MATHS

BOOKS - ARIHANT MATHS

MATHEMATICAL INDUCTION

Examples

1. Prove the following by using the principle of mathematical induction for all $n \in N$:- $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

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2. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

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3. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

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4. Prove by mathematical induction that

$$\sum_{r=0}^{n} r^{n}C_{r} = n.2^{n-1}, \forall n \in N.$$

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5. Use the principle of mathematical induction to show that

$5^{2n+1} + 3^{n+2}.2^{n-1}$ divisible by 19 for all natural numbers n.

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6. Use the principle of mathematical induction to show that $a^n - b^n)$ is divisible by $a - b$ for all natural numbers n.

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7. If first term is 3 and common ratio is 3 then find the 6th term of G.P

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8. Using mathematical induction prove that $n^3 - 7n + 3$ is divisible by $3, \ \forall n \in N$

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9. if $a+b=c+d$ and $a^2+b^2=c^2+d^2$, then show by mathematical induction $a^n+b^n=c^n+d^n$

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10. Evaluate $I=\int_0^{\frac{\pi}{4}}\left(\frac{\sin x}{\sin x\cos^2 x}\right)dx$

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11. Given that $u_{n+1}=3u_n-2u_{n-1}$, and $u_0=2, u_1=3$, then prove that $u_n=2^n+1$ for all positive integer of n

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12. How many numbers between 99 to 999 which are divisible by 11.

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13. If p is a natural number, then prove that $p^{n+1}+(p+1)^{2n-1}$ is divisible by p^2+p+1 for every positive integer n.

A. P

B. P^2+P

C. P^2+P+1

D. P^2-1

Answer:

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14. Let P(n) denote the statement that n^2+n is odd . It is seen that $P(n) \Rightarrow P(n+1), P(n)$ is true for all

A. $n>1$

B. n

C. $n > 2$

D. None of these

Answer:

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15. For a positive integer n let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots . + \frac{1}{(2^n) - 1}$. Then

A. $a(100) > 100$

B. $a(100) < 200$

C. $a(200) \leq 100$

D. $a(200) > 100$

Answer: D

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16. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true ?

A. Principle of mathematical induction can be used to prove the formula

B. $S(k) \Rightarrow S(k+1)$

C. $S(k) \not\Rightarrow S(k+1)$

D. S(1) is correct

Answer:

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17. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

A. 7

B. 5

C. 9

D. 8

Answer:

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18. Statement-1 For all natural number n, $1+2+\ldots.+n<(2n+1)^2$ Statement -2 For all natural numbers , $(2n+3)^2-7(n+1)<(2n+3)^3$.

A. Statement -1 is true , Statement -2 is true Statement -2 is correct explanation for Statement -1.

B. Statement -1 is true , Statement -2 is true , Statement -2 is not the correct explanation for Statement -1

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer: B

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19. Prove the following by the principle of mathematical induction:
$7 + 77 + 777 + \ldots . . + 777 + + \ddot{n} - digits7 = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$
for all $n \in NB$.

A. Statement -1 is true , Statement -2 is true Statement -2 is correct explanation for Statement -1.

B. Statement -1 is true , Statement -2 is true , Statement -2 is not the correct explanation for Statement -2

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer: C

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20. Using the principle of mathematical induction to show that $41^n - 14^n$ is divisible by 27 for all n.

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21. Using mathematical induction , show that $n(n+1)(n+5)$ is a multiple of 3.

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22. Using the principle of mathematical induction to show that $41^n - 14^n$ is divisible by 27 for all n.

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23. Use the principle of mathematical induction to prove that for all $n \in N$

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\ldots+\sqrt{2}}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$$

when the LHS contains n radical signs.

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24. Using the principle of mathematical induction to show that $41^n - 14^n$ is divisible by 27 for all n.

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25. Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x} dx$

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26. Show that for all $n \in N$.

$$\sqrt{a+\sqrt{a+\sqrt{a+....+\sqrt{a}}}} < \frac{1+\sqrt{(4a+1)}}{2}$$

where'a' is fixed positive number and n radical signs are taken on LHS.

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27. Prove that $\sum_{r=0}^{n} \hat{} \, nC_r \sin rx \cos(n-r)x = 2^{n-1}\sin(nx)\cdot$

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Mathematical Induction Exercise 1 Single Option Correct Tpye Questions

1. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+.......}}}\Big)$ having n radical signs then by methods of mathematical induction which is true

A. $a_n > 7, \ \forall n \geq 1$

B. $n_n > 3, \forall n \geq 1$

C. $a_n < 4, \forall n \geq 1$

D. $a_n < 3, \forall n \geq 1$

Answer:

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2. if $2x - 2y = 10$ find the value of $x - y$

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3. Prove the following by using the principle of mathematical induction for all $n \in N$:- $(2n + 7) < (n + 3)^2$.

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Exercise Statement I And Ii Type Questions

1. If $a_1 = 1, a_2 = 5$ find the common difference and 5th term of A.P

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2. Statement -1 for all natural numbers n , $2.7^n + 3.5^n - 5$ is divisible by 24.

Statement -2 if f(x) is divisible by x, then $f(x+1) - f(x)$ is divisible by $x+1, \forall x \in N$.

A. Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -2

B. Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -2

C. Statement -1 is true , Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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3. Statement -1 For all natural numbers n , $0.5 + 0.55 + 0.555 + \ldots\ldots$ upto n terms $= \frac{5}{9}\left\{n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right\}$,

Statement-2 $a + ar + ar^2 + \ldots. + ar^{n-1} = \frac{a(1-r^n)}{(1-r)}$, for $0 < r < 1$.

A. (a)Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -1

B. (b)Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -1

C. (c)Statement -1 is true , Statement -2 is false

D. (d)Statement -1 is false , Statement - 2 is true.

Answer:

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Exercise Subjective Type Questions

1. Prove the following by the principle of mathematical induction: $11^{n+2}+12^{2n+1}$ is divisible 133 for all $n \in N$.

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2. Show that $n^7 - n$ is divisible by 42 .

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3. Prove that $3^{2n}+24n-1$ is divisible by 32 .

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4. Prove using mathematical induction:-$n(n+1)(n+5)$ is divisible by 6 for all natural numbers

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5. Prove that $3^{2n}+24n-1$ is divisible by 32 .

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6. Prove the following by using the principle of mathematical induction for all $n \in N$:- $x^{2n}-y^{2n}$ is divisible by$x+y$.

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7. Prove by induction that if n is a positive integer not divisible by 3, then $3^{2n}+3^n+1$ is divisible by 13.

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8. Prove that the product of three consecutive positive integers is divisible by 6.

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9. Find the sum of A.P first term 3 and common difference 2 and n=5

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10. When the square of any odd number, greater than 1, is divided by 8, it always leaves remainder

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11. Prove the following by using induction for all $n \in N$.

$$1 + 2 + 3 + \ldots.. + n = \frac{n(n+1)}{2}$$

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12. Prove the following by the principle of mathematical induction:

$$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{n(n+1)(2n+1)}{6}$$

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13. Prove the following by the principle of mathematical induction:

$$1.3 + 2.4 + 3.5 + \quad + (2n-1)(2n+1) = \frac{n\left(4n^2 + 6n - 1\right)}{3}$$

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14. If first term is 3 and common ratio is 3 then find the 6th term of G.P

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15. The third term of a GP is 3. What is the product of the first five terms?

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16. If First term of G.P is 1 and common ratio '1/2' then find the infinite sum of G.P

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17. Let $a_0 = 2, a_1 = 5$ and for $n \geq 2, a_n = 5a_{n-1} - 6a_{n-2}$. Then prove by induction that $a_n = 2^n + 3^n \forall n \in Z^+$.

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18. If $a_1 = 1, a_{n+1} = \frac{1}{n+1}a_n, a \geq 1$, then prove by induction that $a_{n+1} = \frac{1}{(n+1)!} n \in N$.

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19. if a,b,c,d,e and f are six real numbers such that $a+b+c=d+e+f$ $a^2+b^2+c^2=d^2+e^2+f^2$ and $a^3+b^3+c^3=d^3+e^3+f^3$, prove by mathematical induction that $a^n+b^n+c^n=d^n+e^n+f^n \forall n \in N$.

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20. The sum of the first ten terms of an AP is four times the sum of the first five terms, the ratio of the first term to the common difference is

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Exercise Questions Asked In Previous 13 Years Exam

1. If $t_1 + t_5 + t_{15} + t_{10} + t_{20} + t_{24} = 225$ Find the sum of first 24th term of that A.P?

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2. Statement -1 For each natural number $n, (n+1)^7 - n^7 - 1$ is divisible by 7.

Statement -2 For each natural number $n, n^7 - n$ is divisible by 7.

A. Statement-1 is false , Statement-2 is true

B. Statement-1 is true , Statement-2 is true , Statement-2 is correct explanation for Statement-1

C. Statement-1 is true , Statement-2 is true , Statement-2 is not a correct explanation for Statement-1

D. Statement-1 is true , Statement-2 is false

Answer:

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