



India's Number 1 Education App

## MATHS

### BOOKS - ARIHANT MATHS

## LIMITS

#### Examples

$$1. \lim_{x \rightarrow 2} \frac{x^6 - 24x - 16}{x^3 + 2x - 12}$$



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$$2. \lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}, \text{ is}$$



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3. Evaluate  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$



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4. Evaluate  $\lim_{x \rightarrow 1} \frac{x^3 - x^2 \log x + \log x - 1}{x^2 - 1}$



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5. Evaluate  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$



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6. The value of  $\lim_{x \rightarrow 2a} \frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$  is



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7. Evaluate  $\lim_{x \rightarrow 2} \frac{x^3 - 2^3}{x - 2}$



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8. Evaluate  $\lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}$



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9. The value of  $\lim_{x \rightarrow 1} \frac{y^3}{x^3 - y^2 - 1}$  as  $(x, y) \rightarrow (1, 0)$  along the line

$y = x - 1$  is

A. 1

B. -1

C. 0

D. Doesn't exist

**Answer:**



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10. Evaluate  $\lim_{x \rightarrow \infty} \frac{x^2 + 5}{x^2 + 4x + 3}$



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11. Evaluate  $\lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$



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12.  $\lim_{x \rightarrow \infty} \left( x - \sqrt{x^2 + x} \right)$



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13. Evaluate :  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 + 1}$



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14. Evaluate  $\lim_{x \rightarrow \infty} \frac{ax^2 + b}{x + 1}$  when  $a \geq 0$



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15. If  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + 1}{x + 1} - ax - b \right) = 0$ , find the values of a and b.



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16. Evaluate  $\lim_{x \rightarrow \infty} \frac{ax^2i + b}{x + 1}$  when  $a \geq 0$



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17. If  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + 1}{x + 1} - ax - b \right) = 0$ , find the values of a and b.



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**18.**

Let

$$S_n = 1 + 2 + 3 + \dots + n$$

and

$P_n = \frac{S_2}{S_2 - 1} \cdot \frac{\dot{S}_3}{S_3 - 1} \cdot \frac{\dot{S}_4}{S_4 - 1} \cdots \frac{\dot{S}_n}{S_n - 1}$  Where  $n \in N, (n \geq 2)$ . Then  
 $(\lim)_{n \rightarrow \infty} P_n = \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$



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**19.** If  $(\lim)_{n \rightarrow \infty} \frac{1}{(\sin^{-1} x)^n + 1} = 1$ , then find the value of  $x$

A.  $(-\sin 1, \sin 1)$

B.  $(-1, 1)$

C.  $(0, 1)$

D.  $(-1, 0)$

**Answer: A::C**



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**20.** Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$



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**21.** Solve  $\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{\sin^4 x}$



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**22.** Evaluate  $\lim_{x \rightarrow \infty} (2^{-x} \sin(2^x))$



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**23.** Evaluate  $\lim_{x \rightarrow \infty} e^x \sin(d/e^x)$ .



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**24.** Evaluate  $\lim_{x \rightarrow \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$



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**25.** the value of  $\lim_{x \rightarrow y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$  equals



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**26.**  $(\lim)_{x \rightarrow \infty} \{x + 5 \tan^{-1}(x + 5) - (x + 1) \tan^{-1}(x + 1)\}$  is equal to

A.  $\pi$

B.  $2\pi$

C.  $\frac{\pi}{2}$

D. None of these

**Answer: B**



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27. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$



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28. Let  $a = \min \{x^2 + 2x + 3, x \in R\}$  and  $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2}$   
then the value of  $\sum_{r=0}^n a^r \cdot b^{n-r}$  is :



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29. Write a note on Co-ordinate isomerism.



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30. Evaluate  $\lim_{x \rightarrow a} \frac{\log\{1 + (x - a)\}}{(x - a)}$



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31. Evaluate  $\lim_{h \rightarrow 0} \frac{\log_{10}(1 + h)}{h}$



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32. Evaluate  $\lim_{x \rightarrow 0} \frac{\log(5 + x) - \log(5 - x)}{x}$ .



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33.  $(\lim)_{x \rightarrow 0} \frac{1n(1 + 2h) - 21n(1 + h)}{h^2} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$



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34. The value of  $\lim_{x \rightarrow \infty} \left( x - x^2 \log_e \left( 1 + \frac{1}{x} \right) \right)$  is \_\_\_\_\_.



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35. Evaluate  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$



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36. Evaluate  $\lim_{x \rightarrow 0} \frac{(ab)^x - a^x - b^x + 1}{x^2}$



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37.  $\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$



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38. Evaluate  $\lim_{x \rightarrow 0} \frac{ae^x - b}{x} = 2$ . Find a and b



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39. Evaluate  $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$



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40. Find the constant  $a$ ,  $b$  and  $c$  such that

$$\lim_{x \rightarrow 0} \frac{axe^x - b \log(1 + x) + cxe^{-x}}{x^2 \sin x} = 2$$



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41. Evaluate  $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$



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42.  $\lim_{x \rightarrow 1} (\log_3 3x)^{\log_x 3} =$



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**43.** Evaluate  $\lim_{x \rightarrow a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$



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**44.** Evaluate  $\lim_{x \rightarrow \infty} \left(x - \sqrt{x^2 + x}\right)$



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**45.** Evaluate  $\lim_{x \rightarrow 0} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$



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**46.** Evaluate  $\lim_{x \rightarrow 0} \left\{ \tan\left(\frac{\pi}{4} + x\right) \right\}^{\frac{1}{x}}$



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**47.** Evaluate  $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$



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**48.** Evaluate  $\lim_{x \rightarrow \infty} \left(\frac{x - 3}{x + 2}\right)^x$



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**49.**  $\lim_{x \rightarrow 0} \sin^2\left(\frac{\pi}{2 - px}\right)^{\sec^2\left(\left(\frac{\pi}{2 - px}\right)\right)}$



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**50.** Evaluate :  $\int \frac{dx}{2x^2 - 5x + 7}$ .

A.  $e^{5/2}$

B.  $e^{-5/2}$

C.  $e^{7/2}$

D.  $e^{3/2}$

**Answer: A::C**



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51. The value of  $\lim_{x \rightarrow 1} \left( \tan\left(\frac{\pi}{4} + \log x\right) \right)^{\frac{1}{\log x}}$  is equal to

A.  $e$

B.  $e^{-1}$

C.  $e^2$

D.  $e^{-2}$

**Answer: A::C**



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52.  $\lim_{x \rightarrow 0} \left( \sin \frac{x}{m} + \cos \frac{3x}{m} \right)^{(2m)/x}$  is

A. e

B. 1

C.  $e^{-1}$

D.  $e^2$

**Answer: A::C::D**



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53. The value of sum  $\sum_{n=1}^{13} (i^n + i^{n+1})$ , where  $i = \sqrt{-1}$  equals

A.  $\sqrt[a]{b}$

B.  $\sqrt[b]{a}$

C.  $\sqrt{b}$

D.  $\sqrt{a}$

**Answer: A::C**



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**54.** Evaluate  $\lim_{x \rightarrow 0^+} (\sin x)^x$



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**55.** Evaluate  $\lim_{n \rightarrow \infty} (\pi n)^{2/n}$



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**56.** evaluate  $\lim_{n \rightarrow \infty} \left( \frac{e^n}{\pi} \right)^{\frac{1}{n}}$



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**57.** Evaluate  $\lim_{x \rightarrow 0} (\cos ex)^x$ .



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58. Slove  $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$



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59. Evaluate the left-and right-hand limits of the function

$$f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x = 4, 0, x = 4 \end{cases}$$



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60. Let  $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2. \end{cases}$

Find  $\lim_{x \rightarrow 1} f(x)$ .



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**61.** Show that  $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$  does not exist.



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**62.**  $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1},$

A. (a) exists and equals to  $\sqrt{2}$

B. (b) exists and equals to  $-\sqrt{2}$

C. (c) does not exist

D. (d) None of these

**Answer:** Hence  $\lim_{x \rightarrow 1} f(x)$  doesn't exist.



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**63.** Solve  $\lim_{x \rightarrow 0} \left[ \frac{\sin x}{x} \right]$

(where  $[.]$  denotes greatest integer function)



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**64.** Solve  $\lim_{x \rightarrow \infty} [\tan^{-1} x]$

(where  $[.]$  denotes greatest integer function)



**Watch Video Solution**

**65.** Solve  $\lim_{x \rightarrow 0} \left[ \frac{\tan x}{x} \right]$

(where  $[.]$  denotes greatest integer function)



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**66.** Solve  $\lim_{x \rightarrow 1} \sin(\sin^{-1} x)$



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67. Solve (i)  $\lim_{x \rightarrow 0} \cot x$

(ii)  $\lim_{x \rightarrow +\infty} \cot^{-1} x$

(where  $[.]$  denotes greatest integer function)



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68. Solve  $\lim_{x \rightarrow 0} \left[ \frac{\sin|x|}{|x|} \right]$ , where  $[.]$  denotes greatest integer function.



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69. Solve  $\lim_{x \rightarrow 0} \left[ \sin \frac{|x|}{x} \right]$ , where  $e[.]$  denotes greatest integer function.



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70.  $\lim_{x \rightarrow 0} \left[ \frac{-2x}{\tan x} \right]$ , where  $[.]$  denotes greatest integer function is

A. -1

B. 4

C. 5

D. None of these

**Answer: A::C::D**



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$$71. \text{ Evaluate } \lim_{x \rightarrow 0} x^3 \cos \frac{2}{x}.$$



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$$72. \text{ Evaluate } \lim_{x \rightarrow \infty} \frac{x^2(2 + \sin^2 x)}{x + 100}$$



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73.  $\lim_{n \rightarrow \infty} \left( \frac{1}{n^2 + 1} + \frac{2}{n^2 + 2} + \frac{3}{n^2 + 3} + \dots + \frac{n}{n^2 + n} \right)$



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74. The value of the  $\lim_{x \rightarrow 0} \frac{x}{a} \left[ \frac{b}{x} \right]$  ( $a \neq 0$ ) (where  $[ \cdot ]$  denotes the greatest integer function) is

A. a

B. b

C.  $\frac{b}{a}$

D.  $1 - \frac{b}{a}$

Answer: A::C



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75. If  $[x]$  denotes the greatest integer less than or equal to  $x$ , then

evaluate  $\lim_{n \rightarrow \infty} \frac{1}{n^2} ([1 \cdot x] + [2 \cdot x] + [3 \cdot x] + \dots + [n \cdot x]).$



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76.  $\lim (x \rightarrow 0) \left( \frac{1}{x^5} \int_0^x e^{-t^2} dt - \frac{1}{x^4} + \frac{1}{3x^2} \right)$  is equal to



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77. Evaluate  $\lim_{x \rightarrow 0} \frac{x - (\int_0^x \cos t dt)}{x^3 - 6x}$



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78. Evaluate  $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos^2 t dt}{x \sin x}$



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**79.** Let  $S$  be the sum of all possible determinants of order 2 having 0,1,2 and 3 as their elements,. Find the common root  $\alpha$  of the equations

$$x^2 + ax + [m + 1] = 0,$$

$$x^2 + bx + [m + 4] = 0$$

$$\text{and } x^2 - cx + [m + 15] = 0$$

such that  $\alpha > S$  where  $a+b+c=0$  and

$$m = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$$

and  $[.]$  denotes the greatest integer function.



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**80.** The value of  $\lim_{x \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{(n^2 + 1)^2}$



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**81.** Evaluate:  $(\lim)_{n \rightarrow \infty} \left[ \frac{n!}{n^n} \right]^{1/n}$

A. (a)  $\frac{1}{e}$

B. (b)  $e^0$

C. (c)  $e^2$

D. (d)  $\frac{1}{e^2}$

**Answer: A::C**



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82. The value of  $\lim_{x \rightarrow 0} \frac{x \sin x}{x^2}$  equals

A. (a)

B. (b)

C. (c)

D. (d)

**Answer: A::B::C**



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83. If  $\lim_{x \rightarrow 0} \frac{\log_e \cot\left(\frac{\pi}{4} - K_1 x\right)}{\tan K_2 x} = 1$ , then find relationship between the 2 constants.

A.  $K_1 = K_2$

B.  $2K_1 = K_2$

C.  $K_1 = 2K_2$

D.  $K_1 = 4K_2$

**Answer: A::B**



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84.

Let

$$f(\theta) = \frac{1}{\tan^2 \theta} \left\{ (1 + \tan \theta)^3 + (2 + \tan \theta)^3 + \dots + (10 + \tan \theta)^3 \right\}$$

$$10 \tan \theta$$

Then,  $\lim_{\theta \rightarrow \frac{\pi}{2}} f(\theta)$  is equal to

- A. 170
- B. 166
- C. 165
- D. None of these

**Answer: A**



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**85.** The numbers 1, 3, 6, 10, 15, 21, 28... are called triangular numbers.

Let  $t_n$  denote the  $n^{th}$  triangular number such that

$$t_n = t_{n-1} + n, \forall n \geq 2.$$

The value of  $t_{50}$  is:

A.  $\frac{2}{\pi^2}$

B.  $\frac{4}{\pi^2}$

C.  $\frac{8}{\pi^2}$

D.  $\frac{1}{(2\pi)^2}$

**Answer: B**



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**86.** Evaluate  $\int 29x^2 dx$ .



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**87.** Evaluate:  $(\lim)_{x \rightarrow 2} \frac{\sqrt{(x + 7)} - 3\sqrt{(2x - 3)}}{(x + 6)^{\frac{1}{3}} - 2(3x - 5)^{\frac{1}{3}}}$

A.  $\frac{33}{23}$

B.  $\frac{34}{23}$

C.  $\frac{54}{25}$

D. None of these

**Answer: B::C::D**



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**88.**

Let

$$g(x)|f(x + c)f(x + 2c)f(x + 3c)f(c)f(2c)f(3c)f'(c)f'(2c)f'(3c)|,$$

where  $c$  is constant, then find  $(\lim_{x \rightarrow 0} \frac{g(x)}{x})$

A. 0

B. 1

C. 2

D. 3

**Answer: A::C::D**



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89. Let  $\lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2} = f(a)$ . The value of  $f(101)$  equals

A. 5050

B. 5151

C. 4950

D. 101

**Answer: A::B**



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90.  $(\lim)_{x \rightarrow 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}$ , where  $n = 100$ , is equal to :  
 $\frac{5050}{\pi e}$  (b)  $\frac{100}{\pi e}$  (c)  $-\frac{5050}{\pi e}$  (d)  $-\frac{4950}{\pi e}$

A.  $\frac{5050}{\pi e}$

B.  $\frac{100}{\pi e}$

C.  $-\frac{5050}{\pi e}$

$$D. -\frac{4950}{\pi e}$$

Answer: A



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91. The value of  $\lim_{x \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{(n^2 + 1)^2}$

A.  $\frac{1}{3}$

B.  $\frac{2}{3}$

C.  $\frac{1}{2}$

D.  $\frac{1}{6}$

Answer: A



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92. Evaluate  $\int 30x^2 dx$ .



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93. Find slope of tangent if equation of the curve is  $y = 3x^4 - 4x$  at  $x = 4$



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$$94. \lim_{x \rightarrow \infty} \frac{\cot^{-1}(\sqrt{x+1} + \sqrt{x})}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right\}} =$$

A. 1

B. 0

C.  $\frac{\pi}{2}$

D. None of these

Answer: A



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**95.** Evaluate  $\lim_{n \rightarrow \infty} \cos(\pi\sqrt{n^2 + n})$  when n is an integer.

- A. 1
- B. -1
- C. 0
- D. Doesn't exist

**Answer:**



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**96.** Evaluate  $\int 31x^2 dx$ .



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**97.**  $\lim_{x \rightarrow 0^-} (-\ln(\{x\} + |[x]|))^{\{x\}}$  is equal to

A. 0

B. 1

C.  $\ln 2$

D.  $\ln \frac{1}{2}$

**Answer: A::B**



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98.  $\lim_{x \rightarrow \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$  is equal to

A. 0

B.

C. -1

D. Non-existent

**Answer: A**



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99. The value of  $\lim_{x \rightarrow 0} (\cos ax)^{\cos ec^2 bx}$  is

A. a)  $e\left(-\frac{8b^2}{a^2}\right)$

B. b)  $e\left(-\frac{8a^2}{b^2}\right)$

C. c)  $e^{-\frac{a^2}{2b^2}}$

D. d)  $e^{-\frac{b^2}{2a^2}}$

**Answer: C**



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100.  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{n^2 + n + 4}$  equals

a. 0

b. 1/3

c. 1/2

d. 1

A. 0

B.  $1/3$

C.  $1/2$

D. 1

**Answer: C**



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101. The value of  $\lim_{n \rightarrow \infty} \left( \sqrt{n^2 + n + 1} - [\sqrt{n^2 + n + 1}] \right)$  where  $[.]$

denotes the greatest integer function is

A. 0

B.  $1/2$

C.  $2/3$

D.  $1/4$

**Answer: A::B**



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102.  $\lim_{x \rightarrow 1} \frac{\sin^2(x^3 + x^2 + x - 3)}{1 - \cos(x^2 - 4x + 3)}$  has the value equal to

A. 18

B.  $9/2$

C. 9

D. None of these

Answer: A



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103. Evaluate  $\int 12x^4 dx$



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104. Evaluate  $\int 13x^4 dx$

A.

B.

C.

D.

**Answer: A::C**



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105. Let  $f(x) = \begin{cases} \frac{\tan^2\{x\}}{x^2 - [x]^2} & \text{for } x > 0 \\ \frac{1}{\sqrt{\{x\}\cot\{x\}}} & \text{for } x < 0 \end{cases}$  where  $[x]$  is the step up function and  $\{x\}$  is the fractional part function of  $x$  then



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**106.** Given that the derivative  $f'(a)$  exists. Indicate which of the following statements(s) is/are always true?

- a.  $f'(a) = \lim_{h \rightarrow a} \frac{f(h) - f(a)}{h - a}$
- b.  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a) - f(a - h)}{h}$
- c.  $f'(a) = \lim_{t \rightarrow 0} \frac{f(a + 2t) - f(a)}{t}$
- d.  $f'(a) = \lim_{t \rightarrow 0} \frac{f(a + 2t) - t(a + t)}{2t}$

- A.  $f'(a) = \lim_{h \rightarrow a} \frac{f(h) - f(a)}{h - a}$
- B.  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a) - f(a - h)}{h}$
- C.  $f'(a) = \lim_{t \rightarrow 0} \frac{f(a + 2t) - f(a)}{t}$
- D.  $f'(a) = \lim_{t \rightarrow 0} \frac{f(a + 2t) - t(a + t)}{2t}$

**Answer:** A::C::D



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**107.** Let  $f(x) = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{x}{(rx + 1)\{(r + 1)x + 1\}}$ . Then

A.  $f(0) = 0$

B.  $f(0) = x$

C.  $f(0^+) = 1$

D.  $f(0^-) = 1$

**Answer:** A



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**108.** The slope of tangent to the curve  $y = 2 - x^2$  at  $x = 1$  is

A.  $2 + \sqrt{3}$

B.  $\sqrt{3}$

C.  $2 + \sqrt{2}$

D.  $2 - \sqrt{2}$

**Answer:** B::C

109. Find  $f'(x)$  if  $f(x) = e^{\{x^2\}}$

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110. Find  $\frac{dy}{dx}$  if  $y = e^{5x}$

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111. Let  $A$  be  $n \times n$  matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that  $a_{24} = 1$ ,  $a_{42} = \frac{1}{8}$  and  $a_{43} = \frac{3}{16}$ . Let  $D_i$  be the common difference

of  $i^{th}$  row

The value of  $\lim_{n \rightarrow \infty} \sum_{i=1}^n D_i$  is equal to

A.  $\frac{1}{4}$

B.  $\frac{1}{8}$

C.  $\frac{1}{16}$

D.  $\frac{1}{32}$

**Answer: D**



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**112.** Let A be  $n \times n$  matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given

that  $a_{24} = 1$ ,  $a_{42} = \frac{1}{8}$  and  $a_{43} = \frac{3}{16}$

Let  $d_i$  be the common difference of the elements in with row then

$$\sum_{i=1}^n d_i \text{ is}$$



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113. Let  $A$  be  $n \times n$  matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that  $a_{24} = 1$ ,  $a_{42} = \frac{1}{8}$  and  $a_{43} = \frac{3}{16}$ . Let  $D_i$  be the common difference of  $i^{th}$  row

The value of  $\lim_{n \rightarrow \infty} \sum_{i=1}^n D_i$  is equal to



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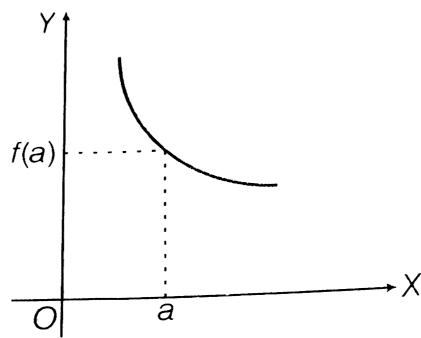
$$114. \text{ Find } \frac{dy}{dx} \text{ if } y = \frac{\sin x}{x}$$



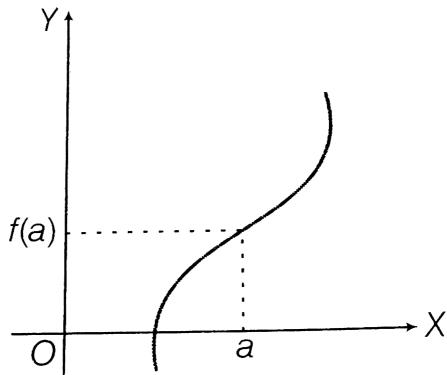
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115. To evaluate  $\lim_{x \rightarrow a} [f(x)]$ , we must analyse the  $f(x)$  in right hand neighbourhood as well as in left hand neighbourhood of  $x = a$ . E.g. In case of continuous function, we may come across followign cases.

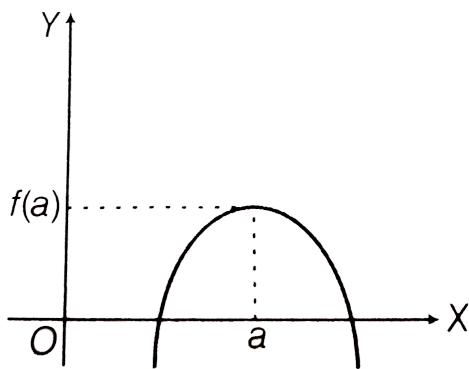
**Case I**



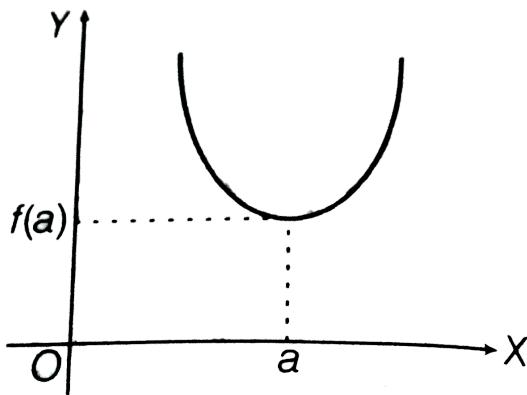
**Case II**



**Case III**



## **Case IV**



If  $f(a)$  is an integer, the limit will exist in Case III and Case IV but not in Case I and Case II.  $\lim_{x \rightarrow 1} \left[ \operatorname{cosec} \frac{\pi x}{2} \right]^{-1/(1-x)}$  is equal to (where  $[.]$  denotes the greatest integer function).

- A. 0
- B. 1
- C.  $\infty$
- D. Doesn't exist

**Answer: B**



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**116. Statement 1**  $\lim_{x \rightarrow \pi/2} \frac{\sin(\cot^2 x)}{(\pi - 2x)^2} = \frac{1}{2}$

**Statement 2:**  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  and  $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1$ , where  $\theta$  is measured in radians.

A. Statement 1 is true, Statement 2 is true, Statement 2 is correct

explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the

correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

**Answer: A::B::D**



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**117. Statement 1**  $\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \{\sin^{2m} n! \pi n\} = 0$ ,  $n \in N$ , when  $x$  is rational.

**Statement 2** When  $n \rightarrow \infty$  and  $x$  is rational  $n!x$  is integer.

- A. (a) Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.
- B. (b) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.
- C. (c) Statement 1 is true, Statement 2 is false
- D. (d) Statement 1 is false, Statement 2 is true

**Answer:**



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**118.** Show that  $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$  does not exist.

- A. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

**Answer: B**



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119. Evaluate  $\int 22x^4 dx$



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120. Match the following

<b>Column I</b>	<b>Column II</b>
(A) $\lim_{x \rightarrow \infty} (\sqrt{x} + \sqrt[3]{x} - \sqrt{1 - \sqrt{x}})$ equals	(p) - 2
(B) The value of the limit $\lim_{x \rightarrow 0^+} \frac{\sin 2x - 2 \tan x}{\ln(1+x^3)}$ is	(q) - 1
(C) $\lim_{x \rightarrow 0^+} (\ln \sin^3 x - \ln(x^4 + ex^3))$ equals	(r) 0
(D) Let $\tan(2\pi  \sin \theta ) = \cot(2\pi  \cos \theta )$ , where $\theta \in R$ and $f(x) = ( \sin \theta  +  \cos \theta )^x$ . The value of $\lim_{x \rightarrow \infty} \left[ \frac{2}{f(x)} \right]$ equals (here, $[ ]$ represents greatest integer function)	(s) 1



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121. If  $\lim_{x \rightarrow 0} (x^{-3} \sin 3x + ax^{-2} + b)$  exists and is equal to zero then



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122. For a certain value of 'c'  $\lim_{x \rightarrow -\infty} [(x^5 + 7x^4 + 2)^c - x]$  is finite

and non-zero. Then the value limit is :



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123. Evaluate  $\lim_{x \rightarrow 0^+} \log_{\sin x} \sin 2x.$



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124.

$$\lim_{x \rightarrow \infty} \left( \sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d} \right) = 4$$



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125. Let  $f(x) = \lim_{m \rightarrow \infty} \left\{ \lim_{n \rightarrow \infty} \cos^{2m}(n! \pi x) \right\}$ , where  $x \in R$ . Then the range of  $f(x)$



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126. If  $a_1 = 1$  and  $a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n}$ ,  $n \geq 1$ , show that  $a_{n+2} \geq a_{n+1}$  and if a limit as  $n \rightarrow \infty$  the evaluate  $\lim_{n \rightarrow \infty} a_n$



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127. Let  $a_1, a_2, a_n$  be sequence of real numbers with  $a_{n+1} = a_n + \sqrt{1 + a_n^2}$  and  $a_0 = 0$ . Prove that  $\lim_{x \rightarrow \infty} \left( \frac{a_n}{2^{n-1}} \right) = \frac{2}{\pi}$



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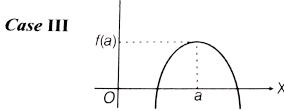
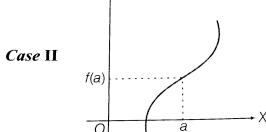
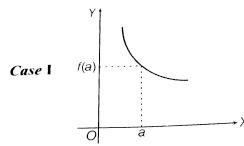
**128.** A square is inscribed in a circle of radius  $R$ , a circle is inscribed in this square then a square in this circle and so on  $n$  times. Find the limit of the sum of areas of all the squares as  $n \rightarrow \infty$ .



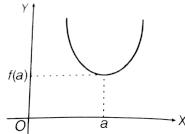
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### Solved Examples

**1.** To evaluate  $\lim_{x \rightarrow a} [f(x)]$ , we must analyse the  $f(x)$  in right hand neighbourhood as well as in left hand neighbourhood of  $x = a$ . E.g. In case of continuous function, we may come across followign cases.



**Case IV**



If  $f(a)$  is an integer, the limit will exist in Case III and Case IV but not in Case I and Case II. If  $f(a)$  is not an integer, the limit exists in all the cases. If  $f'(1) = -3$  and  $\lim_{x \rightarrow 1} \left[ f(x) - \frac{1}{2} \right]$  does not exist, (where  $[.]$  denotes the greatest integer function), then



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### Exercise Single Option Correct Type Questions

1.  $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2}$  is equal to

A.  $\pi$

B.  $\frac{\pi}{4}$

C.  $\frac{\pi}{2}$

D. None of these

**Answer: A**



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2.  $\lim_{t \rightarrow 0} \frac{1 - (1+t)^t}{\ln(1+t) - t}$  is equal to

A.  $\frac{1}{2}$

B.  $-\frac{1}{2}$

C. 2

D. -2

**Answer: C**



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3. If  $I_1 = \lim_{x \rightarrow 0} (\tan^{-1} \pi x - \tan^{-1} x) \cos x$  and  
 $I_2 = \lim_{x \rightarrow 0} (\tan^{-1} \pi x - \tan^{-1} x) \sin x$  then  $(I_1, I_2)$  is

A.  $(0, 0)$

B.  $(0, 1)$

C.  $(1, 0)$

D. None of the above

**Answer: A**



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4. If  $f(x) = 0$  is a quadratic equation such that  $f(-\pi) = f(\pi) = 0$

and  $f\left(\frac{\pi}{2}\right) = -\frac{3\pi^2}{4}$ , then  $\lim_{x \rightarrow -\pi} \frac{f(x)}{\sin(\sin x)}$  is equal to

A. 0

B.  $\pi$

C.  $2\pi$

D. None of these

**Answer: C**



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5.  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + \sin^2 x} - \sqrt[4]{1 - 2 \tan x}}{\sin x + \tan^2 x}$  is equal to:

A.  $-1$

B.  $1$

C.  $\frac{1}{2}$

D.  $-\frac{1}{2}$

**Answer: C**



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6.

For

$n \in N$ ,

let

$$f_n(x) = \tan \frac{x}{2} (1 + \sec x)(1 + \sec 2x)(1 + \sec 4x) \dots \dots (1 + \sec 2^n x),$$

the  $\lim_{x \rightarrow 0} \frac{f_n(x)}{2x}$  is equal to :

A. 0

B.  $2^n$

C.  $2^{n-1}$

D.  $2^{n+1}$

**Answer: C**



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7. Let  $f(x)$  be a real valued function defined for all  $x \geq 1$ , satisfying

$$f(1) = 1 \text{ and } f'(x) = \frac{1}{x^2 + f(x)}, \text{ then } \lim_{x \rightarrow \infty} f(x)$$

a. doesn't exist

b. exists and less than  $\frac{\pi}{4}$

c. exists and less than  $1 + \frac{\pi}{4}$

d. exists and equal to 0

A. (a) doesn't exist

- B. exists and less than  $\frac{\pi}{4}$
- C. exists and less than  $1 + \frac{\pi}{4}$
- D. exists and equal to 0

**Answer:** C



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8. The quadratic equation whose roots are the minimum value of  $\sin^2 \theta - \sin \theta + \frac{1}{2}$  and  $\lim_{x \rightarrow \infty} \sqrt{(x+1)(x+2)} - x$  is

A. (a)  $3x^2 - 7x + 3 = 0$

B. (b)  $8x^2 - 14x + 3 = 0$

C. (c)  $x^2 - 7x + 3 = 0$

D. (d)  $2x^2 - 8x + 3 = 0$

**Answer:** B



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9. If  $x_1 = \sqrt{3}$  and  $x_{n+1} = \frac{x_n}{1 + \sqrt{1 + x_n^2}}$ ,  $\forall n \in N$  then  $\lim_{n \rightarrow \infty} 2^n x_n$  is equal to

A.  $\frac{3}{2\pi}$

B.  $\frac{2}{3\pi}$

C.  $\frac{3\pi}{3}$

D.  $\frac{3\pi}{2}$

Answer: C



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10.  $\lim_{x \rightarrow a^-} \frac{\sqrt{x-b} - \sqrt{a-b}}{(x^2 - a^2)}$ , ( $a > b$ ) is

A.  $\frac{1}{4a}$

- B.  $\frac{1}{a\sqrt{a-b}}$
- C.  $\frac{1}{2a\sqrt{a-b}}$
- D.  $\frac{1}{4a\sqrt{a-b}}$

**Answer: D**



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11.  $\lim_{n \rightarrow \infty} (\sin^n 1 + \cos^n 1)^n$  is equal to



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12. The value of  $\lim_{x \rightarrow 0} \frac{2}{x^3} (\sin^{-1} x - \tan^{-1} x)^{2/x^3}$  equals

A.  $e$

B.  $\sqrt{e}$

C.  $\frac{1}{e}$

D.  $\frac{1}{\sqrt{e}}$

**Answer: D**



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13.  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{n^2 + n + 4}$  equals

a. 0

b. 1/3

c. 1/2

d. 1

A. (a)  $\frac{1}{4}\sin 4 + \frac{1}{15}\cos 4 - \frac{1}{16}$

B. (b)  $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

C. (c)  $\frac{1}{16}(1 - \sin 4)$

D. (d)  $\frac{1}{16}(1 - \cos 4)$

**Answer: D**



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14. Let  $U = \{x : x \in \mathbb{N}, x \leq 9\}$ ;  $A = \{x : x \text{ is an even number, } 0 < x < 10\}$ ;  $B = \{2, 3, 5, 7\}$ . Write the set  $(A \cup B)'$ .



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15.  $\int_{-1}^2 \left[ \frac{[x]}{1+x^2} \right] dx$ , where  $[.]$  denotes the greatest integer function, is equal to



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16. If  $\alpha$  and  $\beta$  are roots of  $x^2 - (\sqrt{1 - \cos 2\theta})x + \theta = 0$ , where  $0 < \theta < \frac{\pi}{2}$ . Then  $\lim_{\theta \rightarrow 0^+} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)$  is

A. (a)  $\frac{1}{\sqrt{2}}$

B. (b)  $-\sqrt{2}$

C. (c)  $\sqrt{2}$

D. (d) None of these

**Answer: C**



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17. If  $f(x) = \frac{1}{3} \left( f(x+1) + \frac{5}{f(x+2)} \right)$  and  $f(x) > 0, \forall x \in R$ , then

$\lim_{x \rightarrow \infty} f(x)$  is

A. 0

B.  $\sqrt{\frac{2}{5}}$

C.  $\sqrt{\frac{5}{2}}$

D.  $\infty$

**Answer: C**



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18. Let  $f: (1, 2) \xrightarrow{R}$  satisfies the inequality

$$\frac{\cos(2x - 4) - 33}{2} < f(x) < \frac{x^2|4x - 8|}{x - 2} \quad \forall x \in (1, 2). \quad \text{Then find}$$

$$\lim_{x \rightarrow 2^-} f(x).$$

A. 16

B. -16

C. does'nt exist

D.  $\theta$

Answer: B



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19. Let  $f(x)$  be polynomial of degree 4 with roots 1,2,3,4 and leading coefficient 1 and  $g(x)$  be the polynomial of degree 4 with roots

$1, \frac{1}{2}, \frac{1}{3}$  and  $\frac{1}{4}$  with leading coefficient 1. Then  $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$  equals

A. (a)  $\frac{1}{24}$

B. (b) – 24

C. (c)  $\frac{1}{12}$

D. (d)  $-\frac{1}{12}$

**Answer: B**



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20.  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$  is equal to

A.  $\sqrt{2}$

B.  $3\sqrt{5}$

C.  $5\sqrt{2}$

D.  $-5\sqrt{2}$

**Answer: C**



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21. If  $\lim_{x \rightarrow \infty} \frac{n \cdot 3^n}{n \cdot (x-1)^n + n \cdot 3^{n+1} - 3^n} = \frac{1}{3}$ , the number of the integral values of  $x$  is .....

A. 3

B. 4

C. 5

D. infinite

**Answer: C**



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22. Let  $f(x) = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{3}{\pi} \tan^{-1} 2x\right)^{2n} + 5}$ . Then the set of values of  $x$

for which  $f(x) = 0$  is

A.  $|2x| > \sqrt{3}$

B.  $|2x| < \sqrt{3}$

C.  $|2x| \geq \sqrt{3}$

D.  $|2x| \leq \sqrt{3}$

**Answer: A::B::D**



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**23.** Find the integral value of  $n$  for which

$(\lim)_{x \rightarrow 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$  is a finite nonzero number

A. 2

B. 3

C. 4

D. None of these

**Answer: C**



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**24.** Find  $\frac{dy}{dx}$  if  $x^7 - e^x = \sin y$



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**25.**  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left[ \frac{x}{2} \right]}{\log_e(\sin x)}$  ([.] denotes greatest integer function)

A. 0

B. 1

C. -1

D. Doesn't exist

**Answer:** A::B::D



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**26.** If  $t_n = \frac{1}{4}(n+2)(n+3)$  for  $n = 1, 2, 3, ,$  then

$$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \dots + \frac{1}{t_{2003}} =$$

A.  $e$

B.  $\frac{e}{2}$

C.  $2e$

D.  $3e$

**Answer:** A::B::D



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**27.** If  $f(x+y) = f(x) + f(y)$  for  $x, y \in R$  and  $f(1) = 1,$  then find the

value of  $\lim_{x \rightarrow 0} \frac{2^{f(\tan x)} - 2^{f(\sin x)}}{x^2 \cdot f(\sin x)}$

A.  $\log 2$

B.  $\log 4$

C.  $\frac{\log(2)}{2}$

D.  $\log 8$

**Answer: C**



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**28.**

**Evaluate**

$$\lim_{n \rightarrow \infty} n^{-n^2} [(n + 2^0)(n + 2^{-1})(n + 2^{-2}) \dots (n + 2^{-n+1})]^n.$$

A.  $e$

B.  $e^2$

C.  $e^3$

D.  $e^4$

**Answer: B**



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29. If  $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & \text{for } [x] \neq 0, \\ 0, & \text{for } [x]=0, \end{cases}$  where  $[x]$  denotes the greatest integer less than or equal to  $x$ , then  $\lim_{x \rightarrow 0} f(x)$  is (a) 1 (b) 0 (c) -1 (d) none of these

A. 1

B. 0

C. -1

D. does not exist

Answer: D



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30. Evaluate:  $\lim_{x \rightarrow 2} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$

A.  $\frac{3}{2}$

B.  $\frac{1}{2}$

C.  $\frac{7}{2}$

D. None of these

**Answer:**



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31. Let  $r^{th}$  term  $t_r$  of a series if given by  $t_c = \frac{r}{1 + r^2 + r^4}$ . Then

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n t_r \text{ is equal to :}$$

A. 2

B.  $\frac{1}{2}$

C. 1

D.  $\frac{1}{4}$

**Answer: B**



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**32.** The value of  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \cot^{-1} \left( \frac{r^3 - r + \frac{1}{r}}{2} \right)$  is

A. A.  $\pi$

B. B.  $\frac{\pi}{4}$

C. C.  $\frac{\pi}{2}$

D. D.  $\pi$

**Answer:** C



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**33.** Let  $x \tan \alpha + y \sin \alpha = \alpha$  and  $x \alpha \cos eca + y \cos \alpha = 1$  be two variable straight lines,  $\alpha$  being the parameter. Let  $P$  be the point of intersection of the lines. In the limiting position when  $a \rightarrow 0$ , the point  $P$  lies on the line :

A. (2, -1)

B.  $(2, 1)$

C.  $(-2, 1)$

D.  $(-2, -1)$

**Answer: A::B::D**



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**34.** The polynomial of least degree such that

$$\lim_{x \rightarrow 0} \left( 1 + \frac{x^2 + f(x)}{x^2} \right)^{1/x} = e^2 \text{ is}$$

A.  $x^2$

B.  $x^3 + 2x^2$

C.  $-x^2 + 2x^3$

D. None of these

**Answer: C**



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35. If  $n$  is a non zero integer and  $[*]$  denotes the greatest integer function then  $\lim_{x \rightarrow 0} \left[ n \frac{\sin x}{x} \right] + \lim_{x \rightarrow 0} \left[ n \frac{\tan x}{x} \right]$  equals

- A.  $n + 1$
- B.  $2n$
- C.  $n - 1$
- D.  $2n - 1$

Answer: D



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36. The value of  $\lim_{x \rightarrow a} [\sqrt{2-x}]$ , where  $a \in \left[0, \frac{1}{2}\right]$  and  $[.]$  denotes the greatest integer function is:

- A. 1

B. 2

C. 3

D. 4

**Answer: B**



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37. Evaluate:  $\lim_{\theta \rightarrow 0} \frac{\cos^2(1 - \cos^2(1 - \cos^2(1 - \cos^2(\theta))))}{\sin\left(\pi \frac{\sqrt{\theta+4}-2}{\theta}\right)}$

A. (a) 2

B. (b)  $\sqrt{2}$

C. (c)  $\frac{1}{2}$

D. (d)  $\frac{1}{\sqrt{2}}$

**Answer: B**



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**38.** The value of  $\lim_{n \rightarrow \infty} a_n$  when  $a_{n+1} = \sqrt{2 + a_n}$ ,  $n = 1, 2, 3, \dots$  is

A. 1

B. 2

C. 3

D. 4

**Answer:** B



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### Exercise More Than One Correct Option Type Questions

**1.** If  $\lim_{x \rightarrow \infty} 4x \left( \frac{\pi}{4} - \tan^{-1} \left( \frac{x+1}{x+2} \right) \right) = y^2 + 4y + 5$  then  $y$  can be equal to

A. 1

B. - 1

C. - 4

D. - 3

**Answer: B::D**



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2.  $\lim_{x \rightarrow 0} \frac{1 - \cos(x^2)}{x^3(4^x - 1)}$  is equal to:

A. (a)  $\frac{1}{2}\ln 2$

B. (b)  $\ln 2$

C. (c)  $\ln 4$

D. (d)  $1 - \frac{1}{2}\ln\left(\frac{e^2}{4}\right)$

**Answer: null**



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3. If  $f(x) = e^{[\cot x]}$ , where  $[y]$  represents the greatest integer less than or equal to  $y$  then

A.  $\lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = 1$

B.  $\lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \frac{1}{e}$

C.  $\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \frac{1}{e}$

D.  $\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = 1$

**Answer: B::D**



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4.  $\lim_{x \rightarrow 0} \left[ m \frac{\sin x}{x} \right]$  is equal to (where  $m \in I$  and  $[.]$  denotes greatest integer function)

a.  $m$  if  $m < 0$

b.  $m - 1$  if  $m > 0$

c.  $m - 1$ , if  $m < 0$

d.  $m$  if  $m > 0$

A.  $m$  if  $m < 0$

B.  $m - 1$  if  $m > 0$

C.  $m - 1$ , if  $m < 0$

D.  $m$  if  $m > 0$

**Answer: A::B**



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5. If  $(\lim)_{x \rightarrow 0} (1 + ax + bx^2)^{\frac{2}{x}} = e^3$ , then find the value of  $a$  and  $b$ .

A.  $a = 3, b = 0$

B.  $a = \frac{3}{2}, b = 1$

C.  $a = \frac{3}{2}, b = 4$

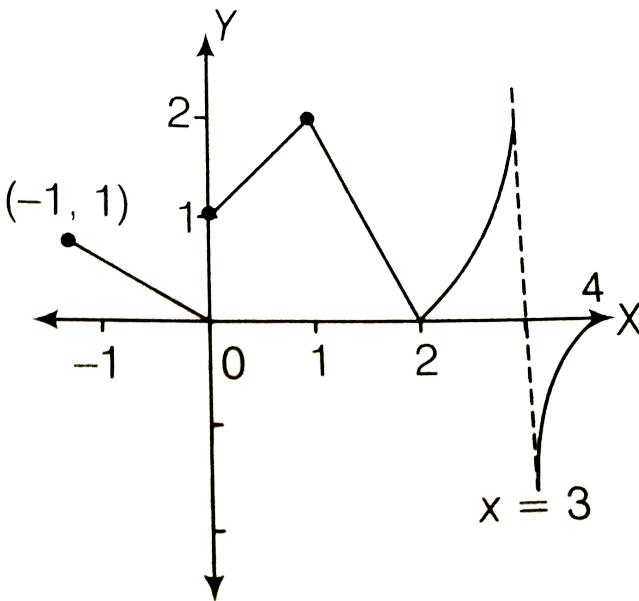
D.  $a = 2, b = 3$

**Answer: B::C**



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6. The graph of the function  $y = f(x)$  is shown in the adjacent figure  
then correct statement is



- A.  $\lim_{x \rightarrow 0^+} f(x) = 1$
- B.  $\lim_{x \rightarrow 1^-} f(x) = 2$
- C.  $\lim_{x \rightarrow 3} f(x) = \text{does not exist}$

D.  $\lim_{x \rightarrow 4} f(x) = 0$

**Answer: A::B::C::D**



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7. Find  $\lim_{x \rightarrow 0^+} \frac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$



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8. If  $l = \lim_{x \rightarrow \infty} \left( \frac{x+1}{x-1} \right)^x$ , the value of  $\{l\}$  and  $[l]$  are

A. 7

B.  $7 - e^2$

C.  $-7$

D.  $e^2 - 7$

**Answer: A::D**



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9. If  $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & \text{for } [x] \neq 0, \\ 0, & \text{for } [x]=0, \end{cases}$  where  $[x]$  denotes the greatest integer less than or equal to  $x$ , then  $\lim_{x \rightarrow 0} f(x)$  is (a) 1 (b) 0 (c) -1 (d) none of these

A.  $\lim_{x \rightarrow 0^-} f(x) = \sin 1$

B.  $\lim_{x \rightarrow 0^+} f(x) = 0$

C. limit does not exist at  $x = 0$

D. limit exists at  $x = 0$

**Answer: A::B::C::D**



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10.  $\lim_{x \rightarrow c} f(x)$  does not exist when

where  $[.]$  and  $\{.\}$  denotes greatest integer and fractional part of  $x$

A.  $f(x) = [x] - [2x - 1]$ ,  $c = 3$

B.  $f(x) = [x] - x$ ,  $c = 1$

C.  $f(x) = \{x\}^2 - \{-x\}^2$ ,  $c = 0$

D.  $f(x) = \frac{\tan(sgnx)}{(sgnx)}$ ,  $c = 0$

**Answer: B::C**



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11. Identify the correct statement.

a.  $\lim_{x \rightarrow \infty} \left[ \sum_{r=1}^n \frac{1}{2^r} \right] = 1$

b. If  $f(x) = (x - 1)\{x\}$ , where  $[.]$  and  $\{.\}$  denotes greatest integer function and fractional part of  $x$  respectively, the limit of  $f(x)$  does not exist at  $x = 1$

c.  $\lim_{x \rightarrow 0^+} \left[ \frac{\tan x}{x} \right] = 1$

d.  $\left[ \lim_{x \rightarrow 0^+} \frac{\tan x}{x} \right] = 1$

A.  $\lim_{x \rightarrow \infty} \left[ \sum_{r=1}^n \frac{1}{2^r} \right] = 1$

B. If  $f(x) = (x - 1)\{x\}$ , where  $[.]$  and  $\{.\}$  denotes greatest

integer function and fractional part of  $x$  respectively, the limit of

$f(x)$  does not exist at  $x = 1$

C.  $\lim_{x \rightarrow 0^+} \left[ \frac{\tan x}{x} \right] = 1$

D.  $\left[ \lim_{x \rightarrow 0^+} \frac{\tan x}{x} \right] = 1$

**Answer: C::D**



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12. For  $a > 0$ , let  $l = \lim_{x \rightarrow \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$  and

$m = \lim_{x \rightarrow -\infty} \left( \sqrt{x^2 + ax} \right) - \left( \sqrt{x^2 - ax} \right)$  then solve it



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13. Consider the function  $f(x) = \left(\frac{ax+1}{bx+2}\right)^x$ , where  $a, b > 0$ , the

$\lim_{x \rightarrow \infty} f(x)$  is

A. exists for all values of a and b

B. zero for  $a < b$

C. non existent for  $a > b$

D.  $e^{-(1/a)}$  or  $e^{-(1/b)}$  if  $a = b$

**Answer: B::C::D**



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14. If  $f(x) = \frac{x \cdot 2^x - x}{1 - \cos x}$  and  $g(x) = 2^x \cdot \sin\left(\frac{\log 2}{2^x}\right)$  then

a.  $\lim_{x \rightarrow 0} f(x) = \log 2$

b.  $\lim_{x \rightarrow \infty} g(x) = \log 4$

c.  $\lim_{x \rightarrow 0} f(x) = \log 4$

d.  $\lim_{x \rightarrow \infty} g(x) = \log 2$

A.  $\lim_{x \rightarrow 0} f(x) = \log 2$

B.  $\lim_{x \rightarrow \infty} g(x) = \log 4$

C.  $\lim_{x \rightarrow 0} f(x) = \log 4$

D.  $\lim_{x \rightarrow \infty} g(x) = \log 2$

**Answer:** C::D



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15. If  $\lim_{x \rightarrow 3} \frac{x^3 + cx^2 + 5x + 12}{x^2 - 7x + 12} = l$  (finite real number), then

A.  $l = 4$

B.  $c = -6$

C.  $c = 4$

D.  $x \in R$

**Answer: A::B**



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### Exercise Passage Based Questions

1. If  $\lim_{x \rightarrow a} f(x) = 1$  and  $\lim_{x \rightarrow a} g(x) = \infty$  then

$$\lim_{x \rightarrow a} \{f(x)\}^{g(x)} = e^{\lim_{x \rightarrow a} (f(x) - 1)g(x)}$$

$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}$  is equal to

A. (a)  $1/e$

B. (b)  $-1/e$

C. (c)  $e$

D. (d)  $-e$

**Answer: A**



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2. If  $\lim_{x \rightarrow a} f(x) = 1$  and  $\lim_{x \rightarrow a} g(x) = \infty$  then  
 $\lim_{x \rightarrow a} \{f(x)\}^{g(x)} = e^{\lim_{x \rightarrow a} (f(x) - 1)g(x)}$   $\lim_{x \rightarrow 0} \left( \frac{x - 1 + \cos x}{x} \right)^{\frac{1}{x}}$  is equal to

A. (a) $e^{1/2}$

B. (b) $e^{-1/2}$

C. (c) $e^1$

D. (d) $\frac{1}{e}$

**Answer: B**



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3. If  $\lim_{x \rightarrow a} f(x) = 1$  and  $\lim_{x \rightarrow a} g(x) = \infty$  then  
 $\lim_{x \rightarrow a} \{f(x)\}^{g(x)} = e^{\lim_{x \rightarrow a} (f(x) - 1)g(x)}$   $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x}{3} \right)^{\frac{2}{x}}$  is equal to

a.  $a^{2/3} + b^{2/3} + c^{2/3}$

b.  $abc$

c.  $(abc)^{2/3}$

d. 1

A.  $a^{2/3} + b^{2/3} + c^{2/3}$

B.  $abc$

C.  $(abc)^{2/3}$

D. 1

**Answer: C**



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4. Find  $\frac{dy}{dx}$  if  $2x - 3y = \tan x$



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5. Find  $|A|$  If  $A = \begin{vmatrix} x^2 & 2x \\ x^4 & 5x \end{vmatrix}$



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6. Let  $f(x) = \lim_{n \rightarrow \infty} \left( \cos \sqrt{\frac{x}{n}} \right)^n$ ,  $g(x) = \lim_{n \rightarrow \infty} (1 + x + x\sqrt[n]{e})^n$

Now consider the function  $y = h(x)$  where

$$h(x) = \tan^{-1}(g^{-1}f^{-1}(x)).$$

Range of the function  $y = h(x)$  is



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7. How many 4 digits numbers can be formed from the digits 1 to 9 if not digit can be repeated.



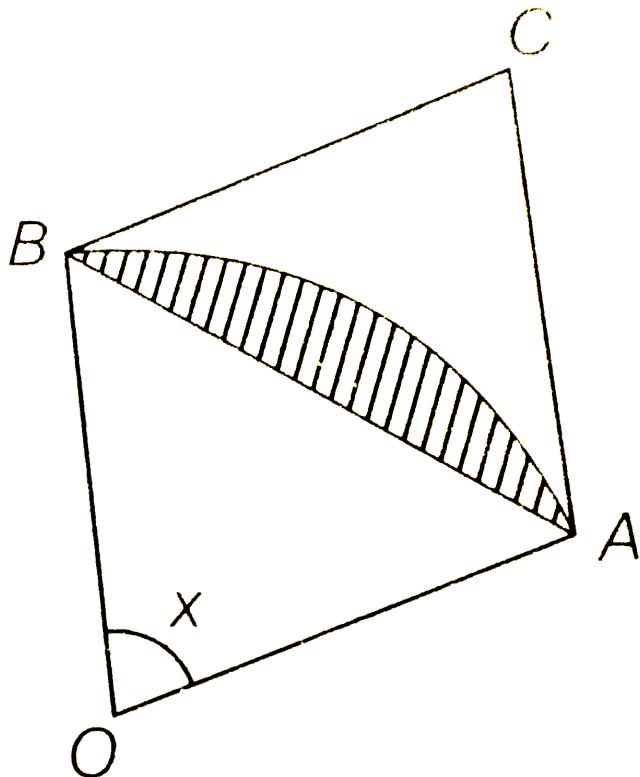
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8. Find  $\frac{dy}{dx}$  if  $2y = e^y - 2x$



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9. A circle arc of radius 1 subtends an angle of  $x$  radians as shown in figure. The centre of the circle is  $O$  and the point  $C$  is the intersection of two tangent lines at  $A$  and  $B$ . Let  $T(x)$  be the area of  $\triangle ABC$  and  $S(x)$  be the area of shaded region.



$$\lim_{x \rightarrow 0} \frac{T(x)}{x^3} \text{ is}$$

A.  $\frac{1}{2}$

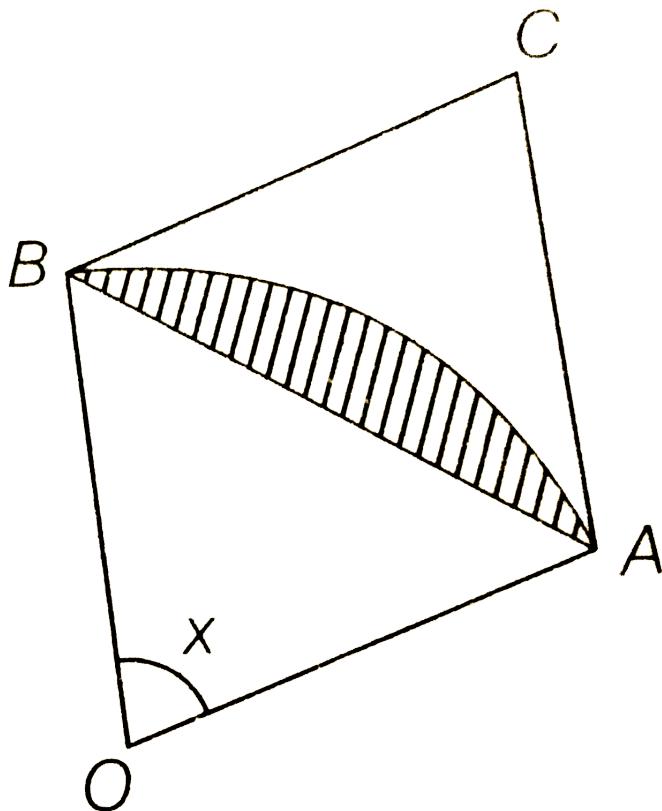
B.  $\frac{1}{3}$

C.  $\frac{1}{4}$

D.  $\frac{1}{8}$

**Answer: D**

10. A circle arc of radius 1 subtends an angle of  $x$  radians as shown in figure. The centre of the circle is  $O$  and the point  $C$  is the intersection of two tangent lines at  $A$  and  $B$ . Let  $T(x)$  be the area of  $\triangle ABC$  and  $S(x)$  be the area of shaded region.



$$\lim_{x \rightarrow 0} \frac{S(x)}{x}$$
 is

A. 0

B.  $\frac{1}{2}$

C. 1

D. None of these

**Answer: A**



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**Exercise Matching Type Questions**

**1. Match the statements of Column I with values of Column II.**

**Column I**

**Column II**

(A)  $\lim_{x \rightarrow \frac{\pi}{2}^+} \tan^{-1}(\tan x)$

(p) 0

(B)  $\lim_{n \rightarrow \infty} \left[ \sum_{r=1}^n \frac{1}{2^r} \right] ([\cdot])$  denotes the greatest integer function

(q) Doesn't exist

(C)  $\lim_{x \rightarrow \infty} \sec^{-1} \left( \frac{x}{x+1} \right)$

(r)  $-\frac{\pi}{2}$

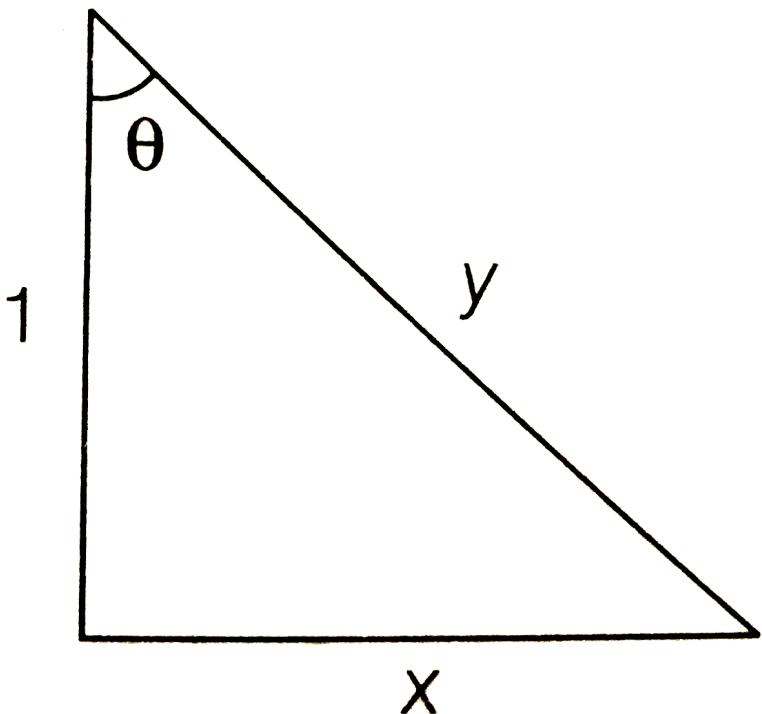
(D)  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{(1 - \sin x)^{2/3}}$

(s)  $\frac{\pi}{2}$



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- 2.** A right angled triangle has legs 1 and  $x$ . The hypotenuse is  $y$  and angle opposite to the side  $x$  is  $\theta$ . Shown as



	<b>Column I</b>		<b>Column II</b>
(A)	$\lim_{\theta \rightarrow \frac{\pi}{2}} (\sqrt{y} - \sqrt{x})$	(p)	0
(B)	$\lim_{\theta \rightarrow \frac{\pi}{2}} (y - x)$	(q)	$\frac{1}{2}$
(C)	$\lim_{\theta \rightarrow \frac{\pi}{2}} (y^2 - x^2)$	(r)	1
(D)	$\lim_{\theta \rightarrow \frac{\pi}{2}} (y^3 - x^3)$	(s)	$\infty$



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3. Find  $\frac{dy}{dx}$  if  $2y = x - \cos 2x$



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### Matching Type Questions

1. Find  $\frac{dy}{dx}$  if  $1 - y = x$



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### Exercise Single Integer Answer Type Questions

1. Let  $L = \lim_{x \rightarrow \infty} \left( x \log x + 2x \cdot \log \sin \left( \frac{1}{\sqrt{x}} \right) \right)$ , then value of  $\left( -\frac{2}{L} \right)$  is .....



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2. If  ${}^{2n+1}P_{n-1}:{ }^{2n-1}P_n = 7:10$ , then  $.^nP_3$  equals



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3. Let  $f(x) = x^4 - x^3$  then find  $f'(2)$



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4. If the arithmetic mean of the product of all pairs of positive integers

whose sum is  $n$  is  $A_n$  then  $\lim_{n \rightarrow \infty} \frac{n^2}{A_n}$  equals to



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5. The value of  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$  is equal to



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6. The value of  $\lim_{x \rightarrow \frac{\pi}{2}} \sqrt{\frac{\tan x - \sin\{\tan^{-1}(\tan x)\}}{\tan x + \cos^2(\tan x)}}$  is .....



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7. Express in the form of complex number if  $z = i^5$



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8. If the two  $AB: \left( \int_0^{2t} \left( \frac{\sin x}{x} + 1 \right) dx \right) x + y = 3t$  and  $AC: 2tx + y = 0$  intersect at a point A the x-coordinate of a point A as  $t \rightarrow 0$ , is equal to  $\frac{p}{q}$  ( $p$  and  $q$  are in their lowest form) the  $(p + q)$  is .....  
.....



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9. Find  $\frac{dy}{dx}$  if  $e^x = \log y - \sin x$



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10. If  $L = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cos \tan^{-1}(\tan x)}{x - \pi/2}$  then  $\cos(2\pi L)$  is



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11. Number of solutions of the equation  
 $\cot(\theta) + \cot\left(\theta + \frac{\pi}{3}\right) + \cot\left(\theta - \frac{\pi}{3}\right) + \cot(3\theta) = 0$ , where  
 $\theta \in \left(0, \frac{\pi}{2}\right)$



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12. If C satisfies the equation  $\lim_{x \rightarrow \infty} \left(\frac{x+c}{x-c}\right)^x = 4$  then  $\left|\frac{e^c}{2}\right|$  is .....



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13. If  $f(x) = \lim_{t \rightarrow 0} \left[ \frac{2x}{\pi} \cdot \tan^{-1} \left( \frac{x}{t^2} \right) \right]$ , then  $f(1)$  is .....



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14. Differentiate  $x^3 - 5 \sin x$  w.r.t  $x$



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15. If  $l = \lim_{x \rightarrow 1^+} 2^{-2^{\frac{1}{1-x}}}$  and  $m = \lim_{x \rightarrow 1^+} \frac{x \sin(x - [x])}{x - 1}$  (where  $[.]$  denotes greatest integer function). Then  $(l + m)$  is .....



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16. The value of  $\lim_{x \rightarrow 0} \left[ \frac{\sin x \cdot \tan x}{x^2} \right]$  is .....

(where  $[.]$  denotes greatest integer function).



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17.  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{1 \times 3 \times 5 \times 7 \times 9 \times \dots \times (2r+1)}$  is equal to

A.  $\frac{1}{2} \left[ 1 - \frac{1}{1 \times 3 \times 5 \dots (2n+1)} \right]$

B.  $\frac{1}{4} \left[ 1 - \frac{1}{1 \times 3 \times 5 \dots (2n-1)} \right]$

C.  $\frac{1}{4} \left[ 1 + \frac{1}{1 \times 3 \times 5 \dots (2n-1)} \right]$

D. None of these

**Answer: 4**



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18. Find  $\frac{dy}{dx}$  if  $y = \sin^4 x$



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19. If  $f(x+y+z) = f(x) + f(y) + f(z)$  with  $f(1) = 1$  and  $f(2) = 2$

and  $x, y, z \in R$  the value of  $\lim_{x \rightarrow \infty} \sum_{r=1}^n \frac{(4r)f(3r)}{n^3}$  is .....



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20. If  $\lim_{n \rightarrow \infty} \frac{n \cdot 3^n}{n(x-2)^n + n \cdot 3^{n+1} - 3^n} = \frac{1}{3}$ , then the range of  $x$  is  
(where  $n \in N$ )



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21. Let  $p(x)$  be a polynomial of degree 4 having extremum at  $x = 1, 2$   
and  $\lim_{x \rightarrow 0} \left(1 + \frac{p(x)}{x^2}\right) = 2$ . Then find the value of  $p(2)$ .



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22. If  $\alpha$  is the number of solution of  $|x| = \log(x - [x])$ , (where  $[.]$   
denotes greatest integer function) and

$$\lim_{x \rightarrow \alpha} \frac{x e^{ax} - b \sin x}{x^3}$$
 is finite, the value of  $(a - b)$  is



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**23.** Suppose  $x_1 = \tan^{-1} 2 > x_2 > x_3 > \dots$  are the real numbers satisfying  $\sin(x_{n+1} - x_n) + 2^{-(n+1)} \cdot \sin x_n \cdot \sin x_{n+1}$  for all  $n > 1$  and  $l = \lim_{x \rightarrow \infty} x_n$ , the value of  $[4l]$  is...where  $[t]$  denotes greatest integer function.



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### Exercise Questions Asked In Previous 13 Years Exam

**1.** Let  $\alpha, \beta \in R$  such that  $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$ . Then  $6(\alpha + \beta)$  equals



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**2.** If  $y = \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$ . Find  $\frac{dy}{dx}$



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3. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous odd function, which vanishes exactly at one point and  $f(1) = \frac{1}{2}$ . Suppose that  $F(x) = \int_{-1}^x f(t) dt$  or all  $x \in [-1, 2]$  and  $G(x) = \int_{-1}^x t|f(f(t))| dt$  or all  $x \in [-1, 2]$ . If  $(\lim)_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$ , Then the value of  $f\left(\frac{1}{2}\right)$  is

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4. The largest value of non negative integer for which

$$\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

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5. Let  $L = (\lim)_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$ . If  $L \in \text{ite}$ , then  
 a = 2 (b)  $a = 1 L = \frac{1}{64}$  (d)  $L = \frac{1}{32}$

A.  $a = 2$

B.  $a = 1$

C.  $L = \frac{1}{64}$

D.  $L = \frac{1}{32}$

**Answer: C**



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6.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$  equals

A.  $\frac{1}{4}$

B.  $\frac{1}{24}$

C.  $\frac{1}{16}$

D.  $\frac{1}{8}$

**Answer: C**



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7. Let  $p = \lim_{x \rightarrow 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ . Then  $\log_e p$  is equal to

A. 2

B. 1

C.  $\frac{1}{2}$

D.  $\frac{1}{4}$

**Answer: B**



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8.  $\lim_{n \rightarrow \infty} \left( \frac{(n+1)(n+2)(n+3) \dots \dots 2n}{n^{2n}} \right)^{\frac{1}{n}}$  is equal to

A.  $\frac{18}{e^4}$

B.  $\frac{27}{e^2}$

C.  $\frac{9}{e^2}$

D.  $3 \log 3 - 2$

**Answer:** D



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9.  $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$  is equal to

A.  $\frac{\pi}{2}$

B. 1

C.  $-\pi$

D.  $\pi$

**Answer:** D



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10.  $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$  is equal to

A. 4

B. 3

C. 2

D.  $\frac{1}{2}$

**Answer: B**



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11. If  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$ , then

A.  $a = 1, b = 4$

B.  $a = 1, b = -4$

C.  $a = 2, b = -3$

D.  $a = 2, b = 3$

**Answer: D**



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12. Let  $\alpha(a)$  and  $\beta(a)$  be the roots of the equation  $\left((1+a)^{\frac{1}{3}} - 1\right)x^2 + \left((1+a)^{\frac{1}{2}} - 1\right)x + \left((1+a)^{\frac{1}{6}} - 1\right) = 0$  where  $a > -1$  then  $\lim_{a \rightarrow 0^+} \alpha(a)$  and  $\lim_{a \rightarrow 0^+} \beta(a)$

A.  $-\frac{5}{2}$  and 1

B.  $-\frac{1}{2}$  and -1

C.  $-\frac{7}{2}$  and 2

D.  $-\frac{9}{2}$  and 3

**Answer: C**



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13.

If

$$\left(\lim\right)_{x \rightarrow 0} [1 + x \ln(1 + b^2)]^{\frac{1}{x}} = 2b \sin^2 \theta, b > 0, \text{ if } \theta \in (-\pi, \pi],$$

then the value of  $\theta$  is  $\pm \frac{\pi}{4}$  (b)  $\pm \frac{\pi}{3}$  (c)  $\pm \frac{\pi}{6}$  (d)  $\pm \frac{\pi}{2}$

A.  $\pm \frac{\pi}{4}$

B.  $\pm \frac{\pi}{3}$

C.  $\pm \frac{\pi}{6}$

D.  $\pm \frac{\pi}{2}$

**Answer: D**



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14. The value of  $\lim_{x \rightarrow 0} \left( (\sin x)^{\frac{1}{x}} + \left( \frac{1}{x} \right)^{\sin x} \right)$ , where  $x > 0$ , is (a) 0

(b) -1 (c) 1 (d) 2

A. 0

B.  $-1$

C.  $1$

D.  $2$

**Answer: D**



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15.  $\lim_{h \rightarrow 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$  given that

$f'(2) = 6$  and  $f'(1) = 4$  then (a) limit does not exist (b) is equal to  $-\frac{3}{2}$  (c) is equal to  $\frac{3}{2}$  (d) is equal to 3

A. does not exist

B. is equal to  $-3/2$

C. is equal to  $3/2$

D. is equal to 3

**Answer: C**



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16.  $\lim_{x \rightarrow 0} \frac{\sin(nx)((a-n)nx - \tan x)}{x^2} = 0$ , when n is a non-zero positive integer, then a is equal to

A. 0

B.  $\frac{n+1}{n}$

C. n

D.  $n + \frac{1}{n}$

Answer: C



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17. The integer n for which  $(\lim)_{x \rightarrow 0} \left( (\cos x - 1) \frac{\cos x - e^{\hat{x}}}{x^n} \right)$  is finite nonzero number is \_\_\_\_\_

A. 1

B. 2

C. 3

D. 4

**Answer: C**



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**18.** Let  $f: R \rightarrow R$  be such that  $f(1) = 3$  and  $f'(1) = 6$ . Then

$$\lim_{x \rightarrow 0} \left( \frac{f(1+x)}{f(1)} \right)^{1/x} =$$

A. 1

B.  $e^{\frac{1}{2}}$

C.  $e^2$

D. None of these

**Answer: C**



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19.  $\lim_{x \rightarrow \infty} \left( \frac{x - 3}{x + 2} \right)$  is equal to :

A.  $e$

B.  $e^{-1}$

C.  $e^{-5}$

D.  $e^5$

**Answer: C**



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**Exercise For Session 1**

1. If  $f(a) = 2$      $f'(a) = 1$      $g(a) = -1$      $g'(a) = 2$     then

$$\lim_{x \rightarrow a} \frac{g(x)f(a) - g(a)f(x)}{x - a} \text{ is}$$

A. -5

B. 3

C. -3

D. 5

**Answer: C**



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2. The value of  $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1 + x)}{x^2}$  is

A. 1

B.  $\frac{1}{4}$

C.  $\frac{1}{2}$

D. None of these

**Answer: C**



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3. Evaluate  $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$

A.  $\frac{3}{2}$

B.  $-\frac{3}{2}$

C.  $\frac{1}{2}$

D.  $-\frac{1}{2}$

**Answer: C**



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4. The value of  $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{\cot x - \cot a}$  is

A.  $-\sin^3 a$

B.  $\cos^2 a$

C.  $\sin^3 a$

D.  $\cot a$

**Answer: A**



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5. The value of  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \cot x \right)$  is

A. 0

B. 1

C.  $\frac{1}{4}$

D. None of these

**Answer: C**



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6. Evaluate  $\lim_{x \rightarrow \infty} [\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1}]$ .

A.  $\frac{1}{2}$

B.  $-\frac{1}{2}$

C. does'nt exist

D. None of these

**Answer: A**



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7. The value of  $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{(n^2 + 1)^2}$

A.  $\frac{1}{4}$

B.  $\frac{1}{2}$

C.  $\frac{1}{2\sqrt{2}}$

D. None of these

**Answer: A**



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8. The value of  $\lim_{n \rightarrow \infty} \frac{1. n + 2. (n - 1) + 3. (n - 2) + \dots + n.1}{1^2 + 2^2 + \dots + n^2}$

A. 1

B. -1

C.  $\frac{1}{\sqrt{2}}$

D.  $\frac{1}{2}$

**Answer:**



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9. The value of  $\lim_{n \rightarrow \infty} \frac{a^n + b^n}{a^n - b^n}$ , (where  $a > b$ ) is

A. 1

B. -1

C.  $\frac{1}{2}$

D.  $\frac{1}{\sqrt{2}}$

**Answer:**



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### Exercise For Session 2

1. If  $(\lim)_{x \rightarrow 0} (x^{-3} \sin 3x + ax^{-2} + b)$  exists and is equal to 0, then

$$a = -3 \text{ and } b = \frac{9}{2} \quad a = 3 \text{ and } b = \frac{9}{2} \quad a = -3 \text{ and } b = -\frac{9}{2}$$

$$a = 3 \text{ and } b = -\frac{9}{2}$$

A.  $a = -3, b = \frac{9}{2}$

B.  $a = 3, b = \frac{9}{2}$

C.  $a = -3, b = \frac{-9}{2}$

D. None of these

**Answer: D**



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2. The value of  $\lim_{x \rightarrow a} \frac{x \sin a - a \sin x}{x - a}$  is

A.  $a \sin a - \cos a$

B.  $\sin a - a \cos a$

C.  $\cos a + a \sin a$

D.  $\sin a + a \cos a$

**Answer: B**



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**3. Evaluate the following limits:**

$$\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

A.  $\frac{1}{4}$

B.  $\frac{1}{2}$

C. 2

D. Doesn't exist

**Answer: A**



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**4. evaluate**  $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - \cos \theta - \sin \theta}{(4\theta - \pi)^2}$

A.  $\frac{1}{16\sqrt{2}}$

B.  $\frac{1}{16}$

C.  $\frac{1}{8\sqrt{2}}$

D.  $\frac{1}{2\sqrt{2}}$

**Answer: D**



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5. The value of  $\lim_{x \rightarrow \pi/4} \frac{(\cos x + \sin x)^3 - 2\sqrt{2}}{1 - \sin 2x}$  is

A.  $\frac{\sqrt{3}}{2}$

B.  $-\frac{3}{\sqrt{2}}$

C.  $\frac{1}{2}$

D.  $-\frac{1}{\sqrt{2}}$

**Answer: B**



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**Exercise For Session 3**

1. Evaluate  $\lim_{x \rightarrow 0} \frac{e^x - e^{x \cos x}}{x + \sin x}$ .

- A. 0
- B. 1
- C. -1
- D. None of these

**Answer: A**



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2. The value of  $\lim_{x \rightarrow y} \frac{x^y - y^x}{x^x - y^y}$  is:

- A. (a)  $\frac{1 - \log x}{1 + \log x}$
- B. (b)  $\frac{1 - \log y}{1 + \log y}$
- C. (c)  $\frac{\log x - \log y}{\log x + \log y}$
- D. (d) None of these

**Answer: B**



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3. The value of  $\lim_{x \rightarrow 0} \frac{p^x - q^x}{r^x - s^x}$  is

A.  $\frac{1 - \log p}{1 + \log p}$

B.  $\frac{\log p - \log q}{\log r - \log s}$

C.  $\frac{\log p \cdot \log q}{\log r \cdot \log s}$

D. None of these

**Answer: B**



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4. The value of  $\lim_{x \rightarrow \infty} (x + 2)\tan^{-1}(x + 2) - (x \tan^{-1} x)$  is

A.  $\frac{\pi}{2}$

B. Doesn't exist

C.  $\frac{\pi}{4}$

D. None of these

**Answer: D**



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5. The value of  $\lim_{x \rightarrow 4} \frac{(\cos \alpha)^x - (\sin \alpha)^x - \cos 2\alpha}{x - 4}$ ,  $\alpha \in \left(0, \frac{\pi}{2}\right)$  is

A.  $\log(\cos \alpha) + (\sin \alpha)^4 \log(\sin \alpha)$

B.  $(\cos^4 \alpha) \log(\cos \alpha) - (\sin \alpha)^4 \log(\sin \alpha)$

C.  $(\cos^4 \alpha) \log(\cos \alpha)$

D. None of these

**Answer: B**



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6. The value of  $\lim_{x \rightarrow 0} \left( \frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right)^{a/x}$  is

- a.  $(n!)^{a/n}$
- b.  $n!$
- c.  $a^n!$
- d. Doesn't exist

A.  $(n!)^{a/n}$

B.  $n!$

C.  $a^n!$

D. Doesn't exist

**Answer:**



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7. If  $\lim_{x \rightarrow 0} (1 + ax + bx^2)^{2/x} = e^3$  then

A.  $a = \frac{3}{2}, b \in R$

B.  $a = \frac{1}{2}, b\varepsilon R$

C.  $a\varepsilon R, b\varepsilon R$

D. None of these

**Answer:**



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8. If  $\alpha$  and  $\beta$  are roots of  $ax^2 + bx + c = 0$  the value for

$$\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{2/x - \alpha}$$
 is

A.  $e^{2a(\alpha - \beta)}$

B.  $e^{a(\alpha - \beta)}$

C.  $e^{\frac{2a}{3}(\alpha - \beta)}$

D. None of these

**Answer:**



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9. The value of  $\lim_{n \rightarrow \infty} \left( (1.5)^n + \left[ (1 + 0.0001)^{10000} \right]^n \right)^{\frac{1}{n}}$ ,

where [.] denotes the greatest integer function is:

A. 1

B.  $\frac{1}{2}$

C. doesn't exist

D. 2

**Answer:**



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10. The value of  $\lim_{x \rightarrow 0} |x|^{\lceil \cos x \rceil}$ , [.] denotes greatest integer function is

A. 0

B. Doesn't exist

C. 1

D. None of these

**Answer:**



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### Exercise For Session 4

1. Evaluate  $\lim_{x \rightarrow 0^+} (\sin x)^x$



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2. Evaluate  $\lim_{x \rightarrow 0^+} (\sin x)^{\tan x}$



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3. Evaluate  $\lim_{n \rightarrow \infty} \left( \frac{e^n}{\pi} \right)^{1/n}$



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4. The value of  $\lim_{x \rightarrow 0} \frac{e - (1 + x)^{1/x}}{\tan x}$  is

A.  $e$

B.  $\frac{11e}{24}$

C.  $\frac{e}{2}$

D. None of these

Answer: D



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Exercise For Session 5

1. The value of  $\lim_{x \rightarrow 1} (\{1 - x + [x - 1] + [1 - x]\})$  (where  $\{.\}$  denotes the greatest integral function) is

- A.  $-1$
- B. Doesn't exist
- C.  $1$
- D. None of these

**Answer:**



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2. The value of  $\lim_{x \rightarrow 0} \frac{\sin[x]}{[x]}$  (where  $[.]$  denotes the greatest integer function) is

- A.  $1$
- B.  $\sin 1$
- C. doesn't exist

D. None of these

**Answer:**



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3. The value of  $\lim_{x \rightarrow 0} \sin^{-1}\{x\}$  (where  $\{.\}$  denotes fractional part of  $x$ ) is

A. 0

B.  $\frac{\pi}{2}$

C. doesn't exist

D. None of these

**Answer:**



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4. If  $[.]$  denotes the greatest integer function then  $\lim_{x \rightarrow 0} \left[ \frac{x^2}{\tan x \cdot \sin x} \right]$

=

A. 0

B. 1

C. does'nt exist

D. None of these

**Answer:**



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### Exercise For Session 6

1. The value of

$$\lim_{x \rightarrow \infty} \left( \frac{1}{n^3} \right) ([1^2 x + 1^2] + [2^2 x + 2^2] + \dots + [n^2 x + n^2])$$

where  $[.]$  denotes the greatest integer function.

A.  $\frac{x}{3}$

B.  $x + \frac{1}{3}$

C.  $\frac{x}{3} + \frac{1}{3}$

D. None of these

**Answer:**



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2.

The value of

$$\lim_{x \rightarrow 0} \left\{ \lim_{n \rightarrow \infty} \frac{[1^2(\sin x)^x] + [2^2(\sin x)^x] + \dots + [n^2(\sin x)^x]}{n^3} \right\}$$

is (where [.] denotes the greatest integer function)

A. A.  $\frac{x}{3} + \frac{\sin x}{3}$

B. B.  $\frac{x}{3} + (\sin x)^x$

C. C.  $\frac{1}{3}$

D. D. 0

**Answer: option 4**



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3. The value of  $\lim_{x \rightarrow 1^+} \frac{\int_1^x |t - 1| dt}{\sin(x - 1)}$  is:

A. (a) 0

B. (b) 1

C. (c) doesn't exist

D. (d) None of these

**Answer:**



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4. The value of  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \log\left(1 + \frac{k}{n}\right)^{\frac{1}{n}}$ , is

A.  $\log_e \left( \frac{e}{4} \right)$

B.  $\log_e \left( \frac{4}{e} \right)$

C.  $\log_e 4$

D. None of these

**Answer:**



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5. Evaluate:  $(\lim)_{n \rightarrow \infty} \left[ \frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \dots + \frac{1}{nb} \right]$

a.  $\log \left( \frac{a}{b} \right)$

b.  $\log \left( \frac{b}{a} \right)$

c.  $\log(ab)$

d. None of these

A.  $\log \left( \frac{a}{b} \right)$

B.  $\log \left( \frac{b}{a} \right)$

C.  $\log(ab)$

D. None of these

**Answer:**



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