MATHS

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AREA

Illustration

1. Find the area of the closed figure bounded by the curves $y = \sqrt{x}, y = \sqrt{4 - 3x}$ and $y = 0$

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2. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside in the parabola $y^2 = 4x$.

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3. Find the area bounded by (a) $y = (\log)_e|x| and y = 0$ (b) $y = |(\log)_e|x| \,|\, and y = 0$

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4. Find the area bounded by $y = \frac{1}{x^2 - 2x + 2}$ and x-axis.

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5. find the area of $y = 3 - 2x - x^2$ and x axis

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6. If the area of bounded between the x-axis and the graph of $y = 6x - 3x^2$ between the ordinates $x = 1 and x = a$ is 19 units, then a

can take the value 4 or -2 two value are in (2,3) and one in $(-1, 0)$

two value are in (3,4) and one in $(-2, -1)$ none of these

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7. Prove that area common to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle $x^2 + y^2 = a^2$ is equal to the area of another ellipse of semi-axis a and a-b.

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8. Let $f(x) =$ maximum $\left\{x^2, (1-x)^2, 2x(1-x)\right\}$ where $x \in [0, 1]$. Determine the area of the region bounded by the curve $y = f(x)$ and the lines $y = 0, x = 0, x = 1$.

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9. Consider the region formed by the lines $x = 0, y = 0, x = 2, y = 2$. If the area enclosed by the curves $y = e^x andy = 1nx$, within this region, is being removed, then find the area of the remaining region.

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10. Draw a rough sketch of the curve $y = \frac{x^2 + 3x + 2}{x^2 - 3x + 2}$ and find the area of the bounded region between the curve and the x-axis.

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11. Find the area bounded by the curve $y = (x - 1)(x - 2)(x - 3)$ lying between the ordinates $x = 0andx = 3$.

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12. Find the area bounded by the curve $x = \begin{cases} -2-y, & y < -1 \\ y^3, & -1 \leq y \leq 1 \\ 2-y, & y > 1 \end{cases}$

and x=0 is

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13. Find the area enclosed by the graph of $y = \log_e(x+1)$, y-axis, and the line y=1

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14. Find the area bounded by the curve $y = \sin^{-1} x$ and the line $x = 0, |y| = \frac{\pi}{2}$.

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15. Find the area of the region bounded by the curves $y = \sqrt{x+2}$ and $y = \frac{1}{x+1}$ between the lines x=0 and x=2.

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16. Find the area bounded by $y = \sin^{-1} x, y = \cos^{-1} x$,and the X-axis.

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17. Find the area bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$.

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18. Find the area bounded by the curves y=sin x and y=cos x between two consecutive points of the intersection.

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19. Find the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$.

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20. Find the area of the figure bounded by the parabolas $x = -2y^2, x = 1 - 3y^2$.

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21. The area common to regions $x^2 + y^2 - 2x \leq 0$ and $y \geq \sin\left(\pi \frac{x}{2}\right)$

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22. Find the area of the region enclosed by the curves $y = x \log x$ and $y = 2x - 2x^2$.

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23. Find the area bounded by $y^2 \leq 4x, x^2 + y^2 \geq 2x, andx \leq y + 2$ in the first quadrant.

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24. Sketch the region bounded by the curves $y = x^2 andy = \frac{2}{1 + x^2}$. Find the area.

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25. Find the area bounded by the curves $y = x^3 - x$ and $y = x^2 + x$.

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26. Find the area bounded by $y = -x^3 + x^2 + 16x$ and $y = 4x$

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27. Find the area of the region enclosed by $y = -5x - x^2$ and $y = x$ on interval $[-1, 5]$

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28. If the area enclosed by curve $y = f(x) and y = x^2 + 2$ between the abscissa $x = 2 and x = \alpha, \alpha > 2,$ is $\left(\alpha^3 - 4\alpha^2 + 8\right) sq.$ unit. It is known that curve $y = f(x)$ lies below the parabola $y = x^2 + 2.$

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29. Let $C_1 C_2$ be the graphs of the functions $y = x^2$ and $y = 2x$, respectively, where $0 \leq x \leq 1$. Let C_3 be the graph of a function $y = f(x)$, where $0 \leq x \leq 1, f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axis, meet C_2 and C_3 at Q and R, respectively (see Figure). If for every position of $P(on C_1)$, the areas of the shaded regions OPQ and ORP are equal, determine the function $f(x)$

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30. If the area bounded by $f(x) = \sqrt{\tan x}, y = f(c), x = 0 \text{ and } x = a, 0 < c < a < \frac{\pi}{2}$ is minimum then find the value of c.

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31. Find the area of the region R which is enclosed by the curve $y \geq \sqrt{1 - x^2}$ and max $\{|x|, |y|\} \leq 4.$

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32. Plot the region in the first quadrant in which points are nearer to the origin than to the line $x = 3.$

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33. Consider a square with vertices at $(1, 1), (-1, 1), (-1, -1), and (1, -1)$. Set S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

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34. Let $O(0, 0), A(2, 0), and B\left(1, \frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside OAB which satisfy $d(P, OA) \leq \min[d(P, OB), d(P, AB)]$, where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area.

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35. Find the area enclosed by $y = g(x)$, x-axis, x=1 and x=37, where g(x) is inverse of $f(x) = x^3 + 3x + 1$.

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36. The area bounded by the curve $f(x) = x + \sin x$ and its inverse function between the ordinates $x = 0 and x = 2\pi$ is $4\pi sq units$ (b) $8\pi sq units$ $4 sq units$ (d) $8 sq units$

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Solved Examples

1. Find the area bounded by the curve $x^2 = y, x^2 = -y$ and $y^2 = 4x - 3$

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2. Find the area of the region enclosed by the curve $y = \left|x - \frac{1}{x}\right|(x > 0)$ and the line y=2

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3. Find the area of the region bounded by the curves $y = x^2, y = \left|2 - x^2\right|, andyl = 2,$ which lies to the right of the line $x = 1.$

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4. The ratio in which the line $x - 1 = 0$ divides the area bounded by the curves $2x + 1 = \sqrt{4y + 1}, y = x \text{ and } y = 2$ is

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5. If $S_0, S_1, S_2, \ldots$ are areas bounded by the x-axis and half-wave of the curve $y = \sin \pi \sqrt{x}, \quad \text{then prove that} \quad S_0, S_1, S_2, \ldots$ are in A.P...

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6. Find the area of the figure enclosed by the curve $5x^2 + 6xy + 2y^2 + 7x + 6y + 6 = 0$.

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7. Find the area bounded by the curves $x^2 + y^2 = 4, x^2 = \sqrt{2}y, andx = y\cdot$

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8. Find the area of the region bounded by the curve C : y=tan x ,tangent drawn to C at x=pi/4, and the x-axis.

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9. Compute the area of the region bounded by the curves $y - ex(\log)_e x and y = \frac{\log x}{ex}$

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10. If A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0,\ y = 0,\ x = \pi/4$, then for $n > 2$.

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Concept Application Exercise 9 1

1. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

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2. Find the area enclosed by the curves $x^2 = y$, $y = x + 2$ and x-axis

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3. A curve is given by $byy = \left\{\left(\sqrt{4 - x^2}\right), 0 \leq x < 1\sqrt{(3x)}, 1 \leq x \leq 3.\right.$ Find the area lying between the curve and x-axis.

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4. Find the area bounded by $x = 2y - y^2$ and the $y - axis.$

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5. Find the area bounded by the x-axis, part of the curve $y = \left(1 - \frac{8}{x^2}\right)$, and the ordinates at $x = 2andx = 4.$ If the ordinate at $x = a$ divides the area into two equal parts, then find $a\cdot$

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6. Find the area of the region bounded by the x-axis and the curves defined by $y = \tan x \left(where - \frac{\pi}{3} \leq x \leq \frac{\pi}{3}\right)$ and $y = \cot x \left(where \frac{\pi}{6} \leq x \leq \frac{3\pi}{2}\right)$.

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7. Find the area bounded by $y = \left|\sin x - \frac{1}{2}\right|$ and $y = 1$ for $x \in [0, \pi]$

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8. If the area bounded by the graph of $y = xe^{-ax}(a > 0)$ and the x-axis is $1/9$ then find the value of a.

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9. The area bounded by the curve xy^2 =a^2 (a−x) and y-axis is

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Concept Application Exercise 9 2

1. Find the area lying in the first quadrant and bounded by the curve $y = x^3$ and the line $y = 4x$.

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2. Find the area bounded by the curve $x^2 = 4y$ and the line x = 4y - 2.

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3. Find the area enclosed by the figure described by the equation $x^4 + 1 = 2x^2 + y^2$.

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4. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2 and y = x^2 - x$?

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5. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola y = 6x is

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6. Find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0, and x = 3.$

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7. Find the area of the region bounded by the limits $x=0, x=\frac{\pi}{2}, and f(x)=\sin x, g(x)=\cos x\cdot$

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8. Find the area bounded by $y=\tan^{-1}x, y=\cot^{-1}x, and y-a\xi s$ in the first quadrant.

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9. Find the area bounded by $y=-(\log)_e x, y=-(\log)_e x, y=(\log)_e(-x), and y=-(\log)_e(-x)\cdot$

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10. Find the area of the region $\{(x,y): y^2 \leq 4x, 4x^2+4y^2 \leq 9\}$

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11. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

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12. Sketch the curves and identity the region bounded by $x = \frac{1}{2}, x = 2, y = 1nx, andy = 2^x$. Find the area of this region.

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13. Find the area bounded by $y = x^2$ and $y = x^{1/3}$ for $x \in [-1, 1]$.

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14. Find the smallest area bounded by the curves $y = x - \sin x, y = x + \cos x$.

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Concept Application Exercise 9 3

1. Find the continuous function f where $(x^4 - 4x^2) \leq f(x) \leq (2x^2 - x^3)$ such that the area bounded by $y = f(x), y = x^4 - 4x^2.$ then y-axis, and the line $x = t,$ where $(0 \leq t \leq 2)$ is k times the area bounded by $y = f(x), y = 2x^2 - x^3$,y-axis , and line $x = t(where 0 \leq t \leq 2).$

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2. If the area bounded by the x-axis, the curve $y = f(x), (f(x) > 0)$ and the lines $x = 1, x = b$ is equal to $\sqrt{b^2 + 1} -$ then find f(x).

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3. The area bounded by the graph of $y = f(x), f(x) > 0$ on [0,a] and x-axis is $\frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$ then find the value of $f\left(\frac{\pi}{2}\right)$.

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4. A curve y = f(x) is such that $f(x) \geq 0$ and $f(0) = 0$ and bounds a curvilinear triangle with the base [0,x] whose area is proportional to $(n+1)^{th}$ power of $f(x) \cdot \quad \text{If} \quad f(1) = 1$ then find f(x).

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5. Find the area of curve enclosed by : $|x+y| + |x-y| \leq 4, |x| \leq 1, y \geq \sqrt{x^2 - 2x + 1}$.

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6. Consider two regions

R_1 : points P are nearer to (1,0) than to $x = -1$.

R_2 : Points P are nearer to (0,0) than to (8,0) Find the area of the region common to R_1 and R_2.

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7. If $f : [-1, 1] \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right] : f(x) = \frac{x}{1+x^2}$, then find the area bounded by $y = f^{-1}(x)$, the x-axis and the lines $x = \frac{1}{2}$, $x = -\frac{1}{2}$.

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Exercises Single Correct Answer Type

1. Area enclosed by the curve $y = f(x)$ defined parametrically as $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2} isequa < o \quad \pi squnits$ (b) $\frac{\pi}{2} squnits$ $\frac{3\pi}{4} squnits$ (d) $\frac{3\pi}{2} squnits$

A. π sq. units

B. $\pi/2$ sq. units

C. $\frac{3\pi}{4}$ sq. units

D. $\frac{3\pi}{2}$ sq. units

Answer: A

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2. Let $f(x) = \min\left(x+1, \sqrt{1-x}\right)$ for all $x \le 1$. Then the area bounded by $y = f(x)$ and the x-axis is (a) $\frac{7}{3}$ sq units (b) $\frac{1}{6}$ sq units (c) $\frac{11}{6}$ sq units (d) $\frac{7}{6}$ sq units

A. $\frac{7}{3}$ sq. units

B. $\frac{1}{6}$ sq. units

C. $\frac{11}{6}$ sq. units

D. $\frac{7}{6}$ sq. units

Answer: D

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3. The area of the closed figure bounded by $x = -1, y = 0, y = x^2 + x + 1,$ and the tangent to the curve $y = x^2 + x + 1$ at $A(1, 3)$ is (a) $\frac{4}{3}$ sq. units (b) $\frac{7}{3}$ sq. units (c) $\frac{7}{6}$ sq. units (d) none of these

A. (a) $4/3$ sq. units

B. (b) $7/3$ sq. units

C. (c) $7/6$ sq. units

D. (d) None of these

Answer: C

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4. The area bounded by the curve $a^2 y = x^2(x+a)$ and the x-axis is $\frac{a^2}{3} squnits$ (b) $\frac{a^2}{4} squnits$ $\frac{3a^2}{4} squnits$ (d) $\frac{a^2}{12} squnits$

A. $a^2 / 3$ sq. units

B. $a^2 / 4$ sq. units

C. $3a^2 / 4$ sq. units

D. $a^2 / 12$ sq. units

Answer: D

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5. The area between the curve $y = 2x^4 - x^2$, the axis, and the ordinates of the two minima of the curve is 11/60 sq. units (b) 7/120 sq. units 1/30 sq. units (d) 7/90 sq. units

A. $11 / 60$ sq. units

B. $7 / 120$ sq. units

C. $1/30$ sq. units

D. $7/90$ sq. units

Answer: B

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6. The area of the closed figure bounded by $x = -1, x = 2, and$ $y = \{-x^2 + 2, x \le 12x - 1, x > 1$and the ascissa axis is (a) $\frac{16}{3} squnits$ (b) $\frac{10}{3} squnits$ (c) $\frac{13}{3} squnits$ (d) $\frac{7}{3} squnits$

A. $16/3$ sq. units

B. $10/3$ sq. units

C. $13/3$ sq. units

D. $7/3$ sq. units

Answer: A

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7. The value of the parameter a such that the area bounded by $y = a^2x^2 + ax + 1,$ coordinate axes, and the line x=1 attains its least value is equal to

A. $\frac{1}{4}$ sq. units

B. $-\frac{1}{2}$ sq. units

C. $\frac{3}{4}$ sq. units

D. -1 sq. units

Answer: C

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8. The positive valu of the parameter 'k' for which the area of the figure bounded by the curve $y = \sin(kx), x = \frac{2\pi}{3k}, x = \frac{5\pi}{3k}$ and x-axis is less than 2 can be

A. $\frac{1}{8} < k < \frac{3}{8}$

B. $0 < k < \frac{1}{8}$

C. $1 < k < 2$

D. $\frac{3}{8} < k < \frac{5}{8}$

Answer: C

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9. The area bounded by the curve $y = x(1 - \log_e x)$ and x-axis is

(a) $\frac{e^2}{4}$ (b) $\frac{e^2}{2}$ (c) $\frac{e^2 - e}{2}$ (d) $\frac{e^2 - e}{4}$

A. $\frac{e^2}{4}$

B. $\frac{e^2}{2}$

C. $\frac{e^2 - e}{2}$

D. $\frac{e^2 - e}{4}$

Answer: A

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10. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is (a) $12\sqrt{3}$ sq units (b) $6\sqrt{3}$ sq units (c) $8\sqrt{3}$ sq units (d) $4\sqrt{3}$ sq units

A. $12\sqrt{3}$ sq. units

B. $6\sqrt{3}$ sq. units

C. $8\sqrt{3}$ sq. units

D. $4\sqrt{3}$ sq. units

Answer: A

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11. Area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is (a) $\frac{3\pi - 8}{3}$ (b) $\frac{\pi - 8}{3}$ (c) $\frac{2\pi - 8}{3}$ (d) None of these

A. $\frac{3\pi - 8}{3}$ sq. units

B. $\frac{\pi - 8}{3}$

C. $\frac{2\pi - 8}{3}$ sq. units

D. None of these

Answer: A

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12. If A_n is the area bounded by $y = x$ and $y = x^n, n \in \mathbb{N}$,then $A_2 A_3 \ldots A_n =$ (a) $\frac{1}{n(n+1)}$ (b) $\frac{1}{2^n n(n+1)}$ (c) $\frac{1}{2^{n-1} n(n+1)}$ (d) $\frac{1}{2^{n-2} n(n+1)}$

A. $\frac{1}{n(n+1)}$

B. $\frac{1}{2^n n(n+1)}$

C. $\frac{1}{2^{n-1} n(n+1)}$

D. $\frac{1}{2^{n-2} n(n+1)}$

Answer: D

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13. The area of the region is 1st quadrant bounded by the y-axis, $y=\frac{x}{4}, y=1+\sqrt{x}$, and $y=\frac{2}{\sqrt{x}}$ is

A. $2/3$ sq. units

B. $8/3$ sq. units

C. $11/3$ sq. units

D. $13/6$ sq. units

Answer: C

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14. The area of the closed figure bounded by $y=\frac{x^2}{2}-2x+2$ and the tangents to it at $\left(1,\frac{1}{2}\right) and (4,2)$ is $\frac{9}{8} squnits$ (b) $\frac{3}{8} squnits$ $\frac{3}{2} squnits$ (d) $\frac{9}{4} squnits$

A. $9/8$ sq. units

B. $3/8$ sq. units

C. $3/2$ sq. units

D. $9/4$ sq. units

Answer: A

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15. The area of the region bounded by $x^2 + y^2 - 2x - 3 = 0$ and $y = |x| + 1$ is

A. $\frac{\pi}{2} - 1$ sq. units

B. 2π sq. units

C. 4π sq. units

D. $\pi/2$ sq. units

Answer: A

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16. The area enclosed by the curve $y=\sqrt{4-x^2}, y \geq \sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$,

and the x-axis is divided by the y-axis in the ratio.

(a) $\frac{\pi^2-8}{\pi^2+8}$

(b) $\frac{\pi^2-4}{\pi^2+4}$

(c) $\frac{\pi-4}{\pi-4}$

(d) $\frac{2\pi^2}{2\pi+\pi^2-8}$

A. $\frac{\pi^2-8}{\pi^2+8}$

B. $\frac{\pi^2-4}{\pi^2+4}$

C. $\frac{\pi-4}{\pi-4}$

D. $\frac{2\pi^2}{2\pi+\pi^2-8}$

Answer: D

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17. The area bounded by the curve $y^2(2-x)=x^3$ and $x=2$ is

A. A. $\frac{3}{\sqrt{2}}-\frac{11}{6}$ sq. units

B. B. $3\sqrt{2}-\frac{11}{4}$ sq. units

C. C. $\frac{6}{\sqrt{2}}-\frac{11}{5}$ sq. units

D. D. None of these

Answer: A

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18. The area bounded by the curves $y=(\log)_e x and y=((\log)_e x)^2$ is

A. $e-2$ sq. units

B. $3-e$ sq. units

C. e sq. units

D. $e-1$ sq. units

Answer: B

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19. The area bounded by $y = 3 - |3 - x|$ and $y = \dfrac{6}{|x + 1|}$ is

A. $\dfrac{15}{2} - 6 \quad \text{In} \quad 2$ sq. units

B. $\dfrac{13}{2} - 3 \quad \text{In} \quad 2$ sq. units

C. $\dfrac{13}{2} - 6 \quad \text{In} \quad 2$ sq. units

D. None of these

Answer: C

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20. The area enclosed between the curves $y = (\log)_e(x + e), x = (\log)_e\left(\dfrac{1}{y}\right),$ and the x-axis is $2squnits$ (b) $1squnits$ $4squnits$ (d) none of these

A. 2 sq. units

B. 1 sq. units

C. 4 sq. units

D. None of these

Answer: A

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21. Find the area enclosed the curve y=sin x and the X-axis between $x = 0 \text{ and } x = \pi$.

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22. The area bounded by $y = x^2, y = [x + 1], 0 \leq x \leq 2$ and the y-axis is where $[.]$ is greatest integer function.

A. $\frac{1}{3}$

B. $\frac{\sqrt{2}}{3}$

C. 1

D. $\frac{7}{3}$

Answer: B

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23. The area of the figure bounded by the parabola $(y-2)^2 = x-1$, the tangent to it at the point with the ordinate $x=3$, and the $x-a\xi s$ is $7sq\dot{u}nites$ (b) $6sq\dot{u}nites$ $9sq\dot{u}nites$ (d) None of these

A. 7 sq. units

B. 6 sq. units

C. 9 sq. units

D. None of these

Answer: C

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24. The area bounded by the curves $y = xe^x, y = xe^{-x}$ and the line $x = 1$ is $\frac{2}{e} squnits$ (b) $1 - \frac{2}{e} squnits$ $\frac{1}{e} squnits$ (d) $1 - \frac{1}{e} squnits$

A. $\frac{2}{e}$ sq. units

B. $1 - \frac{2}{e}$ sq. units

C. $\frac{1}{e}$ sq. units

D. $1 - \frac{1}{e}$ sq. units

Answer: A

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25. The area of the region whose boundaries are defined by the curves y=2 cos x, y=3 tan x, and the y-axis is

A. $1 + 3 \text{ In } \left(\frac{2}{\sqrt{3}}\right)$ sq. units

B. $1+\frac{3}{2}$ In 3 - 3 In 2 sq. units

C. $1+\frac{3}{2}$ In 3 - In 2 sq. units

D. In 3 - In 2 sq. units

Answer: B

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26. The area bounded by $y=\sec^{-1}x, y=\cos ec^{-1}x$, and line $x-1=0$ is $\log(3+2\sqrt{2})-\frac{\pi}{2}squnits$ $\frac{\text{pi}}{2}-\log(3+2\sqrt{2})squnits$ $\pi-(\log)_e 3squnits$ None of these

A. $\log(3+2\sqrt{2})-\frac{\pi}{2}$ sq. units

B. $\frac{\pi}{2}-\log(3+2\sqrt{2})$ sq. units

C. $\pi-\log_e 3$ sq. units

D. None of these

Answer: A

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27. The area bounded by the curve $y = \dfrac{3}{|x|}$ and $y + |2 - x| = 2$ is

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28. The area enclosed by $y = x^2 + \cos x$ and its normal at $x = \dfrac{\pi}{2}$ in the first quadrant is

A. A. $\dfrac{\pi^5}{32} - \dfrac{\pi^4}{64} + \dfrac{\pi^3}{32} + 1$

B. B. $\dfrac{\pi^5}{16} - \dfrac{\pi^4}{32} + \dfrac{\pi^3}{24} - 1$

C. C. $\dfrac{\pi^5}{32} - \dfrac{\pi^4}{32} + \dfrac{\pi^3}{16}$

D. D. $\dfrac{\pi^5}{32} - \dfrac{\pi^4}{32} + \dfrac{\pi^3}{24} + 1$

Answer: D

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29. Given $f(x)=\int_0^x e^t\left(\log_e \sec t-\sec^2 t\right)dt,\ g(x)=-2e^x\tan x$, then the area bounded by the curves $y=f(x)$ and $y=g(x)$ between the ordinates $x=0$ and $x=\frac{\pi}{3}$, is (in sq. units)

A. $\frac{1}{2}e^{\frac{\pi}{3}}\log_e 2$

B. $e^{\frac{\pi}{3}}\log_e 2$

C. $\frac{1}{4}e^{\frac{\pi}{3}}\log_e 2$

D. $e^{\frac{\pi}{3}}\log_e 3$

Answer: B

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30. The area of the loop of the curve $ay^2=x^2(a-x)$ is $4a^2 squnits$ (b) $\frac{8a^2}{15} squnits$ $\frac{16a^2}{9} squnits$ (d) None of these

A. $4a^2$ sq. units

B. $\frac{8a^2}{15}$ sq. units

C. $\frac{16a^2}{9}$ sq. units

D. None of these

Answer: B

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31. Aea of the region nclosed between the curves $x = y^2 - 1$ and $x = |y|\sqrt{1 - y^2}$ is

A. 1 sq. units

B. $4/3$ sq. units

C. $2/3$ sq. units

D. 2 sq. units

Answer: D

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32. The area bounded by the loop of the curve $4y^2 = x^2(4 - x^2)$ is $(a)\frac{7}{3}sq.\ units\ (b)\frac{8}{3}squnites\ (c)\frac{11}{3}squnits\ (d)\frac{16}{3}squnits$

A. $7/3$ sq. units

B. $8/3$ sq. units

C. $11/3$ sq. units

D. $16/3$ sq. units

Answer: D

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33. The area enclosed by the curves $xy^2 = a^2(a - x)and(a - x)y^2 = a^2\xi s$ $(\pi - 2)a^2squnits$ (b) $(4 - \pi)a^2squnits$ $\frac{\pi a^2}{3}squnits$ (d) none of these

A. $(\pi - 2)a^2$ sq. units

B. $(4 - \pi)a^2$ sq. units

C. $\pi a^2 / 3$ sq. units

D. None of these

Answer: A

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34. The area bounded by the two branches of curve $(y-x)^2 = x^3$ and the straight line $x = 1$ is (a) $\frac{1}{5} squnits$ (b) $\frac{3}{5} squnits$ $\frac{4}{5} squnits$ (d) $\frac{8}{4} squnits$

A. $1/5$ sq. units

B. $3/5$ sq. units

C. $4/5$ sq. units

D. $8/4$ sq. units

Answer: C

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35. Draw the graph of $y = \sin^{-1}|\sin x|$ and $y = (\sin^{-1}|\sin x|)^2, 0 \le x \le 2\pi$

A. $\frac{1}{3} + \frac{\pi^2}{4}$ sq. units

B. $\frac{1}{6} + \frac{\pi^3}{8}$ sq. units

C. 2 sq. units

D. None of these

Answer: D

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36. Consider two curves $C_1 : y^2 = 4\left[\sqrt{y}\right] x and C_2 : x^2 = 4\left[\sqrt{x}\right] y$, where [.] denotes the greatest integer function. Then the area of region enclosed by these two curves within the square formed by the lines $x = 1, y = 1, x = 4, y = 4$ is (a) $\frac{8}{3} sq.units$ (b) $\frac{10}{3} sq.units$ (c) $\frac{11}{3} sq.units$ (d) $\frac{11}{4} sq.units$

A. $8/3$ sq. units

B. $10/3$ sq. units

C. $11/3$ sq. units

D. $11/4$ sq. units

Answer: C

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37. The area enclosed between the curve $y^2(2a-x)=x^3$ and the line x=2 above the x-axis is

A. πa^2 sq. units

B. $\dfrac{3\pi a^2}{2}$ sq. units

C. $2\pi a^2$ sq. units

D. $3\pi a^2$ sq. units

Answer: B

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38. The area of the region of the plane bounded by $\max(|x|, |y|) \leq 1$ and $xy \leq \frac{1}{2}$ is

(a) $\frac{1}{2} + \ln 2\ sq.\ units$

(b) $3 + \ln 2\ sq.\ units$

(c) $\frac{31}{4}\ sq.\ units$

(d) $1 + 2\ln 2\ sq.\ units$

A. $1/2$ + In 2 sq. units

B. 3+ In 2 sq. units

C. $31/4$ sq. units

D. 1+2 In 2 sq. units

Answer: B

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39. Find tha area of the region containing the points (x, y) satisfying $4 \leq x^2 + y^2 \leq 2(|x| + |y|)$.

A. 8 sq. units

B. 2 sq. units

C. 4π sq. units

D. 2π sq. units

Answer: A

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40. Let f(x) be a non-negative continuous function such that the area bounded by the curve y=f(x), the x-axis, and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2}\beta$. Then $f'\left(\frac{\pi}{2}\right)$ is

A. $\left(\frac{\pi}{2} - \sqrt{2} - 1\right)$

B. $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$

C. $-\frac{\pi}{2}$

D. $\left(1-\frac{\pi}{2}-\sqrt{2}\right)$

Answer: C

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Multiple Correct Answers Type

1. Let $A(k)$ be the area bounded by the curves $y=x^2-3$ and $y=kx+2$ The range of $A(k)$ is $\left(\frac{10\sqrt{5}}{3},\infty\right)$ The range of $A(k)$ is $\left(\frac{20\sqrt{5}}{3},\infty\right)$ If function $k\overrightarrow{A}(k)$ is defined for $k\in[-2,\infty)$, then $A(k)$ is many-one function. The value of k for which area is minimum is 1.

A. The range of A(k) is $\left[\frac{10\sqrt{5}}{3},\infty\right)$

B. The range of A(k) is $\left[\frac{20\sqrt{5}}{3},\infty\right)$

C. If function $k \to A(k)$ is defined for $k \in [-2, \infty)$, then A(k) is many-one function

D. The value of k for which area is minimum is 1

Answer: B::C

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2. The parabolas $y^2 = 4xandx^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are the areas of these parts numbered from top to bottom, respectively, then $S_1 : S_2 \equiv 1 : 1$ (b) $S_2 : S_3 \equiv 1 : 2$ $S_1 : S_3 \equiv 1 : 1$ (d) $S_1 : (S_1 + S_2) = 1 : 2$

A. $S_1 : S_2 \equiv 1 : 1$

B. $S_2 : S_3 \equiv 1 : 2$

C. $S_1 : S_3 \equiv 1 : 1$

D. $S_1 : (S_1 + S_2) = 1 : 2$

Answer: A::C::D

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3. Find the area bounded by the following curve :

(i) $f(x) = \sin x, g(x) = \sin^2 x, 0 \leq x \leq 2\pi$

(ii) $f(x) = \sin x, g(x) = \sin^4 x, 0 \leq x \leq 2\pi$

A. $f(x) = \sin x, g(x) = \sin^2 x, \quad \text{where} \quad 0 \leq x \leq 10\pi$

B. $f(x) = \sin x, g(x) = |\sin|, \quad \text{where} \quad 0 \leq x \leq 20\pi$

C. $f(x) = |\sin|, g(x) = \sin^3 x, \quad \text{where} \quad 0 \leq x \leq 10\pi$

D. $f(x) = \sin x, g(x) = \sin^4 x, \quad \text{where} \quad 0 \leq x \leq 10\pi$

Answer: A::C::D

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4. If the curve $y = ax^{\frac{1}{2}} + bx$ passes through the point $(1, 2)$ and lies above the x-axis for $0 \leq x \leq 9$ and the area enclosed by the curve, the x-axis, and the line $x = 4$ is 8 sq. units, then (a)$a = 1$ (b) $b = 1$ $a = 3$ (d) $b = -1$

A. $a = 1$

B. $b = 1$

C. $a = 3$

D. $b = -1$

Answer: C::D

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5. The area bounded by the curve $x = a\cos^3 t, , y = a\sin^3 t,$ is :

A. $12a^2 \int_0^{\pi/2} \cos^4 t \sin^2 t dt$

B. $12a^2 \int_0^{\pi/2} \cos^2 t \sin^4 t dt$

C. $2 \int_{-a}^{a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx$

D. $4 \int_0^{a} \left(a^{2/3} - x^{2/3}\right) dx$

Answer: A::C::D

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6. If A_i is the area area bounded by $|x - a_i| + |y| = b_i, i \in N$,where $a_{i+1} = a_i + \frac{3}{2} b_i$ and $b_{i+1} = \frac{b_i}{2}$, $a_1 = 0$ and $b_1 = 32$, then

A. A. $A_3 = 128$

B. B. $A_3 = 256$

C. C. $\lim_{n \to \infty} \sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$

D. D. $\lim_{n \to \infty} \sum_{i=1}^{n} A_i = \frac{4}{3}(16)^2$

Answer: A::C

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7. For which of the following values of m is the area of the regions bounded by the curve $y = x - x^2$ and the line $y = mx$ equal $\frac{9}{2}$? -4 (b) -2 (c) 2 (d) 4

A. $2\int_1^e \sqrt{\log_e y}dy$

B. $2e - \int_{-1}^{1} e^{x^2} dx$

C. $\int_{-1}^{1} \left(e - e^{x^2}\right) dx$

D. $2\int_0^1 \sqrt{x} e^x dx$

Answer: A::B::C::D

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8. The area bounded by the curves $y = |x| - 1 and y = -|x| + 1$ is 1 sq. units (b) 2 sq. units $2\sqrt{2}$ sq. units (d) 4 sq. units

A. $\alpha = e^2 + 1$

B. $\alpha = e^2 - 2$

C. $\beta = 1 + e^{-1}$

D. $\beta = 1 + e^{-2}$

Answer: A::D

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9. Consider curves $S_1 : \sqrt{|x|} + \sqrt{|y|} = \sqrt{a}$, $S_2 : x^2 + y^2 = a^2$ and S_3: $|x| + |y| = a$. If α i

is the area bounded by S_2 and S_3, then

A. $\alpha = a^2\left(\pi - \frac{2}{3}\right)$

B. $\beta = \frac{4a^2}{3}$

C. $\gamma = 2a^2(\pi - 1)$

D. the ratio in which S_3 divides area between S_1 and S_2 is $4 : 3(\pi - 2)$

Answer: A::B::D

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10. Let A(k) be the area bounded by the curves $y = x^2 + 2x - 3$ and $y = kx + 1$. Then

A. the value of k for which A(k) is least is 2

B. the value of k for which A(k) is least is $3/2$

C. least value of A(k) is $32/3$

D. least value of A(k) is $64/3$

Answer: A::C

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11. The area of the region bounded by the curve $y = e^x$ and lines $x = 0 and y = e$ is $e - 1$ (b) $\int_1^e 1n(e + 1 - y)dy$ $e - \int_0^1 e^x dx$ (d)

$$\int_1^e 1nydy$$

A. $e-1$

B. $\int_1^e \text{In} \ (e+1-y)dy$

C. $e-\int_0^1 e^x dx$

D. $\int_1^e \text{In y} \ dy$

Answer: B::C::D

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12. The area of the region $R=\{(x,y):|x|\le|y| \text{ and } x^2+y^2\le 1\}$ is

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13. Let $f:R\to R$ be a differentiable function such that $f(x)=x^2+\int_0^x e^{-t}f(x-t)dt.$

$y = f(x)$ is

A. The curve y=f(x) passes through the point (1,2)

B. The curve y=f(x) passes through the point (2,-1)

C. The area of the region $\left\{(x, y) \in [0, 1] \times R : f(x) \leq y \leq \sqrt{1 - x^2}\right\}$ is $\frac{\pi - 2}{4}$

D. The area of the region $\left\{(x, y) \in [0, 1] \times R : f(x) \leq y \leq \sqrt{1 - x^2}\right\}$ is $\frac{\pi - 1}{4}$

Answer: B::C

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Linkded Comprehension Type

1. Let A_r be the area of the region bounded between the curves $y^2 = (e^{-kr})x (\text{where } k > 0, r \in N)$ and the line $y = mx (\text{where } m \neq 0$

, k and m are some constants

$A_1, A_2, A_3, \ldots$ are in G.P. with common ratio

A. e^{-k}

B. e^{-2k}

C. e^{-4k}

D. None of these

Answer: B

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2. Let A_r be the area of the region bounded between the curves $y^2 = (e^{-kr})x(\text{where } k > 0, r \in N)$ and the line $y = mx(\text{where } m \neq 0$

, k and m are some constants

$\lim_{n \to \infty} \Sigma_{i=1}^{n} A_i = \frac{1}{48(e^{2k} - 1)}$ then the value of m is

A. 3

B. 1

C. 2

D. 4

Answer: C

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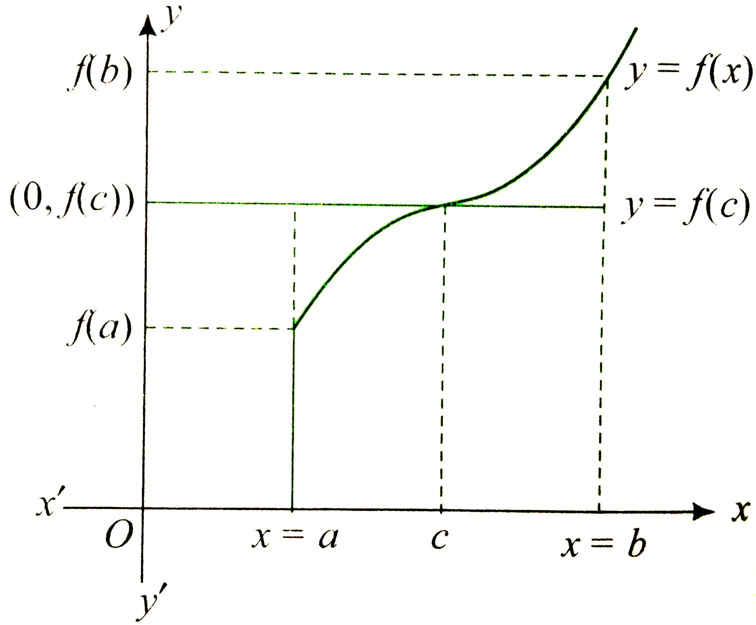
3. If y=f(x) is a monotonic function in (a,b), then the area bounded by the ordinates at $x=a, x=b, y=f(x)$ and $y=f(c)$(where $c \in (a,b)$) is minimum whe

.

$$\text{Proof}: \quad A=\int_a^c (f(c)-f(x))dx+\int_c^b (f(c))dx$$

$$=f(c)(c-a)-\int_a^c (f(x))dx+\int_a^b (f(x))dx-f(c)(b-c)$$

$$\Rightarrow \qquad A=[2c-(a+b)]f(c)+\int_c^b (f(x))dx-\int_a^c (f(x))dx$$



Differentiating w.r.t. c, we get

$$\frac{dA}{dc} = [2c - (a+b)]f'(c) + 2f(c) + 0 - f(c) - (f(c) - 0)$$

For maxima and minima , $\frac{dA}{dc} = 0$

$$\Rightarrow \quad f'(c)[2c - (a+b)] = 0 (as f'(c) \neq 0)$$

Hence, $c = \frac{a+b}{2}$

Also for $c < \frac{a+b}{2}, \frac{dA}{dc} < 0$ and for $c > \frac{a+b}{2}, \frac{dA}{dc} > 0$

Hence, A is minimum when $c = \frac{a+b}{2}$.

If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ and the straight lines x=0, x=2, and the x-axis is minimum, then the value of a is

A. $1/2$

B. 2

C. 1

D. $2/3$

Answer: D

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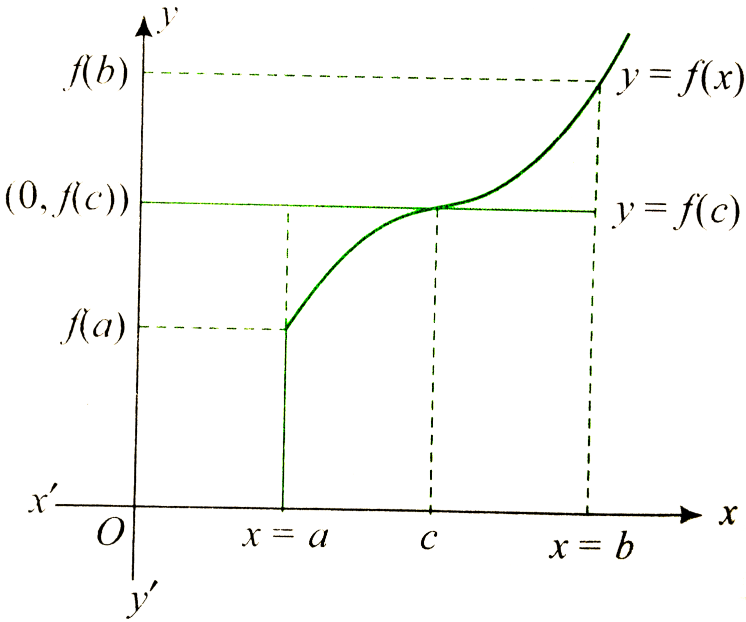
4. If y=f(x) is a monotonic function in (a,b), then the area bounded by the ordinates at $x=a, x=b, y=f(x)$ and $y=f(c)(\text{where } c\in(a,b)) is \min i\mu m when$ $c=\frac{a+b}{2}$.

$$\text{Proof}: \quad A=\int_a^c (f(c)-f(x))dx+\int_c^b (f(c))dx$$

$$=f(c)(c-a)-\int_a^c (f(x))dx+\int_a^b (f(x))dx-f(c)(b-c)$$

$$\Rightarrow \qquad A=[2c-(a+b)]f(c)+\int_c^b (f(x))dx-\int_a^c (f(x))dx$$



Differentiating w.r.t. c, we get

$$\frac{dA}{dc} = [2c - (a+b)]f'(c) + 2f(c) + 0 - f(c) - (f(c) - 0)$$

For maxima and minima , $\frac{dA}{dc} = 0$

$$\Rightarrow \quad f'(c)[2c - (a+b)] = 0(as f'(c) \neq 0)$$

Hence, $c = \frac{a+b}{2}$

Also for $c < \frac{a+b}{2}, \frac{dA}{dc} < 0$ and for $c > \frac{a+b}{2}, \frac{dA}{dc} > 0$

Hence, A is minimum when $c = \frac{a+b}{2}$.

The value of the parameter a for which the area of the figure bounded by the abscissa axis, the graph of the function $y = x^3 + 3x^2 + x + a$, and the straight lines, which are parallel to the axis of ordinates and cut the abscissa axis at the point of extremum of the function, which is the least, is

A. 2

B. 0

C. -1

D. 1

Answer: C

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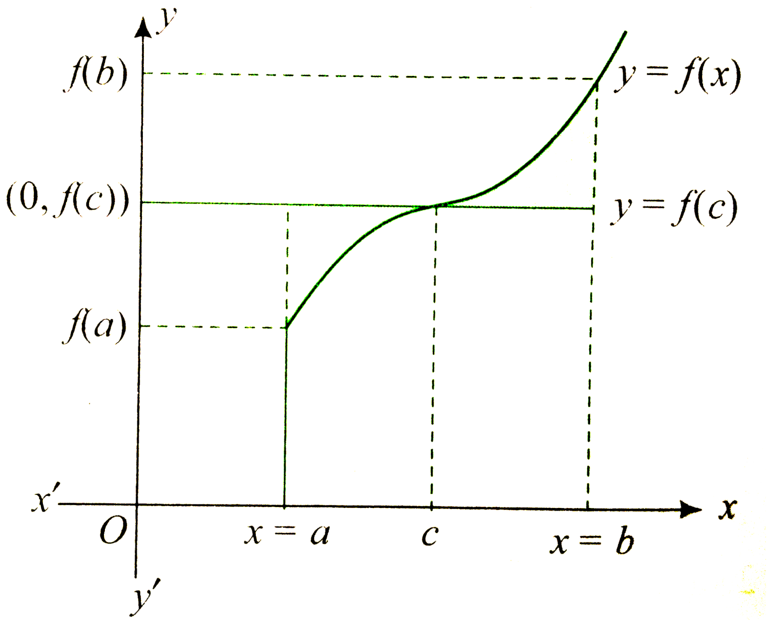
5. If y=f(x) is a monotonic function in (a,b), then the area bounded by the ordinates at $x=a, x=b, y=f(x)$ and $y=f(c)(\text{where } c \in (a,b))$ is minimum whe

.

$$\text{Proof}: \quad A=\int_a^c (f(c)-f(x))dx+\int_c^b (f(c))dx$$

$$=f(c)(c-a)-\int_a^c (f(x))dx+\int_a^b (f(x))dx-f(c)(b-c)$$

$$\Rightarrow \quad A = [2c - (a+b)]f(c) + \int_c^b (f(x))dx - \int_a^c (f(x))dx$$



Differentiating w.r.t. c, we get

$$\frac{dA}{dc} = [2c - (a+b)]f'(c) + 2f(c) + 0 - f(c) - (f(c) - 0)$$

For maxima and minima , $\frac{dA}{dc} = 0$

$$\Rightarrow \quad f'(c)[2c - (a+b)] = 0(as f'(c) \neq 0)$$

Hence, $c = \frac{a+b}{2}$

Also for $c < \frac{a+b}{2}, \frac{dA}{dc} < 0$ and for $c > \frac{a+b}{2}, \frac{dA}{dc} > 0$

Hence, A is minimum when $c = \frac{a+b}{2}$.

If the area enclosed by $f(x) = \sin x + \cos x, y = a$ between two consecutive points of extremum is minimum, then the value of a is

A. 0

B. -1

C. 1

D. 2

Answer: A

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6. If $S_0, S_1, S_2, \ldots$ are areas bounded by the x-axis and half-wave of the curve $y = \sin \pi \sqrt{x}, \quad \text{then prove that} \quad S_0, S_1, S_2, \ldots$ are in A.P...

A. $\frac{1}{2}(1 + e^{\pi})$ sq. units

B. $\frac{1}{2}\left(1 + e^{-\pi}\right)$ sq. units

C. $\frac{1}{2}\left(1 - e^{-\pi}\right)$ sq. units

D. $\frac{1}{2}(e^{\pi} - 1)$ sq. units

Answer: A

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7. Consider the sequence of natural numbers $S_0, S_1, S_2,...$ such that $S_0 = 3, S_1 = 3$ and $S_n = 3 + S_{n-1}S_{n-2}$, then

A. A. S_{1545} is odd and S_{1546} is even.

B. B. S_{1545} is even and S_{1546} is odd.

C. C. both S_{1545} and S_{1546} are even.

D. D. both S_{1545} and S_{1546} are odd.

Answer: C

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8. If $S_0, S_1, S_2, \ldots$ are areas bounded by the x-axis and half-wave of the curve $y = \sin \pi\sqrt{x}$, then prove that $S_0, S_1, S_2, \ldots$ are in A.P...

A. $\dfrac{1+e^{\pi}}{1-e^{-\pi}}$

B. $\dfrac{\frac{1}{2}(1+e^{\pi})}{1-e^{\pi}}$

C. $\dfrac{1}{2(1-e^{-\pi})}$

D. None of these

Answer: B

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9. Two curves $C_1 \equiv [f(y)]^{2/3} + [f(x)]^{1/3} = 0$ and $C_2 \equiv [f(y)]^{2/3} + [f(x)]^{2/3} = 12,$ satisfying the relation $(x-y)f(x+y) - (x+y)f(x-y) = 4xy(x^2-y^2)$

The area bounded by the curve C_1 and C_2 is

A. (a) $2\pi - \sqrt{3}$ sq. units

B. (b) $2\pi + \sqrt{3}$ sq. units

C. (c) $\pi + \sqrt{6}$ sq. units

D. (d) $2\sqrt{3} - \pi$ sq. units

Answer: B

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10. Two curves $C_1 \equiv [f(y)]^{2/3} + [f(x)]^{1/3} = 0$ and $C_2 \equiv [f(y)]^{2/3} + [f(x)]^{2/3} = 12,$ satisfying the relation $(x-y)f(x+y) - (x+y)f(x-y) = 4xy(x^2-y^2)$

The area bounded by the curve C_2 and $|x| + |y| = \sqrt{12}$ is

A. (a) $12\pi - 24$ sq. units

B. (b) $6 - \sqrt{12}$ sq. units

C. (c) $2\sqrt{12} - 6$ sq. units

D. (d) None of these

Answer: A

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11. The area bounded by the curve $y^2(2-x)=x^3$ and $x=2$ is

A. (a) $5/2$ sq. units

B. (b) $7/2$ sq. units

C. (c) $9/2$ sq. units

D. (d) None of these

Answer: C

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12. Consider the two curves $C_1: y=1+\cos x$ and $C_2: y=1+\cos(x-\alpha)$ for $\alpha \in \left(0, \frac{\pi}{2}\right)$, where Also the area of the figure bounded by the curves $C_1, C_2,$ and $x=0$ is same as that of the figure bounded by $C_2, y=1,$ and $x=\pi$.

The value of α is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{8}$

Answer: C

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13. Consider two curves $C_1 : y = \frac{1}{x} and C_2 : y = \log x$ on the xy plane. Let D_1 denotes the region surrounded by $C_1, C_2,$ and the line $x = 1 and D_2$ denotes the region surrounded by C_1, C_2 and the line $x = a.$ If $D_1 = D_2,$ then the sum of logarithm of possible value of a is ____________

A. 1 sq. units

B. 2 sq. units

C. $2 + \sqrt{3}$ sq. units

D. None of these

Answer: B

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14. Consider the function defined implicity by the equation

$y^2 - 2ye^{\sin^{-1} x} + x^2 - 1 + [x] + e^{2 \sin^{-1} x} = 0(\text{where [x] denotes the greates}$

The area of the region bounded by the curve and the line $x = -1$ is

A. $\pi + 1$ sq. units

B. $\pi - 1$ sq. units

C. $\frac{\pi}{2} + 1$ sq. units

D. $\frac{\pi}{2} - 1$ sq. units

Answer: A

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15. Consider the function defined implicity by the equation $y^2 - 2ye^{\sin^{-1}x} + x^2 - 1 + [x] + e^{2\sin^{-1}x} = 0$(where [x] denotes the greates

Line x=0 divides the region mentioned above in two parts. The ratio of area of left-hand side of line to that of right-hand side of line is

A. $1 + \pi : \pi$

B. $2 - \pi : \pi$

C. $1 : 1$

D. $\pi + 2 : \pi$

Answer: D

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16. Consider two functions

$$f(x) = \begin{cases} [x], & -2 \leq x \leq -1 \\ |x| + 1, & -1 < x \leq 2 \end{cases} \text{ and } g(x) = \begin{cases} [x], & -\pi \leq x < 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$$

where [.] denotes the greatest integer function.

The exhaustive domain of $g(f(x))$ is

A. $\frac{\sqrt{3}}{4}+\frac{\pi}{6}$ sq. units

B. $\frac{\sqrt{3}}{2}+\frac{\pi}{6}$ sq. units

C. $\frac{\sqrt{3}}{4}-\frac{\pi}{6}$ sq. units

D. $\frac{\sqrt{3}}{2}-\frac{\pi}{6}$ sq. units

Answer: A

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17. Computing area with parametrically represented boundaries : If the boundary of a figure is represented by parametric equation, i.e., $x=x(t), y=(t),$ then the area of the figure is evaluated by one of the three formulas :

$$S=-\int_{\alpha}^{\beta} y(t)x'(t)dt,$$

$$S=\int_{\alpha}^{\beta} x(t)y'(t)dt,$$

$$S=\frac{1}{2}\int_{\alpha}^{\beta}(xy'-yx')dt,$$

Where α and β are the values of the parameter t corresponding respectively to the beginning and the end of the traversal of the curve corresponding to increasing t.

The area of the region bounded by an are of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ and the x-axis is

A. $6\pi a^2$ sq. units

B. $3\pi a^2$ sq. units

C. $4\pi a^2$ sq. units

D. None of these

Answer: B

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18. Computing area with parametrically represented boundaries : If the boundary of a figure is represented by parametric equation, i.e., $x = x(t), y = (t),$ then the area of the figure is evaluated by one of the three formulas :

$$S = -\int_{\alpha}^{\beta} y(t)x'(t)dt,$$

$$S = \int_{\alpha}^{\beta} x(t)y'(t)dt,$$

$$S = \frac{1}{2}\int_{\alpha}^{\beta} (xy' - yx')dt,$$

Where α and β are the values of the parameter t corresponding respectively to the beginning and the end of the traversal of the curve corresponding to increasing t.

The area of the loop described as

$x = \frac{t}{3}(6 - t), y = \frac{t^2}{8}(6 - t)$ is

A. $\frac{27}{5}$ sq. units

B. $\frac{24}{5}$ sq. units

C. $\frac{27}{6}$ sq. units

D. $\frac{21}{5}$ sq. units

Answer: A

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19. Computing area with parametrically represented boundaries : If the boundary of a figure is represented by parametric equation, i.e., $x = x(t), y = (t),$ then the area of the figure is evaluated by one of the three formulas :

$$S = -\int_{\alpha}^{\beta} y(t)x'(t)dt,$$

$$S = \int_{\alpha}^{\beta} x(t)y'(t)dt,$$

$$S = \frac{1}{2}\int_{\alpha}^{\beta} (xy' - yx')dt,$$

Where α and β are the values of the parameter t corresponding respectively to the beginning and the end of the traversal of the curve corresponding to increasing t.

If the curve given by parametric equation $x = t - t^3, y = 1 - t^4$ forms a loop for all values of $t \in [-1, 1]$ then the area of the loop is

A. $\frac{1}{7}$ sq. units

B. $\frac{3}{5}$ sq. units

C. $\frac{16}{35}$ sq. units

D. $\frac{8}{35}$ sq. units

Answer: C

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20. Let f(x) be a continuous function fiven by

$$f(x) = \begin{cases} 2x, & |x| \le 1 \\ x^2 + ax + b, & |x| > 1 \end{cases}, \text{then}$$

If f(x) is continuous for all real x then the value of $a^2 + b^2$ is

A. 3

B. 4

C. 5

D. 11

Answer: C

Watch Video Solution

21. Let $f(x)$ be continuous function given by $f(x) = \{2x, |x| \leq 1$and $x^2 + ax + b, |x| > 1\}$.

Find the area of the region in the third quadrant bounded by the curves $x = -2y^2 andy = f(x)$ lying on the left of the line $8x + 1 = 0.$

A. sq. units

B. $\frac{257}{192}$ sq. units

C. $\frac{257}{96}$ sq. units

D. $\frac{289}{192}$ sq. units

Answer: B

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Matrix Match Type

1. Match the following lists :

List I	List II
a. The area bounded by the curve $y = x\|x\|$, x-axis and the ordinates $x = 1, x = -1$	**p.** 10/3 sq. units
b. The area of the region lying between the lines $x - y + 2 = 0$, $x = 0$, and the curve $x = \sqrt{y}$	**q.** 64/3 sq. units
c. The area enclosed between the curves $y^2 = x$ and $y = \|x\|$	**r.** 2/3 sq. units
d. The area bounded by parabola $y^2 = x$, straight line $y = 4$, and the y-axis	**s.** 1/6 sq. units

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2. show that the tangents to the curve $y = 2x^3 - 3$ at the point where x=2 and x=-2 are parallel

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3. find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis

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4. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent has the equation $y = x - 11$

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5. find the equation of the tangent to the curve $y = -5x^2 + 6x + 7$ at the point $(1/2, 35/4)$

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Numerical Value Type

1. The area enclosed by the curve $c: y = x\sqrt{9 - x^2}(x \geq 0)$ and the x-axis is_______

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2. Let S be the area bounded by the curve $y = \sin x (0 \leq x \leq \pi)$ and the x-axis and T be the area bounded by the curves $y = \sin x\left(0 \leq x \leq \frac{\pi}{2}\right), y = a \cos x\left(0 \leq x \leq \frac{\pi}{2}\right)$, and the x-axis $\left(wherea \in R^{+}\right)$ The value of $(3a)$ such that $S : T = 1 : \frac{1}{3}$ is______

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3. Let C be a curve passing through $M(2, 2)$ such that the slope of the tangent at anypoint to the curve is reciprocal of the ordinate of the point. If the area bounded by curve C and line x=2 is A ,then the value of $\frac{3A}{2}$ i s__`

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4. The area enclosed by $f(x) = 12 + ax \pm x^{2}$ coordinates axes and the ordinates at $x = 3(f(3) > 0)$ is 45 sq. units. If $mandn$ are the x-axis intercepts of the graph of $y = f(x)$, then the value of $(m + n + a)$ is____

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5. If the area bounded by the curve $f(x) = x^{\frac{1}{3}}(x - 1)$ and the x-axis is A, then the value of $28A$ is____

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6. If the area bounded by the curve $y = x^2 + 1$ and the tangents to it drawn from the origin is A, then the value of $3A$ is__-

Watch Video Solution

7. If the area enclosed by the curve $y = \sqrt{x}$ and $x = -\sqrt{y}$, the circle $x^2 + y^2 = 2$ above the x-axis is A, then the value of $\frac{16}{\pi}A$ is___

Watch Video Solution

8. The value of $a(a > 0)$ for which the area bounded by the curves $y = \frac{x}{6} + \frac{1}{x^2}, y = 0, x = a, andx = 2a$ has the least value is__

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9. Area bounded by the curve $[|x|] + [|y|] = 3,$ where$[.]$ denotes the greatest integer function

Watch Video Solution

10. -q, and d-s, then the correctly bubbled $4x4$ matrix should be as follows: Figure: of the region lying between the lines $x - y + 2 = 0, x = 0$ and the curve $x = \sqrt{y}$, q. 64/3 sq. units The area enclosed between the curves $y^2 = xandy = |x|$, r. 2/3 sq. units The area bounded by parabola $y^2 = x,$ straight line $y = 4, andthey - a\xi s$, s. 1/6 sq. units

Watch Video Solution

Watch Video Solution

11. If the area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$ is A, then the value of $3A - 17$ is___

Watch Video Solution

12. If S is the sum of possible values of c for which the area of the figure bounded by the curves $y = \sin 2x$, the straight lines $x = \frac{\pi}{6}, x = c$, and the abscissa axis is equal to $\frac{1}{2}$, then the value of π / S is___

Watch Video Solution

13. If A is the area bounded by the curves $y = \sqrt{1 - x^2}$ and $y = x^3 - x$, then value of $\frac{\pi}{A}$.

Watch Video Solution

14. Consider two curves $C_1 : y = \frac{1}{x} and C_2 : y = \log x$ on the xy plane. Let D_1 denotes the region surrounded by $C_1, C_2,$ and the line $x = 1 and D_2$ denotes the region surrounded by C_1, C_2 and the line $x = a \cdot$ If $D_1 = D_2,$ then the sum of logarithm of possible value of a is ___________

Watch Video Solution

15. If $'a'(a > 0)$ is the value of parameter for each of which the area of the figure bounded by the straight line $y = \frac{a^2 - ax}{1 + a^4}$ and the parabola $y = \frac{x^2 + 2ax + 3a^2}{1 + a^4}$ is the greatest, then the value of a^4 is___

View Text Solution

16. If S is the sum of cubes of possible value of c for which the area of the figure bounded by the curve $y = 8x^2 - x^5,$ then straight lines $x = 1 and x = c$ and the abscissa axis is equal to $\frac{16}{3},$ then the value of $[S], where [.]$ denotest the greatest integer function, is____

Watch Video Solution

17. For a point P in the plane, let $d_1(P) and d_2(P)$ be the distances of the point P from the lines $x - y = 0 and x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4,$ is

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18. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0, 0), \ Q(1, 1)$ and $R(2, 0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n \ (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of PQR , then the value of n is ______.

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Archives

1. The area of the figure bounded by the parabola $(y-2)^2 = x-1$, the tangent to it at the point with the ordinate $x=3$, and the $x-a\xi s$ is $7squnites$ (b) $6squnites$ $9squnites$ (d) None of these

A. 3

B. 6

C. 9

D. 12

Answer: C

Watch Video Solution

2. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x=0$ and $x = \frac{3\pi}{2}$ is

A. $4\sqrt{2}+1$

B. $4\sqrt{2}-1$

C. $4\sqrt{2}+2$

D. $4\sqrt{2}-2$

Answer: D

Watch Video Solution

3. The area of the region enclosed by the curve $y=x, x=e, y=\frac{1}{x}$ and the positive X-axis is

A. $\frac{5}{2}$ square units

B. $\frac{1}{2}$ square units

C. 1 square units

D. $\frac{3}{2}$ square units

Answer: D

Watch Video Solution

4. The area bounded by the parabolas $y = 4x^2, y = \frac{x^2}{9}$ and the straight line y = 2 is

A. $20\sqrt{2}$

B. $\frac{10\sqrt{2}}{3}$

C. $\frac{20\sqrt{2}}{3}$

D. $10\sqrt{2}$

Answer: C

Watch Video Solution

5. The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x,$ and x-axis in the 1st quadrant is 18 sq. units (b) $\frac{27}{4} s qu n \dot{i} t s$ $\frac{4}{3} s qu n \dot{i} t s$ (d) 9 sq. units

A. 9

B. 36

C. 18

D. $\frac{27}{4}$

Answer: A

Watch Video Solution

6. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is

A. $\frac{\pi}{2} + \frac{4}{3}$

B. $\frac{\pi}{2} - \frac{4}{3}$

C. $\frac{\pi}{2} - \frac{2}{3}$

D. $\frac{\pi}{2} + \frac{2}{3}$

Answer: D

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7. The area (in sq. units) of the region described by $\{x, y) : y^2 \leq 2x$ and $y \geq 4x - 1\}$ is

A. A. $\frac{7}{32}$

B. B. $\frac{5}{64}$

C. C. $\frac{15}{64}$

D. D. $\frac{9}{32}$

Answer: D

Watch Video Solution

8. The area (in sq units) of the region $\{(x, y) : y^2 \geq 2x$ and $x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is

A. $\pi - \frac{8}{3}$

B. $\pi - \frac{4\sqrt{2}}{3}$

C. $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

D. $\pi - \frac{4}{3}$

Answer: A

Watch Video Solution

9. The area (in sq units) of the region bounded by the curve $y = \sqrt{x}$ and the lines $y = 0, y = x - 2,$ is

A. $\frac{5}{2}$

B. $\frac{59}{12}$

C. $\frac{3}{2}$

D. $\frac{7}{3}$

Answer: A

Watch Video Solution

10. Let $g(x) = \cos^2 x, f(x) = \sqrt{x}$ and α, β $(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 - 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $a = \beta$ and $y = 0$ is

A. $\frac{1}{2}(\sqrt{2} - 1)$

B. $\frac{1}{2}(\sqrt{3} - 1)$

C. $\frac{1}{2}(\sqrt{3} + 1)$

D. $\frac{1}{2}(\sqrt{3} - \sqrt{2})$

Answer: B

Watch Video Solution

Single Correct Answer Type

1. Let the straight line x= b divide the area enclosed by $y = (1 - x)^2, y = 0$, and $x = 0$ into two parts

$R_1(0 \leq x \leq b)$ and $R_2(b \leq x \leq 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

A. $3/4$

B. $1/2$

C. $1/3$

D. $1/4$

Answer: B

Watch Video Solution

2. The area enclosed by the curve $y = \sin x + \cos x andy = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is $(a)4(\sqrt{2} - 2)$ (b) $2\sqrt{2}$ ($\sqrt{2}$ -1) $(c)2(\sqrt{2}$ +1) (d) $2\sqrt{2}(\sqrt{2} + 1)$

A. $4(\sqrt{2} - 1)$

B. $2\sqrt{2}(\sqrt{2} - 1)$

C. $2(\sqrt{2} + 1)$

D. $2\sqrt{2}(\sqrt{2}+1)$

Answer: B

Watch Video Solution

3. Area of the region $\left\{(x, y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$ is equal to

A. $\frac{1}{6}$

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. $\frac{5}{3}$

Answer: C

Watch Video Solution

4. The area enclosed between the curve $y = \sin^2 x$ and $y = \cos^2 x$ in the interval $0 \leq x \leq \pi$ is _____ sq. units.

A. 2 sq unit

B. $\frac{1}{2}$ sq unit

C. 1 sq unit

D. None of these

Answer: C

Watch Video Solution

5. The area of the region enclosed by $y = x^2$ and $y = \sqrt{|x|}$ is

A. $1/3$

B. $2/3$

C. $1/6$

D. 1

Answer: B

Watch Video Solution

6. The area of the region bonded by $y = e^x, y = e^{-x}, x = 0$ and x = 1 is

(a) $e + \frac{1}{e}$ (b) $\log\left(\frac{4}{e}\right)$ (c) $4\log\left(\frac{4}{e}\right)$ (d) $e + \frac{1}{e} - 2$

A. $e + \frac{1}{e}$

B. $\log(4/e)$

C. $4\log(4/e)$

D. $e + \frac{1}{e} - 2$

Answer: D

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7. The area bounded by the curve $y = \left|\cos^{-1}(\sin x)\right| - \left|\sin^{-1}(\cos x)\right|$ and axis from $\frac{3\pi}{2} \leq x \leq 2\pi$

A. π^2 sq. units

B. $\pi^2/4$ sq. units

C. $\pi^2/2$ sq. units

D. none of these

Answer: B

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8. If $(a, 0)$, agt 0, is the point where the curve $y = \sin 2x - \sqrt{3}\sin x$ cuts the x-axis first, A is the area bounded by this part of the curve, the origin and the positive x-axis. Then

A. $4A + 8\cos a = 7$

B. $4 + 8\sin a = 7$

C. $4A - 8\sin a = 7$

D. $4A - 8\cos a = 7$

Answer: A

Watch Video Solution

9. The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$ is

(a) $\frac{(\pi^3 - 8)}{4}$ (b) $\frac{\pi^3}{4}$ (c) $\frac{(\pi^3 - 16)}{4}$ (d) $\frac{(\pi^3 - 8)}{2}$

A. $\frac{(\pi^3 - 8)}{4}$

B. $\frac{\pi^3}{4}$

C. $\frac{(\pi^3 - 16)}{4}$

D. $\frac{(\pi^3 - 8)}{2}$

Answer: A

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10. The area bounded by the curves $y = \cos^{-1} x, y = \sin^{-1} x$ and $y = -\pi x^3$, where $-1 \le x \le 1$,is

A. $\frac{3\pi}{2}+1-\sqrt{2}$ sq. units

B. $\frac{3\pi}{4}+1+\sqrt{2}$ sq. units

C. $\frac{3\pi}{4}+2-\sqrt{2}$ sq. units

D. $\frac{3\pi}{4}+1-\sqrt{2}$ sq. units

Answer: D

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11. The area bounded by the curve $y=\sin^2 x-2\sin x$ and the x-axis, where $x\in[0, 2\pi]$, is

A. 4 sq. units

B. 8 sq. units

C. 16 sq. units

D. 20 sq. units

Answer: B

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12. Consider the functions f(x) and g(x), both defined from $R \rightarrow R$ and are defined as $f(x) = 2x - x^2$ and $g(x) = x^n$ where $n \in N$. If the area between f(x) and g(x) is 1/2, then the value of n is

A. 5

B. 6

C. 7

D. 8

Answer: A

Watch Video Solution

13. Let a function $f(x)$ be defined in $[-2, 2]$ as

$$f(x) = \begin{cases} \{x\}, & -2 \leq x < -1 \\ |\text{sgn } x|, & -1 \leq x \leq 1 \\ \{-x\}, & 1 < x \leq 2 \end{cases}$$ where $\{x\}$ and sgn x denote

fractional part and signum functions, respectively. Then find the area bounded by the graph of $f(x)$ an the x-axis.

A. 2 sq. units

B. 3 sq. units

C. 4 sq. units

D. 5 sq. units

Answer: B

Watch Video Solution

14. The area bounded by $y = x^2 + 2$ and $y = 2|x| - \cos \pi x$ is equal to

A. $2/3$

B. $8/3$

C. $4/3$

D. $1/3$

Answer: B

Watch Video Solution

15. Area bounded by $f(x) = \frac{x^2 - 1}{x^2 + 1}$ and the line y = 1 is

A. π sq. units

B. 2π sq. units

C. $\pi / 2$ sq. units

D. none of these

Answer: B

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16. The area bounded by the curve $y = xe^{-x}; xy = 0$and $x = c$ where c is the x-coordinate of the curve's inflection point, is

A. $1 - 3e^{-2}$

B. $1 - 2e^{-2}$

C. $1 - e^{-2}$

D. none of these

Answer: A

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17. Area of region bounded by the curve $y = \frac{16 - x^2}{4}$ and $y = \sec^{-1}\left[-\sin^2 x\right]$ (where [x] denotes the greatest ingeger function) is

A. $\frac{1}{3}(4 - \pi)^{3/2}$

B. $8(4 - \pi)^{3/2}$

C. $\frac{8}{3}(4 - \pi)^{3/2}$

D. $\frac{8}{3}(4 - \pi)^{1/2}$

Answer: C

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18. Suppose $y = f(x)$ and $y = g(x)$ are two continuous functiond whose graphs intersect at the three points $(0, 4), (2, 2)$ and $(4, 0)$ with $f(x) > g(x)$ for $0 < x < 2$ and $f(x) < g(x)$ for $2 < x < 4$. If $\int_0^4 [f(x) - g(x)]dx = 10$ and $\int_2^4 [g(x) - f(x)]dx = 5$ the area between two curves for $0 < x < 2$,is (A) 5 (B) 10 (C) 15 (D) 20

A. 5

B. 10

C. 15

D. 20

Answer: C

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19. The ratio of the areas of two regions of the curve $C_1 \equiv 4x^2 + \pi^2 y^2 = 4\pi^2$ divided by the curve $C_2 \equiv y = -\left(sgn\left(x - \frac{\pi}{2}\right)\right)\cos x$ (where sgn (x) = signum (x)) is

A. $\dfrac{\pi^2 - 2}{\pi^2 - 2\sqrt{2}}$

B. $\dfrac{\pi^2 + 2}{\pi^2 - 2\sqrt{2}}$

C. $\dfrac{\pi^2 + 6}{\pi^2 + 3\sqrt{2}}$

D. $\dfrac{\pi^2 - 1}{\pi^2 - \sqrt{2}}$

Answer: A

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20. The area bounded by the curves

$x\sqrt{3} + y = 2\log_e\left(x - y\sqrt{3}\right) - 2\log_e 2,\ y = \sqrt{3}x,$

$y = -\dfrac{1}{\sqrt{3}}x + 2,$ is

A. $2\log_e 2$ sq. units

B. $2\log_e 2 + 1$ sq. units

C. $2\log_e 2 - 1$ sq. units

D. $4\log_e 2 - 1$ sq. units

Answer: C

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21. Area of region bounded by the curve $y = \frac{4-x^2}{4+x^2}$, $25y^2 = 9x$ and $y = \frac{3}{5}|x| - \frac{6}{5}$ which contains (1, 0) point in its interior is

A. $\left\{\pi - 4\tan^{-1}.\frac{1}{2} + \frac{1}{10}\right\}$ sq. units

B. $\left\{\pi - 2\tan^{-1}.\frac{1}{2} - \frac{1}{5}\right\}$ sq. units

C. $\left\{\pi + 4\tan^{-1}.\frac{1}{2} - \frac{1}{5}\right\}$ sq. units

D. none of these

Answer: A

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22. Area bounded by the min. $\{|x|, |y|\} = 1$ and the max. $\{|x|, |y|\} = 2$ is

A. 4

B. 8

C. 16

D. 9

Answer: A

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23. Consider $f(x) = \begin{cases} \cos x & 0 \leq x < \frac{\pi}{2} \\ \left(\frac{\pi}{2} - x\right)^2 & \frac{\pi}{2} \leq x < \pi \end{cases}$ such that f is periodic with period π. Then which of the following is not true?

A. The range of f is $\left[0, \frac{\pi^2}{4}\right)$.

B. f is discontinuous for infinite values of x.

C. The area bounded by y = f(x) and the X-axis from x = 0 to x = $n\pi$ is $n\left(1+\frac{\pi^3}{24}\right)$ for a given $n \in N$.

D. none of these

Answer: D

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24. The area made by curve $f(x)=[x]+\sqrt{x-[x]}$ and x-axis when $0 \leq x \leq n(n \in N)$ is equal to { where [x] is greatest integer function}

A. $\frac{2n}{3}+\frac{n(n+1)}{2}$

B. $\frac{n}{3}+\frac{n(n+1)}{2}$

C. $\frac{2n}{3}+\frac{n(n-1)}{2}$

D. $\frac{n}{3}+\frac{n(n-1)}{2}$

Answer: C

Watch Video Solution

25. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \leq 100\}$ and $B = | x \quad y | \sin(x + y) > 0\}$ in the plane. Then the area of the region $A \cap B$ is

A. 10π

B. 100

C. 100π

D. 50π

Answer: D

Watch Video Solution

26. Let R be the region containing the point (x, y) on the X-Y plane, satisfying $2 \leq |x + 3y| + |x - y| \leq 4$. Then the area of this region is

A. 5 sq. units

B. 6 sq. units

C. 7 sq. units

D. 8 sq. units

Answer: B

Watch Video Solution

27. If $f(x)=\begin{cases}\sqrt{\{x\}} & \text{for } x \notin Z \\ 1 & \text{for } x \in Z\end{cases}$ and $g(x)=\{x\}^2$ where {.} denotes fractional part of x then area bounded by f(x) and g(x) for $x \in 0, 6$ is

A. $\frac{2}{3}$

B. 2

C. $\frac{10}{3}$

D. 6

Answer: B

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28. Let S is the region of points which satisfies $y^2 < 16x$, $x < 4$ and $\frac{xy(x^2-3x+2)}{x^2-7x+12} > 0$. Its area is

A. $\frac{8}{3}$

B. $\frac{64}{3}$

C. $\frac{32}{3}$

D. none of these

Answer: B

Watch Video Solution

29. The area of the region $\{(x, y) : x^2 + y^2 \leq 5, ||x| - |y|| \geq 1$ is

A. $4\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 4$

B. $5\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 4$

C. $3\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 4$

D. $2\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 1$

Answer: B

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30. The folloiwng figure shows the graph of a continuous function y = f(x) on the interval [1, 3]. The points A, B, C have coordinates (1,1), (3,2),(2,3), respectively , and the lines L_1 and L_2 are parrallel, with L_1 being tangent to the curve at C. If the area under the graph of y = f(x) from x = 1 to x = 3 is 4 square units, then the area of the shaded region is

A. 1

B. 2

C. 3

D. 4

Answer: B

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Multiple Correct Answer Type

1. If the area of bounded between the x-axis and the graph of $y = 6x - 3x^2$ between the ordinates $x = 1 and x = a$ is 19 units, then a can take the value 4 or -2 two value are in (2,3) and one in $(-1, 0)$ two value are in (3,4) and one in $(-2, -1)$ none of these

A. one value in (2, 3)

B. one value in $(-2, -1)$

C. one value in $(-1, 0)$

D. one value in $(3, 4)$

Answer: B::D

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2. Which of the following is the possible value/values of c for which the area of the figure bounded by the curves $y = \sin 2x$, the straight lines $x = \pi/6$, $x = c$ and the abscissa axis is equal to 1/2?

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. none of these

Answer: B

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3. Area of the region bounded by the curve $y = \tan x$ and lines y = 0 and x = 1 is

A. $\int_0^1$ and $(1 - x)dx$

B. $\tan 1 - \int_0^{\tan 1} \tan^{-1} y dy$

C. $\int_0^{\tan 1} \tan^{-1} y dy$

D. $\int_0^{1} \tan^{-1} x dx$

Answer: A::B

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Comprehension Type

1. In the following figure, the graphs of two functions y = f(x) and y = sin x are givne. They intersect at origin, $A(a, f(a)), B(\pi, 0)$ and $C(2\pi, 0)$. $A_i(i = 1, 2, 3)$ is the area bounded by the curves as shown in the figure, respectively, for $x \in (0, a), x \in (a, \pi), x \in (\pi, 2\pi)$.

If $A_1 = 1 + (a - 1)\cos a -$ sin a, then

The function f(x) is

A. $x^2 \sin x$

B. $x \sin x$

C. $2x \sin x$

D. $x^3 \sin x$

Answer: B

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2. In the following figure, the graphs of two functions y = f(x) and y = sin x are givne. They intersect at origin, $A(a, f(a)), B(\pi, 0)$ and $C(2\pi, 0)$. $A_i (i = 1, 2, 3)$ is the area bounded by the curves as shown in the figure, respectively, for $x \in (0, a), x \in (a, \pi), x \in (\pi, 2\pi)$.

If $A_1 = 1 + (a - 1)\cos a -$ sin a, then

The function f(x) is

A. $(\pi - 1)\text{units}^2$

B. $(\pi/2-1)\text{units}^2$

C. $(\pi-\sin 1-1)\text{units}^2$

D. $\pi/2\text{units}^2$

Answer: C

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