MATHS

BOOKS - CENGAGE PUBLICATION

BINOMIAL THEOREM

Illustration

1. Prove that

$$\sum_{r=0}^{n} \hat{}\ nC_r(-1)^r\left[i + i^{2r} + i^{3r} + i^{4r}\right] = 2^n + 2^{\frac{n}{2}+1}\cos(n\pi/4), where i =$$

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2. Find the term in $\left(3\sqrt{\left(\frac{a}{\sqrt{b}}\right)} + \left(\sqrt{\frac{b}{3\sqrt{a}}}\right)\right)^{21}$ which has the same power of a and b.

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3. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

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4. If a and b are distinct integers,Using Mathematical Induction prove that $a-b$ is a factor of a^n-b^n, whenever n is a positive integer.

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5. The number of terms in the expansion of $(a+b+c)^n$ where $n \in N$ is

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6. Find the number of terms which are free from radical signs in the expansion of $\left(y^{1/5}+x^{1/10}\right)^{55}$.

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7. If $kandn$ are positive integers and $s_k = 1^k + 2^k + 3^k + \quad + n^k,$ then prove that $\sum_{r=1}^{m} \hat{} \ (m+1)C_r s_r = (n+1)^{m+1} - (n+1).$

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8. Prove that $\sum_{r=0}^{n} {}^nC_r \sin rx. \cos(n-r)x = 2^{n-1} \times \sin nx.$

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9. Prove that $2 \leq \left(1 + \frac{1}{n}\right)^n < 3$ for all $n \in N.$

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10. Find the positive integer just greater than $(1 + 0.0001)^{10000}.$

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11. Prove that $\sqrt{10}\left[\left(\sqrt{10}+1\right)^{100}-\left(\sqrt{10}-1\right)^{100}\right]$.

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12. If 9^7-7^9 is divisible by 2^n, then find the greatest value of $n, where n \in N$.

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13. Find the degree of the polynomial

$$\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1+\sqrt{4x+1}}{2}\right)^7\right\}$$

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14. If $T_0, T_1, T_2, , T_n$ represent the terms in the expansion of $(x+a)^n$, then find the value of $(T_0 - T_2 + T_4 -)^2 + (T_1 - T_3 + T_5 -)^2 n \in N.$

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15. If $n = 12m(m \in N)$, prove that

$$.^n C_0 - \frac{.^n C_2}{\left(2+\sqrt{3}\right)^2} + \frac{.^n C_4}{\left(2+\sqrt{3}\right)^4} - \frac{.^n C_6}{\left(2+\sqrt{3}\right)^6} + \ldots\ldots = (-1)^m \left(\frac{2\sqrt{2}}{1+\sqrt{3}}\right)^n.$$

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16. Prove that $\sum_{r=1}^{k} (-3)^{r-1} (3n)^{C} \_ (2r-1) = 0$, where $k = 3n/2$ and n is an even integer.

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17. If the middle term in the expansion of $\left(x^2 + 1/x\right)^n$ is $924\ x^6$, then find the value of n.

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18. Show that the middle term in the expansion of $(x+1)^{2n}$ is $\frac{1.3.5....(2n-1)}{n!}2^n.x^n.$

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19. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is α and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are β and γ then relate α, β and γ.

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20. The coefficient of a^3b^4c in the expansion of $(1 + a - b + c)^9$ is equal to

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21. The co-efficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$ is

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22. Find the coefficient of x^7 in the expansion of $\left(1 + 3x - 2x^3\right)^{10}$.

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23. Find the approximation of $(0.99)^5$ using the first three terms of its expansion.

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24. Using binomial theorem, evaluate : $(101)^5$

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25. Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

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26. Find the last three digits of 17^{256}.

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27. Using binomial theorem prove that $6^n - 5n$ always leaves remainder I when divided by 25.

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28. Using binomial theorem, prove that $2^{3n} - 7n - 1$ is divisible by 49 , where $n \in N$.

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29. Find the remainder and the fractional part when 5^{99} is divided by 13. Also, prove that the integral part of the number $\frac{5^{99}}{13}$ is odd.

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30. Find the remainder when 27^{40} is divided by 12.

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31. Find the remainder when $1690^{2608} + 2608^{1690}$ is divided by 7.

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32. If $\left(2+\sqrt{3}\right)^n = I + f$, where I and n are positive integers and $0 < f < 1$,

show that I is an odd integer and $(1-f)(1+f) = 1$

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33. Statement 1: If p is a prime number $(p \neq 2)$, then $\left[\left(2+\sqrt{5}\right)^p\right] - 2^{p+1}$ is always divisible by $p(where[.]$ denotes the greatest integer function). Statement 2: if n prime, then $\hat{} \ nC_1,^n C_2,^n C_2,^n C_{n-1}$ must be divisible by $n\cdot$

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34. Find the coefficient of x^{18} in $\left(x^2+2+\frac{1}{x^2}\right)^{-5}\left(1+x^2\right)^{40}$.

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35. Find the coefficient of x^4 in the expansion of $\left(x/2 - 3/x^2\right)^{10}$.

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36. Find the coefficient of x^{13} in the expansion of $(1-x)^5 \times \left(1+x+x^2+x^3\right)^4$.

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37. Find the coefficient of x^{25} in expansion of expression $\sum_{r=0}^{50} \hat{} (50)C_r(2x-3)^r(2-x)^{50-r}$.

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38. Find the coefficient of $x^k \in 1+(1+x)+(1+x)^2 + \; + (1+x)^n (0 \leq k \leq n)$.

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39. Find the coefficient of x^{50} in the expansion of $(1+x)^{101} \times (1-x+x^2)^{100}$.

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40. Find the term independent of x in the expansion of $(2^x + 2^{-x} + \log_e e^{x+2}))^{30}$.

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41. Find the coefficient of x^4 in the expansion of $(2-x+3x^2)^6$.

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42. The term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}(x^2)-\frac{1}{3x}\right)^9$

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43. Prove that in the expansion of $(1+x)^{n}(1+y)^{n}(1+z)^{n}$, the sum of the coefficients of the terms of degree $ris^{3n}C_{r}$.

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44. The coefficients of x^{n} in $\left(1+\frac{x}{1!}+\frac{x^{2}}{2!}+\ldots\ldots+\frac{x^{n}}{n!}\right)^{2}$ is

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45. Find the sum $.^{n}C_{1}+2\times .^{n}C_{2}+3\times .^{n}C_{3}+\ldots\ldots+n\times .^{n}C_{n}$.

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46. Find the sum $\sum_{k=0}^{15}.^{30}C_{k}$.

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47. Find value of the series $.^{10}C_1 +^{10}C_2 + ... +^{10}C_9.$

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48. If n is a positive integer, prove that $1 - 2n + \frac{2n(2n-1)}{2!} - \frac{2n(2n-1)(2n-2)}{3!} + ... + (-1)^{n-1}\frac{2n(2n$

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49. Prove that $\sum_{k=0}^{n}(-1)^k . {}^{3n}C_k = (-1)^n . {}^{3n-1}C_n$

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50. The sum of the coefficients of all the integral powers of x in the expansion of $(1+2\sqrt{x})^{40}$ is

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51. Find the sum $\left(\sum\sum\right)_{0 \le i < j \le n} {}^nC_i \cdot {}^nC_j$.

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52. Find the value of $\sum\sum_{0 \le i < j \le n} \left({}^nC_i + {}^nC_j\right)$.

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53. Find the value of $\left(\sum\sum\right)_{0 \le i < j \le n} (i + j)\left({}^nC_i + {}^nC_j\right)$.

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54. Find the following sums :

(i) $.^n C_0 - .^n C_2 + .^n C_4 - .^n C_6 + \ldots..$

(ii) $.^n C_1 - .^n C_3 + .^n C_5 - .^n C_7 + \ldots.$

(iii) $.^n C_0 + .^n C_4 + .^n C_8 + .^n C_{12} + \ldots..$

(iv) $.^{n}C_2 + .^{n}C_6 + .^{n}C_{10} + .^{n}C_{14} + \ldots\ldots$

(v) $.^{n}C_1 + .^{n}C_5 + .^{n}C_9 + .^{n}C_{13} + \ldots.$

(vi) $.^{n}C_3 + .^{n}C_7 + .^{n}C_{11} + .^{n}C_{15} + \ldots..$

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55. Find the sum $\sum_{r=0}^{5} {}^{32}C_{6r}$.

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56. If the sum of the coefficient in the expansion of $\left(\alpha^2x^2 - 2\alpha x + 1\right)^{51}$ vanishes, then find the value of α

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57. If $\left(1 + x - 2x^2\right)^{20} = a_0a_1x = a_2x^2 + a_3x^3 + \quad + a_{40}x^{40}$, then find the value of $a_1 + a_3 + a_5 + \quad + a_{39}$.

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58. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \quad + a_{2n}x_{2n},$ find the value of $a_0 + a_6 + \quad + \,, n \in N.$

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59. Prove that $\sum_{\alpha+\beta+\gamma=10} \frac{10!}{\alpha!\beta!\gamma!} = 3^{10}.$

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60. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:7:42, then find the value of $n.$

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61. In the coefficients of rth, $(r+1)th, and(r+2)th$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then prove that

$m^2 - m(4r+1) + 4r^2 - 2 = 0.$

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62. Prove that $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4) \ldots\ldots\ldots (C_{n-1} + C_n)$ = $\frac{C_0 C_1 C_2 \ldots C_{n-1}(n+1)^n}{n!}$

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63. If a_1, a_2, a_3, a_4 be the coefficient of four consecutive terms in the expansion of $(1+x)^n$, then prove that: $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}.$

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64. Find the sum of $\sum_{r=1}^{n} r \cdot \frac{nC_r}{nC_{r-1}}$

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65. Find the numerically greatest term in the expansion of $(2x+5y)^{34} when x = 3$ and $y = 2$.

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66. The largest term in the expansion of $(3+2x)^{50}$, where x=1/5, is

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67. Find the greatest coefficient in the expansion of $(1+2x/3)^{15}\cdot$.

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68. Given that the 4th term in the expansion of $[2+(3x/8)]^{10}$ has the maximum numerical value. Then find the range of value of $x\cdot$

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69. Prove that

$.^n C_1 + 2 \times .^n C_2 + 3 \times .^n C_3 + \ldots. + n \times .^n C_n = n2^{n-1}.$

Hence, prove that

$.^n C_1 . (.^n C_2)^2 . (.^n C_3)^3 ......(.^n C_n)^n \leq \left(\frac{2^n}{n+1}\right)^{.^{n+1} C_2} \forall n \in N.$

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70. Find the sum $.^n C_1 + 2 \times .^n C_2 + 3 \times .^n C_3 + \ldots\ldots + n \times .^n C_n.$

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71. If $(1 + x + x^2 + \ + x^p)^n = a_0 + a_1 x + a_2 x^2 + \ + a_{np} x^{np},$ then find the value of $a_1 + 2a_2 + 3a_3 + \ \ddot{+}\ np a_{np}.$

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72. Find the sum

$1 \times 2 \times .^n C_1 + 2 \times 3 \times .^n C_2 + \ldots .. + n \times (n+1) \times .^n C_n.$

A. A. $n(n+1)2^{n-1}$

B. B. $n(n+3)2^{n-2}$

C. C. $2^n .^{2n} C_n$

D. D. none of this

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73. If $n > 2,$ then prove that

$C_1(a-1) - C_2 \times (a-2) + \; + (-1)^{n-1} C_n(a-n) = a, where C_r =^n C_r$

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74. Find the sum

$3^nC_0 - 8^nC_1 + 13^nC_2 - 18^nC_3 + + \dots . + (n+1) terms$

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75. If $x + y = 1$, prove that $\sum_{r=0}^{n} r^nC_r x^r y^{n-r} = nx.$

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76. Prove that $.^n C_0 + \frac{.^n C_1}{2} + \frac{.^n C_2}{3} + \dots\dots + \frac{.^n C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}.$

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77. Prove that $\frac{.^n C_1}{2} + \frac{.^n C_3}{4} + \frac{.^n C_5}{6} + \dots . = \frac{2^n - 1}{n+1}.$

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78. Find the sum

$$2.\,{}^{10}C_0 + \frac{2^2}{2}.{}^{10}C_1 + \frac{2^3}{3}.{}^{10}C_2 + \frac{2^4}{4}.{}^{10}C_3 + \dots + \frac{2^{11}}{11}.{}^{10}C_{10}.$$

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79. Prove that

$$\frac{C_1}{1} - \frac{C_2}{2} + \frac{C_3}{3} - \frac{C_4}{4} + \dots + \frac{(-1)^{n-1}}{n}C_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

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80. Prove that $\sum_{r=1}^{n}(-1)^{r-1}\left(1+\frac{1}{2}+\frac{1}{3}+\;+\frac{1}{r}\right)({}^{n}C_r) = \frac{1}{n}$.

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81. Prove that $\frac{3!}{2(n+3)} = \sum_{r=0}^{n}(-1)^r\left(\frac{nC_r}{(r+3)C_3}\right)$

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82. There are two bags can each of which contains n balls. A man has to select an equal number of balls from both the bags. Prove that the number of ways in which a man can choose at least one ball from each bag is $\hat{} \; 2nC_n - 1$.

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83. Find the $\sum_{r=0}^{r} {}^{n_1}C_{(r-i)} {}^{n_2}C_i$.

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84. Prove that $\sum_{r=0}^{2n} r. \left({}^{2n}C_r\right)^2 = 2n. {}^{4n-1}C_{2n-1}$.

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85. Using binomial theorem (without using the formula for $.^n C_r$) , prove that

$$.^n C_4 +^m C_2 -^m C_1 .^n C_2 = .^m C_4 -^{m+n} C_1 .^m C_3 +^{m+n} C_2 .^m C_2 -^{m+n} C_3^m$$

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86. Find the following sum:

$$\sum\sum_{i \neq j} {}^nC_i . {}^nC_j$$

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87. Prove that

$$.^{100} C_0^{100} C_2 +^{100} C_2^{100} C_4 +^{100} C_4^{100} C_6 + +^{100} C_{98}^{100} C_{100} = \frac{1}{2}[.^{200} C_{98} -^{100}$$

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88. Let m $\in$ N and $C_r = {}^nC_r$, for $0 \leq r \leq n$

Statement-1:

$$\frac{1}{m!}C_0 + \frac{n}{(m+1)!}C_1 + \frac{n(n-1)}{(m+2)!}C_2 + \ldots + \frac{n(n-1)(n-2)\ldots.2.1}{(m+n)!}C_n = \frac{(m+n+1)(m+n+2)\ldots(m+2n)}{(m+n)!}$$

Statement-2: For r $\leq$ 0

$${}^mC_r{}^nC_0 + {}^mC_{r-1}{}^nC_1 + {}^mC_{r-2}{}^nC_2 + \ldots + {}^mC_0{}^nC_r = {}^{m+n}C_r.$$

(a) Statement-1 and Statement-2 both are correct and Statement-2 is the correct explanation for the Statement-1.

(b) Statement-1 and Statement-2 both are correct and Statement-2 is not the correct explanation for the Statement-1.

(c) Statement-1 is correct but Statement-2 is wrong.

(d) Statement-2 is correct but Statement-1 is wrong.

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89. If $n \in N$such that is not a multiple of 3 and $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r.\ X^r$, then find the value of $\sum_{r=0}^{n} (-1)^r.\ a_r.\ {}^nC_r$.

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90. Find the sum $\sum_{r=0} .^{n+r}C_r$.

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91. Prove that $\hat{}\ nC_0{}^{2n}C_n -^n C_1^{2n-2}C_n +^n C_2^{2n-4}C_n = 2^n$

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92. Prove that

$\hat{}\ mC_1^nC_m -^m C_2^{2n}C_m +^m C_3^{3n}C_m - \ldots. = (-1)^{m-1}n^m.$

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93. If every pair from among the equations $x^2 + ax + bc = 0.\ x^2 + bx + ca = 0, and x62 + cx + ab = 0$ has a common root, then the sum of the three common roots is $-1/2(a+b+c)$ the sum of the three common roots is $2(a+b+c)$ the product of the three common roots is abc the product of the three common roots is $a^2b^2c^2$

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94. Find the value of x, for which $\frac{1}{\sqrt{5+4x}}$ can be expanded as infinite series.

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95. Prove that $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n.$

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96. Find the third term in the expansion of $(1-2x)^{3/2}$.

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97. Prove that the coefficient of x^r in the expansion of $(1-2x)^{1/2} is (2r)!/\left[2^r(r!)^2\right]$.

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98. Find the sum: $1-\frac{1}{8}+\frac{1}{8}\times\frac{3}{16}-\frac{1\times3\times5}{8\times16\times24}+\ldots$

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99. Find the coefficient of x^n in the expansion of $\left(1-9x+20x^2\right)^{-1}$.

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100. Assuming x to be so small that x^2 and higher power of x can be neglected, prove that

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101. If x is very large as compare to y, then prove that

$$\sqrt{\frac{x}{x+y}}\sqrt{\frac{x}{x-y}} = 1 + \frac{y^2}{2x^2}.$$

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102. Prove that the coefficient of x^n in the expansion of

$$\frac{1}{(1-x)(1-2x)(1-3x)} is \frac{1}{2}\left(3^{n+2} - 2^{n+3} + 1\right).$$

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103. Prove that

$${}^{n}C_0 - {}^{n}C_1 + {}^{n}C_2 - {}^{n}C_3 + \ldots\ldots + (-1)^r \cdot {}^{n}C_r + \ldots\ldots = (-1)^r \times$$

.

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104. Find the value of

$.^{20}C_0 - .^{20}C_1 + .^{20}C_2 - .^{20}C_3 + \ldots\ldots - \ldots.. + .^{20}C_{10}.$ is

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Example

1. If $U_n = \left(\sqrt{3}+1\right)^{2n} + \left(\sqrt{3}-1\right)^{2n}$, then prove that $U_{n+1} = 8U_n - 4U_{n-1}.$

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2. Prove that

$$\frac{{}^nC_0}{x} - \frac{{}^nC_1}{x+1} + \frac{{}^nC_2}{x+2} - \ldots. + (-1)^n \frac{{}^nC_n}{x+n} = \frac{n!}{x(x+1)\ldots(x-n)},$$

where n is any positive integer and x is not a negative integer.

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3. Find the coefficients of x^{50} in the expression $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\ldots+1001x^{1000}$.

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4. Given, $s_n=1+q+q^2+ \ +q^n, S_n=1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^2+ \ +\left(\frac{q+1}{2}\right)^n, q$

prove that $.^{n+1}C_1+^{n+1}C_2 s_1+^{n+1}C_3 s_2+ \ +^{n+1}C_{n+1}s_n=2^n S_n.$

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5. Prove that $.^nC_1-\left(1+\frac{1}{2}\right).^nC_2+\left(1+\frac{1}{2}+\frac{1}{3}\right).^nC_3+\ldots$
$+(-1)^{n-1}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right).^nC_n=\frac{1}{n}$

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6. Prove that $.^n C_0 + .^n C_5 + .^n C_{10} + \ldots..$

$$= \frac{2^n}{5}\left(1 + 2\cos^n{}'\frac{\pi}{5}\cos{}'\frac{n\pi}{5} + 2\cos{}'\frac{\pi}{5}\cos{}'\frac{2n\pi}{5}\right).$$

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7. Find the sum $\left(\sum\sum\right)_{0 \le i < j \le n} {}^nC_i . {}^nC_j.$

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8. If for $n \in N, \sum_{k=0}^{2n} (-1)^k (\hat{} (2n) C_k)^2 = A,$ then find the value of $\sum_{k=0}^{2n} (-1)^k (k - 2n)(\hat{} (2n) C_k)^2.$

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9. Prove that $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + = \frac{2^n - 1}{n+1}.$

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10. Prove that $\sum_{r=0}^{n} {}^{n}C_{r}.\ (n-r)\cos\left(\frac{2r\pi}{n}\right) = -n.2^{n-1}.\ \cos^{n}{}' \frac{\pi}{n}.$

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Concept Application Exercise 8.1

1. The first three terms in the expansion of $(1+ax)^{n}(n \neq 0)$ are $1,\ 6x$ and $16x^{2}$. Then find the value of a and n.

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3. The two successive terms in the expansion of $(1+x)^{24}$ whose coefficients are in the ratio 1:4 are

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4. If the number of terms in the expansion of $(x+y+z)^{n}$ are 36, then find the value of n.

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5. Find the value of $\frac{1}{81^{n}} - \left(\frac{10}{81^{n}}\right)^{2n} C_{1} + \left(\frac{10^{2}}{81^{n}}\right)^{2n} C_{2} - \left(\frac{10^{3}}{81^{n}}\right)^{2n} C_{3} + + \frac{10^{2n}}{81^{n}}$.

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6. prove that $\sum_{r=0}^{n}(-1)^{r} \hat{}\ nC_{r}.\ [\frac{1}{2^{r}} + \frac{3^{r}}{2^{2r}} + \frac{7^{r}}{2^{3r}} + \frac{15^{r}}{2^{4r}} +$ ......up to m terms] = $\frac{2^{mn}-1}{2^{mn}(2^{n}-1)}$

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7. In $\left(33+\frac{1}{33}\right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

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8. If the coefficients of $(r-5)th$ and $(2r-1)th$ terms in the expansion of $(1+x)^{34}$ are equal, find r.

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9. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.

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10. Represent $\cos 6\theta$ in terms of $\cos \theta$.

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11. Find the number of nonzero terms in the expansion of $\left(1+3\sqrt{2}x\right)^9+\left(1-3\sqrt{2}x\right)^9$.

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12. Find the value of $\left(\sqrt{2}+1\right)^4-\left(\sqrt{2}-1\right)^4$.

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13. Find the degree of the polynomial $\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1+\sqrt{4x+1}}{2}\right)^7\right\}$

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14. Let $R = \left(5\sqrt{5} + 11\right)^{2n+1}$ and f=R-[R] where [] is the greatest integer function. Prove that Rf=4^{2n+1}

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15. If the middle term in the binomial expansion of $\left(\frac{1}{x} + x\sin x\right)^{10}$ is equal to $\frac{63}{8}$, find the value of $x\cdot$

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16. Find the middle term in the expansion of $\left(x^2 + \frac{1}{x^2} - 2\right)^n$.

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17. If the number of terms in the expansion $(1 + 2x - 3y + 4z)^n$ is 286, then find the coefficient of term containing xyz.

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Concept Application Exercise 8.2

1. Let n be an odd natural number greater than 1. Then , find the number of zeros at the end of the sum $99^n + 1$.

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2. Using the principle of mathematical induction, prove that $\left(2^{3n} - 1\right)$ is divisible by 7 for all $n \in N\cdot$

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3. Find the last two digits of the number 27^{27}.

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4. If 10^m divides the number $101^{100} - 1$ then, find the greatest value of m.

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5. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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6. Show that $2^{4n+4} - 15n - 16, where$ n $\in N$ is divisible by 225.

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7. Find remainder when 7^{103} is divided by 25.

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8. Find the value of $\{3^{2003}/28\}$, $where\{.\}$ denotes the fractional part.

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9. Statement 1: Remainder when 3456^{2222} is divided by 7 is 4. Statement 2: Remainder when 5^{2222} is divided by 7 is 4.

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10. Show that the integer next above $(\sqrt{3}+1)^{2m}$ contains 2^{m+1} as a factor.

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Concept Application Exercise 8.3

1. If x^4 occurs in the rth term in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, then find the value of $r \cdot$

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2. If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, prove that its coefficient is $\frac{(2n)!}{\left[\frac{1}{3}(4n-p)\right]!\left[\frac{1}{3}(2n+p)\right]!}$.

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3. Find the coefficient of t^8 in the expansion of $\left(1 + 2t^2 - t^3\right)^9$.

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4. The coefficient of x^4 in the expansion of $\left(1 + x + x^2 + x^3\right)^{11}$ is

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5. The coefficient of the term independent of x in the expansion of $\left[\frac{(x+1)}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10}$ is

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6. In the expansion of $(1+3x+2x^2)^6$, the coefficient of x^{11} is a. 144 b. 288 c. 216 d. 576

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7. The coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is $\hat{}\ 12C_6+3$ b. $\hat{}\ 12C_6+1$ c. $\hat{}\ 12C_6$ d. $\hat{}\ 12C_6+2$

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Concept Application Exercise 8.4

1. In the expansion of $(1+x)^{50}$, find the sum of coefficients of odd powers of $x\cdot$

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2. $\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+\ldots\ldots$ is equal to

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3. Find the sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of $x\cdot$

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4. Find the sum $\sum_{j=0}^{n}\left(\hat{}\,(4n+1)C_j+{}^{4n+1}C_{2n-j}\right)$.

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5. The remainder when $\left(\sum_{k=1}^{5} {}^{20}C_{2k-1}\right)^6$ is divided by 11, is :

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6. Prove that

$.^n C_0 + 5 \times .^n C_1 + 9 \times .^n C_2 + \ldots. + (4n+1) \times .^n C_n = (2n+1)2^n.$

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7. Prove that $.^n C_0 +^n C_3 +^n C_6 + \; = \frac{1}{3}\left(2^n + 2\cos\left(\frac{n\pi}{3}\right)\right)$.

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8. Find the value of $.^{4n} C_0 +^{4n} C_4 +^{4n} C_8 + \ldots + {}^{4n}C_{4n}$.

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9. Prove that $\sum_{\substack{r=0 \\ r \le s}}^{s} \sum_{s=1}^{n} {}^{n}C_{s} {}^{s}C_{r} = 3^{n} - 1.$

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10. Find the sum of coefficients in $\left(1 + x - 3x^2\right)^{4163}$.

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11. If the sum of coefficients in the expansion of $(x - 2y + 3z)^n$ is 128, then find the greatest coefficient in the expansion of $(1 + x)^n$.

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12. Find the sum of the coefficients in the expansion of $\left(1 + 2x + 3x^2 + nx^n\right)^2$.

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13. If $\left(1+x-2x^2\right)^6 = 1 + a_1x + a_2x^2 + \ldots\ldots + a_{12}x^{12}$ then

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Concept Application Exercise 8.5

1. In the expansion of $(1+x)^n$, 7th and 8th terms are equal. Find the value of (7/x $+6)^2$.

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2. Find the sum $\sum_{r=1}^{n} r^2 \frac{{}^nC_r}{{}^nC_{r-1}}$.

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3. Show that no three consecutive binomial coefficients can be in G.P.

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4. If the 3rd, 4th. 5th and sixth term in the expansion of $(x+\alpha)^n$ are a,b,c,d respectively, then prove that $\left(\frac{b^2-ac}{c^2-bd}\right)=\frac{5a}{3c}$.

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5. The largest term in the expansion of $(3+2x)^{50}$, where x=1/5, is

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6. If $x=1/3$, find the greatest tem in the expansion of $(1+4x)^8$.

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7. If n is an even positive integer, then find the value of x if the greatest term in the expansion of $1+x$ ^n may have the greatest coefficient also.

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8. If in the expansion of $(2x+5)^{10}$, the numerically greatest term in equal to the middle term, then find the values of x

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Concept Application Exercise 8.6

1. Find the value of $(.^{10}C_{10})+(.^{10}C_0+.^{10}C_1)+(.^{10}C_0+.^{10}C_1+.^{10}C_2)+....+(.^{10}C_0+$

.

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2. Prove that $\frac{1^2}{3}{}^nC_1+\frac{1^2+2^2}{5}{}^nC_2+\frac{1^1+2^2+3^2}{7}{}^nC_3+\ldots+\frac{1^2+2^2++n^2}{2n+1}{}^nC_n$ = $\left(\frac{n(n+3)}{6}\right)\cdot 2^{n-2}$.

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3. If p+q=1, then show that $\sum_{r=0}^{n} r^2 \hat{} nC_r p^r q^{n-r} = npq + n^2p^2$

Watch Video Solution

4. Prove that

$1 -^n C_1 \frac{1+x}{1+nx} +^n C_2 \frac{1+2x}{(1+nx)^2} -^n C_3 \frac{1+3x}{(1+nx)^3} + ... (n+1)terms =$

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5. Prove that $\frac{.^n C_0}{1} + \frac{.^n C_2}{3} + \frac{.^n C_4}{5} + \frac{.^n C_6}{7} + \ldots = \frac{2^n}{n+1}.$

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6. If $(1+x)^{15} = C_0 + C_1x + C_2x^2 + \; + C_{15}x^{15}$, then find the sum of $C_2 + 2C_3 + 3C_4 + \; + 14C_{15}.$

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7. Find the coefficient of x^n in the polynomial $(x+^n C_0)(x+3^n C_1)\times(x+5^n C_2)\left[x+(2n+1)^n C_n\right]$.

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8. Find the value of $.^{20}C_0-\frac{.^{20}C_1}{2}+\frac{.^{20}C_2}{3}-\frac{.^{20}C_3}{4}+\dots$

Watch Video Solution

9. Prove that $:^{10}C_1(x-1)^2-^{10}C_2(x-2)^2+^{10}C_3(x-3)^2\dots.-^{10}C_{10}(x-10)^2=x^2$

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10. Prove that $\frac{1}{n+1} = \frac{.^{n}C_1}{2} - \frac{2(.^{n}C_2)}{3} + \frac{3(.^{n}C_3)}{4} - \ldots + (-1^{n+1})\frac{n\cdot(.^{n}C_n)}{n+1}$

.

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Concept Application Exercise 8.7

1. Prove that $\sum_{r=0}^{n} r(n-r)(.^{n}C_r)^2 = n^2(.^{2n-2}C_n).$

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2. Prove that $(.^{2n}C_0)^2 - (.^{2n}C_1)^2 + (.^{2n}C_2)^2 - .. + (.^{2n}C_{2n})^2$ = $(-1)^n.^{2n}C_n.$

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3. Find the sum of the series $.^{84}C_4 + 6 \times .^{84}C_5 + 15 \times .^{84}C_6 + 20 \times .^{84}C_7 + 15 \times .^{84}C_8$
$+ 6 \times .^{84}C_9 + .^{84}C_{10}.$

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4. Evaluate.$^{n}C_0.^{n}C_2 + .^{n}C_1.^{n}C_3 + .^{n}C_2.^{n}C_4 + ... + .^{n}C_{n-2}.^{n}C_n.$

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5. Prove that $C_0 + 2C_1 + 4C_2 + 8C_3 + \dots . + 2^n C_n = 3^n$.

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6. Find the value of $\sum\sum_{0 \le i < j \le n} (-1)^{i-j+1} (.^{n}C_i \cdot .^{n}C_j).$

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7. Prove that $.^n C_0 .^n C_0 -^{n+1} C_1 .^n C_1 +^{n+2} C_2 .^n C_2 - .. = (-1)^n$.

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8. Prove that $\hat{} \ nC_0{}^{2n}C_n -^n C_1^{2n-2}C_n +^n C_2^{2n-4}C_n = 2^n$

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9. Find the value of $\sum_{p=1}^{n}\left(\sum_{m=p}^{n} .^n C_m .^m C_p\right)$. And hence, find the value of $\lim_{n \to \infty} \frac{1}{3^n}\sum_{p=1}^{n}\left(\sum_{m=p}^{n} .^n C_m .^m C_p\right)$.

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Concept Application Exercise 8.8

1. If the third term in the expansion of $(1+x)^m is -\frac{1}{8}x^2,$ then find the value of m.

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2. Find the cube root of 217 upto two decimal places

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3. Show that $\sqrt{3}$ =1+$\frac{1}{3}$+($\frac{1}{3}$).($\frac{3}{6}$)+($\frac{1}{3}$).($\frac{3}{6}$).($\frac{5}{9}$)+.......

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4. Find the coefficient of x^2 in $\left(\frac{a}{a+x}\right)^{1/2}+\left(\frac{a}{a-x}\right)^{1/2}$

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5. If $|x| < 1,$ then find the coefficient of x^n in the expansion of $\left(1 + x + x^2 + \ldots\ldots\right)^2.$

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6. If $|x| > 1,$ then expand $(1 + x)^{-3}.$

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7. If $|x| < 1,$ then find the coefficient of x^n in the expansion of $\left(1 + 2x + 3x^2 + 4x^3 + \right)^{1/2}.$

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8. If $(r + 1)th$ term is the first negative term in the expansion of $(1 + x)^{7/2},$ then find the value of $r.$

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9. Prove that the coefficient of x^n in the expansion of $\frac{1}{(1-x)(1-2x)(1-3x)}$ is $\frac{1}{2}\left(3^{n+2}-2^{n+3}+1\right)$

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10. Prove that

$.^n C_0 - .^n C_1 + .^n C_2 - .^n C_3 + \ldots\ldots + (-1)^r .^n C_r + \ldots\ldots = (-1)^r \times$

.

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Single Correct Answer

1. If the coefficients of 5th, 6th , and 7th terms in the expansion of $(1+x)^n$ are in A.P., then $n=$ a. 7 only b. 14 only c. 7 or 14 d. none of these

A. 7 only

B. 14 only

C. 7 or 14

D. none of these

Answer: C

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2. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same, if α equals

A. $-\frac{5}{3}$

B. $\frac{10}{3}$

C. $-\frac{3}{10}$

D. $\frac{3}{5}$

Answer: C

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3. If $(1+x)^5 = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5,$ then the value of $(a_0 - a_2 + a_4)^2 + (a_1 - a_3 + a_5)^2$ is equal to 243 b. 32 c. 1 d. 2^{10}

A. 243

B. 32

C. 1

D. 2^{10}

Answer: B

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4. The expression $\left(\sqrt{2x^2+1} + \sqrt{2x^2-1}\right)^6 + \left(\frac{2}{\sqrt{2x^2+1} + \sqrt{2x^2-1}}\right)^6$ is polynomial of degree

A. 6

B. 8

C. 10

D. 12

Answer: A

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5. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600, then x equals

A. 1

B. $\log_e 10$

C. 10

D. x does not exist

Answer: C

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6. If in the expansion of $(a-2b)^n$, the sum of 5^{th} and 6^{th} terms is 0, then the values of $\frac{a}{b}$ is

A. $\frac{n-4}{5}$

B. $\frac{2(n-4)}{5}$

C. $\frac{5}{n-4}$

D. $\frac{5}{2(n-4)}$

Answer: B

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7. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x > 0$ is

A. n

B. n+1

C. n-1

D. 2n

Answer: A

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8. The sum of rational in $\left(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5}\right)^{10}$ is equal to

A. 12632

B. 1260

C. 126

D. 11792

Answer: D

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9. The value of x for which the sixth term in the expansion of $\left[2^{\log}2\sqrt{9^{x-1}+7}+\frac{1}{2^{\frac{1}{5}}(\log)_2\left(3^{(x-1)+1}\right)}\right]^7$ is 84 is a. 4 b. 1 or 2 c. 0 or 1 d. 3

A. 4

B. 1 or 2

C. 0 or 1

D. 3

Answer: B

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10. The number of distinct terms in the expansion of $\left(x+\frac{1}{x}+x^2+\frac{1}{x^2}\right)^{15}$ is/are (with respect to different power of x is a) 255 b. 61 c. 127 d. none of these

A. 255

B. 61

C. 127

D. none of these

Answer: B

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11. Find the sum

$1 \times 2 \times .^n C_1 + 2 \times 3 \times .^n C_2 + \ldots.. + n \times (n+1) \times .^n C_n.$

A. $\dfrac{3^n + (-1)^n}{2}$

B. $\dfrac{3^n - (-1)^n}{2}$

C. $\dfrac{3^n + 1}{2}$

D. $\dfrac{3^n - 1}{2}$

Answer: B

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12. If $(4x^2+1)^n = \sum_{r=0}^{n} a_r (1+x^2)^{n-r} x^{2r}$, then the value of $\sum_{r=0}^{n} a_r$ is

A. 3^n

B. 4^n

C. 5^n

D. 6^n

Answer: B

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13. The fractional part of $= \frac{2^{4n}}{15}$ is

A. $\frac{1}{15}$

B. $\frac{2}{15}$

C. $\frac{4}{15}$

D. none of these

Answer: A

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14. If $p = (8 + 3\sqrt{7})^n andf = p - [p], where[.]$ denotes the greatest integer function, then the value of $p(1 - f)$ is equal to 1 b. 2 c. 2^n d. 2^{2n}

A. 1

B. 2

C. 2^n

D. 2^{2n}

Answer: A

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15. The remainder when the number $3^{256} - 3^{12}$ is divided by 8 is (a) 0 (b) 3 (c) 4 (d) 7

A. 0

B. 3

C. 4

D. 7

Answer: A

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16. The smallest integer larger than $(\sqrt{3}+\sqrt{2})^6$ is

A. 969

B. 970

C. 971

D. 972

Answer: B

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17. The coefficient of x^5 in the expansion of $\left(1+x^2\right)(1+x)^4$ is

A. 12

B. 5

C. 4

D. 56

Answer: C

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18. Coefficient of x^2in the expansion of $\left(x^3+2x^2+x+4\right)^{15}$ is

A. Prime

B. Composite

C. 0

D. Perfect square

Answer: D

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19. If the coefficients of rth and $(r+1)th$ terms in the expansion of $(3+7x)^{29}$ are equal, then r is equals to a. 15 b. 21 c. 14 d. none of these

A. 15

B. 21

C. 14

D. none of these

Answer: B

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20. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n, n \in N$, if the sum of the coefficients of $x^5 and x^{10}$ is zero , then n is a. 25 b. 20 c. 15 d. none of these

A. 25

B. 20

C. 15

D. None of these

Answer: C

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21. If $\left(1 + 2x + x^2\right)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$

A. (a) $(.^n C_r)^2$

B. (b) $.^n C_r . .^n C_{r+1}$

C. (c) $.^{2n} C_r$

D. (d) $.^{2n}C_{r+1}$

Answer: C

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22. If the term independent of x in the $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then k equals $2, -2$ b. $3, -3$ c. $4, -4$ d. $1, -1$

A. $2, -2$

B. $3, -3$

C. $4, -4$

D. $1, -1$

Answer: B

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23. The coefficient of x^{53} in the expansion of $\sum_{m=0}^{100} \hat{} \, 100C_m(x-3)^{100-m}2^m$ is equal to

A. $.^{100}C_{47}$

B. $.^{100}C_{53}$

C. $-.^{100}C_{53}$

D. $-.^{53}C_{100}$

Answer: C

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24. If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ then

A. $a+b=1$

B. $a-b=1$

C. $ab = 1$

D. $\frac{a}{b} = 1$

Answer: C

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25. The coefficient of x^3 in the expansion of $\left(1 - x + x^2\right)^5$ is

A. -83

B. 0

C. $.^{30} C_{10}$

D. none of these

Answer: D

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26. The term independent of a in the expansion of $\left(1+\sqrt{a}+\frac{1}{\sqrt{a}-1}\right)^{-30}$ is $\hat{}\ 30C_{20}$ b. 0 c. $\hat{}\ 30C_{10}$ d. none of these

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27. The coefficient of x^{10} in the expansion of $\left(1+x^2-x^3\right)^8$ is 476 b. 496 c. 506 d. 528

A. 476

B. 496

C. 506

D. 528

Answer: A

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28. The coefficient of x^n in $(1+x)^{101}\left(1-x+x^2\right)^{100}$ is nonzero, then n cannot be of the form

A. $3r+1$

B. $3r$

C. $3r+2$

D. none of these

Answer: C

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29. The coefficient of x^{28} in the expansion of $\left(1+x^3-x^6\right)^{30}$ is 1 b. 0 c. $\hat{}\ 30C_6$ d. $\hat{}\ 30C_3$

A. 1

B. 0

C. $.^{30}C_6$

D. $.^{30}C_3$

Answer: B

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30. The coefficient of $a^8b^4c^9d^9$ in $(abc+abd+acd+bcd)^{10}$ is

A. $10!$

B. $\frac{10!}{8!4!9!9!}$

C. 2520

D. none of these

Answer: C

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31. In the expansion of $\left(1+x+\frac{7}{x}\right)^{11}$ find the term not containing x.

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32. The coefficient of x^7 in the expansion of $\left(1-x-x^3+x^4\right)^8$ is equal to

A. (a) -648

B. (b) 792

C. (c) -792

D. (d) 648

Answer: C

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33. Sum of the coefficients of terms of degree 13 in the expansion of $(1+x)^{11}\left(1+y^2-z\right)^{10}$ is

A. A. $.^{10}C_3$

B. B. . $^{10}C_4$

C. C. . $^{11}C_3$

D. D. . $^{11}C_4$

Answer: B

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34. The coefficient of x^2y^3 in the expansion of $(1 - x + y)^{20}$ is (a) $\frac{20!}{213!}$ b. $-\frac{20!}{213!}$ c. $\frac{20!}{5!2!3!}$ d. none of these

A. $\frac{20!}{2!3!}$

B. $-\frac{20!}{2!3!}$

C. $\frac{20!}{5!2!3!}$

D. $\frac{20!}{15!2!3!}$

Answer: D

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35. If coefficient of $a^2b^3c^4 \in (a+b+c)^m (wherem \in N) isL(L \neq 0)$, then in same expansion coefficient of $a^4b^4c^1$ will be L b. $\frac{L}{3}$ c. $\frac{mL}{4}$ d. $\frac{L}{2}$

A. L

B. $\frac{L}{3}$

C. $\frac{mL}{4}$

D. $\frac{L}{2}$

Answer: D

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36. The coefficient of $x^r [0 \leq r \leq (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots.$ $+ (x+2)^{n-1}$ is $a.^n C_r(3^r - 2^n)$ $b.^n C_r\left(3^{n-r} - 2^{n-r}\right)$ $c.^n C_r\left(3^r + 2^{n-r}\right)$

d. none of these

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37. If $\left(1+2x+3x^2\right)^{10} = a_0 + a_1x + a_2x^2 + \ldots. + a_{20}x^{20}, then a_1$ equals a.10 b. 20 c. 210 d. none of these

A. 10

B. 20

C. 210

D. none of these

Answer: B

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38. If $f(x) = 1 - x + x^2 - x^3 + .. - x^{15} + x^{16} - x^{17}$, then the coefficient of $x^2 \in f(x-1)$ is a. 826 b. 816

c. 822 d. none of these

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39. Let $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ and $\dfrac{f(x)}{1-x} = b_0 + b_1x + b_2x^2 + \dots + b_nx^n$, then a. $b_n + b_{n-1} = a_n$ b. $b_n - b_{n-1} = a_n$ c. $\dfrac{b_n}{b_{n-1}} = a_n$ d. none of these

A. $b_n + b_{n-1} = a_n$

B. $b_n - b_{n-1} = a_n$

C. $b_n / b_{n-1} = a_n$

D. none of these

Answer: B

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40. Statement 1: The coefficient of x^n in $\left(1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + + \dfrac{x^n}{n!}\right)^3$ is $\dfrac{3^n}{n!}$. Statement 2: The coefficient of x^n in e^{3x} is $\dfrac{3^n}{n!}$

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41. In the expansion of $\left(3^{-x/4}+3^{5x/4}\right)^{n}$ the sum of binomial coefficient is 64 and term with the greatest binomial coefficient exceeds the third by $(n-1)$, the value of x must be 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: A

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42. The sum of the coefficients of even power of x in the expansion of $\left(1+x+x^{2}+x^{3}\right)^{5} is\ 256$ b. 128 c. 512 d. 64

A. 256

B. 128

C. 512

D. 64

Answer: C

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43. Maximum sum of coefficient in the expansion of $(1 - x\sin\theta + x^2)^n$ is

A. 1

B. 2^n

C. 3^n

D. 0

Answer: C

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44. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is a. 924 b. 792 c. 1594 d. none of these

A. 924

B. 792

C. 1594

D. none of these

Answer: A

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45. The value of $.^{20}C_0 + .^{20}C_1 + .^{20}C_2 + .^{20}C_3 + .^{20}C_4 + .^{20}C_{12} + .^{20}C_{13} + .^{20}C_{14} + .^{20}$ is

A. A. $2^{19} - \frac{(.^{20}C_{10} + .^{20}C_9)}{2}$

B. B. $2^{19} - \frac{(.^{20}C_{10} + 2 \times .^{20}C_9)}{2}$

C. C. $2^{19} - \frac{.^{20}C_{10}}{2}$

D. D. none of these

Answer: B

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46. The sum of series $.^{20}C_0 -^{20}C_1 +^{20}C_2 -^{20}C_3 + +^{20}C_{10}$ is $\frac{1}{2}$ $.^{20}C_{10}$ b. 0 c. $.^{20}C_{10}$ d. $-^{20}C_{10}$

A. $\frac{1}{2}.^{20}C_{10}$

B. 0

C. $.^{20}C_{10}$

D. $-.^{20}C_{10}$

Answer: A

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47. If $\left(3+x^{2008}+x^{2009}\right)^{2010}=a_0+a_1x^2+\ldots.+a_nx^n$, $a_0-\frac{1}{2}a_1-\frac{1}{2}a_2+a_3$ ..... is

A. 3^{2010}

B. 1

C. 2^{2010}

D. none of these

Answer: C

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48. Value of $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}\left(\hat{}\ kC_r\right)$ is $\frac{2}{3}$ b. $\frac{4}{3}$ c. 2 d. 1

A. $\frac{2}{3}$

B. $\frac{4}{3}$

C. 2

D. 1

Answer: C

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49. The value of $\sum_{r=0}^{10} (r)^{20}C_r$ is equal to $20(2^{18} +^{19} C_{10})$ b. $10(2^{18} +^{19} C_{10})$ c. $20(2^{18} +^{19} C_{11})$ d. $10(2^{18} +^{19} C_{11})$

A. $20(2^{18} + .^{19} C_{10})$

B. $10(2^{18} + .^{18} C_{10})$

C. $20(2^{16} + .^{19} C_{11})$

D. $10(2^{18} + .^{19} C_{11})$

Answer: A

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50.

$$\left[\left(.^{n}C_0+^{n}C_3+\right)-\frac{1}{2}\left(.^{n}C_1+^{n}C_2+^{n}C_4+^{n}C_5\right]^2+\frac{3}{4}\left(.^{n}C_1-^{n}C_2+^{n}C\right.\right.$$

equals to a.3 b. 4 c. 2 d. 1

A. 3

B. 4

C. 2

D. 1

Answer: D

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51. The value of $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n}\right.$ ^kC_(r-1))$(wherer$, k, n in N`) is equal to

A. $2^{n+1}-2$

B. $2^{n+1}-1$

C. 2^{n+1}

D. none of these

Answer: A

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52. The sum $\sum \sum_{0 \le i \le j \le 10} \left({}^{10}C_j\right)\left({}^{j}C_{i-1}\right)$ is equal to

A. A. $2^{10} - 1$

B. B. 2^{10}

C. C. $3^{10} - 1$

D. D. 3^{10}

Answer: C

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53. The value of the sum $^{1000}C_{50}+^{999}C_{49}+^{998}C_{48}+\ldots.^{950}C_0$ is

A. (a) $^{1001}C_{50}$

B. (b) $^{1002}C_{951}$

C. (c) $^{1001}C_{950}$

D. (d) $^{1002}C_{50}$

Answer: A

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54. If $\sum_{r=0}^{n}\{a_r(x-\alpha+2)^r-b_r(\alpha-x-1)^r\}=0$, then

A. A. $b_n=1+a_n$

B. B. $b_n=(-1)^n\times a_n$

C. C. $b_n=(-1)^{n-1}\times a_n$

D. D. $b_n+1=a_n$

Answer: B

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55. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r and a_k = 1$ for all $k \geq n,$ then show that $b_n = ^{2n+1} C_{n+1}$.

A. $.^{2n+1} C_{n-1}$

B. $.^{2n} C_{n+1}$

C. $.^{2n} C_n$

D. $.^{2n+1} C_{n+1}$

Answer: D

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56. The value of $\sum_{r=2}^{10} {}^rC_2 . {}^{10}C_r$ is

A. 10460

B. 11240

C. 11520

D. 12640

Answer: C

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57. If $.^{n+1}C_{r+1} :^{n}C_{r} :^{n-1}C_{r-1} = 11:6:3,$ then $nr =$ 20 b. 30 c. 40 d. 50

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58. If a, b and c are three consecutive coefficients terms in the expansion of $(1+x)^n$, then find n.

A. $\dfrac{ac+ab+bc}{b^2+ac}$

B. $\dfrac{2ac+ab+bc}{b^2-ac}$

C. $\frac{ab+ac}{b^2-ac}$

D. none of these

Answer: B

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59. Which term in the expansion of $(2-3x)^{19}$ has algebraically the last coefficients ?

A. A. 10^{th}

B. B. 11^{th}

C. C. 12^{th}

D. D. 13^{th}

Answer: C

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60. The value of $\frac{.^{n}C_0}{n} + \frac{.^{n}C_1}{n+1} + \frac{.^{n}C_2}{n+2} + \dots + \frac{.^{n}C_n}{2n}$

A. A. $\int_0^1 x^{n-1}(1-x)^n dx$

B. B. $\int_1^2 x^n (x-1)^{n-1} dx$

C. C. $\int_1^2 (1+x)^n dx$

D. D. $\int_0^1 (1-x)^n x^{n-1} dx$

Answer: B

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61. The value of $\sum_{r=1}^{n} (-1)^{r+1} \frac{^{n}C_r}{r+1}$ is equal to

A. (a) $-\frac{1}{n+1}$

B. (b) $-\frac{1}{n}$

C. (c) $\frac{1}{n+1}$

D. (d) $\frac{n}{n+1}$

Answer: D

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62. If $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right).^{n}C_{r}=\frac{2^{8}-1}{6}$, then n is (A) 8 (B) 4 (C) 6 (D) 5

A. 8

B. 4

C. 6

D. 5

Answer: D

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63. The value of $\sum_{r=0}^{3}.^{8}C_{r}\left(.^{5}C_{r+1}-.^{4}C_{r}\right)$ is ______.

A. (a) ${}^{30}C_{10} \times 2^{10}$

B. (b) ${}^{30}C_9 \times 4^{10}$

C. (c) ${}^{30}C_{10} \times 3^{10}$

D. (d) ${}^{30}C_9 \times 4^{10}$

Answer: C

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64. Find $\sum_{r=0}^{10} r\,{}^{10}C_r.3^r.(-2)^{10-r}$

A. 20

B. 10

C. 300

D. 30

Answer: D

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65.

The value of ${}^{15}C_0^2 - {}^{15}C_1^2 + {}^{15}C_2^2 \ldots\ldots\ldots {}^{15}C_{15}^2$ is

a. 15

b. -15

c. 0

d. 51

A. 15

B. -15

C. 0

D. 51

Answer: C

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66. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + ... + C_{n-2}C_n =$ a. $\frac{(2n)!}{(n!)^2}$ b. $\frac{(2n)!}{(n-1)!(n+1)!}$ c. $\frac{(2n)!}{(n-2)!(n+2)!}$ d. none of these

A. $\frac{(2n)!}{(n!)^2}$

B. $\frac{(2n)!}{(n-1)!(n+1)^!}$

C. $\frac{(2n)!}{(n-2)!(n+2)!}$

D. none of these

Answer: C

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67.

${}^{404}C_4 - {}^4C_1 {}^{303}C_4 + {}^4C_2 {}^{202}C_4 - {}^4C_3 {}^{101}C_4$ is equal to

A. $(401)^4$

B. $(101)^4$

C. 0

D. $(201)^4$

Answer: B

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68. The value of $\sum_{r=0}^{40} r^{40}C_r{}^{30}C_r$ is (a) $40.^{69}C_{29}$ (b) $40.^{70}C_{30}$ (c) $.^{60}C_{29}$ (d) $.^{70}C_{30}$

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69. The value of $\sum_{r=1}^{15} \frac{r2^r}{(r+2)!}$ is (a). $\frac{(17)!-2^{16}}{(17)!}$ (b). $\frac{(18)!-2^{17}}{(18)!}$ (c). $\frac{(16)!-2^{15}}{(16)!}$ (d). $\frac{(15)!-2^{14}}{(15)!}$

A. $\frac{(17)!-2^{16}}{(17)!}$

B. $\frac{(18)!2^{17}}{(18)!}$

C. $\frac{(16)!-2^{15}}{(16)!}$

D. $\frac{(15)!-2^{14}}{(15)!}$

Answer: A

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70. $(n+2)C_0(2^{n+1})-(n+1)C_1(2^n)+(n)C_2(2^{n-1})-....$ is equal to

A. 4

B. 4n

C. 4(n+1)

D. 2(n+2)

Answer: C

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71. The value of $\sum_{r=0}^{20}(-1)^r\frac{{}^{50}C_r}{r+2}$ is equal to

A. $\frac{1}{50\times51}$

B. $\frac{1}{52\times50}$

C. $\frac{1}{52\times51}$

D. none of these

Answer: C

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72. $\sum_{r=0}^{300}a_rx^r=\left(1+x+x^2+x^3\right)^{100}$. If $a=\sum_{r=0}^{300}a_r$, then $\sum_{r=0}^{300}ra_r$ is equal to

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73. $\sum_{r=0}^{300} a_r x^r = (1 + x + x^2 + x^3)^{100}$. If $a = \sum_{r=0}^{300} a_r$, then $\sum_{r=0}^{300} r a_r$ is equal to

A. 300a

B. 100a

C. 150a

D. 75a\

Answer: C

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74. The value of $\sum_{r=0}^{20} r(20 - r)$(^(20)C_r)^2` is equal to ?

A. $400.^{39} C_{20}$

B. $400.^{40} C_{19}$

C. $400.^{39} C_{19}$

D. $400.^{38}C_{20}$

Answer: D

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75. If

$f(x) = {}^{40}C_1 . x(1-x)^{39} + 2. {}^{40}C_2 . x^2(1-x)^{38} + 3. {}^{40}C_3 . x^3(1-x)^{37} + \ldots\ldots + 40. {}^{40}C_{40} . x^{40}$, then the value of $f(3)$ is

A. a. 120

B. b. 150

C. c. 200

D. d. 240

Answer: A

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76. Find the value of $\left(\sum\sum\right)_{0\le i<j\le n}(i+j)\left({}^{n}C_{i}+{}^{n}C_{j}\right)$.

A. an^2

B. $\frac{a^2n}{2}$

C. a^2n

D. $\frac{n^2a}{2}$

Answer: D

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77. In the expansion of $[(1+x)/(1-x)]^2$, the coefficient of x^n will be

a.$4n$ b. $4n-3$ c. $4n+1$ d. none of these

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78. The sum of $1+n\left(1-\frac{1}{x}\right)+\frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2+\ldots..\infty$

A. x^n

B. x^{-n}

C. $\left(1-\frac{1}{x}\right)^n$

D. none of these

Answer: A

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79. $\sum_{k=1}^{\infty} k\left(1-\frac{1}{n}\right)^{k-1}$ =?

A. $n(n-1)$

B. $n(n+1)$

C. n^2

D. $(n+1)^2$

Answer: C

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80. The coefficient of x in the expansion of $\left\{\sqrt{1+x^2}-x\right\}^{-1}$ in ascending powers of x, when $|x|<1, is$ a. 1 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. $-\frac{1}{8}$

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. $-\frac{1}{8}$

Answer: D

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81. $1+\frac{1}{3}x+\frac{1\times 4}{3\times 6}x^2+\frac{1\times 4\times 7}{3\times 6\times 9}x^3+$ ------ is equal to

A. x

B. $(1+x)^{1/3}$

C. $(1-x)^{1/3}$

D. $(1-x)^{-1/3}$

Answer: D

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82. $1+\frac{1}{4}+\frac{1\times3}{4\times8}+\frac{1\times3\times5}{4\times8\times12}+\ldots..$ is equal to

A. a. $\sqrt{2}$

B. b. $\frac{1}{\sqrt{2}}$

C. c. $\sqrt{3}$

D. d. $\frac{1}{\sqrt{3}}$

Answer: A

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83. If $|x| < 1, then 1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \ldots\ldots$ is equal to

A. $\left(\frac{2x}{1+x}\right)^n$

B. $\left(\frac{1+x}{2x}\right)^n$

C. $\left(\frac{1-x}{1+x}\right)^n$

D. $\left(\frac{1+x}{1-x}\right)^n$

Answer: D

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84. The coefficient of $x^5 \in \left(1 + 2x + 3x^2 + \right)^{-3/2} is(|x| < 1)$

A. 21

B. 25

C. 26

D. none of these

Answer: D

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85. If $|x| < 1,$ then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 +)^2$ is a. n b. $n - 1$ c. $n + 2$ d. $n + 1$

A. n

B. n-1

C. n+2

D. n+1

Answer: D

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86. If x is positive, the first negative term in the expansion of $(1+x)^{27/5} is (|x|<1)$

A. 5^{th} term

B. 8^{th} term

C. 6^{th} term

D. 7^{th} term

Answer: B

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87. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2x}\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

A. $3x+\dfrac{3}{8}x^2$

B. $1-\dfrac{3}{8}x^2$

C. $\frac{x}{2} - \frac{3}{x^2}$

D. $-\frac{3}{8}x^2$

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88. If the expansion in power of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots,$ then a_n is

A. $\frac{b^n - a^n}{b-a}$

B. $\frac{a^n - b^n}{b-a}$

C. $\frac{a^{n+1} - b^{n+1}}{b-a}$

D. $\frac{b^{n+1} - a^{n+1}}{b-a}$

Answer: D

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89. If $\frac{x^2+x+1}{1-x} = a_0 + a_1x + a_2x^2 + \ldots.,$ then $\sum_{r=1}^{50} a_r$ equal to

A. 148

B. 146

C. 149

D. none of these

Answer: C

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90. If

$$(1-x)^{-n} = a_0 + a_1 x + a_2 x^2 + \quad + a_r x^r + , then a_0 + a_1 + a_2 + \quad + a_r$$

is equal to

a. $\dfrac{n(n+1)(n+2)(n+r)}{r!}$

b. $\dfrac{(n+1)(n+2)(n+r)}{r!}$

c. $\dfrac{n(n+1)(n+2)(n+r-1)}{r!}$

d. none of these

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91. The coefficient of x^{103} in $\left(1+x+x^2+x^3+x^4\right)^{199}(x-1)^{201}$ is ____.

A. 26

B. 28

C. 30

D. 35

Answer: B

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92. The term independent of x in the product $\left(4+x+7x^2\right)\left(x-\frac{3}{x}\right)^{11}$ is (a) $7 \cdot {}^{11}C_6$ (b) $(3)^6 \cdot {}^{11}C_6$ (c) $3^5 \cdot {}^{11}C_5$ (d) $-12 \cdot 2^{11}$

A. $7 \cdot {}^{11}C_6$

B. $36 \cdot {}^{11}C_6$

C. $3^5 \cdot {}^{11}C_5$

D. $-12 \cdot 2^{11}$

Answer: B

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93. The 13^{th} term in the expanion of $\left(x^2 + 2/x\right)^n$ is independent of x then the sum of the divisiors of n is

A. 36

B. 37

C. 38

D. 51

Answer: D

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94. Coefficient of x^{2009} in $(1+x+x^2+x^3+x^4)^{1001}(1-x)^{1002}$ is

A. 0

B. $4.\ ^{1001}C_{501}$

C. -2009

D. none of these

Answer: A

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95. If the constant term in the binomial expansion of $\left(x^2-\frac{1}{x}\right)^n, n \in N$ is 15, then find the value of n.

A. 6

B. 9

C. 12

D. 15

Answer: A

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96. If $p^4 + q^3 = 2(p > 0, q > 0)$, then the maximum value of term independent of x in the expansion of $\left(px^{\frac{1}{12}} + qx^{-\frac{1}{9}}\right)^{14}$ is (a) ${}^{14}C_4$ (b) ${}^{14}C_6$ (c) ${}^{14}C_7$ (d) ${}^{14}C_{12}$

A. ${}^{14}C_4$

B. ${}^{14}C_6$

C. ${}^{14}C_7$

D. ${}^{14}C_{12}$

Answer: B

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97. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n, n \in N$, if the sum of the coefficients of $x^5 and x^{10}$ is zero , then n is a. 25 b. 20 c. 15 d. none of these

A. 25

B. 20

C. 15

D. None of these

Answer: C

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98. Find the coefficient of t^8 in the expansion of $\left(1 + 2t^2 - t^3\right)^9$.

A. 1680

B. 2140

C. 2520

D. 2730

Answer: C

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99. The term idependent of $'x'$ in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, is α times the corresponding binomial coefficient. Then $'\alpha'$ is

A. 3

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. 1

Answer: D

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100. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\leq\theta\leq\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\leq\theta\leq\frac{\pi}{8}$, then the value of $\frac{l_2}{l_1}$ is

A. 8

B. 32

C. 16

D. 64

Answer: C

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101. If $A_{i,j}$ be the coefficient of $a^i b^j c^{2010-i-j}$ in the expansion of $(a+b+c)^{2010}$, then

(a) $A_{i,i}$ is defined for $i\geq 1010$ (b) $A_{i,j}=A_{j,i}$ (c) $A_{2i,3i}$ is defined for $i\geq 405$ (d) $A_{0,1}=2000$

A. (a) $A_{i,i}$ is defined for $i \geq 1010$

B. (b) $A_{i,j} = A_{j,i}$

C. (c) $A_{2i,3i}$ is defined for $i \geq 405$

D. (d) $A_{0,1} = 2000$

Answer: B

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102. The coefficient of x^{301} ub the expansion of $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$ is

A. ${}^{501}C_{301}$

B. ${}^{500}C_{301}$

C. ${}^{501}C_{300}$

D. none of these

Answer: A

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103. The coefficient of x^7 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is

A. 4

B. 6

C. 8

D. 12

Answer: A

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104. Given $\left(1-x^3\right)^n=\sum_{k=0}^{n} a_k x^k (1-x)^{3n-2k}$ then the value of $3\cdot a_{k-1}+a_k$ is (a) ${}^{n}C_k\cdot 3^k$ (b) ${}^{n+1}C_k\cdot 3^k$ (c) ${}^{n+1}C_k\cdot 3^{k-1}$ (d) ${}^{n}C_{k-1}\cdot 3^k$

A. ${}^{n}C_k\cdot 3^k$

B. ${}^{n+1}C_k \cdot 3^k$

C. ${}^{n+1}C_k \cdot 3^{k-1}$

D. ${}^{n}C_{k-1} \cdot 3^k$

Answer: B

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105. Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$. (a) 2000 (b) 2001 (c) 1000 (d) 500

A. 2000

B. 2001

C. 1000

D. 500

Answer: D

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106. If the 4^{th} term of $\left\{\sqrt{x^{\frac{1}{1+\log_{10}x}}}+\sqrt[12]{x}\right\}^6$ is equal to 200, $x>1$and the logarithm is common logarithm, then$x$ is not divisible by

(a)2 (b)5 (c)10 (d)4

A. 2

B. 5

C. 10

D. 4

Answer: D

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107. The number of distinct terms in the expansion of $\left(x+y^2\right)^{13}+\left(x^2+y\right)^{14}$ is (a) 27 (b) 29 (c) 28 (d) 25

A. 27

B. 29

C. 28

D. 25

Answer: C

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108. The value of $\sum_{r=1}^{n}\left(\sum_{p=0}^{r-1}\hat{}\ nC_r^r C_p 2^p\right)$ is equal to (a) $4^n - 3^n + 1$ (b) $4^n - 3^n - 1$ (c) $4^n - 3^n + 2$ (d) $4^n - 3^n$

A. $4^n - 3^n + 1$

B. $4^n - 3^n - 1$

C. $4^n - 3^n + 2$

D. $4^n - 3^n$

Answer: D

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109. If in the expansion of $\left(x^3-\frac{2}{\sqrt{x}}\right)^n$ a term like x^2 exists and $'n'$ is a double digit number, then least value of $'n'$ is (a) 10 (b) 11 (c) 12 (d) 13

A. 10

B. 11

C. 12

D. 13

Answer: A

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110. In $\left(33+\frac{1}{33}\right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

A. 6

B. 9

C. 12

D. 15

Answer: B

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111. The number of distinct terms in the expansion of is $\left(x^3 + \frac{1}{x^3} + 1\right)^n$ is (a) 2n (b) 3n (c) 2n+1 (d) 3n+1

A. $2n$

B. $3n$

C. $2n + 1$

D. $3n + 1$

Answer: C

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112. If r^{th} and $(r+1)^{th}$ term in the expansion of $(p+q)^n$ are equal, then $\frac{(n+1)q}{r(p+q)}$ is (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 0

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. 1

D. 0

Answer: C

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113. If $(3+a\sqrt{2})^{100}+(3+b\sqrt{2})^{100}=7+5\sqrt{2}$ number of pairs (a, b) for which the equation is true is, (a, b are rational numbers) (a) 1 (b) 6 (c) 0 (d) infinite

A. 1

B. 6

C. 0

D. infinite

Answer: C

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114. The middle term in the expansion of $\left(1-3x+3x^2-x^3\right)^{2n}$ is

A. $\frac{(6n)!x^n}{(3n)!(3n)!}$

B. $\frac{(6n)!x^{3n}}{(3n)!}$

C. $\frac{(6n)}{(3n)!(3n)!}(-x)^{3n}$

D. None of these

Answer: C

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115. The algebraically second largest term in the expansion of $(3-2x)^{15}$ at $x=\frac{4}{3}$. is (a) 5 (b)7 (c)9 (d)11

A. 5

B. 7

C. 9

D. 11

Answer: B

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116. If 6^{th} term in the expansion of $\left(\frac{3}{2}+\frac{x}{3}\right)^{n}$ is numerically greatest, when $x=3$, then the sum of possible integral values of $'n'$ is (a) 23 (b) 24 (c) 25 (d) 26

A. 23

B. 24

C. 25

D. 26

Answer: C

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117. Let $\left(5+2\sqrt{6}\right)^n = I + f$, where n, $I \in N$ and $0 < f < 1$, then prove that the value of $I = \frac{1}{1-f} - f$.

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118. The sum of last 3 digits of 3^{100} is

A. 1

B. 2

C. 3

D. 4

Answer: A

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119. The remainder when $27^{10} + 7^{51}$ is divided by 10

A. 4

B. 6

C. 9

D. 2

Answer: D

Watch Video Solution

120. Consider the sequence $\frac{{}^{n}C_0}{1.2.3}, \frac{{}^{n}C_1}{2.3.4}, \frac{{}^{n}C_2}{3.4.5}, \dots,$ if $n = 50$ then greatest term is

A. 30^{th}

B. 24^{th}

C. 26^{th}

D. 27^{th}

Answer: B

Watch Video Solution

121. If P_n denotes the product of all the coefficients of $(1+x)^n$ and $9!P_{n+1} = 10^9 P_n$ then n is equal to

A. 10

B. 9

C. 19

D. none of these

Answer: B

Watch Video Solution

122. If N is a prime number which divides $S = ^{39}P_{19} + ^{38}P_{19} + ^{37}P_{19} + \ldots + ^{20}P_{19}$, then the largest possible value of N among following is

A. 41

B. 31

C. 37

D. 19

Answer: A

Watch Video Solution

123. If $\sum_{r=0}^{n} \left\{ \frac{^nC_{r-1}}{^nC_r + ^nC_{r-1}} \right\}^3 = \frac{25}{24}$, then n is equal to (a) 3 (b) 4 (c) 5 (d) 6

A. 3

B. 4

C. 5

D. 6

Answer: C

Watch Video Solution

124. If a, b, c, d be four consecutive coefficients in the binomial expansion of $(1+x)^n$, then value of the expression $\left(\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right)$ (where $x > 0$ and $n \in N$) is

A. A. positive

B. B. negative

C. C. zero

D. D. depends on n

Answer: A

Watch Video Solution

125.

$({}^mC_0 +^m C_1 -^m C_2 -^m C_3) + ({}^m C_4 +^m C_5 -^m C_6 -^m C_7) + .. = 0$

if and only if for some positive integer $k, m =$ (a) 4k (b) 4k+1 (c) 4k-1 (d) 4k+2

A. $4k$

B. $4k + 1$

C. $4k - 1$

D. $4k + 2$

Answer: C

Watch Video Solution

126. The value of $\sum_{r=0}^{3n-1}(-1)^r.{}^{6n}C_{2r+1}3^r$ is

A. 2^{3n}

B. 2^{2n-1}

C. 2^{6n-1}

D. 0

Answer: D

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127. The coefficient of x^{50} in $\left(x+{}^{101}C_0\right)\left(x+{}^{101}C_1\right)\ldots.\left(x+{}^{101}C_{50}\right)$ is

A. 4^{50}

B. 2^{50}

C. $2^{101}-1$

D. 2^{101}

Answer: A

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128. In the expansion of $(1+x)^{70}$, the sum of coefficients of odd powers of x is

A. 0

B. 2^{69}

C. 2^{70}

D. 2^{71}

Answer: B

Watch Video Solution

129. The sum of all the coefficients of the terms in the expansion of $(x+y+z+w)^6$ which contain x but not y, is (a) 3^6 (b) 2^6 (c) 3^6-2^6 (d) none of these

A. 3^6

B. 2^6

C. $3^6 - 2^6$

D. none of these

Answer: C

Watch Video Solution

130. The value of ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \dots + {}^{999}C_{989}$ is

A. ${}^{1000}C_{11} - 12$

B. ${}^{1000}C_{11} + 12$

C. ${}^{900}C_{11} - 12$

D. ${}^{1000}C_{989}$

Answer: A

Watch Video Solution

131. If $\left(1+ax+bx^2\right)=a_0+a_1x+a_2x^2++a_8x^8, wherea, b, a_0, a_1, , a_8 \in R$ such that $a_0+a_1+a_2 \neq 0and\left|a_0a_1a_2a_1a_2a_0a_2a_0a_1\right|=0,$ then the value of $5\frac{a}{b}$ is ______.

A. even

B. odd and of the form $3n$

C. odd and of the form $(3n-1)$

D. odd and of the form $(3n+1)$

Answer: A

Watch Video Solution

132. If the sum of the coefficients in the expansion of $(q+r)^{20}(1+(p-2)x)^{20}$ is equal to square of the sum of the

coefficients in the expansion of $[2rqx - (r+q)\cdot y]^{10}$, where p, r,q are positive constants, then

A. $\leq P$

B. $\frac{r+q}{2} \geq p$

C. r, p and q are in $G.P.$

D. $1/r, 1/p$ an $1/q$ are in $H.P.$

Answer: B

Watch Video Solution

133. The sum $S_n = \sum_{k=0}^{n} (-1)^k \cdot {}^{3n}C_k$, where $n = 1, 2, \ldots.$ is

A. $(-1)^n \cdot {}^{3n-1}C_{n-1}$

B. $(-1)^n \cdot {}^{3n-1}C_n$

C. $(-1)n \cdot {}^{3n-1}C_{n+1}$

D. None of these

Answer: B

Watch Video Solution

134. If for $n \in I, n > 10; 1 + (1 + x) + (1 + x)^2 + \quad + (1 + x)^n = \sum_{k=0}^{n} a_k \cdot x^k, x \neq 0$

then

A. $a_{n-2} = \frac{n(n+1)}{2}$

B. $a_9^2 - a_8^2 = {}^{n+2}C_{10}\left({}^{n+1}C_{10} - {}^{n+1}C_9\right)$

C. $a_p > a_{p-1}$ for $p < \frac{n}{2}$

D. $\sum_{k=0}^{n} a_k = 2^{n+1}$

Answer: D

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135. Given

$^{8}C_1 x(1-x)^7 + 2 \cdot ^{8}C_2 x^2(1-x)^6 + 3 \cdot ^{8}C_3 x^3(1-x)^5 + \ldots + 8 \cdot x^8 = ax$

, then $a+b$ is

A. (a) 4

B. (b) 6

C. (c) 8

D. (d) 10

Answer: C

Watch Video Solution

136. The value of $99^{50} - 99.98^{50} + \dfrac{99 \cdot 98}{1 \cdot 2}(97)^{50} - \ldots + 99$ is

A. 0

B. -1

C. -2

D. -3

Answer: A

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137. Let $f(n) = \sum_{k=1}^{n} k^2 \hat{} (n)C_k \Big)^2$ then the value of f(5) equals

A. 1000

B. 1250

C. 1750

D. 2500

Answer: C

Watch Video Solution

138. The value of $\sum_{r=1}^{n}(-1)^{r-1}\left(\frac{r}{r+1}\right).^{n}C_r$ is (a) $\frac{1}{n+1}$ (b) $\frac{1}{n}$ (c) $\frac{1}{n-1}$ (d) 0

A. $\frac{1}{n+1}$

B. $\frac{1}{n}$

C. $\frac{1}{n-1}$

D. 0

Answer: A

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139. The value of $\binom{100}{0}\binom{200}{150}+\binom{100}{1}\binom{200}{151}+\ldots\ldots+\binom{100}{50}\binom{200}{200}$ equals (where $\binom{n}{r}={}^{n}C_r$)

A. $\binom{300}{50}$

B. $\binom{100}{50}\binom{200}{150}$

C. $\binom{100}{50}^2$

D. $\binom{300}{50}^2$

Answer: A

Watch Video Solution

140. Let $t_{100} = \sum_{r=0}^{100} \frac{1}{\left({}^{100}C_r\right)^5}$ and $S_{100} = \sum_{r=0}^{100} \frac{r}{\left({}^{100}C_r\right)^5}$, then the value of $\frac{100 t_{100}}{S_{100}}$ is (a) 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

141. Let $S_1 = \sum_{0 \leq i < j \leq 100}\sum C_i C_j$, $S_2 = \sum_{0 \leq j < i \leq 100}\sum C_i C_j$ and $S_3 = \sum_{0 \leq i = j \leq 100}\sum C_i C_j$ where C_r represents cofficient of x^r in the binomial expansion of $(1 + x)^{100}$

If $S_1 + S_2 + S_3 = a^b$ where $a, b \in N$, then the least value of $(a + b)$ is

A. 66

B. 72

C. 46

D. 52

Answer: A

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142. $^{74}C_{37} - 2$ is divisible by a. 37^2 b. 38. c. 36 d. none of these

A. 37^2

B. 38

C. 36

D. none of these

Answer: A

Watch Video Solution

143. If ${}^nC_0 -{}^n C_1 +{}^n C_2 -{}^n C_3 + \ldots + (-1)^r \cdot{}^n C_r = 28$, then n is equal to ......

A. 7

B. 8

C. 9

D. 11

Answer: C

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144. If the value of ${}^{n}C_0 + 2 \cdot {}^{n} C_1 + 3 \cdot {}^{n} C_2 + ... + (n+1) \cdot {}^{n} C_n = 576$,

then n is (a) 7 (b) 5 (c) 6 (d) 9

A. 7

B. 5

C. 6

D. 9

Answer: A

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145. The value of

$$\binom{50}{6} - \binom{5}{1}\binom{40}{6} + \binom{5}{2}\binom{30}{6} - \binom{5}{3}\binom{20}{6} + \binom{5}{4}\binom{10}{6}$$

where $\binom{n}{r}$ denotes ${}^{n}C_r$, is

A. 15625

B. 0

C. 1000000

D. 2250000

Answer: D

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146. The value of the expansion $\left(\sum\sum\right)_{0\le i<j\le n}(-1)^{i+j-1}{}^{n}C_i\cdot{}^{n}C_j=$

A. ${}^{2n-1}C_n$

B. ${}^{2n}C_n$

C. ${}^{2n+1}C_n$

D. None of these

Answer: A

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147. Value of $\sum_{m=1}^{n}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{n}C_{m}\cdot{}^{m}C_{p}\cdot{}^{p}C_{k}\right)\right)=$

A. $3^n - 2^n$

B. $4^n - 3^n$

C. $3^n + 2^n$

D. $4^n - 1$

Answer: B

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148. If $\tan(A-B)=1 and \sec(A+B)=\frac{2}{\sqrt{3}}$, then the smallest positive values of B, respectively, is (a) $\frac{25\pi}{24}$ (b) $\frac{19\pi}{24}$ (c) $\frac{31\pi}{24}$ (d) $\frac{13\pi}{24}$

A. $\left(1, \frac{35}{24}\right)$

B. $\left(1, -\frac{35}{24}\right)$

C. $\left(2, \frac{35}{12}\right)$

D. $\left(2, -\frac{35}{12}\right)$

Answer: B

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149. The sum of the series $\frac{9}{5^2 \cdot 2 \cdot 1} + \frac{13}{5^3 \cdot 3 \cdot 2} + \frac{17}{5^4 \cdot 4 \cdot 3} + \dots$ upto infinity

A. $\left(\frac{3}{2}\right)^{\frac{1}{3}}$

B. $\left(\frac{5}{4}\right)^{\frac{1}{3}}$

C. $\left(\frac{3}{2}\right)^{\frac{1}{6}}$

D. None of these

Answer: A

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150. Coefficient of $x^{2^{m+1}}$ in the expansion of $\frac{1}{(1+x)(1+x^2)(1+x^4)(1+x^8)\dots.(1+x^{2^m})}(|x| < 1)$ is

A. 0

B. 1

C. 2^m

D. none of these

Answer: B

Watch Video Solution

151. Let $\left(2x^2 + 3x + 4\right)^{10} = \sum_{r=0}^{20} a_r x^r$, then the value of $\frac{a_7}{a_{13}}$ is (a) 6 (b) 8 (c) 12 (d) 16

A. 6

B. 8

C. 12

D. 16

Answer: B

Watch Video Solution

152. ${}^{30}C_0 \cdot {}^{20}C_{10} + {}^{31}C_1 \cdot {}^{19}C_{10} + {}^{32}C_2 \cdot 18C_{10} + \ldots {}^{40}C_{10} \cdot {}^{10}C_{10}$ is equal to

A. ${}^{51}C_{41}$

B. ${}^{50}C_{40}$

C. ${}^{51}C_{21}$

D. ${}^{50}C_{40}$

Answer: A

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Multiple Correct Answer Type

1. The value of x in the expression $\left(x + x^{(\log)_{10} x}\right)^5$ if third term in the expansion is 10,00,000 is/are

a. 10 b. 100 c. $10^{-5/2}$ d. $10^{-3/2}$

A. 10

B. 100

C. $10^{-5/2}$

D. $10^{-3/2}$

Answer: A::C

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2. The value of x for which the sixth term in the expansion of

$$\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2\left(3^{x-1}+1\right)}}\right]^7$$ is 84 is

A. 4

B. 3

C. 2

D. 1

Answer: C::D

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3. If the 4th term in the expansion of $(ax+1/x)^n$ is 5/2, then a. $a=\frac{1}{2}$ b. $n=8$ c. $a=\frac{2}{3}$ d. $n=6$

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4. In the expansion of $(x+a)^n$ if the sum of odd terms is P and the sum of even terms is $Q,$ then

A. $P^2-Q^2=(x^2-a^2)^n$

B. $4PQ=(x+a)^{2n}-(x-a)^{2n}$

C. $2(P^2+Q^2)=(x+a)^{2n}+(x-a)^{2n}$

D. none of these

Answer: A::B::C

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5. If $\left(4+\sqrt{15}\right)^{n}=I+f$, where n is an odd natural number, I is an integer and ,then

A. I is an odd integer

B. I is an even nteger

C. $(I+f)(I-f)=1$

D. $I-f=\left(4-\sqrt{15}\right)^{n}$

Answer: A::C::D

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6. If the coefficients of x^{39} and x^{40} are equal in the expansion of $(p+qx)^{49}$. then the possible values of p and q are

A. 4,15

B. 1,4

C. 5,18

D. 3,9

Answer: A::B::C

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7. The sum of the coefficient in the expansion of $\left(1 + ax - 2x^2\right)^n$ is

A. positive, when $a < 1$ and $n = 2k, k \in N$

B. negative, when $a < 1$ and $n = 2k + 1, k \in N$

C. positive, when $a > 1$ and $n \in N$

D. zero, when $a = 1$

Answer: A::B::C::D

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8. Let $\left(1+x^2\right)^2(1+x)^n = \sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

A. 5

B. 4

C. 3

D. 2

Answer: B::C::D

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9. For natural numbers $m, n,$ if $(1-y)^m(1+y)^n = 1 + a_1 y + a_2 y^2 + \ldots,$ and $a_1 = a_2 = 10, t$

A. $m < n$

B. $m > n$

C. $m + n = 80$

D. $m - n = 20$

Answer: A::C

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10. The middle term in the expansion of $\left(\frac{x}{2} + 2\right)^8$ is 1120, then $x \in R$ is equal to a. -2 b. 3 c. -3 d. 2

A. -2

B. 3

C. -3

D. 2

Answer: A::D

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11. In the expansion of $\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{15}$ the 11th term is a

A. an irrational number

B. a rational number

C. a positive integer

D. a negative integer

Answer: A

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12. For the expansion $\left(x\sin p+x^{-1}\cos p\right)^{10}, (p\in R)$,

A. A. the greatest value of the term independent of x is $10!/2^5(5!)^2$

B. B. the least value of sum of coefficient is zero

C. C. the greatest value of sum of coefficient is 12

D. D. the least value of the term independent of x occurs when

$$p = (2n+1)'\frac{\pi}{4}, n \in Z$$

Answer: A::B::C

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13. For which of the following values of x, $5th$ term is the numerically greatest term in the expansion of $(1 + x/3)^{10}$

A. -2

B. 1.8

C. 2

D. -19

Answer: A::B::C::D

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14. Which of the following is/are true ?

A. (a) $1000^{1000} > 1002^{999}$

B. (b) $1000^{1000} < 1002^{999}$

C. (c) $1000^{1002} < 1002^{1000}$

D. (d) $1000^{1002} > 1002^{1000}$

Answer: A::D

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15. If $s_n = \sum_{r=0}^{n} \frac{1}{\hat{} nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$ then $\frac{t_n}{s_n}$ is equal to

A. a. $n = 1$

B. b. $n = 2$

C. c. $n = 3$

D. d. none of these

Answer: A::C

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16. The value of $.^{n} C_1 + .^{n+1} C_2 + .^{n+2} C_3 + \dots.. + .^{n+m-1} C_m$ is equal to

A. $.^{m+n} C_n - 1$

B. $.^{m+n} C_{n-1}$

C. $.^{m} C_1 + .^{m+1} C_2 + .^{m+2} C_3 + \dots. + .^{m+n-1} C_n$

D. $.^{m+n} C_m - 1$

Answer: A::C::D

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17. The number of terms in the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n, n \in N$, is:

A. number of terms is $2n + 1$

B. constant term is 2^{n-1}

C. coefficient of x^{2n-2} is n

D. coefficient of x^2 in n

Answer: A::C

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18. In the expansion of $\left(7^{1/3} + 11^{1/9}\right)^{6561}$,

A. there are exactly 730 rational terms

B. there are exactly 5832 irrational terms

C. the term which involves greatest binomial coefficients is irrational

D. the term which involves greatest binomial coefficients is rational

Answer: A::B::C

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19. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots.. + C_nx^n$, then find the value of $C_0 + C_2 + C_4 + C_6 + \ldots\ldots.$

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20. In the expansion of $(a+b)^n$, if two consecutive terms are equal, then which of the following is/are always integer ?

(a) $\frac{(n+1)b}{a+b}$ (b) $\frac{(n+1)a}{a+b}$ (c) $\frac{na}{a-b}$ (d) $\frac{na}{a+b}$

A. $\frac{(n+1)b}{a+b}$

B. $\frac{(n+1)a}{a+b}$

C. $\frac{na}{a-b}$

D. $\frac{na}{a+b}$

Answer: A::B

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21. If for z as real or complex, $(1+z^2+z^4)^8 = C_0 + C_1 z^2 + C_2 z^4 + \ldots + C_{16} z^{32} then$ prove that $C_0 - C_1 + C_2 - C_3 + \ldots\ldots + C_{16} = 1$ and $C_0 + C_3 + C_6 + C_{12} + C_{15} = 3^7$

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22. If $f(m) = \sum_{i=0}^{m} \binom{30}{30-i}\binom{20}{m-i}$ where $\binom{p}{q} = {}^{p}C_q$, then

A. A. maximum value of $f(m)$ is $.^{50}C_{25}$

B. B. $f(0) + f(1) + \ldots\ldots + f(50) = 2^{50}$

C. C. $f(m)$ is always divisible by $50(1 \le m \le 49)$

D. D. The value of $\sum_{m=0}^{50} (f(m))^2 = .^{100}C_{50}$

Answer: A::B::D

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23. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, $n \in N$,then $C_0 - C_1 + C_2 - .... . + (-1)^{n-1}C_{m-1}$, is equal to $(m < n)$

A. A. $\dfrac{(n-1)(n-2).....(n-m+1)}{(m-1)!}(-1)^{m-1}$

B. B. $.^{n-1}C_{m-1}$

C. C. $\dfrac{(n-1)(n-2).....(n-m)}{(m-1)!}(-1)^{m-1}$

D. D $.^{n-1}C_{n-m}(-1)^{m-1}$

Answer: A::B::D

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24. If $\sum_{r=0}^{n}(pr+2).\,^nC_r = (25)(64)$ where $n, p \in N$, then (a) p=3 (b) p=4 (c) n=7 (d) n=6

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25. If

$$\left(x+\frac{1}{x}+1\right)^6 = a_0 + \left(a_1x+\frac{b_1}{x}\right) + \left(a_2x^2+\frac{b_2}{x^2}\right) + \ldots. + \left(a_6x^6+\frac{b}{x}\right.$$

, then

A. $a_0 = 141$

B. $a_5 = 6$

C. $\sum_{i=1}^{6} a_i + b_i = 588$

D. $\sum_{i=1}^{6} a_i + b_i = 3^6$

Answer: A::B::C

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26. Find the value of

$$.^{20}C_0 \times .^{13}C_{10} - .^{20}C_1 \times .^{12}C_9 + .^{20}C_2 \times .^{11}C_8 - \ldots\ldots + .^{20}C_{10}.$$

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27. The sum $2 \times {}^{40}C_2 + 6 \times {}^{40}C_3 + 12 \times {}^{40}C_4 + 20 \times {}^{40}C_5 + \ldots . + 1560 \times {}^{40}C_{40}$ is divisible by

A. (a) 3

B. (b) 5

C. (c) 13

D. (d) 2^{41}

Answer: A::B::C::D

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Linked Comphrension

1. The sixth term in the expansion of $\left[\sqrt{\left\{2^{\log(10-3^x)}\right\}} + 5\sqrt{\left\{2^{(x-2)\log 3}\right\}}\right]^m$ is equal to 21, if it is know that the binomial coefficient of the 2^{nd}, 3^{nd} and 4^{th} terms in the expansion represents, respectively, the first, third and fifth terms of an A.P. (the symbol log stand for logarithm tothe base 10).

The sum of possible values of x is

A. 6

B. 7

C. 8

D. 9

Answer: B

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2. The sixth term in the expansion of $\left(\sqrt{2^{\log(10-3^x)}}+\left(2^{(x-2)\log 3}\right)^{\frac{1}{5}}\right)^m$ is equal to 21, if it is known that the binomial coefficient of the 2nd 3rd and 4th terms in the expansion represent, respectively, the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10) The value of m is

A. 1

B. 3

C. 4

D. 2

Answer: D

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3. The sixth term in the expansion of $\left[\sqrt{\left\{2^{\log\left(10-3^{x}\right)}\right\}}+5\sqrt{\left\{2^{(x-2)\log 3}\right\}}\right]^{m}$ is equal to 21, if it is know that the binomial coefficient of the 2^{nd}, 3^{nd} and 4^{th} terms in the expansion represents, respectively, the first, third and fifth terms of an A.P. (the symbol log stand for logarithm tothe base 10).

The sum of possible values of x is

A. 64

B. 32

C. 128

D. none of these

Answer: C

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4. If the 2nd, 3rd and 4th terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, find $x \text{ and } n$.

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5. If the 2nd, 3rd and 4th terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, find $x \text{ and } n$.

A. 16

B. 160

C. 32

D. 81

Answer: C

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6. If the 2nd, 3rd and 4th terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, find x and n.

A. 1664

B. 2376

C. 1562

D. 1486

Answer: C

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7. An equation $a_0 + a_2x^2 + \ldots\ldots + a_{99}x^{99} + x^{100} = 0$ has roots $.^{99} C_0, .^{99} C_1, C_{99}C_2, \ldots\ldots, .^{99} C_{99}$

the value of a_{98} is equal to

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8. An equation $a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots + a_{99}x^{99} + x^{100} = 0$ has roots $.^{99} C_0, .^{99} C_1, .^{99} C_2 \ldots .^{99} C_{99}$. Find the value of a_{99}.

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9. An equation $a_0 + a_2x^2 + \ldots\ldots + a_{99}x^{99} + x^{100} = 0$ has roots $.^{99} C_0, .^{99} C_1, ^{99} C_2, \ldots\ldots, .^{99} C_{99}$

The value of $(.^{99} C_0)^2 + (.^{99} C_1)^2 + \ldots\ldots + (.^{99} C_{99})^2$ is equal to

A. $2a_{98} - a_{99}^2$

B. $a_{99}^2 - a_{98}$

C. $a_{99}^2 - 2a_{98}$

D. none of these

Answer: C

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10. If $a = .^{20}C_0 + .^{20}C_3 + .^{20}C_6 + .^{20}C_9 + \ldots..,$ $b = .^{20}C_1 + .^{20}C_4 + .^{20}C_7 + \ldots\ldots'$ and $c = .^{20}C_2 + .^{20}C_5 + .^{20}C_8 + \ldots..,$ then

Value of $a^3 + b^3 + c^3 - 3abc$ is

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11. If $a =^{20} C_0 +^{20} C_3 +^{20} C_6 +^{20} C_9 + \ldots.., b =^{20} C_1 +^{20} C_4 +^{20} C_7 + \ldots$ and $c =^{20} C_2 +^{20} C_5 +^{20} C_8 + \ldots..,$ then

Value of $(a-b)^2 + (b-c)^2 + (c-a)^2$ is

A. (a) 1

B. (b) 2

C. (c) 2^{20}

D. (d) 2^{40}

Answer: B

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12. Consider the expansion of $(a+b+c+d)^6$. Then the sum of all the coefficients of the term

Which contains both a and b is

A. 4096

B. 1560

C. 3367

D. 670

Answer: B

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13. The sum of all the coefficients of the expansion of $(a+b+c+d)^8$ which contains b but not c is (a) 729 (b) 6305 (c) 665 (d) 1024

A. 729

B. 3367

C. 665

D. 1024

Answer: C

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14. Consider the expansion of $(a+b+c+d)^6$. Then the sum of all the coefficients of the term

Which contains both a and b is (a) 729 (b) 3367 (c) 2702 (d) 1024

A. 2884

B. 4032

C. 1974

D. 2702

Answer: D

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15. Let $P = \sum_{r=1}^{50} \frac{{}^{50+r}C_r(2r-1)}{{}^{50}C_r(50+r)}, Q = \sum_{r=1}^{50} \left({}^{50}C_r\right)^2,$

Then find the value of $P - Q$

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16. Let $P = \sum_{r=1}^{50} \frac{{}^{50+r}C_r(2r-1)}{{}^{50}C_r(50+r)}, R = \sum_{r=0}^{100} (-1)^r \left({}^{100}C_r\right)^2$

The value of P - R is equal to

A. (a) 1

B. (b) -1

C. (c) 2^{50}

D. (d) 2^{100}

Answer: B

Watch Video Solution

17. Let $Q = \sum_{r=0}^{50} \left({}^{50}C_r\right)^2, R = \sum_{r=0}^{100} (-1)^r \left({}^{100}C_r\right)^2$

Then find the value of Q + R

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18. If $\left(1 + x - 2x^2\right)^6 = 1 + a_1x + a_2x^2 + \ldots\ldots + a_{12}x^{12}$, then find the value of $a_2 + a_4 + a_6 + \ldots\ldots + a_{12}$.

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19. If $\left(1+x+2x^2\right)^{20}=a_0+a_1x......+a_{40}x^{40}$, then following questions.

The value of $a_0+a_2+a_4+......+a_{38}$ is

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20. If $\left(1+x+2x^2\right)^{20}=a_0+a_1x^2......+a_{40}x^{40}$, then following questions.

The value of $a_0+a_2+a_4+......+a_{38}$ is

A. (a) 161×3^{20}

B. (b) 41×3^{40}

C. (c) 41×3^{20}

D. (d) none of these

Answer: C

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1. Facorise the following: $10ab + 4a + 5b + 2.$

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2. Factoris the following: $16x^5 - 144x^3$

Watch Video Solution

3. Find the product of the following: (x + 3) (x + 3)

Watch Video Solution

4. If the second term of the expansion $\left[a^{\frac{1}{13}} + \left(\frac{a}{\sqrt{a^{-1}}}\right]is$14a^(5/2)

and $thevalueof($^nC_3)/("^nC_2) $= \lambda$ then λ is

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Numerical

1. Find the product of the following: (x + 3) (x + 3)

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2. If the three consecutive in the expansion of $(1+x)^n$ are 28, 56, and 70, then the value of n is.

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3. Least positive integer just greater than $(1+0.00002)^{50000}$ is.

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4. If the second term of the expansion $\left[a^{\frac{1}{13}}+\left(\frac{a}{\sqrt{a^{-1}}}\right]is14a^(5/2)$ and $thevalueof(^nC_3)/("^nC_2)=\lambda$ then λ is

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5. If the constant term in the binomial expansion of $\left(x^2-\frac{1}{x}\right)^n, n\in N$ is 15, then find the value of n.

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6. The largest value of x for which the fourth tem in the expansion $\left(\frac{5^2}{3}(\log)_5\sqrt{4^{x+44}}+\frac{1}{5^{\log}\_52^{(x-1)+73}}\right)$ is 336 is.

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7. Let $aandb$ be the coefficients of x^3 in $(1+x+2x^2+3x^3)^4and(1+x+2x^2+3x^3+4x^4)^4,$ then respectively. Then the value of $4a/b$ is.

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8. If R is remainder when $6^{83}+8^{83}$ is divided by 49, then the value of $R/5$ is.

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9. The remainder, if $1+2+2^2+2^3+....+2^{1999}$ in divided by 5 is

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10. Given $\left(1-2x+5x^2-10x^3\right)(1+x)^n=1+a_1x+a_2x^2+....$ and that $a_1^2=2a_2$, then the value of n is

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11. Find the largest real value of x such that $\sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)=\frac{32}{3}$.

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12. The coefficient of x^{103} in $\left(1+x+x^2+x^3+x^4\right)^{199}(x-1)^{201}$ is ____.

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13. The total number of different terms in the product $\left({}^{101}C_0-{}^{101}C_1x+{}^{101}C_2x^2-\ldots\ldots-{}^{101}C_{101}x^{101}\right)\left(1+x+x^2+\ldots+\right.$

is ____.

A. A. 100

B. B. 101

C. C. 102

D. D. 103

Answer: 102

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14. The constant term in the expansion of

$\left(\log\left(x^{\log x}\right) - \log_{x^2} 100\right)^{12}$ is (base of log is 10) ______.

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15. The value of $\sum_{r=0}^{3} {}^{8}C_r\left({}^{5}C_{r+1} - {}^{4}C_r\right)$ is ______.

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16. The sum of the series

$\frac{.^{101}C_1}{.^{101}C_0} + \frac{2. .^{101}C_2}{.^{101}C_1} + \frac{3. .^{101}C_3}{.^{101}C_2} + \ldots.. + \frac{101. .^{101}C_{101}}{.^{101}C_{100}}$ is ______.

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17. Let $a = 3^{\frac{1}{223}} + 1$ and for all $n \geq 3, let f(n) = {}^n C_0 \dot{a}^{n-1} - {}^n C_1 \dot{a}^{n-2} + {}^n C_2 \dot{a}^{n-3} - + (-1)^{n-1} .{}^n C_{n-1} a$

. If the value of $f(2007) + f(2008) = 3^k$ where k in N , then the value of k is.

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18. Let $1 + \sum_{r=1}^{10} \left(3^r .{}^{10} C_r + r .{}^{10} C_r\right) = 2^{10}\left(\alpha . 4^5 + \beta\right)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1.$ If α, β lies between the roots of $f(x) = 0$, then find the smallest positive integral value of k.

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19. The value of $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{5^n} .{}^n C_r \left(\sum_{t=0}^{r-1} .{}^r C_t . 3^t\right)$ is equal to

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20. If $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right).^{n}C_{r}=\frac{2^{8}-1}{6}$. then nis

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21. If $S_{n}=(.^{n}C_{0})^{2}+(.^{n}C_{1})^{2}+\ldots+(.^{n}C_{n})^{n}$, then maximum value of $\left[\frac{S_{n+1}}{S_{n}}\right]$ is ______.

(where $[\cdot]$ denotes the greatest integer function)

A. A. 2

B. B. 3

C. D. 4

D. D. None of these

Answer: 3

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22. The value of $.^{40}C_0 + .^{40}C_1 + .^{40}C_2 + \quad + .^{40}C_{20}$

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23. The value of $\sum_{0 \le i < j \le 5} \sum \left({}^5C_j\right)\left({}^jC_i\right)$ is equal to ______

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24. If $\left(1 - x - x^2\right)^{20} = \sum_{r=0}^{40} a_r . x^r$, then value of $a_1 + 3a_3 + 5a_5 + \ldots.. + 39a_{39}$ is

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25. The value of $\sum_{r=1}^{49} \frac{2r^2 - 48r + 1}{(50 - r).\,{}^{50}C_r}$ is ______.

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1. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is (1) 0 (2) 2 (3) 7 (4) 8

A. 0

B. 2

C. 7

D. 8

Answer: B

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2. Let $S_1 = \sum_{j=1}^{10} j(j-1).\ {}^{10}C_j,\ S_2 = \sum_{j=1}^{10} j.\ {}^{10}C_j$, and $S_3 = \sum_{j=1}^{10} j^2.\ {}^{10}C_j$.

Statement 1 : $S_3 = 55 \times 2^9$.

Statement 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

A. (a) Statement 1 is false, statement 2 is true.

B. (b) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1.

C. (c) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 2.

D. (d) Statement 1 is true, statement 2 is false.

Answer: B

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3. The coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$ is

A. 132

B. 144

C. -132

D. -144

Answer: D

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4. If n is a possible integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is

A. an irrational number

B. an odd positive integer

C. an even positive integer

D. a rational number other than positiveintegers

Answer: A

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5. The coefficient of the term independent of x in the expansion of $\left[\frac{(x+1)}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10}$ is

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6. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to (1) $\left(16, \frac{251}{3}\right)$ (3) $\left(14, \frac{251}{3}\right)$ (2) $\left(14, \frac{272}{3}\right)$ (4) $\left(16, \frac{272}{3}\right)$

A. $\left(16, \frac{251}{3}\right)$

B. $\left(14, \frac{251}{3}\right)$

C. $\left(14, \frac{272}{3}\right)$

D. $\left(16, \frac{272}{3}\right)$

Answer: D

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7. The sum of coefficient of integral powers of x in the binomial expansion of $(1 - 2\sqrt{x})^{50}$ is:

A. $\frac{1}{2}(3^{50} + 1)$

B. $\frac{1}{2}(3^{50})$

C. $\frac{1}{2}(3^{50} - 1)$

D. $\frac{1}{2}(2^{10} + 1)$

Answer: A

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8. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n, x \neq 0,$ is 28, then the sum of the coefficients of all the terms in this expansion, is : (1) 64 (2) 2187 (3) 243 (4) 729

A. 2187

B. 243

C. 729

D. 64

Answer: C

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9. The value of

$\left(.^{21}C_1 - .^{10}C_1\right) + \left(.^{21}C_2 - .^{10}C_2\right) + \left(.^{21}C_3 - .^{10}C_3\right) + \left(.^{21}C_4 - .^{10}C_4\right.$

is

A. $2^{20} - 2^{10}$

B. $2^{21} - 2^{11}$

C. $2^{21} - 2^{10}$

D. $2^{20} - 2^{9}$

Answer: A

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10. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1)$

A. 2

B. -1

C. 0

D. 1

Answer: A

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11. For $r = 0, 1, \ldots ..., 10$, let $A_r, B_r,$ and C_r denote, respectively, the coefficient of x^r in the expansion of $(1+x)^{10}, (1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

A. $B_{10} - C_{10}$

B. $A_1\left(B_{10}^2 - C_{10}A_{10}\right)$

C. 0

D. $C_{10} - B_{10}$

Answer: D

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12. Coefficient of x^{11} in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7\left(1+x^4\right)^{12}$ is

A. 1051

B. 1106

C. 1113

D. 1120

Answer: C

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13. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n= ______

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14. The coefficient of x^9 in the expansion of $(1+x)(16x^2)(1+x^3)(1+x^{100})$ is

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15. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+(1+x)^4+\ldots\ldots.+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1).^{51}C_3$ for some positive integer n. Then find the value of n.

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16. Let $X=\left({}^{10}C_1\right)^2+2\left({}^{10}C_2\right)^2+3\left({}^{10}C_3\right)^2+\cdot\cdot+10\left({}^{10}C_{10}\right)^2$, where ${}^{10}C_r$, $r\in\{1,\ 2,\ ;\ 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is ________.

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Multiple Correct Answer

1. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that

$$C_0 \cdot {}^{2n}C_n - C_1 \cdot {}^{2n-2}C_n + C_2 \cdot {}^{2n-4}C_n - \ldots = 2^n$$

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2. Which of the following is/are correct ?

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots . - {}^{20}C_{15} = -{}^{19}C_{15}$$

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots . - {}^{20}C_{15} = -{}^{20}C_{14}$$

$$16\,{}^{20}C_0 - 15\,{}^{20}C_1 + 14\,{}^{20}C_2 - \ldots . + 2\,{}^{20}C_{14} - {}^{20}C_{15} = {}^{19}C_{14}$$

$$16\,{}^{20}C_0 - 15\,{}^{20}C_1 + 14\,{}^{20}C_2 - \ldots . + 2\,{}^{20}C_{14} - {}^{20}C_{15} = {}^{18}C_{15}$$

A. ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots . - {}^{20}C_{15} = -{}^{19}C_{15}$

B. ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots . - {}^{20}C_{15} = -{}^{20}C_{14}$

C. $16\,{}^{20}C_0 - 15\,{}^{20}C_1 + 14\,{}^{20}C_2 - \ldots . + 2\,{}^{20}C_{14} - {}^{20}C_{15} = {}^{19}C_{14}$

D. $16^{20}C_0 - 15^{20}C_1 + 14^{20}C_2 - \ldots . + 2^{20}C_{14} -^{20}C_{15} =^{18}C_{15}$

Answer: A::D

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3. The value of $\sum_{k=0}^{7}\left[\frac{\binom{7}{k}}{\binom{14}{k}}\sum_{r=k}^{14}\binom{r}{k}\binom{14}{r}\right]$, where $\binom{n}{r}$ denotes $^{n}C_r$ is

A. 6^7

B. greater than 7^6

C. 8^7

D. greater than 7^8

Answer: A::B

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4. The value of the sum ${}^{1000}C_{50} + {}^{999}C_{49} + {}^{998}C_{48} + \dots {}^{950}C_0$ is

A. ${}^{1001}C_{50}$

B. ${}^{1002}C_{951} - {}^{1001}C_{51}$

C. ${}^{1001}C_{951}$

D. ${}^{1002}C_{51} - {}^{1001}C_{95}$

Answer: A::B::C::D

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Comprehension

1. Consider a $G.P.$ with first term $(1+x)^n$, $|x|<1$, common ratio $\dfrac{1+x}{2}$ and number of terms $(n+1)$. Let $'S'$ be sum of all the terms of the $G.P.$, then

The coefficient of x^n is $'S'$ is

A. 2^n

B. 2^{n+1}

C. 2^{2n}

D. 2^{2n+1}

Answer: A

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2. Consider a $G.P.$ with first term $(1+x)^n$, $|x|<1$, common ratio $\frac{1+x}{2}$ and number of terms $(n+1)$. Let S be sum of all the terms of the $G.P.$, then

$\sum_{r=0}^{n} {}^{n+r}C_r\left(\frac{1}{2}\right)^r$ equals (a) $\frac{3}{4}$ (b) 1 (c)2^n (d) 3^n

A. $(3/4)$

B. 1

C. 2^n

D. 3^n

Answer: C

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3. A path of length n is a sequence of points (x_1, y_1), (x_2, y_2),....,(x_n, y_n) with integer coordinates such that for all i between 1 and $n-1$ both inclusive,

either $x_{i+1} = x_i + 1$ and $y_{i+1} = y_i$ (in which case we say the i^{th} step is rightward)

or $x_{i+1} = x_i$ and $y_{i+1} = y_i + 1$ (in which case we say that the i^{th} step is upward).

This path is said to start at (x_1, y_1) and end at (x_n, y_n). Let $P(a, b)$, for a and b non-negative integers, denotes the number of paths that start at $(0, 0)$ and end at (a, b).

The value of $\sum_{i=0}^{10} P(i, 10-i)$ is

A. (a) 1024

B. (b) 512

C. (c) 256

D. (d) 128

Answer: A

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4. A path of length n is a sequence of points (x_1, y_1), (x_2, y_2),....,(x_n, y_n) with integer coordinates such that for all i between 1 and $n-1$ both inclusive,

either $x_{i+1} = x_i + 1$ and $y_{i+1} = y_i$ (in which case we say the i^{th} step is rightward)

or $x_{i+1} = x_i$ and $y_{i+1} = y_i + 1$ (in which case we say that the i^{th} step is upward).

This path is said to start at (x_1, y_1) and end at (x_n, y_n). Let $P(a, b)$, for a and b non-negative integers, denotes the number of paths that start at $(0, 0)$ and end at (a, b).

Number of ordered pairs (i, j) where $i \neq j$ for which $P(i, 100 - i) = P(j, 100 - j)$ is

A. (a) 50

B. (b) 99

C. (c) 100

D. (d) 101

Answer: C

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5. A path of length n is a sequence of points (x_1, y_1), (x_2, y_2),....,(x_n, y_n) with integer coordinates such that for all i between 1 and $n-1$ both inclusive,

either $x_{i+1} = x_i + 1$ and $y_{i+1} = y_i$ (in which case we say the i^{th} step is rightward)

or $x_{i+1} = x_i$ and $y_{i+1} = y_i + 1$ (in which case we say that the i^{th} step is upward).

This path is said to start at (x_1, y_1) and end at (x_n, y_n). Let $P(a, b)$, for a and b non-negative integers, denotes the number of paths that start at

$(0, 0)$ and end at (a, b).

The value of $\sum_{i=0}^{10} P(i, 10-i)$ is

A. (a) $P(4, 48)$

B. (b) $P(3, 49)$

C. (c) $P(4, 47)$

D. (d) $P(5, 47)$

Answer: A

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6. The expansion $1+x, 1+x+x^2, 1+x+x^2+x^3, \ldots .1+x+x^2+\ldots+x^{20}$ are multipled together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0+a_1x+a_2x^2+\ldots$, then,

Number of terms in the product

A. 200

B. 211

C. 231

D. none of these

Answer: B

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7. The expressions $1 + x, 1 + x + x^2, 1 + x + x^2 + x^3, \ldots\ldots\ldots\ldots 1 + x + x^2 + \ldots\ldots\ldots\ldots\ldots + x$

are mutiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the from of $a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots\ldots\ldots,$ then sum of even coefficients?

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8. The expansion $1+x, 1+x+x^2, 1+x+x^2+x^3, \ldots., 1+x+x^2+\ldots+x^{20}$ are multipled together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0+a_1x+a_2x^2+\ldots$ then,

The value of $\frac{a_r}{a_{n-r}}$, where n is the degree of the product.

A. (a) 2

B. (b) 1

C. (c) $1/2$

D. (d) depends on r

Answer: B

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9. If $(1+px+x^2)^n = 1+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$.

Which of the following is true for $1 < r < 2n$

A. (a) $(np+pr)a_r=(r+1)a_{r+1}+(r-1)a_{r-1}$

B. (b) $(np-pr)a_r=(r+1)a_{r+1}+(r-1-2n)a_{r-1}$

C. (c) $(np-pr)a_r=(r+1)a_{r+1}+(r-1-n)a_{r-1}$

D. (d) $(2np+pr)a_r=(r+1+n)a_{r+1}+(r+1-n)a_{r-1}$

Answer: B

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10. If $(1+px+x^2)^n=1+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$.

The remainder obtained when $a_1+5a_2+9a_3+13a_4+\ldots+(8n-3)a_{2n}$ is divided by $(p+2)$ is (a) 1 (b) 2 (c) 3 (d) 0

A. 1

B. 2

C. 3

D. 0

Answer: C

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11. If $(1 + px + x^2)^n = 1 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$.

The value of $a_1 + 3a_2 + 5a_3 + 7a_4 + \ldots. (4n - 1)a_{2n}$ when $p = -3$ and $n \in$ even is

A. n

B. $2n - 1$

C. $2n - 2$

D. $2n$

Answer: D

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