

MATHS

BOOKS - ARIHANT MATHS

PROPERTIES AND SOLUTION OF TRIANGLES

Examples

1. Find the angles of the triangle whose sides are $3 + \sqrt{3}$, $2\sqrt{3}$ and $\sqrt{6}$.

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2. The sides of a triangle are 8 cm, 10 cm and 12 cm. Prove that the greatest angle is double of the smallest angle.



6. Solve

$$b\cos^2rac{C}{2}+c\cos^2rac{B}{2}$$
 in terms of k, where k is permeter of the $\Delta ABC.$

7. In a
$$\Delta ABC, c\cos^2rac{A}{2}+a\cos^2rac{C}{2}=rac{3b}{2}, ext{ then } a,b,c$$
 are in

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8. In a
$$\Delta ABC, a = 2b$$
 and $|A - B| = \frac{\pi}{3}$. Determine the $\angle C$.

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9. In a ΔABC , the tangent of half the difference of two angle is onethird the tangent of half the sum of the angle. Determine the ratio of the sides opposite to the angles.





11. Consider the following statements concerning a ΔABC

(i) The sides a,b,c and area of triangle are rational.

(ii)
$$a, \tan\frac{B}{2}, \tan\frac{C}{2}$$

(iii) $a, \sin A \sin B, \sin C$ are rational .

Prove that $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$

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12. Prove that
$$rac{r_1-r}{a}+rac{r_2-r}{b}=rac{c}{r_3}$$

13. If in a triangle $r_1 = r_2 + r_3 + r$, prove that the triangle is right

angled





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18. Find the ratio of the circum-radius and the inradius of ΔABC , whose

sides are in the ratio 4:5:6.

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19. Find the ratio of IA: IB: IC, where I is the incentre of $\triangle ABC$.

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20. If the sides of a triangle are in GP and its largest angle is twice tha smallset then the common ratio r satisfies the inequality

21. The equation $ax^2 + bx + c = 0$, where a,b,c are the side of a ΔABC , and the equation $x^2 + \sqrt{2}x + 1 = 0$ have a common root. Find measure for $\angle C$.

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22. In triangle ABC, if $\cot A$, $\cot B$, $\cot C$ are in $A\dot{P}$; then a^2, b^2, c^2 are

in _____ progression.

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23. Find the Side of pedal triangle and Circum-Radius of pedal Triangle ?



24. Find the ratio of the circum-radius and the inradius of ΔABC , whose

sides are in the ratio 4:5:6.



25. In a triangle ABC I_1 , I_2 , I_3 are excentre of triangle then show that II_1 . II_2 . $II_3 = 16R^2r$.

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26. Prove that the locus of a point that is equidistant from both axis is

y=x.



27. If g, h, k denotes the side of a pedal triangle, then prove that

$$rac{g}{a^2} + rac{h}{b^2} + rac{k}{c^2} = rac{a^2 + b^2 + c^2}{2abc}$$

28. If x, y, z are respectively perpendiculars from the circumcentre on the

sides of the
$$\Delta ABC$$
, the value of $rac{a}{x}+rac{b}{y}+rac{c}{z}-rac{abc}{4xyz}=$

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29. If O, H and G represents circum centre, orthocentre and centroid respectively, then show

`HG:GO=

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30. In $\triangle ABC$ it is given distance between the circumcentre (O) and orthocentre (H) is $R\sqrt{1-8\cos A\cos B\cos C}$. If Q is the midopoint of OH, then AQ is

31. Find the distance between the circumcentre and the incentre of the

 $\Delta ABC.$



32. Differentiate the following w.r.t x

 $2^{\sin x}$

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33. If the area of circle is A_1 and area of regular pentagon inscibed in the

circle is A_2 , then find the ratio of area of two.



34. The area of a cyclic quadrilateral ABCD is $\frac{(3\sqrt{3})}{4}$. The radius of the circle circumscribing cyclic quadrilateral is 1. $IfAB = 1, BD = \sqrt{3}$, then

find	BC.	CD.
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35. A regular pentagon and a regular decagon have the same perimeter, prove that their areas are as $2:\sqrt{5}$.
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36. If the sides of a cyclic quadrilateral are 3,3,4,4, show that a circle be inscribed in it.

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37. The two adjacent sides of a cyclic quadrilateral are 2and5 and the angle between them is 60^{0} . If the area of the quadrilateral is $4\sqrt{3}$, find the remaining two sides.

38. If a,b,c,d are the side of a quadrilateral, then find the the minimum



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39. In any triangle ABC, the sides are 6 cm, 10 cm and 14 cm. Then the triangle is obtuse angled with the obtuse angle equal to

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40. If a, b and A are given in a triangle and c_1, c_2 are possible values of

the third side, then prove that $c_1^2+c_2^2-2c_1c_2\cos 2A=4a^2\cos^2 A$

 $x^{\log x}$

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42. Two hagstaffs stand on a horizontal plane. A and B are two points on the line joining their feet and between them. The angles of elevation of the tops of the flagstaffs as seen from A are 30° and 60° and as seen from B are 60° and 45° . If AB is 30 m, then the distance between the flagstaffs is

A. $30 + 15\sqrt{3}$ B. $45 + 15\sqrt{3}$ C. $60 - 15\sqrt{3}$ D. $60 + 15\sqrt{3}$

Answer: A::C

43. In a cubicul hall ABCDPQRS with each side 10m, G is the centre of the walls BCRQ and T is the midpoint of the side AB, the angle of elevation of G at the Point T is

A.
$$\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

B. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
C. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
D. $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Answer: A::C



44. Each side of an equilateral triangle subtends an angle of 60° at the top of a tower h m high located at the centre of the triangle. If a is the length of each side of the triangle, then

A.
$$3a^2 = 2h^2$$

B. $2a^2 = 2h^2$
C. $a^2 = 3h^2$
D. $3a^2 = h^2$

Answer: A::B::C



45. A vertical tower PQ subtends th same angle 30° at each of two place A and B, 60 m apart on the ground, AB subtends an angle 120° at the foot of the tower. If h is the height of the tower, then $9h^2 + h + 1$ is equal to

A. 3121

B. 2136

C. 3600

D. None of these

Answer: A::C



46. If
$$rac{ anual ext{tan}^2(\pi-A)}{4}+rac{ anual ext{tan}^2(\pi-B)}{4}+rac{ anual ext{tan}^2(\pi-C)}{4}=1$$
 , then ABC is

equilateral (b) isosceles (c) scalene (d) none of these

A. equilateral

B. isosceles

C. scalene

D. None of these

Answer: A::B::C



47. In
$$\Delta ABC, a^2+c^2=2002b^2$$
 then $rac{\cot A+\cot C}{\cot B}$ is equal to

A.
$$\frac{1}{2001}$$

B. $\frac{2}{2001}$
C. $\frac{3}{2001}$
D. $\frac{4}{2001}$

Answer: B

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48. A triangle has vertices A, B and C, and the respective opposite sides have lengths a, b and c. This triangle is inscribed in a circle of radius R. If b = c = 1 and the altitude from A to side BC has length $\sqrt{\frac{2}{3}}$, then R equals:

A.
$$\frac{1}{\sqrt{3}}$$

B.
$$\frac{2}{\sqrt{3}}$$

C.
$$\frac{\sqrt{3}}{2}$$

D.
$$\frac{\sqrt{3}}{2\sqrt{2}}$$

Answer: B::C



49. In ΔABC , if AC = 8, BC = 7 and D lies between A and B such

that AD = 2, BD = 4, then the length CD equals



B. $\sqrt{48}$

 $\mathsf{C.}\,\sqrt{51}$

D.
$$\sqrt{75}$$

Answer: A



triangle is

(a)isosceles (b)right angled (c)equilateral (d)obtuse angled

A. isosceles

B. right angled

C. equilateral

D. obtuse angled

Answer: A::B::C::D

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51. Consider a ΔABC and let a, b and c denote the lengths of the sides

opposite to vertices A, B and C, repectively. if a=1, b=3 and $C=60^\circ,$ then \sin^2 B is equal to

A.
$$\frac{27}{28}$$

B. $\frac{3}{28}$
C. $\frac{81}{28}$

D.
$$\frac{1}{3}$$

Answer: B



52. In
$$\triangle ABC$$
, if $\cos A + \sin A - \frac{2}{\cos B + \sin B} = 0$, then $\frac{a+b}{c}$ is equal to

A. $\sqrt{2}$

B. 1

C.
$$\frac{1}{\sqrt{2}}$$

D. $2\sqrt{2}$

Answer: A

53. In a triangle ABC, if $\angle A = 30^{\circ}$, b = 10 and a = x, then the values of x for which there are 2 possible triangles is given by(All symbols used have usual meaning in a triangle.) (a)5 < x < 10 (b) $x < \frac{5}{2}$ (c) $\frac{5}{3} < x < 10$ (d) $\frac{5}{2} < x < 10$

B. (b)
$$x < rac{5}{2}$$

C. (c) $rac{5}{3} < x < 10$
D. (d) $rac{5}{2} < x < 10$

A. (a)5 < x < 10

Answer: A::B

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54. In a $\triangle ABC$, AB = AC, P and Q are points on AC and AB respectively such that CB = BP = PQ = QA. if $\angle AQP = \theta$, then $\tan^2 \theta$ is a root of the equation

A.
$$y^3 + 21y^2 - 35y - 12 = 0$$

B. $y^3 - 21y^2 + 35y - 12 = 0$
C. $y^3 - 21y^2 + 35y - 7 = 0$
D. $12y^3 - 35y^2 + 35y - 12 = 0$

Answer: A::B::C



55. The angle of elevation of the top of a tower a point A due south of it is 30° and from a point B due west of it is 45° . If the height of the tower is 100 meters ,then find the distance AB.

(a)150m (b)200m (c)173.2m (d)141.4m

A. 150 m

B. 200 m

C. 173.2 m

D. 141.4 m

Answer: A::B

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56. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60o. After 10 seconds, its elevation is observed to be 30o. Find the speed of the aeroplane in km/hr.

A. 240

B. $240\sqrt{3}$

C. $60\sqrt{3}$

D. None of these

Answer: A::B::C::D

57. In $\triangle ABC$, the ratio $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ is not always equal to

(All symbols used have usual meaning in a triangle.)

a. 2R, where R is the circumradius

b.
$$\frac{abc}{2\Delta}$$
, where Δ is the area of the triangle
c. $\frac{2}{3}(a^2 + b^2 + c^2)^{\frac{1}{2}}$
d. $\frac{(abc)^{\frac{2}{3}}}{(h_1h_2h_3)^{\frac{1}{3}}}$

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D. $\frac{(abc)^{\frac{2}{3}}}{(h_1h_2h_3)^{\frac{1}{3}}}$

Answer: A::B::C::D

58. Let ABCD be a cyclic quadrilateral such that $AB = 2, BC = 3, \angle B = 120^{\circ}$ and area of quadrilateral $= 4\sqrt{3}$. Which of the following is/are correct ?

A. The value of $(AC)^2$ is equal to 19

B. The sum of all positive value of product AC. BD I is equal to 35

C. The sum of all posible value of $(AD)^2$ is equal to 29

D. The value of $\left(CD \right)^2$ can be 4

Answer: B

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59. In a triangle ABC, which of the following quantities denote the area of

the triangles $rac{a^2-b^2}{2}rac{\sin A \sin B}{\sin (A-B)}$

60. Consider the system of equations

 $\sin x \cos 2y = \left(a^2-1\right)^2+1, \cos x \sin 2y = a+1$

The number of values of $x\in [0,2\pi]$, when the system has solution for permissible values of a, is/are

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61. In a triangle ABC, let $2a^2 + 4b^2 + c^2 = 2a(2b+c)$, then which of the

following holds good?

A.
$$\cos B = rac{-7}{8}$$

B. $\sin(A-C) = 0$
C. $rac{r}{r_1} = rac{1}{5}$

 $\mathsf{D.} \sin A : \sin B : \sin C = 1 : 2 : 1$

Answer: A::B::C

62. In AABC, angle A, B and C are in the ratio 1:2:3, then which of the following is (are) correct? (All symbol used have usual meaning in a triangle.)

A. Circum-radius of
$$\Delta ABC=c$$

B.
$$a:b:c = 1:\sqrt{3}:2$$

C. Permimeter of $\Delta ABC = 3 + \sqrt{3}$

D. Area of
$$\Delta ABC = rac{\sqrt{3}}{8}c^2$$

Answer: A::B::C

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63. If the length of tangents from A, B, C to the incircle of ΔABC are 4, 6, 8 then which of the following is(are) correct? (All symbols used have usual meaning in a triangle.

A. Area of $\Delta ABCis12\sqrt{6}$

B. r_1, r_2, r_3 are in HP

C. a,b,c are in AP

D.
$$r=rac{4\sqrt{6}}{3}$$

Answer: B

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64. In a right angle triangle ABC, let b = 10, c = $10\sqrt{2}$ find a.

A. Area of triangle ABC is 50.

B. Distance between orthocentre and circumcetre iss $5\sqrt{2}$.

C. Sum of circum-radius and in-radius of $\ riangle ABC$ is equal to 10.

D. Length of internal angle bisector of $\angle ABC$ of $\triangle ABC$ is $\frac{5}{2\sqrt{2}}$.

Answer: A::B::C

65. Let 'l' is the length of median from the vertex A to the side BC of a ΔABC . Then

A.
$$4l^2 = 2b^2 + 2c^2 - a^2$$

B. $4l^2 = b^2 + x^2 + 2bc \cos A$
C. $4l^2 = a^2 + 4bos A$
D. $4l^2 = (2s - a)^2 - 4bc \sin^2 \frac{A}{2}$

Answer: A::B::C::D

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66. If a right angled ΔABC of maximum area is inscribled within a circle of radius R, then (δ representes area of ΔABC and r, r_1, r_2, r_3 represent in-radius and ex-radii, and s is the semi-perimeter of ΔABC , then

A.
$$\Delta=R^2$$

B.
$$rac{1}{r_1}+rac{1}{r_2}+rac{1}{r_3}=rac{\sqrt{2}+1}{R}$$

C. $r=(\sqrt{2}-1)R$
D. $s=(1+\sqrt{2})R$

Answer: A,B,C,D

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67. Statement I In a ΔABC , if a < b < c and r is inradius and r_1, r_2, r_3 are the exradii opposite to angle A,B,C respectively, then $r < r_1 < r_2 < r_3$.

Statement II For, $\Delta ABCr_1r_2+r_2r_3+r_3r_1=rac{r_1r_2r_3}{r}$

A. Statement I is True, Statement II is True, Statement II is a correct

explanation for Statement I.

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I.

C. Statement I is True, Statement II is False.

D. Statement I is False, Statement II is True.

Answer: A::B::C::D



68. Solve the given quadratic equation

$$-2x^2 + 4x + 1 = 0$$

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69. Statement I In
$$a\Delta ABC$$
, $\sum \frac{\cos^2 \frac{A}{2}}{a}$ has the value equal to $\frac{s^2}{abc}$.
Statement II in $a\Delta ABC$, $\cos \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
 $\cos \frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}, \cos \frac{c}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

A. Statement I is True, Statement II is True, Statement II is a correct

explanation for Statement I.

B. Statement I is True, Statement II is True, Statement II is NOT a

correct explanation for Statement I.

C. Statement I is True, Statement II is False.

D. Statement I is False, Statement II is True.

Answer: C

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70. If side(a) = 4 then area of the equilateral triangle ΔABC is equal to

A. a)
$$\frac{3\sqrt{2}}{2}$$

B. b) 3

C. c) $4\sqrt{3}$

D. d) $3\sqrt{2}$

Answer: A::B::C::D

71. In triangle ABC let tanA=1, tanB= 2, tanC=3 and c= 3, The radius of the circle circumscribing the triangle ABC, is equal to



Answer: A::B::C::D



72. Let Δ denote the area of the ΔABC and Δp be the area of its pedal

triangle. If $\Delta = k \Delta p, \,$ then k is equal to



73. If sinx=1, then x is equal to

A.
$$\frac{\pi}{9}$$

B. $\frac{2\pi}{9}$
C. $\frac{\pi}{3}$

D. None of these

Answer: B



74. If the sum of squares of two sides is equal to square of third side then

 ΔABC is

A. Equilateral

B. Isosceless

C. Scalene

D. Right angled

Answer: D

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75. Consider the differential equation $y^2 dx + \left(x - rac{1}{y}
ight) dy = 0$. If y(1)=1,

then x is given by:

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76. If I, m, n denote the side of a pedal triangle, then $\frac{l}{a^2} + \frac{m}{b^2} + \frac{n}{c^2}$ is

equal to

A. (a)
$$rac{a^2+b^2+c^2}{a^3+b^3+c^2}$$

B. (b)
$$\frac{a^2 + b^2 + c^2}{2abc}$$

C. (c) $\frac{a^3 + b^3 + c^3}{abc(a + b + c)}$
D. (d) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

Answer: B

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77. If R be circum-radius of $\Delta ABC, \,$ then circum-radius of a pedal Δ is

A. R

B.
$$\frac{2R}{3}$$

C. $\frac{R}{3}$
D. $\frac{R}{2}$

Answer: D
78. The in-radius of pedal Δ of a ΔABC is

A.
$$\frac{R}{2}$$

 $\mathsf{B.}\,R\sin A\sin B\sin C$

C. $2R\cos A\cos B\cos C$

D.
$$4R\sin{\frac{A}{2}}\sin{\frac{b}{2}}\sin{\frac{C}{2}}$$

Answer: A::B::C::D

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79. In a triangle ABC, if $r_1+r_3+r=r_2$, then find the value of $\left(\sec^2 A + \cos^2 B - \cot^2 C
ight)$,

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80. In ΔABC , let b=6, c=10 and $r_1=r_2+r_3+r$ then find area of

 $\Delta ABC.$



81. Consider on obtuse angle triangles with side 8 cm, 15 cm and \times cm (largest side being 15 cm). If \times is an integer, then find the number of possible triangels.

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82. let ABC be a right angled triangle at C. If the inscribed circle touches the side AB at D and (AD)(BD) = 11, then find the area of triangle ABC.

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83. Consider a ΔABC and let a,b, and c denote the leghts of the sides opposite to vertices A,B and C, respectively. Suppose a = 2, b = 3, c = 4 and H be the orthocentre. Find $15(HA)^2$.

84. In a triangle ABC, the internal angle bisector of $\angle ABC$ meets AC at K.

If BC = 2, CK = 1 and $BK = \frac{3\sqrt{2}}{2}$, then find the length of side AB.

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85. Triangle ABC has AC = 13, AB = 15 and BC = 14. Let 'O' be the circumcentre of the ΔABC . If the length of perpendicular from the point 'O' on BC can be expressed as a rational $\frac{m}{n}$ in the lowest form then find (m + n).

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86. Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$. The angle between the sides is $\frac{\pi}{3}$. Find the perimeter of Δ .

87. If in $\Delta ABC,$ $\angle A=90^\circ$ and c, sin B cos B are rational numbers, then show a and b are rational .



88. If the sides of a ΔABC are in AP and a is the smallest side, then $\cos A$

equals

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89. If A, B and C are angles of a triangle such that $\angle A$ is obtuse, then

show tan B $\tan C < 1$.



90. If A is the area and 2s is the sum of the sides of a triangle, then (a)

$$A \leq rac{s^2}{4}$$
 (b) $A \leq rac{s^2}{3\sqrt{3}}$ (c) $2R\sin A \sin B \sin C$ (d) none of these

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91. In a triangle, if $r_1 > r_2 > r_3$, then show a > b > c.

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92. D is midpoint of BC in ΔABC such that AD and AC are perpendicular,

Show that
$$\cos A \cos C = rac{2ig(c^2-a^2ig)}{3ac}$$

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93. In any ΔABC , prove that

$$a^3\cos(B-C)+b^3\cos(C-A)+c^3\cos(A-B)=3abc$$

94. If in a triangle of base 'a', the ratio of the other two sides is r (<1). Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$

95.
$$ig(x^2+y^2ig)dy=xydx.$$
 If $y(x_0)=e,$ y (1) = 1, then the value of x_0 =

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96. The internal bisectors of the angles of a ΔABC meet the sides BC, CA, AB in D,E and F, respectively. Show that the area of the ΔDEF is equal to,

 $\frac{2\Delta abc}{(b+c)(c+a)(a+b)}$

97. Prove m:n theorem in a ΔABC , a point D is taken on side BC such that BD:DC is m:n. Then prove that(1) $(m + n)\cot \theta = m \cot \alpha - n \cot \beta$ (2) $(m + n)\cot \theta = n \cot B - m \cot C$

98. The base of a triangle is divided into three equal parts. If t_1, t_2, t_3 are the tangents of the angles subtended by these parts at the opposite vertex, prove that $\left(\frac{1}{t_1} + \frac{1}{t_2}\right)\left(\frac{1}{t_2} + \frac{1}{t_3}\right) = 4\left(1 + \frac{1}{(t_2)^2}\right)$.

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99. Prove that the triangle ABC is equilateral if $\cot A + \cot B + \cot C = \sqrt{3}$

100. In any triangle. if $rac{a^2-b^2}{a^2+b^2}=rac{\sin(A-B)}{\sin(A+B)}$, then prove that the

triangle is either right angled or isosceles.



101. The sides of a triangle are in AP. If the angles A and C are the greatest and smallest angle respectively, then $4(1 - \cos A)(1 - \cos C)$ is equal to

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102. perpendiculars are drawn from the angles A, BandC of an acuteangled triangle on the opposite sides, and produced to meet the circumscribing circle. If these produced parts are α , β , γ , respectively, then show that $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 2(\tan A + \tan B + tanC)$.



104. If p is the produce of the since of angles of a triange, and q the product of their consines then tangents of the angles are roots of the equation

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105. Given the base of a triangle, the opposite angle A, and the product k^2

of the other two sides, show that it is not possible for a to be less than

 $2k\frac{\sin A}{2}$

106. If in a triangle
$$ABC, \angle C = 60^{\circ}$$
, then prove that
 $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$.

107. Let 1 < m < 3. In a triangle ABC , if 2b = (m+1) a & $\cos A = \frac{1}{2}\sqrt{\frac{(m-1)(m+3)}{m}}$ prove that the are two values to the

third side, one of which is m times the other.

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108. In any $\triangle ABC$, if D be any points of the base BC such that $\frac{BD}{DC} = \frac{m}{n}$ and $\angle ABD = \alpha$, $\angle DAC = \beta$, $\angle CDA = \theta$ and AD = x

then prove that

$$(m+n)^2.\ x=(m+n)ig(mb^2+nc^2ig)-mna^2$$

109. ABCD is a trapezium such that $AB \mid |CDandCB$ is perpendicular to them. If $\angle ADB = \theta$, BC = p, andCD = q, show that $AB = \frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ **Watch Video Solution 110.** In a triangle ABC, prove that $\frac{\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right)}{\cot A + \cot B + \cot(C)} = \frac{(a + b + c)^2}{a^2 + b^2 + c^2}$

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111. If the sides of a triangle are in A.P., and its greatest angle exceeds the least angle by α , show that the sides are in the ratio 1 + x : 1 : 1 - x, , where $x = \sqrt{\frac{1 - \cos \alpha}{7 - \cos \alpha}}$

112. In a ABC, if $\frac{\tan A}{2}$, $\frac{\tan B}{2}$, $\frac{\tan C}{2}$ $are \in A\dot{P}$; then show that $\cos A$, $\cos B$, $\cos C$ are in $A\dot{P}$.

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113. If a,b,c are in HP, then prove that $\sin^2 \frac{A}{2}$, $\sin^2 \frac{B}{2}$, $\sin^2 \frac{C}{2}$ are also in

HP.

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114. If r_1, r_2, r_3 are the ex-radii of ΔABC , then prove that

$$rac{bc}{r_1}+rac{ca}{r_2}+rac{ab}{r_3}=2Rigg[igg(rac{a}{b}+rac{b}{a}igg)+igg(rac{b}{c}+rac{c}{b}igg)+igg(rac{c}{a}+rac{a}{c}igg)-3igg]$$

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115. If r and R are radii of the incircle and circum-circle of ΔABC , then

prove that :

$$8 r R ig\{ \cos^2 A \, / \, 2 + \cos^2 B \, / \, 2 + \cos^2 C \, / \, 2 ig\}$$

$$= 2bc + 2ca + 2ab - a^2 - b^2 - c^2.$$

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116. Prove that

 $r_1^2 + r_2^2 + r_3^3 + r^2 = 16R^2 - a^2 - b^2 - c^2.$

where r= in radius, R = circumradius, r_1, r_2, r_3 are ex-radii.

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117. Tangent are parallel to the three sides are drawn to the in-circle. If

x,y,z are the lengths of parts of the tangents with in triangle, then prove

that
$$rac{x}{a}+rac{y}{b}+rac{z}{c}=1.$$

118. If in a triangle ABC, $\cos A + 2\cos B + \cos C = 2$ prove that the

sides of the triangle are in AP



119. In a cyclic quadrilateral ABCD, prove that $\tan^2 \frac{B}{2} = \frac{(s-a)(s-b)}{(s-c)(s-d)}$, a, b, c, and d being the lengths of sides

ABC, CD and DA respectively and s is semi-perimeter of quadrilateral.

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120. If x,y,z are perpendicular from circum centre of the sides of the

$$\Delta ABC$$
 respectively. Prove that $rac{a}{z}+rac{b}{y}+rac{c}{z}=rac{abc}{4xyz}$

121. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, then prove that

$$y^2rac{d^2y}{dx^2}-xrac{dy}{dx}+y=0$$

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122. The circle inscribed in the triangle ABC touches the side BC, CA and AB in the point A_1B_1 and C_1 respectively. Similarly the circle inscribed in the $\Delta A_1B_1C_1$ touches the sieds in A_2 , B_2 , C_2 respectively and so on. If $A_nB_nC_n$ be the nth Δ so formed, prove that its angle are

$$rac{\pi}{3}-(2)^{-n}\Big(A-rac{\pi}{3}\Big), rac{\pi}{3}-(2)^{-n}\Big(B-rac{\pi}{3}\Big), rac{\pi}{3}-(2)^{-n}\Big(C-rac{\pi}{3}\Big).$$

Hence prove that the triangle so formed is ultimately equilateral.

123. In a
$$\Delta ABC, \,\,$$
 prove that: $2r \leq rac{a \cot A + b \cot B + c \cot C}{3} \leq R$

124. If Δ is the area of a triangle with side lengths a, b, and c, then show that $\Delta \leq \frac{1}{4}\sqrt{(a+b+c)abc}$. Also show that equality occurs in the above inequality if and only if a = b = c

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SOLVED EXAMPLES

1. Match the statements of Column I with values of Column II.

Column I	Column II
(A) $\lim_{x \to \frac{\pi^+}{2}} \tan^{-1} (\tan x)$	(p) 0
(B) $\lim_{n \to \infty} \left[\sum_{r=1}^{n} \frac{1}{2^{r}} \right] ([\cdot]] \text{ denotes the greatest integer function})$	(q) Doesn't exist
(C) $\lim_{x \to \infty} \sec^{-1}\left(\frac{x}{x+1}\right)$	$(r) = \frac{\pi}{2}$
(D) $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{(1 - \sin x)^{2/3}}$	(s) $\frac{\pi}{2}$

1. In the given figure, if $AB = AC, \angle BAD = 30^{\circ}$ and AE = AD, then

x is equal to



A. $15^{\,\circ}$

B. 10°

C.
$$12\frac{1}{2}$$

D. $7\frac{1}{2}$

2. In $\Delta ABC, \, a=4, \, b=12 \, ext{ and } \, B=60^{\,\circ}, \,$ thn the vlaue of sin A is

A.
$$\frac{1}{2\sqrt{3}}$$

B.
$$\frac{1}{3\sqrt{2}}$$

C.
$$\frac{2}{\sqrt{3}}$$

D.
$$\frac{\sqrt{3}}{2}$$

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3. Let ABC be a triangle such that $\angle A = 45^{\circ}, \angle B = 75^{\circ}, ext{ then } a + c\sqrt{2}$

is equal to

A. (a) 0

B. (b) b

C. (c) 2b

D. (d) -b

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4. If the angles A, B, C of ΔABC are in A.P. and $b\!:\!c=\sqrt{3}\!:\!\sqrt{2}$, show that

 $A=75^{\circ}.$

A.
$$\frac{\pi}{6}$$

B. $\frac{\pi}{4}$
C. $\frac{5\pi}{12}$
D. $\frac{\pi}{2}$

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5. If
$$\cot rac{A}{2} = rac{b+c}{a},$$
 then ΔABC is

A. (a) Isosceles

B. (b) Equilateral

- C. (c) Right angled
- D. (d) None of these

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6. In any triangle. if
$$\frac{a^2-b^2}{a^2+b^2}=rac{\sin(A-B)}{\sin(A+B)}$$
 , then prove that the

triangle is either right angled or isosceles.

A. Right angled or isosceles

- B. Right angled and isosceles
- C. Equiliateral
- D. None of these



7. In any triangle
$$ABC$$
, prove that:
 $\frac{a^2 \sin(B-C)}{\sin B + \sin C} + \frac{b^2 \sin(C-A)}{\sin C + \sin A} + \frac{c^2 \sin(A-B)}{\sin A + \sin B} = 0$
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8. In any ΔABC , is $2 \cos B = \frac{a}{c}$, then the triangle is
A. right angled
B. equilateral
C. isosceles
D. None of these

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9. Prove that

$$\sin^2\Bigl(rac{\pi}{4}\Bigr)+\cos^2\Bigl(rac{\pi}{6}\Bigr)- an^2\Bigl(rac{\pi}{4}\Bigr)=rac{1}{4}$$

10. If a $\cos A = b \cos B$, then the triange is

A. Isosceles

B. Right angled

C. Isosceles or right angled

D. Right angled isosceles

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11. In
$$\Delta ABC$$
, $(a+b+c)(b+c-a) = kbc$ if

A. $\lambda < 0$

B. $\lambda > 0$

 $\mathsf{C}.\,0<\lambda<4$

D. $\lambda < 4$

12. If : a = 9, b = 8 and c = x satisfies 3 cos C = 2, then : x =

A. x = 5

- $\mathsf{B.}\,x=6$
- $\mathsf{C.}\,x=4$
- $\mathsf{D.}\, x=7$

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13. In ΔABC , if $\sin^2 A + \sin^2 B = \sin^2 C$, then the triangle is

A. equilateral

B. isosceles

C. right angled

D. None of these

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14. The lengths of the sides of a triangle are $\alpha - \beta, \alpha + \beta$ and $\sqrt{3\alpha^2 + \beta^2}, (\alpha > \beta > 0)$. Its largest angle is

A. (a)
$$\frac{2\pi}{3}$$

B. (b) $\frac{\pi}{2}$
C. (c) $\frac{3\pi}{4}$
D. (d) $\frac{5\pi}{6}$

15. In any triangle ABC , prove that: $rac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}=rac{a^2+b^2}{a^2+c^2}$

A.
$$\frac{a^2 + b^2}{a^2 + c^2}$$

B. $\frac{b^2 + c^2}{b^2 - c^2}$
C. $\frac{c^2 - a^2}{a^2 + b^2}$

D. None of these

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16. If the sides of a triangle ABC are in AP and 'a' is the smallest side, then

express cos A in term of b and c.

A.
$$\frac{3c-4b}{2c}$$
B.
$$\frac{3c-4b}{2b}$$
C.
$$\frac{4c-3b}{2c}$$

D. None of these

17. In a $\Delta ABC, \, a^2\cos 2B + b^2\cos$ 2A+2ab cos (A-B) is equal to?

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18. In any $\Delta ABC, 2[bc\cos A + ca\cos B + ab\cos C] =$

A.
$$a^2 + b^2 + c^2$$

B. $a^2 + b^2 - c^2$
C. $a^2 - b^2 + c^2$

 $\mathsf{D}.\,a^2-b^2+c^2$

19. In a
$$\Delta ABC, anrac{1}{2}(A+B). ext{ cot } rac{1}{2}(A-B)$$
 is equal to ?

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20. If in a $\Delta ABC, \, b=\sqrt{3}, \, c=1 \, ext{ and } \, B-C=90^\circ, \,$ then $\angle A$ is

A. 30°

B. 45°

C. 75°

D. $15^{\,\circ}$

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Exercise For Sesssion 2

1. If in a triangle ABC, (s-a)(s-b) = s(s-c), then angle C is equal to

A. 90°

B. 45°

C. 30°

D. $60^{\,\circ}$

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2. In any $\triangle ABC$, If $\cot \frac{A}{2}$, $\cot \frac{B}{2}$, $\cot \frac{C}{2}$ are in AP, then a, b, c are in (a) AP (b) GP (c) HP (d) None of these

3. In any ΔABC , $\frac{\tan \frac{A}{2} - \tan \frac{B}{2}}{\tan \frac{A}{2} + \tan \frac{B}{2}}$ is equal to: A. (a) $\frac{a-b}{a+b}$ B. (b) $\frac{a-b}{c}$ C. (c) $\frac{a-b}{a+b+c}$ D. (d) $\frac{c}{a+b}$

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4. Prove that
$$bc\cos^2$$
. $\frac{A}{2} + ca\cos^2$. $\frac{B}{2} + ab\cos^2$. $\frac{C}{2} = s^2$

A. $(s - a)^2$ B. $(s - b)^2$ C. $(s - c)^2$ D. s^2 5. In a $\Delta ABC, ~~ ext{if}~~\cos A + \cos B = 4 \sin^2 igg(rac{C}{2} igg), ~ ext{then a,b,c are in}$

(a) AP

- (b) GP
- (c) HP

(d) None of these

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6. In a riangle ABC, if $b^2+c^2=3a^2$, then $\cot B+\cot C-\cot A$ is equal to

A. 1

B. $\sqrt{3}$

C. 2

D. None of these



7. In any
$$\Delta ABC, \left(\frac{b-c}{a}\right)\cos^2\left(\frac{A}{2}\right) + \left(\frac{c-a}{b}\right)\cos^2\left(\frac{b}{2}\right) + \left(\frac{a-b}{2}\right)\cos^2\left(\frac{C}{2}\right)$$

is equal to?

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8. In a ΔABC , the tangent of half the difference of two angle is onethird the tangent of half the sum of the angle. Determine the ratio of the sides opposite to the angles.

A. 2:3

B. 1:3

C.2:1

D. 3:4

9. If in a triangle
$$a\cos^2$$
. $\frac{C}{2} + c\cos^2$. $\frac{A}{2} = \frac{3b}{2}$, then find the relation

between the sides of the triangles

A. AP

B. GP

C. HP

D. AGP



10. In the adjacent figure 'P' is any interior point of the equilateral triangle ABC is side length 2 unit . If x_a, x_b and x_c represent the distance of P from the sides BC, CA and AB respectively then $x_a + x_b + x_c$ is equal to

B. $\sqrt{3}$

C.
$$\frac{\sqrt{3}}{2}$$

D. $2\sqrt{3}$

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11. If
$$c^2=a^2+b^2$$
, then prove that $4s(s-a)(s-b)(s-c)=a^2b^2$

A. s^4 B. $b^2 c^2$ C. $c^2 a^2$

D. a^2b^2

12. The number of possible $\angle ABC$ in which $BC = \sqrt{11}cm, CA = \sqrt{13}cm$ and $A = 60^{\circ}$ is A. 0 B. 1 C. 2 D. None of these

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13. If two sides a, b and the $\angle A$ be such that two triangles are formed then sum of two values of the third side is

A. $b^2 - a^2$

 $\mathrm{B.}\,2b\cos A$

 $\mathsf{C.}\,2b\sin A$

$$\mathsf{D}.\,\frac{b-c}{b+c}$$

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14. If in a ΔABC , $\sin A = \sin^2 B$ and $2\cos^2 A = 3\cos^2 B$, then the ΔABC is

A. right angled

B. obtuse angled

C. isosceles

D. equilateral



15. If a $\cos A = b \cos B$, then the triange is

A. (a)equliateral

B. (b)right angled

C. (c)isosceles

D. (d) isosceles or right angled

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16. Point D,E are taken on the side BC of an acute angled triangle ABC,, such that BD = DE = EC. If $\angle BAD = x, \angle DAE = y$ and $\angle EAC = z$ then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is _____ A.1 B.2 C.4

D. None of these
17. If the base angles of triangle are $22\frac{1}{2}and112\frac{1}{2}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

A. half the base

B. the base

C. twice the base

D. four times the base

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18. In a $\Delta ABC, a=1$ and the perrimeter is six times the AM of the sine

of the angles. The measure of $\angle A$ is

A.
$$\frac{\pi}{3}$$

B.
$$\frac{\pi}{2}$$

C. $\frac{\pi}{6}$
D. $\frac{\pi}{4}$

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19. In a ΔABC , if median AD is perpendicular to AB, the an A + 2 an B

is equal to

A. (a) 1

B. (b) 3

C. (c) 0

D. (d)
$$\frac{1}{2}$$

20. The product of the sines of the angles of a triangle is p and the product of their cosines is q. Show that the tangents of the angles are the roots of the equation $qx^3 - px^2 + (1+q)x - p = 0$.

A.
$$qx^2 - px^2 + (1+q)x - p = 0$$

B.
$$px^3-qz^2+(1-p)x-q=0$$

C.
$$(1+q)x^3-px^2+qx-p=0$$

D. None of these

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Exercise For Sesssion 3

1. The side of a triangle are 22 cm, 28 cm, and 36 cm So, find the area of a

the circumscribed circle.

2. If the lengths of the side of a triangle are 3,4 and 5 units, then find the

circum radius R.



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4. If $8R^2 = a^2 + b^2 + c^2$. then prove that the Δ is right angled.

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5. In a ΔABC , show that $2R^2 \sin A \sin B \sin C = \Delta$.



9. In any ΔABC , find $\sin A + \sin B + \sin C$.

10. In any ΔABC , show that $\cos A + \cos B + \cos C = \Big(1 + rac{r}{R}\Big).$



11. Prove that
$$rac{r_1-r}{a}+rac{r_2-r}{b}=rac{c}{r_3}$$

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12. Prove that
$$r_1r_2 + r_2r_3 + r_3r_1 = rac{1}{4}(a+b+c)^2$$

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13. In triangle ABC .lf $(r_1+r_2)(r_2+r_3)(r_3+r_1)=4Rs^2$ Then prove it

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14. If $r_2 + r_3 + r = r_1$, then show that Δ is right angled.

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15. In an equilateral triangle, the inradius, circumradius, and one of the exradii are in the ratio
Vatch Video Solution
16. Show that $igg(rac{1}{r_1}+rac{1}{r_2}igg)igg(rac{1}{r_2}+rac{1}{r_3}igg)igg(rac{1}{r_3}+rac{1}{r_1}igg)=rac{64R^3}{a^2b^2c^2}$

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17. The ex-radii r_1, r_2, r_3 of a ΔABC are in Hp, show that its sides a,b,c,

are in AP.

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18. In a $\Delta ABC,\,$ show that $r_1\cdot r_2\cdot r_3\cdot r=\Delta^2.$

19. If l_1, l_2, l_3 are respectively the perpendicular from the vertices of a

triangle on the opposite side, then show that $l_1 l_2 l_3 = rac{a^2 b^2 c^2}{8 R^3}$.

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20. If the angle opf a triangle are in the ratio 1:2:3, then show that the

sides opposite to the respective angle are in the ratio $1:\sqrt{3}:2$.

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21. Show that,
$$4Rr\cos{rac{A}{2}}\cos{rac{B}{2}}\cos{rac{C}{2}}=\Delta$$

22. If in
$$\Delta ABC, \, (a-b)(s-c)=(b-c)(s-a),$$
 prove that r_1,r_2,r_3

are in A.P.



23. In a triangle ABC,if
$$rac{1}{r_1^2}+rac{1}{r_2^2}+rac{1}{r_3^2}+rac{1}{r^2}=$$

D Watch Video Solution

24. Prove that
$$(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$$

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25. Show that
$$igg(rac{1}{r_1}+rac{1}{r_2}igg)igg(rac{1}{r_2}+rac{1}{r_3}igg)igg(rac{1}{r_3}+rac{1}{r_1}igg)=rac{64R^3}{a^2b^2c^2}$$

26. Prove that (r+r1)tan((B-C)/2)+(r+r2)tan((C-A)/2)+(r+r3)tan((A-B)/2)=0



27. Show that
$$\displaystyle rac{b-c}{r_1} + \displaystyle rac{c-a}{r_2} + \displaystyle rac{a-b}{r_3} = 0$$

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28. If the sides be a,b,c, than find $(r_1 - r)(r_2 + r_3)$.

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29. If a, b, c are in AP, then show that r_1, r_2, r_3 are in HP.

30. In
$$\Delta ABC$$
 with usual notation $rac{r_1}{bc}+rac{r_2}{ca}+rac{r_3}{ab}$ is

31. Show that
$$r_1+r_2=c\cot\left(rac{C}{2}
ight)$$

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32. Show that
$$Rr(\sin A + \sin B + \sin C) = \Delta$$

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33. Show that
$$16R^2rr_1r_2r_3=a^2b^2c^2$$

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34. If in a triangle
$$rac{r}{r_1}=rac{r_2}{r_3}$$
, then

1. H is orthocenter of the triangle ABC, then AH is equal to :



4. If the altitudes of a triangle be 2,4,6, then find its in-radius.

5. In a $\triangle ABC$, if a = 3, b = 4, c = 5, then find the distance between its incentre and circumcentre.

6. If the lengths of the perpendiculars from the vertices of a triangle ABC

on the opposite sides are p_1, p_2, p_3 then prove that $rac{1}{p_1} + rac{1}{p_2} + rac{1}{p_3} = rac{1}{r} = rac{1}{r_1} + rac{1}{r_2} + rac{1}{r_3}.$

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7. Prove that the distance between the circumcenter and the incenter of

triangle ABC is $\sqrt{R^2-2Rr}$

8. Prove that the distance between the circucentre and the orthocentre of

a triangle ABC is $R\sqrt{1-8\cos A\cos B\cos C}$.



9. In a triangle ABC, AD, BE and CF are the altitudes and R is the circum radius, then the radius of the circle DEF is

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10. If l_1, l_2 and l_3 be respectively the radii of the in-circle and the three

escribed circles of a $\triangle ABC$, then find $l_1l_2l_3 - l(l_1l_2 + l_2l_3 + l_1l_3)$.

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Exercise For Sesssion 5

1. Find the sum of the radii of the circles, which are respectively inscribed and circumscribed about the a regular polygon of n sides.

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2. Find the radius of the circumscribing circle of a regular polygon of n
sides each of length is a.

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3. Find the area of a right angle ΔABC , whose base is 10 and perpendicular is 12.



4. A polygon of nine sides, each side of length 2, is inscribed in a circle.

The radius of the circle is _____.



5. The area of the circle and the area of a regular polygon of n sides and the perimeter of polygon equal to that of the circle are in the ratio of (a)

$$\tan\left(\frac{\pi}{n}\right): \frac{\pi}{n}$$
 (b) $\cos\left(\frac{\pi}{n}\right): \frac{\pi}{n} \frac{\sin \pi}{n}: \frac{\pi}{n}$ (d) $\cot\left(\frac{\pi}{n}\right): \frac{\pi}{n}$

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6. Let $A_1, A_2, A_3, \ldots, A_n$ be the vertices of an n-sided regular polygon

such that $rac{1}{A_1A_2}=rac{1}{A_1A_3}+rac{1}{A_1A_4}.$ Find the value of n.

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7. If I_n is the area of n - sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given

circle, prove that
$$I_n = rac{O_n}{2} igg(\sqrt{1 + igg(rac{2I_n}{n}igg)^2} igg)$$

1. The area of a cyclic quadrilateral ABCD is $\frac{3\sqrt{3}}{4}$. The radius of the circle circumscribing cyclic quadrilateral is 1.If AB = 1 and $BD = \sqrt{3}$, then $BC \cdot CD$ is equal to



2. If two adjacent sides of a cyclic quadrilateral aré 2 and 5 and the angle between them is 60°. If the third side is 3, then the remaining fourth side is (a) 2 (b) 3 (c) 4 (d) 5



3. The ratio of the area of a regular polygon of n sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is 3: 4. Then the value of n is



Exercise For Sesssion 7

1. In triangle $ABC, a\!:\!b\!:\!c=(1+x)\!:\!1\!:\!(1-x)$ where $x\in(0,1)$ If

$$igtriangle A = rac{\pi}{2} + igtriangle C$$
 , then x equal to





3. If in a $\Delta ABC, \angle C = 90^{\circ}$, then the maximum value of $\sin A \sin B$ is



4. If the area of a triangle is 81 square cm and its perimeter is 27 cm, then

find its in-radius in centi-metres.



5. In triangle ABC, if $r_1 = 2r_2 = 3r_3$, then a : b is equal to

6. The radius of the larger circle lying in the first quadrant and touching the line 4x + 3y - 12 = 0 and the coordinate axes, is

A. (a) 5

B. (b) 6

C. (c) 7

D. (d) 8

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7. In a ΔPQR as show in figure given that x: y: z: :2:3:6, then find value of $\angle QPR$.



8. In a ΔABC if $rac{R}{r} \leq 2$, then show that the triangle is equilateral.

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9. If the angle of a right angled triangle are in A.P. then the ratio of the in

-radius and the perimeter, is



10. If in a triangle $igg(1-rac{r_1}{r_2}igg)igg(1-rac{r_1}{r_3}igg)=2$ then the triangle is right

angled (b) isosceles equilateral (d) none of these



Exercise For Sesssion 8

1. If a tower subtends angles θ , 2θ and 3θ at three points A,B,and C respectively, lying on the same side of a horizontal line through the foot of the tower, show that $\frac{AB}{BC} \frac{\cot \theta - \cot 2\theta}{\cot 2\theta - \cot 3\theta}$. **Vatch Video Solution**

2. A person stands at a point A due south of a tower of height h and observes that its elevation is 60° . He then walks westwards towards B, where the elevation is 45° . At a point C on AB produced, show that if he find it to be 30° . OA, OB, OC are in GP.

3. A train travelling on one of two intersecting railway lines, subtends at a certain station on the other line, an/ angle α when the front of the carriage reaches the junction and an angle β when the end of the carriage reaches it. Then, the two lines are inclined to each other at an angle θ , show that $2 \cot \theta = \cot \alpha - \cot \beta$



4. The angle of elevation of the top of the tower observed from each of three points A,B, C on the ground, forming a triangle is observed to be ' θ '. If R is the circum-radius of the triangle ABC, then find the height of the tower

5. The length of the shadow of a pole inclined at 10° to the vertical towards the sun is 2.05 metres, when the elevation of the sun is 38° . The length of the pole is

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PROPERTIES AND SOLUTIONS OF TRIANGLES EXERSISE 1: SINGLE OPTION CORRECT TYPE QUESTIONS

1. In the adjoining figure, the circle meets the sides of an equilateral triangle at six points.



If $AG=2,\,GF=13,\,FC=1\,$ and $\,HJ=7,\,$ then DE equals to

A. $2\sqrt{22}$

B. $7\sqrt{3}$

C. 9

D. 10

Answer: A

1. In a $\triangle ABC$, if a = 13, b = 14 and c = 15, then $\angle A$ is equal to (All symbols used have their usual meaning in a triangle.)

A.
$$\sin^{-1} \frac{4}{5}$$

B. $\sin^{-1} \frac{3}{5}$
C. $\sin^{-1} \frac{3}{4}$
D. $\sin^{-1} \frac{2}{3}$

Answer: A

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2. In a ΔABC , if $b=ig(\sqrt{3}-1ig)a\,$ and $\,\angle C=30^{\,\circ}\,,\,$ then the value of (A-B)

is equal to (All symbols used have usual meaning in the triangel.)

A. 30°

B. $45^{\,\circ}$

C. 60°

D. $75^{\,\circ}$

Answer: C

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3. In $a\Delta ABC,\,\,$ if $\angle C=105^{\,\circ},\, \angle B=45^{\,\circ}$ and length of side AC =2 units,

then the length of th side AB is equal to

A. $\sqrt{2}$

B. $\sqrt{3}$

 $\mathsf{C}.\,\sqrt{2}+1$

D. $\sqrt{3} + 1$

Answer: C

4. If P is a point on the altitude AD of the triangle ABC such the

$$\angle CBP = \frac{B}{3}, \text{ then AP is equal to}$$
A. $2a \sin \frac{C}{3}$
B. $2b \sin \frac{A}{3}$
C. $2c \sin \frac{B}{3}$
D. $2c \sin \frac{C}{3}$

Answer: C

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5. In ΔABC , if 2b = a + c and $A - C = 90^{\circ}$, then sin B equal

All symbols used have usual meaning in ΔABC .

A.
$$\frac{\sqrt{7}}{5}$$

B.
$$\frac{\sqrt{5}}{8}$$

$$\mathsf{C}.\frac{\sqrt{7}}{4}$$
$$\mathsf{D}.\frac{\sqrt{5}}{3}$$

Answer: C

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6. Let ABC be a right triangle with length of side AB = 3 and hyotenus AC = 5. If D is a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$, then AD is equal

to

A.
$$\frac{4\sqrt{3}}{3}$$

B.
$$\frac{3\sqrt{5}}{2}$$

C.
$$\frac{4\sqrt{5}}{3}$$

D.
$$\frac{5\sqrt{3}}{4}$$

Answer: B

7. Two medians drawn from the acute angles of a right angled triangle intersect at an angle $\frac{\pi}{6}$. If the length of the hypotenuse of the triangle is 3units, then the area of the triangle (in sq. units) is $\sqrt{3}$ (b) 3 (c) $\sqrt{2}$ (d) 9



$$\mathsf{B.}\,\frac{3\sqrt{5}}{2}$$

C. $\sqrt{3}$

D. None of these

Answer: C

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8. If in a right angle $\Delta ABC, 4\sin A\cos B - 1 = 0$ and tan A is finite,

then

A. angles are in AP

B. angles are in GP

C. angles are in HP

D. None of these

Answer: A

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9. Find
$$rac{d^2y}{dx^2}, \quad ext{if} \quad y = \cos^{-1}x$$

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10. If in a $\triangle ABC$, the incricle passing through the point of intersection of perpendicular bisector of sides BC, AB, then $4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ equal

to

A.
$$\sqrt{2}$$

B. $\sqrt{2}-1$

C.
$$\sqrt{2}+1$$

D. $\frac{1}{2}$

Answer: B

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11. If two sides of a triangle are roots of the equation $x^2 - 7x + 8 = 0$ and the angle between these sides is 60^0 then the product of inradius and circumradius of the triangle is $\frac{8}{7}$ (b) $\frac{5}{3}$ (c) $\frac{5\sqrt{2}}{3}$ (d) 8



D. 8

Answer: B

12. If median AD of a triangle ABC makes angle $\frac{\pi}{6}$ with side BC, then the valur of $(\cot B - \cot C)^2$ is equal to

A. 6 B. 9 C. 12

D. 15

Answer: C

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13. If the perimeter of the triangle formed by feet of altitudes of the triangle ABC is equal to four times the circumradius of ΔABC , then identify the type of ΔABC

A. isosceles triangle

B. equilateral triangle

C. right angled triangle

D. None of these

Answer: D

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14. In a triangle with one angle $\frac{2\pi}{3}$, the lengths of the sides form an A.P. If the length of the greatest side is 7 cm, the radius of the circumcircle of the triangle is

A.
$$\frac{7\sqrt{3}}{3}cm$$

B. $\frac{5\sqrt{3}}{3}cm$
C. $\frac{2\sqrt{3}}{3}cm$

D. $\sqrt{3}cm$

Answer: A



15. If the sides of a ΔABC are in AP and a is the smallest sie, then $\cos A$ equals

A.
$$\frac{3c - 4b}{2b}$$
B.
$$\frac{3c - 4b}{2c}$$
C.
$$\frac{4c - 3b}{2b}$$
D.
$$\frac{4c - 3b}{2c}$$

Answer: D

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16. If p is the produce of the since of angles of a triange, and q the product of their consines then tangents of the angles are roots of the equation

A.
$$qx^2 - px^2 + (1-x)x - p = 0$$

B. $qx^3 - px^2 - (1-q)x - p = 0$
C. $qx^3 - px^2 + (1+q)x - p = 0$

D. None of these

Answer: A



17. Let C be incircle of $\triangle ABC$. If the tangents of lengths t_1, t_2 and t_3 are drawn inside the given triangle parallel to side a,b, and c, respectively, then $\frac{t_1}{a} + \frac{t_2}{b} + \frac{t_3}{c}$ is equal to

A. 0

B. 1

C. 2

D. 3
Answer: B



18. If the sine of the angles of ΔABC satisfy the equation $c^3x^3-c^2(a+b+c)x^2+lx+m=0$

(where a,b,c are the sides of ΔABC), then ΔABC is

A. A. always right angled for any I, m

B. B. right angled only when $l = c(ab + bc + ca) = c\sum ab, m = -abc$ C. C. right angled only when $l = \frac{c\sum ab}{4}, m = -\frac{abc}{8}$ D. D. never right angled

Answer: B

19. In triangle ABC, medians AD and CE are drawn AD = 5, $\angle DAC = \pi/8$, and $\angle ACE = \pi/4$, then the area of the triangle ABC is equal to

A.
$$\frac{50}{9}$$

B. $\frac{25}{9}$
C. $\frac{25}{3}$
D. $\frac{27}{7}$

Answer: C



20. In a triangle
$$ABC, a \ge b \ge c$$
. If $rac{a^3+b^3+c^3}{\sin^3 A+\sin^3 B+\sin^3 C}=8, ext{ then the maximum value of a}$

A.
$$rac{1}{2}$$

B. 2

C. 8

D. 64

Answer: B



22. In a δABC , a,c, A are given and b_1, b_2 are two values of third side b such that $b_2=2b_1$. Then, the value of sin A.

A.
$$\sqrt{rac{9a^2-c^2}{8a^2}}$$

B. $\sqrt{rac{9a^2-c^2}{8c^2}}$
C. $\sqrt{rac{9a^2-c^2}{8b^2}}$

D. None of these

Answer: B



23. In a triangle ABC, if
$$\cot A = (x^3 + x^2 + x)^{\frac{1}{2}}, \cot B = (x + x^{-1} + 1)^{\frac{1}{2}}$$
 and $\cot C = (x^{-3} + x^{-1})^{\frac{1}{2}}$ then the triangle is

A. equilateral

B. isosceles

C. right angled

D. obtuse anguled

Answer: C

24. In a $\triangle ABC$, a, b, A are given and c_1 , c_2 are two values of the third side c. The sum of the areas two triangles with sides a,b, c_1 and a, b, c_2 is

A.
$$\frac{1}{2}a^2 \sin 2A$$

B. $\frac{1}{2}b^2 \sin 2A$

 $\mathsf{C}.\,b^2\sin 2A$

D. None of these

Answer: A

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25. In $\triangle ABC$, if a = 10 and $b \cot B + c \cot C = 2(r+R)$ then the

maximum area of ΔABC will be

A. 50

B. $\sqrt{50}$

C. 25

Answer: C



26. Find the second order derivative of the function

 $x^3 + 4x^2 + 2$

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27. Let a,b,c be the sides of a triangle. Now two of them are equal to $\lambda \varepsilon R$.

If the roots of the equation $x^2+2(a+b+c)x+3\lambda(ab+bc+ca)=0$ are real then

A.
$$\lambda < rac{4}{3}$$

B. $\lambda > rac{5}{3}$
C. $\lambda \in \left(rac{1}{3}, rac{5}{3}
ight)$

D.
$$\lambda \in \left(rac{4}{3},rac{5}{3}
ight)$$

Answer: A



28. In triangle ABC, if $P\dot{Q}$, R divides sides BC, AC, and AB, respectively, in the ratio k: 1 (in order).

If the ratio
$$\left(\frac{areaPQR}{areaABC}\right)$$
 is $\frac{1}{3}$, then k is equal to
A. $\frac{1}{3}$
B. 2
C. 3

D. None of these

Answer: B

29. Let f(x + y) = f(x). f(y) for all x and y f(1) = 2 If in a triangle `ABC, a =f (3),b=f(1)+f (3), c=f (2)+f (3), then 2A is equal to

A. C

B. 2C

C. 3C

D. 4C

Answer: A

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30. Differentiate the following function w.r.t x

 $\sin^3 x + \cos^6 x$

31. If tangents PA and PB from a point to a circle with centre O are inclined to each other at angle of 80 \circ , then \angle POA is equal to ,

A. (a) 50

B. (b) 60

C. (c) 70

D. (d) 80

Answer: B

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32. Differentiate the following w.r.t x

$$\left(3x^2-9x+5
ight)^9$$

33. In a triangle $a=\sqrt{5}, b=2, \angle A=rac{\pi}{6}$ and the two possible values of

third side are c_1 and c_2 , then

A.
$$|c_1-c_2|=2\sqrt{6}$$

B. $|c_1-c_2|=4\sqrt{6}$
C. $|c_1-c_2|=4$

D.
$$|c_1 - c_2| = 6$$

Answer: C

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34. If R_1 is the circumradius of the pedal triangle of a given triangle ABC, and R_2 is the circumradius of the pedal triangle of the pedal triangle formed, and so on R_3 , R_4 ..., then the value of $\sum_{i=1}^{\infty} R_i$, where R (circumradius) of ΔABC is 5 is B. 10

C. 12

D. 15

Answer: B

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35. If in a triangle
$$\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$$
. then show that the

triangle is right angled.

A. right angled

B. isosceles

C. equilateral

D. None of these

Answer: A

36. If the median AD of a triangle ABC makes an angle θ with side, AB, then

 $\sin(A- heta)$ is equal to

A.
$$\frac{b}{c}\sin\theta$$

B. $\frac{c}{b}\sin\theta$
C. $\frac{c}{b}\cos\theta$

D. None of these

Answer: B



37. Differentiate the following w.r.t x

 $(\sin x - \cos x)^{\sin x - \cos x}$

38. In a $\Delta ABC, \, (a+b+c)(b+c-a)=\lambda bc, \,$ if

A. $\lambda < 0$

 $\mathrm{B.}\,\lambda>6$

 $\mathsf{C}.\,0<\lambda<4$

D. $\lambda > 4$

Answer: C

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39. In the $\triangle ABC$, if $(a^2 + b^2)\sin(A - B) = (a^2 - b^2)\sin(A + B)$.

Prove that the triangle is either isosceles or right angled.

A. either isosceles or right angled

B. only right angled

C. only isosceles triangle

D. None of the above

Answer: A



41. If a, b,c be the sides of a triangle ABC and if roots of equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 90 \quad \text{are equal then}$ $\frac{\sin^2 A}{2}, \frac{\sin^2 B}{2}, \frac{\sin^2 C}{2} \text{ are in}$

A. (a)AP

B. (b)GP

C. (c)HP

D. (d)none of these

Answer: C



42. The ratio of the area of a regular polygon of n sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is 3: 4. Then the value of n is

A. 6

B. 4

C. 8

D. 12

Answer: A

43. In any triangle ABC $\sum \frac{\sin^2 A + \sin A + 1}{\sin A}$ is always greater than or equal A. 9 B. 3 C. 27 D. None of these

Answer: C

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44. If the incircel of the triangle ABC, through it's circumcentre, then the

 $\cos A + \cos B + \cos C$ is

- $\mathsf{A.}-2$
- $\mathsf{B.}\,\sqrt{2}$
- C. $-\sqrt{2}$

D. None of these

Answer: B



45. The perimeter of a triangle ABC is six times the arithmetic mean of the sines of its angles. If the side ais1 then find angle A.

A. 30°

B. $60\,^\circ$

C. 90°

D. 120°

Answer: A

46. If there are only two linear functions f and g which map [1, 2]on[4, 6]and in a ΔABC , c = f(1) + g(1) and a is the maximum valur of r^2 , where r is the distance of a variable point on the curve $x^2 + y^2 - xy = 10$ from the origin, then sin A: sin C is

A. A. 1:2

B. B. 2:1

C. C. 1:1

D. D. None of these

Answer: C

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47. A circle is inscribed in an equilateral triangle of side a. The area of any square inscribed in this circle is _____.

A.
$$a^2$$

B.
$$\frac{a^2}{4}$$

C. $\frac{a^2}{3}$
D. $\frac{a^2}{6}$

Answer: D

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48. In any triangle ABC, if sin A , sin B, sin C are in AP, then the maximum B

value of $an \frac{B}{2}$ is

A.
$$-\frac{1}{\sqrt{3}}$$

B. $\frac{1}{\sqrt{3}}$
C. $\frac{1}{3}$

D. None of these

Answer: B

49. In a $\triangle ABC$, $2\cos A = \frac{\sin B}{\sin C}$ and $2^{\tan^2 B}$ is a solution of equation $x^2 - 9x + 8 = 0$, then $\triangle ABC$ is

A. equilateral

B. isosceles

C. scalene

D. right angled

Answer: A

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50. Differentiate the following w.r.t x

$$\sin^{-1} \left(x^{rac{3}{2}}
ight)$$

51. If a,b and c are the sides of a traiangle such that $b. c = \lambda^2$, then the relation is a, λ and A is

 $egin{aligned} \mathsf{A}.\,c &\geq 2\lambda \sin\!\left(rac{C}{2}
ight) \ \mathsf{B}.\,b &\geq 2\lambda \sin\!\left(rac{A}{2}
ight) \ \mathsf{C}.\,a &\geq 2\lambda \sin\!\left(rac{A}{2}
ight) \end{aligned}$

D. None of these

Answer: C

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52. Differentiate the following w.r.t x

cos(acosx + bsinx), for some constant a and b

53. In triangle ABC, a = 5, b = 4 and $\cos(A + B) = \frac{31}{32}$ In this triangle,c =A.3 B.6 C.7 D.9

Answer: B

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54. In a ABC, if $AB = x, BC = x + 1, \angle C = \frac{\pi}{3}$, then the least

integer value of x is 6 (b) 7 (c) 8 (d) none of these

A. 6

B. 7

C. 8

D. None of these

Answer: B

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55. In an equilateral triangle, three coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. The area of the triangle is 2sqrt(3)(b)6+4sqrt(3)12+(7sqrt(3))/4(d)3+(7sqrt(3))/4

A.
$$(4 + 2\sqrt{3})cm^2$$

B. $\frac{1}{4}(12 + 7\sqrt{3})cm^2$
C. $\frac{1}{4}(48 + 7\sqrt{3})cm^2$
D. $(6 + 4\sqrt{3})cm^2$

Answer: D



58. In a triangle ABC , the line joining the circumcentre and incentre is parallel to BC, then Cos B + Cos C is equal to:

A.
$$\frac{3}{2}$$

B. 1
C. $\frac{3}{4}$
D. $\frac{1}{2}$

Answer: B





 $\angle COA = 60^{\circ}, AB = 2r, Ac = d \hspace{0.1 cm} ext{and} \hspace{0.1 cm} CD = l, \hspace{0.1 cm} ext{then} \hspace{0.1 cm} ext{l} ext{ is equal to}$



A. $d\sqrt{3}$

B.
$$\frac{d}{\sqrt{3}}$$

 $\mathsf{C.}\, 3d$

D.
$$\frac{\sqrt{3}d}{2}$$

Answer: A

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60. In a triangle ABC, AD, BE and CF are the altitudes and R is the circum radius, then the radius of the circle DEF is

A. 2R

B. R

 $\mathsf{C}.\,\frac{R}{2}$

D. None of these

Answer: C

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61. Find $\frac{dy}{dx}$, If x = a(sint - t cost) and y = a(cost + t sint)

62. Differentiate the following w.r.t x

$$(x-4)^{x^3}$$

63. Find
$$\frac{dy}{dx}$$
, if x = 6(t-sint), y= 4(1-cost)

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64. Using the fact that $\cos (A + B) = \cos A \cos B - \sin A \sin B$ and the

differentiation, obtain the sum formula for sine.



65. Find
$$rac{dy}{dx}, \hspace{1em} ext{if} \hspace{1em} y = \sin^{-1} \left(1 - x^2
ight)^{rac{1}{2}}$$

1. If area of $\Delta ABC(\Delta)$ and angle C are given and if c opposite to given

angle is minimum, then

A.
$$a = \sqrt{\frac{2\Delta}{\sin C}}$$

B. $b = \sqrt{\frac{2\Delta}{\sin C}}$
C. $a = \frac{4\Delta}{\sin C}$
D. $b = \frac{4\Delta}{\sin^2 C}$

Answer: A



2. Differentiate the following w.r.t x

 $x^a + a^x$

3. In $\Delta ABC, ext{ the value of } c\cos(A- heta) + a\cos(C+ heta) =$



4. In
$$\triangle ABC$$
, If $a=4,b=3$ and $COS(A-B)=rac{3}{4},$ then

A. measure of $\angle A$ is $\frac{\pi}{2}$

B. measuere of $\angle Bis\frac{\pi}{2}$

C.
$$\cot{rac{C}{2}}=\sqrt{7}$$

D. circumradius of $\Delta ABCis rac{2}{7^{1/14}}$

Answer: B::C::D

5. Find the second order derivative of the function

 $2{\log x^2} + 4x$

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6. Is it possible to have $a\Delta ABC$ with a=3, b=5, c=7 and $C=rac{2\pi}{3}$?

A. (a)
$$(a+c-b)(a-c+b)=4bc$$

B. (b)
$$b^2 \sin 2C + \cos^2 \sin 2B = ab$$

C. (c)
$$a = 3, b = 5, c = 7$$
 and $C = \frac{2\pi}{3}$
D. (d) $\cos\left(\frac{A-C}{2}\right) = \cos\left(\frac{A+C}{2}\right)$

Answer: B::C

7. Find the second order derivative of the function

x.cosx



8. Differentiate the following w.r.t x

 $(x+3)^2$. $(x+4)^3$

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9. Find the second order derivative of the function

 x^6+2



10. Given an isoceles triangle with equal side of length b and angle

 $lpha < \pi/4$, then

the inradius r is given by

A.
$$R = rac{1}{2}b\cos eclpha$$

B. $\Delta = 2b^2\sin 2lpha$
C. $r = rac{b\sin 2lpha}{2(1+\cos lpha)}$
D. $OI\left|rac{b\cos\left(rac{3lpha}{2}
ight)}{2\sin lpha\left(rac{lpha}{2}
ight)}
ight|$

Answer: A::C::D



11. There can exist a triangle ABC satisfying the condition $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{7}$

A. an A + an B + an C = 0

B.
$$\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{7}$$

C. $(a+b)^2 = c^2 + ab$ and $\sqrt{2}(\sin A + \cos A) = \sqrt{3}$

D.

$$\sin A + \sin B = \left(\frac{\sqrt{3}+1}{2}\right)\cos A\cos B = \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4} = \sin A\sin B$$

Answer: C::D

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12. Find
$$rac{d^2y}{dx^2}, \hspace{1em} ext{if} \hspace{1em} y = \log x$$

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13. If in $\Delta ABC,\,A=90^{\,\circ}$ and c, sin B and cos B are rational number, then

A. a is rational

B. a is irrational

C. b is rational

D. b is irational

Answer: A::C



15. Let 'l' is the length of medians from the vertex A to the side BC of a ΔABC , then

A.
$$4l^2 = 2b^2 + 2c^2 - a^2$$

B.
$$4l^2=b^2+c^2+2bc\cos A$$

$$\mathsf{C.}\,4l^2=b^2+4bc\cos A$$

D.
$$4l^2-(2s-a)^2-4bc\sin^2\left(rac{A}{2}
ight)$$

Answer: A::B::C::D

16. If A, A_1 , A_2 and A_3 are the areas of the inscribed and escribed circles of a triangle, prove that $\frac{1}{\sqrt{A}} = \frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}}$ A. (a) $\sqrt{A_1} + \sqrt{A_2} + \sqrt{A_3} = \sqrt{\pi}(r_1 + r_2 + r_3)$ B. (b) $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \frac{1}{\sqrt{A}}$ C. (c) $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \frac{s^2}{\sqrt{\pi}r_1r_2r_3}$ D. (d) $\sqrt{A_1} + \sqrt{A_2} + \sqrt{A_3} = \sqrt{\pi}(4R + r)$

Answer: A::B::C::D

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17. If a, b, A be given in a triangle and c_1 and c_2 be two possible value of the third side such that $c_1^2 + c_1c_2 + c_2^2 = a^2$, then a is equal to

A. (a) 30°

B. (b) $60^{\,\circ}$

C. (c) 90°

D. (d) 120°

Answer: B::C

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18. D,E and F are the middle points of the sides of the triangle ABC, then

A. centroid of the triangle DEF is the same as that of ABC

B. orthocentre of the triangle DEF is the circumcentre of ABC

C. orthocentre of the triangle DEF is the incentre of ABC

D. centroid of the triangle DEF is not the same as that of ABC

Answer: A::B
19. The sides of ABC satisfy the equation $2a^2 + 4b^2 + c^2 = 4ab + 2ac$ Then a) the triangle is isosceles b) the triangle is obtuse c) $B = \cos^{-1}\left(\frac{7}{8}\right)$ d) $A = \cos^{-1}\left(\frac{1}{4}\right)$

A. the triangle is isosceles

B. the triangle is obtuse

C.
$$B = \cos^{-1} \left(rac{7}{8}
ight)$$

D. $A = \cos^{-1} \left(rac{1}{4}
ight)$

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Answer: A::C::D

20. If
$$\Delta$$
 represents the area of acute angled triangle ABC, then
 $\sqrt{a^2b^2 - 4\Delta^2} + \sqrt{b^2c^2 - 4\Delta^2} + \sqrt{c^2a^2 - 4\Delta^2} =$ (a) $a^2 + b^2 + c^2$ (b)
 $\frac{a^2 + b^2 + c^2}{2}$ (c) $ab\cos C + bc\cos A + ca\cos B$ (d)
 $ab\sin C + bc\sin A + ca\sin B$

A. `(a^(2) +b^(2)+c^(2))

$$\mathsf{B}.\,\frac{a^2+b^2+c^2}{2}$$

 $\mathsf{C}.\,ab\cos C+bc\cos A+ca\cos B$

 $\mathsf{D}.\,ab\sin C + bc\sin A + ca\sin B$

Answer: B::C

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21. In triangle, ABC if $2a^2b^2 + 2b^2c^2 = a^2 + b^4 + c^4$, then angle B is equal to 45^0 (b) 135^0 120^0 (d) 60^0

A. $45^{\,\circ}$

B. 135°

C. 120°

D. $60^{\,\circ}$

Answer: A::B

22. If H is the orthocentre of triangle ABC, R = circumradius and P = AH + BH + CH, then

A. p = 2(R+r)

B. max, of P is 3R

C. min. of P is 3R

D. P = 2(R - r)

Answer: A::B

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23. If inside a big circle exactly $n(n \le 3)$ small circles, each of radius r, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles, then the radius of big

circle is (a)
$$r\left(1 + \cos ec\frac{\pi}{n}\right)$$
 (b) $\left(\frac{1 + \frac{\tan \pi}{n}}{\frac{\cos \pi}{\pi}}\right)$ (c) $r\left[1 + \cos ec\frac{2\pi}{n}\right]$ (d) $\frac{r\left[s \in \frac{\pi}{2n} + \frac{\cos\left(2\pi\right)}{n}\right]^2}{\frac{\sin \pi}{n}}$

A.
$$r\left(1 + \cos ec \frac{\pi}{n}\right)$$

B. $\left(\frac{1 + \tan \frac{\pi}{n}}{\cos \frac{\pi}{n}}\right)$
C. $r\left[1 + \cos ec \frac{2\pi}{n}\right]$
D. $\frac{r\left[\sin \frac{\pi}{2n} + \cos \frac{2\pi}{n}\right]^2}{\sin \frac{\pi}{n}}$

Answer: A::D



24. If in triangle ABC, a, c and angle A are given and $c \sin A < a < c$, then

(b_1 and b_2 are values of b)

A. (a) $b_1+b_2=2c\cos A$

B. (b) $b_1 + b +_2 = c \cos A$

C. (c)
$$b_1 b_2 = c^2 - a^2$$

D. (d)
$$b_1 b_2 = c^2 + a^2$$

Answer: A::C

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Exercise (Statement I And Ii Type Questions)

1. In a triangle ABC, $a^3 + b^3 + c^3 = c^2(a + b + c)$ (All symbol used have usual meaning in a triangle.) Statement-1: The value of $\angle C = 60^\circ$. Statement-2: $\triangle ABC$ must be equilateral.

A. (a)Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. (b)Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. (c)Statement I is correct but Statement II is incorrect

D. (d)Statement I is correct but Statement I is incorrect

Answer: C

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2. Let a = 6, b = 3 and
$$\cos(A - B) = \frac{4}{5}$$

Value of $\sin A$ is equal to

A. Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement I is correct but Statement I is incorrect

Answer: D

3. Statement I: If in a triangle $ABC, \sin^2 A + \sin^2 B + \sin^2 C = 2$, then one of the angles must be 90°.

Statement II: In any triangle ABC $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

A. (a) Both Statement I and Statement II are correct and Statement II

is the correct explanation of Statement I

B. (b) Both Statement I and Statement II are correct but Statement II is

not the correct explanation of Statement I

- C. (c) Statement I is correct but Statement II is incorrect
- D. (d) Statement II is correct but Statement I is incorrect

Answer: A

4. Statement I: If A, B, C, D are angles of a cyclic quadrilateral then $\sum \sin A = 0.$

Statement II: If A, B, C, D are angles of cyclic quadrilateral then $\sum \cos A = 0.$

A. (a) Both Statement I and Statement II are correct and Statement II

is the correct explanation of Statement I

B. (b) Both Statement I and Statement II are correct but Statement II is

not the correct explanation of Statement I

- C. (c) Statement I is correct but Statement II is incorrect
- D. (d) Statement II is correct but Statement I is incorrect

Answer: D



5. Statement I In any triangle ABC, the square of the length of the bisector AD is $bc\left(1 - \frac{a^2}{(b+c)^2}\right)$. Statement II In any triangle ABC length of bisector AD is

$$\frac{2bc}{(b+c)}\cos\bigg(\frac{A}{2}\bigg).$$

A. (a)Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. (b)Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

- C. (c)Statement I is correct but Statement II is incorrect
- D. (d)Statement I is correct but Statement I is incorrect

Answer: A

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6. Prove that $\sin 4x = 4 \sin x \cos^3 x - 4 \cos x \sin^3 x$

7. All the notations used in statemnt I and statement II are usual.

Statement I: In triangle ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$. then value of $\frac{r_1 + r_2 + r_3}{r}$ is equal to 9. Statement II: In $\Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is circumradius.

A. a) Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. b) Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. c) Statement I is correct but Statement II is incorrect

D. d) Statement II is correct but Statement I is incorrect

Answer: A

8. Statement I In a triangle ABC if an A : an B : an C = 1 : 2 : 3, then $A = 45^{\circ}$

Statement II If p:q:r=1:2:3, then p=1

A. A. Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. B. Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. C. Statement I is correct but Statement II is incorrect

D. D. Statement I is correct but Statement I is incorrect

Answer: C

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9. Statement I: In any right angled triangle $rac{a^2+b^2+c^2}{R^2}$ is always equal

to 8.

Statement II: $a^2 = b^2 + c^2$

A. a) Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. b) Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. c) Statement I is correct but Statement II is incorrect

D. d) Statement I is incorrect but Statement II is correct

Answer: A



10. Statement I perimeter of a regular pentagon inscribed in a circle with centre O and radius a cm equals $10a\sin 36^\circ$ cm.

Statement II Perimeter of a regular polygon inscribed in a circle with centre O and radius a cm equals $(3n-5)\sin\left(\frac{360^{\circ}}{2n}\right)cm$, then it is n sided, where $n \geq 3$.

A. (a)Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. (b)Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

- C. (c)Statement I is correct but Statement II is incorrect
- D. (d)Statement I is correct but Statement I is incorrect

Answer: C

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11. Statement I In any triangle ABC

 $a\cos A + b\cos B + c\cos C \le s.$

Statement II In any triangle ABC

$$\sin{\left(rac{A}{2}
ight)}{\sin{\left(rac{B}{2}
ight)}}{\sin{\left(rac{C}{2}
ight)}}\leqrac{1}{8}$$

A. Both Statement I and Statement II are correct and Statement II is

the correct explanation of Statement I

B. Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement I is correct but Statement I is incorrect

Answer: A

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12. Statement I In a ΔABC , if $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = y\left(x^2 + \frac{1}{x^2}\right)$ then the maximum value of $yis\frac{9}{8}$. Statement II In a ΔABC , $\sin \frac{A}{2} \cdot \sin \frac{B}{2} \sin \frac{C}{2} \le \frac{1}{8}$

A. Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct and Statement II is

not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement I is correct but Statement I is incorrect

Answer: A



13. Find the second order derivative of the given function

 $e^y(x+1) = 1$

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14. Find
$$\frac{dy}{dx}$$
, if $y = 3\cos(\log x) + 4\sin(\log x)$
A.

Β.

C.

D.

Answer: A



Exercise (Passage Based Questions)

1. Find
$$rac{dy}{dx}, \hspace{1em} ext{if} \hspace{1em} y = 500 e^{7x} + 600 e^{-7x}$$

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2. Find the second order derivative of the function

 $Ae^{mx} + Be^{nx}$



3. If x and y are connected parameterically by the equation , without

eliminating the parameter , find $\displaystyle rac{dy}{dx}$

 $x=a\cos(heta), y=b\sin(heta)$

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4. Let ABC to be an acute triangle with BC = a, CA = b and AB = c, where $a \neq b \neq c$. From any point 'p' inside $\Delta ABC \leq D$, E, F denot foot of perpendiculars form 'p' noto the sides, BC, CA and AB, respectively. Now, answer the following equations.

If ΔDEF is equilateral, then 'P'

A. the incircle of ΔABC

B. line of intternal angle bisectors from A, B and C

C. arcs of 3 circles

D. None of the above

Answer: C

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5. Let ABC to be an acute triangle with BC = a, CA = b and AB = c, where $a \neq b \neq c$. From any point 'p' inside $\Delta ABC \leq D$, E, F denot foot of perpendiculars form 'p' noto the sides, BC, CA and AB, respectively. Now, answer the following equations.

If ΔDEF is equilateral, then 'P'

A.
$$x = 4 ext{ or } x + yy = 7 ext{ or } 4x = 3y$$

B.
$$x = 4$$
 or $x^2 + y^2 = 4x + 4y$

C.
$$3(x^2+y^2)+196=49(x+y)$$

D. None of the above

Answer: C

6. Let ABC to be an acute triangle with BC = a, CA = b and AB = c, where $a \neq b \neq c$. From any point 'p' inside $\Delta ABC \leq D$, E, F denot foot of perpendiculars form 'p' noto the sides, BC, CA and AB, respectively. Now, answer the following equations.

If ΔDEF is equilateral, then 'P'

A. coincides with incentre of ΔABC

B. coincides with orthocentre of ΔABC

C. lies on padal Δ of ΔABC

D. None of the above

Answer: D

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7. In an acute angled triangle ABC, let AD, BE and CF be the perpendicular opposite sides of the triangle. The ratio of the product of the side lengths of the triangles DEF and ABC, is equal to

A. A.
$$rac{3(abc)^{rac{1}{3}}}{4(a+b+c)}$$

B. B. $rac{1}{4}$

C. C. $\cos A \cos B \cos C$

D. D.
$$\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$

Answer: C



8. In an acute angle ΔABC , let AD, BE and CF be the perpendicular from A, B and C upon the opposite sides of the triangle. (All symbols used have usual meaning in a tiangle.)

The orthocentre of the ΔABC , is the

A. centroid of the ΔDEF

B. circum-centre of the ΔDEF

C. incentre of the ΔDEF

D. orthocentre of the ΔDEF

Answer: C



9. In an acute angle ΔABC , let AD, BE and CF be the perpendicular from A, B and C upon the opposite sides of the triangle. (All symbols used have usual meaning in a tiangle.)

The circum-radius of the ΔDEF can be equal to



Answer: A::B::C::D

10. Let a,b, c are the sides opposite to angles A, B , C respectively in a

$$\Delta ABC \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2} \text{ and } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

If $a = 6, b = 3$ and $\cos(A-B) = \frac{4}{5}$

Angle C is equal to

A.
$$\frac{\pi}{4}$$

B. $\frac{\pi}{2}$
C. $\frac{3\pi}{4}$
D. $\frac{2\pi}{3}$

Answer: B

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11. Let a,b, c are the sides opposite to angles A, B, C respectively in a $\Delta ABC \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2} \text{ and } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$ If a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

Area of the trianlge is equal to

A. 8	
B. 9	
C. 10	
D. 11	

Answer: B

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12. Let a,b, c are the sides opposite to angles A, B, C respectively in a $\Delta ABC \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2} \text{ and } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$ If a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

Valus of sin A is equal to

A.
$$\frac{1}{\sqrt{5}}$$

B.
$$\frac{2}{\sqrt{5}}$$

C.
$$\frac{1}{2\sqrt{5}}$$

D.
$$\frac{1}{\sqrt{3}}$$

Answer: B



13. Find
$$rac{dy}{dx}$$
, if $x=rac{4}{t}$ and $y=4t$

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14. If x and y are connected parametrically by the equation , without eliminating parameter , find $\frac{dy}{dx}$ $x = a \sec(\theta), y = b \tan(\theta)$

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15. Find
$$rac{dy}{dx}$$
 , if $x=a\sin t, y=aigg(\cos t+\logigg(anigg(rac{t}{2}igg)igg)$

16. Find
$$\frac{dy}{dx}$$
, if $x = \cos(\theta)$ and $y = \sin(2\theta)$

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17. Differentiate the following function w.r.t x

 $\frac{\cos^{-1}x}{(2x+7)^{\frac{1}{2}}}$

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18. Consider a triangle ABC, where x,y,z are the length of perpendicular drawn from the vertices of the triangle to the opposite sides a,b, c respectively. Let the letters R, rS, Δ denote the circumradius, inradius semi-perimeter and area of the triangle respectively.

If
$$\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b} = \frac{a^2 + b^2 + c^2}{k}$$
, then the value of k is A. R
B. S

C. 2R

D.
$$\frac{3}{2}R$$

Answer: C

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19. Consider a triangle ABC, where x,y,z are the length of perpendicular drawn from the vertices of the triangle to the opposite sides a,b, c respectively. Let the letters R, rS, Δ denote the circumradius, inradius semi-perimeter and area of the triangle respectively.

If $\cot A + \cot B + \cot C = k \bigg(rac{1}{x^2} + rac{1}{y^2} + rac{1}{z^2} \bigg)$, then the value of k is

A. R^2

B.rR

 $\mathsf{C}.\,\Delta$

D. $a^2+b^2+c^2$

Answer: C

20. Consider a triangle ABC, where x,y,z are the length of perpendicular drawn from the vertices of the triangle to the opposite sides a,b, c respectively. Let the letters R, r, S, Δ denote the circumradius, inradius semi-perimeter and area of the triangle respectively.

The value of $rac{c\sin B + b\sin C}{x} + rac{a\sin C + c\sin A}{y} + rac{b\sin A + a\sin B}{z}$ is equal to

A.
$$\frac{R}{r}$$

B. $\frac{S}{R}$
C. 2

D. 6

Answer: D

21. AL, BM and CN are perpendicular from angular points of a triangle ABC on the opposite sides BC, CA and AB respectively. Δ is the area of triangle ABC, (r) and R are the inradius and circumradius.

If perimeters of ΔLMN and $\Delta ABCan\lambda$ and μ , then the value of $\frac{\lambda}{\mu}$ is

A.
$$\frac{r}{R}$$

B. $\frac{R}{r}$
C. $\frac{rR}{\Delta}$
D. $\frac{\Delta}{rR}$

Answer: B

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22. The angle of triangle A, B, C are in AP and C=3A . Find angle B



23. AL, BM and CN are perpendicular from angular points of a triangle ABC on the opposite sides BC, CA and AB respectively. Δ is the area of triangle ABC, (r) and R are the inradius and circumradius.

If area of ΔLMN is Δ' , then the value of $\frac{\Delta'}{\Delta}$ is

A. $2\sin A \sin B \sin C$

B. $2\cos A\cos B\cos C$

 $\mathsf{C.} \sin A \sin B \sin C$

D. $\cos A \cos B \cos C$

Answer: D



24. AL, BM and CN are perpendicular from angular points of a triangle ABC on the opposite sides BC, CA and AB respectively. Δ is the area of triangle ABC, (r) and R are the inradius and circumradius.

Radius is the circum circle of ΔLMN is

A. 2R

B. R C. $\frac{R}{2}$ D. $\frac{R}{4}$

Answer: B

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25. Differentiate the following w.r.t x

$$\left(2x^2-6x+4
ight)^3$$

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PROPERTIES AND SOLUTIONS OF TRIANGLES EXERCISE 5: MATCHING TYPE QUESTIONS

1. Match the statement of Colume I with values of Column II.

	Column I	Column II
(A)	In a $\triangle ABC$, let $\angle C = \frac{\pi}{2}$, $r = \text{inradius}$, $R =$	(p) $a + b + c$
	circumradius then $2(r + R)$	
(B)	If l, m, n are perpendicular drawn from the vertices of triangle having sides a, b and c then	(q) $a-b$
	$\sqrt{2R\left(\frac{bl}{c}+\frac{cm}{a}+\frac{an}{b}\right)+2ab+2bc+2ca}$	
(C)	In a $\triangle ABC$, $R(b^2 \sin 2C + c^2 \sin 2B)$ equals	(r) $a + b$
(D)	In a right angle triangle <i>ABC</i> , $\angle C = \frac{\pi}{2}$,	(s) <i>abc</i>
	then $4R\sin\frac{A+B}{2}\sin\frac{(A-B)}{2}$	

2. Find the radian measure of the following degree measure

40°20`

3. Match the statement of Column I with values of Column II.

	Column I	Column II
(<u>A</u>)	In a $\triangle ABC$, $(a + b + c)(b + c - a) = \lambda bc$, (p) where $\lambda \in I$, then greatest value of λ is	3
(B)	In a $\triangle ABC$, $\tan A + \tan B + \tan C = 9$. If (q) $\tan^2 A + \tan^2 B + \tan^2 C = k$, then least value of k satisfying is	9(3) ^{1/3}
(C) In a triangle ABC , then line joining the circumcentre to the incentre is parallel to BC , then value of $\cos B + \cos C$ is	(r) 1
(D)	If in a $\triangle ABC$, $a = 5$, $b = 4$ and $\cos(A - B) = \frac{31}{32}$, then the third side c is	(s) 6
	14-	

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4. Find the radian measure corresponding to the degree measure

300°

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Exercise (Single Integer Answer Type Questions)

1. If in
$$\triangle ABC$$
, $\angle C = \frac{\pi}{8}$, $a = \sqrt{2}$ and $b = \sqrt{2 + \sqrt{2}}$ then find the measure of angle A (in degree).

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2. If in
$$ABC, A=rac{\pi}{7}, B=rac{2\pi}{7}, C=rac{4\pi}{7}$$
 then $a^2+b^2+c^2$ must be (a) R^2 (b) $3R^2$ (c) $4R^2$ (d) $7R^2$

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3. Find the radian measure corresponding to the following degree measure 15°



4. The orthocenter of the triangle formed by (0,0),(5,-1) and (-2,3) is

A. (a) (4,-7)

B. (b) (-4,7)

C. (c) (-4,-7)

D. (d) none of above

Answer: 4

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5. With usual notation in ΔABC , the numerical value of

$$iggl(rac{a+b+c}{r_1+r_2+r_3}iggr)iggl(rac{a}{r_1}+rac{b}{r_2}+rac{c}{r_3}iggr)$$
 is

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6. ABC is a triangle and D is the middle point of BC. If AD is perpendicular

to AC, then prove that $\cos A. \cos C = rac{2ig(c^2-a^2ig)}{3ac}$

7. The rational number which equals the number 2. 357 with recurring decimal is $\frac{2355}{1001}$ b. $\frac{2379}{997}$ c. $\frac{2355}{999}$ d. none of these Watch Video Solution

8. In triangle $ABC, r = rac{R}{6}$ and $r_1 = 7r$. Then the measure of angle A =

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9. In a
$$\Delta ABC$$
, the maximum value of $120\left(\frac{\sum a\cos^2\left(\frac{A}{2}\right)}{a+b+c}\right)$ must be

10. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smalles one. Determine the largest and smallest sides of the triangle.

11. In $\triangle ABC, \angle C = 2\angle A$, and AC = 2BC, then the value of $\frac{a^2 + b^2c^2}{B^2}$ (where R is circumradius of triangle) is _____

 R^2

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12. If a,b and A are given in a triangle and C_1, C_2 are the possible values

of the third side, prove that : $C_1^2+C_2^2-2C_1C_2\cos 2A=4a^2\cos^2 A$

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13. In ΔABC , a=5, b=4, c=3. G is the centroid of triangle. If R_1 be the

circum radius of triangle GAB then the value of $rac{a}{65}R_1^2$ must be
14. A triangle ABC is inscribed in a circle with centre at O, The lines AO, BOandCO meet the opposite sides at D, E, andF, respectively. Prove that $\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{a \cos A + b \cos B + osC}{\triangle}$ Watch Video Solution

15. In a triangle
$$ABC, a \ge b \ge c$$
. If $rac{a^3+b^3+c^3}{\sin^3 A+\sin^3 B+\sin^3 C}=8, ext{ then the maximum value of a}$

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16. In a cyclic quadrilateral PQRS, PQ= 2 units, QR= 5 units, RS=3 units and

 $\angle PQR = 60^{\circ}$, then what is the measure of SP?



PROPERTIES AND SOLUTIONS OF TRIANGLES EXERCISE 6 : SINGLE INTEGER ANSWER TYPE QUESTIONS





Exercise (Subjective Type Questions)

1. In a ΔABC , the angles A and B are two values of θ satisfying $\sqrt{3}\cos\theta + \sin\theta = k$, where |K| < 2, then show triangles is obtuse angled.

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2. In an obtuse angled triangle, the obtuse angle is $\frac{3\pi}{4}$ and the other two angles are equal to two values of heta satisfying $a \tan heta + b \sec heta = c$, where $|b| \leq \sqrt{a^2 + c^2}$, then $a^2 - c^2$ is equal to

3. If in trianlge ABC, a,b,c and angle A are given and $c\sin A < a < c, then(b_1 ext{ and } b_2 ext{ are values of b })$

A. (a) $b_1+b_2=2c\cos A$

B. (b)
$$b_1+b_2=c\cos A$$

C. (c)
$$b_1 b_2 = c^2 - a^2$$

D. (d)
$$b_1b_2=c^2+a^2$$

Answer:
$$\sqrt{rac{9a^2-c^2}{8c^2}}$$

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4. If P is a point on the altitude AD of the triangle ABC such the $\angle CBP = \frac{B}{3}$, then AP is equal to

5. If R denotes circumradius, then in $\Delta ABC, \frac{b^2-c^2}{2aR}$ is equal to

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6.

$$\Delta ABC, A=rac{2\pi}{3}, b-c=3\sqrt{3}cm ~~ ext{and}~~~ ext{area of}~~ \Delta ABC=rac{9\sqrt{3}}{2}cm^2,$$

then a =

7. If
$$\Delta = a^2 - \left(b-c
ight)^2, \Delta$$
 is the area of the ΔABC then $an A=~?$

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8. In a $\Delta ABC, B=90^\circ, AC=h$ and the length of perpendicular from

B to AC is p such that h = 4p. If AB < BC, then measure $\angle C$.

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In

9. If in a $\Delta ABC, \sin^3 A + \sin^3 B + \sin^3 C$

 $= 3 \sin A \cdot \sin B \cdot \sin C$, then find the value of determinant

 $\begin{array}{c|ccc} a & b & c \\ b & c & a \\ c & a & b \end{array}$



$$(\tan)\frac{1}{2} + (\tan)\frac{3}{2} = \frac{1}{3}(\cot)\frac{1}{2}$$

12. The sides of a triangle are in A.P. and its area is $\frac{3}{5}$ th of an equilateral

triangle of the same perimeter. Find the greatest angle of the triangle



13. If AD, BE and CF are the medians of a ΔABC , then evaluate $\left(AD^2+BE^2+CF^2
ight):\left(BC^2+CA^2+AB^2
ight).$

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14. Let AD be a median of the ΔABC .

If AE and AF are medians of the triangle ABD and ADC, respectively, and

$$BD=rac{a}{2}AD=m_1, AE=m_2, AF=m_3, thenrac{a^2}{8}$$
 is equal to

15. Find the radian measure corresponding to following degree measure

340°



16. ΔABC is equilateral triangle of side a. P lies on AB such that A is midpoint of PB. If r_1 is inradius of PAC and r_2 is ex radius of PBC opposite to P, then $r_1 + r_2 =$

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17. The base of a triangle is divided into three equal parts.

If $heta_1, heta_2, heta_3$ be the angles subtended by these parts at the vertex,

then prove that $(\cot \theta_1 + \cot \theta_2)(\cot \theta_2 + \cot \theta_3) = 4 \cos ec^2 \theta_2$



PROPERTIES AND SOLUTIONS OF TRIANGLES EXERCISE 7 : SUBJECTIVE TYPE QUESTIONS

1. A hexagon is inscribed in a circle of radius r. Two of its sides have length 1, two have length 2 and the last two have length 3. Prove that r is a root of the equation $2r^3 - 7r - 3 = 0$.

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2. Prove that $a^2 + b^2 + c^2 + 2abc < 2$, where a,b,c are the sides of triangle ABC such that a + b + c = 2.

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3. Find the radian measure corresponding to the following degree measure 25°

4. A polygon of n sides, inscribed in a circle, is such that its sides subtend angle $2\alpha, 4\alpha, \ldots, 2n\alpha$ at the centre of the circles. Prove that its area A_1 , is to the area A_2 of the regular polygon of n sides inscribed in the same circle, as $\sin n\alpha : n \sin \alpha$.

5.	Find	the	radian	measure	corresponding	to	the	following	degree
measure									

50°

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6. Find the value of the trigonometric function $\sin\left(\frac{5\pi}{6}\right)$

7. Find the radian measure of the following degree measure

75°



8. If in a ΔABC , $\frac{a\cos A + b\cos B + c\cos C}{a\sin B + b\sin C + c\sin A} = \frac{a+b+c}{9R}$, then prove

that Δ is equilateral.

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9. In $\triangle ABC$, 'h' is the length of altitude drawn from vertex A on the side BC. Prove that:

 $2ig(b^2+c^2ig)\geq 4h^2+a^2.$ Also, discuss the case when equality holds true.

10. Find the value of other five trigonometric functions if $\cos x = -\frac{3}{5}$, x

lies in third quadrant.



11. If in a \triangle A B C, b = $\sqrt{3}$, c = 1 and B - C = 90 \circ , then \angle A is

A. (a) 30

B. (b) 60

C. (c) 90

D. (d) 120

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12. An equilateral triangle PQR is circumscribed about a given ΔABC .

Prove that the maximum area of ΔPQR is $2\Delta + rac{a^2+b^2+c^2}{2\sqrt{3}}$. Where



(c) 3sqrt(3) (d) none of these

16. Find the value of trigonometric function $\cot\left(\frac{-15\pi}{4}\right)$



17. Prove that
$$\sin^2(30^\circ) + \cos^2(60^\circ) - \tan^2(45^\circ) = -\frac{1}{2}$$

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18. Prove that
$$rac{2\cos^3 x - \cos x}{\sin x - 2\sin^3 x} = \cot x$$

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Exercise (Questions Asked In Previous 13 Years Exam)

1. Prove that
$$2\sin^2\Bigl(rac{\pi}{6}\Bigr) - \sec^2\Bigl(rac{\pi}{3}\Bigr) = -rac{7}{2}$$

2. In a triangle the sum of two sides is x and the product of the same is y. If $x^2 - c^2 = y$ where c is the third side. Determine the ration of the inradius and circum-radius

A.
$$\displaystyle rac{3y}{2x(x+c)}$$

B. $\displaystyle rac{3y}{2c(x+c)}$
C. $\displaystyle rac{3y}{4x(x+c)}$
D. $\displaystyle rac{3y}{4c(x+c)}$

Answer: B

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3. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B, and C, respectively. Suppose a = 6, b = 10, and the area of triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and

if r denotes the radius of the incircle of the triangle, then the value of r^2

is



where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R, respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

A.
$$\frac{3}{4Detla}$$

B.
$$\frac{45}{4\Delta}$$

C.
$$\left(\frac{3}{4\Delta}\right)^{2}$$

D.
$$\left(\frac{45}{4\Delta}\right)^{5}$$

Answer: B::C::D



6. If the angle A, BandC of a triangle are in an arithmetic propression and if a, bandc denote the lengths of the sides opposite to A, BandCrespectively, then the value of the expression $rac{a}{c} \sin 2C + rac{c}{a} \sin 2A$ is (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$ A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. 1 D. $\sqrt{3}$

Answer: D

7. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the side opposite to A, B, and C respectively. The value(s) of x for which $a = x^2 + x + 1, b = x^2 - 1$, and c = 2x + 1 is(are) $-(2 + \sqrt{3})$ (b) $1 + \sqrt{3}$ (c) $2 + \sqrt{3}$ (d) $4\sqrt{3}$

A. $-\left(2+\sqrt{3}
ight)$ B. $1+\sqrt{3}$ C. $2+\sqrt{3}$

D. $4\sqrt{3}$

Answer: B

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8. A triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\left(\frac{A}{2}\right)$. If a, b and c, denote the length of the sides of the triangle opposite to the angles A, B, andC, respectively, then

(a)b + c = 4a

(b) b+c=2a

(c) the locus of point A is an ellipse

(d) the locus of point A is a pair of straight lines

A.
$$b+c=4a$$

 $\mathsf{B}.\,b+c=2a$

C. locus of point A is an ellipes

D. locus of point A is a pair of straight line

Answer: B::C



9. Let ABCandABC' be two non-congruent triangles with sides $AB = 4, AC = AC' = 2\sqrt{2}$ and angle $B = 30^0$. The absolute value of the difference between the areas of these triangles is

10. A straight line through the vertex P of a triangle PQR intersects the side QR at the points S and the cicumcircle of the triangle PQR at the point T. If S is not the center of the circumcircle, then $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{OS \times SR}}$ $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{OR} \frac{1}{PS} + \frac{1}{ST} > \frac{4}{OR}$ A. $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ $\mathsf{B}.\,\frac{1}{PS}+\frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$ $\mathsf{C}.\frac{1}{PS} + \frac{1}{ST} < \frac{4}{OB}$ $\mathsf{D}.\frac{1}{PS} + \frac{1}{ST} > \frac{4}{OB}$

Answer: D

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11. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and fourth quadrant respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S. The radius of the circumcircle of the triangle PRS is-

A. 4 B. 3 C. $\frac{8}{3}$

D. 2

Answer: B

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12. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and fourth quadrant respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S. The radius of the circumcircle of the triangle PRS isB. $3\sqrt{3}$

C. $3\sqrt{2}$

D. $2\sqrt{3}$

Answer: B

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13. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in first and fourth quadrant respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents at the parabola at P and Q intersect the x-axis at S. The radius of the circumcircle of the triangle PRS is-

A. 1: $\sqrt{2}$

 $\mathsf{B}.\,1\!:\!2$

C.1:4

D.1:8

Answer: C



14. Internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of $\triangle ABC$, then

A. AE is HM of b and a

B.
$$AD = rac{2bc}{b+c} \cos rac{A}{2}$$

C. $EF = rac{4bc}{b+c} \sin rac{A}{2}$

D. ΔAEF is isosceles

Answer: A::B::C::D

15. One angle of an isosceles triangle is 120^0 and the radius of its incricle is $\sqrt{3}$. Then the area of the triangle in sq. units is (a) $7 + 12\sqrt{3}$ (b) $12 - 7\sqrt{3}$ $12 + 7\sqrt{3}$ (d) 4π

A. $4\sqrt{3}$

B. $12 - 7\sqrt{3}$

C. $12 + 7\sqrt{3}$

D. None of the above

Answer: C

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16. In ΔABC , which one is true among the following ?

A.
$$(b+c)\cos{rac{A}{2}} = a\sin{\left(rac{B+C}{2}
ight)}$$

B. $(b+c)\cos{\left(rac{B+C}{2}
ight)} = a\sin{rac{A}{2}}$

C.
$$(b-c)\cos\left(rac{B-C}{2}
ight) = a\cos\left(rac{A}{2}
ight)$$

D. $(b-c)\cosrac{A}{2} = a\sin\left(rac{B-C}{2}
ight)$

Answer: D

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17. Let a vertical tower AB have its end A on the level ground. Let C be the mid point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan \beta$ is equal to : (1) $\frac{2}{9}$ (2) $\frac{4}{9}$ (3) $\frac{6}{7}$ (4) $\frac{1}{4}$ A. $\frac{6}{7}$ B. $\frac{1}{4}$ C. $\frac{2}{9}$ D. $\frac{4}{9}$

Answer: C



18. ABCD is a trapezium such that AB and CD are parallel and $BC\perp CD.$

If
$$\angle ADB = \theta$$
, $BC = p$ and $CD = q$, then AB is equal to

(a)
$$\frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

(b)
$$\frac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$$

(c)
$$\frac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$$

(d)
$$\frac{(p^2 + q^2)\sin\theta}{(p\cos\theta + \sin\theta)^2}$$

A.
$$\frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

B.
$$\frac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$$

C.
$$\frac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$$

D.
$$\frac{(p^2 + q^2)\sin\theta}{(p\cos\theta + \sin\theta)^2}$$

Answer: A

19. For a regular polygon, let r and R be the radii of the inscribed and the cirumscribed circles, respectively. A false statement among the following is

(a)there is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (b)there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$ (c)there is a regular plygon with $\frac{r}{R} = \frac{2}{3}$ (d)there is a regular plygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

A. there is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ B. there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$ C. there is a regular plygon with $\frac{r}{R} = \frac{2}{3}$ D. there is a regular plygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Answer: A::B::D

20. In triangle ABC, let $\angle c = \frac{\pi}{2}$. If r is the inradius and R is circumradius of the triangle, then 2(r+R) is equal to

A. c + aB. a + b + cC. a + b

 $\mathsf{D}.\,b+c$

Answer: A

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21. If in a ΔABC , the altitudes from the vertices A, B, C on opposite sides

are in H.P, then sin A, sin B, sin C are in

A. HP

B. AGP

C. AP

D. GP

Answer: C

