MATHS

BOOKS - ARIHANT MATHS

PRODUCT OF VECTORS

Example

1. If θ is the angle between the vectors $a = 2\hat{i} + 2\hat{j} - \hat{k}$ and $b = 6\hat{i} - 3\hat{j} + 2\hat{k}$, then

A. $\cos\theta = \frac{4}{21}$

B. $\cos\theta = \frac{3}{19}$

C. $\cos\theta = \frac{2}{19}$

D. $\cos\theta = \frac{5}{21}$

Answer: A

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2. $\left(a \cdot \hat{i}\right)\hat{i} + \left(a \cdot \hat{j}\right)\hat{j} + \left(a \cdot \hat{k}\right)\hat{k}$ is equal to

A. a

B. 2a

C. 3a

D. 0

Answer: A

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3. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4,$ then the value 'lambda' for which $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{a} - \lambda \vec{b}$, is

A. $9/16$

B. $3/4$

C. $3/2$

D. $-3/4$

Answer: B

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4. The projection of vectors $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $vacb = \hat{i} + 2\hat{j} + \hat{k}$ is :

A. $\dfrac{1}{\sqrt{14}}$

B. $\dfrac{2}{\sqrt{14}}$

C. $\sqrt{14}$

D. $\dfrac{-2}{\sqrt{14}}$

Answer: B

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5. If $\overrightarrow{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\overrightarrow{b} = \hat{i} - \hat{j} + \lambda\hat{k}$. Find λ such that $\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - \overrightarrow{b}$ are orthogonal.

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6. Let $\overrightarrow{a}, \overrightarrow{b}\, and\, \overrightarrow{c}$ be unit vectors such that $\overrightarrow{a} + \overrightarrow{b} - \overrightarrow{c} = 0$. If the area of triangle formed by vectors $\overrightarrow{a}$ and $\overrightarrow{b}\, is A$, then what is the value of $4A^2$?

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7. If $\overrightarrow{a}, \overrightarrow{b}$, and $\overrightarrow{c}$ are mutually perpendicular vectors of equal magnitudes, then find the angle between vectors $\overrightarrow{a}$ and $\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}$.

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8. If $a + 2b + 3c = 4$, then find the least value of $a^2 + b^2 + c^2$.

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9. Prove using vectors: The median to the base of an isosceles triangle is perpendicular to the base.

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10. In ΔABC, prove that $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ by vector method.

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11. In any triangle ABC, prove the projection formula $a = b \cos C + c \cos B$ using vector method.

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12. If $\overrightarrow{a} = 4\hat{i} + 6\hat{j}$ and $\overrightarrow{b} = 3\hat{j} + 4\hat{k}$ find the vector component of $\overrightarrow{a}$ along $\overrightarrow{b}$.

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13. Express the vector $\overrightarrow{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\overrightarrow{b} = 3\hat{i} + \hat{k}$ and other is perpendicular to $\overrightarrow{b}\cdot$

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14. Two forces $f_1 = 3\hat{i} - 2\hat{j} + \hat{k}$ and $f_2 = \hat{i} + 3\hat{j} - 5\hat{k}$ acting on a particle at A move it to B. find the work done if the position vector of A and B are $-2\hat{i} + 5\hat{k}$ and $3\hat{i} - 7\hat{j} + 2\hat{k}$.

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15. Forces of magnitudes 5 and 3 units acting in the directions $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + 6\hat{k}$ respectively act on a particle which is displaced from the point (2,2,-1) to (4,3,1) . The work done by the forces, is

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16. If $\overrightarrow{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \overrightarrow{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{0}$ then find (m,n)

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17. Show that $\left(\overrightarrow{a} - \overrightarrow{b}\right) \times \left(\overrightarrow{a} + \overrightarrow{b}\right) = 2\left(\overrightarrow{a} \times \overrightarrow{b}\right)$.

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18. If $\overrightarrow{a}$ is any vector, then $\left(\overrightarrow{a} \times \hat{i}\right)^2 + \left(\overrightarrow{a} \times \hat{j}\right)^2 + \left(\overrightarrow{a} \times \hat{k}\right)^2 =$

A. $|a|^2$

B. 0

C. $3|a|^2$

D. $2|a|^2$

Answer: D

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19. If $\vec{a}.\vec{b}=0$ and $\vec{a}\times\vec{b}=0$, prove that $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$.

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20. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}\dot{\vec{b}}=\vec{a}\dot{\vec{c}}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, $\vec{a}\neq\vec{0}$, then show that $\vec{b}=\vec{c}$

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21. Let $\vec{a}, \vec{b}, \vec{c}$, be three non-zero vectors. If $\vec{a}.\left(\vec{b} \times \vec{c}\right) = 0$ and $\vec{b}$ and $\vec{c}$ are not parallel, then prove that $\vec{a} = \lambda \vec{b} + \mu \vec{c}, where \lambda$ are some scalars.

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22. If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0} \, and \, \vec{b} \neq \vec{c}$, show that $\vec{b} = \vec{c} + t\vec{a}$ for some scalar t.

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23. For any two vectors $\vec{u} \, and \, \vec{v}$ prove that $\left(\vec{u} \cdot \vec{v}\right)^2 + \left|\vec{u} \times \vec{v}\right|^2 = \left|\vec{u}\right|^2 \left|\vec{v}\right|^2 and$

$$\left(\vec{1} + \left|\vec{u}\right|^2\right)\left(\vec{1} + \left|\vec{v}\right|^2\right) = \left(1 - \vec{u} \cdot \vec{v}\right)^2 + \left|\vec{u} + \vec{v} + \left(\vec{u} \times \vec{v}\right)\right|^2$$

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24. The sine of the angle between the vector $a = 3\hat{i} + \hat{j} + \hat{k}$ and $b = 2\hat{i} - 2\hat{j} + \hat{k}$ is

A. $\sqrt{\frac{74}{99}}$

B. $\sqrt{\frac{55}{99}}$

C. $\sqrt{\frac{37}{99}}$

D. $\frac{5}{\sqrt{41}}$

Answer: A

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25. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 5\ and \left|\vec{a} \times \vec{b}\right| = 8$, find $\vec{a} \dot{\vec{b}}$.

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26. The unit vector perpendicular to the vectors $6\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-6\hat{j}-2\hat{k}$, is

A. $\dfrac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

B. $\dfrac{2\hat{i}-3\hat{j}-6\hat{k}}{7}$

C. $\dfrac{2\hat{i}+3\hat{j}-6\hat{k}}{7}$

D. $\dfrac{2\hat{i}+3\hat{j}+6\hat{k}}{7}$

Answer: C

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27. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1)$ and $(0, 2, 1)$.

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28. Let A,B and C be unit vectors. Suppose $A \cdot B = A \cdot C = 0$ and the angle betweenn B and C is $\frac{\pi}{4}$. Then,

a. $A = \pm 2(B \times C)$

b. $A = \pm \sqrt{2}(B \times C)$

c. $A = \pm 3(B + C)$

d. $A = \pm \sqrt{3}(B \times C)$.

A. $A = \pm 2(B \times C)$

B. $A = \pm \sqrt{2}(B \times C)$

C. $A = \pm 3(B + C)$

D. $A = \pm \sqrt{3}(B \times C)$.

Answer: b

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29. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right-handed system, then find $\vec{c}$.

A. $z\hat{i} - x\hat{k}$

B. 0

C. $y\hat{j}$

D. $-z\hat{i} + x\hat{k}$

Answer: A

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30. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \, and \, \vec{b} \times \vec{c} = \vec{a}$; prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually at righ angles such that $\left|\vec{b}\right| = 1 and \left|\vec{c}\right| = \left|\vec{a}\right|$.

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31. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a triangle ABC, show that the area triangle

$ABC is \frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$. Deduce the condition for points $\vec{a}, \vec{b}, \vec{c}$ to be collinear.

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32. Show that perpendicular distance of the point $\vec{c}$ from the line joining $\vec{a}$ and $\vec{b}$ is $\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right|}{\left|\vec{b}-\vec{a}\right|}$

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33. find the area of a parallelogram whose diagonals are $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}+4\hat{k}$.

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34. Find the moment about $(1, -1, -1)$ of the force $3\hat{i}+4\hat{j}-5\hat{k}$ acting at $(1, 0, -2)$.

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35. Three forces $\hat{i}+2\hat{j}-3\hat{k}, 2\hat{i}+3\hat{j}+4\hat{k}$ and $-\hat{i}-\hat{j}+\hat{k}$ acting on a particle at the point (0,1,2) the magnitude of the moment of the forces about the point (1,-2,0) is

A. $2\sqrt{35}$

B. $6\sqrt{10}$

C. $4\sqrt{7}$

D. none of these

Answer: B

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36. The moment about a line through the origin having the direction of $30\left(12\hat{i}-4\hat{j}-3\hat{k}\right)$ is

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37. The moment of the couple formed by the forces $5\hat{i}+\hat{j}$ and $-5\hat{i}-\hat{k}$ acting at the points (9 ,-1,2) and (3,-2,1) respectively, is

A. $-\hat{i}+\hat{j}+5\hat{k}$

B. $\hat{i}-\hat{j}-5\hat{k}$

C. $2\hat{i}-2\hat{j}-10\hat{k}$

D. $-2\hat{i}+2\hat{j}+10\hat{k}$

Answer: B

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38. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)$. Find the velocity of the particle at point $P(3, 6, 4)$.

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39. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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40. Find the volume of the parallelopiped whose edges are represented by $a = 2\hat{i} - 3\hat{j} + 4\hat{k}, b = \hat{i} + 2\hat{j} - \hat{k}$ and $c = 3\hat{i} - \hat{j} + 2\hat{k}$.

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41. Let $a = x\hat{i} + 12\hat{j} - \hat{k}, b = 2\hat{i} + 2x\hat{j} + \hat{k}$ and $c = \hat{i} + \hat{k}$. If b,c,a in that order form a left handed system, then find the value of x.

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42. For any three vectors a, b, c prove that
$$\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]=2\left[\vec{a}\ \vec{b}\ \vec{c}\right].$$

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43. Show that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar.

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44. For any three vectors a,b and c prove that
$$[\mathrm{a}\ \mathrm{b}\ \mathrm{c}]^2=\begin{vmatrix} a\cdot a & a\cdot b & a\cdot c \\ b\cdot a & b\cdot b & b\cdot c \\ c\cdot a & c\cdot b & c\cdot c \end{vmatrix}$$

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45. If a,b,c,l and m are vectors, prove that

$$[\text{a b c}]\,(l \times m) = \begin{vmatrix} a & b & c \\ a \cdot l & b \cdot l & c \cdot l \\ a \cdot m & b \cdot m & c \cdot m \end{vmatrix}$$

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46. If a and b are non-zero and non-collinear vectors, then show that

$$a \times b = [\text{a b i}]\hat{i} + [\text{a b j}]\hat{j} + [\text{a b k}]\hat{k}$$

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47. If a,b and c are any three vectors in space, then show that

$$(c+b) \times (c+a) \cdot (c+b+a) = [\text{a b c}]$$

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48. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then prove that

$$\left(\vec{u} + \vec{v} - \vec{w}\right) \cdot \left[\left(\vec{u} - \vec{v}\right) \times \left(\vec{v} - \vec{w}\right)\right] = \vec{u} \cdot \left(\vec{v} \times \vec{w}\right)$$

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49. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + \mu\vec{c}$ and $(2\lambda - 1)\vec{c}$ are coplanar when

A. no value of λ

B. all except one value of λ

C. all except two values of λ

D. all values of λ

Answer: C

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50. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non -coplanar and l, m, n are distinct scalars such that

$\left[l\vec{a} + m\vec{b} + n\vec{c} \quad l\vec{b} + m\vec{c} + n\vec{a} \quad l\vec{c} + m\vec{a} + n\vec{b}\right] = 0$ then

A. $l+m+n=0$

B. $lm+mn+nl=0$

C. $l^3+m^3+n^3=0$

D. $l^2+m^2+n^2=0$

Answer: A

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51. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar unit vectors each inclined with other at an angle of 30°, then the volume of tetrahedron whose edges are $\vec{a}, \vec{b}, \vec{c}$ is (in cubic units)

A. $\dfrac{\sqrt{3\sqrt{3}-5}}{12}$

B. $\dfrac{3\sqrt{3}-5}{12}$

C. $\dfrac{5\sqrt{2}+3}{12}$

D. none of these

Answer: A

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52. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i}$ and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$, then $\lambda + \mu =$

A. 0

B. 1

C. 2

D. 3

Answer: A

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53. Q8) Ifa, b, c (b, care non-parallel) are unit vectors such that a x $(b \times c)$ = (1/2) then the angle which a makes with b and are en the angle which a makes with b and c are A. 30, 60 B. 600, 90º C. 90, 60 D. 60º, 30º

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54. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 0\hat{j} + \hat{k}$ then find vector $\vec{c}$ satisfying the following conditions, (i) that it is coplaner with $\vec{a}$ and $\vec{b}$, (ii) that its projection on $\vec{b}$ is 0.

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55. Prove that

$$a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0$$

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56. Show that the vectors $\vec{a} \times \left(b \overrightarrow{\times} \vec{c}\right), \vec{b} \times \left(\vec{c} \times \vec{a}\right)$ and $\vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are coplanar.

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57. If $[a \times bb \times cc \times a] = \lambda[abc]^2$, then λ is equal to

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58. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then show that $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are also coplanar.

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59. If A,B and C are vectors such that $|B| = |C|$, prove that $\{(A + B) \times (A + C)\} \times (B \times C) \cdot (B + C) = 0$

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60. If $\vec{b}$ and $\vec{c}$ are two non-collinear such that $\vec{a} \mid \mid \left(\vec{b} \times \vec{c}\right)$. Then prove that $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to $\left|\vec{a}\right|^2\left(\vec{b}.\vec{c}\right)$`

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61. Find the set of vector reciprocal to the set off vectors $2\hat{i}+3\hat{j}-\hat{k},\hat{i}-\hat{j}-2\hat{k},\ -\hat{i}+2\hat{j}+2\hat{k}.$

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62. Find a set of vector reciprocal to the vectors a,b and $a\times b.$

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63. If $a'=\frac{b\times c}{[\mathrm{a\ b\ c}]},b'=\frac{c\times a}{[\mathrm{a\ b\ c}]},c'=\frac{a\times b}{[\mathrm{a\ b\ c}]}$

then show that

$a\times a'+b\times b'+c\times c'=0$, where a,b and c are non-coplanar.

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64. If (e_1, e_2, e_3) and (e'_1, e'_2, e'_3) are two sets of non-coplanar vectors such that $i = 1, 2, 3$ we have $e_i . e'_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$ then show that $[e_1 e_2 e_3][e'_1 e'_2 e'_3] = 1$

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65. Solve the vector equation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{r} . \vec{c} = 0$ provided that $\vec{c}$ is not perpendicular to $\vec{b}$

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66. Solve for X, such that $A \cdot X = C$ and $A \times X = B$ with $C \neq 0$.

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67. Solve the vector equation $\vec{r} \times \vec{a} + k\vec{r} = \vec{b}$, where $\vec{a}, \vec{b}$ are two non-collinear vectors and k is any scalar.

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68. Solve for vectors A and B, where

$A + B = a, A \times B = b, A \cdot a = 1.$

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69. If $\left|\vec{a}\right| = 5, \left|\vec{a} - \vec{b}\right| = 8 \text{ and } \left|\vec{a} + \vec{b}\right| = 10$, then find $\left|\vec{b}\right|$.

A. 1

B. $\sqrt{57}$

C. 3

D. none of these

Answer: B

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70. Find the area of a parallelogram whose adjacent sides are given by $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

A. $\cos^{-1}\left(\frac{1}{3}\right)$

B. $\cos^{-1}\left(\frac{1}{2}\right)$

C. $\cos^{-1}\left(\frac{4}{9}\right)$

D. $\cos^{-1}\left(\frac{5}{9}\right)$

Answer: A

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71. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length $3, 4, 5$ respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is :

A. $2\sqrt{5}$

B. $2\sqrt{2}$

C. $10\sqrt{5}$

D. $5\sqrt{2}$

Answer: D

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72. Let $a, b > 0$ and $\alpha = \frac{\hat{i}}{a} + \frac{4\hat{j}}{b} + b\hat{k}$ and $\beta = b\hat{i} + a\hat{j} + \frac{\hat{k}}{b}$, then the maximum value of $\frac{30}{5 + \alpha \cdot \beta}$ is

A. 3

B. 2

C. 4

D. 8

Answer: A

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73. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\left|\vec{a}-\vec{b}\right|<1$ and $0\le\theta\le\pi$, then θ lies in the interval

A. $\left[0, \frac{\pi}{6}\right)$

B. $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

C. $\left(\frac{5\pi}{6}, \pi\right]$

D. $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$.

Answer: A

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74. If $\vec{a}=3\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ are given vectors. A vector $\vec{c}$ which is perpendicular to z-axis satisfying $\vec{c}.\vec{a}=9$ and $\vec{c}.\vec{b}=-4$. If inclination of $\vec{c}$ with x-axis and y-axis and y-axis is α and β respectively, then which of the following is not true?

A. $\alpha>\frac{\pi}{4}$

B. $\beta > \frac{\pi}{2}$

C. $\alpha > \frac{\pi}{2}$

D. $\beta < \frac{\pi}{2}$

Answer: C

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75. If A is 3×3 matrix and u is a vector. If Au and u are orthogonal for all real u, then matrix A is a

A. singular

B. non-singular

C. symmetric

D. skew-symmetric

Answer: A

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76. Let the cosine of angle between the vectors p and q be λ such that $2p + q = \hat{i} + \hat{j}$ and $p + 2q = \hat{i} - \hat{j}$, then λ is equal to

A. $\frac{5}{9}$

B. $-\frac{4}{5}$

C. $\frac{3}{9}$

D. $\frac{7}{9}$

Answer: B

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77. The three vectors a, b and c with magnitude 3, 4 and 5 respectively and $a + b + c = 0,$ then the value of $a.\,b + b.\,c + c.\,a$ is

A. 47

B. 25

C. 50

D. -25

Answer: D

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78. Let $\overrightarrow{u}, \overrightarrow{v} and \overrightarrow{w}$ be such that $\left|\overrightarrow{u}\right| = 1, \left|\overrightarrow{v}\right| = 2 and \left|\overrightarrow{w}\right| = 3.$ If the projection of $\overrightarrow{v}$ along $\overrightarrow{u}$ is equal to that of $\overrightarrow{w}$ along $\overrightarrow{u}$ and vectors $\overrightarrow{v} and \overrightarrow{w}$ are perpendicular to each other, then $\left|\overrightarrow{u} - \overrightarrow{v} + \overrightarrow{w}\right|$ equals a 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. $\sqrt{14}$

B. $\sqrt{7}$

C. 2

D. 14

Answer: A

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79. If $\vec{a}, \vec{b}, \vec{c}\, areunit \overrightarrow{\longrightarrow} rs, then \left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}^2-\vec{a}^2\right|^2$ does not exceed

A. 4

B. 9

C. 8

D. 6

Answer: B

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80. The value of x for which the angle between $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $\vec{b}=7\hat{i}-2\hat{j}+\hat{k}$ is obtuse and the angle between $\vec{b}$ and the z-axis is acute and less then $\pi/6$

A. $0 < \lambda < \frac{1}{2}$

B. $\lambda > \sqrt{159}$

C. $-\frac{1}{2} < \lambda < 0$

D. null set

Answer: D

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81. The locus of a point equidistant from two points with position vectors $\vec{a}$ and $\vec{b}$ is

A. $\left[r - \frac{1}{2}(a+b)\right] \cdot (a+b) = 0$

B. $\left[r - \frac{1}{2}(a+b)\right] \cdot (a-b) = 0$

C. $\left[r - \frac{1}{2}(a+b)\right] \cdot a = 0$

D. $[r - (a+b)] \cdot b = 0$

Answer: B

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82. If A is (x_1, y_1) where $x_1 = 1$ on the curve $y = x^2 + x + 10$. The value of $\overline{OA}.\ \overline{AB}$ is

A. $-\dfrac{520}{3}$

B. -148

C. 140

D. 12

Answer: B

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83. In a tetrahedron OABC, the edges are of lengths, $|OA| = |BC| = a, |OB| = |AC| = b, |OC| = |AB| = c.$ Let G_1 and G_2 be the centroids of the triangle ABC and AOC such that $OG_1 \perp BG_2,$ then the value of $\dfrac{a^2 + c^2}{b^2}$ is

A. 2

B. 3

C. 6

D. 9

Answer: B

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84. The OABC is a tetrahedron such that $OA^2 + BC^2 = OB^2 + CA^2 = OC^2 + AB^2$,then

A. $OA \perp BC$

B. $OB \perp AC$

C. $OC \perp AB$

D. $AB \perp AC$

Answer: D

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85. If a,b,c and A,B,C $\in$ R-{0} such that $aA + bB + cD + \sqrt{(a^2 + b^2 + c^2)(A^2 + B^2 + C^2)} = 0$, then value of $\frac{aB}{bA} + \frac{bC}{cB} + \frac{cA}{aC}$ is

A. 3

B. 4

C. 5

D. 6

Answer: A

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86. The unit vector in XOZ plane and making angles 45° and 60° respectively with $\overrightarrow{a} = 2i + 2j - k$ and $\overrightarrow{b} = 0i + j - k$, is

A. $\frac{1}{\sqrt{2}}\left(-\hat{i}+\hat{k}\right)$

B. $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{k}\right)$

C. $\frac{\sqrt{3}}{2}\left(\hat{i}+\hat{k}\right)$

D. none of these

Answer: B

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87. the unit vector orthogonal to vector $-\hat{i}+2\hat{j}+2\hat{k}$ and making equal angles with the x- and y-axes is

A. $\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

B. $\frac{1}{3}\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

C. $-\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

D. $\frac{1}{3}\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

Answer: A

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88. If $(a+3b)\cdot(7a-5b)=0$ and $(a-4b)\cdot(7a-2b)=0$. Then, the angle between a and b is

A. 60°

B. 30°

C. 90°

D. none of these

Answer: A

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89. Let two non-collinear vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $\left|\overrightarrow{a}\right|=3$ and $\left|\overrightarrow{b}\right|=2$. If a point P moves so that at any time t its position vector $\overrightarrow{OP}$ (where O is the origin) is given as

$\overrightarrow{OP} = \left(t + \frac{1}{t}\right)\vec{a} + \left(t - \frac{1}{t}\right)\vec{b}$ then least distance of P from the origin is

A. $\sqrt{2\sqrt{133} - 10}$

B. $\sqrt{2\sqrt{133} + 10}$

C. $\sqrt{5 + \sqrt{133}}$

D. none of these

Answer: B

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90. If a,b,c be non-zero vectors such that a is perpendicular to b and c and $|a| = 1, |b| = 2, |c| = 1, b \cdot c = 1$ and there is a non-zero vector d coplanar with a+b and 2b-c and $d \cdot a = 1$, then minimum value of |d| is

A. $\frac{2}{\sqrt{13}}$

B. $\frac{3}{\sqrt{13}}$

C. $\dfrac{4}{\sqrt{5}}$

D. $\dfrac{4}{\sqrt{13}}$.

Answer: D

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91. A groove is in the form of a broken line ABC and the position vectors fo the three points are respectively $2\hat{i} - 3\hat{j} + 2\hat{k}, 3\hat{i} - \hat{k}, \hat{i} + \hat{j} + \hat{k},$ A force of magnitude $24\sqrt{3}$ acts on a particle of unit mass kept at the point A and moves it angle the groove to the point C. If the line of action of the force is parallel to the vector $\hat{i} + 2\hat{j} + \hat{k}$ all along, the number of units of work done by the force is

A. $144\sqrt{2}$

B. $144\sqrt{3}$

C. $72\sqrt{2}$

D. $72\sqrt{3}$

Answer: C

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92. For any vectors $a, b, |a \times b|^2 + (a \cdot b)^2$ is equal to

A. $|a|^2|b|^2$

B. $|a+b|$

C. $|a|^2 - |b|^2$

D. 0

Answer: A

Watch Video Solution

93. If $a = \hat{i} + \hat{j} + \hat{k}, b = \hat{i} + \hat{j} - \hat{k}$, then vectors perpendicular to a and b is/are

A. $\lambda\left(\hat{i}+\hat{j}\right)$

B. $\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

C. $\lambda\left(\hat{i}+\hat{k}\right)$

D. none of these

Answer: C

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94. If $a \times b = b \times c \neq 0$, then the correct statement is

a. $b||c$

b. $a||b$

c. $(a+c)||b$

d. none of these

A. $b \mid \mid c$

B. $a \mid \mid b$

C. $(a+c) \mid \mid b$

D. none of these

Answer: C

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95. If $a = \hat{i} + 2\hat{j} + 3\hat{k}, b = -\hat{i} + 2\hat{j} + \hat{k}$ and $c = 3\hat{i} + \hat{j}$. If $(a + tb) \perp c$, then t is equal to

A. 5

B. 4

C. 3

D. 2

Answer: A

Watch Video Solution

96. If $a = 2\hat{i} - 3\hat{j} + \hat{k}$, $b = -\hat{i} + \hat{k}$, $c = 2\hat{k}j - \hat{k}$, then the area (in sq units) of parallelogram with diagonals a+b and b+c will be

A. $\sqrt{21}$

B. $2\sqrt{21}$

C. $\frac{1}{2}\sqrt{21}$

D. none of these

Answer: C

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97. The coordinates of the mid-points of the sides of ΔPQR, are $(3a, 0, 0)$, $(0, 3b, 0)$ and $(0, 0, 3c)$ respectively, then the area of ΔPQR is

A. $18\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

B. $9\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

C. $\frac{9}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

D. $18\sqrt{ab + bc + ca}$

Answer: A

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98. In a parallelogram $ABCD, AB = \hat{i} + \hat{j} + \hat{k}$ and diagonal $AC = \hat{i} - \hat{j} + \hat{k}$ and area of parallelogram is $\sqrt{8}sq$ units, $\angle BAC$ is equal to

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\sin^{-1}\left(\frac{\sqrt{8}}{3}\right)$

D. $\cos^{-1}\left(\frac{\sqrt{8}}{3}\right)$

Answer: C

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99. Let $\triangle ABC$ be a given triangle. If $\left|\overrightarrow{BA} - t\overrightarrow{BC}\right| \geq \left|\overrightarrow{AC}\right|$ for any $t \in R$,then $\triangle ABC$ is

A. Equilateral

B. Right angled

C. Isosceles

D. none of these

Answer: B

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100. If $a^2 + b^2 + c^2 = 1$ where, a,b,$c \in R$, then the maximum value of $(4a - 3b)^2 + (5b - 4c)^2 + (3c - 5a)^2$ is

A. 25

B. 50

C. 144

D. none of these

Answer: B

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101. If a, b, c are then p^{th}, q^{th}, r^{th}, terms of an HP and $\overrightarrow{u} = (q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$ and $\overrightarrow{v} = \frac{\hat{i}}{a} + \frac{\hat{j}}{b} + \frac{\hat{k}}{c}$ then

A. u and v are parallel vectors

B. u and v are orthogonal vectors

C. $u \cdot v = 1$

D. $u \times v = \hat{i} + \hat{j} + \hat{k}$.

Answer: B

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102. If the vector product of a constant vector $\overrightarrow{O}A$ with a variable vector $\overrightarrow{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is (a)a straight line perpendicular to $\overrightarrow{O}A$ (b)a circle with centre O and radius equal to $\left|\overrightarrow{O}A\right|$ (c) a straight line parallel to $\overrightarrow{O}A$ (d) none of these

A. a straight line perpendicular to OA

B. a circle with centre O radius equal to |OA|

C. a straight line parallel to OA

D. none of these

Answer: C

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103. Unit vector perpendicular to the plane of ΔABC with position vectors a,b,c of the vertices A,B,C is

A. $\dfrac{a \times b + b \times c + c \times a}{\Delta}$

B. $\dfrac{a \times b + b \times c + c \times a}{2\Delta}$

C. $\dfrac{a \times b + b \times c + c \times a}{4\Delta}$

D. none of these

Answer: B

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104. The vector r satisfying the conditions that I. it is perrpendicular to $3\hat{i} + 2\hat{j} + 2\hat{k}$ and $18\hat{i} - 22\hat{j} - 5\hat{k}$ II. It makes an obtuse angle with Y-axis III. $|r| = 14$.

A. $2\left(-2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

B. $2\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

C. $4\hat{i} + 6\hat{j} - 12\hat{k}$

D. none of these

Answer: A

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105. Let a,b,c denote the lengths of the sides of a triangle such that

$(a-b)\overrightarrow{u}+(b-c)\overrightarrow{v}+(c-a)\left(\overrightarrow{u}\times\overrightarrow{v}\right)=\overrightarrow{0}$

For any two non-collinear vectors $\overrightarrow{u}$ and $\overrightarrow{v}$,then the triangle is

A. right angled

B. equilateral

C. isosceles

D. scalene

Answer: B

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106. The value of the following expression

$\hat{i}.\left(\hat{j}\times\hat{k}\right)+j.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{j}\times\hat{i}\right)$is

A. 3

B. 2

C. 1

D. 0

Answer: A

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107. For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}, \left|\left(\vec{a} \times \vec{b}\right). \vec{c}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$ holds if and only if

A. $a \cdot b = 0, b \cdot c = 0$

B. $b \cdot c = 0, c \cdot a = 0$

C. $c \cdot a = 0, a \cdot b = 0$

D. $a \cdot b = b \cdot c = c \cdot a = 0$

Answer: D

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108. The position vectors of three vertices A,B,C of a tetrahedron OABC with respect to its vertex O are $6\hat{i}, 6\hat{j}, \hat{k}$, then its volume (in cu units) is

A. 3

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. 6

Answer: D

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109. A parallelepiped is formed by planes drawn parallel to coordinate axes through the points A=(1,2,3) and B=(9,8,5). The volume of that parallelepiped is equal to (in cubic units)

A. 192

B. 48

C. 32

D. 96

Answer: D

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110. If $|a| = 1, |b| = 3 \text{ and } |c| = 5,$ then the value of $[a - b \quad b - c \quad c - a]$ is

A. 0

B. 1

C. -1

D. none of these

Answer: A

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111. If a,b,c are three non-coplanar vectors, then $3a-7b-4c, 3a-2b+c$ and $a+b+\lambda c$ will be coplanar, if λ is

A. -1

B. 1

C. 3

D. 2

Answer: D

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112. Find the number of terms in the AP -3,1,5,9.......,237

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113. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lies in a plane then c is

A. HM of a and b

B. 0

C. AM of a and b

D. GM of a and b

Answer: D

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114. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number, then $\left[\lambda\left(\vec{a} + \vec{b}\right) \quad \lambda^2 \vec{b} \quad \lambda \vec{c}\right] = \left[\vec{a} \quad \vec{b} + \vec{c} \quad \vec{b}\right]$ for

A. exactly two values of λ

B. exactly one value of λ

C. exactly three values of λ.

D. no value of λ

Answer: C

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115. In a regular tetrahedron, let θ be angle between any edge and a face not containing the edge. Then the value of $\cos^2\theta$ is

A. $1/6$

B. $1/9$

C. $1/3$

D. none of these

Answer: C

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116. DABC be a tetrahedron such that AD is perpendicular to the base ABC and $\angle ABC = 30^{\circ}$. The volume of tetrahedron is 18. if value of $AB + BC + AD$ is minimum, then the length of AC is

A. $6\sqrt{2-\sqrt{3}}$

B. $3(\sqrt{6}-\sqrt{2})$

C. $6\sqrt{2+\sqrt{3}}$

D. $3(\sqrt{6}+\sqrt{2})$.

Answer: A

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117. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \hat{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then find the value of $\begin{vmatrix} \vec{a} . \vec{a} & \vec{a} . \vec{b} & \vec{a} . \vec{c} \\ \vec{b} . \vec{a} & \vec{b} . \vec{b} & \vec{b} . \vec{c} \\ \vec{c} . \vec{a} & \vec{c} . \vec{b} & \vec{c} . \vec{c} \end{vmatrix}$

A. 2

B. 4

C. 16

D. 64

Answer: C

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118. Find the value of a so that the volume of the parallelopiped formed by vectors $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum.

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: C

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119. If a,b and c be any three non-zero and non-coplanar vectors, then any vector r is equal to

where, $x = \frac{[rbc]}{[abc]}, y = \frac{[rca]}{[abc]}, z = \frac{[rab]}{[abc]}$

A. $za + xb + yc$

B. $xz + yb + zc$

C. $ya + zb + xc$

D. none of these

Answer: B

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120. The position vectors of vertices of ΔABC are a,b,c and $a \cdot a = b \cdot b = c \cdot c = 3.$ if [a b c]=0, then the position vectors of the orthocentre of ΔABC is

A. $a + b + c$

B. $\frac{1}{3}(a + b + c)$

C. 0

D. none of these

Answer: A

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121. Find the discriminant of the following quadratic equation :

$2x^2 + 5x + 7 = 0$

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122. Let $\vec{b} = -\vec{i} + 4\vec{j} + 6\vec{k}$, $\vec{c} = 2\vec{i} - 7\vec{j} - 10\vec{k}$. If $\vec{a}$ be a unit vector and the scalar triple product $\left[\vec{a}\vec{b}\vec{c}\right]$ has the greatest value then $\vec{a}$ is

A. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

B. $\frac{1}{\sqrt{5}}\left(\sqrt{2}\hat{i}-\hat{j}-\sqrt{2}\hat{k}\right)$

C. $\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

D. $\frac{1}{\sqrt{59}}\left(3\hat{i}-7\hat{j}-\hat{k}\right)$

A. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

B. $\frac{1}{\sqrt{5}}\left(\sqrt{2}\hat{i}-\hat{j}-\sqrt{2}\hat{k}\right)$

C. $\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

D. $\frac{1}{\sqrt{59}}\left(3\hat{i}-7\hat{j}-\hat{k}\right)$

Answer: C

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123. Prove that vectors

$\vec{u}=(al+a_1l_1)\hat{i}+(am+a_1m_1)\hat{j}+(an+a_1n_1)\hat{k}$

$\vec{v}=(bl+b_1l_1)\hat{i}+(bm+b_1m_1)\hat{j}+(bn+b_1n_1)\hat{k}$

$\vec{w}=(cl+c_1l_1)\hat{i}+(cm+c_1m_1)\hat{j}+(cn+c_1n_1)\hat{k}$ are coplanar.

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124. Find the discriminant of the following quadratic equation :

$16x^2 = 40x - 25$

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125. If a,b and c are three mutually perpendicular vectors, then the projection of the vectors $l\frac{a}{|a|} + m\frac{b}{|b|} + n\frac{(a \times b)}{|a \times b|}$ along the angle bisector of the vectors a and b is

A. $\frac{l+m}{\sqrt{2}}$

B. $\sqrt{l^2 + m^2 + n^2}$

C. $\frac{\sqrt{l^2 + m^2}}{\sqrt{l^2 + m^2 + b^2}}$

D. none of these

Answer: A

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126. If the volume of the parallelopiped formed by the vectors $\vec{a}, \vec{b}, \vec{c}$ as three coterminous edges is 27 units, then the volume of the parallelopiped having $\vec{\alpha} = \vec{a} + 2\vec{b} - \vec{c}, \vec{\beta} = \vec{a} - \vec{b}$ and $\vec{\gamma} = \vec{a} - \vec{b} - \vec{c}$ as three coterminous edges, is

A. 27

B. 9

C. 81

D. none of these

Answer: C

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127. If V is the volume of the parallelepiped having three coterminous edges as $\vec{a}, \vec{b}$ and $\vec{c}$, then the volume of the parallelepiped having three coterminous edges as

$\vec{\alpha} = (\vec{a}.\vec{a})\vec{a} + (\vec{a}.\vec{b})\vec{b} + (\vec{a}.\vec{c})\vec{c}$,

$\vec{\beta} = (\vec{b}.\vec{a})\vec{a} + (\vec{b}.\vec{b}) + (\vec{b}.\vec{c})\vec{c}$

and $\vec{\lambda} = (\vec{c}.\vec{a})\vec{a} + (\vec{c}.\vec{b})\vec{b} + (\vec{c}.\vec{c})\vec{c}$ is

A. V^3

B. $3V$

C. V^2

D. 2V

Answer: A

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128. The nth term of an AP is 5-6n.Find a and d.

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129. The 4th term of AP is 7 and 10th term is 16. Find a and d.

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130. let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2, respectively, if $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is ______

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. none of these

Answer: C

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131. Let $a = 2\hat{i} + \hat{j} + \hat{k}, b = \hat{i} + 2\hat{j} - \hat{k}$ and c is a unit vector coplanar to them. If c is perpendicular to a, then c is equal to

A. $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$

B. $-\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

C. $\frac{1}{\sqrt{5}}\left(\hat{i} - 2\hat{j}\right)$

D. $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

Answer: A

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132. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, and $\vec{b} = \hat{i} + \hat{j}$ if c is a vector such that $\vec{a} . \vec{c} = \left|\vec{c}\right|, \left|\vec{c} - \vec{a}\right| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{i} s30^\circ$, then $\left|\left(\vec{a} \times \vec{b}\right)\right| \times \vec{c}\,\Big|$ is equal to

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 2

D. 3

Answer: B

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133. Let $\overrightarrow{a} = \hat{i} - \hat{j}, \overrightarrow{b} = \hat{j} - \hat{k}, \overrightarrow{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\overrightarrow{a} . \hat{d} = 0 = \left[\overrightarrow{b} \overrightarrow{c} \overrightarrow{d}\right]$ then $\hat{d}$ equals

A. $\pm \frac{\left(\hat{i} + \hat{j} + 2\hat{k}\right)}{\sqrt{6}}$

B. $\pm \frac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}}$

C. $\pm \frac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}}$

D. $\pm \hat{k}$

Answer: A

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134. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non coplanar and unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer: A

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135. The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is (a) $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ (c) $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

A. $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\dfrac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

C. $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$

D. $\dfrac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$.

Answer: C

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136. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that no two are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\dfrac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$ if θ is the acute angle between vectors $\vec{b}$ and $\vec{c}$ then find value of $\sin\theta$.

A. $\dfrac{2\sqrt{2}}{3}$

B. $\dfrac{\sqrt{2}}{3}$

C. $\dfrac{2}{3}$

D. $\dfrac{1}{3}$

Answer: A

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137. The value for $[a \times (b+c), b \times (c-2a), c \times (a+3b)]$ is equal to

A. $[abc]^2$

B. $7[abc]^2$

C. $-5[a \times b \quad b \times c \quad c \times a]$

D. none of these

Answer: B

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138. If a,b,c and p,q,r are reciprocal systemm of vectors, then $a \times p + b \times q + c \times r$ is equal to

A. $[abc]$

B. $[p+q+r]$

C. 0

D. a+b+c

Answer: C

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139. Find the AP whose third term is 4 times first term and 6 th term is 17.

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140. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $Re(z_1 \bar{z}_2) = 0$, then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies

a. $|\omega_1| = 1$

b. $|\omega_2| = 1$

c. $Re(\omega_1 \overline{\omega}_2) = 0$

d. None of these

A. $|w_1| = r$

B. $|w_2| = r$

C. $w_1 \cdot w_2 = 0$

D. none of these

Answer: A::B::C

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141. If unit vectors $\hat{i}$ and $\hat{j}$ are at right angles to each other and $p = 3\hat{i} + 4\hat{j}, q = 5\hat{i}, 4r = p + q$ and $2s = p - q$, then

A. $|r + ks| = |r - ks|$ for all real k

B. r is perpendicular to s

C. $r + s$ is perpendicular to r-s

D. $|r| = |s| = |p| = |q|$

Answer: A::B::C

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142. Let $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}.\vec{a}=\vec{b}.\vec{b}=\vec{c}.\vec{c}=3$ and $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=$ then

A. a,b and c are necessarily coplanar

B. a,b and c represent sides of a triangle in magnitude and direction

C. $a \cdot b+b \cdot c+c \cdot a$ has the least value $-9/2$

D. a,b and c represent orthogonal triad of vectors

Answer: A::B::C

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143. Find the discriminant of the following quadratic equation :

$6x^2-4x-7=0$

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144. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\frac{3\left|\overrightarrow{a}+\overrightarrow{b}\right|}{2}+2\left|\overrightarrow{a}-\overrightarrow{b}\right|$

A. 2

B. 3

C. 4

D. 5

Answer: B::C::D

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145. Which of the following expressions are meaningful?

A. $u \cdot (v \times w)$

B. $(u \cdot v) \cdot w$

C. $(u \cdot v)w$

D. $u \times (v \cdot w)$

Answer: A::C

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146. If $a + 2b + 3c = 0$, then $a \times b + b \times c + c \times a$ is equal to

A. $2(a \times b)$

B. $6(b \times c)$

C. $3(c \times a)$

D. 0

Answer: A::B::C

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147. Let $\alpha = a\hat{i} + b\hat{j} + c\hat{k}, \overrightarrow{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\overrightarrow{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplanar vectors with $a \neq b$ and $\overrightarrow{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\overrightarrow{v}$ is perpendicular to

A. α

B. β

C. γ

D. none of these

Answer: A::B::C

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148. If $\overrightarrow{a}$ is perpendicular to $\overrightarrow{b}$ and $\overrightarrow{r}$ is non-zero vector such that $p\overrightarrow{r} + \left(\overrightarrow{r}\dot{\overrightarrow{a}}\right)\overrightarrow{b} = \overrightarrow{c}$, then $\overrightarrow{r} = \dfrac{\overrightarrow{c}}{p} - \dfrac{\left(\overrightarrow{a}\dot{\overrightarrow{c}}\right)\overrightarrow{b}}{p^2}$ (b) $\dfrac{\overrightarrow{a}}{p} - \dfrac{\left(\dot{\overrightarrow{}}\ \overrightarrow{b}\right)\overrightarrow{a}}{p^2}$ $\dfrac{\overrightarrow{a}}{p} - \dfrac{\left(\overrightarrow{a}\dot{\overrightarrow{b}}\right)\overrightarrow{c}}{p^2}$ (d) $\dfrac{\overrightarrow{c}}{p^2} - \dfrac{\left(\overrightarrow{a}\dot{\overrightarrow{c}}\right)\overrightarrow{b}}{p}$

A. $[rac] = 0$

B. $p^2 r = pa - (c \cdot a)b$

C. $p^2 r = pb - (a \cdot b)c$

D. $p^2 r = pc - (b \cdot c)a$

Answer: A::D

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149. If $\alpha(a \times b) + \beta(b \times c) + \gamma(c \times a) = 0$, then

A. a,b,c are coplanar if all of $\alpha, \beta, \gamma \neq 0$

B. a,b,c are coplanar if any one of $\alpha, \beta, \gamma \neq 0$

C. a,b,c are non-coplanar for any $\alpha, \beta, \gamma \neq 0$

D. none of these

Answer: A::B

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150. If $a=\hat{i}+\hat{j}+\hat{k}$ and $b=\hat{i}-\hat{j}$, then vectors $\left(\left(a\cdot\hat{i}\right)\hat{i}+\left(a\cdot\hat{j}\right)\hat{j}+\left(a\cdot\hat{k}\right)\hat{k}\right),\left\{\left(b\cdot\hat{i}\right)\hat{i}+\left(b\hat{j}\right)\hat{j}+\left(b\cdot\hat{k}\right)\hat{k}\right\}$ and $\left(\hat{i}\right.$

A. are mutually perpendicular

B. are coplanar

C. form a parallepiped of volume 6 units

D. form a parallelopiped of volume 3 units

Answer: A::C

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151. The volume of the parallelepiped whose coterminous edges are represented by the vectors $2\vec{b}\times\vec{c}, 3\vec{c}\times\vec{a}$ and $4\vec{a}\times\vec{b}$ where

$\vec{b}=\sin\left(\theta+\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta+\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta+\frac{4\pi}{3}\right)\hat{k},$

$\vec{c}=\sin\left(\theta-\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta-\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta-\frac{4\pi}{3}\right)\hat{k}$

is 18 cubic units, then the values of θ, in the interval $\left(0,\frac{\pi}{2}\right)$, is/are

A. $\frac{\pi}{9}$

B. $2\frac{\pi}{9}$

C. $\frac{\pi}{3}$

D. $4\frac{\pi}{9}$

Answer: A::B::D

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152. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, $\text{then } \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

(a)parallel to $(y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$ (b)orthogonal to $\hat{i} + \hat{j} + \hat{k}$ (c)orthogonal to $(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$

(d)orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

A. parallel to $(y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$

D. parallel to $\hat{i} + \hat{j} + \hat{k}$

Answer: A::B::C

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153. If a, b, c are three non-zero vectors, then which of the following statement(s) is/are ture?

A. $a \times (b \times c), b \times (c \times a), c \times (a \times b)$ from a right handed system.

B. c, $(a \times b) \times c$, $a \times b$ from a right handed system.

C. $a \cdot b + b \cdot c + c \cdot a < 0$, iff a+b+c=0

D. $\dfrac{(a \times b) \cdot (b \times c)}{(b \times c) \cdot (a \times c)} = -1$, if a+b+c=0.

Answer: B::C::D

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154. Unit vectors $\vec{a}$ and $\vec{b}$ ar perpendicular , and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c} = \alpha \vec{a} + \beta \vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$ then.

A. α=β

B. $\gamma^2 = 1 - 2\alpha^2$

C. $\gamma^2 = -\cos 2\theta$

D. $\beta^2 = \dfrac{1 + \cos 2\theta}{2}$

Answer: A::B::C::D

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155. $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is perpendicular to which vectors:

A. $(a \cdot c)|b|^2 = (a \cdot b)(b \cdot c)$

B. $a \cdot b = 0$

C. $a \cdot c = 0$

D. $b \cdot c = 0$

Answer: A::C

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156. If $\left(\vec{a} \times \vec{b}\right) = 0$ then what can we say about vectors ?

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157. Find the angle of triangle, two of whose angles are 25 and 60 degree.

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158. Let the vectors PQ,OR,RS,ST,TU and UP represent the sides of a regular hexagon.

Statement I: $PQ \times (RS + ST) \neq 0$

Statement II: $PQ \times RS = 0 \text{ and } PQ \times ST \neq 0$

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: C

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159. p,q and r are three vectors defined by $p = a \times (b + c), q = b \times (c + a)$ and $r = c \times (a + b)$

Statement I: p,q and r are coplanar.

Statement II: Vectors p,q,r are linearly independent.

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: C

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160. Statement I. If in a $\triangle ABC, \overrightarrow{BC} = \frac{\overrightarrow{p}}{\left|\overrightarrow{p}\right|} - \frac{\overrightarrow{q}}{\left|\overrightarrow{q}\right|}$ and $\overrightarrow{AC} = \frac{2\overrightarrow{p}}{\left|\overrightarrow{p}\right|}, \left|\overrightarrow{p}\right| \ne \left|\overrightarrow{q}\right|$ then the value of $\cos 2A + \cos 2B + \cos 2C$ is -1.,

Statement II. If in $\triangle ABC, \angle C = 90^0 then \cos 2A + \cos 2B + \cos 2C = -1$

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: B

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161. Statement I: If a is perpendicular to b and c, then $a \times (b \times c) = 0$

Statement II: if a is perpendicular to b and c, then $b \times c = 0$

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: C

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162. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. Find their dot product.

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163. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then

$\vec{a}_2$ is equal to

A.

B.

C.

D.

Answer: A

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164. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then

$\vec{a}_2$ is equal to

A. a and a_2 are collinear

B. a_1 and c are collinear

C. a, a_1 and b are coplanar

D. a, a_1 and a_2 are coplanar

Answer: C

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165. Let a, b be two vectors perependicular to each other and $|a| = 2, |b| = 3$ and $c \times a = b.$ Q. The least value of |c-a| is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{3}{2}$

Answer: D

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166. Let a, b be two vectors perependicular to each other and $|a| = 2, |b| = 3$ and $c \times a = b.$ Q. The least value of |c-a| is

A. $\tan^{-1}(2)$

B. $\dfrac{\tan^{-1}(3)}{4}$

C. $\cos^{-1}\left(\dfrac{2}{3}\right)$

D. None of these

Answer: B

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167. Let a, b be two vectors perependicular to each other and $|a| = 2, |b| = 3$ and $c \times a = b$. Q. The least value of |c-a| is

A. $\dfrac{1}{2}$

B. $\dfrac{7}{2}$

C. $\dfrac{5}{2}$

D. 4

Answer: C

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168. Consider a triangular pyramid ABCD the position vectors of whose angular points are $A(3, 0, 1)$, $B(-1, 4, 1)$, $C(5, 2, 3)$ and $D(0, -5, 4)$. Let G be the point of intersection of the medians of triangle BCD. Q. Area of triangle ABC in sq. units is

A. $\sqrt{17}$

B. $\frac{\sqrt{51}}{3}$

C. $\frac{3}{\sqrt{6}}$

D. $\frac{\sqrt{59}}{4}$

Answer: B

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169. Consider a triangulat pyramid ABCD the position vector of whose angular points are $A(3, 0, 1)$, $B(-1, 4, 1)$, $C(5, 2, 3)$ and $D(0, -5, 4)$

. Let G be the point of intersection of the medians of the $\triangle (BCD)$.

Q. Area of the $\triangle (ABC)$ (in sq. units) is

A. 24

B. $8\sqrt{6}$

C. $4\sqrt{6}$

D. None of these

Answer: C

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170. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of AG is

A. $\frac{14}{\sqrt{6}}$

B. $\frac{2}{\sqrt{6}}$

C. $\frac{3}{\sqrt{6}}$

D. None of these

Answer: A

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171. If AP, BQ and CR are the altitudes of acute $\triangle ABC$ and $9AP + 4BQ + 7CR = 0$ $\angle ABC$ is equal to

A. a. $\frac{\cos^{-1}(2)}{\sqrt{7}}$

B. b. $\frac{\pi}{2}$

C. c. $\cos^{-1}\left(\frac{\sqrt{7}}{3}\right)$

D. d. $\frac{\pi}{3}$

Answer: A

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172. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$ Q. Volume of parallelopiped with edges a, b, c is

A. $p + (q + r)\cos\theta$

B. $(p + q + r)\cos\theta$

C. $2p - (q + r)\cos\theta$

D. None of these

Answer: A

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173. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$ Q. The value of $\left(\frac{q}{p} + 2\cos\theta\right)$ is

A. (a) 1

B. (b) 0

C. (c) $2[abc]$

D. (d) None of these

Answer: B

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174. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$ Q. The value of $\left(\frac{q}{p} + 2\cos\theta\right)$ is

A. $(1 + \cos\theta)\left(\sqrt{1 - 2\cos\theta}\right)$

B. $2\frac{\sin(\theta)}{2}\sqrt{(1 + 2\cos\theta)}$

C. $(1 - \sin\theta)\sqrt{1 + 2\cos\theta})$

D. None of these

Answer: B

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175. Given that $\overrightarrow{u} = \hat{i} - 2\hat{j} + 3\hat{k}; \overrightarrow{v} = 2\hat{i} + \hat{j} + 4\hat{k}; \overrightarrow{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $\left(\overrightarrow{u}\dot{\overrightarrow{R}} - 15\right)\hat{i} + \left(\overrightarrow{v}\dot{\overrightarrow{R}} - 30\right)\hat{j} + \left(\overrightarrow{w}\dot{\overrightarrow{R}} - 20\right)\hat{k} = 0.$ Then find the greatest integer less than or equal to $\left|\overrightarrow{R}\right|.$

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176. The position vector of a point P is $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k},$ where $x, y, z \in N$ and $\overrightarrow{a} = \hat{i} + 2\hat{j} + \hat{k}.$ If $\overrightarrow{r} \cdot \overrightarrow{a} = 20$ and the number of possible of P is 9λ, then the value of λ is:

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177. Let $\overrightarrow{u}$ be a vector on rectangular coordinate system with sloping angle 60^0. Suppose that $\left|\overrightarrow{u}-\hat{i}\right|$ is geometric mean of $\left|\overrightarrow{u}\right| and \left|\overrightarrow{u}-2\hat{i}\right|, where \hat{i}$ is the unit vector along the x-axis. Then find the value of $(\sqrt{2}+1)\left|\overrightarrow{u}\right|$.

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178. Let $A\left(2\hat{i}+3\hat{j}+5\hat{k}\right), B\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ and $C\left(\lambda\hat{i}+5\hat{j}+\mu\hat{k}\right)$ are vertices of a triangle and its median through A is equally inclined to the positive directions of the axes, the value of $2\lambda-\mu$ is equal to

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179. Three vectors $a(|a|\neq 0)$, b and c are such that $a\times b=3a\times c$, also $|a|=|b|=1$ and $|c|=\frac{1}{3}$. If the angle between b and c is 60° and $|b-3c|=\lambda|a|$, then the value of λ is

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180. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}\dot{\vec{b}} = 0 = \vec{a}\dot{\vec{c}}$ and the angle between $\vec{b}\, and\, \vec{c}$ is $\pi/3$, then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is $1/2$ b. 1 c. 2 d. none of these

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181. The area of the triangle whose vertices are A (1,-1,2) , B (1,2, -1) ,C (3, -1, 2) is _______.

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182. Let $\vec{O}A = \vec{a}, \hat{O}B = 10\vec{a} + 2\vec{b}\, and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq,$ then find$k\cdot$

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183. If , $\vec{x}$, $\vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a-2)\alpha^2+(b-3)\alpha+c\right]\vec{x}+\left[(a-2)\beta^2+(b-3)\beta+c\right]\vec{y}$ + $\left[(a-2)\gamma^2+(b-3)\gamma+c\right]\left(\vec{x}\times\vec{y}\right)=0, \text{ where } \alpha, \beta, \gamma$ are three distinct distinct real numbers, then find the value of $\left(a^2+b^2+c^2-4\right)$

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184. Let $\vec{V}=2\hat{i}+\hat{j}-\hat{k}and\vec{W}=\hat{i}+3\hat{k}\cdot$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is a.-1 b. $\sqrt{10}+\sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

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185. Let $a=\alpha\hat{i}+2\hat{j}-3\hat{k}, b=\hat{i}+2\alpha\hat{j}-2\hat{k}$ and $c=2\hat{i}-\alpha\hat{j}+\hat{k}$. Then the value of 6α, such that $\{(a\times b)\times(b\times c)\}\times(c\times a)=a$, is

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186. If $\widehat{a}$ and $\hat{b}$ are unit vectors such that $\left(\widehat{a}+\hat{b}\right)$ is a unit vector, what is the angle between $\widehat{a}$ and $\hat{b}$?

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187. Determine the value of c so that for all real x, vectors $cx\hat{i}-6\hat{j}-3\hat{k}$ and $x\hat{i}+2\hat{j}+2cx\hat{k}$ make an obtuse angle with each other.

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188. A, B, C and D are four points in space. Then, $AC^2+BD^2+AD^2+BC^2 \geq$

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189. Prove that the perpendicular let fall from the vertices of a triangle to the opposite sides are concurrent.

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190. Using vector method, prove that the angel in a semi circle is a right angle.

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191. The corner P of the square OPQR is folded up so that the plane OPQ is perpendicular to the plane OQR, find the angle between OP and QR.

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192. In a $\triangle ABC$, prove by vector method that $\cos 2A + \cos 2B + \cos 2C \geq \frac{-3}{2}$.

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193. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy- plane. All vectors in the sme plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$., respectively, are given by _______

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194. If a, b and c are three coplanar vectors. If a is not parallel to b, show that $c = \dfrac{\begin{vmatrix} c\cdot a & a\cdot b \\ c\cdot b & b\cdot b \end{vmatrix} a + \begin{vmatrix} a\cdot a & c\cdot a \\ a\cdot b & c\cdot b \end{vmatrix} b}{\begin{vmatrix} a\cdot a & a\cdot b \\ a\cdot b & b\cdot b \end{vmatrix}}$.

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195. In $\triangle ABC$, D is the mid point of the side AB and E is centroid of $\triangle CDA$. If $OE \cdot CD = 0$, where O is the circumcentre of $\triangle ABC$, using vectors prove that AB=AC.

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196. Let I be the incentre of $\triangle ABC$. Using vectors prove that for any point P $a(PA)^2 + b(PB)^2 + c(PC)^2 = a(IA)^2 + b(IB)^2 + c(IC)^2 + (a+b+c)(IP)^2$ where a, b, c have usual meanings.

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197. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

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198. Prove by vector method that $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

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199. A circle is inscribed in an n-sided regular polygon $A_1, A_2, \ldots. A_n$ having each side a unit for any arbitrary point P on the circle, pove that

$$\sum_{i=1}^{n}(PA_i)^2 = n\frac{a^2}{4}\frac{1+\cos^2\left(\frac{\pi}{n}\right)}{\sin^2\left(\frac{\pi}{n}\right)}$$

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200. If a,b and c are position vectors off the vertices A,B and C of ΔABC, show that the area of ΔABC is $\frac{1}{2}|a\times b+b\times c+c\times a|$.

Deduce the condition for points a,b and c to be collinear.

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201. In a $\triangle ABC$points D,E,F are taken on the sides BC,CA and AB respectively such that $\frac{BD}{DC}=\frac{CE}{EA}=\frac{AF}{FB}=n$ prove that

$$\triangle DEF = \frac{n^2-n+1}{(n+1)^2}\triangle ABC$$

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202. Let the area of a given triangle ABC be Δ. Points $A_1, B_1,$ and C_1 , are the mid points of the sides BC,CA and AB respectively. Point A_2 is the mid point of CA_1. Lines C_1A_1 and AA_2 meet the median BB_1 points E and D respectively. If Δ_1 be the area of the quadrilateral A_1A_2DE, using vectors or otherwise find the value of $\frac{\Delta_1}{\Delta}$

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203. If $\vec{a}, \vec{b},$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b},$ then prove that $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$.

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204. If a, b, c and d are four coplanr points, then prove that $[abc] = [bcd] + [abd] + [cad]$.

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205. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w}+\vec{w}\times\vec{u}=\vec{v}$, then prove that $\left|\left(\vec{u}\times\vec{v}\right).\vec{w}\right|\leq\frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

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206. Prove that

$$\vec{R}+\frac{\left[\vec{R}\vec{\beta}\times\left(\dot{\vec{\beta}}\times\vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha}\times\vec{\beta}\right|^{2}}+\frac{\left[\vec{R}\vec{\alpha}\times\left(\dot{\vec{\alpha}}\times\vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha}\times\vec{\beta}\right|^{2}}=\frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\ldots\right.}{\left|\vec{\alpha}\times\ldots\right.}$$

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207. Prove that the formula for the volume V of a tetrahedron, in terms of the lengths of three coterminous edges and their mutul inclinations is

$$V^2=\frac{a^2b^2c^2}{36}\begin{vmatrix}1 & \cos\phi & \cos\psi\\ \cos\phi & 1 & \cos\theta\\ \cos\psi & \cos\theta & 1\end{vmatrix}$$

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208. A pyramid with vertex at point P has a regular hexagonal base ABCDEF. Position vectors of points A and B are $\hat{i}$ and $\hat{i}+2\hat{j}$, respectively. The centre of the base has the position vector $\hat{i}+\hat{j}+\sqrt{3}\hat{k}$. Altitude drawn from P on the base meets the diagonal AD at point G. Find all possible vectors of G. It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP is 5 units.

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209. Let $\widehat{a}$, $\hat{b}$,and $\hat{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c}$ is α , between $\hat{c}$ and $\widehat{a}$ is β and between $\widehat{a}$ and $\hat{b}$ is γ . If $A(\widehat{a}\cos\alpha, 0), B\left(\hat{b}\cos\beta, 0\right)$ and $C(\hat{c}\cos\gamma, 0)$, then show that in triangle ABC, $\dfrac{\left|\widehat{a}\times\left(\hat{b}\times\hat{c}\right)\right|}{\sin A}=\dfrac{\left|\hat{b}\times(\hat{c}\times\widehat{a})\right|}{\sin B}=\dfrac{\left|\hat{c}\times\left(\widehat{a}\times\hat{b}\right)\right|}{\sin C}$

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210. Let a and b be given non-zero and non-collinear vectors, such that $c \times a = b - c$. Express c in terms for a, b and aXb.

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JEE Type Solved Examples: Passage Based Type Questions

1. Let A, B, C respresent the vertices of a triangle, where A is the origin and B and C have position b and c respectively. Points M, N and P are taken on sides AB, BC and CA respectively, such that `(AM)/(AB)=(BN)/(BC)=(CP)/(CA)=alpha Q. AN+BP+CM is

A. a. $3\alpha(b+c)$

B. b. $\alpha(b+c)$

C. c. $(1-\alpha)(b+c)$

D. d. 0

Answer: D

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2. Let A, B, C respresent the vertices of a triangle, where A is the origin and B and C have position b and c respectively. Points M, N and P are taken on sides AB, BC and CA respectively, such that `(AM)/(AB)=(BN)/(BC)=(CP)/(CA)=alpha Q. AN+BP+CM is

A. concurrent

B. sides of a triangle

C. non coplanar

D. None of these

Answer: B

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3. Let A, B, C represent the vertices of a triangle, where A is the origin and B and C have position b and c respectively.* Points M, N and P are taken

on sides AB, BC and CA respectively, such that $\frac{AM}{AB}=\frac{BN}{BC}=\frac{CP}{CA}=\alpha$. If $\triangle$ represent the area enclosed by the three vectors AN, BP and CM, then the value of α, for which $\triangle$ is least

A. a. does not exist

B. b. $\frac{1}{2}$

C. c. $\frac{1}{4}$

D. d. None of these

Answer: B

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JEE Type Solved Examples: Match Type Questions

1. Match the items of column I with items of column II

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2. Match the items of column I with items of column II

Watch Video Solution

3. Match the items of column I with items of column II

Watch Video Solution

4. Match the items of column I with items of column II

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JEE Type Solved Examples: Subjective Type Questions

1. If the angles of a right angle triangle are in AP then the angles of triangle are:

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2. Find the slope of the normal to the curve x = 1 − sin 2 θ , y = cos 2 θ at θ = π .

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Exercise For Session 1

1. The angle between the vectors $\overrightarrow{a} = \hat{i} - 2\hat{j} + 3\hat{k}$and $\hat{b} = 3\hat{i} - 2\hat{j} - \hat{k}$ is :

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2. Find the angle between two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ with magnitudes $\sqrt{3}$ and 2,respectively having $\overrightarrow{a} \cdot \overrightarrow{b} = \sqrt{6}$

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3. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$ are at right angles.

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4. If $\vec{r}.\hat{i} = \vec{r}.\hat{j} = \vec{r}.\hat{k} and \left|\vec{r}\right| = 3,$ then find the vector $\vec{r}$.

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5. Find the angle between the vectors a+b and a-b, if $a = 2\hat{i} - \hat{j} + 3\hat{k}$ and $b = 3\hat{i} + \hat{j} - 2\hat{k}$.

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6. Find the angle between the vectors $\hat{i} + 3\hat{j} + 7\hat{k}$ and $7\hat{i} - \hat{j} + 8\hat{k}$.

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7. If the scalar projection of vector $x\hat{i}-\hat{j}+\hat{k}$ on vector $2\hat{i}-\hat{j}+5\hat{k}, is \dfrac{1}{\sqrt{30}}$,then find the value of x.

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8. If $\left|\vec{a}\right|+\left|\vec{b}\right|=\left|\vec{c}\right|$ and $\vec{a}+\vec{b}=\vec{c}$, then find the angle between $\vec{a}$ and $\vec{b}$.

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9. If three unit vectors $\vec{a}, \vec{b},$ and $\vec{c}$ satisfy $\vec{a}+\vec{b}+\vec{c}=0,$ then find the angle between $\vec{a}$ and $\vec{b}$.

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10. If $\vec{a}=x\hat{i}+(x-1)\hat{j}+\hat{k}$ and $\vec{b}=(x+1)\hat{i}+\hat{j}+a\hat{k}$ make an acute angle $\forall x \in R,$ then find the values of a.

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11. Find the component of $\widehat{a}$ in the direction of vector $\hat{i} + \hat{j} + 2\hat{k}$.

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12. Find the vector component of a vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ along and perpendicular to the non-zero vector $2\hat{i} + \hat{j} + 2\hat{k}$.

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13. A particle acted by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \hat{k}$. find the total work done by the forces in units.

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Exercise For Session 2

1. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

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2. Find the values of γ and μ for which $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \gamma\hat{j} + \mu\hat{k}\right) = \vec{0}$

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3. If $a = 2\hat{i} + 3\hat{j} - \hat{k}, b = -\hat{i} + 2\hat{j} - 4\hat{k}, c = \hat{i} + \hat{j} + \hat{k}$, then find the value of $(a \times b) \cdot (a \times c)$.

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4. Prove that $\left(\vec{a} . \hat{i}\right)\left(\vec{a} \times \hat{i}\right) + \left(\vec{a} . j\right)\left(\vec{a} \times \hat{j}\right) + \left(\vec{a} . \hat{k}\right)\left(\vec{a} \times \hat{k}\right) = 0.$

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5. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, provided $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$

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6. If $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} . \vec{b}\right)^2 = 144$ and $\left|\vec{a}\right| = 4,$ then find the value of $\left|\vec{b}\right|$.

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7. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 7 \, and \, \vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k},$ find the angle between $\vec{a} \, and \, \vec{b}$.

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8. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, *then*, $\vec{a} \times \vec{b}$ is a unit vector, if the angel between $\vec{a}$ and $\vec{b}$ is?

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9. If $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$, $f \in d\,\vec{a}\,.\,\vec{b}$

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10. Find a unit vector perpendicular to the plane of two vectors $a = \hat{i} - \hat{j} + 2\hat{k}$ and $b = 2\hat{i} + 3\hat{j} - \hat{k}$.

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11. Find a vector of magnitude 15, which is perpendicular to both the vectors $\left(4\hat{i} - \hat{j} + 8\hat{k}\right)$ and $\left(-\hat{j} + \hat{k}\right)$.

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12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}$. $\vec{d}$ =15.

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13. Let A,B and C be unit vectors . Suppuse that A.B=A.c=O and that the angle between Band C is $\pi/6$ then prove that

$A = \pm 2(B \times C)$

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14. Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a} = \left(-2\hat{i} - 5\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2\hat{j} - \hat{k}\right)$.

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15. Find the area of parallelogram whose adjacent sides are represented by the vectors $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} - \hat{k}$.

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16. Find the area of the parallelogram whose diagonals are $a = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

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17. A force $F = 2\hat{i} + \hat{j} - \hat{k}$ acts at point A whose position vector is $2\hat{i} - \hat{j}$. Find the moment of force F about the origin.

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18. Find the moment of $\overrightarrow{F}$ about point (2, -1, 3), where force $\overrightarrow{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is acting on point (1, -1, 2).

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19. Forces $2\hat{i} + \hat{j}, 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$ act at a point P, with position vector $4\hat{i} - 3\hat{j} - \hat{k}$. Find the moment of the resultant of these force about the point Q whose position vector is $6\hat{i} + \hat{j} - 3\hat{k}$.

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Exercise For Session 3

1. If $\vec{a} \, and \, \vec{b}$ are two vectors such that $\left|\vec{a} \times \vec{b}\right| = 2,$ then find the value of $\left[\vec{a} \, \vec{b} \, \vec{a} \times \vec{b}\right]$.

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2. If the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k} and 3\hat{i} - \hat{k}$ form three concurrent edges of a parallelepiped, then find the volume of the parallelepiped.

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3. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)$.

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4. The position vectors of the four angular points of a tetrahedron are $A(\hat{j} + 2\hat{k}), B(3\hat{i} + \hat{k}), C(4\hat{i} + 3\hat{j} + 6\hat{k}) and D(2\hat{i} + 3\hat{j} + 2\hat{k})$. Find the volume of the tetrahedron $ABCD$.

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5. Find the altitude of a parallelopiped whose three conterminous edges are verctors $A = \hat{i} + \hat{j} + \hat{k}, B = 2\hat{i} + 4\hat{j} - \hat{k}$ and $C = \hat{i} + \hat{j} + 3\hat{k}$ with A and B as the sides of the base of the parallelopiped.

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6. Examine whether the vectors $a = 2\hat{i} + 3\hat{j} + 2\hat{k}$, $b = \hat{i} - \hat{j} + 2\hat{k}$ and $c = 4\hat{i} + 2\hat{j} + 4\hat{k}$ form a left handed or a right handed system.

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7. Show that the vectors $\hat{i} - \hat{j} - 6\hat{k}$, $\hat{i} - 3\hat{j} + 4\hat{k}$ and $2\hat{i} - 5\hat{j} + 3\hat{k}$ are coplanar.

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8. Prove that $[abc][uvw] = \begin{vmatrix} a \cdot u & b \cdot u & c \cdot u \\ a \cdot v & b \cdot v & c \cdot v \\ a \cdot w & b \cdot w & c \cdot w \end{vmatrix}$

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9. If $[abc] = 2$, then find the value of $[(a + 2b - c)(a - b)(a - b - c)]$.

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10. If $\vec{a}, \vec{b}, and \vec{c}$ are three non-coplanar vectors, then find the value of

$$\frac{\vec{a}\,\vec{b}\overset{.}{}\times\vec{c}}{\vec{b}\,\vec{c}\overset{.}{}\times\vec{a}}+\frac{\vec{b}\,\vec{c}\overset{.}{}\times\vec{a}}{\vec{c}\,\vec{a}\overset{.}{}\times\vec{b}}+\frac{\vec{c}\,\vec{b}\overset{.}{}\times\vec{a}}{\vec{a}\,\vec{b}\overset{.}{}\times\vec{c}}.$$

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Exercise For Session 4

1. Find the value of $\alpha \times (\beta \times \gamma)$, where $\alpha = 2\hat{i} - 10\hat{j} + 2\hat{k}, \beta = 3\hat{i} + \hat{j} + 2\hat{k}, \gamma = 2\hat{i} + \hat{j} + 3\hat{k}.$

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2. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k} and 2\hat{k} - 3\hat{j}$.

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3. Show that

$(b \times c) \cdot (a \times d) + (a \times b) \cdot (c \times d) + (c \times a) \cdot (b \times d) = 0$

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4. Prove that $\hat{i} \times \left(\vec{a} \times \hat{i}\right) + \hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}$.

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5. Prove that $[a \times b, a \times c, d] = (a \cdot d)[a, b, c]$

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6. If $\vec{a}, \vec{b}$, and $\vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}, \vec{b}$ and $\vec{c}$ are non-parallel, then prove that the angel between $\vec{a}$ and $\vec{b}, is 3\pi/4$.

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7. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, \hat{i}+j+\hat{k}$.

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8. If a, b, c and a', b', c' are recoprocal system of vectors, then prove that $a' \times b' + b' \times c' + c' \times a' = \frac{a+b+c}{[abc]}$.

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9. Solve: $\vec{r} \times \vec{b} = \vec{a}, where \vec{a}$ and $\vec{b}$ are given vectors such that $\vec{a} . \vec{b} = 0.$

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10. Find vector $\overrightarrow{r}$ if $\overrightarrow{r} . \overrightarrow{a} = m$ and $\overrightarrow{r} \times \overrightarrow{b} = \overrightarrow{c}$, where $\overrightarrow{a} . \overrightarrow{b} \neq 0$

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Exercise (Single Option Correct Type Questions)

1. If a has magnitude 5 and points North-East and vector b has magnitude 5 and point North-West, then |a-b| is equal to

A. 25

B. 5

C. $7\sqrt{3}$

D. $5\sqrt{2}$

Answer: D

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2. If $|a+b| > |a-b|$, then the angle between a and b is

A. acute

B. obtuse

C. $\frac{\pi}{2}$

D. π

Answer: A

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3. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} = \vec{b} + \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{2}$, then

A. $a^2 = b^2 + c^2$

B. $b^2 = a^2 + c^2$

C. $c^2 = a^2 + b^2$

D. $2a^2 - b^2 = c^2$

Answer: A

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4. If the angle between the vectors a and b be θ and $a \cdot b = \cos\theta$ then the true statement is

A. a and b are equal vectors

B. a and b are like vectors

C. a and b are unlike vectors

D. a and b are unit vectors

Answer: D

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5. If the vectors $\hat{i} + \hat{j} + \hat{k}$ makes angle α, β and γ with vectors $\hat{i}, \hat{j}$ and $\hat{k}$ respectively, then

A. $\alpha = \beta \neq \gamma$

B. $\alpha = \gamma \neq \beta$

C. $\beta = \gamma \neq \alpha$

D. $\alpha = \beta = \gamma$

Answer: D

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6. $\left(r \cdot \hat{i}\right)^2 + \left(r \cdot \hat{j}\right)^2 + \left(r \cdot \hat{k}\right)^2$ is equal to

A. $3r^2$

B. r^2

C. 0

D. None of these

Answer: B

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7. If $\widehat{a}$ and $\hat{b}$ are two unit vectors inclined at an angle θ, then $\sin\left(\frac{\theta}{2}\right)$

A. $\frac{1}{2}|a - b|$

B. $\frac{1}{2}|a + b|$

C. $|a - b|$

D. $|a + b|$

Answer: A

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8. If $\overrightarrow{A} = 4\hat{i} + 6\hat{j}$ and $\overrightarrow{B} = 3\hat{j} + 4\hat{k}$, then find the component of $\overrightarrow{A}\,along\overrightarrow{B}$

A. $\frac{18}{10\sqrt{3}}\left(3\hat{j} + 4\hat{k}\right)$

B. $\frac{18}{25}\left(3\hat{j} + 4\hat{k}\right)$

C. $\frac{18}{\sqrt{3}}\left(3\hat{j} + 4\hat{k}\right)$

D. $\left(3\hat{j} + 4\hat{k}\right)$

Answer: B

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9. If vectors $a = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and vector $b = -2\hat{i} + 2\hat{j} - \hat{k}$, then (projection of vector a on b vectors)/(projection of vector b on a vector) is equal to

A. $\frac{3}{7}$

B. $\frac{7}{3}$

C. 3

D. 7

Answer: B

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10. If $\vec{a}$ and $\vec{b}$ are two vectors, then prove that

$$\left(\vec{a}\times\vec{b}\right)^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b}\end{vmatrix}$$

A. $\begin{vmatrix}a\cdot b & a\cdot a\\ b\cdot b & b\cdot a\end{vmatrix}$

B. $\begin{vmatrix}a\cdot a & a\cdot b\\ b\cdot a & b\cdot b\end{vmatrix}$

C. $\begin{vmatrix}a\cdot b\\ b\cdot a\end{vmatrix}$

D. None of these

Answer: b

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11. The moment of the force F acting at a point P, about the point C is

A. $F\times CP$

B. $CP\cdot F$

C. a vector having the same direction as F

D. $CP\times F$

Answer: D

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12. The moment of a force represented by $F = \hat{i} + 2\hat{j} + 3\hat{k}$ about the point $2\hat{i} - \hat{j} + \hat{k}$ is equal to

A. $5\hat{i} - 5\hat{j} + 5\hat{k}$

B. $5\hat{i} + 5\hat{j} - 6\hat{k}$

C. $-5\hat{i} - 5\hat{j} + 5\hat{k}$

D. $-5\hat{i} - 5\hat{j} + 2\hat{k}$

Answer: D

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13. A force of magnitude 6 acts along the vector $(9, 6, -2)$ and passes through a point $A(4, -1, -7)$. Then moment of force about the point

$O(1, -3, 2)$ is

A. $\frac{150}{11}\left(2\hat{i} - 3\hat{j}\right)$

B. $\frac{6}{11}\left(50\hat{i} - 75\hat{j} + 36\hat{k}\right)$

C. $150\left(2\hat{i} - 3\hat{k}\right)$

D. $6\left(50\hat{i} - 75\hat{j} + 36\hat{k}\right)$

Answer: A

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14. A force $F = 2\hat{i} + \hat{j} - \hat{k}$ acts at point A whose position vector is $2\hat{i} - \hat{j}$. Find the moment of force F about the origin.

A. $\hat{i} + 2\hat{j} - 4\hat{k}$

B. $\hat{i} - 2\hat{j} - 4\hat{k}$

C. $\hat{i} + 2\hat{j} + 4\hat{k}$

D. $\hat{i} - 2\hat{j} + 4\hat{k}$

Answer: C

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15. If a, b and c are any three vectors and their inverse are a^{-1}, b^{-1} and c^{-1} and $[abc] \neq 0$, then $\left[a^{-1} b^{-1} c^{-1}\right]$ will be

A. zero

B. one

C. non-zero

D. [a b c]

Answer: C

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16. If a, b and c are three non-coplanar vectors, then find the value of $\frac{a \cdot (b \times c)}{b \cdot (c \times a)} + \frac{b \cdot (c \times a)}{c \cdot (a \times b)} + \frac{c \cdot (a \times b)}{a \cdot (b \times c)}$.

A. a) 0

B. b) 2

C. c) -2

D. d) None of these

Answer: A

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17. $a \times (b \times c)$ is coplanar with

A. b and c

B. a and c

C. a and b are unlike vectors

D. a, b and c

Answer: A

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18. If $u = \hat{i}\left(a \times \hat{i}\right) + \hat{j}\left(a \times \hat{j}\right) + \hat{k}\left(a \times \hat{k}\right)$, then

A. $u = 0$

B. $u = \hat{i} + \hat{j} + \hat{k}$

C. $u = 2a$

D. $u = a$

Answer: a

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19. If $a = \hat{i} + 2\hat{j} - 2\hat{k}, b = 2\hat{i} - \hat{j} + \hat{k}$ and $c = \hat{i} + 3\hat{j} - \hat{k}$, then $a \times (b \times c)$ is equal to

A. $20\hat{i} - 3\hat{j} + 7\hat{k}$

B. $20\hat{i} - 3\hat{j} - 7\hat{k}$

C. $20\hat{i} + 3\hat{j} - 7\hat{k}$

D. None of these

Answer: A

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20. If $a \times (b \times c) = 0$, then

A. $|a| = |b| \cdot |c| = 1$

B. $b \mid \mid c$

C. $a \mid \mid b$

D. $b\dot{c}$

Answer: B

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21. A vectors which makes equal angles with the vectors $\frac{1}{3}\left(\hat{i}-2\hat{j}+2\hat{k}\right), \frac{1}{5}\left(-4\hat{i}-3\hat{k}\right), \hat{j}$ is:

A. a) $5\hat{i}+5\hat{j}+\hat{k}$

B. b) $5\hat{i}+\hat{j}-5\hat{k}$

C. c) $5\hat{i}+\hat{j}+5\hat{k}$

D. d) $\pm\left(5\hat{i}-\hat{j}-5\hat{k}\right)$

Answer: D

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22. Find by vector method the horizontal force and the force inclined at an angle of 60° to the vertical whose resultant is a vertical force P.

A. $P, 2P$

B. $P, P\sqrt{3}$

C. $2P, P\sqrt{3}$

D. None of these

Answer: D

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23. If $x+y+z=0, |x|=|y|=|z|=2 \text{ and } \theta$ is angle between y and z, then the value of $\cos ec^2\theta+\cot^2\theta$ is equal to

A. $\frac{4}{3}$

B. $\frac{5}{3}$

C. $\frac{1}{3}$

D. 1

Answer: B

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24. Find the least positive integral value of x for which the angle betweenn vectors $a = x\hat{i} - 3\hat{j} - \hat{k}$ and $b = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute.

A. $x > 0$

B. $x < 0$

C. $x > 1 only$

D. $x < \ -1$ only

Answer: B

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25. If a, b and c are non-coplanar vectors and $d = \lambda \vec{a} + \mu \vec{b} + \nu \vec{c}$, then λ is equal to

A. $\dfrac{[dbc]}{[bac]}$

B. $\dfrac{[bcd]}{[bca]}$

C. $\dfrac{[bdc]}{[abc]}$

D. $\frac{[cbd]}{[abc]}$

Answer: B

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26. If the vectors $3\vec{p}+\vec{q}$; $5\vec{p}-3\vec{q}$ and $2\vec{p}+\vec{q}$; $4\vec{p}-2\vec{q}$ are pairs of mutually perpendicular then sin($\vec{p}, \vec{q}\Big)$ is :

A. a) $\frac{\sqrt{55}}{4}$

B. b) $\frac{\sqrt{55}}{8}$

C. c) $\frac{3}{16}$

D. d) $\frac{\sqrt{247}}{16}$

Answer: B

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27. Let $\overrightarrow{u}=\hat{i}+\hat{j}, \overrightarrow{v}=\hat{i}-\hat{j}$ and $\overrightarrow{w}=\hat{i}+2\hat{j}+3\hat{k}$.If $\widehat{n}$ is a unit vector such that $\overrightarrow{u}\cdot\widehat{n}=0$ and $\overrightarrow{v}\cdot\widehat{n}=0$ then $\left|\overrightarrow{w}\cdot\widehat{n}\right|$ is equal to

A. 1

B. 2

C. 3

D. 0

Answer: C

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28. Given a parallelogram $ABCD$. If $\left|\overrightarrow{AB}\right|=a, \left|\overrightarrow{AD}\right|=b \& \left|\overrightarrow{AC}\right|=c$, then $\overrightarrow{DB}.\ \overrightarrow{AB}$ has the value

A. $\dfrac{3a^2+b^2-c^2}{2}$

B. $\dfrac{a^2+3b^2-c^2}{2}$

C. $\dfrac{a^2-b^2+3c^2}{2}$

D. None of these

Answer: A

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29. For two particular vectors $\vec{A}$ and $\vec{B}$ it is known that $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$. What must be true about the two vectors?

A. Atleast one of the two vectors must be the zero vector

B. $A \times B = B \times A$ is true for any two vectors

C. One of the two vectors is a scalar multiple of the other vector

D. The two vectors must be perpendicular to each other

Answer: C

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30. For some non zero vector $\vec{v}$, if the sum of $\vec{v}$ and the vector obtained from $\vec{v}$ by rotating it by an angle 2α equals to the vector obtained from $\vec{v}$ by rotating it by α then the value of α, is

A. $2n\pi \pm \frac{\pi}{3}$

B. $n\pi \pm \frac{\pi}{3}$

C. $2n\pi \pm \frac{2\pi}{3}$

D. $n\pi \pm \frac{2\pi}{3}$

Answer: A

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31. In isosceles triangles ABC, $\left|\vec{A}B\right| = \left|\vec{B}C\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}E and \vec{C}A\left(where\left|\vec{C}A\right| = 12\right)$.

A. $\frac{-3\sqrt{7}}{8}$

B. $\dfrac{3\sqrt{8}}{17}$

C. $\dfrac{3\sqrt{7}}{8}$

D. $\dfrac{-3\sqrt{8}}{17}$

Answer: C

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32. Given an equilateral triangle ABC with side length equal to 'a'. Let M and N be two points respectivelyABIn the side AB and AC such that $\overrightarrow{AN} = K\overrightarrow{AC}$ and $\overrightarrow{AM} = \dfrac{\overrightarrow{AB}}{3}$ If $\overrightarrow{BN}$ and $\overrightarrow{CM}$ are orthogonalthen the value of K is equal to

A. $\dfrac{1}{5}$

B. $\dfrac{1}{4}$

C. $\dfrac{1}{3}$

D. $\dfrac{1}{2}$

Answer: A

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33. In a quadrilateral ABCD, AC is the bisector of the (AB, AD) which is $\frac{2\pi}{3}$, $15|AC| = 3|AB| = 5|AD|$, then $\cos(BA, CD)$ is equal to

A. $\frac{-\sqrt{14}}{7\sqrt{2}}$

B. $-\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\frac{2}{\sqrt{7}}$

D. $\frac{2\sqrt{7}}{14}$

Answer: C

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34. If the distance from the point P(1, 1, 1) to the line passing through the points $Q(0, 6, 8)$ and $R(-1, 4, 7)$ is expressed in the form $\sqrt{\frac{p}{q}}$, where p and q are co-prime, then the value of $\frac{(q+p)(p+q-1)}{2}$ is equal to

A. 4950

B. 5050

C. 5150

D. None of these

Answer: A

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35. Given the vectors $\overrightarrow{u}=2\hat{i}-\hat{j}-\hat{k}$ and $\overrightarrow{v}=\hat{i}-\hat{j}+2\hat{k}$ and $\overrightarrow{w}=\hat{i}-\hat{k}$ If the volume of the parallelopiped having $-c\overrightarrow{u}$, $\overrightarrow{v}$ and $c\overrightarrow{w}$ as concurrent edges, is 8 then c can be equal to

A. a) ± 2

B. b) 4

C. c) 8

D. d) cannot be determine

Answer: A

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36. Vector $\vec{c}$ is perpendicular to vectors $\vec{a}=(2,-3,1) and \vec{b}=(1,-2,3)$ and satisfies the condition $\vec{c}\hat{i}+2\hat{j}-7\hat{k}=10.$ Then vector $\vec{c}$ is equal to $(7,5,1)$ b. $-7,-5,-1$ c. $1,1,-1$ d. none of these

A. $(7,5,1)$

B. $(-7,,-5,-1)$

C. $(1,1,-1)$

D. None of these

Answer: B

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37. Let $\overrightarrow{a} = \hat{j} + \hat{j}$, $\overrightarrow{b} = \hat{j} + \hat{k}$ and $\overrightarrow{c} = \alpha\overrightarrow{a} + \beta\overrightarrow{b}$. If the vectors, $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{c}$ are coplanar then $\dfrac{\alpha}{\beta}$ is

A. 1

B. 2

C. 3

D. -3

Answer: D

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38. A rigid body rotates about an axis through the origin with an angular velocity $10\sqrt{3}$rad/s. If ω points in the direction of $\hat{i} + \hat{j} + \hat{k}$, then the equation to the locus of the points having tangential speed 20m/s.

A. $x^2 + y^2 + z^2 - xy - yz - xz - 1 = 0$

B. $x^2 + y^2 + z^2 - 2xy - 2yz - 2xz - 1 = 0$

C. $x^2 + y^2 + z^2 - xy - yz - xz - 2 = 0$

D. $x^2 + y^2 + z^2 - 2xy - 2yz - 2xz - 2 = 0$

Answer: C

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39. A rigid body rotates with constant angular velocity ω about the line whose vector equation is, $r = \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. The speed of the particle at the instant it passes through the point with position vector $\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$ is equal to

A. $\omega\sqrt{2}$

B. 2ω

C. $\dfrac{\omega}{\sqrt{2}}$

D. None of these

Answer: A

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40. Consider ΔABC with $A=\left(\vec{a}\right); B=\left(\vec{b}\right)$ and $C=\left(\vec{c}\right)$. If $\vec{b}.\left(\vec{a}+\vec{c}\right)=\vec{b}.\vec{b}+\vec{a}.\vec{c}; \left|\vec{b}-\vec{a}\right|=3; \left|\vec{c}-\vec{b}\right|=4$ then the angle between the medians $\overrightarrow{AM}$ and $\overrightarrow{BD}$ is

A. $\pi-\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$

B. $\pi-\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$

C. $\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$

D. $\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$

Answer: A

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41. Given unit vectors m, n and p such that angle between m and n. Angle between p and $(m\times n)=\frac{\pi}{6}$, then [n p m] is equal to

A. $\frac{\sqrt{3}}{4}$

B. $\frac{3}{4}$

C. $\frac{1}{4}$

D. None of these

Answer: A

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42. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)$ is parallel to the vector

A. $a+b$

B. $a-b$

C. $2a-b$

D. `a+2b

Answer: B

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43. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r} \times \vec{a}$ is equal to

A. $\left[r\hat{a}\hat{b}\right]\left(\hat{a} \times \hat{b}\right)$

B. $\left[r\hat{a}\hat{b}\right]\hat{a} + (r \cdot \hat{a})\left(\hat{a} \times \hat{b}\right)$

C. $\left[r\hat{a}\hat{b}\right]\hat{b} + \left(r \cdot \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$

D. $\left[r\hat{a}\hat{b}\right]\hat{b} + (r \cdot \hat{a})\left(\hat{a} \times \hat{b}\right)$

Answer: C

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44. If vector $\vec{i} + 2\vec{j} + 2\vec{k}$ is rotated through an angle of 90°, so as to cross the positivedirection of y-axis, then the vector in the new position is

A. $-\frac{2}{\sqrt{5}}\hat{i} + \sqrt{5}\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$

B. $-\frac{2}{\sqrt{5}}\hat{i}-\sqrt{5}\hat{j}+\frac{4}{\sqrt{5}}\hat{k}$

C. $4\hat{i}-\hat{j}-\hat{k}$

D. None of these

Answer: A

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45. 10 different vectors are lying on a plane out of which four are parallel with respect to each other. Probability that three vectors chosen from them will satisfy the equation $\lambda_1 a+\lambda_2 b+\lambda_3 c=0,$ where $\lambda_1, \lambda_2 \text{ and } \lambda_3 \neq \; = 0$ is

A. (a) $\frac{.^6 C_2 \times .^4 C_1}{.^{10} C_3}$

B. (b) $\frac{(.^6 C_3 \times .^4 C_1)+,^6 C_3}{.^{10} C_3}$

C. (c) $\frac{(.^6 C_3 + \times .^4 C_1)+,^4 C_3}{.^{10} C_3}$

D. (d) $\frac{(.^6 C_3 + .^4 C_1)+,^6 C_2 \times .^4 C_1}{.^{10} C_3}$

Answer: D

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46. If $\widehat{a}$ is a unit vector and projection of x along $\widehat{a}$ is 2 units and $(\widehat{a} \times x) + b = x,$ then x is equal to

A. $\dfrac{1}{2}(\widehat{a} - b + (\widehat{a} \times b))$

B. $\dfrac{1}{2}(2\widehat{a} + b + (\widehat{a} \times b))$

C. $(\widehat{a} + (\widehat{a} \times b))$

D. None of these

Answer: B

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47. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are any three non-zero vectors, then the component of $\overrightarrow{a} \times \left(\overrightarrow{b} \times \overrightarrow{c}\right)$ perpendicular to $\overrightarrow{b}$ is

A. $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \dfrac{\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{a}\right)}{|b|^2} \vec{b}$

B. $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \dfrac{\left(\vec{a} \times \vec{c}\right) \cdot \left(\vec{a} \times \vec{b}\right)}{|b|^2} \vec{b}$

C. $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \dfrac{\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{b} \times \vec{a}\right)}{|b|^2} \vec{b}$

D. $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \dfrac{\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{b} \times \vec{c}\right)}{|b|^2} \vec{b}$

Answer: D

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48. The position vector of a point P is $r = x\hat{i} + y\hat{j} + \hat{k}z$, where $x, y, z \in N$ and $a = \hat{i} + 2\hat{j} + \hat{k}. Ifr \cdot a = 20$ and the number of possible of P is 9λ, then the value of λ is

A. a) 81

B. b) 9

C. c) 100

D. d) 36

Answer: A

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49. Let $a, b > 0$ and $\alpha = \frac{\hat{i}}{a} + \frac{4\hat{j}}{b} + b\hat{k}$ and $\beta = b\hat{i} + a\hat{j} + \frac{\hat{k}}{b}$, then the maximum value of $\frac{30}{5 + \alpha \cdot \beta}$ is

A. 3

B. 2

C. 4

D. 8

Answer: A

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50. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors forming a linearly independent system, then $\forall \theta \in R$

$\vec{p} = \vec{a} \cos\theta + \vec{b} \sin\theta + \vec{c} (\cos 2\theta)$

$\vec{q} = \vec{a} \cos\left(\frac{2\pi}{3} + \theta\right) + \vec{b} \sin\left(\frac{2\pi}{3} + \theta\right) + \vec{c} (\cos 2)\left(\frac{2\pi}{3} + \theta\right)$

and $\vec{r} = \vec{a} \cos\left(\theta - \frac{2\pi}{3}\right) + \vec{b} \sin\left(\theta - \frac{2\pi}{3}\right) + \vec{c} \cos 2\left(\theta - \frac{2\pi}{3}\right)$

then $\left[\vec{p}\,\vec{q}\,\vec{r}\right]$

A. $[abc] \sin\theta$

B. [a b c]$\cos 2\theta$

C. [a b c]$\cos 3\theta$

D. None of these

Answer: D

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51. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{A}D = -\hat{i} + 2\hat{j} + 2\hat{k}\cdot$ The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: B

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52. If in a $\triangle ABC, BC = \frac{e}{|e|} - \frac{f}{|f|}$ and $AC = \frac{2e}{|e|} : |e| \neq |f|,$ then the value of $\cos 2A + \cos 2B + \cos 2C$ must be

A. (a)-1

B. (b)0

C. (c)2

D. (d) $\frac{-3}{2}$

Answer: A

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53. Unit vectors $\vec{a}$ and $\vec{b}$ ar perpendicular , and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c} = \alpha \vec{a} + \beta \vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$ then.

A. $\alpha = \beta = -\cos\theta, y^2 = \cos 2\theta$

B. $\alpha = \beta = \cos\theta, y^2 = \cos 2\theta$

C. $\alpha = \beta = \cos\theta, y^2 = -\cos 2\theta$

D. $\alpha = \beta = -\cos\theta, y^2 = -\cos 2\theta$

Answer: C

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54. In triangle ABC the mid point of the sides AB, BC and AC respectively (l, 0, 0), (0, m, 0) and (0, 0, n). Then, $\frac{AB^2 + BC^2 + CA^2}{l^2 + m^+ n^2}$ is equal to

A. 2

B. 4

C. 8

D. 16

Answer: C

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55. Find the angle between the two lines whose direction cosines are given by the equation

$l + m + n = 0, 2l + 2m - mn = 0$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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56. A line makes an angle θ both with x-axis and y-axis. A possible range of θ is

A. $\left[0, \frac{\pi}{4}\right]$

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$.

Answer: C

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57. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\dfrac{2}{3}$

C. $\dfrac{3}{5}$

D. $\dfrac{3}{4}$

Answer: D

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58. Find the perpendicular distance of a corner of a cube of unit side length from a diagonal not passing through it.

A. $\sqrt{\dfrac{3}{2}}$

B. $\sqrt{\frac{2}{3}}$

C. $\sqrt{\frac{3}{4}}$

D. $\sqrt{\frac{4}{3}}$

Answer: B

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59. If p,q are two non-collinear and non-zero vectors such that $(b-c)p\times q+(c-a)p+(a-b)q=0$ where a, b, c are lengths of sides of a triangle, then the triangle is

A. right angled

B. obtuse

C. equilateral

D. right angled isosceles triangle

Answer: C

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60. Let

$a = \hat{i} + \hat{j} + \hat{k}, b = -\hat{i} + \hat{j} + \hat{k}, c = \hat{i} - \hat{j} + \hat{k}$ and $d = \hat{i} + \hat{j} - \hat{k}$.

Then, the line of intersection of planes one determined by a, b and other determined by c, d is perpendicular to

A. X-axis

B. Y-axis

C. Both X and Y axes

D. Both y and z-axes

Answer: D

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61. A parallelopiped is formed by planes drawn parallel to coordinate axes through the points $A = (1, 2, 3)$ and $B = (9, 8, 5)$. The volume of that

parallelopiped is equal to (in cubic units)

A. 192

B. 48

C. 32

D. 96

Answer: D

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62. Let a,b,c be three non-coplanar vectors and d be a non-zerro vector, which is perrpendicular to a+b+c. now, if $d = (\sin x)(a \times b) + (\cos y)(b \times c) + 2(c \times a)$, then the minimum value of $\left(x^2 + y^2\right)$ is

A. π^2

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{4}$

D. $\frac{5\pi^2}{4}$

Answer: D

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63. If $\alpha(a \times b) + \beta(b \times c) + \gamma(c \times a) = 0$, then

A. a, b, c are coplanar if all of $\alpha, \beta, \gamma \neq 0$

B. a, b, c are non-coplanar if any one $\alpha, \beta\gamma = 0$

C. a, b, c are non-coplanar for any α, β, γ.

D. None of these

Answer: A

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64. Given four non zero vectors $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$. The vectors $\bar{a}, \bar{b}$ and $\bar{c}$ are coplanar but not collinear pair by pairand vector $\bar{d}$ is not coplanar with vectors $\bar{a}, \bar{b}$ and $\bar{c}$ and $\widehat{\bar{a}\bar{b}} = \widehat{\bar{b}\bar{c}} = \frac{\pi}{3}, (\bar{d}\,\bar{b}) = \beta$,If $(\bar{d}\,\bar{c}) = \cos^{-1}(m\cos\beta + n\cos\alpha)$ then $m - n$ is :

A. $\cos^{-1}(\cos\beta - \cos\alpha)$

B. $\sin^{-1}(\cos\beta - \cos\alpha)$

C. $\sin^{-1}(\sin\beta - \sin\alpha)$

D. $\cos^{-1}(\tan\beta - \tan\alpha)$

Answer: A

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65. The shortest distance between a diagonal of a unit cube and the edge skew to it, is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{6}}$

Answer: A

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66. Let $v = 2\hat{i} + \hat{j} - \hat{k}$ and $w = \hat{i} + 3\hat{k}$. If $\widehat{u}$ is unit vector and the maximum value of $[uvw] = \sqrt{\lambda}$, then the value of $(\lambda - 51)$ is

A. -1

B. $\sqrt{35}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: B

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67. The length of the edge of the regular tetradedron ABCD is 'a'. Points E and F are taken on the edges AD and BD respectively such that 'E' divides DA and 'F' divides BD in the ratio of 2:1 each. Then, area of $\triangle CEF$ is

A. $\dfrac{5a}{12\sqrt{3}}$ sq. units

B. $\dfrac{a}{12\sqrt{3}}$ sq. units

C. $\dfrac{a^2}{12\sqrt{3}}$ sq. unit

D. $\dfrac{5a^2}{12\sqrt{3}}$ sq. units

Answer: D

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68. If the two adjacent sides of two rectangles are represented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} +$ respectively, then the angel between the vector $\vec{x} = \frac{1}{3}\left(\vec{p} + \vec{r} + \vec{s}\right)$ and $\vec{y} = \frac{1}{5}\left(\vec{r} + \vec{s}\right)$ is

A. $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. $\pi - \cos^{-1}\left(\frac{19}{\sqrt{43}}\right)$

Answer: B

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69. Let $\vec{a}, \vec{b}, \vec{c}$ are three vectors along the adjacent edges ofa tetrahedron, if $|\vec{a}| = |\vec{b}| = |\vec{c}| = 2$ and $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 2$ then volume of tetrahedron is (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $2\frac{\sqrt{2}}{3}$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2\sqrt{2}}{3}$

Answer: D

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70. If the angle between the vectors $\vec{a} = \hat{i} + (\cos x)\hat{j} + \hat{k}$ and $\vec{b} = (\sin^2 x - \sin x)\hat{i} - (\cos x)\hat{j} + (3 - 4\sin x)\hat{k}$ is obutse and x in $\left(0, \frac{\pi}{2}\right)$, then the exhaustive set of values of 'x' is equal to-

A. $x \in \left(0, \frac{\pi}{6}\right)$

B. $x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

C. $x \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

D. $x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Answer: B

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71. If position vectors of the points A, B and C are a, b and c respectively and the points D and E divides line segment AC and AB in the ratio 2:1 and 1:3, respectively. Then, the points of intersection of BD and EC divides EC in the ratio

A. $2:1$

B. $1:3$

C. $1:2$

D. $3:2$

Answer: D

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Exercise (More Than One Correct Option Type Questions)

1. If vectors $\vec{a}$ and $\vec{b}$ are non collinear then $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ is (A) a unit vector (B) in the plane of $\vec{a}$ and $\vec{b}$ (C) equally inclined to $\vec{a}$ and $\vec{b}$ (D) perpendicular to $\vec{a} \times \vec{b}$

A. a unit vector

B. in the plane of a and b

C. equally inclined to a and b

D. perpendicular to $a \times b$

Answer: B::C::D

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2. If $a \times (b \times c) = (a \times b) \times c$, then

A. $(c \times a) \times b = 0$

B. $c \times (a \times b) = 0$

C. $b \times (c \times a) = 0$

D. $b \times (c \times a) = 0$

Answer: A::C::D

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3. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a} . \vec{b}\right)\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $|u|$

B. $|u| + |u \cdot a|$

C. $|u| + |u \cdot b|$

D. $|u| + u \cdot (a + b)$

Answer: A::C

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4. The scalars l and m such that $l\vec{a}+m\vec{b}=\vec{c}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are given vectors, are equal to

A. $l=\dfrac{(c\times b)\cdot(a\times b)}{(a\times b)^2}$

B. $l=\dfrac{(c\times sa)\cdot(b\times a)}{(b\times a)^2}$

C. $m=\dfrac{(c\times a)\cdot(b\times a)}{(b\times a)^2}$

D. $n=\dfrac{(c\times sa)\cdot(b\times a)}{(b\times a)^2}$

Answer: A::C

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5. Let $\vec{r}$ be a unit vector satisfying $\vec{r}\times\vec{a}=\vec{b}$, where $\left|\vec{a}\right|=\sqrt{3}$ and $\left|\vec{b}\right|=\sqrt{2}$, then

A. $\hat{r}=\dfrac{2}{3}(a+a\times b)$

B. $\hat{r}=\dfrac{1}{3}(a+a\times b)$

C. $\hat{r}=\dfrac{2}{3}(a-a\times b)$

D. $\hat{r} = \frac{1}{3}(-a + a \times b)$

Answer: B::D

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6. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2 \cos 2x + a_3, \sin^2 x = 0$ for all x is

A. vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b = -\hat{i} + \hat{j} + \hat{k}$ are perpendicular to each other

C. if vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then one of the ordered triplet $(a_1, a_2, a_3) = (1, -1, -2)$

D. if vectors $2a_1 + 3a_2 + 6a_3$, then $\left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right|$ is $2\sqrt{6}$.

Answer: A::B::C::D

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7. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ , then

a. $|a \times b|^2 + (a \cdot b)^2 = |a|^2 |b|^2$

b. $|a \times b| = (a \cdot b), \quad \text{if} \quad \theta = \frac{\pi}{4}$

c. $a \times b = (a \cdot b)\widehat{n}$, (where $\widehat{n}$ is a normal unit vector), if $\theta = \frac{\pi}{4}$

d. $|a \times b| \cdot (a + b) = 0$

A. $|a \times b|^2 + (a \cdot b)^2 = |a|^2 |b|^2$

B. $|a \times b| = (a \cdot b), \quad \text{if} \quad \theta = \frac{\pi}{4}$

C. $a \times b = (a \cdot b)\widehat{n}$, (where $\widehat{n}$ is a normal unit vector), if $\theta = \frac{\pi}{4}$

D. $|a \times b| \cdot (a + b) = 0$

Answer: A::B::C::D

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8. If the unit vectors e_1 and e_2 arke inclined at an angle 2θ and $|e_1 - e_2| < 1$, the for $\theta \in [0, \pi]$, θ may lie in the interval

A. $\left[0, \frac{\pi}{6}\right]$

B. $\left(\frac{5\pi}{6}, \pi\right]$

C. $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

D. $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right]$

Answer: A::B

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9. $\vec{b}, \vec{c}$ being non-collinear if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a} \cdot \vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}$ and $\left(\vec{c} \cdot \vec{c}\right)\vec{a} = \vec{c}$, then

A. A. x=1

B. B. $x = -1$

C. C. $y=(4n+1)\frac{\pi}{2}, n \in I$

D. D. $y=(2n+1)\frac{\pi}{2}, n \in I$

Answer: A::C

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10. If in triangle $ABC, \vec{A}B=\frac{\vec{u}}{|\vec{u}|}-\frac{\vec{v}}{|\vec{v}|} and \vec{A}C=\frac{2\vec{u}}{|\vec{u}|}, where |\vec{u}| \neq |\vec{v}|$, then

a. $1+\cos 2A+\cos 2B+\cos 2C=0$

b.$\sin A=\cos C$

c. projection of AC on BC is equal to BC

d. projection of AB on BC is equal to AB

A. $1+\cos 2A+\cos 2B+\cos 3C=0$

B. $\sin A=\cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: A::B::C

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11. If a, b and c be the three non-zero vectors satisfying the condition $a \times b = c$ and $b \times c = a$, then which of the following always hold(s) good?

A. a, b and c are orthogonal in pairs

B. [a b c]=|b|

C. [a b c] $= \left|c^2\right|$

D. $|b| = |c|$

Answer: A::C

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12. Given the following informations about the non-zero vectors A, B and C

$(i)(A \times B) \times A = 0 : (ii) B \cdot B = 4$

$(iii) A \cdot B = -6 : (iv) B \cdot C = 6$

which one of the following holds good?

A. $A \times B = 0$

B. $A \cdot (B \times C) = 0$

C. $A \cdot A = 8$

D. $A \cdot C = -1$

Answer: A::B

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13. Find the area of the triangle having all side equal to 8.

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14. Given vectors $U = 2\hat{i} + 3\hat{j} - 6\hat{k}, V = 6\hat{i} + 2\hat{j} + 2\hat{k}$. Find their dot product.

A. U, V and W are linearly dependent

B. $(U \times V) \times W = 0$

C. U, V and W form a triplet of mutually perpendicular vectors

D. $U \times (V \times W) = 0$

Answer: B::C::D

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15. Let $\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} + \hat{k}$ and $\overrightarrow{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\overrightarrow{b}$ and $\overrightarrow{c}$, whose projection on $\overrightarrow{a}$ is of magnitude $\sqrt{2/3}$, is

A. (a) $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. (b) $0\hat{i} + \hat{j} + 3\hat{k}$

C. (b) $-2\hat{i}-\hat{j}+5\hat{k}$

D. (d) $2\hat{i}+\hat{j}+5\hat{k}$

Answer: A::C

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16. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a}\times\vec{b}=3\left(\vec{a}\times\vec{c}\right)$ Also $\left|\vec{a}\right|=\left|\vec{b}\right|=1, \left|\vec{c}\right|=\frac{1}{3}$ If the angle between $\vec{b}$ and $\vec{c}$ is 60° then

A. $b=3c+a$

B. $b=3c-a$

C. $a=6c+2b$

D. $a=6c-2b$

Answer: A::B

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17. Let a, b and c be non-zero vectors and $|a| = 1$ and r is a non-zero vector such that $r \times a = b$ and $r \cdot a = 1$, then

a. $a \perp b$

b. $r \perp b$

c. $r \cdot a = \dfrac{1 - [abc]}{a \cdot b}$

d. [r a b]=0

A. $a \perp b$

B. $r \perp b$

C. $r \cdot a = \dfrac{1 - [abc]}{a \cdot b}$

D. [r a b]=0

Answer: A::B::C

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18. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors perpendicular to each other and $\overrightarrow{c} = \lambda_1 \overrightarrow{a} + \lambda_2 \overrightarrow{b} + \lambda_3 \left(\overrightarrow{a} \times \overrightarrow{b} \right)$ then the following is (are) true

A. (a)$\lambda_1 = a \cdot c$

B. (b)$\lambda_2 = |a \times b|$

C. (c)$\lambda_3 = |(a \times b) \times c|$

D. (d)$\lambda_1 + \lambda_2 + \lambda_3 = (a + b + a \times b) \cdot c$

Answer: A::D

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19. Given three non-coplanar vectors OA=a, OB=b, OC=c. Let S be the centre of the sphere passing through the points O, A, B, C if OS=x, then

A. x must be linear combination of a, b, c

B. x must be linear combination of $b \times c, c \times a$ and $a \times b$

C. $x = \dfrac{a^2(b \times c) + b^2(c \times a) + c^2(a \times b)}{2[abc]}, a = |a|, b = |b|. C = |c|$

D. $x = a + b + c$

Answer: A::B::C

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20. If $a = \hat{i} + \hat{j} + \hat{k}$ and $b = \hat{i} - \hat{j}$, then $\left(a \cdot \hat{i}\right)\hat{i} + \left(a \cdot \hat{j}\right)\hat{j} + \left(a \cdot \hat{k}\right)\hat{k}, \left(b \cdot \hat{i}\right)\hat{i} + \left(b \cdot \hat{j}\right)\hat{j} + \left(b \cdot \hat{k}\right)\hat{k}$ is equal to

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21. If $\overrightarrow{a} = x\hat{i} + y\hat{j} + z\hat{k}, \overrightarrow{b} = y\hat{i} + z\hat{j} + x\hat{k}$. The dot product of vectors is

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. parallel to $\hat{i} + \hat{j} + \hat{k}$

Answer: A::B::C

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22. Which of the following statement(s) is/are true?

A. $a \times (b \times c), b \times (c \times a), c \times (a \times b)$ form a right handed system

B. c, $(a \times b) \times$, $a \times b$ form a right handed system

C. $a \cdot b + b \cdot c + c \cdot a < 0, \quad \text{if} \quad a + b + c = 0$

D. $\dfrac{(a \times b) \cdot (b \times c)}{(b \times c) \cdot (a \times c)} = -1, \quad \text{if} \quad a + b + c = 0$

Answer: B::C::D

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23. Unit vectors $\vec{a}$ and $\vec{b}$ ar perpendicular , and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c} = \alpha \vec{a} + \beta \vec{b} + \gamma \left(\vec{a} \times \vec{b}\right)$ then.

A. $l = m$

B. $n^2 = 1 - 2l^2$

C. $n^2 = -\cos 2\alpha$

D. $m^2 = \frac{1+\cos 2\alpha}{2}$

Answer: A::B::C::D

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24. If a,b and c are three non-zero vectors, then which of the following statement(s) is/are true?

A. $a \times (b \times c), b \times (c \times a), c \times (a \times b)$ form a right handed system

B. c, $(a \times b) \times$, $a \times b$ form a right handed system

C. $a \cdot b + b \cdot c + c \cdot a < 0, \quad \text{if} \quad a+b+c=0$

D. $\frac{(a \times b) \cdot (b \times c)}{(b \times c) \cdot (a \times c)} = -1, \quad \text{if} \quad a+b+c=0$

Answer: C::D

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25. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

a $.b - \dfrac{a \times b}{|b|^2}$

b. $2b - \dfrac{a \times b}{|b|^2}$

c. $|a|b - \dfrac{a \times b}{|b|^2}$

d. $|b|b - \dfrac{a \times b}{|b|^2}$

A. $b - \dfrac{a \times b}{|b|^2}$

B. $2b - \dfrac{a \times b}{|b|^2}$

C. $|a|b - \dfrac{a \times b}{|b|^2}$

D. $|b|b - \dfrac{a \times b}{|b|^2}$

Answer: A::B::C::D

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26. If a and b are any two unit vectors, then the possible integers in the range of $\frac{3|a+b|}{2}+2|a-b|$, is/are

A. 2

B. 3

C. 4

D. 5

Answer: B::C::D

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27. If a is perpendicular to b and p is non-zero scalar such that $pr+(r\cdot b)a=c$, then r =

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28. In a four-dimensional space where unit vectors along the axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{l}$, and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as a linear combination of others and

$$(\lambda - 1)\left(\vec{a}_1 - \vec{a}_2\right) + \mu\left(\vec{a}_2 + \vec{a}_3\right) + \gamma\left(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2\right) + \vec{a}_3 + \delta \vec{a}$$

then

A. $\lambda = 1$

B. $\mu = \frac{-2}{3}$

C. $\lambda = \frac{2}{3}$

D. $\delta = \frac{1}{3}$

Answer: A::B::D

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29. A vector(d) is equally inclined to three vectors $a = \hat{i} - \hat{j} + \hat{k}, b = 2\hat{i} + \hat{j}$ and $c = 3\hat{j} - 2\hat{k}$. Let x, y, z be three vectors in the plane a, b:b, c:c, a respectively, then

A. $x \cdot d = 14$

B. $y \cdot d = 3$

C. $z \cdot d = 0$

D. $r \cdot d = 0$, where $r = \lambda x + \mu y + \delta z$

Answer: C::D

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30. Find the nth term of AP whose first term is 12 and common difference is 7.

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31. Given three vectors a,b and c are non-zero and non-coplanar vectors. Then which of the following are coplanar.

A. $a + b, b + c, c + a$

B. $a-b, b+c, c+a$

C. $a+b, b-c, c+a$

D. $a+b, b+c, c-a$

Answer: B::C::D

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32. Find the nth term of GP if the first term is 7 and common ratio is 3.

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33. If vectors $\vec{a}$ and $\vec{b}$ are two adjecent sides of a paralleogram, then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is

A. $b+\dfrac{b\times a}{|a|^2}$

B. $\dfrac{a\cdot b}{|b|^2}b$

C. $b - \frac{a \cdot b}{|b|^2} b$

D. $\frac{a \times (b \times a)}{|a|^2}$

Answer: C::D

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34. Let a, b, c be three vectors such that each of them are non-collinear, a+b and b+c are collinear with c and a respectively and a+b+c=k. Then (|k|, |k|) lies on

A. $y^2 = 4ax$

B. $x^2 + y^2 - ax - by = 0$

C. $x^2 - y^2 = 1$

D. $|x| + |y| = 1$

Answer: A::B

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35. If a, b and c are non-collinear unit vectors also b, c are non-collinear and $2a \times (b \times c) = b + c$, then

A. angle between a and c is 60°

B. angle between b and c is 30°

C. angle between a and b is 120°

D. b is perpendicular to c

Answer: A::C

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36. If

$a = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) : b = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right) : c = c_1\hat{i} + c_2\hat{j} + c_2\hat{k}$

and matrix $A = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \\ c_1 & c_2 & c_3 \end{bmatrix}$ and $AA^T = I$, then c

A. (a) $\frac{3\hat{i} + 6\hat{j} + 2\hat{k}}{7}$

B. (b) $\dfrac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$

C. (c) $\dfrac{-3\hat{i}+6\hat{j}-2\hat{k}}{7}$

D. (d) $-\dfrac{3\hat{i}+6\hat{j}+2\hat{k}}{7}$

Answer: B::C

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Exercise (Statement I And Ii Type Questions)

1. Statement 1: A component of vector $\overrightarrow{b}=4\hat{i}+2\hat{j}+3\hat{k}$ in the direction perpendicular totehdirectin of vector $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k} is \hat{i}-\hat{j}.$

Statement 2: A component of vector in the direction of $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k} is 2\hat{i}+2\hat{j}+2\hat{k}.$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: C

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2. Statement-I $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ are three mutually perpendicular unit vector, then $a_1\hat{i}+b_1\hat{j}+c_1\hat{k}, a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ and $a_3\hat{i}+b_3\hat{j}+3\hat{k}$ may be mutually perpendicular unit vectors.

Statement-II Value of determinant and its transpose are the same .

A. (a)Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. (b)Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. (c)Statement-I is correct but Statement-II is incorrect

D. (d)Statement-II is correct but Statement-I is incorrect

Answer: A

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3. Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ Statement 1 $\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right). \vec{b}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right). \vec{b}\right)\hat{j} + \left(\left(\hat{k} \times \vec{a}\right). \vec{b}\right)\hat{k}$
Statement 2: $\vec{c} = \left(\hat{i}. \vec{c}\right)\hat{i} + \left(\hat{j}. \vec{c}\right)\hat{j} + \left(\hat{k}. \vec{c}\right)\hat{k}$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

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4. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: D

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5. Statement 1: If $\vec{A}=2\hat{i}+3\hat{j}+6\hat{k}, \vec{B}=\hat{i}+\hat{j}-2\hat{k} and \vec{C}=\hat{i}+2\hat{j}+\hat{k}$, then $\left|\vec{A}\times\left(\vec{A}\times\left(\vec{A}\times\vec{B}\right)\right)\dot{\vec{C}}\right|=243.$ Statement 2: $\left|\vec{A}\times\left(\vec{A}\times\left(\vec{A}\times\vec{B}\right)\right)\dot{\vec{C}}\right|=\left|\vec{A}\right|^2\left|\left[\vec{A}\vec{B}\vec{C}\right]\right|.$

A. (a) Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. (b) Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. (c) Statement-I is correct but Statement-II is incorrect

D. (d) Statement-II is correct but Statement-I is incorrect

Answer: D

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6. Statement-I The number of vectors of unit length and perpendicular to both the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is zero.

Statement-II a and b are two non-zero and non-parallel vectors it is true that $a \times b$ is perpendicular to the plane containing a and b

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: D

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7. Statement-I (S_1): If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are non-collinear points. Then, every point (x, y) in the plane of $\triangle ABC$, can be expressed in the form $\left(\frac{kx_1 + lx_2 + mx_3}{k + l + m}, \frac{ky_1 + ly_2 + my_3}{k + l + m}\right)$

Statement-II (S_2) The condition for coplanarity of four A(a), B(b), C(c), D(d) is that there exists scalars l, m, n, p not all zeros such that $la + mb + nc + pd = 0$ where $l + m + n + p = 0$.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

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8. If a, b are non-zero vectors such that $|a+b|=|a-2b|$, then

Statement-I Least value of $a\cdot b+\frac{4}{|b|^2+2}$ is $2\sqrt{2}-1$.

Statement-II The expression $a\cdot b+\frac{4}{|b|^2+2}$ is least when magnitude of b is $\sqrt{2\tan\left(\frac{\pi}{8}\right)}$.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

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9. Statement-I If $a=3\hat{i}-3\hat{j}+\hat{k}, b=-\hat{i}+2\hat{j}+\hat{k}$ and $c=\hat{i}+\hat{j}+\hat{k}$ and $d=2\hat{i}-\hat{j}$

, then there exist real numbers α, β, γ such that $a = \alpha b + \beta c + \gamma d$

Statement-II a, b, c, d are four vectors in a 3-dimensional space. If b, c, d are non-coplanar, then there exist real numbers α, β, γ such that $a = \alpha b + \beta c + \gamma d.$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: B

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10. Statement 1: Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be the position vectors of four points $A, B, C and D$ and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0$. Then points

$A, B, C, and D$ are coplanar. Statement 2: Three non-zero, linearly dependent coinitial vector $\left(\vec{P}Q, \vec{P}R and \vec{P}S\right)$ are coplanar. Then $\vec{P}Q = \lambda\vec{P}R + \mu\vec{P}S, where \lambda and \mu$ are scalars.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

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11. If $a = \hat{i} + \hat{j} - \hat{k}, b = 2\hat{i} + \hat{j} - 3\hat{k}$ and r is a vector satisfying $2r + \times a = b.$

Statement-I r can be expressed in terms of a, b and $a \times b$.

Statement-II $r = \frac{1}{7}\left(7\hat{i} + 5\hat{j} - 9\hat{k} + a \times b\right)$.

A. (a)Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. (b)Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. (c)Statement-I is correct but Statement-II is incorrect

D. (d)Statement-II is correct but Statement-I is incorrect

Answer: A

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12. Let $\widehat{a}$ and $\hat{b}$ be unit vectors at an angle $\frac{\pi}{3}$ with each other. If $\left(\widehat{a} \times \left(\hat{b} \times \hat{c}\right)\right) \cdot (\widehat{a} \times \hat{c}) = 5$ then

Statement-I $\left[\widehat{a}\hat{b}\hat{c}\right] = 10$

Statement-II [x y z]=0, if x=y or y=z or z=x

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: B

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Exercise (Passage Based Questions)

1. Consider three vectors $\overrightarrow{p} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{q} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\overrightarrow{r} = \hat{i} + \hat{j} + 3\hat{k}$ and let $\overrightarrow{s}$ be a unit vector, then $\overrightarrow{p}$, $\overrightarrow{q}$ and $\overrightarrow{r}$ are

a. linealy dependent

b. can form the sides of a possible triangle

c. such that the vectors (q-r) is orthogonal to p

d. such that each one of these can be expressed as a linear combination of the other two

A. linealy dependent

B. can form the sides of a possible triangle

C. such that the vectors (q-r) is orthogonal to p

D. such that each one of these can be expressed as a linear combination of the other two

Answer: C

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2. Consider three vectors $p = \hat{i} + \hat{j} + \hat{k}, q = 2\hat{i} + 4\hat{j} - \hat{k}$ and $r = \hat{i} + \hat{j} + 3\hat{k}$ and let s be a unit vector, then

Q. If $(p \times q) \times r = up + vq + wr$, then (u+v+w) is equal to

A. 8

B. 2

C. -2

D. 4

Answer: B

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3. Consider three vectors $p = \hat{i} + \hat{j} + \hat{k}, q = 2\hat{i} + 4\hat{j} - \hat{k}$ and $r = \hat{i} + \hat{j} + 3\hat{k}$ and let s be a unit vector, then Q. The magnitude of the vector $(p \cdot s)(q \times r) + (q \cdot s)(r \times p) + (r \cdot s)(p \times q)$ is

A. A. 4

B. B. 8

C. C. 18

D. D. 2

Answer: A

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4. Consider the three vectors p, q, r such that $p=\hat{i}+\hat{j}+\hat{k}$ and $q=\hat{i}-\hat{j}+\hat{k}, p\times r=q+cp$ and $p\cdot r=2$

Q.The value of [p q r] is

A. $\frac{5\sqrt{2}c}{|r|}$

B. $-\frac{8}{3}$

C. 0

D. greater than 0

Answer: B

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5. Consider the three vectors p, q, r such that $p = \hat{i} + \hat{j} + \hat{k}$ and $q = \hat{i} - \hat{j} + \hat{k}, p \times r = q + cp$ and $p \cdot r = 2$

Q.The value of [p q r] is

A. $c\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

B. a unit vector

C. independent, as [p q r]

D. $-\dfrac{\hat{i} - 2\hat{j} + \hat{k}}{2}$

Answer: D

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6. Consider the three vectors p, q, r such that $p = \hat{i} + \hat{j} + \hat{k}$ and $q = \hat{i} - \hat{j} + \hat{k}, p \times r = q + cp$ and $p \cdot r = 2$

Q.The value of [p q r] is

A. are collinear

B. are coplanar

C. represent the coterminus edges of a tetrahedron whose volume is c cu. Units

D. represent the coterminus edges of a parallelopiped whose volume is c cu. Units

Answer: C

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7. Let P, Q are two points on the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2 \sqrt{4x^2 4x + 1}$ and P is also on the $x^2 + y^2 = 10$, Q lies inside the given circle such that its abscissa is an integer.

a. $(1, 2)$

b. $(2, 4)$

c. $(3, 1)$

d. $(3, 5)$

A. $(1, 2)$

B. $(2, 4)$

C. $(3, 1)$

D. $(3, 5)$

Answer: C

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8. Let P and Q are two points on the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2 \sqrt{4x^2 - 4x + 1}$ and P is also on the circle $x^2 + y^2 = 10$. Q lies inside the given circle such that its abscissa is an integer.

Q. $OP \cdot OQ$, O being the origin is

a. 4 or 7

b. 4 or 2

c. 2 or 3

d. 7 or 8

A. 4 or 7

B. 4 or 2

C. 2 or 3

D. 7 or 8

Answer: A

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9. Let P, Q are two points on the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2 \sqrt{4x^2 - 4x + 1}$ and P is also on the $x^2 + y^2 = 10$, Q lies inside the given circle such that its abscissa is an integer.so x coordinate of P are

A. 1

B. 4

C. 0

D. 3

Answer: D

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10. If a, b, c are three given non-coplanar vectors and any arbitratry vector r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$

Q. The vector r is expressible in the form

A. (a)$r = \dfrac{\Delta_1}{2\Delta}a + \dfrac{\Delta_2}{2\Delta}b + \dfrac{\Delta_3}{2\Delta}c$

B. (b)$r = \dfrac{2\Delta_1}{\Delta}a + \dfrac{2\Delta_2}{\Delta}b + \dfrac{2\Delta_3}{\Delta}c$

C. (c)$r = \dfrac{\Delta}{\Delta_1}a + \dfrac{\Delta}{\Delta_2}b + \dfrac{\Delta}{\Delta_3}c$

D. (d)$r = \dfrac{\Delta_1}{\Delta}a + \dfrac{\Delta_2}{\Delta}b + \dfrac{\Delta_3}{\Delta}c$

Answer: D

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11. If a, b, c are three given non-coplanar vectors and any arbitratry vector r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$

Q. The vector r is expressible in the form

A. $r = \frac{[rbc]}{2[abc]}a + \frac{[rbc]}{2[abc]}b + \frac{[rbc]}{2[abc]}c$

B. $r = \frac{2[rbc]}{[abc]}a + \frac{2[rbc]}{[abc]}b + \frac{2[rbc]}{[abc]}c$

C. $r = \frac{1}{[abc]}([rbc]a + [rca]b + [rab]c)$

D. None of these

Answer: D

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12. If a, b, c are three given non-coplanar vectors and any arbitratry vector r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$$

Q. The vector r is expressible in the form

A. $a = \frac{1}{[abc]}[(a\cdot a)(b\times c) + (b\cdot b)(c\times a) + c\cdot c(a\times b)]$

B. $a = \frac{1}{[abc]}[(a\cdot a)(b\times c) + (b\cdot a)(c\times a) + (a\cdot a)(a\times b)]$

C. $a = [(a\cdot a)(b\times c) + (a\cdot b)(c\times a) + (c\cdot a)(a\times b)]$

D. None of these

Answer: C

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13. If a, b, c are three given non-coplanar vectors and any arbitratry vector r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$$

Q. The vector r is expressible in the form

A. $(p \times q)[a \times bb \times cc \times a]$

B. $2(p \times q)[a \times bb \times cc \times a]$

C. $4(p \times q)[a \times bb \times cc \times a]$

D. $(p \times q)\sqrt{[a \times bb \times cc \times a]}$

Answer: B

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14. Let $g(x) = \int_0^x (3t^2 + 2t + 9)dt$ and $f(x)$ be a decreasing function $\forall x \geq 0$ such that $AB = f(x)\hat{i} + g(x)\hat{j}$ and $AC = g(x)\hat{i} + f(x)\hat{j}$ are the two smallest sides of a triangle ABC whose circumcentre lies outside the triangle $\forall c > 0.$ Q. Which of the following is true (for $x \geq o$)

A. (a) $f(x) > 0, g(x) < 0$

B. (b) $f(x) < 0, g(x) < 0$

C. (c) $f(x) > 0, g(x) > 0$

D. (d) $f(x) < 0, g(x) > 0$

Answer: D

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15. Let $g(x) = \int_0^x (3t^2 + 2t + 9)dt$ and $f(x)$ be a decreasing function $\forall x \geq 0$ such that $AB = f(x)\hat{i} + g(x)\hat{j}$ and $AC = g(x)\hat{i} + f(x)\hat{j}$ are the two smallest sides of a triangle ABC whose circumcentre lies outside the triangle $\forall c > 0$ then,

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16. Let x, y, z be the vector, such that $|x| = |y| = |z| = \sqrt{2}$ and x, y, z make angles of 60° with each other also, $x \times (y \times z) = a$ and $y \times (z \times x) = bx \times y = c$, . The value of z is

A. $(a + b) \times x - (a + b)$

B. $(a + b) - (a + b) \times c$

C. $\frac{1}{2}\{(a + b) \times c - (a + b)\}$

D. None of these

Answer: C

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17. Let x,y,z be the vector, such that $|x| = |y| = |z| = \sqrt{2}$ and x,y,z make angles of 60° with each other. If $x \times (y \times z) = a$, $y \times (z \times x) = b$ and $(x \times y) = c$

The value of y is:

A. $\frac{1}{2}[(a+b)+(a+b)\times c]$

B. $2[(a+b)+(a+b)\times c]$

C. $4[(a+b)+(a+b)\times c]$

D. None of these

Answer: A

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18. Let x, y, z be the vector, such that $|x| = |y| = |z| = \sqrt{2}$ and x, y, z make angles of 60° with each other also, $x \times (y \times z) = a$ and $y \times (z \times x) = bx \times y = c,$. The value of z is

A. $\frac{1}{2}[(b-a) \times c + (a+b)]$

B. $\frac{1}{2}[(b-a) + c \times (a+b)]$

C. $[(b-a) \times c + (a+b)]$

D. None of these

Answer: B

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19. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$ Q. Volume of parallelopiped with edges a, b, c is

A. $p+(q+r)\cos\theta$

B. $(p+q+r)\cos\theta$

C. $2p-(q+r)\cos\theta$

D. None of these

Answer: A

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20. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$ Q. The value of $\left(\frac{q}{p} + 2\cos\theta\right)$ is

A. 1

B. 2[a b c]

C. 0

D. None of these

Answer: C

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21. a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non-zero scalars satisfying $a \times b + b \times c = pa + qb + rc$. Now, answer the following questions. Q. $|(q+p)\cos\theta + r|$ is equal to

a. $(1+\cos\theta)\left(\sqrt{1-2\cos\theta}\right)$

b. $2\sin^2\left(\frac{\theta}{2}\right)\sqrt{(1+2\cos\theta)}$

c. $(1-\sin\theta)\sqrt{1+2\cos\theta})$

d. None of these

A. $(1+\cos\theta)\left(\sqrt{1-2\cos\theta}\right)$

B. $2\sin^2\left(\frac{\theta}{2}\right)\sqrt{(1+2\cos\theta)}$

C. $(1-\sin\theta)\sqrt{1+2\cos\theta})$

D. None of these

Answer: B

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Product of Vectors Exercise 5 : Matching Type Questions

1. Given two vectors $a = \hat{i} + 2\hat{j} + 2\hat{k}$ and $b = \hat{i} + \hat{j} + 2\hat{k}$. Find their dot product.

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2. Volume of parallelopiped formed by vectors $a \times b, b \times c$ and $c \times a$ is 36 sq.units. then the volumn formed by the vector a b and c is

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3. Match the statements of Column I with values of Column II.

	Column I		Column II
(A)	A circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(p)	4
(B)	If an edge of a cube increases by 1%, then percentage increase in volume is	(q)	0.6π
(C)	If the rate of decrease of $\frac{x^2}{2} - 2x + 5$ is twice the rate of decrease of x, then x is equal to (rate of decrease is non-zero)	(r)	3
(D)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(s)	$\frac{3\sqrt{3}}{4}$

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4. Find the solutions of the trigonometric equation tan3x=1.

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5. Find the solution of trigonometric equation sin 5x=1.

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Exercise (Single Integer Answer Type Questions)

1. Let $\widehat{u}, \hat{v}$ and $\widehat{w}$ are three unit vectors, the angle between $\widehat{u}$ and $\hat{v}$ is twice that of the angle between $\widehat{u}$ and $\widehat{w}$ and $\hat{v}$ and $\widehat{w}$, then $[\widehat{u}\hat{v}\widehat{w}]$ is equal to

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2. If a, b and c are three vectors such that [a b c]=1, then find the value of [a+b b+c c+a]+[$a \times b \, b \times c \, c \times a$]

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3. If $\widehat{a}, \hat{b}$ and $\hat{c}$ are the three unit vector and α, β and γ are scalars such that $\hat{c} = \alpha\widehat{a} + \beta\hat{b} + \gamma\left(\widehat{a} \times \hat{b}\right)$. If is given that $\widehat{a} \cdot \hat{b} = o$ and $\hat{c}$ makes equal angle with both $\widehat{a}$ and $\hat{b}$, then evaluate $\alpha^2 + \beta^2 + \gamma^2$.

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4. The three vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume: ______________

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5. Let $\hat{c}$ be a unit vector coplanar with $a=\hat{i}-\hat{j}+2\hat{k}$ and $b=2\hat{i}-\hat{j}+\hat{k}$ such that $\hat{c}$ is perpendicular to a . If P be the projection of $\hat{c}$ along, where $p=\dfrac{\sqrt{11}}{k}$ then find k.

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6. Let a, b and c are three vectors hacing magnitude 1, 2 and 3 respectively satisfying the relation [a b c]=6. If $\hat{d}$ is a unit vector coplanar with b and c such that $b \cdot \hat{d}=1$, then evaluate $|(a \times c) \cdot d|^2+\left|(a \times c) \times \hat{d}\right|^2$.

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7. Let $A\left(2\hat{i}+3\hat{j}+5\hat{k}\right), B\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ and $C\left(\lambda\hat{i}+5\hat{j}+\mu\hat{k}\right)$ are vertices of a triangle and its median through A is equally inclined to the positive directions of the axes, the value of $2\lambda-\mu$ is

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8. If V is the volumes of the parallelopiped having three coterminous edges as a,b and c, then the volume of parallelopipied having three coterminous edges as $\alpha=(a\cdot a)a+(a\cdot b)b+(a\cdot c)c, \beta=(a\cdot b)a+(b\cdot b)b+(b\cdot c)c$ and γ =
is

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9. If $\vec{a}, \vec{b}$ are vectors perpendicular to each other and $\left|\vec{a}\right|=2, \left|\vec{b}\right|=3, \vec{c}\times\vec{a}=\vec{b}$, then the least value of $2\left|\vec{c}-\vec{a}\right|$ is

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10. M and N are mid-point of the diagnols AC and BD respectivley of quadrilateral ABCD, then $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD}$=

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11. If $a \times b = c, b \times c = a, c \times a = b$. If vectors a, b and c are forming a right handed system, then the volume of tetrahedron formed by vectors $3a - 2b + 2c, \ - a - 2c$ and $2a - 3b + 4c$ is

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12. Let $\vec{a}$ and $\vec{c}$ be unit vectors inclined at $\pi/3$ with each other. If $\left(\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right).\left(\vec{a} \times \vec{c}\right) = 5$, then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ is equal to

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13. Volume of parallelopiped formed by vectors $a \times b, b \times c$ and $c \times a$ is 36 sq.units. then the volumn formed by the vector a b and c is

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14. If α and β are two perpendicular unit vectors such that $x = \widehat{\beta} - (\alpha \times x)$, then the value of $4|x|^2$ is.

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15. The volume of the tetrahedron whose vertices are the points with position vectors $\hat{i} + \hat{j} + \hat{k}, -\hat{i} - 3\hat{j} + 7\hat{k}, \hat{i} + 2\hat{j} - 7\hat{k}$ and $3\hat{i} - 4\hat{j} + \lambda\hat{k}$ is 22, then the digit at unit place of λ is.

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16. The volume of a tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, and \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a} + \vec{b}, \vec{b} + \vec{c} and \vec{c} + \vec{a}$ is

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Exercise (Subjective Type Questions)

1. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $\left|\vec{a} \cdot \vec{b}\right| \leq \left|\vec{a}\right|\left|\vec{b}\right|$ Also write the name of this inequality.

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2. P and Q are two points on the curve $y = 2^{x+2}$ in the rectangular cartesian coordinate system such that $\overline{OP}.\overline{C} = -1$ and $\overline{OQ}.\overline{C} = 2.$ where $\bar{c}$ is the unit vector along the positive direction of the x-axis. Then $\overline{OQ} - 4\overline{OP} =$

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3. O is the origin and A is a fixed point on the circle of radius 'a' with centre O.The vector $\overrightarrow{O}A$ is denoted by $\overrightarrow{a}$. A variable point P lie on the tangent at A and $\overrightarrow{O}P = \overrightarrow{r}$. Show that $\overrightarrow{a}\overrightarrow{r} = a^2$. Hence if $P(x, y)$ and $A(x_1, y_1)$, deduce the equation of tangent at A to this circle.

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4. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4}\tan A + a\tan B + \sqrt{a^2 + 4}\tan c = 6a,$ then the least vale of $\tan^2 A + \tan^2 b + \tan^2 Cis$ 6 b. 10 c. 12 d. 3

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5. Given , the edges A, B and C of triangle ABC. Find $\cos\angle BAM$, where M is mid-point of BC.

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6. Distance of point $A(1, 4, -2)$ is the distance from BC, where B and C The coordinates are respectively $(2, 1, -2)$ and $(0, -5, 1)$, respectively

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7. Given, the angles A, B and C of $\triangle ABC$. Let M be the mid-point of segment AB and let D be the foot of the bisector of $\angle C$. Find the ratio of $\dfrac{AreaOf \triangle CDM}{Areaof \triangle ABC}$

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8. In $\triangle ABC$, a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP$, using vector method, find the area of ABC if the area of BRC is 1 unit

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9. Which of the following statements are True or False :

If the diaonals of a quadilateral are equal and bisect each other at right angles then the quadrilateral is a square.

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10. Two forces $F_1 = \{2, 3\}$ and $F_2 = \{4, 1\}$ are specified relative to a general cartesian form. Their points of application are respectivel, A=(1, 1) and B=(2, 4). Find the coordinates of the resultant and the equation of the straight line l containing it.

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11. A non zero vector $\overrightarrow{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The angle between $\overrightarrow{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ can be

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12. Vector $\overrightarrow{O}A = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is $\dfrac{4\hat{i} - \hat{j} - \hat{k}}{\sqrt{2}}$.

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13. Let $\overrightarrow{u} and \overrightarrow{v}$ be unit vectors such that $\overrightarrow{u} \times \overrightarrow{v} + \overrightarrow{u} = \overrightarrow{w}$ and $\overrightarrow{w} \times \overrightarrow{u} = \overrightarrow{v}$. Find the value of $\left[\overrightarrow{u}\overrightarrow{v}\overrightarrow{w}\right]$.

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14. A, B and C are three vectors given by $2\hat{i} + \hat{k}, \hat{i} + \hat{j} + \hat{k}$ and $4\hat{i} - 3\hat{j} + 7\hat{k}$. Then, find R, which satisfies the relation $R \times B = C \times B$ and $R \cdot A = 0$.

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15. If $x \cdot a = 0, x \cdot b = 1,$ [x a b]=1 and $a \cdot b \neq 0$, then find x in terms of a and b.

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16. Let $\widehat{x}, \hat{y}$ and $\hat{z}$ be unit vectors such that $\widehat{x} + \hat{y} + \hat{z} = a. \widehat{x} \times (\hat{y} \times \hat{z}) = b, (\widehat{x} \times \hat{y}) \times \hat{z} = c, a \cdot \widehat{x} = \frac{3}{2}, a \cdot \hat{y} = \frac{7}{4}$ and $|a| = 2$. Find x, y and z in terms of a, b and c.

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17. Let p, q, r be three mutually perpendicular vectors of the same magnitude. If a vector x satisfies th equation $px((x-q)xp) + qx((x-r)xq) + rx((x-p)xr) = 0$ Then x is given by :

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18. Given vectors $\overline{C}B = \bar{a}, \overline{C}A = \bar{b}$ and $\overline{C}O = \bar{x}$ where O is the centre of circle circumscribed about ΔABC, then find vector $\bar{x}$

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Exercise (Questions Asked In Previous 13 Years Exam)

1. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overline{OP}\cdot\overline{OQ}+\overline{OR}\cdot\overline{OS}=\overline{OR}\cdot\overline{OP}+\overline{OQ}\cdot\overline{OS}=\overline{OQ}\cdot\overline{OR}+\overline{OP}\cdot\overline{OS}$$

Then the triangle PQR has S as its

A. centroid

B. orthogonal

C. incentre

D. circumcentre

Answer: B

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2. Let O be the origin and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vector in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$ respectively , of a triangle PQR.

if the triangle PQR varies , then the manimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is

A. $\frac{-3}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{3}$

D. $\frac{-5}{3}$

Answer: A

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3. Let O be the origin, and OX, OY, OZ be three unit vectors in the direction of the sides QR , RP , PQ , respectively of a triangle PQR.

$|OX \times OY| =$ $(a)\sin(P+R)$ (b) $\sin 2R$ $(c)\sin(Q+R)$ (d) $\sin(P+Q)$.

A. $\sin(P+Q)$

B. $\sin(P+R)$

C. $\sin(Q+R)$

D. $\sin 2R$

Answer: A

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4. Let a, b and c be three unit vectors such that $a \times (b \times c) = \frac{\sqrt{3}}{2}(b+c)$. If b is not parallel to c , then the angle between a and b is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: D

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5. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$ then a value of $\sin\theta$ is : (1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{-\sqrt{2}}{3}$

C. $\frac{2}{3}$

D. $-\frac{2\sqrt{3}}{3}$

Answer: (a)

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6. If $\overrightarrow{a}, \overrightarrow{b}\, and\, \overrightarrow{c}$ are unit vectors satisfying $\left|\overrightarrow{a}-\overrightarrow{b}\right|^2+\left|\overrightarrow{b}-\overrightarrow{c}\right|^2+\left|\overrightarrow{c}-\overrightarrow{a}\right|^2=9,$ then $\left|2\overrightarrow{a}+5\overrightarrow{b}+5\overrightarrow{c}\right|$ is.

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7. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k},$ and perpendicular to vector $\hat{i}+\hat{j}+\hat{k},$ is/are

A. $\hat{j}-\hat{k}$

B. $-\hat{i}+\hat{j}$

C. $\hat{i}-\hat{j}$

D. $-\hat{j}+\hat{k}$

Answer: A

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8. Let $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}, \overrightarrow{b}=\hat{i}-\hat{j}+\hat{k} and \overrightarrow{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\overrightarrow{v}$ in the plane of $\overrightarrow{a} \text{ and } \overrightarrow{b}$, whose projection on $\overrightarrow{c}$ is $\frac{1}{\sqrt{3}}$ is given by a.$\hat{i}-3\hat{j}+3\hat{k}$ b. $-3\hat{i}-3\hat{j}+3\hat{k}$ c. $3\hat{i}-\hat{j}+3\hat{k}$ d. $\hat{i}+3\hat{j}-3\hat{k}$

A. $\hat{i}-3\hat{j}+3\hat{k}$

B. $-3\hat{i}-3\hat{j}-\hat{k}$

C. $3\hat{i}-\hat{j}+3\hat{k}$

D. $\hat{i}+3\hat{j}-3\hat{k}$

Answer: C

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9. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{A}B=2\hat{i}+10\hat{j}+11\hat{k} and \overrightarrow{A}D=-\hat{i}+2\hat{j}+2\hat{k}\cdot$ The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD

becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: B

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10. Let P,Q R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$, respectively. The quadrilateral PQRS must be:

A. parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square

Answer: (a)

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11. If $a and b$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}} and \ \vec{b} = \frac{\hat{2i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then find the value of $\left(2\vec{a} + \vec{b}\right)\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right].$

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12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a} . \vec{c} = \frac{1}{2}$, then

A. a, b, c are non-coplanar

B. a, b, d are non-coplanar

C. b, d are non-parallel

D. a, d are parallel and b, c are parallel

Answer: C

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13. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=\frac{1}{2}$.

Then, the volume of the parallelopiped is

A. a) $\frac{1}{\sqrt{2}}$ cu units

B. b) $\frac{1}{2\sqrt{2}}$ cu units

C. c) $\frac{\sqrt{3}}{2}$ cu units

D. d) $\frac{1}{\sqrt{3}}$ cu units

Answer: A

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14. Let two non-collinear unit vectors $\vec{a}$ and $\vec{b}$ form an acute angle. A point P moves so that at any time t, time position vector, $\overrightarrow{OP}$ (where O is the origin) is given by $\widehat{a}\cot t+\hat{b}\sin t$. When p is farthest fro origing o, let M be the length of $\overrightarrow{OP}$ and $\widehat{u}$ be the unit vector along $\overrightarrow{OP}$ .then

A. $\widehat{u}=\dfrac{\widehat{a}+\hat{b}}{\left|\widehat{a}+\hat{b}\right|}$ and $M=\left(1+\widehat{a}\cdot\hat{b}\right)^{\frac{1}{2}}$

B. $\widehat{u}=\dfrac{\widehat{a}-\hat{b}}{\left|\widehat{a}-\hat{b}\right|}$ and $M=\left(1+\widehat{a}\cdot\hat{b}\right)^{\frac{1}{2}}$

C. $\widehat{u}=\dfrac{\widehat{a}+\hat{b}}{\left|\widehat{a}+\hat{b}\right|}$ and $M=\left(1+2\widehat{a}\cdot\hat{b}\right)^{\frac{1}{2}}$

D. $\widehat{u}=\dfrac{\widehat{a}-\hat{b}}{\left|\widehat{a}-\hat{b}\right|}$ and $M=\left(1+2\widehat{a}\cdot\hat{b}\right)^{\frac{1}{2}}$

Answer: A

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15. Let the vectors PQ,OR,RS,ST,TU and UP represent the sides of a regular hexagon.

Statement I: $PQ\times(RS+ST)\neq 0$

Statement II: $PQ\times RS=0$ and $PQ\times ST\neq 0$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: C

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16. The number of distinct real values of λ , for which the vectors $-\lambda^2\hat{i}+\hat{j}+k, \hat{i}-\lambda^2\hat{j}+\hat{k} and \hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar is

A. 0

B. 1

C. $\pm\sqrt{2}$

D. 3

Answer: C

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17. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

A. $a \times b = b \times c = c \times a = 0$

B. $a \times b = b \times c = c \times a \neq 0$

C. $a \times b = b \times c = a \times c = 0$

D. $a \times b, b \times c, c \times a$ are mutually perpendicular

Answer: B

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18. Let A be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and that P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector A and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{3\pi}{4}$

Answer: B::D

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19. Let $\overrightarrow{a} = \hat{i} + 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} - \hat{j} + \hat{k} and \overrightarrow{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\overrightarrow{a} and \overrightarrow{b}$ whose projectionof c is $1/\sqrt{3}$ is $4\hat{i} - \hat{j} + 4\hat{k}$ b. $3\hat{i} + \hat{j} + 3\hat{k}$ c. $2\hat{i} + \hat{j} - 2\hat{k}$ d. $4\hat{i} + \hat{j} - 4\hat{k}$

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $4\hat{i} + \hat{j} - 4\hat{k}$

C. $2\hat{i} + \hat{j} + 2\hat{k}$

D. None of these

Answer: A

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20. Find the 7th term of GP if the first term is 7 and common ratio is 2.

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21. The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is (a) $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ (b) $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ (c) $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$ (d) $\dfrac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

A. $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

C. $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$

D. $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$.

Answer: C

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22. The value of a so that the volume of parallelepiped formed by $\hat{i}+a\hat{j}+\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ is minimum is a.-3 b. 3 c. $1/\sqrt{3}$ d. $\sqrt{3}$

A. -3

B. 3

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Answer: C

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23. If $\overrightarrow{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \overrightarrow{a}.\overrightarrow{b} = 1 and \overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}$,then $\hat{b}$ is

a. $\hat{i} - \hat{j} + \hat{k}$ b. $2\hat{j} - \hat{k}$ c. $\hat{i}$ d. $2\hat{i}$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{j} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: C

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24. Let $v = 2\hat{i} + \hat{j} - \hat{k}$ and $w = \hat{i} + 3\hat{k}$. If $\widehat{u}$ is unit vector and the maximum value of $[uvw] = \sqrt{\lambda}$, then the value of $(\lambda - 51)$ is

A. -1

B. $\sqrt{10} + \sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: C

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25. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicualar to each other, then the angle between $\vec{a}$ and $\vec{b}$ is

A. 45°

B. 60°

C. $\cos^{-1}\left(\frac{1}{3}\right)$

D. $\cos^{-1}\left(\frac{2}{7}\right)$

Answer: B

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26. Let $a = 2\hat{i} + \hat{j} - 2\hat{k}, b = \hat{i} + \hat{j}$ and c be a vectors such that $|c - a| = 3, |(a \times b) \times c| = 3$ and the angle between c and $a \times b$ is 30°. Then a. c is equal to

A. $\frac{25}{8}$

B. 2

C. 5

D. $\frac{1}{8}$

Answer: B

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27. If $[a \times b b \times c c \times a] = \lambda[abc]^2$, then λ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: C

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28. If a and b_1 are two unit vectors such that $a+2b$ and $5a-4b$ are perpendicular to each other, then the angle between a and b is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

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29. Let ABCD be a parallelogram such that $\overrightarrow{A}B=\overrightarrow{q}, \overrightarrow{A}D=\overrightarrow{p}\, and \angle BAD$ be an acute angle. If $\overrightarrow{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\overrightarrow{r}$ is given by (1) $\overrightarrow{r}=3\overrightarrow{q}-\frac{3\left(\overrightarrow{p}\cdot\overrightarrow{q}\right)}{\left(\overrightarrow{p}\cdot\overrightarrow{p}\right)}\overrightarrow{p}$ (2) $\overrightarrow{r}=-\overrightarrow{q}+\left(\frac{\overrightarrow{p}\cdot\overrightarrow{q}}{\overrightarrow{p}\cdot\overrightarrow{p}}\right)\overrightarrow{p}$ (3) $\overrightarrow{r}=\overrightarrow{q}+\left(\frac{\overrightarrow{p}\cdot\overrightarrow{q}}{\overrightarrow{p}\cdot\overrightarrow{p}}\right)\overrightarrow{p}$ (4) $\overrightarrow{r}=-3\overrightarrow{q}+\frac{3\left(\overrightarrow{p}\cdot\overrightarrow{q}\right)}{\left(\overrightarrow{p}\cdot\overrightarrow{p}\right)}\overrightarrow{p}$

A. $r=3p+\frac{3(q\cdot p)}{p\cdot p}p$

B. $r=-p+\frac{(q\cdot p)}{p\cdot p}p$

C. $r=p-\frac{(q\cdot p)}{p\cdot p}p$

D. $r=-3p+\frac{3(q\cdot p)}{p\cdot p}p$

Answer: B

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30. $\vec{a}=\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\vec{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\vec{a}-\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is:

A. -3

B. 5

C. 3

D. -5

Answer: D

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31. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}X\vec{c}=\vec{b}Xd\vec{} = and\vec{a}\dot{\vec{d}}=0$. Then the vector $\vec{d}$ is equal to : (1) $\vec{b}-\left(\frac{\vec{b}\dot{\vec{c}}}{\vec{a}\dot{\vec{d}}}\right)\vec{c}$ (2) $\vec{c}+\left(\frac{\vec{a}\dot{\vec{c}}}{\vec{a}\dot{\vec{b}}}\right)\vec{b}$ (3) $\vec{b}+\left(\frac{\vec{b}\dot{\vec{c}}}{\vec{a}\dot{\vec{b}}}\right)\vec{c}$ (4) $\vec{c}-\left(\frac{\vec{a}\dot{\vec{c}}}{\vec{a}\dot{\vec{b}}}\right)\vec{b}$

A. $c+\left(\frac{a \cdot c}{a \cdot b}\right) b$

B. $b+\left(\frac{b \cdot c}{a \cdot b}\right) c$

C. $c-\left(\frac{a \cdot c}{a \cdot b}\right) b$

D. $b-\left(\frac{b \cdot c}{a \cdot b}\right) c$

Answer: C

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32. If the vectors $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}, \hat{i}+\hat{j}+c\hat{k}$, where a, b, c are coplanar, then $a+b+c-abc=$

A. -2

B. 2

C. 0

D. -1

Answer: A

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33. Let $\overrightarrow{a}=\hat{j}-\hat{k}$ and $\overrightarrow{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector b satisfying $\overrightarrow{a}\times\overrightarrow{b}+\overrightarrow{c}=0$ and $\overrightarrow{a}.\overrightarrow{b}=3,$ is

A. $-\hat{i}+\hat{j}-2\hat{k}$

B. $2\hat{i}-\hat{j}+2\hat{k}$

C. $\hat{i}-\hat{j}-2\hat{k}$

D. $\hat{i}+\hat{j}-2\hat{k}$

Answer: D

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34. If the vectors $\overrightarrow{a}=\hat{i}-\hat{j}+2\hat{k}.\overrightarrow{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\overrightarrow{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ,μ)

A. $(-3, 2)$

B. $(2, -3)$

C. $(-2, 3)$

D. $(3, -2)$

Answer: A

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35. If $\vec{u}, \vec{v}, \vec{w}$ are non -coplanar vectors and $p, q,$ are real numbers then the equality

$\left[3\vec{u}\,p\vec{v}\,p\vec{w}\right] - \left[p\vec{v}\,\vec{w}\,q\vec{u}\right] - \left[2\vec{w}\,q\vec{v}\,q\vec{u}\right] = 0$ holds for

A. exactly two values of (p, q)

B. more than two but not all values of (p, q)

C. all values of (p,q)

D. exactly one value of (p, q)

Answer: D

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36. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{\text{i}} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of $\alpha \text{and} \beta$? (1) $\alpha = 2, \beta = 2$ (2) $\alpha = 1, \beta = 2$ (3) $\alpha = 2, \beta = 1$ (4) $\alpha = 1, \beta = 1$

A. $\alpha = 1, \beta = 1$

B. $\alpha = 2, \beta = 2$

C. $\alpha = 1, \beta = 2$

D. $\alpha = 2, \beta = 1$

Answer: D

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37. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are unit vectors and θ is the acute angle between them, then $2\overrightarrow{u} \times 3\overrightarrow{v}$ is a unit vector for

A. exactly two values of θ

B. more than two but not all values of θ

C. no value of θ

D. exactly one value of θ

Answer: D

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38. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}, b = \hat{i} - \hat{j} + 2\hat{k}$ and $\bar{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector c lies in the plane of a and b , then x equals

A. 0

B. 1

C. -4

D. -2

Answer: D

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39. If $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right), Where \vec{a}, \vec{b}$ and $\vec{c}$ and any three vectors such that $\vec{a}.\vec{b} = 0, \vec{b}.\vec{c} = 0,$ then $\vec{a}$ and $\vec{c}$ are

A. inclined at an angle of $\frac{\pi}{6}$ between them

B. perpendicular

C. parallel

D. inclined at an angle $\frac{\pi}{3}$ between them

Answer: C

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40. Show that the points with position vectors $\overrightarrow{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\overrightarrow{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{c} = \hat{i} - 3\hat{j} - 5\hat{k}$

respectively form the vertices of a right angled triangle.

A. $-2 \text{ and } -1$

B. $-2 \text{ and } 1$

C. $2 \text{ and } -1$

D. $2 \text{ and } 1$

Answer: D

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41. The distance between the line $r = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and the plane $r \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$, is

A. $\frac{10}{3}$

B. $\frac{3}{10}$

C. $\frac{10}{3\sqrt{3}}$

D. $\frac{10}{9}$

Answer: C

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42. If a is any vector, then

$\left(a \times \hat{i}\right)^2 + \left(a \times \hat{j}\right)^2 + \left(a \times \hat{k}\right)^2$ is equal to

A. $4a^2$

B. $2a^2$

C. a^2

D. $3a^2$

Answer: B

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43. If a,b and c are non-coplanar vectors and λ is a real number, then

$\left[\lambda(a+b)\middle|\lambda^2 b\middle|\lambda c \mid \lambda c\right] = [a \quad a+c \quad b]$ fforr

A. (a)exactly two values of λ

B. (b)exactly three values λ

C. (c)no value of λ

D. (d)exactly one value of λ

Answer: C

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44. If $\overrightarrow{a} = \hat{i} - \hat{k}, \overrightarrow{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$

$\overrightarrow{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}.$

then $\overrightarrow{a}.\left(\overrightarrow{b} \times \overrightarrow{c}\right)$ depends on

A. neither x nor y

B. both x and y

C. only x

D. only y

Answer: A

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45. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2 and |\vec{w}| = 3.$ If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals a 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: C

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46. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then the value of $\sin\theta$ is:

A. $\frac{1}{3}$

B. $\frac{\sqrt{2}}{3}$

C. $\frac{2}{3}$

D. $\frac{2\sqrt{2}}{3}$

Answer: D

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47. A particle acted by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{9}j - \hat{k}$ is displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \hat{k}$. find the total work done by the forces in units.

A. 40 units

B. 30units

C. 25 units

D. 15 units

Answer: A

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48. If $\overrightarrow{u}, \overrightarrow{v}$ and $\overrightarrow{w}$ are three non-coplanar vectors, then prove that $\left(\overrightarrow{u}+\overrightarrow{v}-\overrightarrow{w}\right)\cdot\left[\left(\overrightarrow{u}-\overrightarrow{v}\right)\times\left(\overrightarrow{v}-\overrightarrow{w}\right)\right]=\overrightarrow{u}\cdot\left(\overrightarrow{v}\times\overrightarrow{w}\right)$

A. 0

B. $u\cdot v\times w$

C. $u\cdot w\times v$

D. $3u\cdot v\times w$

Answer: B

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49. a, b, c are three vectors, such that $a+b+c=0, |a|=1, |b|=2, |c|=3,$ then $a \cdot b+b \cdot c+c \cdot a$ is equal to

A. 0

B. -7

C. 7

D. 1

Answer: B

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50. A tetrahedron has vertices of $O(0, 0, 0), A(1, 2, 1), B(2, 1, 3)$ and $C(-1, 1, 2)$. Then, the angle between the faces OAB and ABC will be

A. $\cos^{-1}\left(\frac{19}{35}\right)$

B. $\cos^{-1}\left(\frac{17}{31}\right)$

C. 30°

D. 90°

Answer: A

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51. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$.If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$ then $\left|\vec{w} \cdot \hat{n}\right|$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: D

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52. Given, two vectors are $\hat{i}-\hat{j}$ and $\hat{i}+2\hat{j}$, the unit vector coplanar with the two vectors and perpendicular to first is

A. $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

B. $\frac{1}{\sqrt{5}}\left(2\hat{i}+\hat{j}\right)$

C. $\pm\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

D. None of these

Answer: (a)

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