MATHS

BOOKS - CENGAGE PUBLICATION

APPLICATION OF DERIVATIVES

Illustration

1. Find the length of tangent to the curve $y = 4x^3 - 2x^5$ at $(-1, 1)$

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Exercises

1. The equation of the line tangent to the curve x siny + ysinx = π at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

A. $3x + y = 2\pi$

B. $x - y = 0$

C. $2x - y = \pi / 2$

D. $x + y = \pi$

Answer: D

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2. The x-intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^2} + \frac{b}{y^2} = 1$ is proportional to square of the abscissa of the point of tangency square root of the abscissa of the point of tangency cube of the abscissa of the point of tangency cube root of the abscissa of the point of tangency

A. square of the abscissa of the point of tangency

B. square root of the absciss of the point of tangency

C. cube of the abscissa of the point of tangency

D. cube root of the abscissa of the point of tangency

Answer: C

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3. At any point on the curve $2x^2y^2 - x^4 = c,$ the mean proportional between the abscissa and the difference between the abscissa and the sub-normal drawn to the curve at the same point is equal to (a)Ordinate (b) radius vector (c)x-intercept of tangent (d) sub-tangent

A. ordinate

B. radius vector

C. x-intercect of tangent

D. sub-tangent

Answer: A

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4. Given g(x) $\dfrac{x+2}{x-1}$ and the line $3x+y-10=0$. Then the line is

A. tangent to g(x)

B. normal to g(x)

C. chord ofg(x)

D. none of these

Answer: A

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5. If the length of sub-normal is equal to the length of sub-tangent at any point (3,4) on the curve $y = f(x)$ and the tangent at (3,4) to $y = f(x)$ meets the coordinate axes at $AandB$, then the maximum area of the triangle $OAB,$ where O is origin, is 45/2 (b) 49/2 (c) 25/2 (d) 81/2

A. 45/2

B. 49/2

C. 25/2

D. 81/2

Answer: B

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6. The number of point in the rectangle $\{(x, y)\} - 12 \leq x \leq 12 and - 3 \leq y \leq 3\}$ which lie on the curve $y = x + \sin x$ and at which in the tangent to the curve is parallel to the x-axis is 0 (b) 2 (c) 4 (d) 8

A. 0

B. 2

C. 4

D. 8

Answer: A

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7. Tangent of acute angle between the curves $y = |x^2 - 1|$ and $y = \sqrt{7 - x^2}$ at their points of intersection is $\frac{5\sqrt{3}}{2}$ (b) $\frac{3\sqrt{5}}{2}$ $\frac{5\sqrt{3}}{4}$ (d) $\frac{3\sqrt{5}}{4}$

A. $\frac{5\sqrt{3}}{2}$

B. $\frac{3\sqrt{5}}{2}$

C. $\frac{5\sqrt{3}}{4}$

D. $\frac{3\sqrt{5}}{4}$

Answer: C

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8. The line tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^2 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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9. The two curves $x = y^2, xy = a^3$ cut orthogonally at a point. Then a^2 is equal to $\frac{1}{3}$ (b) 3 (c) 2 (d) $\frac{1}{2}$

A. $\frac{1}{3}$

B. 3

C. 2

D. $\frac{1}{2}$

Answer: D

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10. The tangent to the curve $y = e^{kx}$ at a point (0,1) meets the x-axis at (a,0) where $a \in [-2, -1]$, then k

$\in$:

A. $[-1/2, 0]$

B. $[-1, -1/2]$

C. $[0, 1]$

D. $[1/2, 1]$

Answer: D

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11. The curves $4x^2 + 9y^2 = 72$ and $x^2 - y^2 = 5at(3, 2)$ Then (a) touch each other (b) cut orthogonally intersect at 45^0 (d) intersect at 60^0

A. touch each other

B. cut orthogonally

C. intersect at 45°

D. intersect at 60°

Answer: B

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12. The coordinates of a point on the parabola $y^2 = 8x$ whose distance from the circle $x^2 + (y+6)^2 = 1$ is minimum is $(2, 4)$ (b) $(2, -4)$ $(18, -12)$ (d) $(8, 8)$

A. $(2, 4)$

B. $(2, \ -4)$

C. $(18, \ -12)$

D. $(8, 8)$

Answer: B

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13. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant at normal is drawn. The normal intersects the Y-axis at the point (0, b). If $\lim_{a \to 0} b = \frac{1}{2}$, then n equals ....

A. 1

B. 3

C. 2

D. 4

Answer: C

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14. Let f be a continuous, differentiable, and bijective function. If the tangent to $y = f(x) at x = a$ is also the normal to $y = f(x) at x = b,$ then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$ (b) $f'(c) > 0$ $f'(c) < 0$ (d) none of these

A. $f'(c) = 0$

B. $f'(c) > 0$

C. $f'(c) < 0$

D. none of these

Answer: A

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15. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is (a) (2,6) (b) $(2, -6)$ $\left(\frac{9}{8}, -\frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

A. $(2, 6)$

B. $(2, \ -6)$

C. $\left(\frac{9}{8}, \frac{9}{2}\right)$

D. $\left(\frac{9}{8}, \frac{9}{2}\right)$

Answer: D

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16. Find the rate of change of volume of a sphere with respect to its surface area when the radius is 2cm.

A. 1

B. 2

C. 3

D. 4

Answer: A

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17. If there is an error of $k\%$ in measuring the edge of a cube, then the percent error in estimating its volume is k (b) $3k$ $\frac{k}{3}$ (d) none of these

A. k

B. 3k

C. $\frac{k}{3}$

D. none of these

Answer: B

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18. A lamp of negligible height is placed on the ground l_1 away from a wall. A man $l_2 m$ tall is walking at a speed of $\frac{l_1}{10} m/s$ from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is $-\frac{5l_2}{2} m/s$ (b) $-\frac{2l_2}{5} m/s$ $-\frac{l_2}{2} m/s$ (d) $-\frac{l_2}{5} m/s$

A. $-\frac{5l_2}{2} m/s$

B. $-\frac{2l_2}{5} m/s$

C. $-\frac{l_2}{2}m/s$

D. $-\frac{l_2}{5}m/s$

Answer: B

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19. The function $f(x) = x(x+3)e^{-\left(\frac{1}{2}\right)x}$ satisfies the conditions of Rolle's theorem in (-3,0). The value of c, is

A. -2

B. -1

C. 0

D. 3

Answer: A

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20. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in $cm^2/m \in$, when the radius is $2cm$ and the height is 3cm is (a) $-2p$ (b) $-\frac{8\pi}{5}$ $-\frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

A. -2π

B. $-\frac{8\pi}{5}$

C. $16/6$

D. $-8/15$

Answer: D

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21. A cube of ice melts without changing its shape at the uniform rate of $4\frac{cm^3}{\min}$. The rate of change of the surface area of the cube, in $\frac{cm^2}{\min}$, when the volume of the cube is $125cm^3$, is (a) -4 (b) $-\frac{16}{5}$ (c) $-\frac{16}{6}$ (d) $-\frac{8}{15}$

A. -4

B. $-16/5$

C. $-16/6$

D. $-8/15$

Answer: B

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22. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface when the radius is 7 cm and altitude is 24cm is (a) $108\pi cm^2/\text{ min}$ (b) $7\pi cm^2/\text{ min}$ (c) $27\pi cm^2/\text{ min}$ (d) none of these

A. $108\pi cm^2/\text{ min}$

B. $7\pi cm^2 / \min$

C. $27\pi cm^2 / \min$

D. none of these

Answer: A

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23. If $f(x) = x^3 + 7x - 1$, then $f(x)$ has a zero between $x = 0 and x = 1$. The theorem that best describes this is (a) mean value theorem (b) maximum-minimum value theorem (c) intermediate value theorem (d) none of these

A. mena value theorem

B. maximum-minimum value theorem

C. intermediate value theorem

D. none of these

Answer: C

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24. Consider the function

$$f(x) = \begin{cases} x\frac{\sin(\pi)}{x} & \text{for} x > 0 \\ 0 & \text{for} x = 0 \end{cases}$$

Then, the number of points in $(0, 1)$ where the derivative f'(x) vanishes is

A. 0

B. 1

C. 2

D. infinite

Answer: D

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25. Let $f(x) and g(x)$ be differentiable for $0 \leq x \leq 1$, such that $f(0) = 0, g(0) = 0, f(1) = 6.$ Let there exists real number c in (0,1) such that $f'(c) = 2g'(c)$. Then the value of $g(1)$ must be (a) 1 (b) 3 (c) -2 (d) -1

A. 1

B. 3

C. -2

D. $1-$

Answer: B

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26. If $3(a+2c)=4(b+3d)$, then the equation $ax^3+bx^2+cx+d=0$ will have (a) no real solution (b) at least one real root in $(-1,0)$ (c) at least one real root in $(0,1)$ (d) none of these

A. no real solution

B. at least one real root in $(-1, 0)$

C. at least one real root in $(0, 1)$

D. none of these

Answer: B

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27. A value of c for which the conclusion of Mean value theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

A. $\frac{1}{2}\log_e 3$

B. $\log_3 e$

C. $\log_e 3$

D. $2\log_3 e$

Answer: D

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28. For $f(x) = 4x^3 + 3x^2 - x - 1,$ the range of vaues of $\dfrac{f(x_1) - f(x_2)}{x_1 - x_2} is$

A. $\left(-\infty, -\frac{5}{4}\right)$

B. $\left(-\infty, -\frac{7}{4}\right)$

C. $\left[-\frac{7}{4}, \infty\right)$

D. $\left[-\frac{5}{4}, \infty\right)$

Answer: C

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29. Let $f(x)$ be a twice differentiable function for all real values of x and satisfies $f(1)=1, f(2)=4, f(3)=9$. Then which of the following is definitely true? (a) $f^{''}=2 \forall x \in (1,3)$ (b) $f^{''}=f(x)=5f \text{ or } some x \in (2,3)$ (c) $f^{''}=3 \forall x \in (2,3)$ (d) $f^{''}=2f \text{ or } some x \in (1,3)$

A. $f''(x)=2 \forall x \in (1,3)$

B. $f''(x)=f(x)5\text{for some}x \in (2,3)$

C. $f''(x) = 3 \forall x \in (2, 3)$

D. $f''(x) = 2 \text{for some} x \in (1, 3)$

Answer: D

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30. The value of c in Largrange's theorem for the function $f(x) = \log_e \sin x$ in the interval $[\pi / 6, 5\pi / 6]$ is

A. $\pi / 4$

B. $\pi / 2$

C. $2\pi / 3$

D. none of these

Answer: B

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31. In which of the following function Rolle's theorem is applicable ?

A. $f(x) = \begin{cases} x & 0 \leq x < 1 \\ 0 & x = 1 \end{cases} on[0, 1]$

B. $f(x) = \begin{cases} \frac{\sin x}{x} & -\pi \leq x < 0 \\ 0 & x = 0 \end{cases} on[-\pi, 0]$

C. $f(x) \frac{x^2 - x - 6}{x - 1} on[-2, 3]$

D. $f(x) = \begin{cases} \frac{x^3 - 2x^3 - 5x + 6}{x - 1} & \text{if } x \neq 1 \\ -6 & \text{if } x = 1 \end{cases} on[-2, 3]$

Answer: D

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32. Let $f'(x) = e^x \,\hat{}\, 2$ and $f(0) = 10.$ If A< f(1)< B can be concluded from mean value theorem then the value of A-B will be___

A. e

B. $1 - e$

C. $e - 1$

D. $1 + e$

Answer: B

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33. If $f(x) and g(x)$ are differentiable functions for $0 \leq x \leq 1$ such that $f(0) = 10, g(0) = 2, f(1) = 2, g(1) = 4,$ then in the interval $(0, 1)$. (a) $f'(x) = 0 f$ or $all x$ (b) $f'(x) + 4g'(x) = 0$ for at least one x (c) $f(x) = 2g'(x)$ for at most one x (d) none of these

A. $f(x) = 0$ for all x

B. $f(x) + 4g'(x) = 0$ for at least one x

C. $f(x) = 2g'(x)$ for at most one x

D. none of these

Answer: B

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34. A continuous and differentiable function $y = f(x)$ is such that its graph cuts line $y = mx + c$ at n distinct points. Then the minimum number of points at which $f^{''}(x) = 0$ is/are $n - 1$ (b) $n - 3$ (c) $n - 2$ (d) cannot say

A. $n - 1$

B. $n - 3$

C. $n - 2$

D. cannot say

Answer: C

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35. A man 2m tall, walks at the rate of $1\frac{2}{3}m/sec$ towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{13}m$ from the base of the light?

A. $\frac{1}{4}$

B. $\frac{-1}{2}$

C. $-1\frac{1}{4}$

D. $-\frac{1}{8}$

Answer: D

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36. A man 2m tall, walks at the rate of $1\frac{2}{3}m/sec$ towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{13}m$ from the base of the light?

A. c^2

B. $2c^2$

C. $-3c^2$

D. $12c^2$

Answer: D

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Multiple Correct Answer Type

1. Points on the curve $f(x)=\frac{x}{1-x^2}$ where the tangent is inclined at an angle of $\frac{\pi}{4}$ to the x-axis are (a) (0,0) (b) $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$ (c) $\left(-2, \frac{2}{3}\right)$ (d) $\left(-\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

A. $(0, 0)$

B. $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$

C. $\left(-2, \frac{2}{3}\right)$

D. $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$

Answer: A::B::D

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2. For the curve $y = ce^{x/a}$, which one of the following is incorrect?

A. sub-tangent is constant

B. sub-normal varies as the square of the ordinate

C. tangent at (x_1, y_1) on the curve intersects the x-axis at a distance of $(x_1 - a)$ from the origin

D. equaltion of the normal at the point where the curve cuts $y - \text{axis is} cy + ax = c^2$

Answer: A::B::C::D

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3. Let the parabolas $y = x(c - x) and y = x^2 + ax + b$ touch each other at the point (1,0). Then (a) $a + b + c = 0$ (b) $a + b = 2$ (c) $b - c = 1$ (d) $a + c = -2$

A. $a + b + c = 0$

B. $a + b = 2$

C. $b - c = 1$

D. $a + c = -2$

Answer: A::C::D

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4. The angle formed by the positive $y - a\xi s$ and the tangent to $y = x^2 + 4x - 17at\left(\frac{5}{2}, -\frac{3}{4}\right)$ is $\tan^{-1}(9)$ (b) $\frac{\pi}{2} - \tan^{-1}(9)$ $\frac{\pi}{2} + \tan^{-1}(9)$ (d) none of these

A. $\tan^{-1}(9)$

B. $\frac{\pi}{2} - \tan^{-1}(9)$

C. $\frac{\pi}{2} + \tan^{-1}(9)$

D. none of these

Answer: B::C

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5. Which of the following pair(s) of curves is/are ortogonal? (a) $y^2 = 4ax, y = e^{-x/2a}$ (b) $y^2 = 4ax, x^2 = 4ayat(0, 0)$ (c) $xy = a^2, x^2 - y^2 = b^2$ (d) $y = ax, x^2 + y^2 = c^2$

A. $y^2 = 4ax, y = e^{-x/2a}$

B. $y^2 = 4ax, x^2 = 4ayat(0, 0)$

C. $xy = a^2, x^2 - y^2 = b^2$

D. $y = ax, x^2 + y^2 = c^2$

Answer: A::B::C::D

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6. The coordinates of the point(s) on the graph of the function $f(x) = \frac{x^3}{x} - \frac{5x^2}{2} + 7x - 4$, where the tangent drawn cuts off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, are (a) $\left(2, \frac{8}{3}\right)$ (b) $\left(3, \frac{7}{2}\right)$ (c) $\left(1, \frac{5}{6}\right)$ (d) none of these

A. $(2, 8/3)$

B. $(3, 7/2)$

C. $(1, 5/6)$

D. none of these

Answer: A::B

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7. The abscissa of a point on the curve $xy = (a+x)^2$, the normal which cuts off numerically equal intercepts from the coordinate axes, is $-\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}a$ (c) $\frac{a}{\sqrt{2}}$ (d) $-\sqrt{2}a$

A. $-\frac{a}{\sqrt{2}}$

B. $\sqrt{2}a$

C. $\frac{a}{\sqrt{2}}$

D. $-\sqrt{2}a$

Answer: A::C

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8. The angle between the tangents at any point P and the line joining P to the orgin, where P is a point on the curve ln $\left(x^2+y^2\right) = k\tan^{1-}\frac{y}{x}$, c is a constant, is

A. independent of x

B. independent of y

C. independent of x but dependent on y

D. independent of y but dependent on x

Answer: A::B

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9. If OT and ON are perpendiculars dropped from the origin to the tangent and normal to the curve $x = a \sin^3 t, y = a \cos^3 t$ at an arbitrary point, then

A. $4OT^2 + ON^2 = a^2$

B. $\left|\frac{y}{\cos t}\right|$

C. the length of the normal is $\left|\frac{y}{\sin t}\right|$

D. none of these

Answer: A::B::C

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10. Let

$C_1 : y = x^2 \sin 3x, C_2 : y = x^2$ and $C_3 : y = -y^2,$

then

A. C_1 touches C_2 at infinite points

B. C_1 touches C_3 at infinite points

C. C_1 and C_2 and C_1 and C_3 meet at alternate points

D. none of these

Answer: A::B

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11. If the line x $\cos\theta + y\sin\theta = P$ is the normal to the curve $(x+a)y = 1, then\theta$ may lie in

A. I quadrant

B. II quadrant

C. III quadrant

D. IV quadrant

Answer: B::D

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12. Common tagent (s) to $y = x^3$ and $x = y^3$ is/are

A. $x - y = \frac{1}{\sqrt{3}}$

B. $x - y = -\frac{1}{\sqrt{3}}$

C. $x - y = \frac{2}{3\sqrt{3}}$

D. $x - y = \frac{-2}{3\sqrt{3}}$

Answer: C::D

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13. Given

$f(x) = 4 - \left(\frac{1}{2} - x\right)^{\frac{2}{3}}, g(x) = \left\{\frac{\tan[x]}{x}, x \neq 01, x = 0\right.$

$h(x) = \{x\}, k(x) = 5^{(\log)_2(x+3)}$ Then in [0,1], lagranges mean value theorem is not applicable to (where [.] and {.} represents the greatest integer functions and fractional part functions, respectively). f (b) g (c) k (d) h

A. f

B. g

C. k

D. h

Answer: A::B::D

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14. Let $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x$, where a_i's are real and $f(x) = 0$ has a positive root α_0. Then $f'(x) = 0$ has a positive root α_1 such that `0

A. f'(x) = 0 has a root α_1 such that $< \alpha_1 < \alpha_0$

B. f' (x) = 0 has at least one real root

C. f''(x) = 0 has at least one real root

D. none of these

Answer: A::B::C

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