MATHS

BOOKS - ARIHANT MATHS

DIFFERENTIAL EQUATION

Example

1. Find the order and degree (if defined) of the following differential equations $\left(\dfrac{d^3y}{dx^3}\right)^3 = \dfrac{dy}{dx} + 2$

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2. Write order and degree (if defined) of each of the following differential equations.

$$\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$$

3. Form the differential equation, if $y^2 = 4a(x + a)$, where a is arbitary constant.

4. Find the differential equation of $xy = ae^x + be^{-x}$.

5. Find the differential equation whose solution represents the family :

$c(y + c)^2 = x^3$

6. Find the differential equation whose solution represents the family $y = ae^x + be^x$.

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7. Find the order of the family of curves $y = (c_1 + c_2)e^x + c_3 e^{x + c_4}$

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8. The differential equation of all non-horizontal lines in a plane is given by (i) $\dfrac{d^2y}{dx^2} = 0$ (ii) $\dfrac{d^2y}{dy^2} = 0$ (iii) $\dfrac{d^2y}{dx^2} = 0$ and $\dfrac{d^2y}{dy^2} = 0$ (iv) All of these

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9. The differential equation of all non-vectical lines in a plane is given by

(i) $\dfrac{d^2y}{dx^2} = 0$

(ii) $\dfrac{d^2x}{dy^2} = 0$

(iii) $\dfrac{d^2x}{dy^2} = 0$ and $\dfrac{d^2y}{dx^2} = 0$

(iv) All of these

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10. The differential equation of all straight lines which are at a constant distance p from the origin, is

(a) $(y + xy_1)^2 = p^2 (1 + y_1^2)$

(b) $(y - xy_1^2) = p^2 (1 + y_1)^2$

(c) $(y - xy_1)^2 = p^2 (1 + y_1^2)$

(d) None of these

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11. The differintial eauation of all circle of radius r, is given by

(a) $\left\{ 1 + (y_1)^2 \right\}^2 = r^2 y_2^3$

(b) $\left\{ 1 + (y_1)^2 \right\}^3 = r^2 y_2^3$

(c) $\left\{1 + (y_1)^2\right\}^3 = r^2 y_2^2$

(d) None of these

12. The differential equations of all circle touching the x-axis at orgin is

(a) $(y^2 - x^2) = 2xy\left(\dfrac{dy}{dx}\right)$

(b) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$

(c) $(x^2 - y^2) = 2xy\left(\dfrac{dy}{dx}\right)$

(d) None of these

13. The differential equation of all circle in the first quadrant touch the

coordinate is

(a) $(x - y)^2(1 + y')^2 = (x + yy')^2$

(b) $(x - y)^2(1 + y')^2 = (x + y')^2$

(c) $(x - y)^2(1 + y') = (x + yy')^2$

(d) None of these

A. (x−y)2 (1+(y')^2=(x+yy')2

B. (x−y)2(+y')2)=(x+y')2

C. (x−y)2(1+(y')^2)=(x+yy')^2

D. None of these

Answer:

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14. The differential equation satisfying the curve $\dfrac{x^2}{a^2 + \lambda} + \dfrac{y^2}{b^2 + \lambda} = 1$ when λ begin arbitary uknowm, is

(a) $(x + yy_1)(xy_1 - y) = (a^2 - b^2)y_1$

(b) $(x + yy_1)(xy_1 - y) = y_1$

(c) $(x - yy_1)(xy_1 + y)(a^2 - b^2)y_1$

(d) None of these

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15. The differential equation of all conics whose centre k lies at origin, is given by

(a) $\left(3xy_2 + x^2 y_3\right)\left(y - xy_1\right) = 3xy_2\left(y - xy_1 - x^2 y_2\right)$

(b) $\left(3xy_1 + x^2 y_2\right)\left(y_1 - xy_3\right) = 3xy_1\left(y - xy_2 - x^2 y_3\right)$

(c) $\left(3xy_2 + x^2 y_3\right)\left(y_1 - xy\right) = 3xy_1\left(y - xy_1 - x^2 y_2\right)$

(d) None of these

16. Solve $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

17. Solve the differential equation: $\dfrac{dy}{dx} = e^{x+y} + x^2 e^y$

18. Solve the following differential equation:

$$\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy\frac{dy}{dx} = 0$$

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19. Solve $y - x\dfrac{dy}{dx} = a\left(y^2 + \dfrac{dy}{dx}\right)$

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20. Solve $e^{\frac{dy}{dx}} = x + 1$, given that when $x = 0, y = 3$.

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21. Solve $\dfrac{dy}{dx} = \sin^2(x + 3y) + 5$

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22. Solve $(x + y)^2 \dfrac{dy}{dx} = a^2$

23. Solve $(2x + 3y - 1)dx + (4x + 6y - 5)dy = 0.$

24. The solution of $\dfrac{xdx + ydy}{xdy - ydx} = \sqrt{\dfrac{a^2 - x^2 - y^2}{x^2 + y^2}}$, is given by

25. Solve $\left[2\sqrt{xy} - x\right]dy + ydx = 0$

26. Solve the differential equation $(x^2 + y^2)\,dx = 2xy\,dy$

27. Solve the differential equation $2\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{y^2}{x^2}$

28. Solve $\left(1 + 2e^{x/y}\right)dx + 2e^{x/y}(1 - x/y)dy = 0.$

29. Solve that any equation of the fomr $yf'(xy)dx + xf'(xy)dy = 0$

30. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2y - 6x - 4}{y - 3x + 3}.$

31. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x + 2y - 1}{x + 2y + 1}$.

32. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{\sin y + x}{\sin 2y - x \cos y}$ is

(a) $\sin^2 y = x \sin y + \dfrac{x^2}{2} + C$

(b) $\sin^2 y = x \sin y - \dfrac{x^2}{2} + C$

(c) $\sin^2 y = x + \sin y + \dfrac{x^2}{2} + C$

(d) $\sin^2 y = x - \sin y + \dfrac{x^2}{2} + C$

33. solve the differential equation given by

$\dfrac{dy}{dx} = \dfrac{-3x - 2y + 5}{2x + 3y + 5}$

34. Solve $\dfrac{dy}{dx} + 2y = \cos x.$

35. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = \log x.$

36. Solve $\dfrac{dy}{dx} = \dfrac{y}{2yIny + y - x}.$

37. Solution of the equation $\cos^2 x \dfrac{dy}{dx} - (\tan 2x)y = \cos^4 x,$ where $|x| < \dfrac{\pi}{4}$ and $y\left(\dfrac{\pi}{6}\right) = \dfrac{3\sqrt{3}}{8}$ is

38. Solve $\dfrac{dy}{dx} + y\phi'(x) = \phi(x).\,\phi'(x)$, where $\phi(x)$ is a given function.

39. Solve (y log x-1) y dx=xdy.

40. Solve $\dfrac{dy}{dx} + xy = xy^2$

41. Solve $\dfrac{dy}{dx} = \dfrac{y\phi'(x) - y^2}{\phi(x)}$, where $\phi(x)$ is a given function.

42. $\sec^2 y \dfrac{dy}{dx} + 2x \tan y = x^3$

43. Solve $\dfrac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$.

44. Solve $\sin y. \dfrac{dy}{dx} = \cos y(1 - x \cos y)$.

45. Find the orthogonal trajectories of $xy = c$

46. The orthogonal trajectories of the family of curves $y = Cx^2$, (C is an arbitrary constant), is

47. The differential equation representing all possible curves that cut each member of the family of circles $x^2 + y^2 - 2Cx = 0$ (C is a parameter) at right angle, is

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48. The orthogonal trajectories of the circle $x^2 + y^2 - ay = 0$, (where a is a parameter), is

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49. Solve $(x^2 - ay) dx + (y^2 - ax) dy = 0$.

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50. Solve $(2x \log y) dx + \left(\dfrac{x^2}{y} + 3y^2 \right) dy = 0$.

51. Solve $\left(x^2 - ay\right)dx + \left(y^2 - ax\right)dy = 0.$

52. Solve $x\,dx + y\,dy = x\,dy - y\,dx.$

53. Solve

$$\frac{y + \sin x \cos^2(xy)}{\cos^2(xy)}dx + \left(\frac{x}{\cos^2(xy)} + \sin y\right)dy = 0.$$

54. Solve $\dfrac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \dfrac{y^4}{x^2}$

55. The solution of

$$e^{x \cdot \frac{(y^2 - 1)}{y}} \left\{ xy^2 dy + y^3 dx \right\} + \left\{ ydx - xdy \right\} = 0, \text{ is}$$

A. $e^{xy} + e^{x/y} + c = 0$

B. $e^{xy} - e^{x/y} + c = 0$

C. $e^{xy} + e^{y/x} + c = 0$

D. $e^{xy} - e^{y/x} + c = 0$

Answer:

56. The solution of $x^2 dy - y^2 dx + xy^2 (x - y) dy = 0$, is

A. $\log \left| \dfrac{x - y}{xy} \right| = \dfrac{y^2}{2} + c$

B. $\log \left| \dfrac{xy}{x - y} \right| = \dfrac{x^2}{2} + c$

C. $\log\left|\dfrac{x-y}{xy}\right| = \dfrac{x^2}{2} + c$

D. $\log\left|\dfrac{x-y}{xy}\right| = x + c$

Answer:

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57. The solution of the differential equation $y\,dx - x\,dy + xy^2\,dx = 0,$ is

A. $\dfrac{x}{y} + x^2 = \lambda$

B. $\dfrac{x}{y} - \dfrac{x^2}{2} = \lambda$

C. $\dfrac{x}{2y^2} + \dfrac{x^2}{4} = \lambda$

D. None of these

Answer: 4

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58. The solution of differential equation

$$xdy\left(y^2e^{xy} + e^{x/y}\right) = ydx\left(e^{x/y} - y^2e^{x/y}\right), \text{ is}$$

A. $xy = \log(e^x + \lambda)$

B. $x^2/y = \log\left(e^{x/y} + \lambda\right)$

C. $xy = \log\left(e^{x/y} + \lambda\right)$

D. $xy^2 = \log\left(e^{x/y} + \lambda\right)$

Answer:

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59. The solution of the differential equation

$$\left(y + x\sqrt{xy}(x + y)\right)dx + \left(y\sqrt{xy}(x + y) - x\right)dy = 0$$

A. $\dfrac{x^2 + y^2}{2} + 2\tan^{-1}\sqrt{\dfrac{x}{2y}} = C$

B. $\dfrac{x^2 + y^2}{2} + 2\tan^{-1}\sqrt{\dfrac{x}{y}} = C$

C. $\dfrac{x^2 + y^2}{\sqrt{2}} + 2\tan^{-1}\sqrt{\dfrac{x}{y}} = C$

D. None of these

Answer: B

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60. Solve $(p - x)(p - e^x)(p - 1/y) = 0$, where $P = \dfrac{dy}{dx}$.

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61. Solve $x^2 p^2 + xpy - 6y^2 = 0$.

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62. Solve $xy^2 (p^2 + 2) = 2py^3 + x^3$.

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63. Solve $xp^2 - 2yp + ax = 0$.

64. Solve $y = 2px - p^2 \, where \, p = \dfrac{dy}{dx}$

65. Solve $y = 2px + y^2 p^3$.

66. Form the differential equation of $y = px + \dfrac{p}{\sqrt{1 + p^2}}.$

67. Solve $\sqrt{1 + p^2} = \tan(px - y).$ when $p = \dfrac{dy}{dx}$

68. Solve $y^2 \log y = pxy + p^2$. where $p = \dfrac{dy}{dx}$

69. The population of a certain is known to increase at a rate proportional to the number of people presently living in the country. If after two years the population has doubled, and after three years the population is 20,000 estimates the number of people initially living in the country.

70. A certain radioactive material is known to decay at a rate proportional to the amount present. Initially there is $50kg$ of the material present and after $2h$ it is observed that the material has lost 10% of its original mass. Based on these data answer the following questions.

The time at which the material has decayed to one half of its initial mass is

71. Five mice in a stable population of 500 are intentionally infected with a contagious disease to test a theory of epidemic spred that postulates the rate of change in the infected population is proportional to the product of the number of mice who have the disease with the number that are disease free. Assuming the theory is correct, how long will it take half the popullation to contract the disease?

72. Find the curve for which area of triangle formed by x-axis, tangent drawn at any point on the curve and radius vector of point of tangency is constant, equal to a^2

73. Find the curve for which the intercept cut off by any tangent on y-axis is proportional to the square of the ordinate of the point of tangency.

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74. If $\dfrac{d^2x}{dy^2}\left(\dfrac{dy}{dx}\right)^3 + \dfrac{d^2y}{dx^2} = k$, then k is equal to

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75. The solution of $y^2 = 2y\dfrac{dy}{dx} - \left(\dfrac{dy}{dx}\right)^2$, is

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76. The solution of $\left(\dfrac{dy}{dx}\right)^2 + (2x + y)\dfrac{dy}{dx} + 2xy = 0$, is

A. $\left(y + x^2 - c_1\right)\left(x + \log y + y^2 + c_2\right) = 0$

B. $\left(y + x^2 - c_1\right)\left(x - \log y - c_2\right) = 0$

C. $\left(y + x^2 - c_1\right)\left(x + \log y - c_2\right) = 0$

D. None of these

Answer:

77. A curve $y = f(x)$ passes through the origin. Through any point (x, y) on the curve, lines are drawn parallel to the co-ordinate axes. If the curve divides the area formed by these lines and co-ordinates axes in the ratio $m : n$, find the curve.

A. $y = cx^{m/n}$

B. $my^2 = cx^{m/n}$

C. $y^3 = cx^{m/n}$

D. None of these

Answer:

78. Solve the differential equation, $\dfrac{a^2}{xy} \cdot \dfrac{dx}{dy} = \dfrac{x}{y} + \dfrac{y}{x} - 2$, is

79. Find the family of curves, the subtangent at any point of which is the arithmetic mean of the co-ordinate point of tangency.

A. $(x - y)^2 = cy$

B. $(y - x)^2 = cy$

C. $(x - y)^2 = cxy$

D. None of these

Answer:

80. The order of the differential equation of family of curver $y = C_1 \sin^{-1} x + C_2 \cos^{-1} x + C^3 \tan^{-1} x + C^4 \cot^{-1} x$ (where C_1, C_2, C_3 and C_4 are arbitrary constants) is

A. 2

B. 3

C. 4

D. None of these

Answer:

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81. Differentiate the following function w.r.t x

$\cot(x^2)$

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82. The curve satisfying the equation $\dfrac{dy}{dx} = \dfrac{y(x + y^3)}{x(y^3 - x)}$ and passing through the point $(4, \ -2)$ is

A. $y^2 = -2x$

B. y=-2x

C. $y^3 = -2x$

D. None of these

Answer:

83. Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $K > 0$ is

A. $\dfrac{dr}{dt} + k = 0$

B. $\dfrac{dr}{dt} - k = 0$

C. $\dfrac{dr}{dt} - kr$

D. None of these

Answer:

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84. A function $y - f(x)$ satisfies the differential equation

$f(x)\sin 2x - \cos x + (1 + \sin^2 x)f'(x) = 0$ with $f(0) = 0.$ The value

of $f\left(\dfrac{\pi}{6}\right)$ equals to :
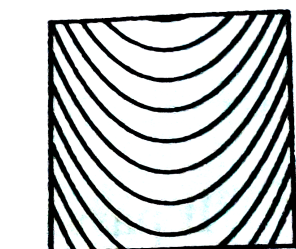
A. $1/5$
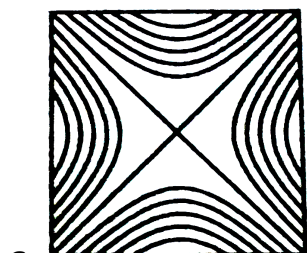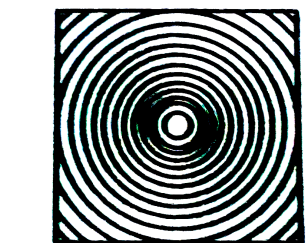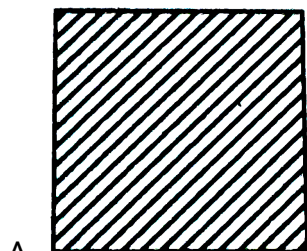
B. $3/5$

C. $4/5$

D. $2/5$

Answer:

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85. The general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1-x}{y}$ is a family of curves which look like which of the following :

A.

B.

C.

D.

Answer:

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86. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \dfrac{1}{15}$, then the time to drain the tank if the water is 4 m deep to start with is

   A. 30 min

   B. 45 min

   C. 60 min

   D. 80 min

Answer:

87. Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is

A. 1

B. 2

C. 3

D. 4

Answer:

88. Consider the two statements Statement-I: y=sin kt satisfies the differential equation $y'' + 9y = 0$. Statement 2 : y= e^{kt} satisfy the differential equation $y'' + y' - 6y = 0$. The value of k for which both the statements are correct is

A. -3

B. 0

C. 2

D. 3

Answer:

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89. If $y = \dfrac{x}{\log|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x} + \varphi\left(\dfrac{x}{y}\right)$, then the function $\varphi\left(\dfrac{x}{y}\right)$ is

A. $\dfrac{x^2}{y^2}$

B. $-\dfrac{x^2}{y^2}$

C. $\dfrac{y^2}{x^2}$

D. $-\dfrac{y^2}{x^2}$

Answer:

90. If $\int_a^x ty(t)dt = x^2 + y(x)$, then find $y(x)$

A. $y = 2 - (2 + a^2)e^{\frac{x^2 - a^2}{2}}$

B. $y = 1 - (2 + a^2)e^{\frac{x^2 - a^2}{2}}$

C. $y = 2 - (1 + a^2)e^{\frac{x^2 - a^2}{2}}$

D. None of these

Answer:

91. Differentiate the following function w.r.t x

$\cos\left(x^{\frac{1}{2}}\right)$

92. The differential equation $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{y}$ determines a family of circle with

    A. varible radii and a fixed centre at (0,1)

    B. variable radii and fixed centre at (0,-1)

    C. fixed radius 1 and varible centres along the x-axis

    D. fixed radius 1 and varible centres along the y-axis

Answer:

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93. A differentiable function satisfies

$$f(x) = \int_{0}^{x} (f(t)\cos t - \cos(t-x))dt.$$

Which of the following hold(s) good?

a. f(x) has a minimem value 1-e

b. f(x) has a maximum value $1 - e^{-1}$

c. $f''\left(\dfrac{\pi}{2}\right) = e$

d. $f'(0) = 1$

A. f(x) has a minimem value 1-e

B. f(x) has a maximum value $1 - e^{-1}$

C. $f''\left(\dfrac{\pi}{2}\right) = e$

D. $f'(0) = 1$

Answer:

94. Let $\dfrac{dy}{dx} + y = f(x)$ where y is a continuous function of x with y(0) = 1
and $f(x) = \begin{cases} e^{-x} & \text{if } o \le x \le 2 \\ e^{-2} & \text{if } x > 2 \end{cases}$ Which of the following hold(s) good ?

A. $y(1) = 2e^{-1}$

B. $y'(1) = -e^{-1}$

C. $y(3) = -2e^{-3}$

D. $y'(3) = -2e^{-3}$

Answer:

95. A curve $y = f(x)$ passes through $(1, 1)$ and tangent at $P(x, y)$ cuts the x-axis and y-axis at A and B , respectively, such that $BP : AP = 3$, then

(a) equation of curve is $xy' - 3y = 0$

(b) normal at $(1, 1)$ is $x + 3y = 4$

(c) curve passes through $2, \dfrac{1}{8}$

(d) equation of curve is $xy' + 3y = 0$

A. equation of curve is xy'-3y=0

B. normal at (1,1) is x+3y=4

C. curve passes through $\left(2, \dfrac{1}{8}\right)$

D. equation of curve is xy'+3y=0

Answer:

96. Let a solution y = y(x) of the differential equation

$$x\sqrt{x^2 - 1}dy - y\sqrt{y^2 - 1}dx = 0, \text{satisfy } y(2) = \frac{2}{\sqrt{3}}$$

A. Statement I is true, Statement II is also true, Statement

B. Statement I is true, Statement II is also true, Statement II is not the

   correct explanation of Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is flase, Statement II is true.

Answer:

97. A curve y=f(x) satisfy the differential equation

$\left(1 + x^2\right)\dfrac{dy}{dx} + 2yx = 4x^2$ and passes through the origin.

The function y=f(x)

    A. (a) is strictly increasing , $\forall x \in R$

    B. (b) is such that it has a minima but no maxima

    C. (c) is such that it has a maxima but no minima

    D. (d) has no inflection point

Answer:

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98. A curve y=f(x) satisfy the differential equation

$\left(1 + x^2\right)\dfrac{dy}{dx} + 2yx = 4x^2$ and passes through the origin.

The area enclosed by $y = f^{-1}(x)$, the x-axis and the ordinate at

$x = 2/3$ is

    A. 2ln2

B. $\frac{4}{3}In2$

C. $\frac{2}{3}In2$

D. $\frac{1}{3}In2$

Answer:

99. A curve y=f(x) satisfy the differential equation $(1 + x^2)\frac{dy}{dx} + 2yx = 4x^2$ and passes through the origin.

The function y=f(x)

A. (a) is strictly increasing

B. (b) is such that it has a minima but no maxima

C. (c) is such that it has a maxima but no minima

D. (d) has no point of inflection

Answer:

100. Let y=f(x) be a curve passing through (4,3) such that slope of normal at any point lying in the first quadrant is negative and the normal and tangent at any point P cuts the Y-axis at A and B respectively such that the mid-point of AB is origin, then the number of solutions of $y = f(x)$ and $y = |5 - |x| \ |$, is

101. If a function satisfies the relation

$$f(x)f''(x) - f(x)f'(x) = (f'(x))^2 \, \forall x \in R \text{ and } f(0) = f'(0) = 1,$$

then

The value of $\lim_{x \to -\infty} f(x)$ is

102. For certain curve $y = f(x)$ satisfying $\dfrac{d^2y}{dx^2} = 6x - 4$, $f(x)$ has local minimum value 5 when $x = 1$

Global maximum value of $y = f(x)$ for $x \in [0, 2]$ is

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103. If $\phi(x)$ is a diferential real-valuted function satisfying $\phi'(x) + 2\phi(x) \le 1$. prove that $\phi(x) - \dfrac{1}{2}$ is a non-incerasing function of x.

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104. Determine all the curves for which the ratio of the length of the segment intercepted be any tangent on the Y-axis to the length of the radius vector is constant.

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105. Let u(x) and v(x) be two continous functions satisfying the differential equations $(du)(dx) + p(x)u = f(x)$ and $\frac{dv}{dx} + p(x)v = g(x)$, respectively. If $u(x_1) > v(x_1)$ for some x_1 and $f(x) > g(x)$ for all $x > x_1$, prove that any point (x, y),where $x > x_1$, does not satisfy the equations $y = u(x)$ and $y = v(x)$ simultaneously.

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106. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q. If PQ has constant length k, then show that the differential equation describing such curves is $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$. Find the equation of such a curve passing through $(0, k)$.

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107. Find the equation of a curve passing through the point (1.1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x-axis.

108. A country has a food deficit of 10%. Its population grows continuously at the rate of 3% per year. Its annual food production every year is 4% more than that of the last year Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\dfrac{\log_e 10 - \log_e 9}{(\log_e 1.04) - 0.03}$

109. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant $= k > 0$). Find the time after which the cone will be empty.

110. For $x \in R, x \neq 0,$ if $y(x)$ differential function such that

$$x \int_1^x y(t)dt = (x+1) \int_1^x ty(t)dt, \text{ then } y(x) \text{ equals:}$$

111. Find a pair of curves such that (a) the tangents drawn at points with equal abscissas intersect on the y-axis. (b) the normal drawn at points with equal abscissas intersect on the x-axis. (c) one curve passes through (1,1) and other passes through (2, 3).

112. Given two curves: $y = f(x)$ passing through the point $(0, 1)$ and $g(x) = \int_{-\infty}^x f(t)dt$ passing through the point $\left(0, \dfrac{1}{n}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersect on the x-axis. Find the curve $y = f(x)$.

113. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis and the y-axis in point A AND B, respectively, such that $\dfrac{1}{OA} + \dfrac{1}{OB}$ =1, where O is the origin. Find the equation of such a curve passing through $(5. 4)$

114. A line is drawn from a point $P(x, y)$ on the curve $y = f(x)$, making an angle with the x-axis which is supplementary to the one made by the tangent to the curve at $P(x, y)$. The line meets the x-axis at A. Another line perpendicular to it drawn from $P(x, y)$ meeting the y-axis at B. If $OA = OB$, where O is the origin, the equation of all curves which pass through $(1, 1)$ is

115. Find the equation of a curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point (x, y) is $2\dfrac{x}{y^2}$

116. Let $f(x)$ be positive continuous and differentiable on the interval (a, b) and $\lim\limits_{x \to a+} f(x) = 1, \ \lim\limits_{x \to b-} f(x) = 3^{1/4}.$ If $f'(x) \geq f^3(x) + \dfrac{1}{f(x)}$ then the greatest value of $b - a$ is

A. $\dfrac{\pi}{48}$

B. $\dfrac{\pi}{36}$

C. $\dfrac{\pi}{24}$

D. $\dfrac{\pi}{12}$

Answer:

1. If the area bounded by y=f(x), $x = \dfrac{1}{2}$, $x = \dfrac{\sqrt{3}}{2}$ and the X-axis is A sq units where $f(x) = x + \dfrac{2}{3}x^3 + \dfrac{2}{3} \cdot \dfrac{4}{5}x^5 + \dfrac{2}{3} \cdot \dfrac{4}{5} \cdot \dfrac{6}{7}x^7 + ...\infty$, $|x| < 1$,

Then the value of [4A] is (where[.] is G.I.F)

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1. If (y_1, y_2) are two solutions of the differential $\dfrac{dy}{dx} + p(x).y = Q(x)$

Then prove that $y = y_1 + C(y_1 - y_2)$ is the genral solution of the equation where C is any constant. For what relation between the constant α, β will the linear combination $\alpha y_1 + \beta y_2$ also be a Solution.

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1. The differential equation of all parabolas whose axis of symmetry is along X-axis is of order.

A. 2

B. 3

C. 1

D. None of these

Answer: A

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2. The order and degree of the differential equation of all tangent lines to the parabola $x^2 = 4y$ is

A. 1,2

B. 2,2

C. 3,1

D. 4,1

Answer: A

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3. The degree of the differential equation

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 = x^2 \log\left(\frac{d^2y}{dx^2}\right), \text{ is}$$

A. 1

B. 2

C. 3

D. Not defined

Answer: D

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4. For the differential equation, find the general solution:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

5. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

A. 1

B. 2

C. 3

D. Nor defined

Answer: D

6. Form the differential equation of the family of circles touching the y-axis at origin.

A. $y^2 - x^2 = 2xy\dfrac{dy}{dx}$

B. $y^2 - x^2 = 2xy\dfrac{dx}{dy}$

C. $y^2 - x^2 = 2xy\dfrac{dy}{dx}$

D. $y^2 - x^2 = 2xy\dfrac{dx}{dy}$

Answer: A

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7. Show that the differential equation that represents the family of all parabolas having their axis of symmetry coincident with the axis of $xisyy_2 + y1^2 = 0$.

A. $yy_2 + y_1^2 - y + y_1$

B. $yy_2 + y_1^2 = 0$

C. $yy_2 + y_1^2 = y_1$

D. None of these

Answer: B

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8. The differential equation of all conics whose axes coincide with the coordinate axes, is

A. (a)$xyy_2 + xy_1^2 - yy_1 = 0$

B. (b)$yy_2 + y_1^2 - yy_1 = 0$

C. (c)$xyy_2 + (x - y)y_1 = 0$

D. (d)None of these

Answer: A

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9. The differential equation having $y = \left(\sin^{-1} x\right)^2 + A\left(\cos^{-1} x\right) + B$, where A and B are abitary constant , is

A. $\left(1 - x^2\right) y_2 - x y_1 = 2$

B. $\left(1 - x^2\right) y_2 + y y_1 = 0$

C. $\left(1 - x^2\right) y_2 + x y_1 = 0$

D. None of these

Answer: A

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10. The differential equation of circles passing through the points of intersection of unit circle with centre at the origin and the line bisecting the first quadrant, is

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1. Solve $\dfrac{dy}{dx} = 4x$

A. $(x + 2)^4 \left(1 + 2\dfrac{(y - 2)}{x + 2}\right) = k^{\frac{2(y-2)}{x+2}}$

B. $(x + 2)^4 \left(1 + \dfrac{(y - 2)}{x + 2}\right) = k^{\frac{2(y-2)}{x+2}}$

C. $(x + 2)^3 \left(1 + 2\dfrac{(y - 2)}{x + 2}\right) = k^{\frac{2(y-2)}{x+2}}$

D. None of these

Answer: B

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2. If $\left(y^3 - 2x^2 y\right) dx + \left(2xy^2 - x^3\right) dy = 0$, then the value of

$xy\sqrt{y^2 - x^2}$, is

A. $y^2 + x$

B. xy^2

C. any contant

D. None of these

Answer: C

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3. The solution of $\dfrac{dy}{dx} = \cos(x+y) + \sin(x+y)$, is given by

A. $\log\left|1 + \tan\left(\dfrac{x+y}{2}\right)\right| = x + c$

B. $\log|1 + \tan((x+y)| = x + c$

C. $\log|1 - \tan((x+y)| = x + c$

D. None of these

Answer: A

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4. The solution of $\dfrac{dy}{dx} = (x + y - 1) + \dfrac{x + y}{\log(x + y)}$, is given by

A. $\{1 + \log(x + y)\} - \log\{1 + \log(x + y)\} = x + c$

B. $\{1 - \log(x + y)\} - \log\{1 - \log(x + y)\} = x + c$

C. $\{1 - \log(x + y)\}^2 - \log\{1 + \log(x + y)\} = x + c$

D. None of these

Answer: A

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5. $(2x^2 + 3y^2 - 7)\, x\, dx - (3x^2 + 2y^2 - 8)\, y\, dy = 0$

A. $(x^2 + y^2 - 1) = (x^2 + y^2 - 3)^5 C$

B. $(x^2 + y^2 - 1)^2 = (x^2 + y^2 - 3)^5 C$

C. $(x^2 + y^2 - 3)^2 = (x^2 + y^2 - 1)^5 C$

D. None of these

Answer: C

6. The solution of $\dfrac{dy}{dx} = \dfrac{(x-1)^2 + (y-2)^2 \tan^{-1}\left(\frac{y-2}{x-1}\right)}{(xy - 2x - y + 2)\tan^{-1}\left(\frac{y-2}{x-1}\right)}$ is equal to

A. (a)

$$\left\{(x-1)^2 + (y-1)^2\right\}\tan^{-1}\left(\frac{y-2}{x-1}\right) - 2(x-1)(y-2) = 2(x-1)$$

B. (b)

$$\left\{(x-1)^2 + (y-1)^2\right\} - 2(x-1)(y-2)\tan^{-1}\left(\frac{y-1}{x-1}\right) = 2(x-1)$$

C. (c)

$$\left\{(x-1)^2 + (y-1)^2 \tan^{-1}\left(\frac{y-2}{x-1}\right) - 2(x-1)(y-2) = \log C(x)\right.$$

D. (d) None of these

Answer: A

7. Differentiate the following w.r.t x

log(logx) , x > 1

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8. Differentiate the following w.r.t x

$e^{\sin^{-1}x}$

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9. Find $\dfrac{dy}{dx}$, if $3xy + y^2 = \tan x + 2y$

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10. Find $\dfrac{dy}{dx}$,if x+2y = cosx

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11. Differentiate the following function w.r.t x

$\cos\left(x^3\right). \sin^2\left(x^5\right)$

12. A curve passes through $(2, 1)$ and is such that the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal. Then the equation of curve is

A. a. $x^2 + y^2 = 9x$

B. b. $4x^2 + y^2 = 9x$

C. c. $4x^2 + 2y^2 = 9x$

D. d. All of these

Answer: C

13. Find $\dfrac{dy}{dx}$, if $2\sin^2 x + 3\cos^2 y = 5$

14. Determine all the curves for which the ratio of the length of the segment intercepted be any tangent on the Y-axis to the length of the radius vector is constant.

A. $\left(y + \sqrt{x^2 - y^2} \right) x^{k-1} = c$

B. $\left(y + \sqrt{x^2 + y^2} \right) x^{k-1} = c$

C. $\left(y - \sqrt{x^2 - y^2} \right) x^{k-1} = c$

D. $\left(y - \sqrt{x^2 + y^2} \right) x^{k-1} = c$

Answer: B

15. A point $P(x, y)$ nores on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}, a > 0$ for each position (x, y) of p, perpendiculars are drawn from origin upon the tangent and normal at P, the length (absolute valve) of them being $P_1(x)$ and $P_2(x)$ brespectively, then

A. $\dfrac{dp_1}{dx} \cdot \dfrac{dp_2}{dx} < 0$

B. $\dfrac{dp_1}{dx} \cdot \dfrac{dp_2}{dx} \leq 0$

C. $\dfrac{dp_1}{dx} \cdot \dfrac{dp_2}{dx} \geq 0$

D. $\dfrac{dp_1}{dx} \cdot \dfrac{dp_2}{dx} \geq 0$

Answer: B

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Exercise For Session 3

1. $(1 + x^2) \dfrac{dy}{dx} + y = e^{\tan^{-1} x}$

A. $2ye^{\tan^{-1}x} = e^{2\tan^{-1}x} + C$

B. $ye^{\tan-1x} = e^{2\tan-1x} + C$

C. $2ye^{\tan-1x} = e^{2\tan-1x} + C$

D. None of these

Answer: A

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2. Solution of differential equation $\dfrac{dy}{dx} + \dfrac{x}{1-x^2}y = x\sqrt{y}$ is

A. a) $3\sqrt{y} + (1 - x^2) = c(1 - x^2)^{1/4}$

B. b) $\dfrac{3}{2}\sqrt{y} + (1 - x^2) = c(1 - x^2)^{3/2}$

C. c) $3\sqrt{y} - (1 - x^2) = c(1 - x^2)^{1/4}$

D. d) None of these

Answer: A

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3. Solve $\dfrac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

A. $e^{x2} = (x^2 - 1)e^{x2} \tan y + c$

B. $e^{x2} \tan y = \dfrac{1}{2}(x^2 - 1)e^{x2} + c$

C. $e^{x2} \tan y = (x^2 - 1)\tan y + c$

D. None of these

Answer: B

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4. The solution of $(x^{2-1})dy/dx \cdot \sin y - 2x \cdot \cos y = 2x - 2x^3$ is

A. $y^2 = ax + c\sqrt{1 - x^2}$

B. $y^3 = ax + cx\sqrt{1 - x^3}$

C. $y^2 = ax + c\sqrt{1 - x^2}$

D. None of these

Answer: B

---

5. The solution of $\dfrac{dy}{dx} + \dfrac{y}{x}\log y = \dfrac{y}{x^2}(\log y)^2$, is

A. a) $x = \dfrac{1}{2x}\log y + C$

B. b) $x^2 + \log y = C$

C. c) $\dfrac{1}{x\log y} = \dfrac{1}{2x^2} + C$

D. d) None of these

Answer: C

---

6. The solution of $\dfrac{dy}{dx} + yf'(x) - f(x).f'(x) = 0, y \neq f(x)$ is

A. $y = f(x) + 1 + ce^{-f(x)}$

B. $y - ce^{-f(x)}$

C. $y = f(x) - 1 + ce^{-f(x)}$

D. None of these

Answer: C

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7. The solution of $(x^2 - 1) \dfrac{dy}{dx} \cdot \sin y - 2x \cdot \cos y = 2x - 2x^3$ is

A. $(x^2 - 1) \cos y = \dfrac{x^4}{2} - x^2 + C$

B. $(x^2 - 1) \sin y = \dfrac{x^4}{2} - x^2 + C$

C. $(x^2 - 1) \cos y = \dfrac{x^4}{2} - \dfrac{x^2}{2} + C$

D. $(x^2 - 1) \sin y = \dfrac{x^4}{2} - \dfrac{x^2}{2} + C$

Answer: A

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8. The Curve possessing the property that the intercept made by the tangent at any point of the curve on they-axis is equal to square of the abscissa of the point of tangency, is given by

A. $y^2 = x + c$

B. $y = 2x^2 + cx$

C. $Y = -x^2 + cx$

D. None of these

Answer: C

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9. The tangent at a point P of a curve meets the y-axis at A, and the line parallel to y-axis at A, and the line parallel to y-axis through P meets the x-axis at B. If area of ΔOAB is constant (O being the origin), Then the curve is

A. $cx^2 - xy + k = 0$

B. $y^2 + 2x^2 = cx$

C. $3x^2 + 4y^2 = k$

D. $xy - x^2y^2 + kx = 0$

Answer: A

10. The value of k such that the family of parabolas $y = cx^2 + k$ is the orthogonal trajectory of the family of ellipse $x^2 + 2y^2 - y = c$, is

A. 1

B. $\dfrac{1}{2}$

C. $\dfrac{1}{3}$

D. $\dfrac{1}{4}$

Answer: D

Exercise For Session 4

1. The solution of $xdy + ydx + 2x^3 dx = 0$ is

A. $xy + x^4 = c$

B. $xy + \dfrac{1}{2}x^4 = c$

C. $\dfrac{x^2}{y} + \dfrac{x^4}{4} = c$

D. None of these

Answer: B

2. Find the solution of $ydx - xdy + (1 + x^2)dx + x^2 \sin ydy = 0$

3. Solve the differential equation : $y\sqrt{1-x^2}dy - \sqrt{1+y^2}dx = 0$

A. $\left(1 + x\sqrt{x^2+y^2}\right)dx + \left(-1 + \sqrt{x^2+y^2}\right)ydy = 0,$

B. $2x - y + \dfrac{2}{3}\left(x^2+y^2\right)^{3/2} = c$

C. $2y - x^2 + \dfrac{2}{3}\left(x^2+y^2\right)^{3/2} = c$

D. None of these

Answer: A

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4. Solution of the differential equation $\dfrac{xdy}{x^2+y^2} = \left(\dfrac{y}{x^2+y^2} - 1\right)dx$, is

A. $\tan^{-1}\left(\dfrac{x}{y}\right) + x = c$

B. $\tan^{-1}\left(\dfrac{y}{x}\right) + x = c$

C.

D. $\tan^{-1}\left(\dfrac{y}{x}\right) + x^2 = c$

Answer: B

---

5. Solve the differential equation $\left(y e^{\frac{x}{y}} \right) dx = \left(x \left(e^{\frac{x}{y}} \right) + y^2 \right) dy (y \neq 0)$

A. $e^{x/y} = -\cos y + c$

B. $e^{x/y} = +2\cos y = c$

C. $e^{x/y} = x\cos y + c$

D. $e^{x/y} = 2\cos y + c$

Answer: A

---

6. The solution of $x \sin\left(\frac{y}{x} \right) dy = \left\{ y \sin\left(\frac{y}{x} \right) - x \right\} dx$, is given by

A. $\log x + \cos\left(\frac{y}{x} \right) = \log C$

B. $\log x - \cos\left(\dfrac{y}{x}\right) = \log C$

C. $\log\left(\dfrac{x}{y}\right) - \cos\left(\dfrac{y}{x}\right) = \log C$

D. None of these

Answer: B

7. Solve $\dfrac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\dfrac{a^2 - x^2 - y^2}{x^2 + y^2}}$

A. $\sin^{-1}\left(\sqrt{x^2 + y^2}\right) = a\tan^{-1} + c$

B. $\sin^{-1}\left(\sqrt{x^2 + y^2}\right) = \dfrac{1}{a}\tan^{-1}\left(\dfrac{y}{x}\right) + c$

C. $\sin^{-1}\left(\sqrt{x^2 + y^2}\right) = \tan^{-1}\left(\dfrac{y}{x}\right) + c$

D. None of these

Answer: D

8. Solve the following differential equation:

$$\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$$

A. $x - ye^{x/y} = c$

B. $x + ye^{x/y} = c$

C. $y - \frac{x}{y}e^{x/y} = c$

D. None of these

Answer: B

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9. Solution of the differential equation $\dfrac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = \dfrac{x\sin^2\left(x^2 + y^2\right)}{y^3}$.

A. $-\cot\left(x^2 + y^2\right) = \left(\dfrac{x}{y}\right)^2 + C$

B. $\tan\left(x^2 + y^2\right) = x^2y^2 + C$

C. $\cot\left(x^2 + y^2\right) = \dfrac{x}{y} + C$

D. None of these

Answer: A

10. The solution of $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{1}{(1 + \log x + \log y)^2}$ is given by

A. $xy(1 + \log(xy)) = C$

B. $xy^2(1 + \log(xy)) = C$

C. $xy(1 + \log(xy))^2 = C$

D. $xy\left(1 + (\log xy)^2\right) - \dfrac{x^2}{2} = C$

Answer: D

Exercise For Session 5

1. Show that the curve for which the normal at every point passes through a fixed point is a circle.

    A. a circle

    B. an ellipse

    C. a hyperbola

    D. Nono of these

Answer: A

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2. If the tangent at any point P of a curve meets the axis of $X \in T$ find the curve for which $OP = PT$, O being the origin.

    A. $x = cy^2$

    B. $x = cy^2$ or $x = c/y^2$

    C. $x = cy$ or $x = c/y$

D. None of these

Answer: C

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3. Differentiate the following w.r.t x

$$\frac{e^x}{\sin x}$$

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4. A curve passing through the point (1,1) has the porperty that the perpendicular distance of the normal at any point P on the curve from the origin is equal to the distance of P from x-axis Determine the equation of the curve.

A. $x^2 + y^2 = 5x$

B. $x^2 - y^2 = 5x$

C. $x^2 y^2 = 5x$

D. All of these

Answer: A

---

5. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \dfrac{1}{2}\right)$

any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable $, \forall x \in$ R

through $(0, 1)$ Tangents drawn to both the curves at the points with

equal abscissae intersect on the same point on the X- axists

The function $f(x)$ is

A. $f(x) = x^2 + x + 1$

B. $f(x) = \dfrac{x^2}{e^2}$

C. $f(x) = e^{2x}$

D.

Answer: C

6. Find the equation of all possible curves such that length of intercept made by any tangent on x-axis is equal to the square of X-coordinate of the point of tangency. Given that the curve passes through (2,1)

A. $x = e^{\pm 2\sqrt{y}/x}$

B. $x = e^{\pm \sqrt{y}/x}$

C. $x = e^{\pm \sqrt{y}/x} - 1$

D. $xy + e^{\pm y/x} - 1 = 0$

Answer: A

7. Differentiate the following w.r.t x

$$\log(\cos(e^x))$$

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8. Find the 5th term of GP if the first term is 4 and common ratio is 2.

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Exercise Single Option Correct Type Questions

1. Find the sum of n terms of the AP 6,13,20......

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2. If the 5th term of AP is 16 and the common difference is 4. Find the 20th term of the AP.

3. The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1,1) is

A. $ye^{\frac{x}{y}} = e$

B. $xe^{\frac{x}{y}} = e$

C. $xe^{\frac{y}{x}} = e$

D. $ye^{\frac{y}{x}} = e$

Answer: A

4. Differentiate the following w.r.t x

$e^x + e^{x^3} + e^{x^5}$

5. A curve is such that the area of the region bounded by the co-ordinate axes, the curve & the coordinate of any point on it is equal to the cube of that ordinate. The curve represents

    A. a pair of straight lines

    B. a circle

    C. a parabola

    D. am ellipse

Answer: C

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6. The value of the constant 'm' and 'c' for which y = mx + c is a solution of the differential equation $\left(d^2\dfrac{y}{dx^2} \right)$ - 3 (dy/dx) -4y = -4x is:

    A. is m =-1,c $= 3/4$

    B. is m =1,c $= -3/4$

C. no such real m,c

D. is m =1,c $= 3/4$

Answer: B

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7. Find the real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4 y \dfrac{dy}{dx} + y^4 = 4x^6$ in to a homogeneous equation.

A. m=0

B. m=1

C. 1m=3//2`

D. No value of m

Answer: C

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8. The solution of the differential equation,

$$x^2 \frac{dy}{dx} \cos \frac{1}{x} - y \sin \frac{1}{x} = -1, \text{ where } y \to -1 \text{ as } x \to \infty \text{ is}$$

A. $y = \sin \dfrac{1}{x} - \cos \dfrac{1}{x}$

B. $y = \dfrac{x + 1}{x \sin \frac{1}{x}}$

C. $y = \sin \dfrac{1}{x} + \cos \dfrac{1}{x}$

D. $y = \dfrac{x + 1}{x \cos \frac{1}{x}}$

Answer: A

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9. A wet porous subtance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half its moisture during the first hour, then the time when it would have lost 99.9% of its moisture is (weather conditions remaining same)

A. (a) more than 100 h

B. (b) more than 10 h

C. (c) approximately 10 h

D. (d) None of these

Answer: C

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10. The equation of tangent in slope form to the the parabola $y^2 = 4x$ is

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11. A function $y = f(x)$ satisfies

$(x + 1)f'(x) - 2(x^2 + x)f(x) = \dfrac{e^{x^2}}{x + 1}$, $A a x > -1$. If $f(0) = 5$,

then f(x) is

A. $\left(\dfrac{3x + 5}{x + 1}\right) \cdot e^{x^2}$

B. $\left(\dfrac{6x + 5}{x + 1}\right) \cdot e^{x^2}$

C. $\left(\dfrac{6x + 5}{(x + 1)^2}\right) \cdot e^{x^2}$

D. $\left(\dfrac{5x - 6x}{x + 1}\right) \cdot e^{x^2}$

Answer: B

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12. The curve with the property that the projection of the ordinate on the normal is constant and has a length equal to a is

A. $x - aIn\left(\sqrt{y^2 - a^2} + y\right) = C$

B. $x + a\sqrt{a^2 - y^2} = C$

C. $(y - a)^2 = Cx$

D. $ay = \tan^{-1}(x + c)$

Answer: A

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13. The differential equation corresponding to the family of curves $y = e^x(ax + b)$ is

A. $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - y = 0$

B. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 0$

C. $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$

D. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - y = 0$

Answer: B

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14. The equation to the orthogonal trajectories of the system of parabolas $y = ax^2$ is

A. $\dfrac{x^2}{2} + y^2 = C$

B. $x^2 - \dfrac{x^2}{2} = C$

C. $\dfrac{x^2}{2} - y^2 = C$

D. $x^2 - \dfrac{y^2}{2} = C$

Answer: A

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15. A function satisfying $\displaystyle\int_0^1 f(tx)\,dt = nf(x)$, where $x > 0$ is

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16. The substituion $y = z^\alpha$ transforms the differential equation $(x^2 y^2 - 1)\,dy + 2xy^3\,dx = 0$ into a homogeneous differential equation for

A. $\alpha = -1$

B. 0

C. $\alpha = 1$

D. No value of α

Answer: A

17. A curve passing through $(2, 3)$ and satisfying the differential equation
$$\int_0^t ty(t)d = x^2 y(x), (x > 0)$$ is

A. $x^2 + y^2 = 13$

B. $y^2 = \dfrac{9}{2}x$

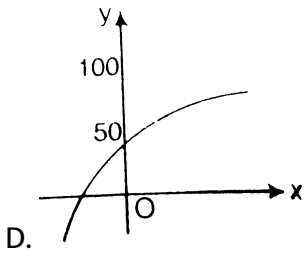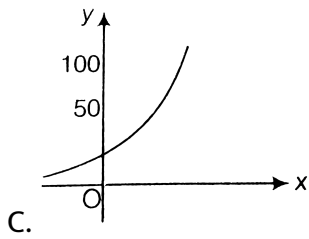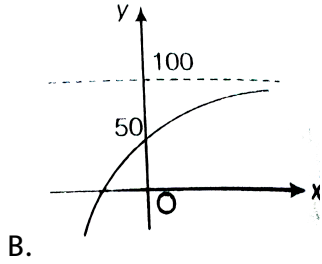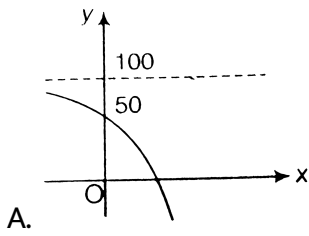C. $\dfrac{x^2}{8} + \dfrac{y^2}{18} = 1$

D. xy=6

Answer: D

18. Which one of the following curves represents the solution of the initial value problem $\dfrac{dy}{dx} = 100 - y$, where $y(0) = 50$

A.

B.

C.

D.

Answer: B

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1. The differential equation $x \dfrac{dy}{dx} + \dfrac{3}{\frac{dy}{dx}} = y^2$

    A. is of order 1

    B. is of degree 2

    C. is linear

    D. is non-linear

Answer: A::B::D

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2. The function $f(x)$ satisfying the equation
$f^2(x) + 4f'(x)f(x) + (f'(x))^2 = 0$

    A. $f(x) = C . e^{\left(2 - \sqrt{3}\right)x}$

    B. $f(x) = C . e^{\left(2 + \sqrt{3}\right)x}$

C. $f(x) = C \cdot e^{\left(\sqrt{3}-2\right)x}$

D. $f(x) = C \cdot e^{-\left(2+\sqrt{3}\right)x}$

Answer: C::D

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3. The set of all the points on the circumference of a circle.

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4. Family of curves whose tangent at a point with its intersection with the curve $xy = c^2$ form an angle of $\dfrac{\pi}{4}$ is

a. $y^2 - 2xy - x^2 = k$

b. $y^2 + 2xy - x^2 = k$

c. $y = x - 2c\tan^{-1}\left(\dfrac{x}{c}\right) + k$

d. $y = cIn\left|\dfrac{c+x}{c-x}\right| - x + k$

A. $y^2 - 2xy - x^2 = k$

B. $y^2 + 2xy - x^2 = k$

C. $y = x - 2c\tan^{-1}\left(\dfrac{x}{c}\right) + k$

D. $y = cIn\left|\dfrac{c+x}{c-x}\right| - x + k$

Answer: B::C::D

---

5. The general solution of the differential equation,

$$x\left(\dfrac{dy}{dx}\right) = y.\log\left(\dfrac{y}{x}\right) \text{ is}$$

A. $y = xe^{1-c}$

B. $y = xe^{1+c}$

C. $y = ex.\,e^{Cx}$

D. $y = xe^{Cx}$

Answer: C

6. Differentiate the following w.r.t x

cosx.cos2x.cos3x

7. The equation of the curve satisfying the differential equation

$$y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0 \text{ can be a (a) circle (b) Straight line (c)}$$

Parabola (d) Ellipse

8. Identify the statement(s) which is/are true.

A. $f(x, y) = e^{y/x} + \dfrac{\tan(y)}{x}$ is homogeneous of degree zero.

B. $x . \log\dfrac{y}{x}dx + \dfrac{y^2}{x}\sin^{-1}\dfrac{y}{x}dy = 0$ is homogeneous deffierntial

equation.

C. $f(x, y) = x^2 + \sin x. \cos y$ is not homogenous.

D. $(x^2 + y^2) dx - (xy^2 - y^3) dy = 0$ is a homogeneous defferential

equation.

Answer: A::B::C::D

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9. The graph of the function $y = f(x)$ passing through the point (0,1) and satisfying the differential equation $\dfrac{dy}{dx} + y \cos x = \cos x$ is such that

A. it is a constanat function

B. it is periodic

C. it is neither an even nor an odd function

D. it is continuous and differrntiable for all x

Answer: A::B::D

10. Differentiate the following w.r.t x

$\cos(\log x + e^x)$, x > 0

11. Differentiate the following w.r.t x

$\sin\left(\tan^{-1}\left(e^{-x}\right)\right)$

12. Let $y = (A + Bx)e^{3x}$ is a Solution of the differential equation $\dfrac{d^2y}{dx^2} + m\dfrac{dy}{dx} + ny = 0, m, n \in I,$ then

    A. m=-6

    B. n=-6

    C. m=9

D. n=9

Answer: A::D

13. A curve C has the property that its intial ordinate of any tangent drawn is less the abscissa of the point of tangency by unity.

Statement I. Differential equation satisfying tha curve is linear.

Statement II. Degree of differential equation is one.

   A. Statement I is true ,and Statement II is the correct explanation for

   Statement I.

   B. Statement I is true, Statement II is true and Statement II is the

   correct explanation for Statment I

   C. Statement I is true, Statement II is false.

   D. Statement I is false, Statement II is true.

Answer: b

Exercise Statement I And Ii Type Questions

1. Statement I Differential equation corresponding to all lines, ax+by+c=0

has the order 3.

Statement II Gereral solution of a differential equation of nth order

contains n independent arbitaray constanis.

    A. Statement I is true ,and Statement II is the correct explanation for

       Statement I.

    B. Statement I is true, Statement II is true and Statement II is the

       correct explanation for Statment I

    C. Statement I is true, Statement II is false.

    D. Statement I is false, Statement II is true.

Answer: d

2. Statement I Integral curves denoted by the first order linear differential equation $\dfrac{dy}{dx} - \dfrac{1}{x}y = -x$ are family of parabolas passing throught the origin.

Statement II Every differential equation geomrtrically represents a family of curve having some common property.

   A. Statement I is true ,and Statement II is the correct explanation for

      Statement I.

   B. Statement I is true, Statement II is true and Statement II is the

      correct explanation for Statment I

   C. Statement I is true, Statement II is false.

   D. Statement I is false, Statement II is true.

Answer: d

3. Statement I The solution of $(ydx - xdy)\cos\left(\dfrac{x}{y}\right) = ny^2 dx$ is $\sin\left(\dfrac{x}{y}\right) = Ce^{nx}$ Statement II Such type of differential equation can only be solved by the substitution x=vy.

A. Statement I is true ,and Statement II is the correct explanation for Statement I.

B. Statement I is true, Statement II is true and Statement II is the correct explanation for Statment I

C. Statement I is true, Statement II is false.

D. Statement I is false, Statement II is true.

Answer: d

4. Statement 1 : The order of the differential equation whose general solution is $y = = c_1 \cos 2x + c_2 \sin^2 x + c_3 \cos^2 x + c_4 e^{2x} + c_5 e^{2x + c_6}$ is

3. Statement 2 : Total number of arbitrary parameters in the given general solution in the statement (1) is 3.

A. Statement I is true ,and Statement II is the correct explanation for Statement I.

B. Statement I is true, Statement II is true and Statement II is the correct explanation for Statment I

C. Statement I is true, Statement II is false.

D. Statement I is false, Statement II is true.

Answer: A

5. Consider differential equation $(x^2 + 1). \dfrac{d^2y}{dx^2} = 2x. \dfrac{dy}{dx}$

Statement I For many member of this family $y \to \infty$ as $x \to \infty$.

Statement II Any solution of this differential equation is a polynomial of odd degree with positive coefficient of maximum power.

A. Statement I is true ,and Statement II is the correct explanation for Statement I.

B. Statement I is true, Statement II is true and Statement II is the correct explanation for Statment I

C. Statement I is true, Statement II is false.

D. Statement I is false, Statement II is true.

Answer: a

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6. Statement I Order of differential equation of family of parabola whose axis is parallel to y-axis and latusrectum is fixed is 2. Statement II Order of first equation is same as actual number of arbitary constant present in differential equation.

    A. Statement I is true ,and Statement II is the correct explanation for

       Statement I.

    B. Statement I is true, Statement II is true and Statement II is the

       correct explanation for Statment I

    C. Statement I is true, Statement II is false.

    D. Statement I is false, Statement II is true.

Answer: A

7. Statement I The differential equation of all non-vertical lines in a plane

is $\dfrac{d^2x}{dy^2} = 0$.

Satement II The general equation of all non-vertical lines in a plane is

ax+by=1, where $b \neq 0$.

   A. Statement I is true ,and Statement II is the correct explanation for

      Statement I.

   B. Statement I is true, Statement II is true and Statement II is the

      correct explanation for Statment I

   C. Statement I is true, Statement II is false.

   D. Statement I is false, Statement II is true.

Answer: d

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8. Statement I The order of differential equation of all conics whose centre lies at origin is , 2.

Statement II The order of differential equation is same as number of arbitary unknowns in the given curve.

   A. Statement I is true ,and Statement II is the correct explanation for

   Statement I.

   B. Statement I is true, Statement II is true and Statement II is the

   correct explanation for Statment I

   C. Statement I is true, Statement II is false.

   D. Statement I is false, Statement II is true.

Answer: d

9. Statement I y=asin x+bcos x is general solution of y"+y=0.

Statement II y=a sin x+b cos x is a trigonometic function.

   A. Statement I is true ,and Statement II is the correct explanation for

      Statement I.

   B. Statement I is true, Statement II is true and Statement II is not the

      correct explanation for Statment I

   C. Statement I is true, Statement II is false.

   D. Statement I is false, Statement II is true.

Answer: b

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Exercise Passage Based Questions

1. Differentiate the following w.r.t x

e^{x^3}

2. Let $y = f(x)$ satisfies the equation

$$f(x) = \left(e^{-x} + e^{x}\right)\cos x - 2x + \int_{0}^{x} (x - t)f'(t)dt$$

y satisfies the differential equation

A. -1

B. 2

C. 1

D. 0

Answer: D

3. Let y=f(x) satisfies the equation

$$f(x) = \left(e^{-e} + e^x\right)\cos x - 2x - \int_0^x (x - t)f'(t)dt.$$

The value of f'(0)+f''(0)equals to

A. $e^{-x}(\cos x - \sin x) + \dfrac{e^x}{5}(3\cos + \sin x) + \dfrac{2}{5}e^{-x}$

B. $e^{-x}(\cos x + \sin x) + \dfrac{e^x}{5}(3\cos - \sin x) - \dfrac{2}{5}e^{-x}$

C. $e^{-x}(\cos x - \sin x) + \dfrac{e^x}{5}(3\cos - \sin x) + \dfrac{2}{5}e^{-x}$

D. $e^{-x}(\cos x + \sin x) + \dfrac{e^x}{5}(3\cos - \sin x) - \dfrac{2}{5}e^{-x}$

Answer: C

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4. For a certain curve y=f(x) satisfying

$\dfrac{d^2y}{dx^2} = 6x - 4$, f(x) has a local minimum value 5 when x=1, Find the

equation of the curve and also the gobal maximum and global minimum

values of f(x) given that $0 \le x \le 2$.

A. 0

B. 1

C. 2

D. 3

Answer: C

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5. For certain curve $y = f(x)$ satisfying $\dfrac{d^2y}{dx^2} = 6x - 4$, $f(x)$ has local minimum value 5 when $x = 1$

Global maximum value of $y = f(x)$ for $x \in [0, 2]$ is

A. 5

B. 7

C. 8

D. 9

Answer: A

6. If any differentisl equation in the form

$$f(f_1(x, y)d(f_1(x, y) + \phi(f_2(x, y)d(f_2(x, y)) + ... = 0$$

then each term can be intergrated separately.

For example,

$$\int \sin xyd(xy) + \int \left(\frac{x}{y}\right)d\left(\frac{x}{y}\right) = -\cos xy + \frac{1}{2}\left(\frac{x}{y}\right)^2 + C$$

The solution of the differential equation

$$(xy^4 + y)dx - xdy = 0 \text{ is}$$

A. (a)$Cx = e^{\sin^{-1}\frac{y}{x}}$

B. (b)$xe^{\sin^{-1}\frac{y}{x}} = c$

C. (c)$x + e^{\sin^{-1}\frac{y}{x}} = c$

D. (d)None of these

Answer: A

7. If any differentisl equation in the form

$$f(f_1(x, y)d(f_1(x, y) + \phi(f_2(x, y)d(f_2(x, y)) + \dots = 0$$

then each term can be intergrated separately.

For example,

$$\int \sin xy d(xy) + \int \left(\frac{x}{y}\right)d\left(\frac{x}{y}\right) = -\cos xy + \frac{1}{2}\left(\frac{x}{y}\right)^2 + C$$

The solution of the differential equation

$$\left(xy^4 + y\right)dx - xdy = 0 \text{ is}$$

A. $\dfrac{x^3}{4} + \dfrac{1}{2}\left(\dfrac{x}{y}\right)^2 = C$

B. $\dfrac{x^4}{4} + \dfrac{1}{3}\left(\dfrac{x}{y}\right)^3 = C$

C. $\dfrac{x^4}{4} - \dfrac{1}{2}\left(\dfrac{x}{y}\right)^3 = C$

D. $\dfrac{x^4}{4} - \dfrac{1}{2}\left(\dfrac{x}{y}\right)^2 = C$

Answer: B

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8. If any differential equation in the form

$$f(f_1(x, y)d(f_1(x, y) + \phi(f_2(x, y)d(f_2(x, y)) + \ldots = 0$$

then each term can be intergrated separately.

For example,

$$\int \sin xyd(xy) + \int \left(\frac{x}{y}\right)d\left(\frac{x}{y}\right) = -\cos xy + \frac{1}{2}\left(\frac{x}{y}\right)^2 + C$$

Solution of differential equation

$$\left(2x\cos y + y^2 \cos x\right)dx + \left(2y\sin x - x^2 \sin y\right)dy = 0 \text{ is}$$

A. $x^2 \cos y + y^2 \sin x = C$

B. $x\cos y + y\sin x = C$

C. $x^2 \cos^2 y + y^2 \sin^2 x = C$

D. None of the above

Answer: A

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9. Differential equation $\dfrac{dy}{dx} = f(x)g(y)$ can be solved by separating

variable $\dfrac{dy}{g(y)} = f(x)dx$.

The equation of the curve to the point (1,0) which satsifies the differential

equation $\left(1 + y^2\right)dx = xydy = 0$ is

A. $x^2 + y^2 = 1$

B. $x^2 - y^2 = 1$

C. $x^2 + y^2 = 2$

D. None of these

Answer: B

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10. Differential equation $\dfrac{dy}{dx} = f(x)g(x)$ can be solved by separating

variable $\dfrac{dy}{g(y)} = f(x)dx$.

Solution of the differential equation $\dfrac{dy}{dx} + \dfrac{1 + y^2}{\sqrt{1 - x^2}} = 0$ is

A. $\tan^{-1} y + \sin^{-1} x = C$

B. $\tan^{-1} x + \sin^{-1} y = C$

C. $\tan^{-1} y \cdot \sin^{-1} x = C$

D. $\tan^{-1} y - \sin^{-1} x = C$

Answer: A

11. Differential equation $\dfrac{dy}{dx} = f(x)g(x)$ can be solved by separating

variable $\dfrac{dy}{g(y)} = f(x)dx.$

If $\dfrac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0,$ then y is equal to

A. $e^{\frac{1-x^2}{2}}$

B. $e^{\frac{1+x^2}{2}} - 1$

C. In (1+x)-1

D. None of these

Answer: B

12. Let C be the set of curves having the property that the point of intersection of tangent with y-axis is equidistant from the point of tangency and origin (0,0)

If $C_1, C_2 \in C$ such that

C_1: Curve is passing through $(1, 0)$

C_2: Curve is passing through $(-1, 0)$

The number of common tangents for C_1 and C_2 is

A. 1

B. 2

C. 3

D. None of these

Answer: C

13. Let C be the set of curves having the property that the point of intersection of tangent with y-axis is equidistant from the point of tangency and origin (0,0)

If $C_3 \in C$

C_3 : is passing through (2,4). If $\dfrac{x}{a} + \dfrac{y}{b} = 1.$ is tangent to C_3, then

    A. $25a + 10b^2 - ab^2 = 0$

    B. $25a + 10b - 13ab = 0$

    C. $13a + 25b - 16ab = 0$

    D. $29a + b - 13ab = 0$

Answer: A

Differential Equations Exerise 5

1. Prove that $3 \tan^2\left(\dfrac{\pi}{6}\right) + \cos esc\left(\dfrac{\pi}{6}\right) + 2 \tan^2\left(\dfrac{\pi}{3}\right) = 9$

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2. Match the following :

Column I		Column II
(A) Circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(p)	4
(B) Side of cube increasing by 1%, then percentage increase in volume is	(q)	0.6π
(C) If the rate of decrease of $\dfrac{x^2}{2} - 2x + 5$ is twice the rate of decrease of x, then x is equal to	(r)	3
(D) Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm s: is	(s)	$\dfrac{3\sqrt{3}}{4}$

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3. Match the following :

(i)	sin (90°–A)	(a)	Sin A
(ii)	Cos 0°	(b)	0
(iii)	Sin 0°	(c)	1
(iv)	Cas (90°–A)	(d)	Cos A

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1. Find the constant of intergration by the general solution of the differential equation $(2x^2y - 2y^4)dx + (2x^3 + 3xy^3)dy = 0$ if curve passes through (1,1).

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2. A tank initially contains 50 gallons of fresh water. Brine contains 2 pounds per gallon of salt, flows into the tank at the rate of 2 gallons per minutes and the mixture kept uniform by stirring, runs out at the same rate. If it will take for the quantity of salt in the tank to increase from 40 to 80 pounds (in seconds) is 206λ, then find λ

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3. If $f: R - \{-1\} \to R$ and f is differentiable function satisfies:

$$f((x) + f(y) + xf(y)) = y + f(x) + yf(x) \, \forall x,$$

$y \in R - \{-1\}$ Find f(x).

4. If $\phi(x)$ is a differential real-valued function satisfying $\phi'(x) + 2\phi(x) \leq 1,$ then the value of $2\phi(x)$ is always less than or equal to ....

5. The degree of the differential equation satisfying by the curves $\sqrt{1+x} - a\sqrt{1+y} = 1,$ is ....

6. Let $f(x)$ be a differentiable bounded function satisfying $2f^5(x) \cdot f'(x) + 2(f'(x))^3 \cdot f^5(x) = f''(x)$. If $f(x)$ is bounded in between $y = k_1$, and $y = k_2$, then the number of intergers between k_1 and k_2 is/are (where $f(0) = f'(0) = 0$).

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7. Let $y(x)$ be a function satisfying $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + e^{2x} = 0, y(0) = 2$ and $y'(0) = 1$. If maximum value of $y(x)$ is $y(a)$, then integral part of $2a$ is..................

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8. Find the time required for a cylindrical tank of radius r and height H to empty through a round hole of area a at the bottom. The flow through the hole is according to the law $v(t) = k\sqrt{2gh(t)}$, where $v(t)$ and $h(t)$, are respectively, the velocity of flow through the hole and the height of

the water level above the hole at time t, and g is the acceleration due to gravity.

1. A hemi-spherical tank of radius 2 m is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = 0.6\sqrt{2gh(t)}$, where $v(t)$ and $h(t)$ are, respectively, the velocity of the flow through the outlet and the height of water level above the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank.

2. Let $f: R^+ \to R$ satisfies the functional equation $f(xy) = e^{xy - x - y}\{e^y f(x) + e^x f(y)\}$, $\forall x, y \in R^+$. If f'(1) = e, determine f(x).

3. Let y=f(x)be curve passing through $(1, \sqrt{3})$ such that tangent at any point P on the curve lying in the first quadrant has positive slope and the tangent and the normal at the point P cut the x-axis at A and B respectively so that the mid-point of AB is origin. Find the differential equation of the curve and hence determine f(x).

4. If y_1 and y_2 are the solutions of the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alond and $y_2 = y_1 z$, then move that $z = 1 + ce^{-\int \left(\frac{Q}{y_1}\right) dx}$, where c is an arbitrary constant.

1. Let y=f(x) be a differentiable function $\forall x \in R$ and satisfies:

$$f(x) = x + \int_0^1 x^2 z f(z) dz + \int_0^1 x z^2 f(z) dz.$$

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Exercise Questions Asked In Previous 13 Years Exam

1. If $f: R - \{-1\} \to R$ and f is differentiable function satisfies:

$$f((x) + f(y) + xf(y)) = y + f(x) + yf(x) \,\forall x,$$

$y \in R - \{-1\}$ Find f(x).

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2. A solution curve of the differential equation $\left(x^2 + xy + 4x + 2y + 4\right)\left(\dfrac{dy}{dx}\right) - y^2 = 0$ passes through the point $(1, 3)$ Then the solution curve is

A. intersects $y = x + 2$ exactly at one points

B. intersects $y = x + 2$ exactly at two points

C. intersects $y = (x + 2)^2$

D. does not intersect $y = (x + 3)^2$

Answer: A::D

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3. Let $f : (0, \infty) \to R$ be a differentiable function such that $f'(x) = 2 - \dfrac{f(x)}{x}$ for all x in $(0, \infty)$ and $f(1) \neq 1.$ Then

A. $\lim\limits_{x \to 0+} \displaystyle\int f'\left(\dfrac{1}{x}\right) = 1$

B. $\lim\limits_{x \to 0+} x \displaystyle\int f\left(\dfrac{1}{x}\right) = 1$

C. $\lim\limits_{x \to 0+} x^2 f'((x)) = 0$

D. $|f(x)| \le 2$ for all $x \in (0, 2)$

Answer: A::D

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4. Let $y(x)$ be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then watch of the following statements is (are) true?

A. y(-4)=0

B. y(-2)=0

C. y(x) has a critical point in the interval (-1,0)

D. y(x) has no critical point in the interval (-1,0)

Answer: A::C::D

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5. Let y(x) be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. if y(0)=2, then which of the following statement(s) is/are true?

   A. P=y+x

   B. P=y-x

   C. P+Q=1-x+y+y'+(y')^(2)`

   D. P-Q=x+y-y'-(y')^(2)`

Answer: B::C::D

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6. Solve the differential equation : $y\sqrt{1 - x^2}dy - \sqrt{1 + y^2}dx$=0

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7. Let $f: \left[\dfrac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x) < 2f(x)$ $and f\left(\dfrac{1}{2}\right) = 1$. Then the value of $\displaystyle\int_{\frac{1}{2}}^{1} f(x)\,dx$ lies in the interval (a) $(2e - 1, 2e)$ (b) $(3 - 1, 2e - 1)$ $\left(\dfrac{e - 1}{2}, e - 1\right)$ (d) $\left(0, \dfrac{e - 1}{2}\right)$

A. (2e-1,2e)

B. (e-1,2e-1)

C. $\left(\dfrac{e - 1}{2}, e - 1\right)$

D. $\left(0, \dfrac{e - 1}{2}\right)$

Answer: D

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8. A curve passes through the point $\left(1, \dfrac{\pi}{6}\right)$. Let the slope of the curve at each point (x,y) be $\dfrac{y}{x} + \sec\left(\dfrac{y}{x}\right), x > 0.$ Then, the equation of the curve

is

A. $\sin\left(\dfrac{y}{x}\right) = \log x + \dfrac{1}{2}$

B. $\cos ec\dfrac{y}{x} = \log x + 2$

C. $\sec\left(\dfrac{2y}{x}\right) = \log x + 2$

D. $\cos\left(\dfrac{2y}{x}\right) = \log x + \dfrac{1}{2}$

Answer: A

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9. Let $f:[0,1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0,1]$ If   the   function $e^{-x}f(x)$ assumes its min in the interval $[0,1]$ at $x = \dfrac{1}{4}$, which of the following is true?

A. $f'(x) < f(x), \dfrac{1}{4} < x < \dfrac{3}{4}$

B. $f'(x) > f(x), 0 < x < \dfrac{1}{4}$

C. $f'(x) < f(x), 0 < x < \dfrac{1}{4}$

D. $f'(x) < f(x), \dfrac{3}{4} < x < 1$

Answer: C

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10. Let $f : [0, 1] \to R$ be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0, 1]$ Which of the following is true for $0 < x < 1$?

A. (a) $0 < f(x) < \infty$

B. (b) $-\dfrac{1}{2} < f(x) < \dfrac{1}{2}$

C. (c) $-\dfrac{1}{4} < f(x) < 1$

D. (d) $-\infty < f(x) < 0$

Answer: D

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11. Let $f: [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0, 1]$ Which of the following is true for $0 < x < 1$? (A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(0, 1)$

C. g is increasing on (1,2) and decreasing on $(2, \infty)$

D. g is decreasing on (1,2) and increasing on $(2, \infty)$

Answer: B

12. Consider the statements : P : There exists some x IR such that f(x) + 2x

= 2(1+x^2) Q : There exists some x IR such that 2f(x) +1 = 2x(1+x). Then

    A. Both I and II are true

    B. I is true and II is false

    C. I is false and II is true

    D. Both I and II are false

Answer: C

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13. Differentiate the following w.r.t x

$(\log x)^{\cos x}$

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14. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0), x \in R,$ where $f'(x)$ denotes $\dfrac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0.$ Then the value of $y(2)$ is_____

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15. Let $f: R \to R$ be a continuous function which satisfies $f(x) = \displaystyle\int_0^x f(t)dt.$ Then the value of $f(1n6)$ is_____

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16. Let a solution y=y(x) of the differential equation $x\sqrt{x^2 - 1}dy - y\sqrt{y^2 - 1}dx = 0$ satisfy $y(2) = \dfrac{2}{\sqrt{3}}$

Statement I $y(x) = \sec\left(\sec^{-1} x - \dfrac{\pi}{6}\right)$

Statement II y(x) is given by $\dfrac{1}{y} = \dfrac{2\sqrt{3}}{x} - \sqrt{1 - \dfrac{1}{x^2}}$

A. Statement I is true, Statement II is also true, Statement II is the

correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the

correct explanation of Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is false, Statement II is true.

Answer: C

17. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation , $y(1 + xy)dx = xdy$, then $f\left(-\dfrac{1}{2}\right)$ is equal to:

A. $-\dfrac{2}{5}$

B. $-\dfrac{4}{5}$

C. $\dfrac{2}{5}$

D. $\dfrac{4}{5}$

Answer: D

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18. Let y(x) be the solution of the differential equation $(x \log x) \dfrac{dy}{dx} + y = 2x \log x, (x \geq 1)$, Then y(e) is equal to

A. e

B. O

C. 2

D. 2e

Answer: C

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19. Let the population of rabbits surviving at a time t be governed by the differential equation $\dfrac{dp(t)}{dt} = \dfrac{1}{2}p(t) - 200$. If $p(0) = 100$, then p(t) equals (1) $400 - 300e^{t/2}$ (2) $300 - 200e^{-t/2}$ (3) $600 - 500e^{t/2}$ (4) $40 - 300e^{-t/2}$

A. $400 - 300e^{\frac{t}{2}}$

B. $300 - 200e^{\frac{t}{2}}$

C. $600 - 500e^{\frac{t}{2}}$

D. $400 - 300e^{\frac{t}{2}}$

Answer: A

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20. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\dfrac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is (1) 3000 (2) 3500 (3) 4500 (4) 2500

A. 2500

B. 3000

C. 3500

D. 4500

Answer: C

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21. The population p(t) at time t of a certain mouse species satisfies the differential equation $d\dfrac{p(t)}{dt} = 0.5p(t) - 450$ If $p(0) = 850$, then the time at which the population becomes zero is (1) 2 In 18 (2) In 9 (3) $\dfrac{1}{2}$ In 18 (4) In 18

A. 2log18

B. log9

C. $\dfrac{1}{2}\log 18$

D. log18

Answer: A

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22. If $\dfrac{dy}{dx} = y + 3$ and y(0)=2, then y(log2) is equal to

A. 5

B. 13

C. -2

D. 7

Answer: D

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23. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\dfrac{dV(t)}{dt}$ =-k(T-t) , where $k > 0$ is a constant

and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is : (1) $T^2 - \dfrac{1}{k}$ (2) $I - \dfrac{kT^2}{2}$ (3) $I - \dfrac{k(T-t)^2}{2}$ (4) e^{-kT}

A. $I - \dfrac{kT^2}{2}$

B. $I - \dfrac{k(T-t^2)}{2}$

C. e^{-kT}

D. $T^2 - \dfrac{1}{k}$

Answer: A

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24. Solution of the differential equation

$\cos x\, dy = y(\sin x - y)dx, 0 < x < \dfrac{\pi}{2}$ is

A. secx=(tanx+C)y

B. ysecx=tanx+C

C. ytanx=secx+C

D. tanx=(secx+C)y

Answer: A

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25. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$ where c_1 and c_2 are arbitary constants is

A. $y' = y^2$

B. y''=y'y

C. yy''=y'

D. $yy'' = (y')^2$

Answer: D

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26. The differential equation of the family of circles with fixed radius 5 units and centre on the line y=2 is

A. (a)$(x - 2)y'^2 = 25 - (y - 2)^2$

B. (b)$(y - 2)y'^2 = 25 - (y - 2)^2$

C. (c)$(y - 2)y'^2 = 25 - (y - 2)^2$

D. (d)$(x - 2)y'^2 = 25 - (y - 2)^2$

Answer: C

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