{k=0}^{n} \frac{{}^nCk}{n^k(k+3)} = e - 2$

151. Let $I = \displaystyle\int_0^{\pi/2} \frac{\cos x}{a \cos x + b \sin x} dx$ and $J = \displaystyle\int_0^{\pi/2} \frac{\sin x}{a \cos x + b \sin x} dx$, where $a > 0$ and $b > 0$ Compute the values of I and j

152. Evaluate $\displaystyle\int_0^{\infty} \frac{\ln x \, dx}{x^2 + 2x + 4}$

153. Find a function f, continous for all x(and not zero everywhere) such that $f^2(x) = \displaystyle\int_0^x \frac{f(t)\sin t}{2 + \cos t} dt$

154. Evaluate $\displaystyle\int_0^\infty \frac{\tan^{-1} ax - \tan^{-1} x}{x} dx$, where a is a parameter.

155. Evaluate $\displaystyle\int_0^\infty \frac{\tan^{-1} ax - \tan^{-1} x}{x} dx$ where a is a parameter.

156. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos x \; dx$

Exercise For Session 1

1. $\displaystyle\int_0^{\pi/4} \cos^2 x\, dx$

2. Evaluate $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + \cos x}$

3. Evaluate $\displaystyle\int_0^{\pi/2} \sqrt{1 + \cos x}\, dx$

4. $\displaystyle\int_0^{\frac{\pi}{6}} \sin 2x . \cos x\, dx$

5. $\int_{(1)}^{(2)} \dfrac{dx}{(\sqrt{x} - \sqrt{x-1})}$

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6. find $\displaystyle\int_0^1 x\,dx$

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7. $\displaystyle\int_0^{\pi/4} \dfrac{\sin x + \cos x}{9 + 16\sin 2x}$

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8. The value of $\displaystyle\int \dfrac{dx}{\sqrt{(x-a)(b-x)}}$, is

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9. $\displaystyle\int_a^b \sqrt{\dfrac{x-a}{b-x}}\,dx$ is equal to

10. Evaluate the definite integral: $\displaystyle\int_0^{\frac{\pi}{4}} (\tan x)\,dx$

11. $\displaystyle\int_0^{\pi} \cos 2x.\log(\sin x)\,dx$

12. $\displaystyle\int_0^{\pi/4} e^{\sin x}\left(\dfrac{(x\cos^3 x - \sin x)}{\cos^2 x}\right)dx$

13. If $f(x)$ is a function satisfying $f\left(\dfrac{1}{x}\right) + x^2 f(x) = 0$ for all nonzero x, then evaluate $\displaystyle\int_{\sin\theta}^{\cos ec\theta} f(x)\,dx$

14. If the first and fifth term of AP are 12 and 60. Find the 15th term of this AP.

15. The true set values of 'a' for which the inequality $\displaystyle\int_{a}^{0} \left(3^{-2x} - 2.3^{-x}\right)dx \geq 0$ is true, is

A. $[0,\ 1]$

B. $[-\infty,\ -1]$

C. $[0,\ \infty]$

D. $[-\infty,\ -1] \cup [1,\ \infty]$

Answer: D

Exercise For Session 2

1. Evaluate $\displaystyle\int_{0}^{\pi/4} \log(1 + \tan\theta)d\theta$

A. $\dfrac{\pi}{2}\log 2$

B. $-\dfrac{\pi}{4}\log 2$

C. $\dfrac{\pi}{8}\log 2$

D. None of these

Answer: C

2. Q. $\displaystyle\int_{0}^{\pi} e^{\cos^2 x}\left(\cos^3(2n + 1)x dx, n \in I\right)$

A. 0

B. 1

C. − 1

D. None of these

Answer: A

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3. The value of $\int_2^3 \dfrac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is _____

A. $1/2$

B. $1/3$

C. $1/4$

D. None of these

Answer: A

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4. Evaluate: $\displaystyle\int_0^2 \frac{dx}{(17 + 8x - 4x^2)\left[e^{6(1-x)} + 1\right]}$

A. $-\dfrac{1}{8\sqrt{21}}\log\left|\dfrac{2 - \sqrt{21}}{2 + \sqrt{21}}\right|$

B. $-\dfrac{1}{8\sqrt{21}}\log\left|\dfrac{2 - \sqrt{21}}{\sqrt{21} - 2}\right|$

C. $-\dfrac{1}{8\sqrt{21}}\left\{\log\left|\dfrac{2 - \sqrt{21}}{2 + \sqrt{21}}\right| - \log\left|\dfrac{2 + \sqrt{21}}{\sqrt{21} - 2}\right|\right\}$

D. None of these

Answer: C

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5. If f **is an odd function, then evaluate** $I = \displaystyle\int_{-a}^{a} \frac{f(\sin x)\,dx}{f(\cos x) + f\left(\sin^2 x\right)}$

A. 0

B. $f(\cos x) + f(\sin x)$

C. 1

D. None of these

6. If [x] stands for the greatest integer function, the value of

$$\int\limits_{4}^{10} \frac{\left[x^2\right]}{\left[x^2 - 28x + 196\right] + \left[x^2\right]} dx, \text{ is}$$

A. 1

B. 2

C. 3

D. 4

7. Evaluate $\displaystyle\int_0^\pi \frac{x\,dx}{1 + \cos\alpha\sin x}$, where $0 < \alpha < \pi$.

A. $\dfrac{\pi}{\sin\alpha}$

B. $\dfrac{\pi\alpha}{\sin\alpha}$

C. $\dfrac{\alpha}{\sin\alpha}$

D. $\dfrac{\sin\alpha}{\alpha}$

Answer: B

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8. $f, g, h,$ are continuous in $[0, a]$, $f(a - x) = f(x)$, $g(a - x) = -g(x)$, $3h(x) - 4h(a - x) = 5$. Then $\displaystyle\int_0^a f(x)g(x)h(x)\,dx =$

A. 0

B. 1

C. a

D. -a

Answer: A

9. If $2f(x) + f(-x) = \dfrac{1}{x}\sin\left(x - \dfrac{1}{x}\right)$ then the value of $\displaystyle\int_{\frac{1}{e}}^{e} f(x)\,dx$ is

A. 0

B. e

C. $1/e$

D. $e + 1/e$

Answer: A

10. Show that $\displaystyle\int_{0}^{\pi} x f(\sin x)\,dx = \dfrac{\pi}{2}\int_{0}^{\pi} \text{fr}(\sin x)\,dx.$

11. Evaluate $\displaystyle\int_0^{\pi} (\sin 2x)\,dx$.

12. The numbers of possible continuous $f(x)$ defined in $[0, 1]$ for which

$$I_1 = \int_0^1 f(x)\,dx = 1,\ I_2 = \int_0^1 x f(x)\,dx - a,\ I_3 = \int_0^1 x^2 f(x)\,dx = a^2 \text{ is } / a$$

1 (b) ∞ (c) 2 (d) 0

13. Let $\displaystyle l_1 = \int_0^1 \frac{e^x}{1+x}\,dx$ and $\displaystyle l_2 = \int_0^1 \frac{x^2}{e^{x^3}(2-x^3)}\,dx$. Then $\dfrac{l_1}{l_2}$ is equal to

A. $\dfrac{3}{e}$

B. $\dfrac{3}{e}$

C. 3e

D. $\dfrac{1}{3e}$

Answer: C

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14.   If $\quad f(x) = \dfrac{e^x}{1 + e^x}, I_1 = \displaystyle\int_{f(-a)}^{f(a)} xg(x(1-x))dx,\quad$ and

$I_2 = \displaystyle\int_{f(-a)}^{f(a)} g(x(1-x))dx,\ $ then the value of $\dfrac{I_2}{I_1}$ is

A. 1

B. -3

C. -1

D. 2

Answer: D

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1. The value of $\int_{-1}^{3} \{|x - 2| + [x]\}dx$, where [.] denotes the greatest integer function, is equal to

A. 5

B. 6

C. 3

D. None of these

Answer: D

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2. The value of $\int_{-1}^{3} (|x| + |x - 1|)dx$ is equal to

A. 9

B. 6

C. 3

D. None of these

Answer: A

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3. Let $f(x) = x - [x]$, for every real number x, where [x] is integral part of x. Then $\displaystyle\int_{-1}^{1} f(x)\,dx$ is

A. 0

B. 1

C. 2

D. None of these

Answer: B

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4. The value of $\int_0^2 [x + [x + [x]]]dx$ (where, [.] denotes the greatest integer function)is equal to

A. 2

B. 3

C. -3

D. None of these

Answer: B

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5. The value of $\int_0^{[x]} \dfrac{2^x}{2^{[x]}}dx$ is equal to (where, [.] denotes the greatest integer function)

a. $\dfrac{[x]}{\log 2}$

b. $\dfrac{[x]}{2\log 2}$

c. $\dfrac{[x]}{4\log 2}$

d. None of these

A. $\dfrac{[x]}{\log 2}$

B. $\dfrac{[x]}{2\log 2}$

C. $\dfrac{[x]}{4\log 2}$

D. None of these

Answer: A

6. The value of $\displaystyle\int_0^4 \{x\}dx$ (where , {.} denotes fractional part of x) is equal to

A. $\dfrac{4}{3}$

B. $\dfrac{5}{3}$

C. $\dfrac{7}{3}$

D. None of these

Answer: D

7. The value of $\int_1^4 \{x\}^{[x]} dx$ (where , [.] and {.} denotes the greatest integer and fractional part of x) is equal to

A. (a) $\dfrac{11}{12}$

B. (b) $\dfrac{13}{12}$

C. (c) $\dfrac{70}{12}$

D. (d) $\dfrac{19}{12}$

Answer: B

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8. The value of $\int_0^x (4t)^3 dt$ is equal to

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9. The value of $\int_0^{10\pi} \left[\tan^{-1} x\right] dx$ (where, [.] denotes the greatest integer functionof x) is equal to

A. $\tan 1$

B. 10π

C. $10\pi - \tan 1$

D. None of these

Answer: C

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10. If $f(x) = \min \{|x - 1|, |x|, |x + 1|\}$, then the value of $\int_{-1}^{1} f(x) dx$ is equal to

A. 1

B. $\dfrac{1}{2}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{8}$

Answer: c

---

11. The value of $\displaystyle\int_{0}^{\infty}\left[2e^{-x}\right]dx$ (where ,[.] denotes the greatest integer function of x) is equal to

A. 1

B. $\log_e 2$

C. 0

D. $\dfrac{1}{e}$

Answer: B

12. The value of $\int_1^{10\pi} \left(\left[\sec^{-1} x\right]\right) dx$ (where ,[.] denotes the greatest integer function) is equal to

A. (a)$(\sec 1) - 10\pi$

B. (b)$10\pi - \sec 1$

C. (c)$\pi - \sec 1$

D. (d)None of these

Answer: B

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13. The value of $\int_{-\pi/2}^{\pi/2} \left[\cot^{-1} x\right] dx$ (where ,[.] denotes greatest integer function) is equal to

A. (a)$\pi + \cot 1$

B. (b)$\pi + \cot 2$

C. (c)$\pi + \cot 1 + \cot 2$

D. (d)$\cot 1 + \cot 2$

Answer: C

14. The value of $\int_0^{\frac{\pi}{4}} \left(\tan^n(x - [x]) + \tan^{n-2}(x - [x]) \right) dx$ (where, [ ]

denotes greatest integer function) is equal to

A. (a)$\dfrac{1}{n}$

B. (b)$\dfrac{1}{n-1}$

C. (c)$\dfrac{1}{n(n-1)}$

D. (d)$\dfrac{1}{n(+1)}$

Answer: B

15. The value of $\int_0^2 \left[x^2 - x + 1\right] dx$ (where , [.] denotes the greatest integer function) is equal to

A. $\dfrac{5 + \sqrt{5}}{2}$

B. $\dfrac{1 + \sqrt{5}}{2}$

C. $\dfrac{1 - \sqrt{5}}{2}$

D. $\dfrac{5 - \sqrt{5}}{2}$

Answer: D

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16. Evaluate $\int_0^a \left[x^n\right] dx$, (where,[*] denotes the greatest integer function).

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17. Prove that $\int_0^x [t] dt = \dfrac{[x]([x] - 1)}{2} + [x](x - [x])$, where [.] denotes the greatest integer function.

18. If $f(n) = \dfrac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ (where,[*] and {*} denotes greatest integer and fractional part of x and $n \in N$). Then, the value of $f(4)$ is...

19. $\displaystyle\int_0^x [\cos t]dt, \; where x \in \left(2n\pi, 2n\pi + \dfrac{\pi}{2}\right), n \in N, and [.]$ denotes the greatest integer function . then the value of $f\left(\dfrac{1}{\pi}\right) is$

20. If $\displaystyle\int_0^x [x]dx = \int_0^{[x]} xdx, \; x \notin$ integer (where, [*] and {*} denotes the greatest integer and fractional parts respectively,then the value of $4\{x\}$ is equal to ...

1. Let $f: \overrightarrow{RR} \, and g: \overrightarrow{RR}$ be continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) - g(-x)]$ dx is

A. -1

B. 0

C. 1

D. None of these

Answer: B

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2. The value of $\int_{-1}^{1} (x|x|) dx$ is equal to

A. 1

B. $\dfrac{1}{2}$

C. 0

D. None of these

Answer: C

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3. The value of $\int_{-1}^{1} \left(\frac{x^2 + \sin x}{1 + x^2} \right) dx$ is equal to

A. 2π

B. $\pi - 2$

C. $2 - \frac{\pi}{2}$

D. None of these

Answer: C

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4. If f is an odd function, then the value of $\int_{-a}^{a} \dfrac{s(\sin x)}{f(\cos x) + f(\sin^2 x)}\, dx$ is

equal to

A. 0

B. $f(\cos x) + f(\sin)$

C. 1

D. None of these

Answer: A

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5. Evaluate: $\int_{-\left(\frac{1}{\sqrt{3}}\right)}^{\frac{1}{\sqrt{3}}} \dfrac{\cos^{-1}\left(\frac{2x}{1+x^2}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x + 1}\, dx$

A. $\dfrac{\pi}{2}$

B. $\dfrac{\pi}{\sqrt{3}}$

C. $\dfrac{\pi}{2\sqrt{3}}$

D. $\dfrac{\pi}{3\sqrt{3}}$

Answer: C

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6. Find the value of $\displaystyle\int_{-\pi}^{\pi} \dfrac{\cos^2 x}{1 + a^x}\,dx,\ a > 0.$

A. π

B. $a\pi$

C. 2π

D. $\dfrac{\pi}{2}$

Answer: A

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7. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + \ln\left(\frac{1+x}{1-x}\right) \right) dx$ is equal to (where [.]

represents the greatest integer function)

A. $\dfrac{-1}{2}$

B. 0

C. 1

D. $2\dfrac{\log(1)}{2}$

Answer: A

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8. Evaluate: $\int_{-\pi/2}^{\pi/2} \dfrac{1}{1 + e^{\sin x}} dx$

A. 0

B. 1

C. $\dfrac{\pi}{2}$

D. $-\dfrac{\pi}{2}$

Answer: C

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9. If [*] denotes the greatest integer function then the value of the integeral $\displaystyle\int_{-\pi/2}^{\pi/2} \left(\left[\dfrac{x}{\pi}\right] + 0.5\right) dx$, is

A. π

B. $\dfrac{\pi}{2}$

C. 0

D. $-\dfrac{\pi}{2}$

Answer: C

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10. The equation $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left\{ a|\sin x| + \dfrac{b\sin x}{1 + \cos^2 x} + c \right\} dx = 0$ where a, b, c

are constants gives a relation between

A. a, b and c

B. a and c

C. a and b

D. b and c

Answer: B

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11. The value of $\displaystyle\int_{-2}^{2} \dfrac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ where [.] denotes greatest integer

function , is

A. 1

B. 0

C. $4\sin 4$

D. None of these

Answer: B

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12. Let $f(x)$ be a contiuous function such a $\int_{n}^{n+1} f(x)dx = n^3, n \in Z$.

Then, the value of the intergral $\int_{-3}^{3} f(x)dx$ (A) 9 (B) -27 (C) -9 (D) none of these

A. 9

B. -27

C. -9

D. 27

Answer: B

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13. Let $f(x) = \dfrac{e^x + 1}{e^x - 1}$ and $\displaystyle\int_0^1 x^3 \cdot \dfrac{e^x + 1}{e^x - 1} dx = \alpha$ Then, $\displaystyle\int_{-1}^1 t^3 f(t) dt$ is equal to

A. 0

B. α

C. 2α

D. None of these

Answer: C

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14. Let $f: R \to R$ be a continuous function given by $f(x + y) = f(x) + f(y)$ for all $x, y, \in R$, if $\displaystyle\int_0^2 f(x) dx = \alpha$, then $\displaystyle\int_{-2}^2 f(x) dx$ is equal to

A. 2α

B. α

C. 0

D. None of these

Answer: C

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15. The value of $\int_{-2}^{2} x\,dx$ is equal to

A. 0

B. 2

C. 3

D. 4

Answer: D

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16. Find the second order derivative if $y = e^{2x}$

17.   Let

$$f(x) = \{1 - |x|, |x| \le 1 \text{ and } 0, |x| < 1 \text{ and } g(x) = f(x-1) + f(x+1)$$

, for all $x \in R$.Then,the value of $\int_{-3}^{3} g(x)dx$ is

A. 2

B. 3

C. 4

D. 5

Answer: A

18. If $\int_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, $n = 0, 1, 2, \ldots \ldots \ldots$ then

A. $n\pi$

B. π

C. $-\pi$

D. 0

Answer: D

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19. Let $I_n = \int_{-\pi}^{\pi} \frac{\sin x}{(1+\pi^x)\sin x} dx$, $n = 0, 1, 2, \ldots$ The value of $I_{n+2} - I_n$ is equal to

A. 0

B. 5π

C. 10π

D. 0

Answer: C

20. Let $I_n = \int_{-\pi}^{\pi} \dfrac{\sin x}{(1 + \pi^x)\sin x}\,dx,\ n = 0, 1, 2, \ldots$ The value of $l_{n+2} - l_n$ is equal to

A. 0

B. 5π

C. 10π

D. None of these

Answer: A

Exercise For Session 5

1. The value of $\int_{-1}^{10} sgn(x - [x])dx$ is equal to (where, [:] denotes the greatest integer function

A. 9

B. 10

C. 11

D. 12

Answer: C

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2. the value of $\int_{0}^{[x]} dx$ (where , [.] denotes the greatest integer function)

A. [x]

B. $\dfrac{[x]}{2}$

C. $x[x]$

D. None of these

Answer: A

3. Evaluate: $\displaystyle\int_{-\frac{\pi}{4}}^{n\pi-\frac{\pi}{4}} |\sin x + \cos x| dx$

A. $2\sqrt{2n}$

B. $\sqrt{2n}$

C. $\dfrac{1}{2\sqrt{2}}n$

D. None of these

Answer: A

4. Let $f(x) = x - [x]$, for ever real number x, $where[x]$ is the greatest integer less than or equal to x. Then, evaluate $\int_{-1}^{1} f(x)dx$.

A. 1

B. 2

C. 0

D. $\dfrac{1}{2}$

Answer: A

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5. $If \int_{0}^{x} f(t)dt = x + \int_{x}^{1} tf(t)dt$, then the value of $f(1)$ is

A. $\dfrac{1}{2}$

B. 0

C. 1

D. $-\dfrac{1}{2}$

Answer: A

6. The least value of the function

$$\phi(x) = \int\limits_{5\pi/4}^{x} (3\sin t + 4\cos t)\,dt$$

on the integral $[5\pi/4,\, 4\pi/3]$, is

a. $\sqrt{3} + \dfrac{3}{2}$

b. $1 - 2\sqrt{3} + \dfrac{3}{2} + \dfrac{1}{\sqrt{2}}$

c. $\dfrac{3}{2} + \dfrac{1}{\sqrt{2}}$

d. None of these

A. $\sqrt{3} + \dfrac{3}{2}$

B. $1 - 2\sqrt{3} + \dfrac{3}{2} + \dfrac{1}{\sqrt{2}}$

C. $\dfrac{3}{2} + \dfrac{1}{\sqrt{2}}$

D. None of these

Answer: B

---

7. The poins of extremum of $\phi(x) = \int\limits_{1}^{x} e^{-t^2/2}\left(1 - t^2\right)dt$ are

A. $x = 1, \ -1$

B. $x = -1, 2$

C. $x = 2, 1$

D. $x = -2, 1$

Answer: A

---

8. If f(x) is periodic function with period, T, then

A. $\int\limits_{a}^{b} f(x)dx = \int\limits_{a}^{b+T} f(x)dx$

B. $\displaystyle\int_a^b f(x) = \int_{a+T}^{b+T} f(x)dx$

C. $\displaystyle\int_a^b f(x)dx = \int_{a+T}^{b} f(x)dx$

D. $\displaystyle\int_a^b f(x)dx = \int_{a+T}^{b+2T} f(x)dx$

Answer: A

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9. Let $\dfrac{d}{dx}(F(x)) = \dfrac{e^{\sin x}}{x}$, $x > 0$. If $\displaystyle\int_1^4 \dfrac{2e^{\sin x2}}{x}dx = F(K) - F(1)$,

then the possible value of K is

A. 4

B. 8

C. 16

D. 32

Answer: C

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10. Let

$f: (0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt.$ If $F(x^2) = x^2(1+x)$, then

f(4) equals

A. $\dfrac{5}{4}$

B. 7

C. 4

D. 2

Answer: C

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11. Let $T > 0$ be a fixed real number. Suppose f is continuous function

such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$, then the

value of $\int_3^{3+3T} f(2x)dx$ is $\dfrac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\dfrac{3}{2}l$

B. $2l$

C. $3l$

D. $6l$

Answer: C

---

12. Let $f(x) = \displaystyle\int_1^x \sqrt{2 - t^2}\,dt$ Then the real roots of the equation $x^2 - f'(x) = 0$ are

A. ± 1

B. $\pm \dfrac{1}{\sqrt{2}}$

C. $\pm \dfrac{1}{2}$

D. $\pm \sqrt{2}$

Answer: A

13. The points of extremum of $\phi(x) = \int_1^x e^{-t^2/2} (1 - t^2) dt$ are

A. $g(x)$ is an odd function

B. $g(n) = 0$ for all $n \in N$

C. $g(2n) = 0$ for all $n \in N$

D. $g(x)$ is non-peridic

Answer: C

14. Let $f:[2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is

A. $[0, 2]$

B. $\left[-\frac{1}{4}, 4 \right]$

C. $\left[-\dfrac{1}{4}, 2 \right]$

D. None of these

Answer: C

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15. The value of $\displaystyle\lim_{x \to 0} \dfrac{2\int_0^{\cos x} \cos^{-1}(t)}{2x - \sin 2x} dx$ is

A. 0

B. $\dfrac{1}{2}$

C. $-\dfrac{1}{2}$

D. $\dfrac{2}{3}$

Answer: C

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16. If $\displaystyle\int_0^x \frac{bt\cos 4t - a\sin 4t}{t^2}\,dt = \frac{a\sin 4x}{x}$ for all $x \neq 0$, then a and b are

given by

A. $a = \dfrac{1}{4}, b = 1$

B. $a = 2, b = 2$

C. $a = -1b = 4$

D. $a = 2, b = 4$

Answer: A

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17. If $f(x) = \displaystyle\int_0^x \{f(t)\}^{-1}dt$ and $\displaystyle\int_0^1 \{f(t)\}^{-1} = \sqrt{2}$, then

A. $f(x) = \sqrt{2}x$

B. $f(x) = \sqrt{2\log_e x}$

C. $f(x) = \sqrt{3x - 1}$

D. None of these

18. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\,dt$, for all, $x \in (-1, 1)$ and f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

19. Consider the function defined on $[0, 1] \to R, f(x) = \dfrac{\sin x - x \cos x}{x^2}$, if $x \neq 0$ and $f(0) = 0$

$\displaystyle\int_0^1 f(x)$ is equal to

A. A. $1 - \sin(1)$

B. B. $\sin(1) - 1$

C. C. $\sin(1)$

D. D. $-\sin(1)$

Answer: A

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20. Evaluate $\displaystyle\int_0^2 x^3 dx$

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Exercise For Session 6

1. The value of

$$\int_0^{\pi/2} \frac{\log(1 + x \sin^2 \theta)}{\sin^2 \theta} \, d\theta, \; x \geq 0 \text{ is equal to}$$

A. $\frac{1}{\pi} \left(\sqrt{1 + x} - 1 \right)$

B. $\sqrt{\pi} \left(\sqrt{1 + x} - 1 \right)$

C. $\pi \left(\sqrt{1 + x} - 1 \right)$

D. None of these

Answer: C

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2. The value of $\lim\limits_{n \to \infty} \frac{1}{2} \sum\limits_{r-1}^{n} \left(\frac{r}{n + r} \right)$ is equal to

A. $1 - \log 2$

B. $\log 4 - 1$

C. $\log 2$

D. None of these

Answer: A

3. $\lim\limits_{n \to \infty} \left[\dfrac{(n+1)(n+2)\ldots 3n}{n^{2n}} \right]^{1/n}$ is equal to

A. $4e$

B. $\dfrac{e}{4}$

C. $\dfrac{4}{e}$

D. None of these

Answer: C

4. If $m, n \in N,$ then the value of $\displaystyle\int_a^b (x-a)^m (b-x)^n dx$ is equal to

A. $\dfrac{(b-a)^{m+n}.\, M!n!}{(m+n)}!$

B. $\dfrac{(b-a)^{m+n+1}.\, m!n!}{(m+n+1)!}$

C. $\dfrac{(b-a)^{m}.\, m!}{m!}$

D. None of these

Answer: B

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5. The value of $\displaystyle\lim_{n\to\infty}\left(\dfrac{n!}{n^n}\right)^{\frac{2n^4+1}{5n^5+1}}$ is equal

A. e

B. $\dfrac{2}{e}$

C. $\left(\dfrac{1}{e}\right)^{\frac{2}{5}}$

D. None of these

Answer: C

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6. The value of

$$\lim_{n \to \infty} n \left\{ \frac{1}{3n^2 + 8n + 4} + \frac{1}{3n^2 + 16n + 16} + \ldots + n\text{terms} \right\} \text{ is equal to}$$

A. $\frac{1}{2}\log\left(\frac{9}{5}\right)$

B. $\frac{1}{3}\log\left(\frac{9}{5}\right)$

C. $\frac{1}{4}\log\left(\frac{9}{5}\right)$

D. None of these

Answer: C

7. Evaluate $\int_1^3 x^2 dx$

8. The value of $f(k) = \int_0^{\pi/2} \log(\sin^2 \theta + k^2 \cos^2 \theta) d\theta$ is equal to

A. $\pi \log(1 + k) - \pi \log^2$

B. $\pi \log 2 - \log(1 + k)$

C. $\log(1 + k) - \pi \log 2$

D. None of these

Answer: A

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9. If $I(mn) = \int_0^1 x^m (1 - x)^n dx, (m, n\varepsilon I, m, n \geq 0)$, then

A. $\dfrac{m!n!}{(m + n + 2)!}$

B. $\dfrac{2m!n!}{(m + n + 1)!}$

C. $\dfrac{m!n!}{(m + n + 1)!}$

D. None of tese

Answer: C

---

10. The value of $I(n) = \int_0^\pi \dfrac{\sin^2 n\theta}{\sin^2 \theta} d\theta$ is $(\forall n \in N)$

A. (a)$n\pi$

B. (b)$\dfrac{n\pi}{2}$

C. (c)$\dfrac{n\pi}{4}$

D. (d)None of these

Answer: A

Exercise (Single Option Correct Type Questions)

1. $\displaystyle\int_0^4 \frac{(y^2 - 4y + 5)\sin(y - 2)dy}{[2y^2 - 8y + 1]}$ is equal to

A. 0

B. 2

C. -2

D. 2

Answer: A

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2. Let $f(x) = x^2 + ax + b$ and the only solution of the equation $f(x) = \min(f(x))$ is $x = 0$ and $f(x) = 0$ has root α and β, then $\displaystyle\int_\alpha^\beta x^3 dx$ is equal to

A. $\dfrac{1}{4}(\beta^4 + \alpha^4)$

B. $\dfrac{1}{4}(a^2 - b^2)$

C. 0

D. None of these

Answer: C

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3. $If \int_{\frac{\pi}{3}}^{x} \sqrt{\left(3 - \sin^2 t\right)} dt + \int_{0}^{y} \cos t dt = 0, then evaluate \frac{dy}{dx}$

A. $\sqrt{3}$

B. $-\sqrt{2}$

C. $-\sqrt{3}$

D. None of these

Answer: A

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4. $\int_{-4}^{4} \dfrac{\sin^{-1}(\sin x) + \cos^{-1}(\cos x)}{(1 + x^2)\left(1 + \left[\frac{x^2}{17}\right]\right)} dx = \log\left(\dfrac{(1 + \pi^2)}{\sqrt{a}}\right)$

$b\pi \ \tan^{-1}\left(\dfrac{c - \pi}{1 + c\pi}\right)$ (where, [.] denotes greatest inteher function), then

the number of ways in which $a - (2b + c)$ distinct object can distributed

among $\dfrac{a - 5}{c}$ persons equally , is

A. $\dfrac{9!}{(3!)^3}$

B. $\dfrac{12!}{(140^3)}$

C. $\dfrac{15!}{(5!)^3}$

D. $\dfrac{10!}{(6!)^3}$

Answer: A

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5. The value of the definite integral $\displaystyle\int_{0}^{\infty} \dfrac{dx}{(1 + x^a)(1 + x^2)}(a > 0)$ is

A. $\dfrac{\pi}{4}$

B. $\dfrac{\pi}{2}$

C. π

D. some function of a

Answer: A

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6. The value of the definite integral $\displaystyle\int_{0}^{3\pi/4} \left((1+x)\sin x + (1-x)\cos x \right) dx$ is

A. $2\dfrac{\tan(3\pi)}{8}$

B. $2\dfrac{\tan(\pi)}{4}$

C. $2\dfrac{\tan(\pi)}{8}$

D. 0

Answer: A

7. Let $C_n = \int_{1/n+1}^{1/n} \frac{\tan^{-1}(nx)}{\sin^{-1}(nx)} dx$, then $\lim_{n \to \infty} n^n. C_n$ is equal to

A. 1

B. 0

C. -1

D. $\dfrac{1}{2}$

Answer: D

8.　　　　　　　　　　　　　　　　　　　　If

$$\left[\int_0^1 \frac{dt}{t^2 + 2t\cos\alpha + 1}\right] x^2 - \left[\int_{-3}^3 \frac{t^2 \sin 2t}{t^2 + 1} dt\right] x - 2 = 0 (0 < \alpha < \pi)$$

then the value of x is

A. (a)$\pm \left(\dfrac{\sqrt{\alpha}}{2\sin\alpha} \right)$

B. $(b)\pm\sqrt{\dfrac{2\sin\alpha}{\alpha}}$

C. $(c)\pm\sqrt{\dfrac{\alpha}{\sin\alpha}}$

D. $(d)\pm2\sqrt{\dfrac{\sin\alpha}{\alpha}}$

Answer: D

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9. $If f(x) = e^{g(x)} \, and g(x) = \displaystyle\int_{2}^{x} \dfrac{tdt}{1+t^4}$, then find the value of $f'(2)$

A. $2/17$

B. 0

C. 1

D. Cannot be determined

Answer: A

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10. If a, b and c are real numbers, then the value of

$$\lim_{t \to 0} \left(\frac{1}{t} \int_0^t (1 + a \sin bx)^{c/x} dx \right) \text{ equals}$$

A. abc

B. $\dfrac{ab}{c}$

C. $\dfrac{bc}{a}$

D. $\dfrac{ca}{b}$

Answer: A

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11.

$$The value of (\lim)_{n \to \infty} \sum_{r=1}^{4n} \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r} + \sqrt{n})^2} is equal \to \frac{1}{35}(b)\frac{1}{4}(c)\frac{1}{10}(d)\frac{1}{5}$$

A. $\dfrac{1}{35}$

B. $\dfrac{1}{14}$

C. $\dfrac{1}{10}$

D. $\dfrac{1}{5}$

Answer: C

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12. Let $f(x) = \displaystyle\int_{-1}^{x} e^{t^2} dt$ and $h(x) = f(1 + g(x))$, where $g(x)$ is defined for all x, $g'(x)$ exists for all x, and $g(x) < 0 f$ or $x > 0$. If $h'(1) = e$ and $g'(1) = 1$, then the possible values which $g(1)$ can take

A. 0

B. -1

C. -2

D. -4

Answer: C

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13. If f be a function satisfying $f(x + y) = f(x) + f(y)$, $\forall x, y \in R$. If f(1) = k, then f(n), $n \in N$ is equal to

A. $e - \dfrac{1}{2}e^2 - \dfrac{5}{2}$

B. $e - e^2 - 3$

C. $\dfrac{1}{2}(e - 3)$

D. $e - \dfrac{1}{2}e^2 - \dfrac{3}{2}$

Answer: D

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14. Let $f(x) = \displaystyle\int_2^x \dfrac{dt}{\sqrt{1 + t^4}}$ and g be the

inverse of f. Then , the value of g'(0) is

A. (a) $l'(0)$

B. (b) $h'(0^-)$

C. (c) $h'(0^+)$

D. (d) $\displaystyle \lim_{x \to 0} \frac{1 - \cos x}{x \sin x}$

Answer: C

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15. For $f(x) = x^4 + |x|, \ \leq tI_1 = \displaystyle\int_0^\pi f(\cos x)\,dx$ and

$I_2 = \displaystyle\int_0^{\pi/2} f(\sin x)\,dx\,$then$\dfrac{I_1}{I_2}$ has the value equal to

A. 1

B. $1/2$

C. 2

D. 4

Answer: C

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16. Let f be a positive function. Let $I_1 = \displaystyle\int_{1-k}^{k} x f[x(1-x)]dx$,

$I_2 = \displaystyle\int_{1-k}^{k} f[x(1-x)]dx$, where $2k-1 > 0$. Then $\dfrac{I_1}{I_2}$ is

A. k

B. $1/2$

C. 1

D. 2

Answer: D

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17. Suppose that the quadratic function $f(x) = ax^2 + bx + c$ is non-negative on the interval [-1,1] . Then, the area under tha graph of f over the interval [-1,1] and the X - axis is given by the formula

A. $A = f(-1) + f(1)$

B. $A = f\left(-\dfrac{1}{2}\right) + f\left(\dfrac{1}{2}\right)$

C. $A = \dfrac{1}{2}[f(-1) + 2f(0) + f(1)]$

D. $A = \dfrac{1}{3}[f(-1) + 4f(0) + f(1)]$

Answer: D

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18. Let $I(a) = \displaystyle\int_0^\pi \left(\dfrac{x}{a} + a\sin x\right)^2 dx$ where a is positive real.

The value of a for which $I(a)$ attains its minimum value is

A. (a) $\sqrt{\pi\sqrt{\dfrac{2}{3}}}$

B. (b) $\sqrt{\pi\sqrt{\dfrac{3}{2}}}$

C. (c) $\sqrt{\dfrac{\pi}{16}}$

D. (d) $\sqrt{\dfrac{\pi}{13}}$

Answer: A

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19. The set of value of 'a' which satisfy the equation

$$\int_0^2 (t - \log_2 a)\,dt = \log_2\left(\frac{4}{a^2}\right) \text{ is}$$

A. $a \in R$

B. $a \in R^+$

C. $a < 2$

D. $a > 2$

Answer: B

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20. $\lim\limits_{x \to \infty} \left(x^3 \int_{-1/x}^{1/x} \frac{\ln(1 + t^2)}{1 + e^t}\,dt \right)$ is equal to `

A. $\dfrac{1}{3}$

B. $\dfrac{2}{3}$

C. 1

D. 0

Answer: D

---

21. The value of $\sqrt{\pi \left(\int_0^{2008} x|\sin \pi x|dx \right)}$ is equal to

A. $\sqrt{2008}$

B. $\pi\sqrt{2008}$

C. 1004

D. 2008

Answer: D

---

22. $\lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \dfrac{k}{n^2 + k^2}$, $x > 0$ is equal to

A.

B.

C.

D.

Answer: C

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23. Let $a > 0$ and $f(x)$ is monotonic increase such that

$f(0) = 0$ and $f(a) = b$, then $\int_0^a f(x)dx + \int_0^b f^{-1}(x)dx$ is equal to

(a)$a + b$ (b)$ab + b$

(c)$ab + a$ (d)ab

A. (a)$a + b$

B. (b)$ab + b$

C. (c)$ab + a$

D. (d)ab

Answer: D

---

24. Evaluate the definite integral: $\int_0^1 \left(\dfrac{xdx}{1+x} \right)$

A. π

B. 2π

C. 2

D. 1

Answer: A

---

25. The value of $\int_0^\infty e^{-\left(a^2 x\right)} dx$ is equal to

A. (a) 0

B. (b) 1

C. (c) $\dfrac{1}{2}$

D. (d) cannot be evaluated

Answer: A

26. $\lim\limits_{\lambda \to 0} \left(\displaystyle\int_0^1 (1+x)^\lambda dx \right)^{1/\lambda}$ Is equal to

(a) $2In2$ (b) $\dfrac{4}{e}$

(c) $In\dfrac{4}{e}$ (d) 4

A. (a) $2In2$

B. (b) $\dfrac{4}{e}$

C. (c) $In\dfrac{4}{e}$

D. (d) 4

Answer: B

27. If $2f(x) + f(-x) = \dfrac{1}{x}\sin\left(x - \dfrac{1}{x}\right)$, then the value of $\displaystyle\int_{1/e}^{e} f(x)\,dx$ is

A. 48

B. 64

C. 71

D. 52

Answer: A

28. The value of the definite integral $\displaystyle\int_{0}^{\pi/2} \sin x \sin 2x \sin 3x\,dx$ is equal to

A. $\dfrac{1}{3}$

B. $-\dfrac{2}{3}$

C. $-\dfrac{1}{6}$

D. $\dfrac{1}{6}$

Answer: D

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29. If f(x) is differentiable function and $(f(x). g(x))$ is differentiable at x = a, then

    A. g (x) must be differentiable at $x = a$

    B. g(x) may be non-differentiable at $x = a$

    C. g(x) may be discontinous at $x = a$

    D. None of the above

Answer: A

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30. The number of integral solutions of the equation

$$4 \int_0^\infty \frac{\log(t)\,dt}{x^2 + t^2} - \pi Iog2 = 0, \, x > 0, \text{ is}$$

A. 0

B. 1

C. 2

D. 3

Answer: C

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31. $\int_0^{16n^2 / \pi} \cos\frac{\pi}{2}\left[\frac{x\pi}{n}\right] dx$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: A

32. If $\int_{-2}^{-1} (ax^2 - 5) dx = 0$ and $5 + \int_{1}^{2} (bx + c) dx = 0$, then

A. $ax^2 - bx + x = 0$ has aleast one root in $(1, 2)$

B. $ax^2 - bx + c = 0$ has aleast one root in $(-2, -1)$

C. $ax^2 + bx + c = 0$ has atleast one root in $(-2, -1)$

D. None of these

Answer: B

33. The value of $\int_3^6 \left(\sqrt{x + \sqrt{12x - 36}} + \sqrt{x - \sqrt{12x - 36}} \right) dx$ is equal to

A. $6\sqrt{3}$

B. $4\sqrt{3}$

C. $12\sqrt{3}$

D. None of these

Answer: A

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34. Let $I_n = \int_{-n}^{n} \left(\{x + 1\} \cdot \{x^2 + 2\} + \{x^2 + 3\}\{x^2 + 4\} \right) dx$, where { } denote the fractional part of x. Find I_1.

A. $-\dfrac{1}{3}$

B. $-\dfrac{2}{3}$

C. $\dfrac{1}{3}$

D. None of these

Answer: B

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1. Find the locus of a point which moves such that its distance from x axis is five times its distance from y axis.

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Exercise (More Than One Correct Option Type Questions)

1. If $f(x) = \left[\sin^{-1}(\sin 2x)\right]$ (where, [] denotes the greatest integer function), then

A. $\displaystyle\int_0^{\pi/2} f(x)\,dx = \frac{\pi}{2} - \sin^{-1}(\sin 1)$

B. $f(x)$ is periodic with period π

C. $\displaystyle\lim_{x \to \frac{\pi+}{2}} f(x) = -1$

D. None of these

Answer: A::B::C

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2. Which of the following definite integral (s) vanishes ?

A. $\displaystyle\int_0^{\frac{\pi}{2}} \log(\cot x)\,dx$

B. $\displaystyle\int_0^{2\pi} \sin^3 x\,dx$

C. $\displaystyle\int_{\frac{1}{e}}^{e} \frac{dx}{x(\log x)^{1/3}}$

D. $\displaystyle\int_0^{\pi} \sqrt{\frac{1 + \cos 2x}{2}}\,dx$

Answer: A::C

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3. The equation $x^3 - 3x + 1 = 0$ has

   A. atleast one root in $(-1, 0)$

   B. alteast one root in $(0, 1)$

   C. alteast two roots in $(-1, 1)$

   D. no roots in $(-1, 1)$

Answer: A

---

4. Suppose

$$I_1 = \int_0^{\frac{\pi}{2}} \cos\left(\pi \sin^2 x\right) dx \text{ and } I_2 = \int_0^{\frac{\pi}{2}} \cos\left(2\pi \sin^2 x\right) dx \text{ and } I_3 = \int_0^{\frac{\pi}{2}} \cos$$

, then

   A. (a) $I_1 = 0$

   B. (b) $I_2 + I_3 = 0$

C. (c) $I_1 + I_2 + I_3 = 0$

D. (d) $I_2 = I_3$

Answer: A::B::C

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5. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \cos x \, dx$

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6. The function f is continuous and has the property $f(f(x)) = 1 - x$ for all $x \in [0, 1]$ and $J = \displaystyle\int_0^1 f(x) \, dx$. Then which of the following is/are true?

(A) $f\left(\dfrac{1}{4}\right) + f\left(\dfrac{3}{4}\right) = 1$

(B) $f\left(\dfrac{1}{3}\right) \cdot f\left(\dfrac{2}{3}\right) = 1$

(C) the value of J equals to $\dfrac{1}{2}$

(D) $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sin x\,dx}{(\sin x + \cos x)^3}$ has the same value as J

A. $f\left(\dfrac{1}{4}\right) + f\left(\dfrac{3}{4}\right) = 1$

B. $f\left(\dfrac{1}{3}\right) \cdot f\left(\dfrac{2}{3}\right) = 1$

C. the value of J equals to $1/2$

D. $\displaystyle\int_0^{\pi/2} \dfrac{\sin x\,dx}{(\sin x + \cos x)^3}$ has the value of as J

Answer: A::C::D

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7. Let $f(x)$ is a real valued function defined by

$$f(x) = x^2 + x^2 \int_{-1}^{1} tf(t)\,dt + x^3 \int_{-1}^{1} f(t)\,dt$$

then which of the following hold (s) good?

A. $\displaystyle\int_{-1}^{1} tf(t)\,dt = \dfrac{10}{11}$

B. $f(1) + f(-1) = \dfrac{30}{11}$

C. $\displaystyle\int_{-1}^{1} t f(t) dt > \int_{-1}^{1} f(t) dt$

D. $f(1) - f(-1) = \dfrac{20}{11}$

Answer: B::D

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8. Let $f(x)$ and $g(x)$ be differentiable functions such that

$f(x) + \displaystyle\int_0^x g(t) dt = \sin x(\cos x - \sin x)$ and $(f'(x))^2 + (g(x))^2 = 1$, then

respectively , can be

A. $\dfrac{1}{2}\sin 2x,\ \sin 2x$

B. $\dfrac{\cos 2x}{2},\ \cos 2x$

C. $\dfrac{1}{2}\sin 2x,\ -\sin 2x$

D. $-\sin^2 x,\ \cos 2x$

Answer: C

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9. Let $f(x) = \int_{-x}^{x} (t\sin at + bt + c)dt,$ where a,b,c are non-zero real numbers , then $\lim_{x \to 0} \dfrac{f(x)}{x}$ is

A. independent of a

B. independent of a and b , and has the value equals to c

C. independent a, b and c

D. dependent only on c

Answer: A,D

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10. Let $L = \lim_{n \to \infty} \int_{a}^{\infty} \dfrac{ndx}{1 + n^2x^2},$ where $a \in R,$ then L can be

A. π

B. $\pi/2$

C. 0

D. 1

Answer: C

Exercise (Passage Based Questions)

1. $Suppose$ $\displaystyle\lim_{x \to 0} \dfrac{\int_0^x \dfrac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \; > 0, r > 0$ and $b \neq 0$

If l exists and is non- zero, then

    A. $b > 1$

    B. $0 < b < 1$

    C. $b < 0$

    D. $b = 1$

Answer: D

2. Suppose $\lim\limits_{x \to 0} \dfrac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > > 0, r > 0$ and $b \neq 0$

If $p = 3$ and $l = 1$, then the value of 'a' is equal to

A. 8

B. 3

C. 6

D. $3/2$

Answer: A

3. Suppose $\lim\limits_{x \to 0} \dfrac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > > 0, r > 0$ and $b \neq 0$

If $p = 2$ and $a = 9$ and l exists , then the value of l is equal to

A. $\dfrac{3}{2}$

B. $\dfrac{2}{3}$

C. $\dfrac{1}{3}$

D. $\dfrac{7}{9}$

Answer: B

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4. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for

$0 \le x \le 1$.Given,

$f(x) = \displaystyle\int_0^1 e^{x+1}.\ f(t)dt$ and $g(x) = \displaystyle\int_0^1 e^{x+1} \cdot g(t)dt + x$ The value of

$f(1)$ equals

A. 0

B. 1

C. e^{-1}

D. e

Answer: A

5. Suppose $f(x)$ and $g(x)$ are two continuous function defined for $0 \le x \le 1$. Given ,

$$f(x) = \int_0^1 e^{x+1}. F(t)dt \text{ and } g(x) = \int_0^1 e^{x+t}. G(t)dt + x,$$

The value of $f(1)$ equals

A. (a) $\dfrac{2}{3 - e^2}$

B. (b) $\dfrac{3}{e^2 - 2}$

C. (c) $\dfrac{1}{e^2 - 1}$

D. (d) 0

Answer: A

6. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0 \leq x \leq 1$.Given,

$$f(x) = \int_0^1 e^{x+1} . f(t)dt \text{ and } g(x) = \int_0^1 e^{x+1} \cdot g(t)dt + x. \text{ The value}$$

of $f(1)$ equals

A. 0

B. $\dfrac{1}{3}$

C. $\dfrac{1}{e^2}$

D. $\dfrac{2}{e^2}$

Answer: B

7. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \dfrac{1}{2}\right)$

any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable ,$\forall x \in$ R

through $(0, 1)$ Tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the X- axists

The number of solutions $f(x) = 2ex$ is equal to

A. 0

B. 1

C. 2

D. None of these

Answer: B

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8. Evaluate $\int_0^1 3x^2\,dx$

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9. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \dfrac{1}{2}\right)$

any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable , $\forall x \in$ R

through $(0, 1)$ Tangents drawn to both the curves at the points with

equal abscissae intersect on the same point on the X- axists

The function $f(x)$ is

A. increasing for all x

B. non-monotonic

C. decreasing for all x

D. None of these

Answer: A

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10. $f(x) = \int_{0}^{x} \left(4t^4 - at^3\right) dt$ and $g(x)$ is quadratic satifying

$g(0) + 6 = g'(0) - c = g''(c) + 2b = 0.$ $y = h(x)$ intersect in 4

distinct points with abscissae $x_i, i = 1, 2, 3, 4$ such that

$$\sum \frac{i}{x_i} = 8, a, b, c \in R^+ \text{ and } h\text{ (x)=f' (x)`.}$$

'a' is equal to

   A. AP

   B. GP

   C. HP

   D. None of these

Answer: A

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11. $f(x) = \int_0^x \left(4t^4 - at^3 \right) dt$ and $g(x)$ is quadratic satifying

$g(0) + 6 = g'(0) - c = g''(c) + 2b = 0.\, y = h(x)$ intersect in 4

distinct points with abscissae $x_i,\, i = 1, 2, 3, 4$ such that

$$\sum \frac{i}{x_i} = 8, a, b, c \in R^+ \text{ and } h\text{ (x)=f' (x)`.}$$

'a' is equal to

   A. 6

B. 8

C. 20

D. 12

Answer: C

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12. $f(x) = \int_0^x (4t^4 - at^3) dt$ and $g(x)$ is quadratic satifying $g(0) + 6 = g'(0) - c = g''(c) + 2b = 0.$ $y = h(x)$ and y=g(x) intersect in 4 distinct points with abscissae $x_i, i = 1, 2, 3, 4$ such that $\sum \dfrac{i}{x_i} = 8, a, b, c \in R^+$ and h (x)=f' (x)`. 'c' is equal to

A. 25

B. $25/2$

C. $25/4$

D. $25/8$

Answer: A

---

13. Let $y = \int_{u(x)}^{y(x)} f(t)dt,$ let us define $\dfrac{dy}{dx}$ as $\dfrac{dy}{dx} = v'(x)f^2(v(x)) - u'(x)f^2(u(x))$ and the equation of the tangent at (a, b) and $y - b = \left(\dfrac{dy}{dx}\right)(a, b)(x - a)$.

If $f(x) = \int_{x}^{x^2} t^2 dt$ then $\dfrac{d}{dx} f(x) atx = 1$ is

A. $x + y = 1$

B. $y = x - 1$

C. $y = x$

D. $y = x + 1$

Answer: B

14. Let $y = \int_{u(x)}^{y(x)} f(t)dt,$ let us define

$\dfrac{dy}{dx}$ as $\dfrac{dy}{dx} = v'(x)f(v(x)) - u'(x)f(u(x))$ and the equation of the

tangent at (a, b) and $y - b = \left(\dfrac{dy}{dx}\right)(a, b)(x - a).$

If $y = \int_{x^2}^{x^4} (Int)dt,$ then $\lim\limits_{x \to 0^+} \dfrac{dy}{dx}$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: A

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15. Let $y = \int_{u(x)}^{y(x)} f(t)dt,$ let us define

$\dfrac{dy}{dx}$ as $\dfrac{dy}{dx} = v'(x)f^2(v(x)) - u'(x)f^2(u(x))$ and the equation of the

tangent at (a, b) and $y - b = \left(\dfrac{dy}{dx}\right)(a, b)(x - a).$

If $f(x) = \int_{x}^{x^2} t^2 dt$ then $\dfrac{d}{dx}f(x)$ at $x = 1$ is

A. 0

B. 1

C. 2

D. -1

Answer: A

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16. Check the injectivity and surjectivity of the function sinx.

A. Injective but nor surjective

B. Surjective but not bijective

C. Neither injective nor surjective

D. Both injective and surjective

Answer: C

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17. The value of $\displaystyle\int_0^{100} \left[\tan^{-1} x\right] dx$ is equal to (where $[.]$ denotes the greatest integer function)

A. $-\dfrac{\pi}{2}$

B. $\dfrac{\pi}{2}$

C. ∞

D. 1

Answer: C

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1. Let $\displaystyle \lim_{T \to \infty} \frac{1}{T} \int_0^T (\sin x + \sin ax)^2 dx = L$, **then**

Column I	Column II	
(A) For $a = 0$, the value of L is	(p)	0
(B) For $a = 1$, the value of L is	(q)	1/2
(C) For $a = -1$, the value of L is	(r)	1
(D) For $a \in R - \{-1, 0, 1\}$, the value of L is	(s)	2

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2. Let $\displaystyle f(\theta) = \int_0^1 (x + \sin \theta)^2 dx$ **and** $\displaystyle g(\theta) = \int_0^1 (x + \cos \theta)^2 dx$

where $\theta \in [0, 2\pi]$. The quantity $f(\theta) - g(\theta)$, $\forall \theta$ in the interval given in

column I, is

Column I	Column II	
(A) $\left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$	(p)	negative
(B) $\left(\dfrac{3\pi}{4}, \pi\right]$	(q)	positive
(C) $\left[\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right)$	(r)	non-negative
(D) $\left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{7\pi}{4}, 2\pi\right)$	(s)	non-positive

3. Differentiate the following function w.r.t x

$$\sec\left(\tan\left(x^{\frac{1}{2}}\right)\right)$$

4. Match the following

	Column I		Column II	
(A)	If $f(x) = \int_0^{g(x)} \dfrac{dt}{\sqrt{1 + t^3}}$, where $g(x) = \int_0^{\cos x} (1 + \sin t^2)\, dt$, then value of $f'\left(\dfrac{\pi}{2}\right)$ is		(p)	-2
(B)	If $f(x)$ is a non-zero differentiable function such that $\int_0^x f(t)\, dt = \{f(x)\}^2$, $\forall x \in R$, then $f(2)$ is equal to		(q)	2
(C)	If $\int_a^b (2 + x - x^2)\, dx$ is maximum, then $a + b$ is equal to		(r)	1
(D)	If $\lim\limits_{x \to 0} \left(\dfrac{\sin 2x}{x^3} + a + \dfrac{b}{x^2}\right) = 0$, then $3a + b$ has the value		(s)	-1

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Exercise (Single Integer Answer Type Questions)

1. If $f(x) = \sum\limits_{n=1}^{\infty} \dfrac{\sin nx}{4^n}$ and $\displaystyle\int_0^{\pi} f(x)\,dx = \log\left(\dfrac{m}{n}\right)$, then the value of $(m+n)$ is ....

2. The value of $I = \displaystyle\int_{-\pi/2}^{\pi/2} \dfrac{\cos x\,dx}{1 + 2\left[\sin^{-1}(\sin x)\right]}$ (where ,[.] denotes greatest integer function) is ...

3. If $f(x) = \dfrac{1}{\pi}\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sin^2(n\theta)}{\sin^2 \theta}\,d(\theta)$ then evaluate $\dfrac{f(15) + f(3)}{f(15) - f(9)}$

4. Let $f(x) = \displaystyle\int_{-2}^{x} e^{(1+t)^2}\,dt$ and $g(x) = f(h(x))$, where $h(x)$ is defined for all $x \in R$. If $g'(2) = e^4$ and $h'(2) = 1$ then absolute value of sum of

all possible values of $h(2)$, is ___

Exercise (Questions Asked In Previous 13 Years Exam)

1. The total number of distinct x $\in$ R for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10 \text{ is}$$

2. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all

$x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$. Then the correct expression (s) is (are)

A. $\displaystyle\int_0^{\pi/4} x f(x)\,dx = \dfrac{1}{12}$

B. $\displaystyle\int_0^{\pi/4} f(x)\,dx = 0$

C. $\displaystyle\int_0^{\pi/4} x f(x)\,dx = \dfrac{1}{6}$

D. $\displaystyle\int_0^{\pi/4} f(x)dx = \dfrac{1}{12}$

Answer: A::B

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3. Let $f'(x) = \dfrac{192x^3}{2 + \sin^4 \pi x}$ for all $x \varepsilon R$ with $f\left(\dfrac{1}{2}\right) = 0.$ If $m \leq \displaystyle\int_{1/2}^{1} f(x)dx \leq M,$ then the possible values of m and M are

A. $m = 13, M = 24$

B. $m = \dfrac{1}{4}, M = \dfrac{1}{2}$

C. $m = -11, M = 0$

D. $m = 1, M12$

Answer: B

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4. The option(s) with the values of a and L that satisfy the following equation is (are) $\dfrac{\int_0^{4\pi} e^t \left(\sin^6 at + \cos^4 at \right) dt}{\int_0^{\pi} e^t \left(\sin^6 at + \cos^4 at \right) dt} = L$

A. $a = 2,\ L = \dfrac{e^{4\pi} - 1}{e^{\pi}}$

B. $a = 2,\ L = \dfrac{e^{4\pi} + 1}{e^{\pi} + 1}$

C. $a = 4,\ L = \dfrac{e^{4\pi} + 1}{e^{\pi} + 1}$

D. $a = 4,\ L = \dfrac{e^{4\pi} + 1}{e^{\pi} + 1}$

Answer: A::C

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5. Let $F : R \to R$ be a thrice differntiable function. Suppose that $F(1) = 0,\ F(3) = -4$ and $F'(x) < 0$ for all $x \varepsilon (1, 3)$. Let $f(x) = xF(x)$ for all $x \in R$. Then the correct statement(s) is (are)

A. $f'(1) < 0$

B. $f(2) < 0$

C. $f'(x) \neq 0$ for any $x \in (1, 3)$

D. $f'(x) = 0$ for some $x \in (1, 3)$

Answer: A::B::C

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6. Let $F : R \to R$ be a thrice differntiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x\varepsilon(1, 3)$. Let $f(x) = xF(x)$ for all $x \in R$. Then the correct statement(s) is (are)

A. (a) $9f'(3) + f'(1) - 32 = 0$

B. (b) $\displaystyle\int_{1}^{3} f(x)dx = 12$

C. (c) $9f'(3) - f'(1) + 32 = 0$

D. (d) $\displaystyle\int_{1}^{3} f(x)dx = -12$

Answer: C::D

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7. If f(x)=x-[x], where [x] is the greatest integer less than or equal to x, then $f\left(+\dfrac{1}{2}\right)$ is:

8. If $\alpha = \displaystyle\int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\dfrac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1}$ takes only principal values, then the value of $\left((\log)_e|1+\alpha| - \dfrac{3\pi}{4}\right)$ is

9. The integral $\displaystyle\int_{\pi/4}^{\pi/2} (2\cos ecx)^{17}dx$ is equal to

A. $\displaystyle\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u + e^{-u}\right)^{16}du$

B. $\displaystyle\int_0^{\log\left(1+\sqrt{2}\right)} \left(e^u + e^{-u}\right)^{17}du$

C. $\displaystyle\int_0^{\log\left(1+\sqrt{2}\right)} \left(e^u - e^{-u}\right)^{17}du$

D. $\displaystyle\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u - e^{-u}\right)^{16} du$

Answer: a

10. Let $f:[0,2]\overrightarrow{R}$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0) = 1$ $Let: F(x) = \displaystyle\int_0^{x^2} f\left(\sqrt{t}\right) dt$ for x in $[0,2]$. If $F'(x) = f'(x)$. for all $x \in (0, 2)$, then $F(2)$ equals

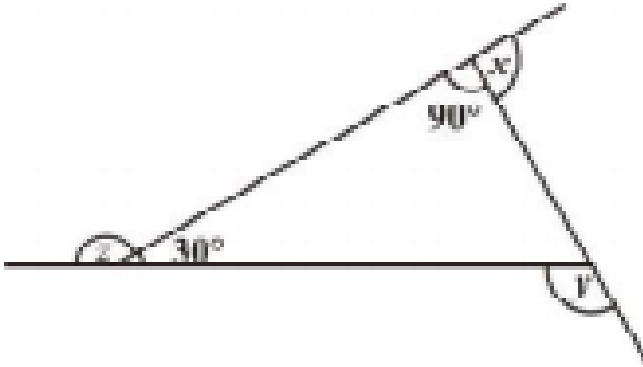A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. e^4

Answer: B

11. Match the conditions/ expressions in Column I with statement in Column II

	Column I		Column II
A.	$\int_{-1}^{1} \frac{dx}{1+x^2}$	p.	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
B.	$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	q.	$2\log\left(\frac{2}{3}\right)$
C.	$\int_{2}^{3} \frac{dx}{1-x^2}$	r.	$\frac{\pi}{3}$
D.	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	s.	$\frac{\pi}{2}$

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12. Find x+y+z

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13. The value of $\int_0^1 4x^3 \left\{ \dfrac{d^2}{dx^2} \left(1 - x^2 \right)^5 \right\} dx$ is

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14. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(x^2 + \log \dfrac{\pi - x}{\pi + x} \right) \cos x dx$

A. 0

B. $\dfrac{\pi^2}{2} - 4$

C. $\dfrac{\pi^2}{2} + 4$

D. $\dfrac{\pi^2}{2}$

Answer: B

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15. The value of $\displaystyle\int_{In2}^{In3} (x \sin x) dx$ is

A. $\dfrac{1}{4} \dfrac{\log(3)}{2}$

B. $\dfrac{1}{2} \dfrac{\log(3)}{2}$

C. $\dfrac{\log(3)}{2}$

D. $\dfrac{1}{6} \dfrac{\log(3)}{2}$

Answer: B::D

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16. Let $f:[1, \infty]$ be a differentiable function such that $f(1) = 2$. If

$$6\int_1^x f(t)dt = 3xf(x) - x^3 \text{ for all } x \geq 1, \text{ then the value of } f(2) \text{ is}$$

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17. The value of $\displaystyle\int_0^1 \dfrac{x^4(1-x)^4}{1+x^2}dx$ is (are)

A. $\dfrac{22}{7} - \pi$

B. $\dfrac{2}{105}$

C. 0

D. $\dfrac{71}{15} - \dfrac{3\pi}{2}$

Answer: A

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18. For $a \varepsilon R$ (the set of all real numbers) $a \neq -1$,

$$\lim_{n \to \infty} \frac{(1^a + 2^a + \ldots\ldots + n^a)}{(n+1)^{a-1}[(na+1) + (n+2) + \ldots\ldots + (na+n)]} = \frac{1}{60}$$

Then $a =$

A. 5

B. 7

C. $\dfrac{-15}{2}$

D. $\dfrac{-17}{2}$

Answer: B::D

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19. Let $f : [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable,f(0)=f(1)=0 and satisfies

$f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0, 1]$ Consider the statements.

I. There exists some $x \in R$ such that, $f(x) + 2x = 2(1 + x^2)$

(II) There exists some $x \in R$ such that, 2f(x)+1=2x(1+x)

A. both P and Q are true

B. P is true and Q is false

C. P is false and Q is true

D. both P and Q are false

Answer: C

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20. Which of the following is true?

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(1, \infty)$

C. g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$

D. g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

Answer: B

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21. $f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$

Then, the value of $\dfrac{\pi^2}{10} \displaystyle\int_{-10}^{10} f(x)\cos \pi \times dx$ is

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22. Let f be a non-negative function defined on the interval $[0, 1]$. If

$$\int_0^x \sqrt{1 - (f'(t))^2}\,dt = \int_0^x f(t)\,dt,\ 0 \le x \le 1,\ and\ f(0) = 0,\ \text{then}$$

a. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

b. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

c. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

d. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

A. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

B. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

C. $f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

D. $f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

Answer: C

---

23. Let $I_n = \int_{-\pi}^{\pi} \frac{\sin x}{(1 + \pi^x)\sin x} dx, n = 0, 1, 2, \ldots$ The value of $l_{n+2} - l_n$ is equal to

A. $I_n = I_{n+2}$

B. $\sum_{k=0}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{n} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: A::B::C

---

24. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for

$n = 1, 2, 3, \ldots$ Then

A. $S_n < \dfrac{\pi}{3\sqrt{3}}$

B. $S_n > \dfrac{\pi}{3\sqrt{3}}$

C. $T_n < \dfrac{\pi}{3\sqrt{3}}$

D. $T_n > \dfrac{\pi}{3\sqrt{3}}$

Answer: D

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25. Then integral $\displaystyle\int_{\pi/4}^{\frac{3\pi}{4}} \dfrac{dx}{1 + \cos x}$ is equal to

A. -1

B. -2

C. 2

D. 4

Answer: C

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26.

Let $I_n = \int \tan^n x\, dx, (n > 1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C i

A. $\left(-\dfrac{1}{5}, 0\right)$

B. $\left(-\dfrac{1}{5}, 1\right)$

C. $\left(\dfrac{1}{5}, 0\right)$

D. $\left(\dfrac{1}{5}, -1\right)$

Answer: C

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27. $\lim\limits_{n \to \infty} \left[\dfrac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right]^{1/n}$ is equal to

A. $\dfrac{18}{e^4}$

B. $\dfrac{27}{e^2}$

C. $\dfrac{9}{e^2}$

D. None of these

Answer: B

28. The integral $\displaystyle\int_2^4 \dfrac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)}dx$ is equal to

A. 2

B. 4

C. 1

D. 6

Answer: C

29. The integral $\int_0^\pi \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\,dx$ is equals to (a)$\pi - 4$ (b) $\frac{2\pi}{3} - 4 - \sqrt{3}$ (c)$\frac{2\pi}{3} - 4 - \sqrt{3}$ (d)$4\sqrt{3} - 4 - \frac{\pi}{3}$

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \pi/3$

Answer: D

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30. Prove that the value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}} = is\pi/6.$

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31. The intercepts on x- axis made by tangents to the curve, $y = \int_0^x |t|dt, x\varepsilon R$ which are parallel to the line $y = 2x$, are equal to

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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32. If $g(x) = \int_0^x \cos^4 tdt$, then $g(x + \pi)$ equals to (a)$\dfrac{g(x)}{g(\pi)}$ (b) $g(x) + g(\pi)$ (c)$g(x) - g(\pi)$ (d)$g(x). g(\pi)$

A. $\dfrac{g(x)}{g(\pi)}$

B. $g(x) + g(\pi)$

C. $g(x) - g(\pi)$

D. $g(x) \cdot g(\pi)$

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33. The value of $\displaystyle\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is

A. $\dfrac{\pi}{8}\log 2$

B. $\dfrac{\pi}{2}\log 2$

C. $\log 2$

D. $\pi \log 2$

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34. For $x\varepsilon\left(0, \dfrac{5\pi}{2}\right)$, definite $f(x) = \displaystyle\int_0^x \sqrt{t}\sin t\, dt.$ Then f has

A. local minimum at π and 2π

B. local minimum at π and local minimum at 2π

C. local minimum at π and local minimum at 2π

D. local maximum at π and 2π

Answer: C

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35. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then, $\int_0^1 p(x)$ equals

A. $\sqrt{41}$

B. 21

C. 41

D. 42

Answer: B

36. $\int_0^\pi [\cos x]\,dx$, $[]$ denotes the greatest integer function , is equal to

A. $\dfrac{\pi}{2}$

B. 1

C. (-1)

D. $-\dfrac{\pi}{2}$

Answer: D

37. Let $I = \int_0^1 \dfrac{\sin x}{\sqrt{x}}\,dx$ and $f = \int_0^1 \dfrac{\cos x}{\sqrt{x}}\,dx$

Then , which one of the following is true ?

A. $I > \dfrac{2}{3}$ and $f > 2$

B. $I < \dfrac{2}{3}$ and $f < 2$

C. $I < \dfrac{2}{3}$ and $f > 2$

D. $I > \dfrac{2}{3}$ and $f < 2$

Answer: B

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