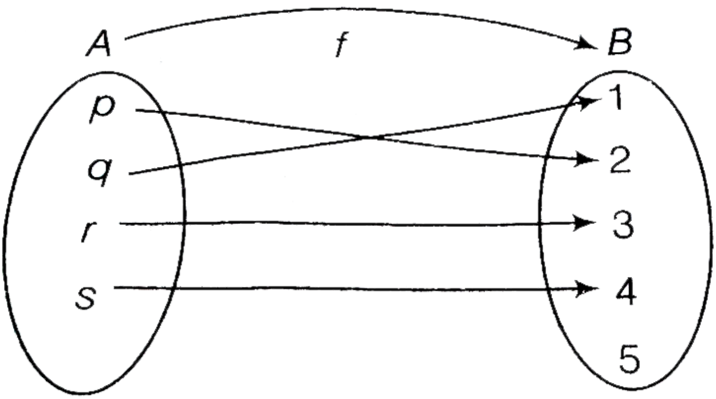
MATHS

BOOKS - ARIHANT MATHS

FUNCTIONS

Example

1. In the given figure, find the domain, codomain and range.



2. The number of functions $f: \{1, 2, 3, ...n\} \rightarrow \{2016, 2017\}$, where ne N, which satisfy thecondition f1)+f(2)+ ...+ f(n) is an odd number are

a. 2^n

b. $n \cdot 2^{n-1}$

c. 2^{n-1}

d. n!

A. 2^n

B. $n \cdot 2^{n-1}$

C. 2^{n-1}

D. n!

Answer: C

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3. Find whether $f(x) = x^3$ forms a mapping or not.

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4. Find whether $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ forms a mapping or not.

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5. Find the domain of the following functions.

$(i) y = \sqrt{5x - 3}$ $(ii) y = \sqrt[3]{5x - 3}$

$(iii) y = \frac{1}{(x-1)(x-2)}$ $(iv) y = \frac{1}{\sqrt[3]{x-1}}$

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6. Find the domain of $f(x) = \sqrt{\left(\frac{1-5^x}{7^{-1} - 7}\right)}$

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7. Draw the graph of polynominal funtions

$(i) y = x + 1 \qquad (ii) y = x^2$

$(iii) y = x^3 + 1 \quad (iv) y = x(x-1)(x-2)$

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8. Find domain of the function $10^x + 10^y = 10$

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9. Find the domain of the function : $f(x) = \dfrac{1}{\sqrt{(\log)_{\frac{1}{2}}(x^2 - 7x + 13)}}$

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10. Find domain of the function $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

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11. Find domain of $f(x) = \log_{10}(1 + x^3)$.

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12. Find domain of $f(x) = \log_{10}(1 + x^3)$.

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13. Find domain of the function $\log_{10} \log_{10} \log_{10} \log_{10} \log_{10} x$

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14. Find the domain of $f(x) = \sqrt{(\log)_{0.4}\left(\frac{x-1}{x+5}\right)}$

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15. Find the domain $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$

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16. The domain of definition of $f(x)=\frac{(\log)_2(x+3)}{x^2+3x+2}$ is $R-\{-1,-2\}$ (b) $-2,\infty)$ $R-\{-1,-2,-3\}$ (d) $(-3,\infty)-\{-1,-2\}$

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17. Find the domain for $f(x)=\sin^{-1}\left(\frac{x^2}{2}\right)$.

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18. The domain of definition of the function $f(x)=\sin^{-1}\left\{\log_2\left(\frac{x^2}{2}\right)\right\}$, is

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19. Find domain for $f(x) = \sqrt{\cos(\sin x)}$

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20. Find the domain for $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

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21. Find range and domain of $f(x) = \sqrt{sin^{-1}(\log_2 x)}$

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22. The domain of the function

$$f(x) = \log_e\left\{\log_{|\sin x|}\left(x^2 - 8x + 23\right) - \frac{3}{\log_2|\sin x|}\right\}$$

contains which of the following interval (s) ?

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23. Solve for x.

$|x-3|+|4-x|=1$

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24. solve $\left|x^2-1+\sin x\right|=\left|x^2-1\right|+|\sin x|$, where $x\in[-2\pi, 2\pi]$.

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25. Solve the equation $\left|\frac{X}{X-1}\right|+|X|=\frac{X^2}{|X-1|}\Bigg|$

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26. Find domain for $y=\frac{1}{\sqrt{|x|-x}}$.

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27. Find domain for

$$y = \cos^{-1}\left(\frac{1-2|x|}{4}\right) + \log(3-x)^{-1}.$$

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28. The domain of the function $f(x) = \frac{1}{\sqrt{4x = |x^2 - 10x + 9}}\Big)$ is $(7-\sqrt{40}, 7+\sqrt{40})$ $(0, 7+\sqrt{40})$ $(7-\sqrt{40}, \infty)$ (d) none of these

A. $(7-\sqrt{40}, 7+\sqrt{40})$

B. $(0, 7+\sqrt{40})$

C. $(7-\sqrt{40}, \infty)$

D. None of these

Answer: D

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29. The domain of the function

$f(x) = \sqrt{\left|\sin^{-1}(\sin x)\right| - \cos^{-1}(\cos x)}$ in $[0, 2\pi]$ is

A. $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

B. $[\pi, 2\pi]$

C. $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

D. $[0, 2\pi] - \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Answer: a

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30. Sketch the graph of

$(i)\, f(x) = sgn\left(x^2 + 1\right) \quad (ii)\, f(x) = sgn(\log_e x)$

$(iii)\, f(x) = sgn(\sin x) \quad (iv)\, f(x) = sgn(\cos x)$

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31. Find domain for, $f(x) = \cos^{-1} x$.

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32. Find the value of

$$\left[\frac{3}{4}\right] + \left[\frac{3}{4} + \frac{1}{100}\right] + \left[\frac{3}{4} + \frac{2}{100}\right] + \dots + \left[\frac{3}{4} + \frac{99}{100}\right].$$

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33. Given that $y = 2[x] + 3$ and $y = 3[x-2] + 5$ then find the value of $[x+y]$

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34. Find domain for $f(x) = [\sin x] \cos\left(\frac{\pi}{[x-1]}\right)$.

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35. The domain of the function

$$f(x) = \frac{\log_4\left(5 - [x-1] - [x]^2\right)}{x^2 + x - 2}$$ is

(where [x] denotes greatest integer function)

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36. Let [x] represent the greatest integer less than or equal to x If $\left[\sqrt{n^2+\lambda}\right] = \left[n^2+1\right] + 2$, where $\lambda, n \in N,$ then λ can assume (a) $(2n+4)$different values (b) $(2n+5)$different values (c) $(2n+3)$ different values (d) $(2n+6)$different values

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37. $f(x) = \frac{1}{\sqrt{[x] - x}}$, where $[\cdot]$ denotes the greatest integeral function less than or equals to x. Then, find the domain of f(x).

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38. The function $f(x)$ is defined in $[0, 1]$. Find the domain of $f(tanx)$.

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39. If the domain of $y = f(x)$ is $[-3, 2]$, then find the domain of $g(x) = f([x])$, where [] denotes the greatest integer function.

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40. Find the domain of function $f(x) = \dfrac{1}{[|x-1|] + [|7-x|] - 6}$ where $[\cdot]$ denotes the greatest integral function .

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41. If the function f(x)=[3.5+bsin x] (where [.] denotes the greatest integer function) is an even function, the complete set of values of b is

A. (-0.5,0.5)

B. [-0.5,0.5]

C. (0,1)

D. [-1,1]

Answer: A

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42. The domain of the function

$$f(x) = \log_3 \log_{1/3}\left(x^2 + 10x + 25\right) + \frac{1}{[x] + 5}$$

(where [.] denotes the greatest integer function) is

A. (-4,-3)

B. (-6,-5)

C. (-6,-4)

D. None of these

Answer: B

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43. If [x] dnote the greatest integer less than or equal to x then the equation $\sin x = [1 + \sin x] + [1 - \cos x][$ has no solution in

A. one solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. one solution in $\left[\frac{\pi}{2}, \pi\right]$

C. One solution in R

D. no solution in R

Answer: d

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44. If $\{x\}$ and $[x]$ represent fractional and integral part of x respectively, find the value of $[x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$

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45. Solve the equation $4[x] = x + \{x\}$

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46. Prove that $[x] + [y] \leq [x + y]$, where x=[x]+{x} and y=[y]+{y} $[\cdot]$ represents greatest integer function and $\{\cdot\}$ represents fractional part of x.

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47. Find the solution set of $(x)^2 + (x + 1)^2 = 25$ where (x) is the least integer greater than or equal to x.

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48. If $[x]$ is the greatest integer less than or equal to x and (x) be the least integer greater than or equal to x and $[x]^2 + (x)^2 > 25$ then x

belongs to

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49. The number of solutions of $|[x] - 2x| = 4$, where $[x]$ denotes the greatest integer $\leq x$ is

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50. Find the range for $y = \frac{2 + x - [x]}{1 - [x] + x}$.

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51. range of $f(x) = \frac{e^x}{[x] + 1}, x \geq 0$

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52. Find the domain and range of the function $y = \log_e(3x^2 - 4x + 5)$.

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53. Find the range of $f(x) = \sqrt{x-1} + \sqrt{5-x}$.

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54. Find the range of $\log_3\left\{\log_{\frac{1}{2}}(x^2 + 4x + 4)\right\}$

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55. Range of the function

$f(x) = \left(\cos^{-1}|1 - x^2|\right)$ is

a. $\left[0, \frac{\pi}{2}\right]$ b. $\left[0, \frac{\pi}{3}\right]$

c. (o, π) d. $\left(\frac{\pi}{2}, \pi\right)$

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[0, \frac{\pi}{3}\right]$

C. (o, π)

D. $\left(\frac{\pi}{2}, \pi\right)$

Answer: A

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56. If x, y and z are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$,then show that each of x,y and z lie in the closed interval $\left[\frac{2}{3}, 2\right]$

A. (-1,1)

B. [0,2]

C. [2,3]

D. $\left[\frac{2}{3}, 2\right]$

Answer: D

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57. The range of the function

$f(x) = \dfrac{1}{|\sin x|} + \dfrac{1}{|\cos x|}$ is

a. $[2\sqrt{2}, \infty)$

b. $(\sqrt{2}, 2\sqrt{2})$

c. $(0, 2\sqrt{2})$

d. $(2\sqrt{2}, 4)$

A. $[2\sqrt{2}, \infty)$

B. $(\sqrt{2}, 2\sqrt{2})$

C. $(0, 2\sqrt{2})$

D. $(2\sqrt{2}, 4)$

Answer: A

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58. If $z = x + iyandx^2 + y^2 = 16,$ then the range of $||x| - |y||$ is

A. [0,4]

B. [0,2]

C. [2,4]

D. None of these

Answer: A

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59. Find the range of f(x)=$\frac{1}{\pi}\sin^{-1}x + \tan^{-1}x + \frac{x+1}{x^2+2x+5}$

A. $\left[-\frac{3}{4}, \frac{1}{5}\right]$

B. $\left[-\frac{5}{4}, \frac{3}{4}\right]$

C. $\left[-\frac{3}{4}, \frac{5}{4}\right]$

D. $\left[-\frac{3}{4}, 1\right]$

Answer: D

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60. The range of the function $\sin^2 x - 5\sin x - 6$ is

A. [-10,0]

B. [-1,1]

C. $[0, \pi]$

D. $\left[-\frac{49}{4}, 0\right]$

Answer: A

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61. If $f(x) = [x^2] - [x]^2$, where [ ] denote the greatestinteger function and $x \varepsilon [0, n], \neq N$ then thenumber of elements In the range of f(x) are

A. (2n+1)

B. 4n-3

C. 3n-3

D. 2n-1

Answer: D

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62. Range of the function

$f(x)=\sqrt{\left|\sin^{-1}|\sin x|\right|-\left|\cos^{-1}|\cos x|\right|}$ is

A. {0}

B. $\left[0, \sqrt{\frac{\pi}{2}}\right]$

C. $\left[0, \sqrt{\pi}\right]$

D. None of these

Answer: A

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63. The number of values of $y \in [-2\pi, 2\pi]$ satisfying the equation $|\sin 2x| + |\cos 2x| = |\sin y|$ is

A. 3

B. 4

C. 5

D. 6

Answer: B

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64. $f(x) = \cot^{-1}(x^2 - 4x + 5)$ then range of $f(x)$ is equal to :

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(0, \frac{\pi}{4}\right]$

C. $\left[0, \frac{\pi}{4}\right)$

D. None of these

Answer: B

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65. Find the range of $f(x) = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$, where x $\in$ R.

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66. If the range of function $f(x) = \frac{x+1}{k+x^2}$ contains the interval [-0,1] , then values of k can be equal to

a. 0

b. 0.5

c. 1.25

d. 1.5

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67. Find the range of the function

$$f(x)=\frac{\sin^2 x+\sin x-1}{\sin^2 x-\sin x+2}.$$

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68. If f is an even function, then find the realvalues of x satisfying the equation $f(x)=f\left(\frac{x+1}{x+2}\right)$

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69. Find out whether the given function is even, odd or neither even nor odd

$$\text{where } \ f(x)=\begin{cases} x|x| & , \quad x\le -1 \\ [1+x]+[1-x] & , \quad -1<x<1 \\ -x|x| & , \quad x\ge 1 \end{cases}$$

where || and [] represent then modulus and greater integer functions.

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70. Find whether the given function is even or odd: $f(x)=\left(x\frac{\sin x+\tan x}{\left[x+\frac{\pi}{\pi}\right]-\frac{1}{2}}\right.;$ whether [] denotes the greatest integer function.

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71. Prove sin x is periodic and find its period.

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72. Prove that f(x)=x-[x] is periodic function. Also, find its period.

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73. Let $f(x)$ be periodic and k be a positive real number such that $f(x+k)+f(x)=0f$ or $allx\in R\cdot$ Prove that $f(x)$ is periodic with period $2k\cdot$

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74. Find periods for

$(i)\cos^4 x. \quad (ii)\sin^3 x. \quad (iii)\cos\sqrt{x}. \quad (iv)\cos x.$

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75. Find the period $f(x) = \sin x + \{x\}$, where {x} is the fractional part of $x\cdot$

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76. Find period of $f(x) = \tan 3x + \sin\left(\frac{x}{3}\right).$

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77. Find the period of $f(x) = \sin x + \tan\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2^2}\right) + tan\left(\frac{x}{2^3}\right) + ... + \sin\left(\frac{x}{2^{n-1}}\right) +$

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78. Find the period of $f(x) = |\sin x| + |\cos x|$.

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79. Period of $f(x) = \sin^4 x + \cos^4 x$

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80. Find the period of $\cos(\cos x) + \cos(\sin x)$.

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81. Find the period of $f(x) = \cos^{-1}(\cos x)$

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82. The period of $f(x) = \cos(|\sin x| - |\cos x|)$ is

A. π

B. 2π

C. $\frac{\pi}{2}$

D. None of these

Answer: C

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83. Period of the function $f(x) = \sin(\sin(\pi x)) + e^{\{3x\}}$, where {.} denotes the fractional part of x is

a. 1 b. 2

c. 3 d. none of these

A. 1

B. 2

C. 3

D. None of these

Answer: B

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84. $\sin \alpha x + \cos \alpha x$ and $|\cos x| + |\sin x|$ are periodic functions of same fundamental period, if 'a' equals

A. 0

B. 1

C. 2

D. 4

Answer: D

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85. Let $f(x) = \sin x + \cos\left(\sqrt{4 - a^2}\right)x$. Then, the integral values of 'a' for which f(x) is a periodic function, are given by

a. {2,-2}
b. (-2,2]
c. [-2,2]
d. none of these

A. {2,-2}

B. (-2,2]

C. [-2,2]

D. None of these

Answer: D

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86. Let $f(x) = \begin{cases} -1 + \sin K_1 \pi x, & \text{x is rational.} \\ 1 + \cos K_2 \pi x, & x \end{cases}$

If f(x) is a periodic function, then

A. either $K_1, K_2 \in$ rational or $K_1, K_2 \in$ irrational

B. $K_1, K_2 \in \text{ rational only}$

C. $K_1, K_2 \in \text{ irrational only}$

D. $K_1, K_2 \in \text{ irrational such that } \frac{K_1}{K_2}$ is rational

Answer: B

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87. If $f(x) = \tan^2\left(\frac{\pi x}{n^2 - 5n + 8}\right) + \cot(n + m)\pi x; (n \in N, m \in Q)$ is a periodic function with 2 as its fundamental period, then m can't belong to

a. $(-\infty, -2) \cup (-1, \infty)$

b. $(-\infty, -3) \cup (-2, \infty)$

c. $(-2, -1) \cup (-3, -2)$

d. $\left(-3, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, -2\right)$

A. $(-\infty, -2) \cup (-1, \infty)$

B. $(-\infty, -3) \cup (-2, \infty)$

C. $(-2, -1) \cup (-3, -2)$

D. $\left(-3, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, -2\right)$

Answer: C

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88. Let $f(x)$ be a periodic function with period 3 and $f\left(-\frac{2}{3}\right) = 7$ and $g(x) = \int_0^x f(t+n)dt$ .where $n = 3k, k \in N$. Then $g'\left(\frac{7}{3}\right) =$

A. $-\frac{2}{3}$

B. 7

C. -7

D. $\frac{7}{3}$

Answer: B

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89. Let $f:[-\pi/2, \pi/2] \to [-1, 1]$ where f(x)=sinx. Find whether f(x) is one-one or not.

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90. If $f(x) = x^3 + 3x^2 + 4x + b\sin x + c\cos x\,\forall x \in R$ is a one-one function then the value of $b^2 + c^2$ is

A. ≥ 1

B. ≥ 2

C. ≤ 1

D. None of these

Answer: C

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91. Show $f: R \rightarrow R$ defined by $f(x) = x^2 + x$ for all $x \in R$ is many-one.

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92. Show that $f: R \rightarrow R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is surjective but not injective.

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93. If $f: R \rightarrow \left[\frac{\pi}{6}, \frac{\pi}{2}\right], f(x) = \sin^{-1}\left(\frac{x^2 - a}{x^2 + 1}\right)$ is an onto function, the set of values a is

A. $\left\{-\frac{1}{2}\right\}$

B. $\left[-\frac{1}{2}, -1\right)$

C. $(-1, \infty)$

D. None of these

Answer: C

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94. Show $f: R \to R$ defined by $f(x) = x^2 + 4x + 5$ is into

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95. Let $A = \{x : -1 \leq x \leq 1\} = B$ be a function $f: A \to B$. Then find the nature of each of the following functions.

(i) $f(x) = |x|$ $\quad$ (ii) $f(x) = x|x|$

(iii) $f(x) = x^3$ $\quad$ (iv) $f(x) = \sin\frac{\pi x}{2}$

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96. The function $f : R \to R$ defined as $f(x) = \frac{1}{2} In\left(\sqrt{\sqrt{x^2+1}+x}+\sqrt{\sqrt{x^2+1}-x}\right)$ is

A. one-one and onto both

B. one-one but not onto

C. onto but not one-one

D. Neither one-one nor onto

Answer: D

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97. If X={1,2,3,4,5} and Y={a,b,c,d,e,f} and $f : X \to Y$, find the total number

of

functions .

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98. Find the number of surjections from A to B, where A={1,2,3,4}, B={a,b}.

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99. $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5.$ Then f(x)=g(x) holds for x belonging to

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100. Let $A = \{1, 2\}, \ B = \{3, 6\}$ and $f: A \to B$ given by $f(x) = x^2 + 2 \, nd \, g: A \to B$ given by $g(x) = 3x$. Then we observe that $f \, and \, g$ have the same domain and co-domain. Also we hve, $f(1) = 3 = g(1) and \, f(2) = 6 = g(2)$. Hence $f = g$.

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101. Which pair of functions is identical?

a. $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1} x)$

b. $\log_e e^x, e^{\log_e x}$

c. $\log_e x^2, 2\log_e x$

d. None of the above

A. $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1} x)$

B. $\log_e e^x, e^{\log_e x}$

C. $\log_e x^2, 2\log_e x$

D. None of the above

Answer: D

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102. Let $f: A \to B$ and $g: B \to C$ be two functions and $gof: A \to C$ is define statement(s) is true?

a. If gof is one-one, then f anf g both are one-one

b. if gof is one-one, then f is one-one

c. If gof is a bijection, then f is one-one and g is onto

d. If f and g are both one-one, then gof is one-one.

A. If gof is one-one, then f anf g both are one-one

B. if gof is one-one, then f is one-one

C. If gof is a bijection, then f is one-one and g is onto

D. If f and g are both one-one, then gof is one-one.

Answer: B::C::D

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103. Let R be the set of real numbes. If $f: R\vec{R}; f(x) = x^2$ and $g: R\vec{R}; g(x) = 2x + 1$. Then, find $fogandgof$. Also, show that $fog \neq gof$.

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104. Let g(x)=1+x-[x]

and $$f(x) = \begin{cases} -1, x < 0 \\ 0, x = 0 \\ 1, x > 0 \end{cases}$$

Then, for all x, find f(g(x)).

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105. Let $f(x)\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}:$

Find fof.

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106. Let

$f(x)=\begin{cases}x+1, & x<1\\ 2x+1, & 1<x\le 2\end{cases}$ and $g(x)=\begin{cases}x^3, & -1\le x<2\\ x+2, & 2\le x\le 3\end{cases}$

find $fog(x)$.

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107. If $f(x)=2x+|x|, g(x)=\frac{1}{3}(2x-|x|)$ and h(x)=f(g(x)), domain of $\underbrace{\sin^{-1}(h(h(h(h...h(x)...))))}_{\text{n times}}$ is

A. [-1,1]

B. $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

C. $\left[-1, -\frac{1}{2}\right]$

D. $\left[\frac{1}{2}, 1\right]$

Answer: A

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108. A function $f: \overrightarrow{RR}$ satisfies $x \cos y(f(2x+2y) - f(2x-2y) = \cos x \sin y(f(2x+2y) + f(2x-2y))$. If $f'(0) = \frac{1}{2}, then$

A. $f(x)''(x) = f(x) = 0$

B. $4f''(x) + f(x) = 0$

C. $f''(x) + f(x) = 0$

D. $4f''(x) - f(x) = 0$

Answer: B

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109. If $f(x) = 3x - 5,$ then $f^{-1}(x)$ is

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110. If $f: [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x},$ find $f^{-1}(x)$ (assume bijection).

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111. Let $f(x) = x^3 + 3$ be bijective, then find its inverse.

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112. The inverse of the function of $f: R \to R$ given by $f(x) = \log_a\left(x + \sqrt{x^2 + 1}(a > 0, a \neq 1)\right.$ is

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113. Let $f: R \to R$ be defined by $f(x) = \left(e^x - e^{-x}\right)/2$. then find its inverse.

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114. Let $f: \left[\frac{1}{2}, \infty\right) \to \left[\frac{3}{4}, \infty\right)$, where $f(x) = x^2 - x + 1$. Find the inverse of f(x).

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115. Let g(x) be the inverse of f(x) and $f'(x) = \frac{1}{1 + x^3}$.Find g'(x) in terms of g(x).

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116. If $f: R \rightarrow R$ be defined by $f(x): x^2 + 1, \ then \ find \ f^{-1}(17) and \ f^{-1}(-3)$.

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117. If the function f and g are defined as $f(x) = e^x$ and g(x)=3x-2, where $f: R \rightarrow R$ and $g: R \rightarrow R$, find the function fog and gof.

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118. If f(x)=ax+b and the equation $f(x) = f^{-1}(x)$ be satisfied by every real value of x, then

A. a=2, b=-1

B. $a = -1, b \in R$

C. $a = 1, b \in R$

D. a=1, b=-1

Answer: B

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119. If g is the inverse function of and f'(x) = sin x then prove that g'(x) = cosec (g(x))

A. sin(g(x))

B. cosec(g(x))

C. tan(g(x))

D. None of these

Answer: B

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120. If A and B are the points of intersection of y=f(x) and $y = f^{-1}(x)$, then

A. (a) A and B necessarity lie on the line y=x

B. (b) A and B must be coincident

C. (c) slope of line AB may be -1

D. (d) None of these above

Answer: C

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121. For $x \in R$, the functions f(x) satisfies $2f(x) + f(1-x) = x^2$. The value of f(4) is equal to

A. $\frac{13}{3}$

B. $\frac{43}{3}$

C. $\frac{23}{3}$

D. None of these

Answer: C

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122. if $f(x) = ax^7 + bx^3 + cx - 5, f(-7) = 7$ then $f(7)$ is

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123. $f(x) + f\left(1 - \frac{1}{x}\right) = 1 + x$ for $x \in R - \{0, 1\}$. Find the value of $4f(2)$.

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124. Draw the graph of the function $f(x) = \max\{x, x^2\}$ and write its equivalent definition.

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125. Let

$f(x) = \max\{1 + \sin x, 1, 1 - \cos x\}, x \in [0, 2\pi],$ and $g(x) = \max\{1, |$

Then

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126. Let $f(x) = \dfrac{a_{2k}x^{2k} + a_{2k-1}x^{2k-1} + \ldots + a_1x + a_0}{b_{2k}x^{2k} + b_{2k-1}x^{2k-1} + \ldots + b_1x + b_0}$, where k is a positive integer, $a_i, b_i \in R$ and $a_{2k} \neq 0, b_{2k} \neq 0$ such that $b_{2k}x^{2k} + b_{2k-1}x^{2k-1} + \ldots + b_1x + b_0 = 0$ has no real roots, then

A. f(x) must be one to one

B. $a_{2k}x^{2k} + a_{2k-1} + \ldots + a_1x + a_0 = 0$

must have real roots

C. f(x) must be many to one

D. Nothing can be said about the above options

Answer: C

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127. If $\log_{10}\left(\sin\left(x+\frac{\pi}{4}\right)\right)=\frac{\log_{10}6-1}{2}$, the value of $\log_{10}(\sin x)+\log_{10}(\cos x)$ is

A. -1

B. -2

C. 2

D. 1

Answer: A

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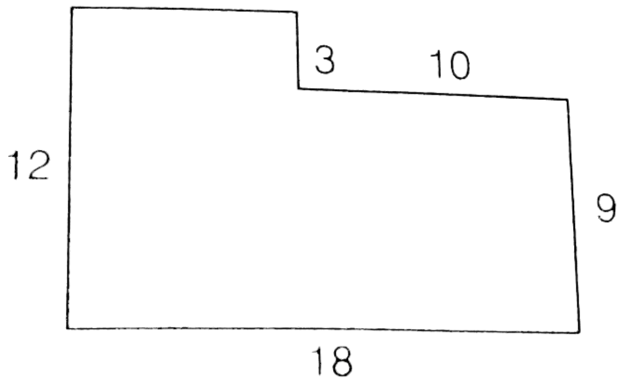
128. The diagram shows the dimensions of the floor of an L-shaped room. (All the angles are right angles). The area of the largest circle that can be drawn on the floor of this room is



A. 16π

B. 25π

C. $\frac{81\pi}{4}$

D. $\frac{145\pi}{4}$

Answer: B

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129. Suppose that the temperature T at every point (x,y) in the plane cartesian is given by the formula $T = 1 - x^2 + 2y^2$. The correct statement about the maximum and minimum temperature along the line

x+y=1 is

A. Minimum is -1. There is no maximum

B. Maximum is -1. There is no minimum

C. Maximum is 0. Minimum is -1

D. There is neither a maximum nor a minimum along the line

Answer: A

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130. The domain of the function f(x)=sin x, is $(-\infty, \infty)$. The range of f(x) is

A. $\left[-\frac{1}{\sqrt{2}}, 1\right]$

B. $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

C. [0,1]

D. [-1,1]

Answer: A

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131. Consider the function $f: A \to A$ where $A: \{1, 2, 3, 4, 5\}$ which satisfy the condition $f(f(x)) = x,$ If the number of such functions are $\lambda,$ then

A. 10

B. 40

C. 41

D. 31

Answer: C

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132. Area bounded by the relation $[2x] + [y] = 5, x, y > 0$ is___

A. (a) 2

B. (b) 3

C. (c) 4

D. (d) 5

Answer: B

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133. If the integers a,b,c,d are in arithmetic progression and $a < b < c < d$ and $d = a^2 + b^2 + c^2$, the value of (a+10b+100c+1000d) is

A. 2008

B. 2010

C. 2099

D. 2016

Answer: C

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134. Let $f(n)$ denotes the square of the sum of the digits of natural number n, where $f^2(n)$ denotes $f(f(n))$. $f^3(n)$ denote $f(f(f(n)))$ and so on.the value of $\frac{f^{2011}(2011) - f^{2010}(2011)}{f^{2013}(2011) - f^{2012}(2011)}$ is....

A. 1

B. 3

C. 5

D. 7

Answer: A

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135. Find $\frac{dy}{dx}$ if $y = \cos^4 x$

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136. If $f(x-y), f(x)f(y),$ and $f(x+y)$ are in A.P. for all x, y, and $f(0) \neq 0,$ then

A. $f(3) + f(-3) = 0$

B. $f'(1) + f'(-1) = 0$

C. $f'(2) - f'(-2) = 0$

D. $f'(3) + f'(-3) = 0$

Answer: B::D

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137. $x^2 + 4 + 3\cos(ax + b) = 2x$ has atleast on solution then the value of a+b is :

A. 5π

B. 3π

C. 2π

D. π

Answer: B::D

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138. Which of following functions have the same graph?

A. $f(x) = \log_e e^x$

B. $g(x) = |x| sgnx$

C. $h(x) = \cot^{-1}(\cot x)$

D. $k(x) = \lim_{n \to \infty} \frac{2|x|}{\pi} \tan^{-1}(nx)$

Answer: A::B::D

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139. Find $\frac{dy}{dx}$ if $y = \sin x.\cos x$

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140. Consider two functions $f(x) = 1 + e^{\cot^2 x}$ and $g(x) = \sqrt{2|\sin x| - 1} + \frac{1 - \cos 2x}{1 + \sin^4 x}$.

Statement I The solutions of the equation f(x)=g(x) is given by $x = (2n + 1)\frac{\pi}{2}, \forall \text{n} \in I.$

Statement II If $f(x) \geq k$ and $g(x) \leq k$ (where k $\in$ R), then solutions of the equation f(x)=g(x) is the solution corresponding to the equation f(x)=k.

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141. Let $a_m (m = 1, 2, , p)$ be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$meets at some point for all real values of b Let $t_r = \prod_{m=1}^{p}(r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r.\ n \in N$ The minimum possible value of a is

A. $\frac{1}{5}$

B. $\frac{5}{26}$

C. $\frac{3}{28}$

D. $\frac{2}{43}$

Answer: A

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142. Let $a_m(m = 1, 2, , p)$ be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$meets at some point for all real values of b Let $t_r = \prod_{m=1}^{p}(r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r.\ n \in N$ The minimum possible value of a is

A. 8

B. 9

C. 10

D. 15

Answer: C

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143. Find $\frac{dy}{dx}$ if $y = 5x^2 - 3bx - a$

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144. Find $\frac{dy}{dx}$ if $y^5 = x$

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145. If f(x) is a polynomial function f:R → R such that f(2x)=f `(x) f "(x)

Then f(x) is

A. a. one- one and onto

B. b. one-one and into

C. c. many-one and onto

D. d. many-one and into

Answer: A

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146. Let $A = \{1, , 2, 3\}$ and $B = \{-2, -1, 0, 1, 2, 3\}$.

The probability of non decreasing functions from A to B, is

A. 120

B. 72

C. 60

D. 56

Answer: D

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147. Let $A = \{1, , 2, 3\}$ and $B = \{-2, -1, 0, 1, 2, 3\}$.

The probability of non decreasing functions from A to B, is

A. 216

B. 540

C. 792

D. 840

Answer: C

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148. Let A={1,2,3,4,5} and B={-2,-1,0,1,2,3,4,5}.

Onto functions from A to A such that $f(i) \neq i$ for all i, is

A. (a)44

B. (b)120

C. (c)56

D. (d)76

Answer: A

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149. Let $f(x) = \sin^{23} x - \cos^{22} x and g(x) = 1 + \frac{1}{2}\tan^{-1}|x|$. Then the number of values of x in the interval $[-10\pi, 8\pi]$ satisfying the equation $f(x) = sgn(g(x))$ is _________

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150. Consider the function g(x) defined as

$g(x)\cdot\left(x^{\left(2^{2008}-1\right)}-1=(x+1)\left(x^2+1\right)\left(x^4+1\right)\ldots\left(x^{2^{2007}}+1\right)-1\right.$

the value of g(2) equals ....

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151. if $f(x) = \left(\frac{9}{\log_2(3-2x)} - 1\right)^{\frac{1}{3}}$ then the value of a which satisfies $f^{-1}(2a-4) = \frac{1}{2}$ is

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152. Let f be defined on the natural numbers as follow: f(1)=1 and for $n > 1,\ f(n) = f[f(n-1)] + f[n - f(n-1)]$, the value of $\frac{1}{30}\sum_{r=1}^{20} f(r)$ is

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153. If a,b,c are real roots of the cubic equation f(x)=0 such that $(x-1)^2$ is a factor of f(x)+2 and $(x+1)^2$ is a factor of f(x)-2, then $|ab+bc+ca|$ is equal to

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154. Find the least positive intergral value of c for which equation $e^x = cx^2$ has three distinct real roots.

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155. $x = \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \dots}}}}$

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156. Let a sequence $x_1, x_2, x_3, \dots$ of complex numbers be defined by $x_1 = 0, x_{n+1} = x_n^2 - i$ for all $n > 1$, where $i^2 = -1$. Find the distance of x_{2000} from x_{1997} in the complex plane.

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157. If a, b, c, d, e are +ve real numbers such that $a + b + c + d + e = 8$ and $a^2 + b^2 + c^2 + d^2 + e^2 = 16$, then the

range of 'e' is

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158. Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$

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159. Solve the equation [x]=x, where [] denote the greatest integer function.

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160. Sum of all the solution of the equation $\frac{[x]}{[x-2]} - \frac{[x-2]}{[x]} = \frac{8\{x\} + 12}{[x-2][x]}$ is (where{*} denotes greatest integer function and {*} represent fractional part function)

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161. If $f(x)$ is a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65, thenf \in df(6)$.

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162. If f(x) satisfies the relation, f(x+y)=f(x)+f(y) for all x,y $\in$ R and f(1)=5, then find $\sum_{n=1}^{m} f(n)$. Also, prove that f(x) is odd.

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163. Let $f(x) = \frac{9^x}{9^x + 3}$. Show $f(x) + f(1 - x) = 1$ and, hence, evaluate. $f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + f\left(\frac{3}{1996}\right) + + f\left(\frac{1995}{1996}\right)$

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164. ABCD is a square of side a. A line parallel to the diagonal BD at a distance x from the vertex A cuts the two adjacent sides. Express the area of the segment of the square with A at a vertex, as a function of x.

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165. If $f: R \to R$, $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$ is onto then $\alpha \in$

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166. What is the general solution of the differential equation

(2x − y + 1) dx + (2y − x + 1) dy = 0?

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167. The real solution of $[x] + 5x + [10x] + [20x] = 36k + 35, k \in I$, if the fractional part of x lies in $\left[\dfrac{1}{10}, \dfrac{1}{5}\right)$

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168. Let $f : N \to N$ be a function such $x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right], \forall x \in N$, where [.] denotes the greatest integer function and [.] denotes the greatest integers function and $1900 < f(1990) < 2000$, then possible value of $f(1990)$ is

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169. Solve the system of equations,

$$\left|x^2 - 2x\right| + y = 1, x^2 + |y| = 1.$$

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170. Let f and g be real - valued functions such that $f(x + y) + f(x - y) = 2f(x) \cdot g(y), \forall$ x , y $\in R$.

Prove that , if f(x) is not identically zero and $|f(x)| \leq 1, \forall x \in R,$ then $|g(y)| \leq 1, \forall y \in R.$

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171. The solution of the differential equation

$\frac{dy}{dx}$ = 1 + x + y + xy is ________.

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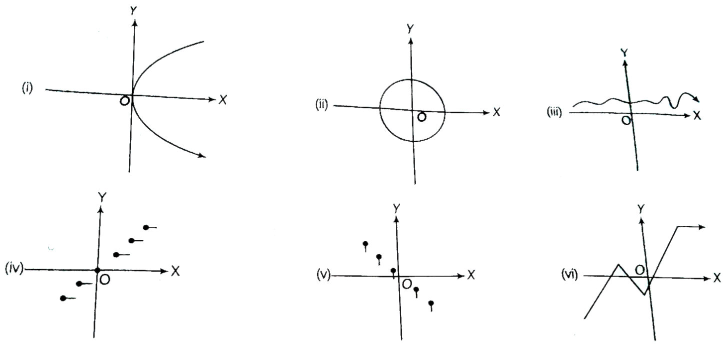
Exercise For Session 1

1. Which of the following graphs are graphs of a function?



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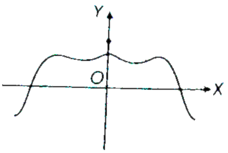

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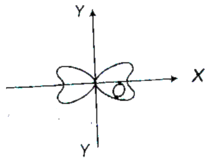

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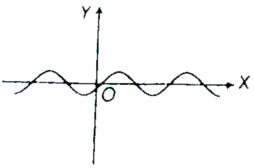

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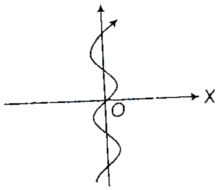

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2. For which of the following, y can be a function of x, $(x \in R, y \in R)$?

$(i)(x-h)^2+(y-k)^2=r^2$ $(ii)y^2=4ax$

$(iii)x^4=y^2$ $(iv)x^6=y^3$

$(v)3y=(\log x)^2$

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3. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at $(0, 0) and (x, g(x))$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is $g(x)=\pm\sqrt{1-x^2}$ $g(x)=\sqrt{1-x^2}$ $g(x)=-\sqrt{1-x^2}$ $g(x)=\sqrt{1+x^2}$

A. $g(x) = \pm\sqrt{1-x^2}$

B. $g(x) = \sqrt{1-x^2}$

C. $g(x) = -\sqrt{1-x^2}$

D. $g(x) = \sqrt{1+x^2}$

Answer: A

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4. Represent all possible functions defined from $\{\alpha, \beta\}$ to $\{1, 2\}$.

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5. The number of functions from $f: \{a_1, a_2, \ldots, a_{10}\} \rightarrow \{b_1, b_2, \ldots, b_5\}$ is

A. 10^5

B. 5^{10}

C. $\frac{10!}{5!}$

D. 5!

Answer: B

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Exercise For Session 2

1. The domain of the function

$f(x)=\sqrt{x^2-5x+6}+\sqrt{2x+8-x^2}$, is

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2. Find domain $f(x)=\sqrt{\frac{2x+1}{x^3-3x^2+2x}}$

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3. Find the domain of $f(x) = \sqrt{1 - \sqrt{1 - \sqrt{1 - x^2}}}$

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4. The exhaustive domain of $f(x) = \sqrt{x^{12} - x^9 + x^4 - x + 1}$ is

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5. The domain of the function $f(x) = {}^{16 - x}C_{2x - 1} + {}^{20 - 3x}P_{4x - 5}$, where the symbols have their usual meanings, is the set

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6. Find the domain? $f(x) = \sqrt{(x^2 + 4x)C_{2x^2 + 3}}$

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Exercise For Session 3

1. The domain of the function

$f(x) = \log_{10}\left(\sqrt{x-4} + \sqrt{6-x}\right)$ is

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2. Find domain of $f(x) = \sqrt{\log_{\frac{1}{2}}\left(\frac{5x - x^2}{4}\right)}$

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3. $f(x) = \sqrt{\log\left(\frac{3x - x^2}{x - 1}\right)}$

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4. Find the domain of definitions of the following function:

$f(x) = \log_{10}\left(1 - \log_{10}\left(x^2 - 5x + 16\right)\right)$

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5. $f(x) = \sin|x| + \sin^{-1}(\tan x) + \sin(\sin^{-1} x)$

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6. The domain of definition of $f(x) = \sqrt{e^{\cos^{-1}(\log_4 x^2)}}$ is

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7. $f(x) = \sin^{-1}\left(\frac{3-2x}{5}\right) + \sqrt{3-x}$.Find the domain of f(x).

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8. Find the domain $f(x) = \frac{\log_{2x} 3}{\cos^{-1}(2x-1)}$

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9. Find the domain of $f(x) = (\log)_{10}(\log)_2(\log)_{\frac{2}{\pi}}\left(tan^{-1}x\right)^{-1}$

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10. $f(x) = \sqrt{\frac{\log(x-1)}{x^2-2x-8}}$. Find the domain of f(x).

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Exercise For Session 4

1. $f(x) = \sqrt{x^2 - |x| - 2}$. Find the domain of f(x).

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2. $f(x) = \sqrt{2 - |x|} + \sqrt{1 + |x|}$. Find the domain of f(x).

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3. $f(x) = \log_e |\log_e x|$. Find the domain of f(x).

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4. $f(x) = \sin^{-1}\left(\frac{2 - 3[x]}{4}\right)$, which $[\cdot]$ denotes the greatest integer function.

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5. $f(x) = \log(x - [x])$, where $[\cdot]$ denotes the greatest integer function. find the domain of f(x).

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6. $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$, where $[\cdot]$ denotes the greatest integer function.

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7. $f(x) = \cos ec^{-1}[1 + \sin^2 x]$, where $[\cdot]$ denotes the greatest integer function. then range of f(x) is

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8. $f(x) = \cos^{-1}\sqrt{\log_{[x]}\frac{|x|}{x}}$, where $[\cdot]$ denotes the greatest integer.

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9. $f(x) = \sqrt{\frac{x-1}{x-2\{x\}}}$, where $\{\cdot\}$ denotes the fractional part.

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10. Domain of $f(x) = \sin^{-1}\left(\frac{[x]}{\{x\}}\right)$, where $[\cdot]$ and $\{\cdot\}$ denote greatest integer and fractional parts.

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11. $f(x) = \sin^{-1}[2x^2 - 3]$, where $[\cdot]$ denotes the greatest integer function. Find the domain of f(x).

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12. Find the domain and range of the following function:

$f(x) = \sin^{-}\left[\log_2\left(\frac{x^2}{2}\right)\right]$, where [.] denotes greatest integer function.

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13. The domain of $f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$, where {.} denotes the fractional part in $[-1, 1]$ is (a) $[-1, 1] - \left(\frac{1}{2}, 1\right)$ (b) $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{0, 1}{2}\right] \cup \{1\}$ (c) $\left[-1, \frac{1}{2}\right]$ (d) $\left[-\frac{1}{2}, 1\right]$

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14. $f(x) = \frac{1}{(x-2)+(x-10)-8}$ Find integration of f(x)

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15. If a function is defined as $f(x) = \sqrt{\log_{h(x)} g(x)}$, where $g(x) = |\sin x| + \sin x, h(x) = \sin x + \cos x, 0 \leq x \leq \pi$. Then find th domain of $f(x)$.

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16. The number of solutions of the equation $[y + [y]] = 2\cos x$, where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$.

where [.] denotes the greatest integer function, is

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17. Prove that for $n = 1, 2, 3...$ $\left[\frac{n+1}{2}\right] + \left[\frac{n+2}{4}\right] + \left[\frac{n+4}{8}\right] + \left[\frac{n+8}{16}\right] + ... = n$ where $[x]$ represents Greatest Integer Function

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18. Find the integral solutions to the equation $[x][y] = x + y$. Show that all the non-integral solutions lie on exactly two lines. Determine these lines. Here [.] denotes greatest integer function.

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Exercise For Session 5

1. $f(x) = \sqrt{9 - x^2}$. find range of f(x).

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2. $f(x) = \frac{x}{1+x^2}$. Find domain and range of f(x).

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3. $f(x) = \sin x + \cos x + 3$. find the range of f(x).

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4. $f(x) = |x-1| + |x-2|, \; -1 \leq x \leq 3$. Find the range of f(x).

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5. $f(x) = \log_3(5 + 4x - x^2)$. find the range of f(x).

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6. $f(x) = \frac{x^2 - 2}{x^2 - 3}$. find the range of f(x).

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7. $f(x) = \frac{x^2 + 2x + 3}{x}$. Find the range of f(x).

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8. $f(x) = |x - 1| + |x - 2| + |x - 3|$. Find the range of f(x).

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9. Find the range of the following function:

$f(x) = \log_{[x-1]} \sin x$, where [ ] denotes greatest integer function.

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10. $f(x) = \cos^{-1} \sqrt{\log_{[x]} \frac{|x|}{x}}$, where $[\cdot]$ denotes the greatest integer function.

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11. Let $f(x)=\sqrt{[\sin 2x]-[\cos 2x]}$ (where I I denotes the greatest integer function) then the range of f(x) will be

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12. The range of $\sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left[x^2-\frac{1}{2}\right]$, where [.] denotes the greatest integer function, is (a)$\left\{\frac{\pi}{2},\pi\right\}$ (b) $\{\pi\}$ (c) $\left\{\frac{\pi}{2}\right\}$ (d) none of these

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13. Range of $f(x)=\sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is

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14. $f(x) = \cos^{-1}\left(\frac{x^2}{\sqrt{1+x^2}}\right)$

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15. Find the range of $f(x) = \sqrt{\log(\cos(\sin x))}$

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16. $f(x) = \frac{x-1}{x^2-2x+3}$ Find the range of f(x).

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17. if: $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} - \frac{\cos x}{\sqrt{1+\cot^2 x}}$, then find the range of $f(x)$

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18. Range of $f(x) = \dfrac{\tan\left(\pi\left[x^2 - x\right]\right)}{1 + \sin(\cos x)}$

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19. range of $f(x) = \dfrac{e^x}{[x] + 1}, x \geq 0$

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20. Find the range of $f(x) = [|\sin x| + |\cos x|]$, where $[\cdot]$ denotes the greatest integer function.

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21. Find domain of $f(x)$ if $f(x) = \sqrt{-x^2 + 4x - 3} + \sqrt{\sin\frac{\pi}{2}\left(\sin\frac{\pi}{2}(x - 1)\right)}$

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22. Find the image of the following sets under the mapping

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 10$

$(-\infty, 1)$

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23. Find the domain and range of $f(x) = \log\left[\cos|x| + \frac{1}{2}\right]$,where [.] denotes the greatest integer function.

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24. Find the domain and range of $f(x) = \sin^{-1}(\log[x]) + \log(\sin^{-1}[x])$, where [.] denotes the greatest integer function.

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25. Find the domain and range of $f(x) = \left[\log\left(\sin^{-1}\sqrt{x^2 + 3x + 2}\right)\right]$.

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Exercise For Session 6

1. Determine whether the following functions are even or odd.

$\left((i) f(x) = \log\left(x + \sqrt{1 + x^2}\right), (ii) f(x) = x\left(\frac{a^x + 1}{a^x - 1}\right)\right), ((iii) f(x) = \text{s}$

$\left((v) f(x) = \log\left(\frac{1 - x}{1 + x}\right), (vi) f(x) = \left\{(sgnx)^{sgnx}\right\}^n, \quad \text{n is an odd integer}\right.$

$((vii) f(x) = sgn(x) + x^2,), ((viii) f(x + y) + f(x - y) = 2f(x) \cdot f(y),$

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2. Determine whether function, $f(x) = (-1)^{[x]}$ is even, odd or neither of two (where $[\cdot]$ denotes the greatest integer function).

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3. A function defined for all real numbers is defined for $x \geq 0$ as follows

$$f(x) = \begin{cases} x|x| & 0 \leq x \leq 1 \\ 2x & x \geq 1 \end{cases}$$

How if f defined for $x \leq 0$.

If (i) f is even ? (ii) f is odd ?

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4. Show the function, $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\frac{x+21\pi}{\pi}\right] - 41}$ is symmetric about origin.

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5. If $f : [-20, 20] \to R$ defined by $f(x) = \left[\dfrac{x^2}{a}\right]\sin x + \cos x$ is an even function, then set of values of a is

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Exercise For Session 7

1. Find $\frac{dy}{dx}$ if $y = \sin 4x$

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2. Find the period of the real-valued function satisfying f(x)+f(x+4)=f(x+2)+f(x+6).

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3. Check whether the function defined by $f(x+\lambda) = 1 + \sqrt{2f(x) - f^2(x)}\ \forall x \in R$ is periodic or not. If yes, then find its period $(\lambda > 0)$.

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4. Let f(x) be a real valued periodic function with domain R such that $f(x+p)=1+\left[2-3f(x)+3(f(x))^2-(f(x))^3\right]^{1/3}$ hold good for all $x \in R$ and some positive constant p, then the periodic of f(x) is

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5. Let $f(x)$ be a function such that : $f(x-1)+f(x+1)=\sqrt{3}f(x)$, for all $x\varepsilon R$. If $f(5)=100$, then prove that the value of $\sum_{r=o}^{99} f(5+12r)$ will be equal to 10000.

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Exercise For Session 8

1. There are exactly two distinct linear functions, which map [-1,1] onto [0,3]. Find the point of intersection of the two functions.

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2. Let f be an injective map with domain $\{x, y, z\}$ and range {1, 2, 3} such that exactly one of the following statements is correct and the remaining are false. $f(x) = 1, \quad f(y) \neq 1, \ f(z) \neq 2$. The value of $f^{-1}(1)$ is

(a) x (b) y (c) z (d) none of these

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3. Let $A = R - \{3\}, B = R - \{1\}$ and $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}$. Is 'f' bijective? Give reasons.

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4. Let $f: R \to R$ defined by $f(x) = \frac{x^2}{1+x^2}$. Proved that f is neither injective nor surjective.

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5. If the function $f: R \rightarrow A$ given by $f(x) = \frac{x^2}{x^2 + 1}$ is a surjection, then $A = R$ (b) [0, 1] (c) (0, 1] (d) [0, 1)

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6. If the function of $f: R \rightarrow A$ is given by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$ is surjection, find A

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7. Let $f(x) = ax^3 + bx^2 + cx + d \sin x$. Find the condition that $f(x)$ is always one-one function.

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8. Let $f: X \rightarrow Y$ be a function defined by f(x) = a sin (x + $\frac{\pi}{4}$) + b cosx + c. If f is both one-one and onto, then find the set X and Y

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Exercise For Session 9

1. $f(x) = In \quad e^x, g(x) = e^{Inx}$. Identical function or not?

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2. $f(x) = \sec x, g(x) = \frac{1}{\cos x}$ Identical or not?

Watch Video Solution

3. f(x) and and g(x) are identical or not ?

$f(x) = \sec^{-1} x + \cos ec^{-1} x, g(x) = \frac{\pi}{2}$

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4. $f(x) = \cot^2 x \cdot \cos^2 x$, and $g(x) = \cot^2 x - \cos^2 x$ prove that $f(x) = g(x)$

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5. $f(x) = sgn\left(\cot^{-1} x\right), g(x) = sgn\left(x^2 - 4x + 5\right)$

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6. $f(x) = \log_e x, g(x) = \dfrac{1}{\log_x e}$. Identical function or not?

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7. $f(x) = \sqrt{1 - x^2}, g(x) = \sqrt{1 - x} \cdot \sqrt{1 + x}$. Identical functions or not?

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8. $f(x) = \frac{1}{|x|}, g(x) = \sqrt{x^{-2}}$. Identical functions or not?

Watch Video Solution

9. Check for identical`f(x)=[{x}],g(x)={[x]} [Note that f(x) and g(x) are constant functions]

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10. $f(x) = e^{\ln \cot}, g(x) = \cot^{-1} x$

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Exercise For Session 10

1. Consider the real -valued function satisfyinig $2f(\sin x) + f(\cos x) = x.$ Then the domain of f(x) is

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2. If f(x) is defined in [-3,2], find the domain of definition of $f([(|x|])$ and $f([2x+3])$.

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3. $f(x)=\begin{cases} x-1, & -1 \leq x \leq 0 \\ x^2, & 0 < x \leq 1 \end{cases}$ and g(x)=sinx. Find $h(x)=f(|g(x)|)+|f(g(x))|$.

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4. If $f(x)$ is defined on $[-2, 2]$ and is given by $f(x)=\begin{cases} -1, & -2 \leq x < 0 \\ x-1, & 0 < x \leq 2 \end{cases}$ and $g(x)=f|x|+|f(x)|$, then $g(x)$ is defined as

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5. Let $f(x) = \begin{cases} x+1, & x \leq 1 \\ 2x+1, & 1 < x \leq 2 \end{cases}$

and $g(x) = \begin{cases} x^2, & -1 \leq x < 2 \\ x+2, & 2 \leq x \leq 3 \end{cases}$

Find (fog).

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Exercise For Session 11

1. Find the inverse of the following function. (i) $f(x) = \sin^{-1}\left(\frac{x}{3}\right), x \in [-3, 3]$ (ii) $f(x) = 5^{\log_e x}, x > 0$

(iii) $f(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

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2. If the function $f: [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ is

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Exercise For Session 12

1. For $x \in R - \{1\}$, the function f(x) satisfies $f(x) + 2f\left(\frac{1}{1-x}\right) = x$. Find $f(2)$.

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2. Let $f(x)$ and $g(x)$be functions which take integers as arguments. Let $f(x+y) = f(x) + g(y) + 8$ for all intege x and y. Let f(x) = x for all negative integers x and let $g(8) = 17$. Find f(0).

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3. The function $f : R \to R$ satisfies the condition $mf(x-1) + nf(-x) = 2|x| + 1$. If $f(-2) = 5$ and $f(1) = 1$ find m and n

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4. Find the equivalent definition of

$$f(x) = \max . \left\{x^2, (1-x)^2, 2x(1-x)\right\} \text{ where } 0 \leq x \leq 1$$

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Exercise (Single Option Correct Type Questions)

1. Show that the function f : X -> Y, such that f(x)= 5x + 7 is one-one.

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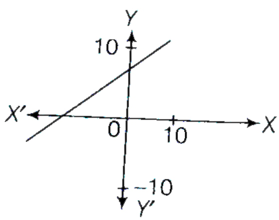
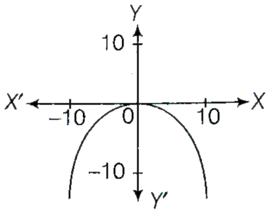
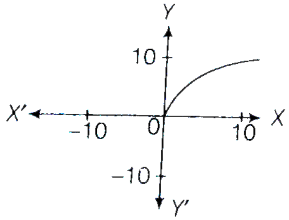
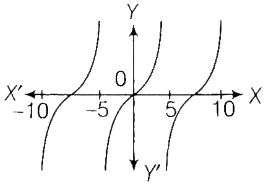
2. Which of the following functions is an odd function?

A. 

B. 

C. 

D. 

Answer: D

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3. Given $f(x) = \sqrt{\frac{8}{1-x} + \frac{8}{1+x}}$ and $g(x) = \frac{4}{f(\sin x)} + \frac{4}{f(\cos x)}$ then $g(x)$ is

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer: A

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4. Let f be a function satisfying of x. Then $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y. If $f(30) = 20$, then find the value of $f(40)$.

A. 15

B. 20

C. 40

D. 60

Answer: A

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5. Let $f(x) = e^{\{e^{|x|sgnx}\}} and g(x) = e^{[e^{|x|sgnx}]}, x \in R,$ where { } and [ ] denote the fractional and integral part functions, respectively. Also, $h(x) = \log(f(x)) + \log(g(x))$. Then for real x, $h(x)$ is

A. an odd function

B. an even function

C. neither odd nor even function

D. both odd as well as even function

Answer: A

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6. Which of the following function is surjective but not injective.

(a) $f: R \to R, f(x) = x^4 + 2x^3 - x^2 + 1$

(b) $f: R \to R, f(x) = x^2 + x + 1$

(c) $f: R \to R^{+}, f(x) = \sqrt{x^2+1}$

(d) $f: R \to R, f(x) = x^3 + 2x^2 - x + 1$

A. $f: R \to R, f(x) = x^4 + 2x^3 - x^2 + 1$

B. $f: R \to R, f(x) = x^3 + x + 1$

C. $f: R \to R^{+}, f(x) = \sqrt{1+x^2}$

D. $f: R \to R, f(x) = x^3 + 2x^2 - x + 1$

Answer: D

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7. If $f(x) = 2x^3 + 7x - 5$ then $f^{-1}(4)$ is :

A. 1

B. 2

C. $1/3$

D. non-existent

Answer: A

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8. The range of the function

$$f(x) = \frac{e^x \cdot \log x \cdot 5^{x^2+2} \cdot (x^2 - 7x + 10)}{2x^2 - 11x + 12} \text{ is}$$

A. $(-\infty, \infty)$

B. $[0, \infty)$

C. $\left(\frac{3}{2}, \infty\right)$

D. $\left(\frac{3}{2}, 4\right)$

Answer: A

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9. If $x = \cos^{-1}(\cos 4)$ and $y = \sin^{-1}(\sin 3)$, then which of the following holds?

A. x-y=1

B. x+y+1=0

C. x+2y=2

D. $x + y = 3\pi - 7$

Answer: D

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10. Let $f(x) = \left(\frac{2\sin x + \sin 2x}{2\cos x + \sin 2x} \cdot \frac{1 - \cos x}{1 - \sin x}\right) : x \in R.$

Consider the following statements.

I. Domain of f is R.

II. Range of f is R.

III. Domain of f is $R - (4n - 1)\frac{\pi}{2}, n \in I.$

IV. Domain of f is $R - (4n + 1)\frac{\pi}{2}, n \in I.$

Which of the following is correct?

A. (a)I and II

B. (b)II and III

C. (c)III and IV

D. (d)II, III and IV

Answer: D

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11. If $f(x) = e^{\sin(x - [x])\cos \pi x}$, where [x] denotes the greatest integer function, then f(x) is

A. non-periodic

B. periodic with no fundamental period

C. periodic with period 2

D. periodic with period π

Answer: C

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12. Find the range of the function $f(x) = \cot^{-1}(\log)_{0.5}\left(x^4 - 2x^2 + 3\right)$

A. $(0, \pi)$

B. $\left(0, \frac{3\pi}{4}\right]$

C. $\left[\frac{3\pi}{4}, \pi\right)$

D. $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

Answer: C

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13. Range of $f(x) = \left[\frac{1}{\log(x^2 + e)}\right] + \frac{1}{\sqrt{1 + x^2}}$, where $[\cdot]$ denotes greatest integer function, is

A. $\left(0, \frac{e+1}{e}\right) \cup \{2\}$

B. (0,1)

C. $(0, 1] \cup \{2\}$

D. $(0, 1) \cup \{2\}$

Answer: D

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14. The period of the function $f(x) = \sin(x + 3 - [x + 3])$where [] denotes the greatest integer function

A. $2\pi + 3$

B. 2π

C. 1

D. 4

Answer: C

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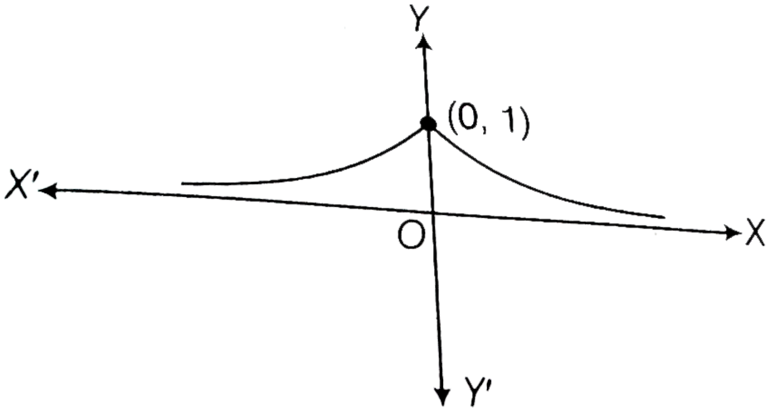
15. Which one of the following function best represents the graphs as shown below?



A. (a) $f(x) = \frac{1}{1+x^2}$

B. (b) $f(x) = \frac{1}{\sqrt{1+|x|}}$

C. (c) $f(x) = e^{-|x|}$

D. (d) $f(x) = a^{|x|}, a > 1$

Answer: C

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16. The solution set for $[x]\{x\} = 1$ (where {x} and [x] are respectively, fractional part function and greatest integer function) is (a) $R^{\pm}(0, 1)$ (b) $r^{\pm}\{1\}$ (c) $\left\{m + \frac{1}{m}m \in I - \{0\}\right\}$ (d) $\left\{m + \frac{1}{m}m \in N - \{1\}\right\}$

A. $R^{+} - (0, 1)$

B. $R^{+} - \{1\}$

C. $\left\{m + \frac{1}{m} : m \in I - \{0\}\right\}$

D. $\left\{m + \frac{1}{m} : m \in N - \{1\}\right\}$

Answer: D

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17. The domain of definition of function

$f(x) = \log\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$, is

A. $(-\infty, -3]$

B. $(-\infty, -3] \cup [8, \infty)$

C. $\left(-\infty, \frac{-28}{9}\right)$

D. None of these

Answer: A

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18. If $f(x)$ is a function $f: R \to R$, we say $f(x)$ has property I. If $f(f(x)) = x$ for all real numbers x. II. $f(-f(x)) = -x$ for all real numbers x. How many linear functions, have both property I and II ?

A. 0

B. 2

C. 3

D. Infinite

Answer: B

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19. Let $f(x) = \frac{x}{1+x}$ and let $g(x) = \frac{rx}{1-x}$, Let S be the set off all real numbers r such that $f(g(x)) = g(f(x))$ for infinitely many real number x. The number of elements in set S is

A. 1

B. 2

C. 3

D. 5

Answer: B

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20. Let f be a linear function with properties $f(1) \leq f(2), f(3) \geq f(4) \text{ and } f(5) = 5,$ then which of the following is true

A. $f(0) < 0$

B. f(0)=0

C. $f(1) < f(0) < f(-1)$

D. f(0)=5

Answer: D

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21. Suppose R is relation whose graph is symmetric to both X-axis and Y-axis and that the point (1,2) is on the graph of R. Which one of the following is not necessarily on the graph of R?

A. (-1,2)

B. (1,-2)

C. (-1,-2)

D. (2,1)

Answer: D

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22. The area between the curve $2\{y\} = [x] + 1, 0 \leq y < 1,$ where $\{.\}$ and $[.]$ are the fractional part and greatest integer functions, respectively and the X-axis is

A. $\frac{1}{2}$

B. 1

C. 0

D. $\frac{3}{2}$

Answer: A

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23. If f(x)=$\sin^{-1} x$ and g(x)=[sin(cosx)]+[cos(sinx)], then range of f(g(x)) is (where $[\cdot]$ denotes greatest integer function)

a. $\left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

b. $\left\{\frac{-\pi}{2}, 0\right\}$

c. $\left\{0, \frac{\pi}{2}\right\}$

d. $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

A. $\left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

B. $\left\{\frac{-\pi}{2}, 0\right\}$

C. $\left\{0, \frac{\pi}{2}\right\}$

D. $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

Answer: C

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24. Find the number of solutions of the equation $e^{2x} + e^x - 2 = \left[\left\{x^2 + 10x + 11\right\}\right]$ is(where, {x} denotes fractional part of x and [x] denotes greatest integer function) (a)0 (b)1 (c)2 (d)3

A. 0

B. 1

C. 2

D. 3

Answer: B

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25. Total number of values of x, of the form $\frac{1}{n}, n \in N$ in the interval $x \in \left[\frac{1}{25}, \frac{1}{10}\right]$ which satisfy the equation $\{x\} + \{2x) + \ldots\ldots + \{12x\} = 78x$ is K. then K is less than,(where { } represents fractional part function) (a)12 (b)13 (c)14 (d)15

A. 12

B. 13

C. 14

D. 15

Answer: B

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26. The sum of the maximum and minimum values of the function $f(x)=\frac{1}{1+(2\cos x-4\sin x)^2} is$

A. (a) $\frac{22}{21}$

B. (b) $\frac{21}{20}$

C. (c) $\frac{22}{20}$

D. (d) $\frac{21}{11}$

Answer: A

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27. Let $f: X \rightarrow Y$ be an invertible function. Show that the inverse of $f^{-}1$ is f, i.e. $(f^{-}1)^{-}1=f$

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28. The range of values of a so that all the roots of the equations $2x^3 - 3x^2 - 12x + a = 0$ are real and distinct, belongs to

A. (a) (7,20)

B. (b) (-7,20)

C. (c) (-20,7)

D. (d) (-7,7)

Answer: B

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29. If f(x) is continuous such that $|f(x)| \leq 1, \forall x \in R$ and $g(x) = \frac{e^{f(x)} - e^{-|f(x)|}}{e^{f(x)} + e^{-|f(x)|}}$, then range of g(x) is

A. [0,1]

B. $\left[0, \frac{e^2 - 1}{e^2 + 1}\right]$

C. $\left[0, \dfrac{e^2+1}{e^2-1}\right]$

D. $\left[\dfrac{1-e^2}{1+e^2}, 0\right]$

Answer: B

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30. Let $f(x)=\sqrt{|x|-\{x\}}(where\{\}$ denotes the fractional part of $(x) and X, Y$ are its domain and range, respectively). Then

A. $f: X \rightarrow Y: y=f(x)$ is one-one function

B. $X \in \left(-\infty, -\dfrac{1}{2}\right] \cup [0, \infty) \text{ and } Y \in \left[\dfrac{1}{2}, \infty\right)$

C. $X \in \left(-\infty, -\dfrac{1}{2}\right] \cup [0, \infty) \text{ and } Y \in [0, \infty)$

D. None of the above

Answer: C

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31. If the graphs of the functions $y = \log_e x$ and $y = ax$ intersect at exactly two points,then find the value of a.

A. (0,e)

B. $\left(\frac{1}{e}, 0\right)$

C. $\left(0, \frac{1}{e}\right)$

D. None of these

Answer: C

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32. A quadratic polynomial maps from [-2,3] onto [0,3] and touches X-axis at x=3, then the polynomial is

A. (a) $\frac{3}{16}\left(x^2 - 6x + 16\right)$

B. (b) $\frac{3}{25}\left(x^2 - 6x + 9\right)$

C. (c) $\frac{3}{25}\left(x^2 - 6x + 16\right)$

D. (d) $\frac{3}{16}\left(x^2-6x+9\right)$

Answer: B

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33. The range of the function $y=\sqrt{2\{x\}-\{x\}^2-\frac{3}{4}}$ (where, denotes the fractional part) is

A. $\left[-\frac{1}{4},\frac{1}{4}\right]$

B. $\left[0,\frac{1}{2}\right)$

C. $\left[0,\frac{1}{4}\right]$

D. $\left[\frac{1}{4},\frac{1}{2}\right]$

Answer: C

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34. Let $f(x)$ be a fourth differentiable function such $f(2x^2-1) = 2xf(x)\,\forall x \in R,$ then $f^{iv}(0)$ is equal

A. 0

B. 1

C. -1

D. Data insufficient]

Answer: A

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35. The number of solutions of the equation $[y+[y]] = 2\cos x,$ where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]].$

where [.] denotes the greatest integer function, is

A. 1

B. 2

C. 3

D. None of these

Answer: D

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36. If a function satisfies $f(x+1)+f(x-1)=\sqrt{2}f(x)$, then period of f(x) can be

A. 2

B. 4

C. 6

D. 8

Answer: D

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37. If x and α are real, then the inequation

$\log_2 x + \log_x 2 + 2\cos\alpha \leq 0$

A. has no solution

B. has exactly two solutions

C. is satisfied for any real α and any real x in (0,1)

D. None of these

Answer: D

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38. The range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of 'x'

A. $(-\infty, -1)$

B. $(-\infty, \infty)$

C. $(-1, 1)$

D. $(-1, \infty)$

Answer: D

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39. Let $f : R \to R$ be a function defined by $f(x) = \{|\cos x|\}$, where {x} represents fractional part of x. Let S be the set containing all real values x lying in the interval $[0, 2\pi]$ for which $f(x) \neq |\cos x|$. The number of elements in the set S is

A. (a) 0

B. (b) 1

C. (c) 3

D. (d) infinite

Answer: C

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40. The domain of the function

$f(x)=\sqrt{\log_{\sin x+\cos x}(|\cos x|+\cos x)}, 0\le x\le\pi$ is

A. $(0,\pi)$

B. $\left(0,\frac{\pi}{2}\right)$

C. $\left(0,\frac{\pi}{3}\right)$

D. None of these

Answer: D

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41. If $f(x)=(x^2+2\alpha x+\alpha^2-1)^{1/4}$ has its domain and range such that their union is set of real numbers, then α satisfies

A. $-1<\alpha<1$

B. $\alpha\le -1$

C. $\alpha\ge 1$

D. $\alpha \leq 1$

Answer: B

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42. If $f : (e, \infty) \to R \& f(x) = \log[\log(\log x)],$ then f is -

A. f is one-one and onto

B. f is one-one but onto

C. f is onto but not one-one

D. the range of f is equal to its domain

Answer: A

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43. The expression $x^2 - 4px + q^2 > 0$ for all real x and also $r^2 + p^2 < qr$ the range of $f(x) = \frac{x+r}{x^2 + qx + p^2}$ is

A. (a) $\left[\frac{p}{2r}, \frac{q}{2r}\right]$

B. (b) $(0, \infty)$

C. (c) $(-\infty, 0)$

D. (d) $(-\infty, \infty)$

Answer: D

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44. Let $f(x) = \frac{x^4 - \lambda x^3 - 3x^2 + 3\lambda x}{x - \lambda}$. If range of f(x) is the set of entire real numbers, the true set in which λ lies is

A. (a)[-2,2]

B. (b)[0,4]

C. (c)(1,3)

D. (d)None of these

Answer: A

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45. Let $a = 3^{1/224} + 1$ and for all $n \geq 3$,

let

$f(n) = {}^nC_0 a^{n-1} - {}^nC_1 a^{n-2} + {}^nC_2 a^{n-3} + \ldots + (\_1)^{n-1} \cdot {}^nC_{n-1} \cdot a^0.$

If the value of f(2016)+f(2017)=3^k, the value of K is

A. 6

B. 8

C. 9

D. 10

Answer: C

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46. The area bounded by $f(x) = \sin^{-1}(\sin x)$ and

$g(x) = \frac{\pi}{2} - \sqrt{\frac{\pi^2}{2} - \left(x - \frac{\pi}{2}\right)^2}$ is

A. $\frac{\pi^3}{8}$ sq units

B. $\frac{\pi^2}{8}$ sq units

C. $\frac{\pi^3}{2}$ sq units

D. $\frac{\pi^2}{2}$ sq units

Answer: A

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47. If $f. R \to R, f(x) = \frac{x^2 + bx + 1}{x^2 + 2x + b}, (b > 1)$ and $f(x), \frac{1}{f(x)}$ have the same bounded set as their range, the value of b is

A. $2\sqrt{3} - 2$

B. $2\sqrt{3} + 2$

C. $2\sqrt{2} - 2$

D. $2\sqrt{2}+2$

Answer: A

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48. The period of $\sin\frac{\pi[x]}{12}+\cos\frac{\pi[x]}{4}+\tan\frac{\pi[x]}{3}$, where [x] represents the greatest integer less than or equal to x is

A. 12

B. 4

C. 3

D. 24

Answer: D

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49. If $f(2x+3y, 2x-7y)=20x, then f(x,y) equals. \ 7x-3y \ 7x+3y$

$3x-7y$ (d) $x-ky$

A. x-y

B. 7x+3y

C. 3x-7y

D. None of these

Answer: B

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50. The range of the function $f(x)=\sqrt{x-1}+2\sqrt{3-x}$ is

A. $[\sqrt{2}, 2\sqrt{2}]$

B. $[\sqrt{2}, \sqrt{10}]$

C. $[2\sqrt{2}, \sqrt{10}]$

D. $[1, 3]$

Answer: B

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51. The domain of the function $f(x) = \cos^{-1}(\sec(\cos^{-1}x)) + \sin^{-1}(\cos ec(\sin^{-1}x))$ is

A. $x \in R$

B. x=1,-1

C. $-1 \leq x \leq 1$

D. $x \in \phi$

Answer: B

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52. Let f(x) be a polynominal one-one function such that

$f(x)f(y) + 2 = f(x) + f(y) + f(xy), \forall x, y \in R - \{0\}, f(1) \neq 1, f'(1) =$

Let $g(x) = \frac{x}{4}(f(x) + 3) - \int_0^x f(x)dx$, then

A. (a) g(x)=0 has exactly one root for $x \in (0, 1)$

B. (b) g(x)=0 has exactly two roots for $x \in (0, 1)$

C. (c) $g(x) \neq 0, x \in R - \{0\}$

D. (d) $g(x) = 0, x \in R - \{0\}$

Answer: D

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53. Let f(x) be a polynominal with real coefficients such that $xf(x) = f'(x) \times f''(x)$. If f(x)=0 is satisfied x=1,2,3 only, then the value of f'(1)f'(2)f'(3) is

A. (a)positive

B. (b)negative

C. (c)0

D. (d)Inadequate data

Answer: C

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54. Let $A = \{1, 2, 3, 4, 5\}$ and $f: A \rightarrow A$ be an into function such that $f(x) \neq x \, \forall x \in A$. Then number of such functions f is:

A. (a) 1024

B. (b) 904

C. (c) 980

D. (d) None of these

Answer: C

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55. If functions $f:\{1, 2, ..., n\} \rightarrow \{1995, 1996\}$ satisfying f(1)+f(2)+...

+f(1996)=odd integer are formed, the number of such functions can be

A. 2^n

B. $2^{n/2}$

C. n^2

D. 2^{n-1}

Answer: D

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56. Find the range of $y = \sin^3 x - 6\sin^2 x + 11\sin x - 6.$

A. [-24,2]

B. [-24,0]

C. [0,24]

D. None of these

Answer: B

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57. Let $f(x) = x^2 - 2x, x \in R, and g(x) = f(f(x) - 1) + f(5 - (x))$.

Show that $g(x) \geq 0 \forall x \in R$.

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58. If f(x) and g(x) are non-periodic functions, then h(x)=f(g(x)) is

A. non-periodic

B. periodic

C. may be periodic

D. always periodic, if domain of h(x) is a proper subset of real numbers

Answer: C

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59. If f(x) is a real-valued function discontinuous at all integral points lying in [0,n] and if $(f(x))^2 = 1, \forall x \in [0, n],$ then number of functions f(x) are

A. 2^{n+1}

B. 6×3^n

C. $2 \times 3^{n-1}$

D. 3^{n+1}

Answer: C

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60. A function f from integers to integers is defined as $f(x) = \begin{cases} n+3 & n \in odd \\ \frac{n}{2} & n \in even \end{cases}$ Suppose $k \in$ odd and $f(f(f(k))) = 27$. Then the sum of digits of k is________

A. 3

B. 6

C. 9

D. 12

Answer: B

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61. If $f: R \to R$ and $f(x) = \frac{\sin(\pi\{x\})}{x^4 + 3x^2 + 7}$, where $\{\}$ is a fractional part of x , then

A. f is injective

B. f is not one-one and non-constant

C. f is a surjective

D. f is a zero function

Answer: B

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62. Show that f: R → R defined as f(a) = 3a – 4 is one to one function.

A. one -one and onto

B. only one-one and not onto

C. only onto but not one-one

D. None of the above

Answer: D

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63. Find $\frac{dy}{dx}$ if $y = 3^x$

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64. Let y be element of the set $A = \{1, 2, 3, , 5, 6, 10, 15, 30\}$ and $x_1, x_2, x_3 be \int egralsolutionsof$ x_(1)x_(2)x_(3)=y`, is

A. 100

B. 150

C. 320

D. 250

Answer: C

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65. If A $>$ 0, c,d,u.v are non-zero constants and the graph of $f(x) = |Ax + c| + d$ and $g(x) = -|Ax + u| + v$ intersect exactly at two points (1,4) and (3,1), then the value of $\frac{u + c}{A}$ equals

A. 4

B. -4

C. 2

D. -2

Answer: B

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66. $f(x) = x^3 + 3x^2 + 4x + b\sin x + c\cos x, \forall x \in R$ is a one-one function, the value of $b^2 + c^2$ is

A. (a)1

B. (b)2

C. (c)$\sqrt{2}$

D. (d)None of these

Answer: A

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67. If two roots of the equation $(p-1)(x^2+x+1)^2-(p+1)(x^4+x^2+1)=0$ are real and distinct and $f(x)=\frac{1-x}{1+x}$ then $f(f(x))+f\left(f\left(\frac{1}{x}\right)\right)$ is equal to

A. (a) p

B. (b) $-p$

C. (c) $2p$

D. (d) $-2p$

Answer: A

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68. If $f(x)=x^{11}+x^9-x^7+x^3+1$ and $f\left(\sin^{-1}(\sin 8)\right)=\alpha$,α is constant, then $f\left(\tan^{-1}(\tan 8)\right)$ is equal to

A. (a) α

B. (b) 2-α

C. (c) 2α

D. (d) α-2

Answer: B

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69. Let $f(x) = \sin^{-1} 2x + \cos^{-1} 2x + \sec^{-1} 2x$. Then the sum of the maximum and minimum values of f(x) is

(a)π

(b)$\frac{\pi}{2}$

(c)2π

(d)$\frac{3\pi}{2}$

A. π

B. $\frac{\pi}{2}$

C. 2π

D. $\frac{3\pi}{2}$

Answer: C

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70. The complete set of values of a for which the function $f(x) = \tan^{-1}(x^2 - 18x + a) > 0 \forall x \in R$ is

a. $(81, \infty)$

b. $[81, \infty)$

c. $(-\infty, 81)$

d. $(-\infty, 81]$

A. $(81, \infty)$

B. $[81, \infty)$

C. $(-\infty, 81)$

D. (-infty,81]`

Answer: A

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71. The domain of the function

$$f(x) = \sin^{-1}\frac{1}{|x^2 - 1|} + \frac{1}{\sqrt{\sin^2 x + \sin x + 1}}$$ is

A. a) $(-\infty, \infty)$

B. b) $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$

C. c) $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right) \cup \{0\}$

D. d) None of the above

Answer: C

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72. The domain of $f(x) = \dfrac{\log\left(\sin^{-1}\sqrt{x^2 + x + 1}\right)}{\log(x^2 - x + 1)}$ is

a. (-1,1)

b. $(-1, 0) \cup (0, 1)$

c. $(-1, 0) \cup \{1\}$

d. None of these

A. (-1,1)

B. $(-1, 0) \cup (0, 1)$

C. $(-1, 0) \cup \{1\}$

D. None of these

Answer: D

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73. The domain of $f(x) = \sqrt{\sin^{-1}(3x - 4x^3)} + \sqrt{\cos^{-1} x}$ is equal to

A. (a) $\left[-1, -\frac{\sqrt{3}}{2}\right] \cup \left[0, \frac{\sqrt{3}}{2}\right]$

B. (b) $\left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right]$

C. (c) $\left[0, \frac{1}{2}\right]$

D. (d) None of these

Answer: A

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74. The domain of the function

$f(x) = \sqrt[6]{4^x + 8^{2/3(x-2)} - 52 - 2^{2(x-1)}}$ is

A. (0,1)

B. $[3, \infty]$

C. [1,0)

D. None of these

Answer: B

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75. The domain of derivative of the function

$f(x) = \left|\sin^{-1}(2x^2 - 1)\right|$, is

A. (-1,1)

B. $(-1, 1) \sim \left\{0, \pm \frac{1}{\sqrt{2}}\right\}$

C. $(-1, 1) \sim \{0\}$

D. $(-1, 1) \sim \left\{\pm \frac{1}{\sqrt{2}}\right\}$

Answer: B

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76. The range of a function

$f(x) = \tan^{-1}\left\{\log_{5/4}\left(5x^2 - 8x + 4\right)\right\}$ is

A. $\left(\frac{-\pi}{4}, \frac{\pi}{2}\right)$

B. $\left[\frac{-\pi}{4}, \frac{\pi}{2}\right)$

C. $\left(\frac{-\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[\frac{-\pi}{4}, \frac{\pi}{2}\right]$

Answer: B

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Exercise (More Than One Correct Option Type Questions)

1. Which of the following function(s) is/are transcendental?

a. $f(x) = 5\sin(\sqrt{x})$

b. $f(x) = \dfrac{2\sin 3x}{x^2 + 2x - 1}$

c. $f(x) = \sqrt{x^2 + 2x + 1}$

d. $f(x) = (x^2 + 3) \cdot 2^x$

A. $f(x) = 5\sin(\sqrt{x})$

B. $f(x) = \dfrac{2\sin 3x}{x^2 + 2x - 1}$

C. $f(x) = \sqrt{x^2 + 2x + 1}$

D. $f(x) = (x^2 + 3) \cdot 2^x$

Answer: A::B

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2. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1}x.$ Then

A. domain of f(x) is $x \geq 1$

B. domain of f(x) is $[1, \infty) - \{2\}$

C. f'(10)=1

D. $f'\left(\frac{3}{2}\right) = -1$

Answer: B::C::D

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3. prove that $f(x) = \cos^2 x + \cos^2\left(\frac{\pi}{3} + x\right) - \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)$ is constant function. find the value of that constant

A. an odd function

B. an even function

C. a periodic function

D. f(0)=f(1)

Answer: B::C::D

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4. If the following functions are defined from $[-1, 1] \to [-1, 1]$, select those which are not bijective. $\sin(s \in^{-1} x)$ (b) $\frac{2}{\pi}\sin^{-1}(\sin x)$ $(sgn(x))1N(e^x)$ (d) $x^3(sgn(x))$

A. $\sin(\sin^{-1} x)$

B. $\frac{2}{\pi} \cdot \sin^{-1}(\sin x)$

C. $sgn(x) \cdot \log(e^x)$

D. $x^3 sgn(x)$

Answer: B::C::D

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5. Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$

and $g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \geq 4 \end{cases}$, which one of the following is/are true?

a) $(f+g)(3.5) = 0$

b) $f(g(3)) = 3$

c) $(fg)(2) = 1$

d) $(f-g)(4) = 0$

A. a) $(f+g)(3.5) = 0$

B. b) $f(g(3)) = 3$

C. c) $f(g(2)) = 1$

D. d) $(f-g)(4) = 0$

Answer: A::B

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6. $f(x) = x^2 - 2ax + a(a+1), f: [a, \infty) \longrightarrow [a, \infty)$. If one of the solution of the equation $f(x) = f^{-1}(x)$ is 5049 , then the other may be

(a)5051 (b) 5048 (c) 5052 (d) 5050

A. 5051

B. 5048

C. 5052

D. 5050

Answer: B::D

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7. The function 'g' defined by $g(x) = \sin\left(\sin^{-1}\{\sqrt{x}\} + \cos\left(\sin^{-1}\{\sqrt{x}\}\right) - 1\right.$ where {x} denotes the functional part function is

A. an even function

B. periodic function

C. odd function

D. neither even or odd

Answer: A::B

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8. The graph of $f: R \rightarrow R$ defined by y=f(x) is symmetric with respect to x=a and x=b. Which of the following is true ?

A. f(2a-x)=f(x)

B. f(2a+x)=f(-x)

C. f(2b+x)=f(-x)

D. f is periodic

Answer: A::B::C::D

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9. Let y=f(x) be a differentiable function such that f(−1)=2,f(2)=−1 and f(5)=3

If the equation f `(x)=2f(x) has real root. Then find f(x)

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10. Let $f(x) = |x-1| + |x-2| + |x-3| + |x-4|,$ then

a. least value of f(x) is 4

b. least value is not attained at unique point

c. the number of integral solution of f(x)=4 is 2

d. the value of $\dfrac{f(\pi-1)+f(e)}{2f\left(\frac{12}{5}\right)}$ is 1

A. least value of f(x) is 4

B. least value is not attained at unique point

C. the number of integral solution of f(x)=4 is 2

D. the value of $\dfrac{f(\pi-1)+f(e)}{2f\left(\frac{12}{5}\right)}$ is 1

Answer: A::B::C::D

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11. Let A={1,2,3,4,5}, B={1,2,3,4} and $f: A \to B$ is a function, then

A. A. number of onto functions, if n(f(A))=4 is 240

B. B. number of onto functions, if n(f(A))=3 is 600

C. C. number of onto functions, if n(f(A))=2 is 180

D. D. number of onto functions, if n(f(A))=1 is 4

Answer: A::B::C::D

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12. In a function

$$2f(x) + xf\left(\frac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\left(\pi\left(x+\frac{1}{4}\right)\right)\right|\right) = 4\cos^2\left[\frac{\pi x}{2}\right] + x\cos\left(\frac{\pi}{x}\right.$$

. Prove that: 1. f(2)+f(1/2)=1 2. f(2)+f(1)=0

A. $f(2)+f\left(\frac{1}{2}\right)=1$

B. f(2)+f(1)=0

C. $f(2)+f(1)=f\left(\frac{1}{2}\right)$

D. $f(1)\cdot f\left(\frac{1}{2}\right)\cdot f(2)=1$

Answer: A::B::C

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13. If f(x) is a differntiable function satisfying the condition $f(100x)=x+f(100x-100), \forall x \in R$ and $f(100)=1$, then $f(10^4)$ is

A. 5049

B. $\sum_{r=1}^{100} r$

C. $\sum_{r=2}^{100} r$

D. 5050

Answer: B::D

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14. If [x] denotes the greatest integer function then the extreme values of the function f(x)=[1+sinx] is:

A. (n-1)

B. n

C. (n+1)

D. (n+2)

Answer: B::C

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15. Which of the following are true?

A. $f(x) = \begin{cases} 1, & \text{if x is rational} \\ 0, & \text{if x is irrational} \end{cases}$

B. $f(x) = \begin{cases} x - [x], & 2n \le x < 2n+1 \\ \frac{1}{2}, & 2n+1 \le x < 2n+2 \end{cases}$ where $[\cdot]$ denotes the greatest integer function

C. $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, where $[\cdot]$ denotes the greatest integer function

D. $f(x) = ax - [ax + a] + \tan\left(\frac{\pi x}{2}\right)$, where $[\cdot]$ denotes the greatest integer function

Answer: B::C::D

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16. If $f(x)$ is a polynomial of degree n such that $f(0) = 0, f(x) = \frac{1}{2}, \dots, f(n) = \frac{n}{n+1}$, ,then the value of $f(n+1)$ is

A. 1, when n is even

B. $\frac{n}{n+2}$, when n is odd

C. 1, when n is odd

D. $\frac{n}{n+2}$, when n is even

Answer: C::D

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17. Let $f: R \rightarrow R$ be a function defined by $f(x+1) = \frac{f(x)-5}{f(x)-3} \forall x \in R$. Then which of the following statement(s) is/are true?

A. f(2008)=f(2004)

B. f(2006)=f(2010)

C. f(2006)=f(2002)

D. f(2006)=f(2018)

Answer: A::B::C::D

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18. Let $f(x) = 1 - x - x^3$.Find all real values of x satisfying the inequality, $1 - f(x) - f^3(x) > f(1 - 5x)$

A. (-2,0)

B. (0,2)

C. $(2, \infty)$

D. $(-2, 0) \cup (2, \infty)$

Answer: A::C

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19. If a function satisfies `(x-y)f(x+y)-(x+y)f(x-y)=2(x^2 y-y^3) AA x, y in R and f(1)=2, then

A. f(x) must be polynominal function

B. f(3)=12

C. f(0)=0

D. f(x) may not be differentiable

Answer: A::B::C

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20. If the fundamental period of function $f(x) = \sin x + \cos\left(\sqrt{4 - a^2}\right)x$ is 4π, then the value of a is/are

A. $\frac{\sqrt{15}}{2}$

B. $-\frac{\sqrt{15}}{2}$

C. $\frac{\sqrt{7}}{2}$

D. $-\frac{\sqrt{7}}{2}$

Answer: A::B::C::D

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21. Let $f(x)$ be a real vaued continuous function such that

$$f(0)=\frac{1}{2} \text{ and } f(x+y)=f(x)f(4-y)+f(y)f(4-x)\ \forall x, y \in R,$$

then for some real a:

a. f(x) is a periodic function

b. f(x) is a constant function

c. $f(x)=\frac{1}{2}$

d. $f(x)=\frac{\cos x}{2}$

A. f(x) is a periodic function

B. f(x) is a constant function

C. $f(x)=\frac{1}{2}$

D. $f(x)=\frac{\cos x}{2}$

Answer: A::B::C

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22. if $f(g(x))$ is one-one function, then

A. g(x) must be one-one

B. f(x) must be one-one

C. f(x) may not be one-one

D. g(x) may not be one-one

Answer: A::C

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23. Which of the following functions have their range equal to R(the set of real numbers)?

A. xsinx

B. $\frac{x}{\tan 2x} \cdot x \in \left(-\frac{\pi}{4} \cdot \frac{\pi}{4}\right) - \{0\}$, where $[\cdot]$ denotes the greatest integer function

C. $\frac{x}{\sin x}$

D. $[x]+\sqrt{\{x\}}$, where $\{\cdot\}$, respectively denote the greatest integer and fractional part functions

Answer: A::D

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24. Which of the following pairs of function are identical?

a. $f(x)=e^{In\sec^{-1}x}$ and $g(x)=\sec^{-1}x$

b. $f(x)=\tan(\tan^{-1}x)$ and $g(x)=\cot(\cot^{-1}x)$

c. $f(x)=sgn(x)$ and $g(x)=sgn(sgn(x))$

d. $f(x)=\cot^2\cdot\cos^2x$ and $g(x)=\cot^2x-\cos^2x$

A. $f(x)=e^{In\sec^{-1}x}$ and $g(x)=\sec^{-1}x$

B. $f(x)=\tan(\tan^{-1}x)$ and $g(x)=\cot(\cot^{-1}x)$

C. $f(x)=sgn(x)$ and $g(x)=sgn(sgn(x))$

D. $f(x)=\cot^2\cdot\cos^2x$ and $g(x)=\cot^2x-\cos^2x$

Answer: B::C::D

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25. Let $f: R \rightarrow R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where {x} denotes fractional part of x. Then, which of the following is/are correct?

A. f is many one but not even function

B. Range of f contains two prime numbers

C. f is non-periodic

D. Graphs of f does not lie below X-axis

Answer: B::D

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Exercise (Statement I And Ii Type Questions)

1. $\textbf{Statement I}$ The function f(x) =xsinx and f'(x)=xcosx+sinx are both non-periodic.

Statement II The derivative of differentiable functions (non-periodic) is non-periodic function.

A. (A) Statement I is true,Statement II is also true

B. (B) Statement I is false,Statement II is also false

C. (C)Statement I is true,Statement II is false

D. (D) Statement I is false ,Statement II is true

Answer: c

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2. **Statement I** The maximum value of $\sin\sqrt{2}x + \sin ax$ cannot be 2 (where a is positive rational number).

Statement II $\frac{\sqrt{2}}{a}$ is irrartional.

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3. Let $f: R \rightarrow R$ be a function defined by $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^{x} + e^{-x}}$. Then, f is a bijection (b) f is an injection only (c) f is surjection on only (d) f is neither an injection nor a surjection

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4. $\textbf{Statement I}$ The range of

$$f(x) = \sin\left(\frac{\pi}{5} + x\right) - \sin\left(\frac{\pi}{5} - x\right) - \sin\left(\frac{2\pi}{5} + x\right) + \sin\left(\frac{2\pi}{5} - x\right)$$

is [-1,1].

$\textbf{Statement II} \quad \cos\frac{\pi}{5} - \cos\frac{2\pi}{5} = \frac{1}{2}$

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5. $\textbf{Statement I}$ The period of

$f(x) = 2\cos\frac{1}{3}(x - \pi) + 4\sin\frac{1}{3}(x - \pi) \quad \text{is} \quad 3\pi.$

$\textbf{Statement II}$ If T is the period of f(x), then the period of f(ax+b) is $\dfrac{T}{|a|}$.

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6. f is a function defined on the interval [-1,1] such that f(sin2x)=sinx+cosx.

Statement I If $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, then $f(\tan^2 x) = \sec x$

Statement II $f(x) = \sqrt{1+x}, \forall x \in [-1, 1]$

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7. **Statement I** The equation $f(x) = 4x^5 + 20x - 9 = 0$ has only one real root.

Statement II $f'(x) = 20x^4 + 20 = 0$ has no real root.

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8. **Statement I** The range of $\log\left(\frac{1}{1+x^2}\right)$ is $(-\infty, \infty)$.

Statement II when $0 < x \leq 1, \log x \in (-\infty, 0]$.

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9. Let $f: X \to Y$ be a function defined by

$$f(x) = 2\sin\left(x + \frac{\pi}{4}\right) - \sqrt{2}\cos x + c.$$

Statement I For set $X, x \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$, f(x) is one-one function.

Statement II $f'(x) \geq 0, x \in \left[0, \frac{3\pi}{2}\right]$

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10. Let $f(x) = \sin x$

Statement I f is not a polynominal function.

Statement II nth derivative of f(x), w.r.t. x, is not a zero function for any positive integer n.

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11. Find the inverse of the function, (assuming onto).

$$y = \log_a\left(x + \sqrt{x^2 + 1}\right), (a > 1).$$

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Exercise (Passage Based Questions)

1. Let $f: R \to R$ be a continuous function such that

$$f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2.$$

f(3) is equal to

A. f(0)

B. 4+f(0)

C. 9+f(0)

D. 16+f(0)

Answer: d

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2. Let $f: R \to R$ be a continuous function such that

$$f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2.$$

The equation f(x)-x-f(0)=0 have exactly

A. no solution

B. one solution

C. two solution

D. infinite solution

Answer: c

Watch Video Solution

3. Let $f: R \to R$ be a continuous function such that

$$f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2.$$

f'(0) is equal to

A. 0

B. 1

C. f(0)

D. $-f(0)$

Answer: a

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4. Consider the equation $x+y-[x][y]=0$, where $[\cdot]$ is the greatest integer function.

Equation of one of the lines on which the non-integral solution of given equation lies is:

Watch Video Solution

5. Consider the equation x+y-[x][y]=0, where $[\cdot]$ is the greatest integer function.

The number of integral solutions to the equation is

A. (a) $x + y = -1$

B. (b) $x + y = 0$

C. (c) $x + y = 1$

D. (d) $x + y = 5$

Answer: b

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6. Let $f(x) = \frac{1}{2}\left[f(xy) + f\left(\frac{x}{y}\right)\right]$ for $x, y \in R^{+}$ such that f(1)=0,f'(1)=2.`

f(x)-f(y) is equal to

A. $f\left(\frac{y}{x}\right)$

B. $f\left(\frac{x}{y}\right)$

C. f(2x)

D. f(2y)

Answer: b

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7. Let $f(x)=\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]$ for $x, y \in R^{+}$ such that f(1)=0,f'(1)=2.`

f'(3) is equal to

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: b

Watch Video Solution

8. Let $f(x)=\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]$ for $x, y \in R^{+}$ such that f(1)=0,f'(1)=2.`

f(e) is equal to

A. 2

B. 1

C. 3

D. 4

Answer: a

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9. If $f: R \rightarrow R$ and f(x)=g(x)+h(x) where g(x) is a polynominal and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is one-one, we need to differentiate f(x). If f'(x) changes sign in domain of f, then f, if many-one else one-one.

If $f: R \rightarrow R$ and f(x) = 2ax

A. one-one into

B. many-one onto

C. one-one onto

D. many-one into

Answer: c

Watch Video Solution

10. If $f: R \to R$ and f(x)=g(x)+h(x) where g(x) is a polynominal and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is one-one, we need to differentiate f(x). If f'(x) changes sign in domain of f, then f, if many-one else one-one.

$f: R \to R$ and $f(x) = \dfrac{x\left(x^4+1\right)(x+1)+x^4+2}{x^2+x+1}$, then f(x) is

A. one-one into

B. many-one onto

C. one-one onto

D. many-one into

Answer: d

Watch Video Solution

11. If $f : R \rightarrow R$ and f(x)=g(x)+h(x) where g(x) is a polynominal and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is one-one, we need to differentiate f(x). If f'(x) changes sign in domain of f, then f, if many-one else one-one.

If $f : R \rightarrow R$ and f(x)=2ax +sin2x, then the set of values of a for which f(x) is one-one and onto is

A. $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

B. $a \in (-1, 1)$

C. $a \in R - \left(-\frac{1}{2}, \frac{1}{2}\right)$

D. $a \in R - (-1, 1)$

Answer: d

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12. Let $g(x)=a_0+a_1x+a_2x^2+a_3x^3$ and $f(x)=\sqrt{g(x)}, f(x)$ have its non-zero local minimum and maximum values at -3 and 3 respectively. If $a_3 \in$ the domain of the function $h(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$

The value of a_1+a_2 is equal to

A. 30

B. -30

C. 27

D. -27

Answer: c

Watch Video Solution

13. Let $g(x)=a_0+a_1x+a_2x^2+a_3x^3$ and $f(x)=\sqrt{g(x)}, f(x)$ have its non-zero local minimum and maximum values at -3 and 3 respectively.

If $a_3 \in$ the domain of the function $h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

The value of a_0 is

A. equal to 50

B. greater than 54

C. less than 54

D. less than 50

Answer: b

Watch Video Solution

14. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, f(x) has its non-zero local minimum and maximum values at -3 and 3, respectively.

If $a_3 \in$ the domain of the function

$h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$.

The value of a_0 is

A. $a_0 > 830$

B. $a_0 < 830$

C. $a_0 = 830$

D. None of these

Answer: d

Watch Video Solution

15. Let $f:[2, \infty) \to \{1, \infty)$ defined by $f(x) = 2^{x^4 - 4x^3}$ and $g:\left[\frac{\pi}{2}, \pi\right] \to A$ defined by $g(x) = \frac{\sin x + 4}{\sin x - 2}$ be two invertible functions, then

$f^{-1}(x)$ is equal to

A. $\sqrt{2 + \sqrt{4 - \log_2 x}}$

B. $\sqrt{2 + \sqrt{4 + \log_2 x}}$

C. $\sqrt{2 - \sqrt{4 + \log_2 x}}$

D. None of these

Answer: b

Watch Video Solution

16. Let $f:[2,\infty)\to\{1,\infty)$ defined by $f(x)=2^{x^4-4x^3}$ and $g:\left[\frac{\pi}{2},\pi\right]\to A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

The set "A" equals to

A. [-5,-2]

B. [2,5]

C. [-5,2]

D. [-3,-2]

Answer: a

Watch Video Solution

17. Let $f:[2,\infty)\to[1,\infty)$ defined by $f(x)=2^{x^4-4x^2}$ and $g:\left[\frac{\pi}{2},\pi\right]\to A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions.

The set A is equal to

A. [-5, sin 1]

B. $\left[-5,\frac{\sin 1}{2-\sin 1}\right]$

C. $\left[-5,-\frac{(4+\sin 1)}{2-\sin 1}\right]$

D. $\left[-\frac{(4+\sin 1)}{2-\sin 1},-2\right]$

Answer: c

Watch Video Solution

18. Let $P(x)$ be a polynomial of degree at most 5 which leaves remainders -1 and 1 upon divison by $(x-1)^3$ and $(x+1)^3$, respectively.

The sum of pairwise product of all roots (real and complex) of $P(x)=0$ is

A. 1

B. 3

C. 5

D. 2

Answer: a

Watch Video Solution

19. Let P(x) be polynominal of degree atmost 5 which leaves remainders -1 and 1 upon division by $(x-1)^3$ and $(x+1)^3$, respectively.

Number of real roots of P(x)=0 is

A. $-\frac{1}{\sqrt{3}}$

B. 0

C. $\frac{1}{\sqrt{3}}$

D. 1

Answer: c

Watch Video Solution

20. Let P(x) be polynominal of degree atmost 5 which leaves remainders -1 and 1 upon division by $(x-1)^3$ and $(x+1)^3$, respectively.

Number of real roots of P(x)=0 is

A. $-\frac{5}{3}$

B. $-\frac{10}{3}$

C. 2

D. -5

Answer: b

Watch Video Solution

21. Consider $\alpha > 1$ and $f: \left[\frac{1}{\alpha}, \alpha\right] \to \left[\frac{1}{\alpha}, \alpha\right]$ be bijective function. Suppose that $f^{-1}(x) = \frac{1}{f(x)}, \quad \text{for all} \quad \in \left[\frac{1}{\alpha}, \alpha\right]$.

Then f(1) is equal to

A. 1

B. 0

C. -1

D. does'nt attain a unique value

Answer: a

Watch Video Solution

22. Consider $\alpha > 1$ and $f: \left[\frac{1}{\alpha}, \alpha\right] \to \left[\frac{1}{\alpha}, \alpha\right]$ be bijective function. Suppose that $f^{-1}(x) = \frac{1}{f(x)}, \quad \text{for all} \quad \in \left[\frac{1}{\alpha}, \alpha\right]$.

Which of the following statements can be concluded about (f(x))?

A. f(x) is discontinuous in $\left[\frac{1}{\alpha}, \alpha\right]$

B. f(x) is increasing in $\left[\frac{1}{\alpha}, \alpha\right]$

C. f(x) is decreasing in $\left[\frac{1}{\alpha}, \alpha\right]$

D. None of the above

Answer: b

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23. Consider $\alpha > 1$ and $f: \left[\frac{1}{\alpha}, \alpha\right] \rightarrow \left[\frac{1}{\alpha}, \alpha\right]$ be bijective function.
Suppose that $f^{-1}(x) = \frac{1}{f(x)}, \quad \text{for all} \quad \in \left[\frac{1}{\alpha}, \alpha\right]$.
Which of the following statements can be concluded about (f(x))?

A. f(f(x)) is discontinuous in $\left[\frac{1}{\alpha}, \alpha\right]$

B. f(f(x)) is increasing in $\left[\frac{1}{\alpha}, \alpha\right]$

C. f(f(x)) is decreasing in $\left[\frac{1}{\alpha}, \alpha\right]$

D. None of the above

Answer: b

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24. Let f be real valued function from N to N satisfying. The relation f(m+n)=f(m)+f(n) for all $m, n \in N$.

The range of f contains all the even numbers, the value of f(1) is

A. 1

B. 2

C. 1 or 2

D. 4

Answer: b

Watch Video Solution

25. Let f be real valued function from N to N satisfying. The relation f(m+n)=f(m)+f(n) for all $m, n \in N$.

If domain of f is first 3m natural numbers and if the number of elements common in domain and range is m, then the value of f(1) is

A. 2

B. 3

C. 6

D. Can't say

Answer: B

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Exercise (Matching Type Questions)

1. Prove that the function f(x) = $x^{2n}+1$ is continuous at x = n , n > 0

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2. Differentiate the following function w.r.t x

cos(sinx)

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FUNCTION EXERCISE 5: Matching Type Questions

1. Find $\frac{dy}{dx}$ if $y = \sin^2 x$

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Exercise (Single Integer Answer Type Questions)

1. A wheel makes 720 revolutions in one minute. Through how many radians does it turn in one second?

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2. If $f: R \rightarrow R$ satisfying f(x-f(y))=f(f(y))+xf(y)+f(x)-1, for all $x, y \in R$, then $\frac{-f(10)}{7}$ is ....

Watch Video Solution

3. Let $f: N \rightarrow R$ be such that f(1)=1 and f(1)+2f(2)+3f(3)+... +nf(n)=n(n+1)f(n), for $n \geq 2, \text{ then }$ (2010f(2010))` is ....

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4. If $f(x) = \frac{2010x + 165}{165x - 2010}, x > 0 \text{ and } x \neq \frac{2010}{165}$, the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is ....

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5. If $\alpha, \beta, \gamma \in R, \alpha + \beta + \gamma = 4 \text{ and } \alpha^2 + \beta^2 + \gamma^2 = 6$, the number of integers lie in the exhaustive range of α is ....

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6. The number or linear functions f satisfying $f(x + f(x)) = x + f(x) \, \forall x \in \mathbb{R}$ is

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7. If A={1,2,3}, B={1,3,5,7,9}, the ratio of number of one-one functions to the number of strictly monotonic functions is ....

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8. If n(A)=4, n(B)=5 and number of functions from A to B such that range contains exactly 3 elements is k, $\frac{k}{60}$ is ....

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9. If a and b are constants, such that

$f(x) = a\sin x + bx\cos x + 2x^2$ and f(2)=15, f(-2) is ....

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10. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

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11. If $f(x) = x^3 - 12x + p, p \in \{1, 2, 3, \ldots, 15\}$ and for each 'p', the number of real roots of equation f(x)=0 is denoted by θ, the $\frac{1}{5}\sum\theta$ is equal to ....

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12. The minimum value of the fuction f(x) given by $f(x) = \frac{x^m}{m} + \frac{x^{-n}}{n} where \frac{1}{m} + \frac{1}{n} = 1$and m>1 is

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13. If $x^2 + y^2 = 4$ then find the maximum value of $\frac{x^3 + y^3}{x + y}$

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14. Let f(n) denotes the square of the sum of the digits of natural number n, where $f^2(n)$ denotes $f(f(n))$, $f^3(n)$ denotes f(f(f(n))) and so on. Then, the value of

$\frac{f^{2017}(2011) - f^{2016}(2011)}{f^{2017}(2011) - f^{2018}(2011)}$, is

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15. If $[\sin x]+\left[\frac{x}{2\pi}\right]+\left[\frac{2x}{5\pi}\right]=\frac{9x}{10\pi}$, where $[\cdot]$ denotes the greatest integer function, the number of solutions in the interval (30,40) is ....

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16. The number of integral solutions of $\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$ with $x \leq y$ is $'\alpha'$. The value of $'\alpha-6'$ is ....

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17. If f(x) is a polynominal of degree 4 with leading coefficient '1' satisfying f(1)=10,f(2)=20 and f(3)=30, then $\left(\frac{f(12)+f(-8)}{19840}\right)$ is ....

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18. If $a+b=3-\cos 4\theta$ and $a-b=4\sin 2\theta$, then ab is always less than or equal to

Watch Video Solution

19. Let 'n' be the number of elements in the domain set of the function $f(x) = \left|\ln \sqrt{{}^{x^2+4x}C_{2x^2+3}}\right|$ and 'Y' be the global maximum value of f(x), then [n+[Y]] is .... (where $[\cdot]$=greatest integer function).

Watch Video Solution

20. Let f(x) be a function such that , $f(x-1)+f(c+1)=\sqrt{3}f(x), \forall x \in R.$ If f(5)=100, find $\sum_{r=0}^{99} f(5+12r)$.

Watch Video Solution

21. If $2f(x)=f(xy)+f\left(\frac{x}{y}\right)$ for all positive values of x and y, $f(1)=0$ and $f'(1)=1$, then $f(e)$ is.

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22. Let f be a function from the set of positive integers to the set of real number such that f(1)=1 and $\sum_{r=1}^{n} rf(r) = n(n+1)f(n), \forall n \geq 2$ the value of 2126 f(1063) is ....

Watch Video Solution

23. If $f(x) = \frac{x^4 + x^2 + 1}{x^2 - x + 1}$, the value of $f(\omega^n)$ (where $'\omega'$ is the non-real root of the equation $z^3 = 1$ and 'n' is a multiple of 3), is ....

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24. If $f^2(x) \cdot f\left(\frac{1-x}{1+x}\right) = x^3, [x \neq -1, 1 \text{ and } f(x) \neq 0],$ then find $|[f(-2)]|$ (where [] is the greatest integer function).

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25. An odd function is symmetric about the vertical line $x = a, (a > 0), and$ if $\sum_{r=0}^{\infty} [f(1+4r]^r = 8$, then find the value of $f(1)$.

Watch Video Solution

26. $Let \frac{e^x - e^{-x}}{e^x + e^{-x}} = \ln \sqrt{\frac{1+x}{1-x}}$,then find x.

Watch Video Solution

27. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

(a) $\frac{17}{7}$

(b) $\frac{1}{4}$

(c) 41

(d) 1

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28. If $f(x)$ satisfies the relation $f(x)+f(x+4)=f(x+2)+f(x+6)$ for all$x,$ then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

29. A non-zero function f (x) is symmetrical about the line $y=x$ then the value of λ (constant) such that $f^2(x)=\left(f^{-1}(x)\right)^2-\lambda x f(x) f^{-1}(x)+3x^2 f(x)$ where all x $\in R^+$

Watch Video Solution

30. Let $f: R \rightarrow R$ and $f(x)=\dfrac{3x^2+mx+n}{x^2+1}$. If the range of this function is $[-4, 3]$, then the value of $\dfrac{m^2+n^2}{4}$ is ....

Watch Video Solution

31. Let f(x)be a monotic ploynomial of degree (2m-1) where $m \in N$ Then the equation

$f(x) - f(3x) + f(5x) + \ldots. + f((2m-1)$ has

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Exercise (Subjective Type Questions)

1. Find $\frac{dy}{dx}$ if $y = \frac{\tan x}{x}$

Watch Video Solution

2. Let n be a positive integer with $f(n) = 1! + 2! + 3! + \ldots\ldots.. + n!$ and $p(x), Q(x)$ be polynomial in x such that $f(n+2) = P(n)f(n+1) + Q(n)f(n)$ for all $n \geq 1,$ Then

(a) $P(x) = x + 3$

(b) $Q(x) = -x - 2$

(c) $P(x) = -x - 2$

(d) $Q(x) = x + 3$

Watch Video Solution

3. If x and y are connected parametrically by the equation ,without eliminating the parameter , find $\frac{dy}{dx}$

$x = 4at^2, y = at^8$

Watch Video Solution

4. Find the domain of the function

$$f(x) = \log_e\left\{\log_{|\sin x|}\left(x^2 - 8x + 23\right) - \frac{3}{\log_2|\sin x|}\right\}$$

Watch Video Solution

5. Solve $\frac{1}{[x]} + \frac{1}{[2x]} = \{x\} + \frac{1}{3}$ where [.] denotes the greatest integers function and{.} denotes fractional part function.

Watch Video Solution

6. Let $f(x) = x^2 + 3x - 3, x \leq 0$. If n points $x_1, x_2, x_3, \ldots., x_n$ are so chosen on the x-axis such that

(1) $\frac{1}{n}\sum f^{-1}(x_i) = f\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$

(2) $\sum_{i=1}^{n} f^{-1}(x_i) = \sum_{i=1}^{n} x_i$ where f^{-1} denotes the inverse of f, Then the AM of xi's is

a)1

b)2

c)3

d)4

Watch Video Solution

7. Let $f(x) = x^2 - 2x, x \in R, and g(x) = f(f(x) - 1) + f(5 - (x))$.

Show that $g(x) \geq 0 \forall x \in R$.

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8. If a function satisfies `(x-y)f(x+y)-(x+y)f(x-y)=2(x^2 y-y^3) AA x, y in R and f(1)=2, then

Watch Video Solution

9. If a+b+c=abc,a,b and c $\in R^+$, prove that $a+b+c \geq 3\sqrt{3}$.

Watch Video Solution

10. Consider the function $f(x) = \begin{cases} x = [x] - \frac{1}{2}, & \text{if } x \in I \\ 0, & \text{if } x \in I \end{cases}$ Where $[.]$ denotes greatest integer function and I is the set of integers, then $g(x) = \max\{x^2, f(x), |x|\}, -2 \leq x \leq 2$ is defined as

Watch Video Solution

11. Let $g(t) = |t-1| - |t| + |t+1|, \forall \quad t \in R.$

Find $f(x) = \max\left\{g(t): -\frac{3}{2} \leq t \leq x\right\}, \forall x \in \left(\frac{-3}{2}, \infty\right)$.]

Watch Video Solution

12. Find the integral solution for

$n_1 n_2 = 2n_1 - n_2, \quad \text{where} \quad n_1, n_2 \in \text{integer}.$

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FUNCTION EXERCISE 7: Subjective Type Questions

1. If f(x) is continuous function in $[0, 2\pi]$ and f(0)=f(2π), then prove that there exists a point $c \in (0, \pi)$ such that $f(x) = f(x+\pi)$.

Watch Video Solution

1. Find all the points of discontinuity, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \geq 1 \\ -1, & \text{if } x \geq 0 \end{cases}$$

A. $\begin{matrix} A & B & C & D \\ r & p & s & q \end{matrix}$

B. $\begin{matrix} A & B & C & D \\ p & r & s & q \end{matrix}$

C. $\begin{matrix} A & B & C & D \\ r & p & q & s \end{matrix}$

D. $\begin{matrix} A & B & C & D \\ p & r & q & s \end{matrix}$

Answer: D

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2. Find the values of x for which f(x) is positive if $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$.

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Exercise (Questions Asked In Previous 13 Years Exam)

1. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Watch Video Solution

2. Let F(x) be an indefinite integral of $\sin^2 x$

Statement-1: The function F(x) satisfies $F(x + \pi) = F(x)$ for all real x. because

Statement-2: $\sin^2(x + \pi) = \sin^2 x$ for all real x.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false.

D. Statement is false, Statement II is true.

Answer: D

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3. Find the range of values of t for which $2\sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$

Watch Video Solution

4. Let $F_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$, where $x \in R$ and $k \geq 1$, then find the value of $F_4(x) - F_6(x)$.

A. $1/6$

B. $1/3$

C. $1/4$

D. $1/12$

Answer: D

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5. Let $f: R - R$ be defined as $f(x) = x^4$, then (a) f is one-one (b) f is many-one onto (c) f is one-one but not onto (d) f is neither one-one nor onto

A. one-one and onto

B. onto but not one-one

C. one-one but not onto

D. neither one-one nor onto

Answer: D

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6. Let $f(x) = x^2 and g(x) = \sin x$ for all x in R Then the set of all x satisfying $(fogogof)(x) = (gogof)(x), where(fog)(x) = f(g(x))$, is

A. (a) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \ldots\}$

B. (b) $\pm\sqrt{n\pi}, n \in \{1, 2, \ldots\}$

C. (c) $\pi/2 + 2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

D. (d) $2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Answer: A

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7. Let $f: (0, 1) \rightarrow R$ be defined by $f(x) = \frac{b-x}{1-bx}$, where b is constant such that $0 < b < 1$ .then ,

A. (a)f is not invertible on (0,1)

B. (b)$f \neq f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$

C. (c)$f = f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$

D. (d) f^{-1} is differentiable on (0,1)

Answer: B

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8. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1}dt$, for all, $x \in (-1, 1)$ and f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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9. If X and Y are two non-empty sets where $f: X \to Y$,is function is defined such that $f(c) = \{f(x): x \in C\}$ for $C \subseteq X$ and $f^{-1}(D) = \{x: f(x) \in D\}$ for $D \subseteq Y$,for any $A \subseteq Y$ and $B \subseteq Y$, then

A. $f^{-1}\{f(A)\} = A$

B. $f^{-1}\{f(A)\} = A$, only if f(X)=Y

C. $f^{-1}\{f(B)\} = B$, only if B $\subseteq$ f(x)

D. $f^{-1}\{f(B)\} = B$

Answer: C

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10. If $f(x) = \{x,$ when x is rational and $0,$ when x is irrational $g(x) = \{0,$ when x is rational and $x,$ when x is irrational then $(f - g)$ is

A. one-one and onto

B. neither one-one nor onto

C. many one and onto

D one-one and into

A. one-one and into

B. neither one-one nor onto

C. many one and onto

D. one-one and onto

Answer: D

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11. If $f(x) = \sin x + \cos x, g(x) = x^2 - 1,$ then g(f(x)) is invertible in the domain

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $[0, \pi]$

Answer: B

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12. The domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real-valued x is $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

A. $\left[-\frac{1}{4}, \frac{1}{2}\right]$

B. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $\left(-\frac{1}{2}, \frac{1}{9}\right)$

D. None of these

Answer: A

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13. The range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}, x \in R,$ is

(a) $(1, \infty)$ (b) $\left(1, \frac{11}{7}\right)$ (c) $\left(1, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{5}\right)$

A. $(1, \infty)$

B. (1,11/7)

C. (1,7/3]

D. (1,7/5)

Answer: C

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14. If $f: [0, \infty) \rightarrow [0, \infty)$ and $f(x) = \frac{x}{1+x}$, then f is

A. one-one and onto

B. one-one but not onto

C. onto but not one-one

D. neither one-one nor onto

Answer: B

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15. If $f: R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then check the nature of the function.

a. one-to-one and onto

b. one-to-one but not onto

c. onto but not one-to-one

d. neither one-to-one nor onto

A. one-to-one and onto

B. one-to-one but not onto

C. onto but not one-to-one

D. neither one-to-one nor onto

Answer: A

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16. Let $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$. If N is the number of onto functions from$E \rightarrow F$, then the value of $N/2$ is

A. 14

B. 16

C. 12

D. 8

Answer: A

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17. Suppose $f(x) = (x+1)^2$ for $x \geq -1$. If g(x) is the function whose graph is the reflection of the graph of f(x) with respect to the line y = x, then g(x) equals

a. $1 - \sqrt{x} - 1, x \geq 0$

b. $\dfrac{1}{(x+1)^2}, x > -1$

c. $\sqrt{x+1}, x \geq -1$

d. $\sqrt{x}-1, x \geq 0$

A. $1-\sqrt{x}-1, x \geq 0$

B. $\frac{1}{(x+1)^2}, x > -1$

C. $\sqrt{x+1}, x \geq -1$

D. $\sqrt{x}-1, x \geq 0$

Answer: D

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18. If $F:[1,\infty) \to [2,\infty)$ is given by $f(x)=x+\frac{1}{x}, then\ f^{-1}(x)$ equals

A. $\frac{x+\sqrt{x^2-4}}{2}$

B. $\frac{x}{1+x^2}$

C. $\frac{x-\sqrt{x^2-4}}{2}$

D. $1+\sqrt{x^2-4}$

Answer: A

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19. Let $f(x)=(1+b^2)x^2+2bx+1$ and let $m(b)$ the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

(a) $[0, 1]$ (b) $\left(0, \frac{1}{2}\right]$ (c) $\left[\frac{1}{2}, 1\right]$ (d) $(0, 1]$

A. [0,1]

B. $\left[0, \frac{1}{2}\right]$

C. $\left[\frac{1}{2}, 1\right]$

D. (0,1]

Answer: D

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20. Find the domain of the following functions.

$$f(x) = \frac{\log_2(x+3)}{(x^2+3x+2)}$$

a. $R/\{-1, -2\}$

b. $(-2, \infty)$

c. $R/\{-1, -2, -3\}$

d. $(-3, \infty)/\{-1, -2\}$

A. $R/\{-1, -2\}$

B. $(-2, \infty)$

C. $R/\{-1, -2, -3\}$

D. $(-3, \infty)/\{-1, -2\}$

Answer: D

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21. Let $f(x) = \frac{\alpha x}{(x+1)}, x \neq -1$. The for what value of α is $f(f(x)) = x$?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 1

D. -1

Answer: D

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22.

Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, x < 0 \\ 0, x = 0 \\ 1, x > 0 \end{cases}$ Then for all $x, f(g(x))$ is equal to

(where $[.]$ represents the greatest integer function)

A. x

B. 1

C. $f(x)$

D. $g(x)$

Answer: B

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23. The domain of definition of the function $f(x)$ given by the equation $2^x = 2$ is

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

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24. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta) is$

A. $\geq$ 0, only when θ ge 0

B. $\leq$ 0, for all real θ

C. $\geq$ 0, for all real θ

D. $\leq$ 0, only when $\theta \leq 0$

Answer: C

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