MATHS

BOOKS - ARIHANT MATHS

PARABOLA

Examples

1. Find the locus of a point , which moves such that its distance from the point $(0, -1)$ is

twice its distance from the line

$3x + 4y + 1 = 0$

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2. What conic does the equation

$25\left(x^2 + y^2 - 2x + 1\right) = (4x - 3y + 1)^2$

represent?

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3. What conic does

$13x^2 - 18xy + 37y^2 + 2x + 14y - 2 = 0$

represent?

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4. Statement I: The conic $\sqrt{a}x + \sqrt{b}y = 1$ represents a parabola.

Statement II: Conic

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

represents a parabola if $h^2 = ab$.

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5. If the equation $x^2 - y^2 - 2x + 2y + \lambda = 0$ represent a degenerate conic . Find the value of λ.


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6. If the equation $x^2 - y^2 - 2x - 2y + c = 0$ represent an empty set , then find the value of c.


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7. IF the equation of conic

$2x^2 + xy + 3y^2 - 3x + 5y + \lambda = 0$

represent a single point, then find the value of

λ

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8. For What value of λ the equation of conic

$2xy+4x-6y+\lambda=0$ represents two intersecting

straight lines, if $\lambda = 17$, then this equation represents?

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9. The centre of the conic $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$, is

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10. Find the equation of the parabola whose focus is at (-1,-2) and the directrix the line $x - 2y + 3 = 0$

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11. Find the equation of the parabola whose focus is (4,-3) and vertex is (4,-1).

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12. The focal distance of a point on a parabola $y^2 = 8x$ is 8. Find it .

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13. PQ is a double ordinate of a parabola $y^2 = 4ax$. Find the locus of its points of trisection.

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14. If y_1, y_2, y_3 be the ordinates of a vertices of the triangle inscribed in a parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$.

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15. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose vertex is at the vertex of the parabola .Find the length of its side.

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16. Prove that the equation of the parabola whose focus is (0, 0) and tangent at the vertex is $x - y + 1 = 0$ is $x^2 + y^2 + 2xy - 4x + 4y - 4 = 0$.

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17. Find the equation of the parabola whose latus-rectum is 4 units , axis is the line

$3x + 4y - 4 = 0$ and the tangent at the vertex is the line $4x - 3y + 7 = 0$.

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18. Find the vertex, focus, and directrix of the following parabolas:

$x^2 + 8x + 12y + 4 = 0$

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19. Prove that the equation $y^2 + 2ax + 2by + c = 0$ represents a parabola whose axis is parallel to the axis of x. Find its vertex.

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20. Derive the equation of the parabola with its vertex at $(3, 2)$ and its focus at $(5, 2)$.

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21. Find the equation of the parabola with latusrectum joining the points (3,6) and (3,-2).

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22. Find the equation to the parabola whose axis parallel to the y-axis and which passes through the points (0,4)(1,9) and (4,5) and determine its latusrectum.

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23. Show that the point (2,3) lies outside the parabola $y^2 = 3x$.

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24. Find the position of the point (-2,2) with respect to the parabola $y^2 - 4y + 9x + 13 = 0$.

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25. If the point $(at^2, 2at)$ be the extremity of a focal chord of parabola $y^2 = 4ax$ then show that the length of the focal chord is $a\left(t + \frac{1}{t}\right)^2$.

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26. Prove that the semi-latus rectum of the parabola y^(2) = 4ax` is the harmonic mean between the segments of any focal chord of the parabola.

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27. Prove that the length of a focal chord of a parabola varies inversely as the square of its distance from the vertex.

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28. If the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax,$ prove that $\ln = am^2$

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29. Show that $x\cos\alpha + a\sin^2\alpha = p$ touches the parabola $y^2 = 4ax$ if $p\cos\alpha + a\sin^2\alpha = 0$ and that the point of contact is $(a\tan^2\alpha,\ -2a\tan\alpha)$.

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30. Prove that the line $\frac{x}{l} + \frac{y}{m} = 1$ touches the parabola $y^2 = 4a(x+b)$, if $m^2(l+b) + al^2 = 0$.

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31. Find the equation of the straight lines touching both $x^2 + y^2 = 2a^2$ and $y^2 = 8ax$.

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32. Find the equation of common tangent of $y^2 = 4ax$ and $x^2 = 4by$.

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33. Two tangents to the parabola $y^2 = 4ax$ make supplementary angles with the x-axis. Then the locus of their point of intersection is

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34. Show that the locus of the point of intersection of mutually perpendicular tangetns to a parabola is its directrix.

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35. The tangents to the parabola $y^2 = 4ax$ at $P\left(at_1^2, 2at_1\right)$, and $Q\left(at_2^2, 2at_2\right)$, intersect at R. Prove that the area of the triangle PQR is $\frac{1}{2}a^2(t_1 - t_2)^3$

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36. Show that normal to the parabols $y^2 = 8x$ at the point (2,4) meets it again at (18.-12) . Find also the length of the normal chord.

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37. Prove that the chord $y - \sqrt{2}x + 4a\sqrt{2} = 0$ is a normal chord of the parabola $y^2 = 4ax$. Also, find the point on the parabola where the given chord is normal to the parabola.

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38. If A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }. Find A – B and B – A.

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39. Prove that the normal chord to a parabola at the point whose ordinate is equal to the abscissa subtends a right angle at the focus.

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40. If the normal to the parabola $y^2 = 4ax$ at point t_1 cuts the parabola again at point t_2 , then prove that $(t_2)^2 \geq 8$.

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41. If two of the three feet of normal drawn from a point to the parabola $y^2 = 4x$ are (1, 2) and (1,-2), then find the third foot.

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42. The locus of the point through which pass three normals to the parabola $y^2 = 4ax$, such that two of them make angles $\alpha \& \beta$ respectively with the axis & $\tan\alpha \cdot \tan\beta = 2$ is$(a > 0)$

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43. The locus of points such that two of the three normals drawn from them to the parabola $y^2 = 4ax$ coincide is

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44. The vertex of the parabola $y^2 = 4ax$ is

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45. Find the point on the axis of the parabola $3y^2 + 4y - 6x + 8 = 0$ from where three distinct normals can be drawn.

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46. Evaluate $\int x^2 \, dx$

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47. Evaluate $\int 2x^2 \, dx$

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48. Evaluate $\int 3x^2\,dx$

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49. Find the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

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50. The locus of the middle points of normal chords of the parabola $y^2 = 4ax$ is-

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51. If the diameter through any point P of a parabola meets any chord in A and the tangent at the end of the chord meets the diameter in B and C, then prove that $PA^2 = PB.PC$

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52. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

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53. A ray of light is coming along the line $y = b$ from the positive direction of x-axis and striks a concave mirror whose intersection with x-plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray and show that it

passes through the focus of the parabola. Both a and b are positive.

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54. Evaluate $\int 4x^2\,dx$

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55. A ray of light travels along a line $y = 4$ and strikes the surface of curves $y^2 = 4(x + y)$. Then the equations of the line along which of

reflected ray travels is $x = 0$ (b) $x = 2$ (c) $x + y$ (d) $2x + y = 4$

A. x=0

B. x=2

C. x+y=4

D. 2x+y=4

Answer: A

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56. A parabola is drawn with focus at (3,4) and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is

A. $x^2 - 6x - 8y + 25 = 0$

B. $y^2 - 8x - 6y + 25 = 0$

C. $x^2 - 6x + 8y - 25 = 0$

D. $x^2 + 6x - 8y - 25 = 0$

Answer: D

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57. Two parabolas have the same focus. If their directrices are the x- and the y-axis, respectively, then the slope of their common chord is ± 1 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) none of these

A. ± 1

B. $\frac{4}{3}$

C. $\frac{3}{4}$

D. None of these

Answer: A

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58. Let us define a region R is xy-plane as a set of points (x,y) satisfying $\left[x^2\right] = [y]$ (where [x] denotes greatest integer $\leq x$),then the region R defines

A. a parabola whose axis is horizontal

B. a parabola whose axis is vertical

C. integer point of the parabola $y = x^2$

D. None of the above

Answer: D

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59. The minimum area of circle which touches the parabolas $y = x^2 + 1$ and $y^2 = x - 1$ is $\frac{9\pi}{16} sq unit$ (b) $\frac{9\pi}{32} sq unit$ $\frac{9\pi}{8} sq unit$ (d) $\frac{9\pi}{4} sq unit$

A. (a) $\frac{9\pi}{16}$ sq units

B. (b) $\frac{9\pi}{32}$ sq units

C. $\frac{9\pi}{8}$ sq units

D. $\frac{9\pi}{4}$ sq units

Answer: B

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60. If a line $x + y = 1$ cut the parabola $y^2 = 4ax$ in points A and B and normals drawn at A and B meet at C. The normals to

the parabola from C other than above two meets the parabola in D, then point D is :

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61. If d is the distance between the parallel tangents with positive slope to $y^2 = 4x$ and $x^2 + y^2 - 2x + 4y - 11 = 0$, then d ` 10

A. $10 < d < 20$

B. $4 < d < 6$

C. $d < 4$

D. None of these

Answer: C

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62. Two parabolas C and D intersect at two different points, where C is $y = x^2 - 3$ and D is $y = kx^2$. The intersection at which the x value is positive is designated Point A, and x=a at this intersection the tangent line l at A to the curve D intersects curve C at point B ,

other than A. IF x-value of point B is 1, then a equal to

A. 1

B. 2

C. 3

D. 4

Answer: C

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63. Evaluate $\int 5x^2\,dx$

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64. The condition that the parabolas $y^2 = 4c(x-d)$ and $y^2 = 4ax$ have a common normal other than X-axis $(a > 0, c > 0)$ is

a. $2a < 2c + d$

b. $2c < 2a + d$

c. $2d < 2a + c$

d. $2d < 2c + a$

A. $2a < 2c + d$

B. $2c < 2a + d$

C. $2d < 2a + c$

D. $2d < 2c + a$

Answer: A

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65. The locus of the midpoint of the focal distance of a variable point moving on theparabola $y^2 = 4ax$ is a parabola whose (a)latus rectum is half the latus rectum of the original parabola (b)vertex is $\left(\frac{a}{2}, 0\right)$ (c)directrix is y-axis. (d)focus has coordinates (a, 0)

A. latusrectum is half the latusrectum of the original parabola

B. Vertex is $\left(\frac{a}{2}, 0\right)$

C. directrix is Y-axis

D. focus has the coordinate (a,0)

Answer: D

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66. IF P_1P_2 and Q_1Q_2 two focal chords of a parabola $y^2 = 4ax$ at right angles, then

A. area of the quadrilateral $P_1Q_1P_2Q_2$ is minimum when the chords are inclined

at an angle $\pi/4$ to the axis of the parabola.

B. minimum area is twice the area of the square on the latusrectum of the parabola.

C. minimum area of quadrilateral $P_1Q_1P_2Q_2$ cannot be found

D. minimum area is thrice the area of the square on the latusrectum of the parabola.

Answer: A

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67. The equation of the line that touches the curves $y = x|x|$ and $x^2 + (y-2)^2 = 4$, where $x \neq 0$, is:

A. $y = 4\sqrt{5}x + 20$

B. $y = 4\sqrt{3}x - 12$

C. $y = 0$

D. $y = -4\sqrt{5}x - 20$

Answer: A

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68. Let V be the vertex and L be the latusrectum of the parabola $x^2 = 2y + 4x - 4$. Then the equation of the parabola whose vertex is at V. Latusrectum $L/2$ and axis s perpendicular to the axis of the given parabola.

A. $y^2 = x - 2$

B. $y^2 = x - 4$

C. $y^2 = 2 - x$

D. $y^2 = 4 - x$

Answer: A,C

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69. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A'.

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70. Find $\frac{dy}{dx} at (6, 1)$ if $y^2 = 4ax$

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71. Find $\frac{dy}{dx}$ if $2x - 3y = e^x$

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72. Tangents are drawn to the parabola $y^2 = 4x$ at the point P which is the upper end

of latusrectum . Area enclosed by the tangent line at, P,X axis and the parabola is

A. $\frac{2}{3}$sq units

B. $\frac{4}{3}$ sq units

C. $\frac{14}{3}$ sq units

D. $\frac{16}{3}$ sq units

Answer: A

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73. Let C_1 and C_2 be parabolas $x^2 = y - 1$ and $y^2 = x - 1$ respectively. Let P be any point on C_1 and Q be any point C_2. Let P_1 and Q_1 be the reflection of P and Q, respectively w.r.t the line y = x then prove that P_1 lies on C_2 and Q_1 lies on C_1 and $PQ \geq [PP_1, QQ_1]$. Hence or otherwise , determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2

A. C_1 and C_2 respectively

B. C_2 and C_1 respectively

C. Cannot be determined

D. None of these

Answer: B

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74. Evaluate $\int 6x^2\,dx$

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75. Let C_1 and C_2 be respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$ Let P be any point on C_1 and Q be any point on C_2 . Let P_1 and Q_1 be the refelections of P and Q, respectively with respect to the line y=x.

Arithemetic mean of $PP_1$and QQ_1 is always less than

A. PQ

B. $\frac{1}{2}$PQ

C. 2PQ

D. $\frac{3}{2}$PQ

Answer: A

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76. If U = {2, 3, 4, 5, 6, 7, 8, 9} X = {3, 5, 7, 9} Y = {2, 4, 6, 8} Show that X = Y' and Y = X'

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77. Find $\frac{dy}{dx}$ if $y^2 = 4ax$

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78. Evaluate $\int 8x^2\,dx.$

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79. Find $\frac{dy}{dx}$ if $y^2 = 2ax$

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80. Parabola $y^2 = 4a(x - c_1)$ and $x^2 = 4a(y - c_2)$, where $c_1 and c_2$ are variable,

are such that they touch each other. The locus of their point of contact is (a)$xy = 2a^2$ (b) $xy = 4a^2$ (c)$xy = a^2$ (d) none of these

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81. The area formed by the normals to $y^2 = 4ax$ at the points t_1, t_2, t_3 is

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82. The condition that the parabolas $y^2 = 4c(x - d)$ and $y^2 = 4ax$ have a common normal other than X-axis $(a > 0, c > 0)$ is

a. $2a < 2c + d$

b. $2c < 2a + d$

c. $2d < 2a + c$

d. $2d < 2c + a$

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83. If on a given base BC, a triangle is described such that the sum of the tangents of the base angles is m, then prove that the locus of the opposite vertex A is a parabola.

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84. A parabola mirror is kept along $y^2 = 4x$ and two light rays parallel to its axis are reflected along one straight line. If one of the incident light rays is at 3 units distance from

the axis, then find the distance of the other incident ray from the axis.

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85. Evaluate $\int 10x^2\,dx$

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86. Find the slope of normal to the curve if equation of the curve is $y = x^2 + x - 2$ at (1,2)

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87. TP and TQ are any two tangents to a parabola and the tangent at a third point R cuts them in P' and Q'. Prove that $\frac{TP'}{TP} + \frac{TQ'}{TQ} = 1$

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88. Prove that on the axis of any parabola there is a certain point 'k' which has the

property that, if a chord PQ of parabola be drawn through it then $\frac{1}{(PK)^2}+\frac{1}{(QK)^2}$ is the same for all positions of the chord.

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89. If the distribution of weight is uniform, then the rope of the suspended bridge takes the form of parabola.The height of the supporting towers is 20m, the distance between these towers is 150m and the height of the lowest point of the rope from the road

is 3m. Find the equation of the parabolic shape of the rope considering the floor of the parabolic shape of the rope considering the floor of the bridge as X-axis and the axis of the parabola as Y-axis. Find the height of that tower which supports the rope and is at a distance of 30 m from the centre of the road.

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90. Evaluate $\int 11x^2\,dx.$

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91. If a chord PQ of the parabola $y^2 = 4ax$ subtends a right angle at the vertex, show that the locus of the point of intersection of the normals at P and Q is $y^2 = 16a(x - 6a)$.

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92. Find $\frac{dy}{dx}$ if $(3x + a)(x + 3a) = y^2$

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93. Find $\frac{dy}{dx}$ if $\frac{x^2}{a^2}+\frac{y^2}{b^2-a^2}=1$

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94. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2=4a(x+a)$ and the other touches $y^2=4b(x+b)$. Their point of intersection lies on the line. (a) $x-a+b=0$ (b) $x+a-b=0$ (c) $x+a+b=0$ (d) $x-a-b=0$

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Jee Type Solved Examples

1. Evaluate $\int 7x^2\, dx$

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Exercise For Session 1

1. The vertex of the parabola $y^2 + 6x - 2y + 13 = 0$ is

A. (-2,1)

B. (2,-1)

C. (1,1)

D. (1,-1)

Answer: A

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2. If the parabola $y^2 = 4ax$ passes through the point (3,2) then find the length of its latus rectum.

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$

Answer: D

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3. Find the value of P such that the vertex of $y = x^2 + 2px + 13$ is 4 units above the x-axis.

A. ± 2

B. 4

C. ± 3

D. 5

Answer: C

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4. The length of the latusrectum of the parbola whose focus is $(3, 3)$ and directrix $3x - 4y - 2 = 0$, is

A. a) 1

B. b) 2

C. c) 4

D. d) 8

Answer: B

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5. If the vertex and focus of a parabola are $(3, 3)$ and $(-3, 3)$ respectively, then its equation is

A. A. $x^2 - 6x + 24y - 63 = 0$

B. B. $x^2 - 6x + 24y - 81 = 0$

C. C. $y^2 - 6y + 24x - 63 = 0$

D. D. $y^2 - 6y - 24x + 81 = 0$

Answer: C

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6. If the vertex of the parabola $y = x^2 - 8x + c$ lies on x-axis, then the value of c, is

A. 4

B. -4

C. 16

D. -16

Answer: C

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7. The parabola having its focus at (3,2) and directrix along the Y-axis has its vertex at

A. $\left(\frac{3}{2}, 1\right)$

B. $\left(\frac{3}{2}, 2\right)$

C. $\left(\frac{3}{2}, \frac{1}{2}\right)$

D. $\left(\frac{3}{2}, -\frac{1}{2}\right)$

Answer: B

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8. Let the set of integer be the universal set and let A = set of whole numbers, then what is A'?

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9. The equation of the latus retum of the parabola $x^2 + 4x + 2y = 0$ is

A. 3y-2=0

B. 3y+2=0

C. 2y-3=0

D. 2y+3=0

Answer: C

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10. The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is

A. $\left(0, -\frac{1}{2}\right)$

B. (4,4)

C. $\left(4, \frac{9}{2}\right)$

D. $\left(-4, -\frac{9}{2}\right)$

Answer: B

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11. The equation of the parabola with the focus (3,0) and directrix x+3=0 is

A. $y^2 = 2x$

B. $y^2 = 3x$

C. $y^2 = 6x$

D. $y^2 = 12x$

Answer: D

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12. Equation of the parabola whose axis is parallel to Y- axis and which passes through the point (1,0),(0,0)and (-2,4) , is

A. $2x^2 + 2y = 3y$

B. $2x^2 - 2x = 3y$

C. $2x^2 + 2x = y$

D. $2x^2 - 2x = y$

Answer: B

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13. Find the equation of the parabola whose focus is (5,3) and directrix is the line 3x-4y+1=0.

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14. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both hot drinks and cold drinks?

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15. Find the vertex , focus, axis , directrix and latusrectum of the parabola $4y^2 + 12x - 20y + 67 = 0$.

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16. Find the name of the conic represented by $\sqrt{\left(\frac{x}{a}\right)} + \sqrt{\left(\frac{y}{b}\right)} = 1.$

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17. The curve described parametrically by $x = t^2 + t + 1$, and $y = t^2 - t + 1$ represents. (a) a pair of straight lines (b) an ellipse (c) a parabola (d) a hyperbola

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18. Prove that the equation of the parabola whose vertex and focus are on the X-axis at a distance a and a'from the origin respectively is $y^2 = 4(a' - a)(x - a)$

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19. Find the equatin to the parabola whose axis is parallel to the y-xis and which passes through the point $(0, 4), (1.9), \text{ and } (-2, 6)$ and determine its latus rectum.

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20. The equation

$$ax^2 + 4xy + y^2 + ax + 3y + 2 = 0$$

represents a parabola. Find the value of a.

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Exercise For Session 2

1. If $2x + y + \lambda = 0$ is a normal to the parabola $y^2 = -8x$, then λ is

(a)12 (b) -12 (c)24 (d) -24

A. -24

B. -16

C. -8

D. 24

Answer: D

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2. The vertex of the parabola $y^2 + 6x - 2y + 13 = 0$ is

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. $-\frac{1}{\sqrt{2}}$

D. $-\sqrt{2}$

Answer: B::D

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3. The common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is

A. (a) x+y+a=0

B. (b) x+y-a=0

C. (c) x-y+a=0

D. (d) x-y-a=0

Answer: A

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4. The circle $x^2 + y^2 + 4\lambda x = 0$ which $\lambda \in R$ touches the parabola $y^2 = 8x$. The value of λ is given by

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5. If the normals at two points P and Q of a parabola $y^2 = 4ax$ intersect at a third point R on the curve, then the product of ordinates of P and Q is

A. $4a^2$

B. $2a^2$

C. $-4a^2$

D. $8a^2$

Answer: D

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6. Evaluate $\int 13x^2\,dx$

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7. The set of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real , is

A. $\lambda, 0), x > 1$

B. $(\lambda, 1), \lambda > 3$

C. $(\lambda, 2), \lambda > 6$

D. $(\lambda, 3), \lambda > 8$

Answer: B

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8. Evaluate $\int 14x^2 \, dx.$

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9. Find the multiplication of 85×0

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10. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to the line $y = 2x - 5$.

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11. Find the equation of the normal to the parabola $y^2 = 4x$ which is perpendicular to the line 2x+6y+5=0.

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12. Find the multiplication of 95×0

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13. Find the multiplication of 27×0

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14. Find the multiplication of 78×0

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15. If m_1, m_2 are the slopes of the two tangents that are drawn from (2,3) to the parabola $y^2 = 4x$, then the value of $\frac{1}{m_1} + \frac{1}{m_2}$ is

A. -3

B. 3

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: B

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16. Find the angle between the tangents drawn from the origin to the parabolas $y^2 = 4a(x - a)$

A. 90°

B. 30°

C. $\tan^{-1}\left(\frac{1}{2}\right)$

D. 45°

Answer: A

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17. If (a, b) is the midpoint of a chord passing through the vertex of the parabola $y^2 = 4x$, then

(a)$a = 2b$ (b) $a^2 = 2b$ (c)$a^2 = 2b$ (d) $2a = b^2$

A. a=2b

B. 2a=b

C. $a^2 = 2b$

D. $2a = b^2$

Answer: D

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18. The diameter of the parabola $y^2 = 6x$ corresponding to the system of parallel chords 3x-y+c=0 is

A. (a)y-1=0

B. (b)y-2=0

C. (c)y+1=0

D. (d)y+2=0

Answer: A

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19. Tangents are drawn from the point (-1, 2) to the parabola $y^2 = 4x$ The area of the triangle for tangents and their chord of contact is

A. 8

B. $8\sqrt{3}$

C. $8\sqrt{2}$

D. None of these

Answer: C

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20. What is the focus of the parabola $x^2 + y^2 + 2xy - 6x - 2y + 3 = 0.$

A. (1,-1)

B. (1,1)

C. (3,1)

D. None of these

Answer: C

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21. Find the locus of the mid-points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

A. $y^2 - 2ax + 8a^2 = 0$

B. $y^2 = a(x - 4a)$

C. $y^2 = 4a(x - 4a)$

D. $y^2 + 3ax + 4a^2 = 0$

Answer: A

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22. A ray of light moving parallel to the x-axis gets reflected form a parabolic mirror whose equation is $(y-2)^2 = 4(x+1)$. Find the

point on the axis of the parabola through which the ray must pass after reflection.

A. (-2,0)

B. (-1,2)

C. (0,2)

D. (2,0)

Answer: C

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23. Evaluate $\int 16x^2\,dx.$

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24. Evaluate $\int 20x^2\,dx.$

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25. Evaluate $\int 17x^2\,dx.$

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Exercise Single Option Correct Type Questions

1. A common tangent is drawn to the circle $x^2 + y^2 = a^2$ and the parabola $y^2 = 4bx$. If the angle which his tangent makes with the axis of x is $\frac{\pi}{4}$, then the relationship between a and b (a,b > 0)

A. A. $b = \sqrt{2}a$

B. B. $a = b\sqrt{2}$

C. C. c=2a

D. D. a=2c

Answer: A

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2. The equation of the parabola whose vertex and focus lie on the axis of x at distances a and a_1 from the origin, respectively, is

A. $y^2 = 4(a_1 - a)x$

B. $y^2 = 4(a_1 - a)(x - a)$

C. $y^2 = 4(a_1 - a)(x - a_1)$

D. $y^2 = 4aa_1x$

Answer: B

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3. If parabolas $y^2 = \lambda x$ and $25\left[(x-3)^2 + (y+2)^2\right] = (3x - 4y - 2)^2$ are equal, then the value of λ is

A. 3

B. 6

C. 7

D. 9

Answer: B

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4. ABCD and EFGC are squares and the curve $y = k\sqrt{x}$ passes through the origin D and the points B and F.The ratio of $\frac{FG}{BC}$ is:

A. A. $\frac{\sqrt{3}+1}{4}$

B. B. $\frac{\sqrt{3}+1}{2}$

C. C. $\frac{\sqrt{5}+1}{4}$

D. D. $\frac{\sqrt{5}+1}{2}$

Answer: D

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5. Find $\frac{dy}{dx}$ if $y^2 = x$

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6. The vertex of the parabola whose parametric equation is $x = t^2 - t + 1, y = t^2 + t + 1, t \in R$, is

A. (1,1)

B. (2,2)

C. (3,3)

D. $\left(\frac{1}{2}, \frac{1}{2}\right)$

Answer: A

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7. The circle $x^2 + y^2 + 2\lambda x = 0, \lambda \in R,$ touches the parabola $y^2 = 4x$ externally. Then,

A. $\lambda > 0$

B. $\lambda < 0$

C. $\lambda > 1$

D. $\lambda > 2$

Answer: A

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8. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then

A. $d^2 + (2b + 3c)^2 = 0$

B. $d^2 + (3b + 2c)^2 = a^2$

C. $d^2 + (2b - 3c)^2 = 0$

D. $d^2 + (2b + 3c)^2 = a^2$

Answer: A

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9. Evaluate $\int 18x^2\, dx.$

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10. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis at P_1 and P_2. If S is the focal of the parabola, Then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to

A. $\frac{1}{4a}$

B. $\frac{1}{a}$

C. $\frac{2}{a}$

D. $\frac{4}{a}$

Answer: B

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11. Evaluate $\int 22x^2\, dx$.`

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12. The normal to the parabola $y^2 = 4ax$ at three points P,Q and R meet at A. If S is the focus, then prove that $SP \cdot SQ \cdot SR = aSA^2$.

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13. Evaluate $\int 23x^2\,dx.$

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14. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y.$

A. $4\sqrt{3}$

B. 4

C. $\frac{4\sqrt{6}}{7}$

D. $2\sqrt{3}$

Answer: D

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15. From a point $(\sin\theta, \cos\theta)$, if three normals can be drawn to the parabola $y^2 = 4ax$ then the value of a is

A. $\left(\frac{1}{2}, 1\right)$

B. $\left[-\frac{1}{2}, 0\right)$

C. $\left[\frac{1}{2}, 1\right]$

D. $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

Answer: D

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16. If two different tangents to $y^2 = 4x$ are the normals to $x^2 = 4$ by, then

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17. The shortest distance between the parabolas $2y^2 = 2x - 1$ and $2x^2 = 2y - 1$ is

$2\sqrt{2}$ (b) $\frac{1}{2}\sqrt{2}$ (c) 4 (d) $\sqrt{\frac{36}{5}}$

A. $\frac{1}{2\sqrt{2}}$

B. $\frac{1}{2}$

C. $2\sqrt{2}$

D. 4

Answer: A

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18. Normals at two points $(x_1 y_1) and (x_2, y_2)$ of the parabola $y^2 = 4x$ meet again on the parabola, where $x_1 + x_2 = 4$. Then $|y_1 + y_2|$ is equal to

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. $8\sqrt{2}$

Answer: C

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19. A line is drawn from A(-2,0) to intersect the curve $y^2 = 4x$ at P and Q in the first quadrent such that $\frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$. Then the slope of the line is always

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20. Find the length of the side of an equilateral traingle inscribed in the parabola

$y^2 = 4ax$, so that one angular point is at the vertex.

A. $a - (2 - \sqrt{3})$

B. $2a(2 - \sqrt{3})$

C. $4a(2 - \sqrt{3})$

D. $8a(2 - \sqrt{3})$

Answer: C

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21. Let C be a circle with center (0,1) and radius unity. P is the parabola $y = ax^2$. The set of values of a for which they meet at a point other than origin is

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22. Evaluate $\int 24x^2\, dx.$

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23. If P be a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot of perpendicular drawn from the point P on the directrix of the parabola, then find length of each sides of an equilateral triangle SMP(where S is the focus of the parabola).

A. 2

B. 4

C. 6

D. 8

Answer: C

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24. Evaluate $\int 25x^2\,dx.$

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25. Through the vertex O of the parabola $y^2 = 4ax$, two chords $OPandOQ$ are drawn and the circles on OP and OQ as diameters intersect at R. If θ_1, θ_2 , and φ are the angles

made with the axis by the tangents at P and Q on the parabola and by OR, then value of $\cot\theta_1 + \cot\theta_2$ is

A. $-2\tan\phi$

B. $2\tan\phi$

C. 0

D. $2\cot\phi$

Answer: A

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26. Evaluate $\int 26x^2\,dx.$

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27. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then find the values of a and n.

A. $q=p$

B. $q>p$

C. $q<p$

D. $pq = 1$

Answer: D

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28. The set of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ find the slope of normal to the curve at (0,0)

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29. The tangent to the parabola $y = x^2$ has been drawn so that the abscissa x_0 of the point of tangency belongs to the interval [1,2]. Find x_0 for which the triangle bounded by the tangent, the axis of ordinates, and the straight line $y = x_0^2$ has the greatest area.

A. 0

B. 1

C. 2

D. 3

Answer: C

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30. if $y = 4x + 3$ is parallel to a tangent to the parabola $y^2 = 12x$, then its distance from the normal parallel to the given line is

A. $\frac{213}{\sqrt{17}}$

B. $\frac{219}{\sqrt{17}}$

C. $\frac{211}{\sqrt{17}}$

D. $\frac{210}{\sqrt{17}}$

Answer: D

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Exercise More Than One Correct Option Type Questions

1. Equation of the common tangent of a circle $x^2 + y^2 = 50$ and the parabola $y^2 = 40x$ can be

A. x+y-10=0

B. x-y+10=0

C. x+y+10=0

D. x-y-10=0

Answer: B::C

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2. Find $\frac{dy}{dx}$. y=sin $\left(\frac{1-x^2}{1+x^2}\right)$

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3. Let $y^2 = 4ax$ be a parabola and $x^2 + y^2 + 2bx = 0$ be a circle. If parabola and circle touch each externally then:

A. $a > 0, b < 0$

B. $a > 0, b > 0$

C. $a < 0, b > 0$

D. $a < 0, b < 0$

Answer: B::D

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4. Tangent is drawn at any point (x_1, y_1) other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point $(x_2, y_2),$ then

A. (a) x_1, a, x_2 are in GP

B. (b) $\frac{y_1}{2}, a, y_2$ are in GP

C. (c) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in GP

D. (d) $x_1x_2 + y_1y_2 = a^2$

Answer: B::C::D

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5. Let P , Q and R are three co-normal points on the parabola $y^2 = 4ax$. Then the correct statement(s) is /at

A. algebraic sum of the slopes of the normals at P,Q and R vanishes

B. algebraic sum of the ordinates of the points P,Q and R vanishes

C. centeroid of the traingle PQR lies on the axis of the parabola

D. Circle cicrcumscribing the traingle PQR passes through the vertex of the parabola.

Answer: A::B::C::D

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6. Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x + 1$ passes through P, then the ordinate of P may be 3 (b) -1 (c) 5 (d) 1

A. 3

B. -1

C. 5

D. 1

Answer: A::C

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7. If a point P on $y^2 = 4x$, the foot of the perpendicular from P on the directrix and the focus form an equilateral traingle , then the coordinates of P may be

A. $(3, -2\sqrt{3})$

B. $(-3, 2\sqrt{3})$

C. $(3, 2\sqrt{3})$

D. $(-3, -2\sqrt{3})$

Answer: A::C

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8. The locus of foot of the perpendiculars drawn from the focus on a variable tangent to the parabola $y^2 = 4ax$ is

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9. The extremities of latus rectum of a parabola are (1,1) and (1,-1). Then the equation

of the parabola can be

A. $y^2 = 2x - 1$

B. $y^2 = 1 - 2x$

C. $y^2 = 2x - 3$

D. $y^2 = 2x - 4$

Answer: A::C

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10. If from the vertex of a parabola $y^2 = 4x$ a pair of chords be drawn at right angles to one another andwith these chords as adjacent sides a rectangle be made, then the locus of the further end of the rectangle is

A. an equal parabola

B. a parabola with focus at (8a,0)

C. a parabola with directrix as x-7a=0

D. not a parabola

Answer: A::C

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11. If two chords drawn from the point $A(4, 4)$ to the parabola $x^2 = 4y$ are bisected by the line $y = mx$, the interval in which m lies is

$(-2\sqrt{2}, 2\sqrt{2})$ $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$(-\infty, -2\sqrt{2} - 2) \cup (2\sqrt{2} - 2, \infty)$ none of these

A. $m \in (-\infty, -\sqrt{3})$

B. $m \in (-\infty, -\sqrt{3} - 1)$

C. $m \in (\sqrt{3}, \infty)$

D. $m \in \left(\sqrt{3}-1, \infty\right)$

Answer: B::C::D

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12. The set of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real , is

A. (3,2)

B. (1,2)

C. (4,2)

D. (5,2)

Answer: A::C::D

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13. Three normals are drawn from the point $(14, 7)$ to the curve $y^2 - 16x - 8y = 0$. Find the coordinates of the feet of the normals.

A. $(0, 0), (8, -16), (3, -4)$

B. $(0, 0), (8, 16), (3, -4)$

C. $(0, 0), (-8, 16), (3, -4)$

D. $(0, 0), (-8, -16), (3, -4)$

Answer: A::B::C

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14. A quadrilateral is inscribed in a parabola. Then

a. the quadrilateral may be cyclic

b. diagonals of the quadrilateral may be equa

c.l allpossible pairs of adjacent side may be perpendicular

d. none of these

A. the quadrilateral may be cyclic

B. diagonals of the quadrilateral may be equal

C. all possible pairs of adjacent sides may be perpendicular

D. None of the above

Answer: A::B

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Exercise Passage Based Questions

1. Consider a parabola P touches coordinate axes at (4,0) and (0,3).

if focus of parabola P is (a,b) then the value of b-a is

A. $\frac{1}{25}$

B. $\frac{3}{25}$

C. $\frac{4}{25}$

D. $\frac{12}{25}$

Answer: D

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2. Consider a parabola P touches coordinate axes at (4,0) and (0,3).

Length of latus rectum of parabola P is

A. `

B.

C.

D.

Answer: D

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3. Consider a parabola P touches coordinate axes at (4,0) and (0,3).

Equation of directrix of parabola P is

A. $4x+3y=0$

B. 3x+4y=12

C. 3x+4y=0

D. 4x+3y=12

Answer: C

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4. Let C be the locus of the circumcentre of a variable traingle having sides Y-axis ,y=2 and ax+by=1, where (a,b) lies on the parabola $y^2 = 4\lambda x$.

For $\lambda = 2$, the product of coordinates of the vertex of the curve C is

A. -8

B. -6

C. 6

D. 8

Answer: B

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5. Find $\frac{dy}{dx}$ if $y^2 = 4\lambda x$

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6. The locus of the circumcenter of a variable triangle having sides the y-axis, y=2, and lx+my=1, where (1,m) lies on the parabola $y^2 = 4x$, is a curve C.

The curve C is symmetric about the line

A. $x = -\frac{3}{2}$

B. $y = -\frac{3}{2}$

C. $x = \frac{3}{2}$

D. $y = \frac{3}{2}$

Answer: D

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7. The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is

A. (2,1)

B. (-2,1)

C. (-2,-1)

D. (2,-1)

Answer: D

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8. The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is

A. $\frac{3}{\sqrt{5}}$

B. $\frac{6}{\sqrt{5}}$

C. $\frac{12}{\sqrt{5}}$

D. $\frac{24}{\sqrt{5}}$

Answer: C

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9. Consider a parabola

$x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0.$

The focus of the parabola (P) is

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10. If l and m are variable real numbers such that $5l^2 - 4lm + 6m^2 + 3l = 0,$ then the variable line $lx + my = 1$ always touches a fixed parabola, whose axis is parallel to the X-axis.

If (c,d) is the focus of the parabola , then the value of $2^{|d-c|}$ is

A. 1

B. 2

C. 4

D. 8

Answer: B

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11. If l and$m$ are variable real numbers such that $5l^2 - 4lm + 6m^2 + 3l = 0$, then the variable line $lx + my = 1$ always touches a fixed parabola, whose axis is parallel to the X-axis.

If ex+f=0 is directrix of the parabola and e,f are prime numbers , then the value of $|e - f|$ is

A. 2

B. 4

C. 6

D. 8

Answer: D

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12. Find the slope of tangent to the curve if equation of the curve is $y^2 = 4x$ at $(5, 6)$

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13. Find the slope of normal to the curve if equation of the curve is $y^2 = 4x$ at $(4, 5)$

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14. Find $\frac{dy}{dx}$ if $y^2 = 10x$

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15. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

The radius of circle when a attains its maximum value is

A. 1

B. 3

C. 5

D. 7

Answer: A

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16. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the Y-axis also touches the circle $x^2 + y^2 = c^2$. It is known that no point of the parabola is below X-axis. The value of $5c^2$ when a attains its maximum value is

A. 1/(rt.10)

B. 1/(rt.5)

C. 1

D. Rt. 5

Answer: B

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17. Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point

of the parabola is below the x-axis.

The minimum area bounded by the tangent and the coordinate axes is

A. 1

B. 2

C. 4

D. 8

Answer: B

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18. Find the slope of tangent to the curve if equation of the curve is $x^2 + xy + y^2 - 2x - 2y + 1 = 0$ at (0,0)

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19. A parabola (P) touches the conic $x^2 + xy + y^2 - 2x - 2y + 1 = 0$ at the points when it is cut by the line x+y+1=0.

The length of latusrectum of parabola (P) is

A. $\sqrt{2}$

B. $3\sqrt{2}$

C. $5\sqrt{2}$

D. $7\sqrt{2}$

Answer: D

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20. A parabola (P) touches the conic $x^2 + xy + y^2 - 2x - 2y + 1 = 0$ at the points when it is cut by the line x+y+1=0.

If (a,b) is the vertex of the parabola (P), then the value of $|a-b|$ is

A. 0

B. $\frac{1}{2}$

C. 1

D. $\frac{3}{2}$

Answer: A

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21. y=3x is tangent to the parabola $2y = ax^2 + b$. The minimum value of a+b is

A. 2

B. 4

C. 6

D. 8

Answer: C

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22. y=3x is tangent to the parabola $2y = ax^2 + b.$

If (2,6) is the point of contact , then the value of 2a is

A. 2

B. 3

C. 4

D. 5

Answer: B

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23. y=3x is tangent to the parabola $2y = ax^2 + ab.$

If b=36,then the point of contact is

A. (1,3)

B. (2,6)

C. (3,9)

D. (6,18)

Answer: D

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Exercise Single Integer Answer Type Questions

1. Two tangent are drawn from the point $(-2, -1)$ to parabola $y^2 = 4x$. if α is the angle between these tangents, then find the value of $\tan\alpha$.

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2. If the distances of two points P and Q from the focus of a parabola $y^2 = 4x$ are 4 and 9,respectively, then the distance of the point of intersection of tangents at P and Q from the focus is

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3. The tangents and normals are drawn at the extremites of the latusrectum of the parabola

$y^2 = 4x$. The area of quadrilateral so formed is λsq units, the value of λ is

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4. Three normals are drawn from the point (a,0) to the parabola $y^2 = x$. One normal is the X-axis . If other two normals are perpendicular to each other , then the value of 4a is

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5. AB is a chord of the parabola $y^2 = 4ax$ with vertex $A\dot{B}C$ is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is a (b) $2a$ (c) $4a$ (d) $8a$

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6. If the points $a + b, a - b$ and $a + kb$ be collinear, then k is equal to

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7. Let n be the number of integral points lying inside the parabola $y^2 = 8x$ and circle $x^2 + y^2 = 16$, then the sum of the digits of number n is

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8. Radius of the largest circle which passes through the focus of the parabola $y^2 = 4x$ and contained in it, is

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9. If the circle $(x-6)^2+y^2=r^2$ and the parabola $y^2=4x$ have maximum number of common chords, then the least integral value of r is _____ .

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10. The slope of line which belongs to family (1+ l) x + (l-1)y + 2(1-l) = 0 and makes shortest intercept on $x^2=4y-4$

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Parabola Exercise 5

1. Find $\frac{dy}{dx}$ if $x = y^2 - 6y + 11$

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2. Find $\frac{dy}{dx}$ if $x^2 - ay + 3 = 0$

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Exercise Statement I And Ii Type Questions

1. Statement I the equation of the common tangent to the parabolas $y^2 = 4x$ and $x^2 = 4y$ is $x + y + 1 = 0$.

Statement II Both the parabolas are reflected to each other about the line $y = x$.

A. Statement I is true, Statement II is true , Statement II is a correct explanation for statement I.

B. Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is false,Statement II is true.

Answer: A

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2. Statement I two perpendicular normals can be drawn from the point $\left(\frac{5}{2}, -2\right)$ to the parabola $(y+1)^2 = 2(x-1)$.

Statement II two perpendicular normals can be drawn from the point (3a,0) to the parabola $y^2 = 4ax$.

A. Statement I is true, Statement II is true , Statement II is a correct explanation for statement I.

B. Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is false,Statement II is true.

Answer: A

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3. Statement I The line $y = mx + \frac{a}{m}$ is tangent to the parabola $y^2 = 4ax$ for all values of m.

Statement II A straight line y=mx+c intersects the parabola $y^2 = 4ax$ one point is a tangent line.

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4. Statement I: The conic $\sqrt{a}x + \sqrt{b}y = 1$ represents a parabola.

Statement II: Conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola if $h^2 = ab$.

A. (a) Statement I is true, Statement II is true and Statement II is the correct explanation for Statement I.

B. (b) Statement I is true and Statement II is true but Statement II is not the correct explanation for Statement I.

C. (c) Statement I is false, Statement II is false.

D. (d) Statement I is false, Statement II is true.

Answer: C

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5. Statement I: The lines from the vertex to the two extremities of a focal chord of the parabola $y^2 = 4ax$ are perpendicular to each

other.

Statement II: If the extremities of focal chord of a parabola are $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$, then $t_1t_2 = -1$.

A. (a) Statement I is true, Statement II is true and Statement II is the correct explanation for Statement I.

B. (b) Statement I is true and Statement II is true but Statement II is not the correct explanation for Statement I.

C. (c) Statement I is true, Statement II is false.

D. (d) Statement I is false, Statement II is true.

Answer: D

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6. Statement 1: The length of focal chord of a parabola $y^2 = 8x$ making on angle of 60^0 with the x-axis is 32. Statement 2: The length of

focal chord of a parabola $y^2 = 4ax$ making an angle with the x-axis is $4a\cos ec^2\alpha$

A. Statement I is true, Statement II is true , Statement II is a correct explanation for statement I.

B. Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is false,Statement II is true.

Answer: C

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7. Statement I Straight line $x + y = k$ touch the parabola $y = x - x^2$, if k=1.

Statement II Discriminant of $(x - 1)^2 = x - x^2$ is zero.

A. Statement I is true, Statement II is true , Statement II is a correct explanation for statement I.

B. Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is false,Statement II is true.

Answer: C

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8. Statement I Length of latusrectum of parabola $(3x + 4y + 5)^2 = 4(4x + 3y + 2)$ is 4. Statement II Length of latusrectum of parabola $y^2 = 4ax$ is 4a.

A. Statement I is true, Statement II is true , Statement II is a correct explanation for statement I.

B. Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.

C. Statement I is true, Statement II is false.

D. Statement I is false,Statement II is true.

Answer: D

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Exercise Subjective Type Questions

1. If a tangent to the parabola $y^2 = 4ax$ meets the axis of the parabola in T and the tangent at the vertex A in Y, and the rectangle

$TAYG$ is completed, show that the locus of G is $y^2 + ax = 0$.

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2. If incident from point $(-1, 2)$ parallel to the axis of the parabola $y^2 = 4x$ strike the parabola, then find the equation of the reflected ray.

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3. Prove that the normal chord to a parabola $y^2 = 4ax$ at the point whose ordinate is equal to abscissa subtends a right angle at the focus.

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4. Find the shortest distance between the parabola $y^2 = 4x$ and circle $x^2 + y^2 - 24y + 128 = 0.$

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5. Let A and B be Two Given Finite Sets Such That n(A) =20, n(B)=28, and n(A ∪ B) =36, Find n(A ∩ B).

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6. Find the slope of tangent to the curve if equation of the curve is $y^2 = 4ax$ at (0,0)

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7. Through the vertex O of the parabola $y^2 = 4ax$, two chords $OPandOQ$ are drawn and the circles on OP and OQ as diameters intersect at R. If θ_1, θ_2 , and φ are the angles made with the axis by the tangents at P and Q on the parabola and by OR, then value of $\cot\theta_1 + \cot\theta_2$ is

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8. In a class of 40 students, 12 enrolled for both English and German. 22 enrolled for

German. If the students of the class enrolled for at least one of the two subjects, then how many students enrolled for only English and not German?

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9. Find the slope of normal to the curve if equation of the curve is $y^2 = 4ax$ at (0 , 0)

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10. If equation of the curve is $y^2 = 4x$ then find the slope of normal to the curve

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11. Evaluate $\int 27x^2\, dx.$

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12. Let P,Q,R be three points on a parabola $y^2 = 4ax$, normals at which are concurrent.

The centroid of the ΔPQR must lie on

A. (a) a line parallel to the directix

B. (b) the axis of parabola

C. (c) a line of slope 1 passing through vertix

D. (d) none of these

Answer: 0

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13. The normal to the parabola $y^2 = 4ax$ at three points P,Q and R meet at A. If S is the focus, then prove that $SP \cdot SQ \cdot SR = aSA^2$.

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14. Find slope of tangent to the curve $x^2 + y^2 = \frac{a^2}{2}$

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15. Prove that any three tangents to a parabola whose slopes are in harmonic progression enclose slopes are in harmonic progression enclose a traingle of constant area .

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Exercise Questions Asked In Previous 13 Years Exam

1. Let A and B be two finite sets such that n(A) =85, n(B) = 65and n(A ∪ B) = 135, find n(A ∩ B)?

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2. Let P be the point (1,0) and Q be a point on the locus $y^2 = 8x$. The locus of the midpoint of PQ is

A. $x^2 - 4y + 2 = 0$

B. $x^2 + 4y + 2 = 0$

C. $y^2 + 4y + 2 = 0$

D. $y^2 - 4y + 2 = 0$

Answer: D

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3. The axis of a parabola is along the line y=x and the distance of its vertex and focus from origin are $\sqrt{2}$ and $2\sqrt{2}$, respectively. If vertex and focus both lie in the first quadrant, then find equation of the parabola.

A. $(x+y)^2 = (x-y-2)$

B. $(x-y)^2 = (x+y+2)$

C. $(x-y)^2 = 4(x+y-2)$

D. $(x-y)^2 = 8(x+y-2)$

Answer: D

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4. In a school, all pupils play either Hockey or Football or both. 400 play Football, 150 play

Hockey, and 130 play both the games. Find the number of pupils who play Football only.

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5. The locus of the vertex of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^{2x}}{2} - 2a$ is

$xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$ $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$

A. $xy = \frac{105}{64}$

B. $xy = \frac{3}{4}$

C. $xy = \frac{35}{16}$

D. $xy = \frac{64}{105}$

Answer: A

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6. The angle between the tangents to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3, 0) is (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

A. $\pi / 3$

B. $\pi/2$

C. $\pi/6$

D. $\pi/4$

Answer: B

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7. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and fourth quadrant respectively. Tangents to the circle at P and Q intersect the

x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S. The radius of the circumcircle of the triangle PRS is-

A. (a)$3\sqrt{2}$

B. (b)$3\sqrt{3}$

C. (c)5

D. (d)2

Answer: C

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8. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and fourth quadrant respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S. The radius of the circumcircle of the triangle PRS is-

A. 5

B. $3\sqrt{3}$

C. $3\sqrt{2}$

D. $2\sqrt{3}$

Answer: B

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9. Find slope of tangent to the curve if equation is $x^2 + y^2 = 9$

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10. Statement 1 : The curve $y = -\frac{x^2}{2} + x + 1$ is symmetric with respect

to the line $x = 1$ Statement 2 : A parabola is symmetric about its axis.

a. Both the statements are true and Statements 1 is the correct explanation of Statement 2.

b. Both the statements are true but Statements 1 is not the correct explanation of Statement 2.

c. Statement 1 is true and Statement 2 is false

d. Statement 1 is false and Statement 2 is true

A. Statement I is true, Statement II is true, Statement II is a correct explanation for

Statement I

B. Statement I is true, Statement II is true, Statement II is not a corrected explanation for Statement I

C. Statement is true , Statement II is false

D. Statement I is false , Statement II is true

Answer: A

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11. The equation of a tangent to the parabola $y^2 = 8xisy = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

A. (-1,1)

B. (0,2)

C. (2,4)

D. (-2,0)

Answer: D

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12. Consider two curves $C1: y^2 = 4x$; $C2 = x^2 + y^2 - 6x + 1 = 0$. Then,

A. C_1 and C_2 touch each other only at one point

B. C_1 and C_2 touch each other exactly at two points

C. C_1 and C_2 intersect (but do not touch) at exactly two points

D. C_1 and C_2 neither intersect nor touch each other

Answer: B

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13. If a parabola has the origin as its focus and the line x = 2 as the directrix, then the coordinates of the vertex of the parabola are

A. $(0, 2)$

B. $(1, 0)$

C. $(0, 1)$

D. $(2, 0)$

Answer: B

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14. In a school, there are 30 teachers who teach Mathematics or Physics. Of these, 18 teach Mathematics and 6 teach both Physics

and Mathematics. How many teach Physics only?

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15. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then find the slope of the line joining A and B .

A. $-\frac{1}{r}$

B. $\frac{1}{r}$

C. $\frac{2}{r}$

D. $-\frac{2}{r}$

Answer: C::D

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16. If two tangents drawn from a point P to the parabola y2 = 4x are at right angles, then the locus of P is

A. $2x+1=0$

B. $x=-1$

C. $2x-1=0$

D. $x=1$

Answer: B

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17. Of the 125 students in an elementary school, 89 students play checkers and 45

students play checkers and chess. How many students in the school play chess?

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18. What is the 5th term of this sequence 9, 13, 17, ...?

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19. In a group of 400 people, 250 can speak Punjabi and 200 can speak English. How many

people can speak only English?

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20. If $f(x) = a \cos(bx + c) + d$, then what is the range of $f(x)$?

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21. If $f(x) = 3x - 5$, then inverse of $f(x)$ is

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22. Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at point lying on the line $y = 2x + a, a < 0$.

If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

A. $\frac{2}{3}\sqrt{7}$

B. $-\frac{2}{3}\sqrt{7}$

C. $\frac{2}{3}\sqrt{5}$

D. $-\frac{2}{3}\sqrt{5}$

Answer: D

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23. Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at point lying on the line $y = 2x + a, a < 0$.

The length of chord PQ is

A. 7a

B. 5a

C. 2a

D. 3a

Answer: B

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24. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

A. $1/8$

B. $2/3$

C. $1/2$

D. $3/2$

Answer: C

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25. Evaluate $\int 28x^2\,dx.$

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26. Let a, r, s, t be non-zero real numbers. Let $P\left(at^2, 2at\right), Q\left(ar^2, 2ar\right)$ and $S\left(as^2, 2as\right)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K the point (2a,0).

The value of r is

A. $-\frac{1}{t}$

B. $\frac{t^2+1}{t}$

C. $\frac{1}{t}$

D. $\frac{t^2-1}{t}$

Answer: D

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27. Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at), Q(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K the point (2a,0).

If st=1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

A. $\frac{\left(t^2+1\right)^2}{2t^3}$

B. $\frac{a\left(t^2+1\right)^2}{2t^3}$

C. $\frac{a\left(t^2+1\right)^2}{t^3}$

D. $\frac{a\left(t^2+2\right)^2}{t^3}$

Answer: B

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28. Let O be the vertex and Q be any point on the parabola,$x^2 = 8y$. It the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :

A. $x^2 = y$

B. $y^2 = x$

C. $y^2 = 2x$

D. $x^2 = 2y$

Answer: D

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29. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$ then the value of r^2 is _____.

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30. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as

diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

A. $(4, 2\sqrt{2})$

B. $(9, 3\sqrt{2})$

C. $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$

D. $(1, \sqrt{2})$

Answer: A::D

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31. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the center C of the circle , $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its canter at P is

A. $x^2 + y^2 - 4x + 8y + 12 = 0$

B. $x^2 + y^2 - x + 4y - 12 = 0$

C. $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

D. $x^2 + y^2 - 4x + 9y + 18 = 0$

Answer: A

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32. The circle C1 :$x^2 + y^2 = 3$, with center at O, intersects the parabola$x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C1 at P touches other two circles C2 and C3 at R2 and R3, respectively. Suppose C2 and C3 have equal radii $2\sqrt{3}$ and centers Q2 and Q3, respectively.If Q_2 and Q_3 lies on the y-axis, then

A. $Q_2Q_3 = 12$

B. $R_2R_3 = 4\sqrt{6}$

C. area of ΔOR_2R_3 is $6\sqrt{2}$

D. area of ΔPQ_2Q_3 is $4\sqrt{2}$

Answer: A::B::C

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33. Let P be the point on the parabola $y^2 4x$ which is at the shortest distance from the center S of the circle

$x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

A. $SP = 2\sqrt{5}$

B. $SQ : QP = \left(\sqrt{5} + 1\right) : 2$

C. the x-intercept of the normal to the parabola at P is 6

D. the slope of the tangent to the circle at Q is $\frac{1}{2}$

Answer: A::C::D

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34. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is :

A. $4\left(\sqrt{2}+1\right)$

B. $2\left(\sqrt{2}+1\right)$

C. $2\left(\sqrt{2}-1\right)$

D. $4\left(\sqrt{2}-1\right)$

Answer: D

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35. If a chord which is not a tangent, of the parabola $y^2 = 16x$ has the equation 2x+y=p, and mid-point (h, k), then which of the following is (are) possible value (s) of p, h and k?

A. $p = 2, h = 3, k = -4$

B. $p = -1, h = 1, k = -3$

C. $p = -2, h = 2, k = -4$

D. $p = 5, h = 4, k = -3$

Answer: A

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