MATHS

BOOKS - ARIHANT MATHS

DETERMINANTS

Examples

1. Find the value of the determinant

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 4 & 9 \\ 2 & -1 & 6 \end{vmatrix}$$

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2. If $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} \text{prove that} 2 \leq \Delta \leq 4.$

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3. Expand $\begin{vmatrix} 3 & 2 & 5 \\ 9 & -1 & 4 \\ 2 & 3 & -5 \end{vmatrix}$ by Sarrus rule.

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4. If a,b,c, $\in$ R, find the number of real root of the equation

$\begin{vmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{vmatrix}$=0

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5. Expand $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 6 & 2 \\ 5 & 9 & 4 \end{vmatrix}$

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6. Find the value of the determinant

$$\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$$

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7. Find the determinants of minors and cofactors of the determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 7 & 2 & -5 \\ 8 & -1 & 3 \end{vmatrix}$$

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8. Find the determinants of minors of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$

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9. If the value of a third order determinant is 11, find the value of the square of the determinat formed by the cofactors.

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10. Evaluate $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$.

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11. Prove that $\begin{vmatrix} \alpha & \beta & \gamma \\ \theta & \phi & \psi \\ \lambda & \mu & \nu \end{vmatrix} = \begin{vmatrix} \beta & \mu & \phi \\ \alpha & \lambda & \theta \\ \gamma & \nu & \psi \end{vmatrix}$.

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12. Using properties of determinants, prove that

$|b + cq + ry + zc + ar + pz + xc + bp + qx + y| = 2\,|apxbqycrz|$

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13. Without expanding as far as possible, prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$= $(x-y)(y-z)(z-x)(x+y+z)$.

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14. Solve for x,

$\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 6x+2 & 9x+3 & 12x \\ 8x+1 & 12x & 16x+2 \end{vmatrix}$=0

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15. Using the properties of determinant, show that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

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16. If a,b, and c are all different and if

$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$=0 Prove that abc =-1.

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17. find the larget value of a third- order determinant whose elements are 0 or 1.

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18. Find the largest value of a third order determinant whose elements are 0 or -1.

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19. Show that the value of a third order determinant whose all elements are 1 or -1 is an even number.

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20. Evaluate $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 3 & 2 \\ 3 & 4 & -4 \end{vmatrix} \times \begin{vmatrix} -2 & 1 & 3 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{vmatrix}$. Using the concept of multiplication of determinants.

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21. If $ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2 = ax_3^2 + by_3^2 + cz_3^2 = d$

$ax_2x_3 + by_2y_3 + cz_2z_3 = ax_3x_1 + by_3y_1 + cz_3z_1 =$

$ax_1x_2 + by_1y_2 + cz_1z_2 = f,$ then prove that

$|x_1y_1z_1\ x_2y_2z_2\ x_3y_3z_3|$=$(d - f)\left\{\dfrac{d + 2f}{abc}\right\}^{1/2}$

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22. Prove that $\begin{vmatrix} a_1\alpha_1 + b_1\beta_1 & a_1\alpha_2 + b_1\beta_2 & a_1\alpha_3 + b_1\beta_3 \\ a_2\alpha_1 + b_2\beta_1 & a_2\alpha_2 + b_2\beta_2 & a_2\alpha_3 + b_2\beta_3 \\ a_3\alpha_1 + b_3\beta_1 & a_3\alpha_2 + b_3\beta_2 & a_3\alpha_3 + b_3\beta_3 \end{vmatrix}$=0.

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23. Prove that

$$\begin{vmatrix} 2 & \alpha+\beta+\gamma+\delta & \alpha\beta+\gamma\delta \\ \alpha+\beta+\gamma+\delta & 2(\alpha+\beta)(\gamma+\delta) & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) \\ \alpha\beta+\gamma\delta & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) & 2\alpha\beta\gamma\delta \end{vmatrix} =$$

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24. For all values of A, B, C and P, Q, R show that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

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25. If α, β, γ are real numbers, then without expanding at any stage, show that

$$|1\cos(\beta-\alpha)\cos(\gamma-\alpha)\cos(\alpha-\beta)1\cos(\gamma-\beta)\cos(\alpha-\gamma)\cos(\beta-\gamma)1| =$$

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26. Solve the following system of equation by Cramer's rule.

$x+y=4$ and $3x-2y=9$

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27. Solve the following system of equation by Cramer's rule.

$x+y+z=9$

$2x+5y+7z=52$

$2x+y-z=0$

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28. For what values of p and q the system of equations

$x+y+z=6$

$x+2y+3z=10$

$x+2y+pz=q$ has

(i) unique solution ?

(ii) an infinitely many solutions ?

(iii) no solution ?

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29. If the following equations

$x+y-3z=0, (1+\lambda)x+(2+\lambda)y-8z=0, x-(1+\lambda)y+(2+\lambda)z$

are consistent then the value of λ is

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30. The equation $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0.$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$ and

$2x+(3\lambda+1)y+3(\lambda-1)z=0$

gives non-trivial solution for some values of λ, then the ratio x : y : z when λ has the smallest of these values :

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31. Given x=cy+bz,y=az+cx and z=bx+ay, then prove $a^2+b^2+c^2$+2abc =1.

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32. If $f(x)=\begin{vmatrix}\sin x & \cos x & \sin x\\ \cos x & -\sin x & \cos x\\ x & 1 & 1\end{vmatrix}$ find the value of $2[f'(0)]+[f'(1)]^2$

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33. Let f(x)=$\begin{vmatrix}\cos x & \sin x & \cos x\\ \cos 2x & \sin 2x & 2\cos 2x\\ \cos 3x & \sin 3x & 3\cos 3x\end{vmatrix}$ then find the value of $f'\left(\frac{\pi}{2}\right)$.

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34. Let α be a repeated root of a quadratic equation $f(x)=0 and A(x), B(x), C(x)$ be polynomials of degrees 3, 4, and 5, respectively, then show that

$|A(x)B(x)C(x)A(\alpha)B(\alpha)C(\alpha)A'(\alpha)B'(\alpha)C'(\alpha)|$ is divisible by $f(x)$

, where prime $(')$ denotes the derivatives.

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35. If $(x) = \begin{vmatrix} \alpha + x & \theta + x & \lambda + x \\ \beta + x & \varphi + x & \mu + x \\ \gamma + x & \psi + x & v + x \end{vmatrix}$ show that $\Delta^x = 0 and \Delta(0) + Sx$

,where S denotes he sum of all the cofactors of all elements in $\Delta(0)$ and

dash denotes the derivative with respect of $x \cdot$

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36. if $f(X) = \begin{vmatrix} x^n & n! & 2 \\ \cos x & \cos.\ \frac{n\pi}{2} & 4 \\ \sin x & \sin.\ \frac{n\pi}{2} & 8 \end{vmatrix}$, then find the value of

$\frac{d^n}{dx^n}[f(x)]_{x=0}.\ (n \in z).$

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37. If $\Delta(x) = \begin{vmatrix} a & b & c \\ 6 & 4 & 3 \\ x & x^2 & x^3 \end{vmatrix}$ then find $\int_0^1 \Delta(x)$dx.

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38. Let f(x)= $\begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cos ecx \\ \cos^2 x & \cos^2 x & \cos ec^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$

Prove that $\int_0^{\pi/2} f(x)dx = -\frac{\pi}{4} - \frac{8}{15}$.

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39. Let $\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix}$

Show that $\sum_{r=1}^{n-1} \Delta_r = 0$

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40. Let n be a positive integer and

$$\Delta_r = \begin{vmatrix} r^2+r & r+1 & r-2 \\ 2r^2+3r-1 & 3r & 3r-3 \\ r^2+2r+3 & 2r-1 & 2r-1 \end{vmatrix} \text{ and}$$

$\sum_{r=1}^{n} \Delta_r = an^2 + bn + c$ find the value of a+b+c.

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41. If a,b,c are comples number and z= $\begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix}$ then show tha z is purely imaginary

A. a non-zero real number

B. purely imaginary

C. 0

D. None of these

Answer:

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42. The equation

$$\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$$

A. no real solution

B. 4 real solution

C. two real and two non-real solutions

D. infinite number of solution real or non-real

Answer:

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43. If X, Y and Z are positive numbers such that Y and Z have respectively 1 and 0 at their unit's place and Δ is the determinant

$$\begin{vmatrix} X & 4 & 1 \\ Y & 0 & 1 \\ Z & 1 & 0 \end{vmatrix}$$

If $(\Delta + 1)$ is divisible by 10, then x has at its unit's place

A. 0

B. 1

C. 2

D. 3

Answer:

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44. The number of distinct values of $a2 \times 2$ determinant whose entries are from the set {-1,0,1}, is

A. 3

B. 4

C. 5

D. 6

Answer:

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45. Consider the polynomial fucntion

$$f(x)=\begin{vmatrix}(1+x)^2 & (1+2x)^b & 1\\ 1 & (1+x)^a & (1+2x)^b\\ (1+2x)^b & 1 & (1+x)^a\end{vmatrix}$$

a,b being positive integers.

Which of the following is true ?

A. constant term in f(x) is 4

B. constant term in f(x) is 0

C. constant term in f(x) is (a-b)

D. constant term in f(x) is (a+b)

Answer:

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46. Let $\Delta_a = \begin{vmatrix} (a-1) & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$ the value of $\sum_{a=1}^{n} \Delta_a$ is

A. 0

B. $\dfrac{(n-1)n}{2}$

C. $\dfrac{(n-1)n^2}{2}$

D. $\dfrac{(n-1)n(2n-1)}{3}$

Answer:

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47. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$ then $\int_0^{\pi/2} \Delta(x)$ dx

is equal to

A. $-\dfrac{1}{2}$

B. 0

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: A

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48. Number of values of a for which the system of equations $ax+(2-a)y=4+a^2$ and ax +(2a-1)y=a^5-2 possess no solution is

A. 0

B. 1

C. 2

D. infinite

Answer:

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49. The determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ is divisible by

A. $a+b+c$

B. $(a+b)(b+c)(c+a)$

C. $a^2+b^2+c^2$

D. $(a-b)(b-c)(c-a)$

Answer:

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50. The value of θ lying between $-\frac{\pi}{4}$ and $\frac{\pi}{2}$ and $0 \leq A \leq \frac{\pi}{2}$ and satisfying the equation

$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix}$ =0 are

A. $A=\frac{\pi}{4}, \theta=-\frac{\pi}{8}$

B. $A = \frac{3\pi}{8} = \theta$

C. $A = \frac{\pi}{5}, \theta = -\frac{\pi}{8}$

D. $A = \frac{\pi}{6}, \theta = -\frac{3\pi}{8}$

Answer:

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51. Statement-1 The digits A,B and C re such that the three digit number A88, 6B8, 86C are divisible by 72, then determinat $\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by 288.

Statement-2 A=B=?

A. 72

B. 144

C. 288

D. 216

Answer:

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52. Find the value of a and b for which the following system of equations has infinite number of solutions. $2x + 3y = 7, (a + b)x + (2a - b)y = 3(a + b + 1)$.

A. a=1,b=-1

B. a=1,b=-2

C. a=-1,b=-1

D. a=-1,b=-2

Answer:

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53. If $\Delta_r = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix}$ where is a natural number, the value of $\sqrt[10]{\sum_{r=1}^{1024} \Delta_r}$ is

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54. If P,Q and R are the angles of a triangle the value of $\begin{vmatrix} \tan P & 1 & 1 \\ 1 & \tan Q & 1 \\ 1 & 1 & \tan R \end{vmatrix}$ is

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55. Expand $\begin{vmatrix} 2 & 0 \\ 3x & 6 \end{vmatrix}$

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56. Suppose a, b and c are distinct and x,y and z are connected by the system of eqations

$x + ay + a^2 z = a^3, x + by + b^2 z = b^3 \text{and} x + cy + c^2 z = c^3.$

	Column I	Column II	
(A)	For $x = 1, y = 2$ and $z = 3, (a + b + c)^{-(ab + bc + ca)}$ is divisible by	(p)	3
(B)	For $x = 4, y = 3$ and $z = 2, (ab + bc + ca)^{abc}$ is divisible by	(q)	6
(C)	For $x = 6, y = 4$ and $z = 2, (abc)^{a + b + c}$ is divisible by	(r)	9
		(s)	12

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57. Consider the determinant

$$f(x) = \begin{vmatrix} 0 & x^2 - a & x^3 - b \\ x^2 + a & 0 & x^2 + c \\ x^4 + b & x - c & 0 \end{vmatrix}$$

Statement -1 f(x) =0 has one root x =0.

Statement -2 The value of skew -symmetric determinant of odd order is always zero.

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58. A determinant of second order is made with the elements 0 and 1. Find the number of determinants with non-negative values.

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59. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ =abc $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

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60. What is 6% Equals to

A. 0.06

B. 0.6

C. 0.006

D. 0.0006

Answer:

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61. Prove that: $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$

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62. if $bc+qr=ca+rp=ab+pq=-1$ then prove that

$\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$

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63. If $\alpha \text{and} \beta$ are the roots of the equations

$x^2-2x+4=0$,find the value of $\begin{vmatrix} \sum \alpha & \sum \alpha^2 & \sum \alpha^3 \\ \sum \alpha^2 & \sum \alpha^3 & \sum \alpha^4 \\ \sum \alpha^3 & \sum \alpha^4 & \sum \alpha^5 \end{vmatrix}$.

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64. If $a^2+b^2+c^2=1$,

$$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & ab(1-\cos\theta) & ac(1-\cos\theta) \\ ba(1-\cos\theta) & b^2(c^2+a^2)\cos\theta & bc(1-\cos\theta) \\ ca(1-\cos\theta) & cb(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$$

then prove that the value of determinant is independent of a,b,c?

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65. If a $\neq$ 0 and a $\neq$ 1, show that

$$\begin{vmatrix} x+1 & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} = a^3\left[1+x\frac{(a^3-1)}{a^2(a-1)}\right].$$

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66. If a $\neq$ 0 and a $\neq$ 1, show that

$$\begin{vmatrix} x+1 & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} = a^3\left[1+x\frac{(a^3-1)}{a^2(a-1)}\right].$$

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67. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$ then evaluate $\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$

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68. Without expanding at any stage, evaluate the value of the determinant

$$\begin{vmatrix} 2 & \tan A \cot B + \cot A \tan B & \tan A \cot C + \cot \\ \tan B \cot A + \cot B \tan A & 2 & \tan B \cot C + \cot \\ \tan C \cot A + \cot C \tan A & \tan B \cot C + \cot B \tan C & 2 \end{vmatrix}$$

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69. What is 11% Equals to

A. 0.11

B. 1.1

C. 0.011

D. 0.0011

Answer:

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70. If $\Delta = \begin{vmatrix} \sin x & \sin(x+h) & \sin(x+2h) \\ \sin(x+2h) & \sin x & \sin(x+h) \\ \sin(x+h) & \sin(x+2h) & \sin x \end{vmatrix}$

find $\lim_{h \to 0} \left(\frac{\Delta}{h^2}\right)$.

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71. If $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ then show that f(x) is linear in x.

Hence deduce $f(0) = \frac{bg(a) - ag(b)}{(b-a)}$ where

$g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

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72. If f(a,b) $= \dfrac{f(b)-f(a)}{b-a}$ and $f(a,b,c)=\dfrac{f(b,c)-f(a,b)}{c-a}$ prove that $f(a,b,c)=\begin{vmatrix} f(a) & f(b) & f(c) \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} \div \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$.

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73. the value of the determinant

$$\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 & (a_1-b_4)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 & (a_3-b_4)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 & (a_3-b_4)^2 \\ (a_4-b_1)^2 & (a_4-b_2)^2 & (a_4-b_3)^2 & (a_4-b_4)^2 \end{vmatrix}$$ is

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Expansion Of Determinant

1. Show that $\begin{vmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+cx & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{vmatrix} = \begin{vmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{vmatrix}^2$.

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2. Prove that

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

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3. Prove that

$$|(a-x)^2(a-y)^2(a-z)^2(b-x)^2(b-y)^2(b-z)^2(c-x)^2(c-y)^2(c-z)^2$$

$$|(1+ax)^2(1+bx)^2(1+cx)^2(1+ay)^2(1+by)^2(1+cy)^2(1+az)^2(1+bz)$$

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4. Find the coefficient of x in the determinant

$$\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$$

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5. Expand $\begin{vmatrix} \sin x & 2 \\ 3x & \sin x \end{vmatrix}$

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Single Option Correct Type Questions

1. If $f_i = \sum_{i=0}^{2} a_{ij} x^i$, j=1,2,3 and f_j and are denoted by $\frac{df}{dx}$ and $\frac{d^2 f_j}{dx^2}$ "respectively then g(x) = $\begin{vmatrix} f_1, f_2, f_3 \\ f_1', f_2', f_3' \\ f_1^{''}, f_2^{''}, f_3^{''} \end{vmatrix}$ is

A. a constant

B. a linear in x

C. a quadratic in x

D. a cubic in x

Answer:

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2. The equation $\frac{2a+3b}{(a-1)(b-3)}$=16 has unique solution if

A. a=2,b=3

B. a=2,b $\neq$ 3

C. a $\neq$ 2,b=3

D. a $\neq$ 2,b $\neq$ 3

Answer:

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3. What is 9% Equals to

A. 0.09

B. 0.9

C. 0.009

D. 0.0009

Answer:

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4. The system of equation $ax - y - z = a - 1$, $x - ay - z = a - 1$, $x - y - az = a - 1$ has no solution if a is,

A. 2

B. -1

C. Either 1 or -2

D. non of this

Answer:

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5. If f(x) is a polynomial of degree $n(> 2)$ and $f(x) = f(\alpha - x)$,

(where α is a fixed real number), then the degree of f'(x) is

A. n

B. n-1

C. n-2

D. 0

Answer:

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6. Expand $\begin{vmatrix} 2a & 3b \\ 5a & 7a \end{vmatrix}$

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7. If A,B and C are the angle of a triangle show that

$$\begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix} = 0.$$

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8. If $g(x) = \frac{f(x)}{(x-a)(x-b)(x-c)}$, where f(x) is a polynomial of degree < 3, then prove that

$$\frac{dg(x)}{dx} = \begin{vmatrix} 1 & a & f(a)(x-a)^{-2} \\ 1 & b & f(b)(x-b)^{-2} \\ 1 & c & f(c)(x-c)^{-2} \end{vmatrix} \div \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}.$$

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9. Let S be the sum of all possible determinants of order 2 having 0,1,2 and 3 as their elements,. Find the common root α of the equations

$x^2 + ax + [m+1] = 0,$

$x^2 + bx + [m+4] = 0$

and $x^2 - cx + [m+15] = 0$

such that $\alpha > S$wherea+b+c=0 and

$$m = \lim_{n \to 00} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$$

and [.] denotes the greates integer function.

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10. Solve $\begin{vmatrix} 0 & 0 & 7 \\ 1 & 2 & 3 \\ 0 & 8 & 2 \end{vmatrix}$

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Exercise For Session 1

1. Sum of real roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is

A. -2

B. -1

C. 0

D. 1

Answer: D

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2. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy,$ then

A. x=3, y=1

B. x=1, y=3

C. x=0,y=3

D. x=0, y=0

Answer: D

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3. Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & -2\lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$

be an indentity in λ p,q, r s and r are constants. Then find the value of t.

A. 7

B. 14

C. 21

D. 28

Answer: C

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4. If one root of the equation $\begin{vmatrix} 7 & 6 & x^2 - 13 \\ 2 & x^2 - 13 & 2 \\ x^2 - 13 & 3 & 7 \end{vmatrix}$ =0 is x=2 the sum of all other five roots is

A. $2\sqrt{15}$

B. -2

C. $\sqrt{20}+\sqrt{15}-2$

D. None of these

Answer: B

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5. What is 22% Equals to

A. 0.22

B. 2.2

C. 0.022

D. 0.0022

Answer: C

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6. If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, the maximum value of Δ is

A. -10

B. $-\sqrt{10}$

C. $\sqrt{10}$

D. 10

Answer: D

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7. If the value of the determinant $|(a, 1, 1)(1, b, 1)(1, 1, c)|$ is positive then

A. abc $>$ 1

B. abc $>$ -8

C. abc $<$ -8

D. abc $>$ -2

Answer: B

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Exercise For Session 2

1. If $\lambda \text{and} \mu$ are the cofactors of 3 and -2 respectively in the determinant $\begin{vmatrix} 1 & 0 & -2 \\ 3 & -1 & 2 \\ 4 & 5 & 6 \end{vmatrix}$ the value of $\lambda + \mu$ is

A. 5

B. 7

C. 9

D. 11

Answer: C

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2. If a, b and c are distinct and $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. then the square of the determinant of its cofactors is divisible by

A. $(a^2 + b^2 + c^2)^2$

B. $(ab + bc + ca)^2$

C. $(a + b + c)^2$

D. $(a + b + c)^4$

Answer: D

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3. An equilateral triangle has each of its sides of length 4 cm. If (x_r, y_r) (r=1,2,3) are its vertices the value of $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$

A. 192

B. 768

C. 1024

D. 128

Answer: A

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4. If the lines ax+y+1=0, x+by+1=0 and x+y+c=0 (a,b and c being distinct and different from 1) are concurrent the value of $\frac{1}{a-1}+\frac{1}{b-1}+\frac{1}{c-1}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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5. if $p+q+r=0=a+b+c$ then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

A. 0

B. pa+qb+rc

C. 1

D. None of these

Answer: A

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6. If p, q, r are in A.P. then value of determinant $\begin{vmatrix} a^2+2^{n+1}+2p & b^2+2^{n+2}+3q & c^2+p \\ 2^n+p & 2^{n+1} & 2q \\ a^2+2^n+p & b^2+2^{n+1} & c^2-r \end{vmatrix}$ is (a) 0 (b) Independent from a, b, c (c) $a^2b^2c^2-2^n$ (d) Independent from n

A. 1

B. 0

C. $a^2 + b^2 + c^2 - 2^n$

D. $(a^2 + b^2 + c^2) - 2^n$q

Answer: B

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7. Let $\{D_1, D_2, D_3, D_n\}$ be the set of third order determinant that can be made with the distinct non-zero real numbers a_1, a_2, a_q. Then $\sum_{i=1}^{n} D_i = 1$ b. $\sum_{i=1}^{n} D_i = 0$ c. $D_i - D_j, \forall i, j$ d. none of these

A. $\sum_{i=1}^{n} D_i = 1$

B. $\sum_{i=1}^{n} D_i = 0$

C. $D_i = D_j, \forall i, j$

D. None of these

Answer: B

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8. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0,$ then x is equal to

A. 0

B. -9

C. 3

D. None of these

Answer: B

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9. If $a+b+c=0,$ one root of $|a-xcbcb-xabac-x|=0$ is $x=1$ b. $x=2$ c. $x=a^2+b^2+c^2$ d. $x=0$

A. 1

B. 2

C. $a^2 + b^2 + c^2$

D. 0

Answer: D

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10. If $a^2 + b^2 + c^2 = -2 and f(x) =$

$\left|1 + a^2 x \left(1 + b^2\right) x \left(1 + c^2\right) x \left(1 + a^2\right) x 1 + b^2 x \left(1 + c^2\right) x \left(1 + a^2\right) x \left(1 + b^2\right) x\right.$

, then $f(x)$ is a polynomial of degree 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: C

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11. If $a, b, c, d, e, and f$ are in G.P. then the value of $\left|a^2 d^2 x b^2 e^2 y c^2 f^2 z\right|$ depends on $x and y$ b. $x and z$ c. $y and z$ d. independent of $x, y, and z$

A. depends on x and y

B. depends on x and z

C. dependes on y and z

D. independent of x,y and z

Answer: D

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Exercise For Session 3

1. Number of second order determiants which have maximum values whose each entry is either -1 or 1 is equal to

A. 2

B. 4

C. 6

D. 8

Answer: B

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2. Minimum value of a second order determinant whose each is either 1 or 2 is equal to

A. 0

B. -1

C. -2

D. -3

Answer: C

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3. If $l_i^2+m_i^2+n_i^2=1$, (i=1,2,3) and $l_i l_j+m_i m_j+n_i n_j=0, (i\ne j, i, j=1,2,3)$ and $\Delta=\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ then

A. $|\Delta|$=3

B. $|\Delta|$=2

C. $|\Delta|$=1

D. $|\Delta|$=0

Answer: C

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4. Let $A = [a_{ij}]$ be a 3×3 matrix and let A_1 denote the matrix of the cofactors of elements of matrix A and A_2 be the matrix of cofactors of elements of matrix A_1 and so on. If A_n denote the matrix of cofactros of elements of matrix A_{n-1}, then $|A_n|$ equals

A. Δ_0^n

B. Δ_0^{2n}

C. $\Delta_0^{n^2}$

D. Δ_0^2

Answer: B

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5. if $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 3$ then find the value of

$$\Delta_c = \begin{vmatrix} x^3 - 1 & 0 & x - x^4 \\ 0 & x - x^4 & x^3 - 1 \\ x - x^4 - 1 & x^3 - 1 & 0 \end{vmatrix}$$

A. 6

B. 9

C. 18

D. 27

Answer: B

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6. If $a_1, a_2, a_3 \text{and} b_1, b_2, b_3 \in$ R and are such that $a_i b_j \ne \text{for} 1 \le I, j \le 3$,

$$\begin{vmatrix} \frac{1-a_1^3 b_1^3}{1-a_{1b_1}} & \frac{1-a_1^3 b_2^3}{1-a_{1b_2}} & \frac{1-a_1^3 b_3^3}{1-a_{1b_3}} \\ \frac{1-a_2^3 b_1^3}{1-a_2 b_1} & \frac{1-a_2^3 b_2^3}{1-a_2 b_2} & \frac{1-a_2^3 b_3^3}{1-a_2 b_3} \\ \frac{1-a_3^3 b_1^3}{1-a_3 b_1} & \frac{1-a_3^3 b_2^3}{1-a_3 b_2} & \frac{1-a_3^3 b_3^3}{1-a_3 b_3} \end{vmatrix} >$$ 0 provided either

$$a_1 < a_2 < a_3 \quad \text{and } b_1 < b_2 < b_3 \quad \text{or } a_1 > a_2 > a_3 \quad \text{and } b_1 > b_2 > b$$

A. depends on a_i, i=1,2,3,4

B. depends on b_i,i=1,2,3,4

C. dependes on c_i,i=1,2,3,4

D. 0

Answer: D

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7. Value of $\begin{vmatrix} 1+x_1 & 1+x_1x & 1+x_1x^2 \\ 1+x_2 & 1+x_2x & 1+x_2x^2 \\ 1+x_3 & 1+x_3x & 1+x_3x^2 \end{vmatrix}$ depends upon

A. only x

B. only x_1

C. only x_2

D. None of these

Answer: D

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8. if the system of linear equations $x+y+z=6, x+2y+3z=14$ and $2x+5y+\lambda z=\mu(\lambda, \mu \in R)$ has a unique solution then

A. $\lambda \neq 8$

B. $\lambda = 8 \text{and} \mu \neq 36$

C. $\lambda = 8 \text{and} \mu = 36$

D. None of these

Answer: A

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9. The system of equations $ax-y-z=a-1, x-ay-z=a-1, x-y-az=a-1$ has no solution if a is:

A. either -2 or 1

B. -2

C. 1

D. not(-2)

Answer: B::C

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10. The system of equations x+2y=3,2x-3y=5 and x -12y =1 has

A. inconsistent solution

B. unique solution

C. infinitely many solutions

D. None of these

Answer: C

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11. if$c < 1$ and the system of equations x+y-1=0,2x-y-c=0 and -bx+3by -c=0 is consistent then the possible real values of b are

A. $b \in \left(-3, \frac{3}{4}\right)$

B. $b \in \left(-\frac{3}{2}, 1\right)$

C. $b \in \left(-\frac{3}{4}, 3\right)$

D. None of these

Answer: B

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12. The equation x+2y=3,y-2x=1 and 7x-6y+a=0 are consistent for

A. a=7

B. a=1

C. a=11

D. None of these

Answer: A

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13. The values of $k \in R$ for which the system of equations

$x + ky + 3z = 0, kx + 2y + 2z = 0, 2x + 3y + 4z = 0$ has nontrivial solution are

A. $\left\{2, \frac{5}{4}\right\}$

B. $\left\{2, \frac{5}{4}\right\}$

C. $\left\{2, -\frac{5}{9}\right\}$

D. $\left\{-2, -\frac{5}{4}\right\}$

Answer: A

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Exercise For Session 4

1. If f(x)= $\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2 & 3 \\ 0 & 1 & x \end{vmatrix}$ $\lim_{x \to 1} f(x)$ is equal to

A. -1

B. 0

C. 1

D. 2

Answer: A

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2. Let f(x) $= \begin{vmatrix} \sec x & x^2 & x \\ 2\sin x & x^3 & 2x^2 \\ \tan 3x & x^2 & x \end{vmatrix}$ $\lim_{x \to 0} \frac{f(x)}{x^4}$ is equal to

A. 0

B. -1

C. 2

D. 3

Answer: B

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3. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$ the value of 5A+4B+3C+2D+E is equal to

A. -16

B. -11

C. 0

D. 16

Answer: B

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4. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is

(a)p (b) $p - p^3$ (c)$p + p^3$ (d) independent of p

A. p

B. p+p^2

C. p+p^3

D. independent of p

Answer: D

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5. if y= sin mx, them the value of the determinant

$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ Where $y_n = \frac{d^n y}{dx^n}$ is

A. m^2

B. m^3

C. m^9

D. None of these

Answer: D

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6. Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$.then the value of $\int_0^{\pi/2} [f(x) + f'(x)]dx$ is

A. $\frac{\pi}{2}$

B. π

C. $\frac{2\pi}{2}$

D. 2π

Answer: B

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7. Find $\frac{dy}{dx}$ if y= sin(ax+b)

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8. Evaluate $\int \sin^2 x dx$

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9. If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 56.$ then n equals

A. 4

B. 6

C. 8

D. None of these

Answer: D

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10. the value of

$$\sum_{r=2}^{n}(-2)^r\begin{vmatrix} .^{n-2}C_{r-2} & .^{n-2}C_{r-1} & .^{n-2}C_r \\ -3 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix}(n>2)$$

A. $2n-1+(-1)^n$

B. $2n+1+(-1)^n$

C. $2n-3+(-1)^n$

D. None of these

Answer: A

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Exercise Single Option Correct Type Questions

1. if $\alpha, \beta, \neq 0$ and $f(n) = \alpha^n + \beta^n$

$$\text{and} \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$ then k is equal to

A. 1

B. -1

C. $\alpha\beta$

D. $\alpha\beta\gamma$

Answer: A

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2. Let $\Delta(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and $\int_0^2 \Delta(x)dx = -16,$

where a, b, c, d are in A.P. then the common difference (i) 1 (ii)2 (iii)3 (iv)4

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: B

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3. If $\Delta(x) = \begin{vmatrix} x & 1+x^2 & x^3 \\ \log(1+x) & e^x & \sin x \\ \cos x & \tan x & \sin^2 x \end{vmatrix}$ then

A. Δ(x) is divisible by x

B. Δ (x)=0

C. Δ'(x)=0

D. None of these

Answer: A

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4. If a, b, c are sides of a triangle and $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$ then

A. Δ ABC is an equilateral triangle

B. Δ ABC is a right angled isosceles triangle

C. Δ ABC is an isosceles triangle

D. None of the above

Answer: C

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5. Find $\frac{dy}{dx}$ if $x^2 + xy + y^2 = 100$

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6. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

A. (1,2)

B. (1,1)

C. (-2,1)

D. (1,0)

Answer: B

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7. If $f(x) = a + bx + cx^2$ and α, β, γ are the roots of the equation $x^3 = 1, then \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is equal to

A. $f(\alpha) + f(\beta) + f(\gamma)$

B. $f(\alpha)f(\beta) + f(\beta)f(\gamma) + f(\gamma)f(\alpha)$

C. $f(\alpha)f(\beta)f(\gamma)$

D. $-f(\alpha)f(\beta)f(\gamma)$

Answer: D

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8. When the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ expanded in powers of sin x, then the constant term in that expression is

A. 1

B. 0

C. -1

D. 2

Answer: C

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9. If $[]$ denotes the greatest integer less than or equal to the real number under consideration, and $-1 \leq x < 0, 0 \leq y < 1, 1 \leq z < 2,$ then the value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is a. $[x]$ b. $[y]$ c. $[z]$

d. none of these

A. [x]

B. [y]

C. [z]

D. None of these

Answer: C

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10. The determinant $\left|y^2 - xyx^2abca'b'c'\right|$ is equal to

A. (a)$\begin{vmatrix} bx+ay & cx+by \\ b'x+a'y & c'x+b'y \end{vmatrix}$

B. (b) $\begin{vmatrix} a'x+b'y & bx+cy \\ ax+by & b'x+c'y \end{vmatrix}$

C. (c) $\begin{vmatrix} bx+cy & ax+by \\ b'x+c'y & a'x+b'y \end{vmatrix}$

D. (d) $\begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

Answer: D

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11. If A, B, C are angles of a triangles, then the value of $e^{2iA}e^{-iC}e^{-iB}e^{-iC}e^{2iB}e^{-iA}e^{-iB}e^{-iA}e^{2iC}$ is

a. 1 b. -1 c. -2 d. -4

A. 1

B. -1

C. -2

D. -4

Answer: D

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12. If $\begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^a & a \\ x^{n+5} & x^{a+6} & x^{2n+5} \end{vmatrix} = 0, \forall x \in R, where. n \in N,$ then value of a is a. n b. $n-1$ c. $n+1$ d. none of these

A. n

B. n-1

C. n+1

D. None of these

Answer: C

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13. If x,y and z are the integers in AP lying between 1 and 9 and x 51, y 41 and z 31 are three digits number the value of $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is

A. x+y+z

B. x-y+z

C. 0

D. None of these

Answer: C

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14. if $a_1b_1c_1, a_2b_2c_2$ and $a_3b_3c_3$ are three-digit even natural numbers and $\Delta = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$ then Δ is

A. divisible by 2 but not necessarily by 4

B. divisible by 4 but not necessarily by 8

C. divisible by 8

D. None of these

Answer: A

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15. Expand $\begin{vmatrix} 4 & 8 \\ 6 & 7 \end{vmatrix}$

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16. If $x_1, x_2 \text{and} y_1, y_2$ are the roots of the equations $3x^2 - 18x + 9 = 0 \text{and} y^2 - 4y + 2 = 0$ the value of the determinant $\begin{vmatrix} x_1x_2 & y_1y_2 & 1 \\ x_1 + x_2 & y_1 + y_2 & 2 \\ \sin(\pi x_1x_2) & \cos(\pi / 2y_1y_2) & 1 \end{vmatrix}$ is

A. (a)0

B. (b)1

C. (c)2

D. (d)None of these

Answer: A

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17. If $\begin{vmatrix} .^9C_4 & .^9C_5 & .^{10}C_r \\ .^{10}C_6 & .^{10}C_7 & .^{11}C_{r+2} \\ .^{11}C_8 & .^{11}C_9 & .^{12}C_{r+4} \end{vmatrix} = 0$, then the value of r is equal to

A. 6

B. 4

C. 5

D. None of these

Answer: C

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18. The value of the determinant $\begin{vmatrix} 1 & \sin(\alpha-\beta)\theta & \cos(\alpha-\beta)\theta \\ a & \sin\alpha\theta & \cos\alpha\theta \\ a^2 & \sin(\alpha-\beta)\theta & \cos(\alpha-\beta)\theta \end{vmatrix}$ is independent of

(a)α

(B)β

(C)θ

(D)a

A. (a)α

B. (B)β

C. (C)θ

D. (D)a

Answer: A

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19. If $f(x), g(x)$ and $h(x)$ are three polynomials of degree 2 , then prove that $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

A. $2(3n + r)$

B. $3(2n - r)$

C. $3(2n + r)$

D. $2n(3n - r)$

Answer: D

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20. If f(x) = $\begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta+\gamma) & \sin(\gamma+\alpha) & \sin(\alpha+\beta) \end{vmatrix}$ then $f(\theta) - 2f(\phi) + f(\psi)$ is equal to

A. (A)0

B. (B)$\alpha - \beta$

C. (C)$\alpha + \beta + \gamma$

D. (D)$\alpha + \beta \pm \gamma$

Answer: A

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21. if $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

where a,b,c are all different then the determinant

$\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$ vanishes when

A. a+b+c=0

B. $x = \frac{1}{3}$(a+b+c)

C. $x = \frac{1}{2}$(a+b+c)

D. x=a+b+c

Answer: B

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22. Let $a, b, c \in R$ such that no two of them are equal and satisfy $\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix} = 0$, then equation $24ax^2 + 4bx + c = 0$ has

A. atleast one root in $[0, 1]$

B. atleast one root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. atleast one root in $[-1, 0]$

D. atleast one root in [0,2]

Answer: A

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23. What is 10% Equals to

A. 0.1

B. 1

C. 0.01

D. 0.001

Answer: B

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24. If f(x)=$ax^2+bx+c, a, b, c \in$ R and eqation f(x)-x=0 has imaginary roots $\alpha, \beta, \gamma \text{and} \ \delta$ be the roots of f(x) -x=0 then $\begin{vmatrix} 1 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$ is

A. 0

B. purely real

C. purely imaginary

D. None of these

Answer: B

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25. If the system of equtions 2x-y+z=0,x-2y+z=0, tx-y+2z=0 has infinitely many solution and f(x) be a continuous function such that f(5+x)+f(x)=2, then $\int_0^{-2t}$ f(x) dx is equal to

A. 0

B. $-2t$

C. 5

D. t

Answer: B

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26. If $\left(1+ax+bx^2\right)^4 = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_8x^8,$ where $a, b, a_0, a_1 \ldots\ldots, a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$ and

$$\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_0 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$$ then the value of $5.\dfrac{a}{b}$ is ____

A. $a = \dfrac{3}{4}, b = \dfrac{5}{8}$

B. $a = \dfrac{1}{4}, b = \dfrac{5}{32}$

C. $a = 1, b = \dfrac{2}{3}$

D. None of these

Answer: B

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27. Given f(x) $= \log_{10}$ x and g(x) = $e^{\pi i x}$.

$$\phi(x) = \begin{vmatrix} f(x).\,g(x) & (f(x))^{g(x)} & 1 \\ f(x^2).\,g(x^2) & (f(x^2))^{g(x^2)} & 0 \\ f(x^3).\,g(x^3) & (f(x^3))^{g(x^3)} & 1 \end{vmatrix}$$ the value of ϕ (10), is

A. 1

B. 2

C. 0

D. None of these

Answer: C

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28. The value of the determinant $\begin{vmatrix} 1 & \left(\alpha^{2x} - \alpha^{-2x}\right)^2 & \left(\alpha^{2x} + \alpha^{-2x}\right)^2 \\ 1 & \left(\beta^{2x} - \beta^{-2x}\right)^2 & \left(\beta^{2x} + \beta^{-2x}\right)^2 \\ 1 & \left(\gamma^{2x} - \gamma^{-2x}\right)^2 & \left(\gamma^{2x} + \gamma^{-2x}\right)^2 \end{vmatrix}$ is (a) 0 (b) $(\alpha\beta\gamma)^{2x}$ (c) $(\alpha\beta\gamma)^{-2x}$ (d) None of these

A. 0

B. $(\alpha\beta)^{2x}$

C. $(\alpha\beta)^{-2x}$

D. None of these

Answer: A

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29. If a, b, c are non-zero real numbers and if the system of equations $(a-1)x = y+z, (b-1)y = z+x, (c-1)z = x+y$ has a non-trivial solution, then prove that $ab+bc+ca = abc$

A. a+b+c=0

B. abc

C. 1

D. None of these

Answer: B

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30. the set of equations $\lambda x - y + (\cos\theta)z = 0, 3x + y + 2z = 0$

$(\cos\theta)x + y + 2z = 0$,$0 \leq \theta < 2\pi$ has non-trivial solution (s)

A. A. for no value of λ and θ

B. B. for all value of $\lambda$and θ

C. C. for all value of λ and only two values of θ

D. D. for only one value of λ and all values of θ

Answer: A

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Exercise More Than One Correct Option Type Questions

1. If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, then find $f'(x)$.

A. x

B. x^2

C. x^3

D. x^4

Answer: A::B::C::D

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2. The value of the determinant $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6}i \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ is (where i=$\sqrt{-1}$

A. complex

B. real

C. irrational

D. rational

Answer: B::D

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3. Let $\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix}$

Show that $\sum_{r=1}^{n-1} \Delta_r = 0$

A. 0

B. independent of n

C. independent of θ

D. independent of x,y, and z

Answer: A::B::C::D

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4. the determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ is equal to zero if

A. a,b and c are in AP

B. a,b,c, are in GP

C. a,b, and c are in HP

D. $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

Answer: B::D

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5. Let f(x) = $\begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then

A. $f\left(\frac{\pi}{3}\right) = -1$

B. $f'\left(\frac{\pi}{3}\right) = \sqrt{3}$

C. $\int_0^{\pi} f(x)dx = 0$

D. $\int_{-\pi}^{\pi} f(x)dx = 0$

Answer: A::C::D

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6. If $\Delta(x) = \begin{vmatrix} x^2-5x+3 & 2x-5 & 3 \\ 3x^2+x+4 & 6x+1 & 9 \\ 7x^2-6x+9 & 14x-6 & 21 \end{vmatrix}$ $=ax^3+bx^2+cx+d$, then

A. a=0

B. b=0

C. c=0

D. d=141

Answer: A::B::C::D

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7. If a,b,and c are the side of a triangle and A,B and C are the angles opposite to a,b,and c respectively, then

$\Delta = \begin{vmatrix} a^2 & b\sin A & C\sin A \\ b\sin A & 1 & \cos A \\ C\sin A & \cos A & 1 \end{vmatrix}$ is independent of

A. (A)a

B. (B)b

C. (C)c

D. (D)A,B,C

Answer: A::B::C::D

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8. $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix}$ is divisible by

A. $(a+b)$

B. $(a+2b)$

C. $(2a+b)$

D. a

Answer: A::B::C::D

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9. Let $f(x)=\begin{vmatrix}\sec^2 x & 1 & 1\\ \cos^2 x & \cos^2 x & \cos ec^2 x\\ 1 & \cos^2 x & \cot^2 x\end{vmatrix}$, then $\int_0^{\frac{\pi}{2}} f(x)dx=$

A. $\frac{1}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{8}$

D. $\frac{3\pi}{16}$

Answer: A::B::C::D

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10. Find the value of x Equation is $x+5=4$

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11. Expand $\begin{vmatrix}2 & 5\\ 4x & 7x\end{vmatrix}$

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12. If $f(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$

then

A. f'(x)=0

B. y=f(x) is a straight line parallel to X-axis

C. $\int_0^2 f(x)dx = 32a^4$

D. None of these

Answer: A::B

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13. If $a > b > c$ and the system of equtions $ax+by+cz=0$, $bx+cy+az=0, cx+ay+bz=0$ has a non-trivial solution then both the roots of the quadratic equation at^2+bt+c are

A. (a) real

B. (b) real & of opposite sign

C. (c) positive

D. (d) complex

Answer: A::B

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14. The values of λ and b for which the equations $x+y+z=3$, $x+3y+2z=6$, and $x+\lambda y+3z=b$ have

A. (a) a uniqe solution if $\lambda\neq 5, b\in R$

B. (b) no solution if $\lambda\neq 5, b=9$

C. (c) infinite many solution $\lambda=5, b=9$

D. (d) None of the above

Answer: A::C

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15. Let λ and α be real. Then the numbers of intergral values λ for which the system of linear equations

$\lambda x + (\sin \alpha)y + (\cos \alpha)z = 0$

$x + (\cos \alpha)y + (\sin \alpha)z = 0$

$-x + (\sin \alpha)y - (\cos \alpha)z = 0$

has non-trivial solutions is

A. $(-1, 1)$

B. $[-\sqrt{2}, \sqrt{2}]$

C. $[1, \sqrt{2}]$

D. $(-2, 2)$

Answer: A::B::C

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Exercise Passage Based Questions

1. Consider the system of equations

x+y+z=5, x+2y+3z=9, x+3y+$\lambda z = \mu$

The sustem is called smart brilliant good and lazy according as it has solution unique solution infinitely many solution respectively .

The system is smart if

A. $\lambda \ne 5 \text{or} \lambda = 5 \text{and} \mu = 13$

B. $\lambda \ne 5$

C. $\lambda \ne 5 \quad \text{and} \quad \mu \ne 13$

D. $\lambda \ne 5 \text{ or } \lambda \quad \text{and} \quad \mu \ne 13$

Answer: A

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2. Consider the system of equations

x+y+z=5, x+2y+3z=9, x+3y+$\lambda z = \mu$

The sustem is called smart brilliant good and lazy according as it has solution unique solution infinitely many solution respectively .

The system is smart if

A. $\lambda \neq 5$ or λ and $\mu \neq 13$

B. $\lambda = 5$ and $\mu = 13$

C. $\lambda = 5$ and $\mu \neq 13$

D. $\lambda \neq 5$and μ is any real number

Answer: B

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3. Consider the system of equations

x+y+z=5, x+2y+3z=9, x+3y+$\lambda z = \mu$

The sustem is called smart brilliant good and lazy according as it has solution unique solution infinitely many solution respectively .

The system is smart if

A. $\lambda \neq 5$or ' lambda $=5 \quad$ and $\mu = 13$

B. $\lambda = 5 \quad$ and $\quad \mu = 13$

C. $\lambda = 5 \quad$ and $\quad \mu \neq 13$

D. $\lambda \neq 5$or $\quad \lambda = 5$ and $\mu \neq 13$

Answer: C

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4. $If \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ then cofactor of a_{23} represented as

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5. Find |A| if $A = \begin{vmatrix} 5 & 2 \\ 6 & 3 \end{vmatrix}$

A. -7

B. 3

C. -2401

D. 2401

Answer: B

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6. $If \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and $C_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is a determinant obtained by deleting ith row and jth column then then $\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \Delta^2.$

Suppose a,b,c, $\in R, a+b+c>0, A = bc - a^2, B = ca - b^2$ and $c = ab - c^2$ and $\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix}$ =49 then the valu of $a^3+b^3+c^3$-3abc is

A. -3

B. 3

C. -7

D. 7

Answer: B

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7. If α, β, γ are the roots of $x^3 + 2x^2 - x - 3 = 0$ The value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{vmatrix}$ is equal to

A. A. 14

B. B. -2

C. C. 10

D. D. -14

Answer: D

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8. Let α, β, γ be the roots of $x^3 + 2x^2 - x - 3 = 0$. If the absolute value of the expression $\frac{\alpha - 1}{\alpha + 2} + \frac{\beta - 1}{\beta + 2} + \frac{\gamma - 1}{\gamma + 2}$ can be expressed as $\frac{m}{n}$ where m and n are co-prime the value of $\begin{vmatrix} m & n^2 \\ m - n & m + n \end{vmatrix}$ is

A. 17

B. 27

C. 37

D. 47

Answer: C

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9. If α, β, γ are the roots of $x^3 + 2x^2 - x - 3 = 0$. If $a = \alpha^2 + \beta^2 + \gamma^2, b = \alpha\beta + \beta\gamma + \gamma\alpha$ the value of $\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$ is

A. A. 14

B. B. 49

C. C. 98

D. D. 196

Answer: D

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10. Suppose f(x) is a function satisfying the folowing conditions:

(i)f(0)=2,f(1)=1

(ii) f(x) has a minimum value at $x = \frac{5}{2}$

(iii) for all x f'(x)= $\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

The value of f(2)+f(3) is

A. 1

B. $\frac{3}{2}$

C. 2

D. $\frac{5}{2}$

Answer: A

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11. Suppose f(x) is a function satisfying the folowing conditions:

(i)f(0)=2,f(1)=1

(ii) f(x) has a minimum value at $x=\frac{5}{2}$

(iii) for all xf'(x)= $\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

The number of solutions of the equation f(x) +1=0 is

A. (A)0

B. (B)1

C. (C)2

D. (D) infinite

Answer: A

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12. Suppose f(x) is a function satisfying the folowing conditions:

(i)f(0)=2,f(1)=1

(ii) f(x) has a minimum value at $x = \frac{5}{2}$

(iii) for all xf'(x)= $\begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$

The number of solutions of the equation f(x) +1=0 is

A. $\left(-\infty, \frac{7}{16}\right]$

B. $\left[\frac{3}{4}, \infty\right)$

C. $\left[\frac{7}{16}, \infty\right)$

D. $\left(-\infty, \frac{3}{4}\right]$

Answer: C

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13.

$$\begin{vmatrix} x & e^{x-1} & (x-1)^3 \\ x-\ln x & \cos(x-1) & (x-1)^2 \\ \tan x & \sin^2 x & \cos^2 x \end{vmatrix} = a_0 + a_1(x-1) + a_2(x-1)^2 \cdots$$

The value of $\cos^{-1}(a_1)$ is:

A. (a) 0

B. (b) $\frac{\pi}{4}$

C. (c) $\frac{\pi}{2}$

D. (d) π

Answer: C

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14. Find |A| if $A = \begin{vmatrix} 4x & 3x \\ 5x & 6x \end{vmatrix}$

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15. Expand $\begin{vmatrix} 8x & 3 \\ 2 & 2 \end{vmatrix}$

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16. Expand $\begin{vmatrix} 7x & 4 \\ x & 1 \end{vmatrix}$

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17. Expand $\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$

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18. Find $\frac{dy}{dx}$ if $x^3 - \lambda x^2 + 11x - 6 = y$

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Exercise Single Integer Answer Type Questions

1. If $\begin{vmatrix} 3^2+k & 4^2 & 3^2+3+k \\ 4^2+k & 5^2 & 4^2+4+k \\ 5^2+k & 6^2 & 5^2+5+k \end{vmatrix} = 0$, then $\sqrt{2^k\sqrt{2^k\sqrt{2^k\cdots\infty}}}$ is _____

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2. Let $\alpha, \beta \text{and} \ \gamma$ are three distinct roots of

$\begin{vmatrix} x-1 & -6 & 2 \\ -6 & x-2 & -4 \\ 2 & -4 & x-6 \end{vmatrix}$=0 the value of $\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^{-1}$ is

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3. Expand $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

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4. If cos α , cos β , cos γ are the direction-cosines of a line, then the value of

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma$ is _____ .

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5. Using properties of determinants, show that:

$$\left|\begin{bmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{bmatrix}\right| = 2abc(a+b+c)^3.$$

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6. If $0 \leq \theta \leq \pi$ and the system of equations

$x = (\sin\theta)y + (\cos\theta)z$

$y = z + (\cos\theta)x$

$z = (\sin\theta)x + y$

has a non-trivial solution then $\dfrac{8\theta}{\pi}$ is equal to

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7. Calculate the value of the determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$

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8. If a,b,c and d are the roots of the equation

$x^4 + 2x^3 + 4x^2 + 8x + 16 = 0$ the value of the determinant

$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$ is

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9. If $a \neq 0, b \neq 0, c \neq 0$ and $\begin{vmatrix} 1+a & 1 & 1 \\ 1+b & 1+2b & 1 \\ 1+c & 1+c & 1+3c \end{vmatrix}$=0

the value of $\left|a^{-1} + b^{-1} + c^{-1}\right|$ is equal to

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10. If the system of equations

ax+hy+g=0 ....(i)

hx+by+f=0...(ii)

and $ax^2 + 2hxy + by^2 + 2gx + 2fy + c + t = 0$....(iii)

has a unique solution and $\frac{abc + 2fgh - af^2 - bg^2 - ch^2}{h^2 - ab} = 8$, find the value of 't'.

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Determinants Exercise 5

1. Find the value of x if $A = \begin{vmatrix} 3x & 2 \\ 5x & 1 \end{vmatrix}$ if |A|= 5

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2. Find $f'(x)$ if $f(x) = \log(\cos e^x)$

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3. Expand $\begin{vmatrix} 2 & 2x \\ 6 & x \end{vmatrix}$

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4. Find |A| If $A = \begin{vmatrix} 7x & 3 \\ 5 & 6 \end{vmatrix}$

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5. If |adj(A)|= 11 and A is a square matrix of order 2then find the value of |A|

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Exercise Statement I And Ii Type Questions

1. If $\Delta(r) = \begin{vmatrix} r & r+1 \\ r+3 & r+4 \end{vmatrix}$ then expand it

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2. Expand $\begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}$

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3. Statement 1: The value of determinant

$\begin{vmatrix} \sin \pi & \cos\left(x+\frac{\pi}{4}\right) & \tan\left(-\frac{\pi}{4}\right) \\ \sin\left(x-\frac{\pi}{4}\right) & -\cos\left(\frac{\pi}{2}\right) & \ln\left(\frac{x}{y}\right) \\ \cot\left(\frac{\pi}{4}+x\right) & \log\left(\frac{y}{x}\right) & \tan(\pi) \end{vmatrix}$ is zero

Statement 2: The value of skew-symetric determinant of odd order equals zero.

A. (a) Both statement 1 and statement 2 is correct and statement 2 is the correct explanation for statement 1

B. (b) Both statement 1 and statement 2 is correct but statement 2 is not the correct explanation for statement 1

C. (c) Statement 1 is correct but statement 2 is incorrect

D. (d) Both statement 1 and statement 2 is incorrect

Answer: A

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4. Statement-1 f(x) = $\begin{vmatrix} (1+x)^{11} & (1+x)^{12} & (1+x)^{13} \\ (1+x)^{21} & (1+x)^{22} & (1+x)^{23} \\ (1+x)^{31} & (1+x)^{32} & (1+x)^{33} \end{vmatrix}$

the cofferent of x in f(x)=0

Statement -2 If P(x)=$a_0 + a_1 x + a_2 x^2 + a_2 x_3 + \cdots + a_n s^n$ then $a_1 = P'(0)$, where dash denotes the differential coefficient.

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5. Statement 1: If system of equations $2x + 3y = a$ and $bx + 4y = 5$ has infinite solutions, then $a = \frac{15}{4}, b = \frac{8}{5}$

Statement 2: Straight lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

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6. Statement -1 The value of the determent $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} \neq 0$

Statement -2 Neither of two rows rows or columns of $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix}$ is identical.

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7. The digits A,B,C are such that the three digit numbers A88, 6B8, 86 C are divisible by 72 the determinant

$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

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Exercise Subjective Type Questions

1. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ = 4 abc

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2. Prove: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

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3. Find the value of determinant $\begin{vmatrix} \sqrt{(13)}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{(15)}+\sqrt{(26)} & 5 & \sqrt{(10)} \\ 3+\sqrt{(65)} & \sqrt{(15)} & 5 \end{vmatrix}$

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4. Find the value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b, and c are respectively, the pth, qth, and rth terms of a harmonic progression.

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5. Without expanding the determinant at any stage prove that $\begin{vmatrix} -5 & 3+5i & \frac{3}{2}-4i \\ 3-5i & 8 & 4+5i \\ \frac{3}{2}+4i & 4-5i & 9 \end{vmatrix}$ has a purely real value.

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6. Prove without expansion that $\begin{vmatrix} ah+bg & g & ab+ch \\ bf+ba & f & hb+bc \\ af+bc & c & bg+fc \end{vmatrix} = a\begin{vmatrix} ah+bg & a & h \\ bf+ba & h & b \\ af+bc & g & f \end{vmatrix}$

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7. In a $\triangle ABC$, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$, then prove that $\triangle ABC$ is an isosceles triangle.

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8. The value of $\begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix}$ is

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9. If $y = \frac{u}{v}$, where u & v are functions of 'x' show that $v^3 \frac{d^2y}{dx^2}$ = $\begin{vmatrix} u & v & 0 \\ u' & v' & v \\ u'' & v'' & 2v' \end{vmatrix}$

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10. Show that the determinant $\Delta(x)$ given by $\Delta(x)$ = $\begin{vmatrix} \sin(x+\alpha) & \cos(x+\alpha) & a + x\sin\alpha \\ \sin(x+\beta) & \cos(x+\beta) & b + x\sin\beta \\ \sin(x+\gamma) & \cos(x+\gamma) & c + x\sin\gamma \end{vmatrix}$ is independent of x.

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11. Evaluate $\begin{vmatrix} .^x C_1 & .^x C_2 & .^x C_3 \\ .^y C_1 & .^y C_2 & .^y C_3 \\ .^z C_1 & .^z C_2 & .^z C_3 \end{vmatrix}$

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12. Find maximum value of

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}.$$

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13. If $f(x) = \begin{vmatrix} x^2-4x+6 & 2x^2+4x+10 & 3x^2-2x+16 \\ x-2 & 2x+2 & 3x-1 \\ 1 & 2 & 3 \end{vmatrix}$ then find the value of $\int_{-3}^{3} \frac{x^2 \sin x}{1+x^6} f(x) dx.$

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14. If |A| = 2 and $A = \begin{vmatrix} 2x & 6 \\ 5x & 1 \end{vmatrix}$ then find the value of x

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15. Find the value of t for which the following system of equations has a non- trival solution

$(a-t)x+by+cz=0,$

$bx+(c-t)y+az=0$ and

$cx+ay+(b-t)z=0.$

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16. Find $\frac{dy}{dx}$ if $y=13x-y^2$

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17. If x, y, z are not all zero & if $ax+by+cz=0, bx+cy+az=0 \& cx+ay+bz=0,$ then prove that $x:y:z=1:1:1$ OR $1:\omega:\omega^2 OR 1:\omega^2:\omega$, where ω is one ofthe complex cube root of unity.

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Determinants Exercise 7

1. If Y=sX and Z =tX all the varibles beings functions of x then prove that

$$\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = X^3 \begin{vmatrix} s_1 & t_1 \\ s_2 & t_2 \end{vmatrix}$$

where suffixes denote the order of differention with respect to x.

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2. If $f, g,$ and h are differentiable functions of x and (δ)=

$$\begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2 f)'' & (x^2 g)'' & (x^2 h)'' \end{vmatrix}$$

prove that δ'= $\begin{vmatrix} f & g & h \\ f' & g' & h \\ (x^3 f'')' & (x^3 g'')' & (x^3 h'')' \end{vmatrix}$

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3. If $|a_1| > |a_2| + |a_3|, |b_2| > |b_1| + |b_3|$ and

$|c_2| > |c_1| + |c_2|$ then show that $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \neq 0.$

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Exercise Questions Asked In Previous 13 Years Exam

1. If $a^2 + b^2 + c^2 = -2$, and $f(x) = \begin{vmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}$, then $f(x)$ is a polynomial of degree

A. 3

B. 2

C. 1

D. 0

Answer: B

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2. The value of $|\alpha|$ for which the system of equation $\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$ $x + y + \alpha z = \alpha - 1$ has no solutions, is ________.

A. not -2

B. 1

C. -2

D. Either -2 or 1

Answer: C

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3. if $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ form a G.P. and $a_1 > 0$, for all $I \geq 1$

$$\begin{vmatrix} \log a_n, & \log a_{n+1}, & \log a_{n+2} \\ \log a_{n+3}, & \log a_{n+4}, & \log a_{n+5} \\ \log a_{n+6}, & \log a_{n+7}, & \log a_{n+8} \end{vmatrix}$$

A. 1

B. 0

C. 4

D. 2

Answer: B

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4. If D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is

A. A. divisible by neither x nor y

B. B. divisible by both x and y

C. C. divisible by x but not y

D. D. divisible by y but not x

Answer: B

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5. Consider the system of equations

x-2y+3z=-1

-x+y-2z=k

x-3y+4z=1

Statement -1 The system of equation has no solutions for $k \ne 3$.

statement -2 The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \ne 0, \text{ for } \quad k \ne 3.$

A. (a)Satement -1 is true Statement -2 is true and Statement -2 is correct explanation for Statement -1.

B. (b)Satement -1 is true Statement -2 is true and Statement -2 is not a correct explanation for Statement -1.

C. (c)Statement -1 is true Statement -2 is false.

D. (d)Statement-1 is false, Statement -2 is true.

Answer: A

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6. Let a,b,c, be any real number. Suppose that there are real numbers x,y,z not all zero such that x=cy+bz,y=az+cx and z=bx+ay. Then $a^2+b^2+c^2$ +2abc is equal to

A. -1

B. 0

C. 1

D. 2

Answer: C

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7. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ =abc $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

A. any integer

B. zero

C. an even integer

D. any odd integer

Answer: D

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8. If $f(\theta)=\begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$ then the set $\left\{f(\theta): 0 \le \theta < \frac{\pi}{2}\right\}$ is

A. $(-\infty, -1) \cup (1, \infty)$

B. $[2, \infty)$

C. $(-\infty, 0] \cup [2, \infty)$

D. $(-\infty, -1] \cup [1, \infty)$

Answer: B

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9. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is

A. zero

B. 3

C. 2

D. 1

Answer: C

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10. If the trivial solution is the only solution of the system of equations

$x - ky + z = 0,$

$kx + 3y - kz = 0,$

$3x + y - z = 0$

Then the set of all values of k is:

A. {2,-3}

B. R-{2,-3}

C. R-{2}

D. R-{-3}

Answer: B

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11. The number of values of k, for which the system of equations $(k+1)x+8y=4k\ kx+(k+3)y=3k-1$ has no solution, is

A. 1

B. 2

C. 3

D. infinite

Answer: A

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12. If f(n)=$\alpha^n+\beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$ $=k(1-\alpha)^2(1-\beta)^2(\alpha=\beta)^2, k^2$ d is equal to

A. 1

B. -1

C. $\alpha\beta$

D. $1/\alpha\beta$

Answer: A

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13. The set of all values of λ for which the system of linear equations

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$

$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$

$$-x_1 + 2x_2 = \lambda x_3$$

has a non-trivial solution,

A. contains two elements

B. contains more than two elements

C. is an empty set

D. is a singleton

Answer: A

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14. Which of the following values of α satisfying the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$?

A. -4

B. 9

C. -9

D. 4

Answer: B::C

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15. The system of linear equations $x+y+z=2, 2x+y-z=3, 3x+2y+kz=4$ has a unique solution if (A) $k \neq 0$ (B) $-1 < k < 1$ (C) $-2 < k < 2$ (D) $k = 0$

A. exactly one-value of λ

B. exactly two values of λ

C. exactly three values of λ

D. infinitely many values of λ

Answer: C

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16. The total number of distinct $x \in R$ for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10 \quad \text{is} \quad \_\_\_\_\_\_$$

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17. Let $a, \lambda, \mu \in R$.Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is (are) correct ?

A. If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

B. If $a \neq -3$, then the system has a unique solution for all values of $\lambda \text{and} \mu$

C. If $\lambda + \mu$ =0 , then the system has infinitely many solutions for $a = -3$

D. If $\lambda + \mu \neq$ 0 then the system has no solution for $a = -3$

Answer: B::C::D

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18. If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution then S is

A. an infinite set

B. a finite set containing two or more elements

C. a singleton

D. an empty set

Answer: C

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