# MATHS

# BOOKS - CENGAGE PUBLICATION

# APPLICATIONS OF DERIVATIVES

## Single Correct Answer Type

1. The equation of the normal to the curve parametrically represented by $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point $P(2, -1)$ is

A. $2x + 3y - 1 = 0$

B. $6x - 7y - 11 = 0$

C. $7x + 6y - 8 = 0$

D. $3x + y - 1 = 0$

**Answer: C**

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**2.** In the curve $y = x^3 + ax$ and $y = bx^2 + c$ pass through the point $(-1, 0)$ and have a common tangent line at this point then the value of $a + b + c$ is

A. 0

B. 1

C. $-3$

D. $-1$

**Answer: D**

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**3.** If the function $f(x) = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of $x$ then the value of $b$ is equal to $-1$ (b) 1 (c) 6 (d) $-6$

A. $-2$

B. $-6$

C. 6

D. 3

**Answer: B**

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**4.** Let $f(x) = x^3 + x + 1$ and let g(x) be its inverse function then equation of the tangent to $y = g(x)$ at x = 3 is

A. $x - 4y + 1 = 0$

B. $x + 4y - 1 = 0$

C. $4x - y + 1 = 0$

D. $4x + y - 1 = 0$

**Answer: A**

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**5.** A curve is represented parametrically by the equations $x = t + e^{at}$ and $y = -t + e^{at}$ when $t \in R$ and $a > 0$. If the curve touches the axis of x at the point A, then the coordinates of the point A are

A. $(1, 0)$

B. $(2e, 0)$

C. $(e, 0)$

D. $(1/e, 0)$

**Answer: B**

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**6.** The equation of the straight lines which are both tangent and normal to the curve $27x^2 = 4y^3$ are

A. $x = \pm\sqrt{2}(y-2)$

B. $x = \pm\sqrt{3}(y-2)$

C. $x = \pm\sqrt{2}(y-3)$

D. $x = \pm\sqrt{3}(y-3)$

**Answer: A**

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**7.** If the tangent at $(1, 1)$ on $y^2 = x(2-x)^2$ meets the curve again at $P,$ then find coordinates of $P.$

A. (4, 4)

B. (2, 0)

C. (9/4, 3/8)

D. $\left(3, 3^{1/2}\right)$

**Answer: C**

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**8.** A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point $(0, 1)$ and also touches the $x-$ axis at the point $(-1, 0)$ then the value of $x$ for which the curve has a negative gradient are: $x \geq -1$ b. $x < 1$ c. $x < -1$ d. $-1 \leq x \leq 1$

A. $x > -1$

B. $x > 1$

C. $x < -1$

D. $-1 \leq x \leq 1$

**Answer: C**

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**9.** find all the lines that pass through the point $(1, 1)$ and are tangent to the curve represent parametrically as $x = 2t - t^2$ and $y = t + t^2$

A. $\frac{2\sqrt{43}}{9}$

B. 2

C. 3

D. $\frac{2\sqrt{53}}{9}$

**Answer: D**

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**10.** The value of parameter t so that the line $(4-t)x+ty+\left(a^3-1\right)=0$ is normal to the curve xy = 1 may lie in the interval

A. $(1, 4)$

B. $(-\infty, 0) \cup (4, \infty)$

C. $(-4, 4)$

D. $[3, 4]$

**Answer: B**

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**11.** The tangent at any point on the curve $x = at^3 . y = at^4$ divides the abscissa of the point of contact in the ratio m:n, then $|n + m|$ is equal to (m and n are co-prime)

A. $1/4$

B. $3/4$

C. $3/2$

D. $2/5$

**Answer: B**

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**12.** The length of the sub-tangent to the hyperbola $x^2 - 4y^2 = 4$ corresponding to the normal having slope unity is $\frac{1}{\sqrt{k}}$, then the value of k is

A. 1

B. 2

C. 3

D. 4

**Answer: C**

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**13.** Cosine of the acute angle between the curve $y = 3^{x-1}\log_e x$ and $y = x^x - 1$, at the point of intersection $(1, 0)$ is

A. 0

B. 1

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

**Answer: B**

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**14.** Acute angle between two curve $x^2 + y^2 = a^2\sqrt{2}$ and $x^2 - y^2 = a^2$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. none of these

**Answer: C**

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**15.** The minimum distance between a point on the curve $y = e^x$ and a point on the curve $y = \log_e x$ is

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. 3

D. $2\sqrt{2}$

**Answer: B**

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**16.** Tangents are drawn from origin to the curve $y = \sin + \cos x$

Â·Then their points of contact lie on the curve

A. $\frac{1}{x^2} + \frac{2}{y^2} = 1$

B. $\frac{2}{x^2} - \frac{1}{y^2} = 1$

C. $\dfrac{2}{x^2} + \dfrac{1}{y^2} = 1$

D. $\dfrac{2}{y^2} - \dfrac{1}{x^2} = 1$

**Answer: D**

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**17.** If $3x + 2y = 1$ is a tangent to $y = f(x)$ at $x = 1/2$, then $\lim_{x \to 0} \dfrac{x(x-1)}{f\left(\frac{e^{2x}}{2}\right) - f\left(\frac{e^{-2x}}{2}\right)}$

A. $1/3$

B. $1/2$

C. $1/6$

D. $1/7$

**Answer: A**

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**18.** Distance of point P on the curve $y = x^{3/2}$ which is nearest to the point M (4, 0) from origin is

A. $\sqrt{\frac{112}{27}}$

B. $\sqrt{\frac{100}{27}}$

C. $\sqrt{\frac{101}{9}}$

D. $\sqrt{\frac{112}{9}}$

**Answer: A**

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**19.** If the equation of the normal to the curve $y = f(x) at x = 0$ is $3x - y + 3 = 0$ then the value of

$\lim_{x \to 0} \frac{x^2}{\{f(x^2) - 5f(4x^2) + 4f(7x^2)\}}$ is

A. $-3$

B. $1/3$

C. 3

D. $-1/3$

**Answer: D**

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**20.** The rate of change of $\sqrt{x^2+16}$ with respect to $\frac{x}{x-1}$ at $x=3$ is

A. 1

B. $\frac{11}{5}$

C. $-\frac{12}{5}$

D. $-3$

**Answer: C**

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**21.** The eccentricity of the ellipse $3x^2 + 4y^2 = 12$ is decreasing at the rate of 0.1 per sec.The time at which it will coincide with auxiliary circle is:

A. 2 seconds

B. 3 seconds

C. 5 seconds

D. 6 seconds

**Answer: C**

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**22.** A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in metres) increases at a rate of 10 m/sec. If the angle of inclination $\theta$ of the line joining the particle to the origin change, when x = 3 m, at the rate of k rad/sec., then the value of k is

A. 1

B. 2

C. $1/2$

D. $1/3$

**Answer: A**

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**23.** The rate of change of volume of a sphere is equal to the rate of change of its radius, then its radius is equal to

A. 1

B. 2

C. 0.5

D. none of these

**Answer: B**

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**24.** Water is dropped at the rate of $2m^2/s$ into a cone of semivertical angel of $45^\circ$. The rate at which periphery of water surface changes when height of water in the cone is 2 m, is

A. $0.5m/s$

B. $2m/s$

C. $3m/s$

D. $1m/s$

**Answer: D**

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**25.** Suppose that water is emptied from a spherical tank of radius 10 cm. If the depth of the water in the tank is 4 cm and is decreasing at the rate of 2 cm/sec, then the radius of the top surface of water is decreasing at the rate of

A. 1

B. $2/3$

C. $-3/2$

D. 2

**Answer: C**

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**26.** The altitude of a cone is 20 cm and its semi-vertical angle is $30^0$ . If the semi-vertical angle is increasing at the rate of $2^0$ per second, then the radius of the base is increasing at the rate of (a) 30 cm/sec (b) $\frac{160}{3}$ cm/sec (c) 10 cm/sec (d) 160 cm/sec

A. 30 cm/sec

B. $\frac{160}{3} cm / \sec$

C. 10 cm/sec

D. 160 cm/sec

**Answer: B**

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**27.** Let the equation of a curve be $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$. If $\theta$ changes at a constant rate k then the rate of change of the slope of the tangent to the curve at $\theta = \frac{\pi}{3}$ is (a) $\frac{2k}{\sqrt{3}}$ (b) $\frac{k}{\sqrt{3}}$ (c) k (d) none of these

A. $2k / \sqrt{3}$

B. $k / \sqrt{3}$

C. k

D. none of these

**Answer: D**

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**28.** Consider $f(x) = |1 - x|, 1 \le x \le 2$ and $g(x) = f(x) + b\sin.\frac{\pi}{2}x, 1 \le x \le 2$

then which of the following is correct?

A. Rolle's theorem is applicable to both f and g with $b = \frac{3}{2}$.

B. LMVT is not applicable to f and Rolle's theorem is applicable to g with $b = \frac{1}{2}$

C. LMVT is applicable to f and Rolle's theorem is applicable to g with b = 1.

D. Rolle's theorem is not applicable to both f and g for any real b.

**Answer: C**

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**29.** If $c = \frac{1}{2}$ and $f(x) = 2x - x^2$, then interval of x in which LMVT is applicable, is

A. $(1, 2)$

B. $( - 1, 1)$

C. $(0, 1)$

D. $(2, 1)$

**Answer: C**

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**30.** If a twice differentiable function f(x) on $(a, b)$ and continuous on [a, b] is such that $f''(x) < 0$ for all $x \in (a, b)$ then for any $c \in (a, b)$, $\dfrac{f(c) - f(a)}{f(b) - f(c)} >$

A. $\dfrac{b - c}{c - a}$

B. $\dfrac{c - a}{b - c}$

C. $(b - c)(c - a)$

D. $\dfrac{1}{(b-c)(c-a)}$

**Answer: B**

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**31.** Let $a, n \in N$ such that $a \geq n^3$. Then $\sqrt[3]{a+1} - \sqrt[3]{a}$ is always

A. less than $\dfrac{1}{3n^2}$

B. less than $\dfrac{1}{2n^3}$

C. more than $\dfrac{1}{n^3}$

D. more than $\dfrac{1}{4n^2}$

**Answer: A**

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**32.** Given $f'(1) = 1$ and $f(2x) = f(x) \forall x > 0. If f'(x)$ is differentiable, then there exists a number $c \in (2, 4)$ such that $f''(c)$ equal

A. $1/4$

B. $-1/2$

C. $-1/4$

D. $-1/8$

**Answer: D**

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**Multiple Correct Answer Type**

**1.** Equation of a line which is tangent to both the curve $y = x^2 + 1 \, and \, y = x^2$ is $y = \sqrt{2}x + \frac{1}{2}$ (b) $y = \sqrt{2}x - \frac{1}{2}$ $y = -\sqrt{2}x + \frac{1}{2}$ (d) $y = -\sqrt{2}x - \frac{1}{2}$

A. $y = \sqrt{2}x - \frac{1}{2}$

B. $y = \sqrt{2}x + \frac{1}{2}$

C. $y = -\sqrt{2}x + \frac{1}{2}$

D. $y = -\sqrt{2}x - \frac{1}{2}$

**Answer: B::C::D**

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**2.** For the functions defined parametrically by the equations

$$f(t) = x = \begin{cases} 2t + t^2 \sin. \frac{1}{t} & t \neq 0 \\ 0 & t = 0 \end{cases}$$ and

$$g(t) = y = \begin{cases} \frac{1}{t}\text{sint}^2 & t \neq 0 \\ 0 & t = 0 \end{cases}$$

A. equation of tangent at t = 0 is $x - 2y = 0$

B. equation of normal at t = 0 is $2x + y = 0$

C. tangent does not exist at t = 0

D. normal does not exist at $t = 0$

**Answer: A::B**

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**3.** Prove that the segment of the normal to the curve $x = 2a\sin t + a\sin t\cos^2 t; y = -a\cos^3 t$ contained between the co-ordinate axes is equal to $2a$.

A. normal is inclined at an angle $\frac{\pi}{2} + t$ with x-axis.

B. normal is inclined at an angle t with x-axis.

C. portion of normal contained between the co-ordinate axes is equal to 2a.

D. portion of normal containned between the co-ordinate axes is equal to 4a.

**Answer: A::C**

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**4.** The curve $y = ax^3 + bx^2 + cx$ is inclined at $45^\circ$ to x-axis at (0,0) but it touches x-axis at (1,0) , then a+b+c+10 is

A. f'(1) = 0

B. f''(1) = 2

C. f'''(2) = 12

D. f(2) = 2

**Answer: A::B::D**

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**5.** Determine the value of p such that the subtsngent and subnormal are equal for the curve $y = e^{px} + px$ at the point (0,1).

A. $\frac{L_{ST}}{2010} = \frac{2010}{L_{SN}}$

B. $\left|\frac{L_T}{L_N}\sqrt{\frac{L_{SN}}{L_{ST}}}\right| = \text{constant}$

C. $1 - L_{ST}L_{SN} = \frac{2000}{2010}$

D. $\left(\frac{L_T + L_N}{L_T - L_N}\right)^2 = \frac{L_{ST}}{L_{SN}}$

**Answer: A::B**

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**6.** Which of the following pair (s) $is / are$ orthogonal ?

A. $16x^2 + y^2 = c$ and $y^{16} = kx$

B. $y = x + ce^{-x}$ and $x + 2 = y + ke^{-y}$

C. $y = cx^2$ and $x^2 + 2y^2 = k$

D. $x^2 - y^2 = c$ and $xy = k$

**Answer: A::B::C::D**

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**7.** Let $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 3 - x & 5 - 3x^2 & 3x^3 - 1 \\ 2x^2 - 1 & 3x^5 - 1 & 7x^8 - 1 \end{vmatrix}$ then the equation of $f(x) = 0$ has

A. f(x) = 0 has at least two real roots

B. f'(x) =0 has at least one real root.

C. f(x) is many-one function

D. none of these

**Answer: A::B::C**

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**8.** Which of the following is correct ?

A. $\dfrac{\tan^{-1}x-\tan^{-1}y}{x-y}\leq 1\forall x, y\in R, (x\neq y)$

B. $\dfrac{\sin^{-1}x-\sin^{-1}y}{x-y}>1\forall x, y\in[-1, 1], x\neq y$

C. $\dfrac{\cos^{-1}x-\cos^{-1}y}{x-y}<1\forall x, y\in[-1, 1], x\neq y$

D. $\dfrac{\cot^{-1}x-\cot^{-1}y}{x-y}<1\forall x, y\in R, x\neq y$

**Answer: A::B**

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## Comprehension Type

**1.** A lamp post of length 10 meter placed at the end A of a ladder AB of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base B is moving at the rate of 5 m/sec. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.

Rate at which $\theta$ decreases, when the base B is 12 m from the vertical wall, is

A. 1 rad/sec

B. 2 rad/sec

C. 5 rad/sec

D. 1/2 rad/sec

**Answer: A**

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**2.** A lamp post of length 10 meter placed at the end A of a ladder AB of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base B is moving at the rate of 5 m/sec. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.

The rate at which the length of shadow of man increases, when the base B is 12 m from vertical wall, is

A. 15 m/sec

B. 40/27 m/sec

C. 15/2 m/sec

D. 5 m/sec

**Answer: B**

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**3.** Let f(x) be a function such that its derovative f'(x) is continuous in [a, b] and differentiable in (a, b). Consider a function $\phi(x) = f(b) - f(x) - (b - x)f'(x) - (b - x)^2$A. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions.

If there exists some unmber c(a lt c lt b) such that $\phi'(c) = 0$ and $f(b) = f(a) + (b - a)f'(a) + \lambda(b - a)^2 f''(c)$, then $\lambda$ is

A. 1

B. 0

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

**Answer: C**

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**4.** Let f(x) be a function such that its derovative f'(x) is continuous in [a, b] and differentiable in (a, b). Consider a function $\phi(x) = f(b) - f(x) - (b - x)f'(x) - (b - x)^2$A. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions.

If there exists some unmber c(a lt c lt b) such that $\phi'(c) = 0 \text{ and } f(b) = f(a) + (b - a)f'(a) + \lambda(b - a)^2 f''(c)$, then $\lambda$ is

A. $1/2$

B. 2

C. 3

D. does not exist

**Answer: C**

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**5.** Let f(x) be a function such that its derovative f'(x) is continuous in [a, b] and differentiable in (a, b). Consider a function $\phi(x) = f(b) - f(x) - (b-x)f'(x) - (b-x)^2 A$. If Rolle's theorem is applicable to $\phi(x)$ on, [a,b], answer following questions.

If there exists some unmber c(a lt c lt b) such that $\phi'(c) = 0 \text{ and } f(b) = f(a) + (b-a)f'(a) + \lambda(b-a)^2 f''(c)$, then $\lambda$ is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

**Answer: B**

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## Subjective Type

**1.** Prove that for $\lambda > 1$, the equation $x \log x + x = \lambda$ has least one solution in $[1, \lambda]$.

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**2.** If $f(x)$ and $g(x)$ are continuous and differentiable functions, then prove that there exists $c \in [a, b]$ such that $\frac{f'(c)}{f(a) - f(c)} + \frac{g'(c)}{g(b) - g(c)} = 1$.

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