MATHS

BOOKS - ARIHANT MATHS

LOGARITHM AND THEIR PROPERTIES

Examples

1. Find the value of the following

$\log_9 27$

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2. $\log_{3\sqrt{2}} 324$

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3. Find the value of the following

$\log_e\left(e^3\right)$

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4. Find the value of the following

$\log_{\left(5+2\sqrt{6}\right)}\left(5-2\sqrt{6}\right)$

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5. Find the value of the following

$\log_{0.2} 0.008$

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6. Find the value of the following

$2^{\log_4 5}$

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7. $(0.4)^{-\log 2.5\left\{\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots.\right\}}$

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8. Find the value of the following $(0.05)^{\log_{\sqrt{20}}(0.\bar{3})}$

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9. Find the value of the following $\log_{\cot 30^\circ} \tan 45^\circ$

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10. Find the value of the following

$\log_{\left(\sec^2 60^\circ - \tan^2 60^\circ\right)} \cos 60^\circ$

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11. Find the value of the following

$\log_{\left(\sin^2 30^\circ + \cos^2 30^\circ\right)} 1$

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12. Find the value of the following

$\log_{30} 1$

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13. If log2=0.301 and log3=0.477, find the number of integers in

(ii) 6^{20}

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14. Find the numbers of zeroes between the decimal point and first significant digit of $(0.036)^{16}$, where $\log 2 = 0.301$ and $\log 3 = 0.477$.

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15. Solve the equation $3 \cdot x^{\log_5 2} + 2^{\log_5 x} = 64$.

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16. If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to

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17. Prove that $a\sqrt{\log_a b} - b\sqrt{\log_b a} = 0$

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18. Prove that $\frac{\log_2 8}{\log_3 9} = \frac{3}{2}$

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19. Solve for a,λ if $\log_\lambda a \cdot \log_5 \lambda \cdot \log_\lambda 25 = 2$.

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20. Arrange in ascending order

$\log_2(x), \log_3(x), \log_e(x), \log_{10}(x)$, if

II.$0 < x < 1$.

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21. Arrange in ascending order

$\log_2(x), \log_3(x), \log_e(x), \log_{10}(x)$, if

II.$0 < x < 1$.

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22. If log 11=1.0414, prove that $10^{11} > 11^{10}$.

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23. If $\log_2(x-2) < \log_4(x-2)$,, find the interval in which x lies.

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24. Prove that $\log_n(n+1) > \log_{n+1}(n+2)$ for any natural number $n > 1$.

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25. For $x > 1$, the minimum value of $2\log_{10}(x) - \log_x(0.01)$ is

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26. Which is smaller 2 or $(\log_\pi 2 + \log_2 \pi)$

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27. The expression $\log_2 5 - \sum_{k=1}^{4} \log_2 \left(\sin \left(\frac{k\pi}{5} \right) \right)$ reduces to $\frac{p}{q}$, where p and q are co-prime, the value of $p^2 + q^2$ is

A. 13

B. 17

C. 26

D. 29

Answer: B

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28. If $3 \leq a \leq 2015, 3 \leq b \leq 2015$ such that $\log_a b + 6 \log_b a = 5$, the number of ordered pairs (a,b) of integers is

A. 48

B. 50

C. 52

D. 54

Answer: C

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29. The lengths of the sides of a traingle are $\log_{10} 12, \log_{10} 75$ and $\log_{10} n$, where $n \in N$. If a and b are the least ad greatest values of n respectively. The value of b-a is divisible by

A. 221

B. 222

C. 223

D. 224

Answer: C

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30. Let $5\log_{abc}\left(a^3+b^3+c^3\right)=3\lambda\left(\frac{1+\log_3(abc)}{\log_3(abc)}\right)$ and $(abc)^{a+b+c}=1$ and $\lambda=\frac{m}{n}$ m, n are relatively prime , find $|m+n|+|m-n|$

A. 8

B. 10

C. 12

D. 14

Answer: B

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31. Find $\frac{dy}{dx}$ if $y=\frac{e^x}{\sin x}$

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32. Number of real roots of equation

$3^{\log 3\left(x^2-4x+3\right)}=(x-3)$ is

A. 0

B. 1

C. 2

D. infinite

Answer: A

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33. If $\log_6 a+\log_6 b+\log_6 c=6$,where a,b,c, $\in$ N and a,b,c are in GP and b-a is a square of an integer, then the value of a+b-c is

A. 21

B. 15

C. 9

D. 3

Answer: B

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34. If $x = \log_{2a}\left(\frac{bcd}{2}\right), y = \log_{3b}\left(\frac{acd}{3}\right), z = \log_{4c}\left(\frac{abd}{4}\right)$ and $w = \log_{5d}\left(\frac{abc}{5}\right.$ and $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} + \frac{1}{w+1} = \log_{abcd} N + 1,$ then value of $\frac{N}{40}$ is

A. 40

B. 80

C. 120

D. 160

Answer: A::B::C

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35. The equation

$(\log_{10} x + 2)^3 + (\log_{10} x - 1)^3 = (2\log_{10} x + 1)^3$ has

A. no natural solution

B. two rational solutions

C. no prime solutions

D. one irritional solution

Answer: A

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36. The value of $\frac{\log_5 9 \cdot \log_7 5 \cdot \log_3 7}{\log_3 \sqrt{6}} + \frac{1}{\log_4 \sqrt{6}}$ is co-prime with

A. 1

B. 3

C. 4

D. 5

Answer: A::C::D

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37. Consider the quadratic equation $(\log_{10} 8)x^2 - (\log_{10} 5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.

A. sum of roots

B. product of roots

C. sum of coefficients

D. discriminant

Answer: A::B

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38. Find the real solutions to the system of equations $\log_{10}(2000xy) - \log_{10} x . \log_{10} y = 4$, $\log_{10}(2yz) - \log_{10} y \log_{10} z = 1$ and $\log_{10} zx - \log_{10} z \log_{10} x = 0$

A. $x_1 + x_2 = 101$

B. $y_1 + y_2 = 25$

C. $x_1 + x_2 = 100$

D. $z_1 + z_2 = 100$

Answer: A

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39. Suppose that $\log_{10}(x-2) + \log_{10} y = 0$ and $\sqrt{x} + \sqrt{y-2} = \sqrt{x+y}$. Then the value of $(x+y)$ is

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40. Suppose that $\log_{10}(x-2)+\log_{10} y=0$ and $\sqrt{x}+\sqrt{y-2}=\sqrt{x+y}$. Then the value of $(x+y)$ is

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41. Suppose that $\log_{10}(x-2)+\log_{10} y=0\ \sqrt{x}+\sqrt{y-2}=\sqrt{x+y}$.

The value of x is

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42. Evaluate $\int_0^{\frac{\pi}{2}} \cos ec^2 x\, dx$

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43. $10^{\log_p(\log_q(\log_r(x)))}=1$ and $\log_q\left(\log_r\left(\log_p(x)\right)\right)=0$, then 'p' is equals

a. $r^{\frac{q}{r}}$

b. rq

c. 1

d. $r^{\frac{r}{q}}$

A. $\log_q\{\log_r(\log_p x)\} = 0.$

B. rq

C. 1

D. $r^{r/p}$

Answer: A

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44. $10^{\log_p(\log_q(\log_r(x)))} = 1$ and $\log_q(\log_r(\log_p(x))) = 0$, then 'p' is equals

a. $r^{\frac{q}{r}}$

b. rq

c. 1

d. $r^{\frac{r}{q}}$

A. $r^{p/r}$

B. $p\log_p r$

C. $r\log_r p$

D. $r^{r/p}$

Answer: C

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45. If x_1 and x_2 are the solutions of the equation $x^{\log_{10} x} = 100x$ such that $x_1 > 1$ and $x_2 < 1$, the value of $\frac{x_1 x_2}{2}$ is

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46. If $(31.6)^a = (0.0000316)^b = 100$, the value of $\frac{1}{a} - \frac{1}{b}$ is

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47. Statement-1(Assertion and Statement -2 (reason) Each of these examples also has four alternative choices, only one of which is the correct answer. You have select the correct choice as given below.

Statement -1 If $N = \left(\frac{1}{0.4}\right)^{20}$, then N contains 7 digit before decimal.

Statement -2 Characteristic of the logarithm of N to the base 10 is 7.

A. Statement -1 if true, Statement-2 is true, Statement-2 is a correct explanation for Statement -1

B. Statement -1 if true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

C. Statement -1 if true, Statement-2 is false

D. Statement -1 if false , Statement-2 is true

Answer: D

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48. Find $\frac{dy}{dx}$ if $y = x^{\sqrt{x}}$

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49. Prove $\log_3(5)$ is irrational.

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50. Find the value of the expressions $(\log 2)^3 + \log 8.\log(5) + (\log 5)^3$.

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51. IF $\lambda^{\log_5 3}$=81 ,find the value of λ.

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52. Find the product of the positive roots of the equation $\sqrt{2009}(x)^{\log_{2009}(x)} = x^2$.

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53. Prove that $\log_7 11$ is greater than $\log_8 5$.

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54. Given $a^2 + b^2 = c^2$. Prove that

$\log_{b+c} a + \log_{c-b} a = 2\log_{c+b} a . \log_{c-b} a,\ \forall a > 0, a \neq 1$

$c - b > 0, c + b > 0$

$c - b \neq 1, c + b \neq 1.$

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55. Let $a > 0, c > 0, b = \sqrt{ac}, a, c$ and $ac \neq 1, N > 0.$

Prove that $\dfrac{\log_a N}{\log_c N} = \dfrac{\log_a N - \log_b N}{\log_b N - \log_c N}$

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56. IF

$a^x = b, b^y = c, c^z = a, x = (\log_b a)^{k1}, y = (\log_c b)^{k2}, z = (\log_a c)^{k3},$

find the minimum value of $3k_1 + 6k_2 + 12k_3$.

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57. If $x = 1 + \log_a bc, y = 1 + \log_b ca, z = 1 + \log_c ab,$ then $\frac{xyz}{xy + yz + zx}$ is equal to

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58. If $\frac{Ina}{(b-c)} = \frac{Inb}{(c-a)} = \frac{Inc}{(a-b)}$,Prove that $a^{b+c}.\, b^{c+a}.\, c^{a+b} = 1$

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59. Simplify $5^{\log 1/5^{(1/2)}} + \log_{\sqrt{2}}\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_{1/2}\left(\frac{1}{10+2\sqrt{21}}\right).$

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60. Find the square of the sum of the roots of the equation $\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_3 x \cdot \log_5 x$

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61. Given that $\log_2 a = \lambda, \log_4 b = \lambda^2$ and $\log_{c^2}(8) = \frac{2}{\lambda^3 + 1}$ write $\log_2\left(\frac{a^2 b^5}{5}\right)$ as a function of λ, $(a, b, c > 0, c \neq 1)$.

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62. Given that $\log_2(3) = a, \log_3(5) = b, \log_7(2) = c,$ express the logrithm of the number 63 to the base 140 in terms of a,b & c.

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63. Show the sum of the roots of the equation $x+1=2\log_2(2^x+3)-2\log_4\left(1980-2^{-x}\right)$ is $\log_2 11$.

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64. Solve the equations $\log_{1000}|x+y|=\frac{1}{2}.\log_{10} y-\log_{10}|x|=\log_{100} 4$ for x and y

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65. Value of x, satisfying $x^2+2x=-1$ is :

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66. The value of x satisfying $x^2+5x+6=0$ is

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67. Find all real numbers x which satisty the equation. $2\log_2\log_2 x + \log_{\frac{1}{2}}\log_2\left(2\sqrt{2}x\right) = 1$

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68. Find x if $(x+1)(2x-3) = 4$.

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69. Solve the following linear equations: $11x = 121$

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Example

1. Evaluate $\int_0^{\frac{\pi}{4}} \cos ec^2 x\, dx$

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2. If α and β is the root of the equation $x^2 - 3x + 2$ then find the ratio of sum of the root and product of the root.

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Exercise For Session 1

1. Find the value of $(\log)_{2\sqrt{3}}1728$.

A. 6

B. 8

C. 3

D. 5

Answer: A

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2. The value of $\log_{\left(8-3\sqrt{7}\right)}\left(8+3\sqrt{7}\right)$ is

A. -2

B. -1

C. 0

D. Not defined

Answer: B

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3. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots . \text{to}\,\infty\right)}$, is

A. 2

B. 4

C. 6

D. 8

Answer: B

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4. If log2=0.301, the number of integers in the expansion of 4^{17} is

A. 9

B. 11

C. 13

D. 15

Answer: B

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5. If log2=0.301, the number of zeroes between the decimal point and the first significant figure of 2^{-34} is

A. 9

B. 10

C. 11

D. 12

Answer: B

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Exercise For Session 2

1. If $a = \log_{24} 12, b = \log_{36} 24, c = \log_{48} 36,$ then show that $1 + abc = 2bc$

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2. The value of $\log_4[\ |\ \log_2\{\log_2(\log_3)81)\}]$ is equal to

A. -1

B. 0

C. 1

D. 2

Answer: B

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3. $\frac{\log_2 \log_2 \left(\sqrt{\sqrt{\ldots \sqrt{\sqrt{2}}}} \right)}{nTimes}$ is equal to

A. 0

B. 1

C. n

D. $-n$

Answer: D

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4. If a $=\log_3(5)$ and b $=\log_{17}(25)$, which one of the following is correct?

A. $a < b$

B. a=b

C. $a > b$

D. None of these

Answer: C

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5. The value of $\sin(\cos 90)$ is equal to

A. -1

B. 0

C. 1

D. None of these

Answer: C

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Exercise For Session 3

1. Solve for x if $x(x-4) = -4$

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2. Prove that $\log_3 81 - 4 = 0$

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3. IF $\log_{\cos ecx} \sin x > 0$ then,

A. $x > 0$

B. $x < 0$

C. $-1 < x < 1$

D. None of these

Answer: D

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4. The value of $\log_{10} 3$ lies in the interval

A. $\left(\frac{2}{5}, \frac{1}{2}\right)$

B. $\left(0, \frac{1}{2}\right)$

C. $\left(0, \frac{2}{5}\right)$

D. None of these

Answer: A

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5. The least value of n in order that the sum of first n terms of the infinite series $1+\frac{3}{4}+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+\ldots$, should differ from the sum of the series by less than 10^{-6}, is $(given \log 2 = 0.30103, \log 3 = 0.47712)$

A. 14

B. 27

C. 53

D. 57

Answer: C

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Exercise Single Option Correct Type Questions

1. Given that $\log(2) = 0.3010$, the number of digits in the number 2000^{2000} is 6601 (b) 6602 (c) 6603 (d) 6604

A. 6601

B. 6602

C. 6603

D. 6604

Answer: C

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2. There exists a positive number k such that $\log_2 x + \log_4 x + \log_8 x = \log_k x$, for all positive real no x. If k=$a^{\frac{1}{b}}$ where (a,b) ε N, the smallest possible value of (a+b)=

A. 12

B. 63

C. 65

D. 75

Answer: D

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3. If $x_1, x_2 \& x_3$ are the three real solutions of the equation $x^{\log_{10}^2 x + \log_{10} x^3 + 3} = \frac{2}{\left(\frac{1}{\sqrt{x+1}-1}\right) - \left(\frac{1}{\sqrt{x+1}+1}\right)}$, where $x_1 > x_2 > x_3$, then these are in

A. AP

B. GP

C. HP

D. $a^{-1} + b^{-1} = c^{-1}$

Answer: B

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4. If $f(n) = \prod_{i=2}^{n-1} \log_i(i+1)$, the value of $\sum_{k=1}^{100} f\left(2^k\right)$ equals

A. 5010

B. 5050

C. 5100

D. 5049

Answer: B

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5. If $x - \frac{1}{x} = m$ then the value of $x^3 - \frac{1}{x^3}$ is :

A. 7^{-3}

B. 3^{-7}

C. 7^3

D. 3^7

Answer: A

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6. In Which quadrant Point $(-2, 3)$ will be lie.

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7. Solve for x if $x(x-4) = -4$

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8. If $x_n > x_{n-1} > \ldots\ldots\ldots > x_3 > x_1 > 1.$ then the value of $\log_{x_1}\left[\log_{x_2}\left\{\log_{x_3}\ldots\ldots\ldots\log_{x_n}(x_n)^{x_{r=i}}\right\}\right]$

A. 0

B. 1

C. 2

D. undefined

Answer: B

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9. If $\frac{x(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y} = \frac{z(x+y-z)}{\log z}$, prove that $x^y y^x = z^y y$

.

A. $z^x x^z$

B. $x^z y^x$

C. $x^y y^z$

D. $x^x y^y$

Answer: A

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10. If $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$, then prove that $x = a^{\frac{1}{1-\log_a z}}$

A. $a^{\frac{1}{1+\log_a z}}$

B. $a^{\frac{1}{2+\log_a z}}$

C. $a^{\frac{1}{1-\log_a z}}$

D. $a^{\frac{1}{2-\log_a z}}$

Answer: C

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11. If $(\log)_{0.3}(x-1) < (\log)_{0.09}(x-1)$, then x lies in the interval

(a)$(2, \infty)$ (b) $(1, 2)$ (c)$(-2, -1)$ (d) None of these

A. $(-\infty.1)$

B. (1,2)

C. $(2, \infty)$

D. None of the above

Answer: C

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12. The value of $a^x - b^y$ is (where $x = \sqrt{\log_a b}$ and $y = \sqrt{\log_b a}$, $a > 0, b > 0$ and $a, b \neq 1$)

A. 1

B. 2

C. 0

D. -1

Answer: C

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13. If $x = 1 + \log_a bc, y = 1 + \log_b ca, z = 1 + \log_c ab,$ then $\frac{xyz}{xy + yz + zx}$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: B

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14. The value of $a^{\frac{\log_b (\log_b N)}{\log_b a}}$, is

A. $\log_a N$

B. $\log_b N$

C. $\log_N a$

D. $\log_N b$

Answer: B

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15. Find the value of $49^{(1-\log_7(2))} + 5^{-\log_5(4)}$ is

A. 10.5

B. 11.5

C. 12.5

D. 13.5

Answer: C

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16. The number of real values of the parameter k for which $(\log_{16} x)^2 - \log_{16} x + \log_{16} k = 0$ with real coefficients will have exactly one solution is

A. 1

B. 2

C. 3

D. 4

Answer: B

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17. Solve the equation $x^{\log_x (x+3)^2} = 16$.

A. 1

B. 0

C. 2

D. 4

Answer: B

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18. The point on the graph $y = \log_2 \log_6 \left\{ 2^{\sqrt{2x+1}} + 4 \right\}$ whose y coordinate is 1 is

A. (1,1)

B. (6,1)

C. (8,1)

D. (12,1)

Answer: D

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19. Find $\frac{dy}{dx}$, if $2x^2 - xy + 4y^2 = 200$

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20. The values of x, satisfying the equation for $\forall a > 0, 2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0$ are

A. one

B. two

C. three

D. four

Answer: B

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Exercise More Than One Correct Option Type Questions

1. If $x^{(\log_2 x)^2 - 6\log_2 x + 11} = 64$ then x is equal to

A. 2

B. 4

C. 6

D. 8

Answer: A::B::D

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2. If $\log_\lambda x . \log_5 \lambda = \log_x 5, \lambda \neq 1, \lambda > 0$, then x is equal to

A. λ

B. 5

C. $\frac{1}{5}$

D. None of these

Answer: B::C

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3. If $S = \left\{x : \sqrt{\log_x \sqrt{3x}}, \text{where } \log_3 x > -1\right\}$, then

A. S is a finite set

B. S is an empty set

C. $S \subset (0, \infty)$

D. S properly contains $\left(\frac{1}{3}, \infty\right)$

Answer: C::D

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4. If x satisfies $\log_2\left(9^{x-1} + 7\right) = 2 + \log_2\left(3^{x-1} + 1\right)$, then

A. $x \in Q$

B. $x \in N$

C. $x \in \{x \notin Q : x < 0\}$

D. $x \in N_e$ (set of even natural number)

Answer: A::B

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5. $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\sqrt{\ldots\ldots \sqrt[p]{p},}}}}}_{\text{n times}}$ $p > 0$ and $p \neq 1$ is equal to

A. n

B. $-n$

C. $\frac{1}{n}$

D. $\log_{1/p}(p^n)$

Answer: B::D

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6. If $\log_a x = \alpha, \log_b x = \beta, \log_c x = \gamma$ and $\log_d x = \delta, x \neq 1$ and a,b,c, $d \neq 1$,then $\log_{abcd}$ x equals

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7. If $\log_{10} 5 = a$ and $\log_{10} 3 = b$,then

A. $\log_{10} 8 = 3(1 - a)$

B. $\log_{40} 15 = \dfrac{(a + b)}{(3 - 2a)}$

C. $\log_{243} 32 = \left(\dfrac{1 - a}{b}\right)$

D. All of these

Answer: A::B::C::D

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8. If x is a positive real number different from 1 such that $\log_a x, \log_b x, \log_c x$ are in A.P then

A. $\log b = 2\dfrac{(\log a)(\log c)}{(\log a + \log c)}$

B. $b = \dfrac{a + c}{2}$

C. $b = \sqrt{ac}$

D. $c^2 = (ac)^{\log_a b}$

Answer: A::D

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9. If $|a| < |b|, b - a < 1$ and a, b are the real roots of the equation $x^2 - |\alpha|x - |\beta| = 0$, then the equation $\log_{|b|}\left|\frac{x}{a}\right| - 1 = 0$ has

A. (a)one root lying in interval $(-\infty, a)$

B. (b)one root lying in interval (b, ∞)

C. (c)one positive root

D. (d) one negative root

Answer: C::D

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Exercise Passage Based Questions

1. Find $\frac{dy}{dx}$ if $y = \cos(\log x)$

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2. Find $\frac{dy}{dx}$ if $\sin 2x - 2y = \cos x$

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3. Let $\log_2 N = a_1 + b_1$, $\log_3 N = a_2 + b_2$ and $\log_5 N = a_3 + b_3$, where $a_1, a_2, a_3 \notin 1$ and $b_1, b_2 b_3 \in [0, 1)$.

If $a_1 = 6, a_2 = 4$ and $a_3 = 3$,the difference of largest and smallest integral values of N, is

A. 2

B. 8

C. 14

D. 20

Answer: A

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4. Let S denotes the antilog of 0.5 to the base 256 and K denotes the number of digits in 6^{10} (given $\log_{10} 2 = 0.301, \log_{10} 3 = 0.477$) and G denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 3.

The value of SKG is

A. 18

B. 24

C. 30

D. 36

Answer: A

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5. Let S denotes the antilog of 0.5 to the base 256 and K denotes the number of digits in 6^{10} (given $\log_{10} 2 = 0.301, \log_{10} 3 = 0.477$) and G denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 3.

The value of SKG is

A. 72

B. 144

C. 216

D. 288

Answer: B

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6. Let S denotes the antilog of 0.5 to the base 256 and K denotes the number of digits in 6^{10} (given $\log_{10} 2 = 0.301, \log_{10} 3 = 0.477$) and G

denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 3.

The value of SKG is

A. 1440

B. 17280

C. 2016

D. 2304

Answer: D

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7. Suppose U denotes the number of digits in the number $(60)^{100}$ and M denotes the number of cyphers after decimal, before a significant figure comes in $(8)^{-296}$. If the fraction U/M is expressed as rational number in the lowest term as p/q (given $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$) .

The value of p is

A. 1

B. 2

C. 3

D. 4

Answer: B

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8. Suppose U denotes the number of digits in the number $(60)^{100}$ and M denotes the number of cyphers after decimal, before a significant figure comes in $(8)^{-296}$. If the fraction U/M is expressed as rational number in the lowest term as p/q (given $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$) .

The value of q is

A. 5

B. 2

C. 3

D. 4

Answer: C

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9. Suppose U denotes the number of digits in the number $(60)^{100}$ and M denotes the number of cyphers after decimal, before a significant figure comes in $(8)^{-296}$. If the fraction U/M is expressed as rational number in the lowest term as p/q (given $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$) .

The equation whose roots are p and q, is

A. $x^2 - 3x + 2 = 0$

B. $x^2 - 5x + 6 = 0$

C. $x^2 - 7x + 12 = 0$

D. $x^2 - 9x + 20 = 0$

Answer: B

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10. Let G,O,E and L be positive real numbers such that log(G.L)+log(G.E)=3,log(E.L)+log(E.O)=4, log(O.G)+log(O.L)=5 (base of the log is 10)

If the value of the product (GOEL) is λ, the value of $\sqrt{\log\lambda\sqrt{\log\lambda\sqrt{\log\lambda....}}}$ is

a. 3

b. 4

c. 5

d. 7

A. 3

B. 4

C. 5

D. 7

Answer: B

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11. If α and β is the root of the equation $x^2 - 3x + 2 = 0$ then find the value of $\alpha^2 + \beta^2$

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12. Let G,O,E and L be positive real numbers such that log(G.L)+log(G.E)=3,log(E.L)+log(E.O)=4, log(O.G)+log(O.L)=5 (base of the log is 10)

If $\log\left(\frac{G}{O}\right)$ and $\log\left(\frac{O}{E}\right)$ are the roots of the equation

A. $x^2 + x = 0$

B. $x^2 - x = 0$

C. $x^2 - 2x + 3 = 0$

D. $x^2 - 1 = 0$

Answer: D

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Exercise Single Integer Answer Type Questions

1. If $x, y \in R^+$ and $\log_{10}(2x) + \log_{10} y = 2$, $\log_{10} x^2 - \log_{10}(2y) = 4$ and $x + y = \frac{m}{n}$,Where m and n are relative prime , the value of $m - 3n^6$ is

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2. A line $x = \lambda$ intersects the graph of $y = \log_5 x$ and $y = \log_5(x + 4)$ .The distance between the point of intersection is 0.5. Given $\lambda = a + \sqrt{b}$,where a and b are integers , the value of (a+b) is

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3. Number of integers satisfying the inequality

$\left(\frac{1}{3}\right)^{\frac{|x+2|}{2-|x|}} > 9$ is

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4. If $x > 2$ is a solution of the equation

$\left|\log_{\sqrt{3}} x - 2\right| + \left|\log_3 x - 2\right| = 2$, then the value of x is

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5. Solve for x if $x^2\left(\frac{1}{x} - 2\right) = -4$

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6. The value of b for which the equation $2\log_{1/25}(bx + 28) = -\log_5\left(12 - 4x - x^2\right)$ has coincident roots is

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7. The value of $\frac{2^{\log_{\frac{1}{2^4}} 2} - 3^{\log_{27} 125} - 4}{\left(7^{4\log_{49} 2}\right) - 3}$ is

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8. Every Natural number is Integer ?

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9. Solve for x if $4x(5-3x) = -2$

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Exercise Matching Type Questions

1. Every Integer is Natural number ?

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2. If α and β is the root of the equation $x^2 - 3x + 2 = 0$ then find the value of $\alpha^2 + \beta^2 + 2\alpha\beta$

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Exercise For Session 8

1.

Column I		Column II	
(A)	The expression $\sqrt{\log_{0.5}^2 8}$ has the value equal to	(p)	1
(B)	The value of the expression $(\log_{10}2)^3 + \log_{10}8 \cdot \log_{10}5 + (\log_{10}5)^3 + 3$, is	(q)	2
(C)	Let $N = \log_2 15 \cdot \log_{1/6} 2 \cdot \log_3\left(\frac{1}{6}\right)$. The value of $[N]$ is (where $[\cdot]$ denotes the greatest integer function)	(r)	3
(D)	If $(52.6)^a = (0.00526)^b = 100$, the value of $\frac{1}{a} - \frac{1}{b}$ is	(s)	4

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Exercise Statement I And Ii Type Questions

1. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 $\log_{10} x < \log_3 x < \log_e x < \log_2 x \ (x > 0, x \neq 1)$

Statment If $0 < x < 1$, then $\log_x a > \log_x b \Rightarrow 0 < a < b$.

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2. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the

correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 The equation $7^{\log_7(x^3+1)} - x^2 = 1$ has two distinct real roots .

Statement -2 $a^{\log_a N} = N$, where $a > 0, a \neq 1$ and $N > 0$

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3. Statement -1 $\left(\frac{1}{2}\right)^7 < \left(\frac{1}{3}\right)^4$

$\Rightarrow 7\log\left(\frac{1}{2}\right) < 4\log\left(\frac{1}{3}\right) \Rightarrow 7 < 4$

Statement-2 If $ax < ay$, where $a < 0$,x,$y > 0$, then $x > y$.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct

explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true.

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4. Find the value of x of the equation $x + m - 7 = 0$ where $m = 3$

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5. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 The equation $(\log x)^2 + \log x^2 - 3 = 0$ has two distinct solutions.

Statement-2 $\log x^2$=2logx.

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6. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 $\log_x 3.\ \log_{x/9} 3 = \log_{81}(3)$ has a solution.

Statement-2 Change of base in logarithms is possible .

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Exercise Subjective Type Questions

1. If $\log_7 12 = a$, $\log_{12} 24 = b$, then find value of $\log_{54} 168$ in terms of a and b.

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2. If $\log_3 4 = a$,$\log_5 3 = b$, then find the value of $\log_3 10$ in terms of a and b.

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3. If $\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$, prove the following .

abc=1

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4. If $\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$, prove the following .

$a^a . b^b . c^c = 1$

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5. If $\frac{\ln a}{b-c} = \frac{\ln b}{c-a} = \frac{\ln c}{a-b}$, prove the following .

$a^{b^2+bc+c^2} . b^{c^2+ca+a^2} . c^{a^2+ab+b^2} = 1$

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6. If $\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$, prove the following .

$a + b + c \geq 3$

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7. If $\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$, prove the following .

$a^a + b^b + c^c \geq 3$

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8. If $\frac{\ln a}{b-c} = \frac{\ln b}{c-a} = \frac{\ln c}{a-b}$, prove the following .

$a^{b^2+bc+c^2} . b^{c^2+ca+a^2} . c^{a^2+ab+b^2} = 1$

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9. Prove that $\log_{10} 2$ lies between $\frac{1}{4}$ and $\frac{1}{3}$.

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10. If log2=0.301 and log3=0.477, find the number of integers in 5^{200}

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11. If log2=0.301 and log3=0.477, find the number of integers in 6^{20}

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12. If log2=0.301 and log3=0.477, find the number of integers in the number of zeroes after the decimal is 3^{-500}.

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13. If log2=0.301 and log3=0.477, find the value of log(3.375).

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14. Find the least value of $\log_2 x - \log_x(0.125)$ for $x > 1$.

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15. Find values of lamda for which $\frac{1}{\log_3 \lambda} + \frac{1}{\log_4 \lambda} > 2$.

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16. Solve the following equations.

(i) $x^{1+\log_{10} x} = 10x$

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17. Solve the following equation.

$\log_2(9 - 2^x) = 3 - x$

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18. Solve the equation $2x^{\log_4^3} + 3^{\log_4^x} = 27$

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19. Solve the following equation.

$\log_4 \log_3 \log_2 x = 0$

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20. Solve the following equations. $x^{\frac{\log_{10} x+5}{3}} = 10^{5+\log_{10} x}$

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21. Solve the following equation.

$$\log_3\left(\log_9 x + \frac{1}{2} + 9^x\right) = 2x$$

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22. Solve the following equation.

$$4^{\log_{10} x+1} - 6^{\log_{10} x} - 2.3^{\log_{10} x^2+2} = 0$$

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23. Solve the following equation.

$$\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$$

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24. Solve the following equations.

$x^{\log_2 x+4} = 32$

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25. Solve the following equations.

$\log_a x = x$, where $a = x^{\log_a x}$

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26. Solve the following equations.

(xi) $\log_{\sqrt{2}\sin x}(1+\cos x) = 2$

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27. A rational number which is 50 times its own logarithm to the base 10, is

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28. $\left[\frac{2}{\log_4 (2000)^6} + \frac{3}{\log_5 (2000)^6}\right]$

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29. Find the value of x satisfying

$\log_a\{1 + \log_b\{1 + \log_c(1 + \log_p x)\}\} = 0.$

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30. The value of $4^{5\log_{4\sqrt{2}}(3-\sqrt{6}) - 6\log_8(\sqrt{3}-\sqrt{2})}$ is

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31. Solve the following inequation .

(i) $\log_{(2x+3)} x^2 < 1$

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32. Solve the following inequation .

(ii) $\log_{2x}\left(x^2 - 5x + 6\right) < 1$

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33. Solve the following inequation .

(iii) $\log_2(2 - x) < \log_{1/2}(x + 1)$

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34. Solve the following inequation .

(iv) $\log_{x^2}(x + 2) < 1$

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35. Solve the following inequation .

(v) $3^{\log_3 \sqrt{(x-1)}} < 3^{\log_3 (x-6)} + 3$

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36. Solve the following inequation .

(vi) $\log_{1/2} (3x-1)^2 < \log_{1/2} (x+5)^2$

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37. Solve the following inequation .

(vii) $\log_{10} x + 2 \leq \log_{10}^2 x$

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38. Solve the following inequation .

(viii) $\log_{10}\left(x^2 - 2x - 2\right) \leq 0$

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39. Solve the following inequation .

(ix) $\log x\left(2x - \frac{3}{4}\right) > 2$

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40. Find the value of x of the equation $x^2 + m - 8 = 0$ where $m = 3$

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41. Solve the inequation $\log_{2x+3} x^2 < \log_{2x+3}(2x + 3)$

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42. Solve the following inequation .

(xii) $(\log_2 x)^2 + 3\log_2 x \geq \frac{5}{2}\log_{4\sqrt{2}} 16$

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43. Solve the following inequation .

(xiii) $\left(x^2 + x + 1\right)^x < 1$

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44. Solve the following inequation .

(xiv) $\log_{(3x^2+1)} 2 < \frac{1}{2}$

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45. Solve the following inequation .

(xv) $x^{(\log_{10} x)^2 - 3\log_{10} x + 1} > 1000$

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46. Solve the following inequation .

(xvi) $\log_4\{14+\log_6(x^2-64)\}\leq 2$

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47. Solve the following inequation .

(xvii) $\log_2(9-2^x)\leq 10^{\log_{10}(3-x)}$

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48. Solve the following inequation: $2x+3<5x-4$

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49. Every Natural number is Whole number?

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50. Find the value of x of $\log_4 x = 2$

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51. $2\left(\sqrt{\log_a (ab)^{1/4} + \log_b (ab)^{1/4}} - \sqrt{\log_a \left(\frac{b}{a}\right)^{1/4} + \log_b \left(\frac{a}{b}\right)^{1/4}}\right)\sqrt{\log_a (b)}$ =

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52. It is known that x=9 is root of the equation. $\log_\lambda\left(x^2 + 15a^2\right) - \log_\lambda(a-2) = \log_\lambda \frac{8ax}{a-2}$ find the other roots of this equation.

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53. Solve $\log_4(\log_3 x) + \log_{1/4}\left(\log_{1/3} y\right) = 0$ and $x^2 + y^2 = \frac{17}{4}$.

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54. Find $\frac{dy}{dx}$ if $\log(4x) + \log(16x) = 4y$.

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55. Find the sum and product of all possible values of x which makes the following statement true.

$\log_6 54 + \log_x 16 = \log_{\sqrt{2}} x - \log_{36}\left(\frac{4}{9}\right)$.

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56. Solve : $\frac{3}{2}\log_4 (x+2)^2 + 3 = \log_4 (4-x)^3 + \log_4 (6+x)^3$.

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57. Find the number of real values of x satisfying the equation.

$\log_2\left(4^{x+1}+4\right)\cdot\log_2\left(4^x+1\right)=3$

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58. Solve the system of equation $2^{\sqrt{x}+\sqrt{y}}=256$ and $\log_{10}\sqrt{xy}-\log_{10}\left(\frac{3}{2}\right)=1.$

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59. Every Integer is Whole number?

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60. Every Whole number Integer ?

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61. Solve $\log_3\left(\sqrt{x}+|\sqrt{x}-1|\right)=\log_9\left(4\sqrt{x}-3+4|\sqrt{x}-1|\right)$.

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62. In the equality

$$(\log_2 x)^4-\left(\log_{1/2}\frac{x^5}{4}\right)^2-20\log_2 x+148<0$$

holds true in (a,b), where a,b $\in$ N. Find the value of ab (a+b).

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63. Find the value of x satisfying the equation,

$$\sqrt{\left(\log_3(3x)^{\frac{1}{3}}+\log_x(3x)^{\frac{1}{3}}\right)\log_3(x^3)}+\sqrt{\left(\log_3\left(\frac{x}{3}\right)^{\frac{1}{3}}+\log_x\left(\frac{3}{x}\right)^{\frac{1}{3}}\right)\log}$$

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64. If P is the number of natural number whose logarithms to the base 10 have the characteristic p and Q is the number of natural numbers

logarithms of whose reciprocals to the base 10 have the characteristic -q, then find the value of $\log_{10} P - \log_{10} Q$.

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Exercise Questions Asked In Previous 13 Years Exam

1. In Which quadrant Point $(-2, -3)$ will be lie.

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2. Let (x_0, y_0) be the solution of the following equations:

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

Then x_0 is

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 6

Answer: C

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3. The value of

$$6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$$ is _____.

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4. If $3^x = 4^{x-1}$, then x equals

A. $\frac{2\log_3 2}{2\log_3 2-1}$

B. $\frac{2}{2-\log_2 3}$

C. $\frac{1}{1-\log_4 3}$

D. $\frac{2\log_2 3}{2\log_2 3 - 1}$

Answer: A::B::C

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