MATHS

BOOKS - CENGAGE PUBLICATION

COMPLEX NUMBERS

Single Correct Answer

1. The value of $\sum_{n=0}^{100} i^n$ equals (where $i = \sqrt{-1}$)

A. -1

B. i

C. 1

D. $-i$

Answer: C

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2. Suppose n is a natural number such that $\left|i + 2i^2 + 3i^3 + \ldots\ldots + ni^n\right| = 18\sqrt{2}$ where i is the square root of -1. Then n is

A. 9

B. 18

C. 36

D. 72

Answer: C

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3. Let $i = \sqrt{-1}$ Define a sequence of complex number by $z_1 = 0, z_{n+1} = (z_n)^2 + i$ for $n \geq 1$. In the complex plane, how far from the origin is z_{111}?

A. 1

B. 2

C. 3

D. 4

Answer: B

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4. The complex number, $z = \dfrac{\left(-\sqrt{3}+3i\right)(1-i)}{\left(3+\sqrt{3}i\right)(i)\left(\sqrt{3}+\sqrt{3}i\right)}$

A. lies on real axis

B. lies on imaginary axis

C. lies in first quadrant

D. lies in second quadrant

Answer: B

5. a,b,c are positive real numbers forming a G.P. If $ax^2 + 2bx + c = 0$and $dx^2 + 2ex + f = 0$ have a common root, then prove that $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

A. $A. P.$

B. $G. P.$

C. $H. P.$

D. None of these

Answer: A

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6. Prove that the equation $Z^3 + iZ - 1 = 0$ has no real roots.

A. three real roots

B. one real roots

C. no real roots

D. no real or complex roots

Answer: C

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7. If a, b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real, whereas the other is purely imaginary, prove that $a^2 - (\bar{a})^2 = 4b.$

A. 2

B. 4

C. 6

D. 8

Answer: B

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8. If Z is a non-real complex number, then find the minimum value of $\left|\frac{Imz^5}{Im^5z}\right|$

A. -1

B. -2

C. -4

D. -5

Answer: C

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9. If z_1, z_2, z_3 are three complex number then prove that

$$z_1 Im(\bar{z}_2 . z_3) + z_2 Im(\bar{z}_3 . z_1) + z_3 Im(\bar{z}_1 . z_2) = 0$$

A. 0

B. $z_1 + z_2 + z_3$

C. $z_1 z_2 z_3$

D. $\left(\dfrac{z_1 + z_2 + z_3}{z_1 z_2 z_3}\right)$

Answer: A

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10. The modulus of $\dfrac{1 + 2i}{1 - (1 - i)^2}$ are

A. $\sqrt{2}$

B. 1

C. 0

D. 2

Answer: C

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11. If the argument of $(z-a)\left(\bar{z}-b\right)$ is equal to that $\left(\left(\sqrt{3}+i\right)\frac{1+\sqrt{3}i}{1+i}\right)$ where a,b,c are two real number and z is the complex conjugate o the complex number z find the locus of z in the rgand diagram. Find the value of a and b so that locus becomes a circle having its centre at $\frac{1}{2}(3+i)$

A. A. $(3, 2)$

B. B. $(2, 1)$

C. C. $(2, 3)$

D. D. $(2, 4)$

Answer: B

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12. If a complex number z satisfies $|z|^2 + \frac{4}{(|z|)^2} - 2\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right) - 16 = 0$, then the maximum value of $|z|$ is

A. $\sqrt{6} + 1$

B. 4

C. $2 + \sqrt{6}$

D. 6

Answer: C

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13. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then $\frac{\sin 3\alpha + \sin 3\beta + \sin 3\gamma}{\sin(\alpha + \beta + \gamma)}$ is equal to

A. 1

B. -1

C. 3

D. -3

Answer: C

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14. The least value of $|z - 3 - 4i|^2 + |z + 2 - 7i|^2 + |z - 5 + 2i|^2$

occurs when z=

A. A. $1 + 3i$

B. B. $3 + 3i$

C. C. $3 + 4i$

D. D. None of these

Answer: D

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15. The roots of the equation $x^4 - 2x^2 + 4 = 0$ are the

vertices of a :

A. square inscribed in a circle of radius 2

B. rectangle inscribed in a circle of radius 2

C. square inscribed in a circle of radius $\sqrt{2}$

D. rectangle inscribed in a circle of radius $\sqrt{2}$

Answer: D

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16. If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 2|, Re(z_2) = |z_2 - 2|$ and $arg(z_1 - z_2) = \pi/3$, then $Im(z_1 + z_2) =$

A. $2/\sqrt{3}$

B. $4/\sqrt{3}$

C. $2/\sqrt{3}$

D. $\sqrt{3}$

Answer: B

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17. If $z = e^{\frac{2\pi i}{5}}$, then $1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 =$

A. 0

B. $4z^3$

C. $5z^4$

D. $-4z^2$

Answer: C

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18. If $z = (3 + 7i)(a + ib)$, where $a, b \in Z - \{0\}$, is purely imaginery, then minimum value of $|z|$ is

A. 74

B. 45

C. 65

D. 58

Answer: D

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19. Let z be a complex number satisfying $|z + 16| = 4|z + 1|$.

Then

A. $|z| = 4$

B. $|z| = 5$

C. $|z| = 6$

D. $3 < |z| < 68$

Answer: A

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20. If $|z| = 1$ and $z' = \frac{1+z^2}{z}$, then

A. z' lie on a line not passing through origin

B. $|z'| = \sqrt{2}$

C. $Re(z') = 0$

D. $Im(z') = 0$

Answer: D

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21. a, b, c are three complex numbers on the unit circle $|z| = 1$, such that $abc = a + b + c$. Then find the value of $|ab + bc + ca|$.

A. 3

B. 6

C. 1

D. 2

Answer: C

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22. If $|z_1| = |z_2| = |z_3| = 1$ then value of $|z_1 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$ cannot exceed

A. 6

B. 9

C. 12

D. none of these

Answer: B

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23. Number of ordered pairs (s), (a, b) of real numbers such that $(a + ib)^{2008} = a - ib$ holds good is

A. A. 2008

B. B. 2009

C. C. 2010

D. D. 1

Answer: C

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24. The region represented by the inequality $|2z - 3i| < |3z - 2i|$ is

A. A. the unit disc with its centre at $z = 0$

B. B. the exterior of the unit circle with its centre at $z = 0$

C. C. the inerior of a square of side 2 units with its centre at $z = 0$

D. D. none of these

Answer: B

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25. If ω is any complex number such that $z\omega = |z|^2$ and $|z - \bar{z}| + |\omega + \bar{\omega}| = 4$, then as ω varies, then the area bounded by the locus of z is

A. 4 sq. units

B. 8 sq. units

C. 16 sq. units

D. 12 sq. units

Answer: B

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26. Show that the equation $az^3 + bz^2 + \bar{b}z + \bar{a} = 0$ has a root α such that $|\alpha| = 1, a, b, z$ and α belong to the set of complex numbers.

A. $1/4$

B. $1/2$

C. $5/4$

D. $3/4$

Answer: D

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27. Let p and q are complex numbers such that $|p| + |q| < 1$. If z_1 and z_2 are the roots of the $z^2 + pz + q = 0$, then which one of the following is correct ?

A. $\left|z_1\right| < 1$ and $\left|z_2\right| < 1$

B. $\left|z_1\right| > 1$ and $\left|z_2\right| > 1$

C. If $\left|z_1\right| < 1$, then $\left|z_2\right| > 1$ and vice versa

D. Nothing definite can be said

Answer: A

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28. If z and w are two complex numbers simultaneously satisfying the equations, $z^3 + w^5 = 0$ and $z^2 . \bar{w}^4 = 1$, then

A. z and w both are purely real

B. z is purely real and w is purely imaginery

C. w is purely real and z is purely imaginery

D. z and w both are imaginery

Answer: A

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29. All complex numbers 'z' which satisfy the relation $|z - |z + 1|| = |z + |z - 1||$ on the complex plane lie on the

A. A. $y = x$

B. B. $y = -x$

C. C. circle $x^2 + y^2 = 1$

D. D. line $x = 0$ or on a line segment joining $(-1, 0)$ to $(1, 0)$

Answer: D

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30. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $iz_1 = Kz_2$, where $K \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is

A. $\tan^{-1}\left(\frac{2K}{K^2 + 1}\right)$

B. $\tan^{-1}\left(\frac{2K}{1-K^2}\right)$

C. $-2\tan^{-1}K$

D. $2\tan^{-1}K$

Answer: D

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31. If $z+\frac{1}{z}=2\cos6^\circ$, then $z^{1000}+\frac{1}{z^{1000}}+1$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: A

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32. Let z_1 and z_2, be two complex numbers with α and β as their principal arguments such that $\alpha + \beta > \pi$ then principal $arg\left(z_1 z_2\right)$ is given by:

A. $\alpha + \beta + \pi$

B. $\alpha + \beta - \pi$

C. $\alpha + \beta - 2\pi$

D. $\alpha + \beta$

Answer: C

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33. Let $arg(z_k) = \frac{(2k+1)\pi}{n}$ where $k = 1, 2, \ldots\ldots\ldots n.$ If $arg(z_1, z_2, z_3, \ldots\ldots\ldots\ldots z_n) = \pi$, then n must be of form $(m \in z)$

A. $4m$

B. $2m$ - 1

C. $2m$

D. None of these

Answer: B

34. Suppose two complex numbers $z = a + ib$, $w = c + id$ satisfy the equation $\frac{z + w}{z} = \frac{w}{z + w}$. Then

A. both a and c are zeros

B. both b and d are zeros

C. both b and d must be non zeros

D. at least one of b and d is non zero

Answer: D

35. If $|z| = 1$ and $z \neq \pm 1$, then one of the possible value of $arg(z) - arg(z + 1) - arg(z - 1)$, is

A. $-\pi/6$

B. $\pi/3$

C. $-\pi/2$

D. $\pi/4$

Answer: C

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36. If $arg\left(z^{3/8}\right) = \frac{1}{2}arg\left(z^2 + \bar{z}^{1/2}\right)$, then which of the following is not possible ?

A. $|z| = 1$

B. $z = \bar{z}$

C. $arg(z) = 0$

D. None of these

Answer: D

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37. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 2|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$, then

A. $-1 \leq Rez \leq 1$

B. $-2 \leq Rez \leq 2$

C. $-3 \leq Rez \leq 3$

D. None of these

Answer: B

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38. If $\alpha, \beta, \gamma \in \{1, \omega, \omega^2\}$ (where ω and ω^2 are imaginery cube roots of unity), then number of triplets (α, β, γ) such that $\left|\frac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha}\right| = 1$ is

A. 3

B. 6

C. 9

D. 12

Answer: C

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39. The value of $\left(\sqrt{3}^{\frac{1}{3}} + \left(3^{5/6}\right)i\right)^3$ is (where $i = \sqrt{-1}$)

A. 24

B. -24

C. -22

D. -21

Answer: B

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40. If $\omega \neq 1$ is a cube root of unity and $a + b = 21$, $a^3 + b^3 = 105$, then the value of $\left(a\omega^2 + b\omega\right)\left(a\omega + b\omega^2\right)$ is be equal to

A. 3

B. 5

C. 7

D. 35

Answer: B

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41. If $z = \frac{1}{2}(\sqrt{3} - i)$, then the least possible integral value of m such that $\left(z^{101} + i^{109}\right)^{106} = z^{m+1}$ is

A. 11

B. 7

C. 8

D. 9

Answer: D

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42. If $y_1 = \max ||z - \omega| - |z - \omega^2||$, where $|z| = 2$ and $y_2 = \max ||z - \omega| - |z - \omega^2||$, where $|z| = \frac{1}{2}$ and ω and ω^2

are complex cube roots of unity, then

A. $y_1 = \sqrt{3}, y_2 = \sqrt{3}$

B. $y_1 < \sqrt{3}, y_2 = \sqrt{3}$

C. $y_1 = \sqrt{3}, y_2 < \sqrt{3}$

D. $y_1 > 3, y_2 < \sqrt{3}$

Answer: C

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43. Let I, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial, with real coefficients having $2\omega^2, 3 + 4\omega, 3 + 4\omega^2$ and $5 - \omega - \omega^2$ as roots is -

A. 4

B. 5

C. 6

D. 7

Answer: B

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44. Number of imaginary complex numbers satisfying the equation, $z^2 = \bar{z}2^{1-|z|}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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45. For the real parameter t,the locus of the complex number $z = \left(1 - t^2\right) + i\sqrt{1 + t^2}$ in the complex plane is

A. $\pi/6$

B. $5\pi/12$

C. $7\pi/12$

D. $11\pi/12$

Answer: B

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46. A root of unity is a complex number that is a solution to the equation, $z^n = 1$ for some positive integer nNumber of roots of unity that are also the roots of the equation $z^2 + az + b = 0$, for some integer a and b is

A. 6

B. 8

C. 9

D. 10

Answer: B

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47. If z is a complex number satisfying the equation $z^6 + z^3 + 1 = 0$. If this equation has a root $re^{i\theta}$ with $90^\circ < 0 < 180^\circ$ then the value of θ is

A. 100°

B. 110°

C. 160°

D. 170°

Answer: C

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48. Suppose A is a complex number and $n \in N$, such that $A^n = (A+1)^n = 1$, then the least value of n is 3 b. 6 c. 9 d. 12

A. 3

B. 6

C. 9

D. 12

Answer: B

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49. If $1, z_1, z_2, z_3, ..., z_{n-1}$ are nth roots of unity, then show that $\left(1 - z_1\right)\left(1 - z_2\right)...\left(1 - z_{n-1}\right) = n$

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50. If $|z - 1 - i| = 1$, then the locus of a point represented by the complex number $5(z - i) - 6$ is

A. circle with centre $(1, 0)$ and radius 3

B. circle with centre $(-1, 0)$ and radius 5

C. line passing through origin

D. line passing through $(-1, 0)$

Answer: B

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51. Let $z \in C$ and if $A = \left\{z : \arg(z) = \frac{\pi}{4}\right\}$ and $B = \left\{z : \arg(z - 3 - 3i) = \frac{2\pi}{3}\right\}$. Then $n(A \quad B) =$

A. 1

B. 2

C. 3

D. 0

Answer: D

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52. $\theta \in [0, 2\pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1 + |\sin\theta|)z_1 + (|\cos\theta| - 1)z_2 - \sqrt{2}z_3 = 0$. If at least one of the complex numbers z_1, z_2, z_3 is nonzero, then number of possible values of θ is

A. Infinite

B. 4

C. 2

D. 8

Answer: B

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53. Let $'z'$ be a comlex number and $'a'$ be a real parameter such that $z^2 + az + a^2 = 0$, then which is of the following is not true ?

A. locus of z is a pair of straight lines

B. $|z| = |a|$

C. $arg(z) = \pm \frac{2\pi}{3}$

D. None of these

Answer: D

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54. Let $z = x + iy$ Then find the locus of $P(z)$ such that

$\frac{1 + z}{z} \in R$

A. union of lines with equations $x = 0$ and $y = -1/2$ but excluding origin.

B. union of lines with equations $x = 0$ and $y = 1/2$but excluding origin.

C. union of lines with equations $x = -1/2$ and $y = 0$ but excluding origin.

D. union of lines with equations $x = 1/2$ and $y = 0$but excluding origin.

Answer: C

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55. Let $A(z_1)$ and $B(z_2)$ are two distinct non-real complex numbers in the argand plane such that $\frac{z_1}{z_2} + \frac{\bar{z}_1}{z_2} = 2$. The value of $|\angle ABO|$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. None of these

Answer: C

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56. It is given that complex numbers z_1 and z_2 satisfy $\left|z_1\right| = 2$and $\left|z_2\right| = 3.$ If the included angled of their corresponding vectors is 60^0 , then find the value of

$$19\left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2.$$

A. 5

B. 6

C. 7

D. 8

Answer: C

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57. If the points A(z),B(-z),C(1-z) are the vertices of an equilateral triangle ABC then Re(z) is

A. $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$

B. $\tan^{-1}\left(\sqrt{15}\right)$

C. $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$

D. $\frac{\pi}{2}$

Answer: A

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58. If $A(z_1)$, $B(z_2)$, $C(z_3)$ are vertices of a triangle such that $z_3 = \frac{z_2 - iz_1}{1 - i}$ and $|z_1| = 3$, $|z_2| = 4$ and $|z_2 + iz_1| = |z_1| + |z_2|$, then area of triangle ABC is

A. $\frac{5}{2}$

B. 0

C. $\frac{25}{2}$

D. $\frac{25}{4}$

Answer: D

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59. Let O, A, B be three collinear points such that $OA. OB = 1$. If O and B represent the complex numbers O and z, then A represents

A. $\frac{1}{\bar{z}}$

B. $\frac{1}{z}$

C. $\bar{z}$

D. z^2

Answer: A

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60. If the tangents at z_1, z_2 on the circle $\left|z - z_0\right| = r$ intersect at z_3, then $\frac{(z_3 - z_1)(z_0 - z_2)}{(z_0 - z_1)(z_3 - z_2)}$ equals

A. 1

B. -1

C. i

D. $-i$

Answer: B

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61. If z_1, z_2 and z_3 are the vertices of ΔABC, which is not right angled triangle taken in anti-clock wise direction and z_0 is the circumcentre, then $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: C

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62. Let P denotes a complex number $z = r(\cos\theta + i\sin\theta)$ on the Argand's plane, and Q denotes a complex number $\sqrt{2|z|^2}\left(\cos\left(\theta + \frac{\pi}{4}\right) + i\sin\left(\theta + \frac{\pi}{4}\right)\right)$. If $'O'$ is the origin, then ΔOPQ is

A. isosceles but not right angled

B. right angled but not isosceles

C. right isosceles

D. equilateral

Answer: C

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Multiple Correct Answer

1. Complex numbers whose real and imaginary parts x and y are integers and satisfy the equation $3x^2 - |xy| - 2y^2 + 7 = 0$, Find the value of x and y.

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2. If $a, b, c, d \in R$ and all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts, then

A. $ab > 0$

B. $bc > 0$

C. $ad > 0$

D. $bc - ad > 0$

Answer: A::B::C

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3. Suppose three real numbers a, b, c are in $G.P.$ Let $z = \frac{a + ib}{c - ib}$. Then

A. $z = \frac{ib}{c}$

B. $z = \frac{ia}{b}$

C. $z = \frac{ia}{c}$

D. $z = 0$

Answer: A::B

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4. w_1, w_2 be roots of $(a+\bar{c})z^2+(b+\bar{b})z+(\bar{a}+c)=0$. If $|z_1|<1, |z_2|<1$, then

A. $|w_1|<1$

B. $|w_1|=1$

C. $|w_2|<1$

D. $|w_2|=1$

Answer: B::D

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5. A complex number z satisfies the equation $\left|Z^2 - 9\right| + \left|Z^2\right| = 41$, then the true statements among the following are

A. $|Z + 3| + |Z - 3| = 10$

B. $|Z + 3| + |Z - 3| = 8$

C. Maximum value of $|Z|$ is 5

D. Maximum value of $|Z|$ is 6

Answer: A::C

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6. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $\left|z_1\right| = 1$. Let P and Q represent the complex numbers z_1 and z_2 in the Argand plane with $\angle POQ = \theta, o^\circ < 180^\circ$ (where O being the origin).Then

A. $b^2 = ac, \theta = \frac{2\pi}{3}$

B. $\theta = \frac{2\pi}{3}, PQ = \sqrt{3}$

C. $PQ = 2\sqrt{3}, b^2 = ac$

D. $\theta = \frac{\pi}{3}, b^2 = ac$

Answer: A::B

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7. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2 - 3 - 4i| = 4$. Then the minimum value of $|Z_1 - Z_2|$ is

A. $|Z_4| = 1$

B. $arg(Z_1 Z_4) = -\pi/2$

C. $\dfrac{Z_5}{\cos(argZ_1)} + \dfrac{Z_6}{\sin(argZ_1)}$ is purely real

D. $Z_5^2 + (\bar{Z}_6)^2$ is purely imaginergy

Answer: A::B::C::D

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8. If $Im\left(\frac{z-1}{e^{\theta i}}+\frac{e^{\theta i}}{z-1}\right)=0$, then find the locus of z

A. a circle with unit radius

B. a circle with radius 3 units

C. a straight line through the point $(3, 0)$

D. a parabola with the vertex $(3, 0)$

Answer: A::C

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9. If α Is the fifth root of unity, then, prove that :

$$\log_2\left|1+\alpha+\alpha^2+\alpha^3-\left(\frac{1}{\alpha}\right)\right|=1$$

10. If z_1, z_2, z_3 are any three roots of the equation $z^6 = (z+1)^6$, then $arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right)$ can be equal to

A. 0

B. π

C. $\frac{\pi}{4}$

D. $-\frac{\pi}{4}$

Answer: A::B

11. Let z_1, z_2, z_3 are the vertices of ΔABC, respectively, such that $\frac{z_3 - z_2}{z_1 - z_2}$ is purely imaginery number. A square on side AC is drawn outwardly. $P\left(z_4\right)$ is the centre of square, then

A. $\left|z_1 - z_2\right| = \left|z_2 - z_4\right|$

B. $arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = +\frac{\pi}{2}$

C. $arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = 0$

D. z_1, z_2, z_3 and z_4 lie on a circle

Answer: C::D

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Matching Column

1. Evaluate :

i^{-47}

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Comprehension

1. Consider the region R in the Argand plane described by the complex number. Z satisfying the inequalities $|Z-2| \leq |Z-4|$, $|Z-3| \leq |Z+3|$, $|Z-i| \leq |Z-3i|$, $|Z+i| \leq |Z+3i|$

Answer the followin questions :

Maximum of $\left|Z_1 - Z_2\right|$ given that Z_1, Z_2 are any two complex numbers lying in the region R is

A. 5

B. 3

C. 1

D. $\sqrt{13}$

Answer: D

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2. Consider the region R in the Argand plane described by the complex number. Z satisfying the inequalities

$|Z - 2| \leq |Z - 4|$, $|Z - 3| \leq |Z + 3|$, $|Z - i| \leq |Z - 3i|$,

$|Z + i| \leq |Z + 3i|$

Answer the followin questions :

The maximum value of $|Z|$ for any Z in R is

A. 5

B. 14

C. $\sqrt{13}$

D. 12

Answer: A

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3. Consider the region R in the Argand plane described by the complex number. Z satisfying the inequalities

$|Z - 2| \leq |Z - 4|$, $|Z - 3| \leq |Z + 3|$, $|Z - i| \leq |Z - 3i|$,

$|Z + i| \leq |Z + 3i|$

Answer the followin questions :

Minimum of $|Z_1 - Z_2|$ given that Z_1, Z_2 are any two complex numbers lying in the region R is

A. 0

B. 5

C. $\sqrt{13}$

D. 3

Answer: A

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4. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The locus of the complex number m is a curve

A. A. straight line

B. B. circle

C. C. ellipse

D. D. hyperbola

Answer: B

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5. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1 x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$. The value of $|m|$,

A. 14

B. $2\sqrt{7}$

C. $7 + \sqrt{41}$

D. $2\sqrt{6} - 4$

Answer: C

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6. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1 x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The maximum value of $|m|$ is

A. $7 + \sqrt{41}$

B. $28 - \sqrt{41}$

C. $\sqrt{41}$

D. $2\sqrt{6} - 4$

Answer: D

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7. The locus of any point $P(z)$ on argand plane is

$$arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}.$$

Then the length of the arc described by the locus of $P(z)$ is

A. $10\sqrt{2}\pi$

B. $\frac{15\pi}{\sqrt{2}}$

C. $\frac{5\pi}{\sqrt{2}}$

D. $5\sqrt{2}\pi$

Answer: B

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8. The locus of any point $P(z)$ on argand plane is

$$arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}.$$

Then the length of the arc described by the locus of $P(z)$ is

A. 62

B. 74

C. 136

D. 138

Answer: C

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9. The locus of any point $P(z)$ on argand plane is

$$arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}.$$

Then the length of the arc described by the locus of $P(z)$ is

A. $75\pi + 50$

B. 75π

C. $\frac{75\pi}{2} + 25$

D. $\frac{75\pi}{2}$

Answer: A

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10. A person walks $2\sqrt{2}$ units away from origin in south west direction $\left(S45\,^\circ W\right)$ to reach A, then walks $\sqrt{2}$ units in south east direction $\left(S45\,^\circ E\right)$ to reach B. From B he travel is 4 units horizontally towards east to reach C. Then he travels along a circular path with centre at origin through an angle of $2\pi/3$ in anti-clockwise direction to reach his destination D.

Position of D in argand plane is (w is an imaginary cube root of unity)

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $-\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: C

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11. A person walks $2\sqrt{2}$ units away from origin in south west direction $\left(S45^{\circ}W\right)$ to reach A, then walks $\sqrt{2}$ units in south east direction $\left(S45^{\circ}E\right)$ to reach B. From B he travel is 4 units horizontally towards east to reach C. Then he travels along a circular path with centre at origin through an angle of $2\pi/3$ in anti-clockwise direction to reach his destination D.

Position of D in argand plane is (w is an imaginary cube root of unity)

A. $(3+i)\omega$

B. $-(1 + i)\omega^2$

C. $3(1 - i)\omega$

D. $(1 - 3i)\omega$

Answer: C

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Illustration

1. Evaluate :

(i) i^{135}

(ii) i^{-47}

(iii) $\left(-\sqrt{-1}\right)^{4n+3}, n \in N$

(iv) $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$

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2. Find the value of $i^n + i^{n-1} + i^{n-2} + i^{n-3}$ for all $n \in N$

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3. Find the value of $1 + i^2 + i^4 + i^6 + i^8$

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4. Evaluate :

i^{124}

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5. Evaluate :

i^{-34}

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6. Evaluate :

i^{75}

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7. Evaluate : i^{103}

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8. Express each of the following in the standerd from $a + ib$

$$\frac{2 + 3i}{3 + 2i}$$

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9. Find the value of $(1 + i)^6 + (1 - i)^6$

A. $16i$

B. 0

C. $-16i$

D. 1

Answer: B

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10. If $\left(\frac{1+i}{1-i}\right)^m = 1,$ then find the least positive integral value of m.

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11. Prove that the triangle formed by the points $1, \frac{1+i}{\sqrt{2}}$,and i as vertices in the Argand diagram is isosceles.

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12. Find real q such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

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13. If the imaginary part of $(2z+1)/(iz+1)$ is -2, then find the locus of the point representing in the complex plane.

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14. If z is a complex number such that $|z - \bar{z}| + |z + \bar{z}| = 4$ then find the area bounded by the locus of z.

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15. If $(x + iy)^5 = p + iq,$ then prove that $(y + ix)^5 = q + ip$

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16. If z = x + iy lies in the third quadrant, then prove that $\frac{\bar{z}}{z}$ also lies in the third quadrant when $y < x < 0$

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17. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If R(z) and I(z), respectively, denote the real and imaginary parts of z, then

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18. Find the relation if z_1, z_2, z_3, z_4 are the affixes of the vertices of a parallelogram taken in order.

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19. Let z_1, z_2, z_3 be three complex numbers and a, b, c be real numbers not all zero, such that

$a + b + c = 0 and a z_1 + b z_2 + c z_3 = 0$. Show that z_1, z_2, z_3 are collinear.

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20. Find real values of x and y for which the complex numbers $-3 + i x^2 y$ and $x^2 + y + 4i$ are conjugate of each other.

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21. about to only mathematics

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22. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$.

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23. Let z be a complex number satisfying the equation $z^2 - (3 + i)z + m + 2i = 0, wherem \in R$. Suppose the equation has a real root. Then find non-real root.

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24. Show that the equation $Z^4 + 2Z^3 + 3Z^2 + 4Z + 5 = 0$ has no root which is either purely real or purely imaginary.

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25. Find the square root of the following: $5 + 12i$

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26. Evaluate :

i^{135}

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27. Solve for $z : z^2 - (3 - 2i)z = (5i - 5)$

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28. Solve the equation $(x - 1)^3 + 8 = 0$ in the set C of all complex numbers.

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29. If n is n odd integer that is greater than or equal to 3 but not a multiple of 3, then prove that $(x + 1)^n - x^n - 1$ is divisible by $x^3 + x^2 + x$.

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30. ω is an imaginary root of unity.

Prove that

If $a+b+c=0$ then prove that

$$\left(a+b\omega+c\omega^2\right)^3+\left(a+b\omega^2+c\omega\right)^3=27abc.$$

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31. Find the complex number ω satisfying the equation z^3-8i and lying in the second quadrant on the complex plane.

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32. $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}+\frac{1}{d+\omega}=\frac{1}{\omega}$ where, a,b,c,d, $\in$ R and ω is a complex cube root of unity then find the value of $\sum\frac{1}{a^2-a+1}$

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33. If sec α and α are the roots of $x^2 - px + q = 0$, then (a) $p^2 = q(q - 2)$ (b) $p^2 = q(q + 2)$ (c)$p^2q^2 = 2q$ (d) none of these

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34. Let $z_1 = \cos 12^\circ + I\sin 12^\circ$ and $z_2 = \cos 48^\circ + i.\sin 48^\circ$. Write complex number $\left(z_1 + z_2\right)$ in polar form. Find its modulus and argument.

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35. Covert the complex number $z = 1 + \frac{\cos(8\pi)}{5} + i.\frac{\sin(8\pi)}{5}$ in polar form. Find its modulus and argument.

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36. Let z and w be two nonzero complex numbers such that $|z| = |w|$ and $arg(z) + arg(w) = \pi$. Then prove that $z = -\bar{w}$.

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37. Find nonzero integral solutions of $|1 - i|^x = 2^x$.

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38. Let z be a complex number satisfying $|z| = 3|z - 1|$. Then prove that $\left|z - \frac{9}{8}\right| = \frac{3}{8}$

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39. If complex number z=x +iy satisfies the equation $Re(z + 1) = |z - 1|$, then prove that z lies on $y^2 = 4x$.

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40. Solve the equation $|z| = z + 1 + 2i$

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41. Find the range of real number α for which the equation $z + \alpha|z - 1| + 2i = 0$ has a solution.

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42. Find the Area bounded by complex numbers $arg|z| \leq \frac{\pi}{4}$ and $|z - 1| < |z - 3|$

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43. Prove that traingle by complex numbers z_1, z_2 and z_3 is equilateral if $\left|z_1\right| = \left|z_2\right| = \left|z_3\right|$ and $z_1 + z_2 + z_3 = 0$

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44. Show that $e^{2mi\theta}\left(\frac{i\cot\theta + 1}{i\cot\theta - 1}\right)^{m} = 1.$

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45. $Z_1 \neq Z_2$ are two points in an Argand plane. If $a\left|Z_1\right| = b\left|Z_2\right|$, then prove that $\frac{aZ_1 - bZ_2}{aZ_1 + bZ_2}$ is purely imaginary.

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46. Find the real part of $(1 - i)^{-i}$

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47. If $\left(\sqrt{8}+i\right)^{50}=3^{49}(a+ib)$, then find the value of a^2+b^2.

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48. Show that $\left(x^2+y^2\right)^4=\left(x^4-6x^2y^2+y^4\right)^2+\left(4x^3y-4xy^3\right)^2$.

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49. If $arg(z_1) = 170^0 and arg(z_2) 70^0$, then find the principal argument of z_1z_2.

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50. Find the value of expression

$$\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)\left(\cos\left(\frac{\pi}{2^2}\right) + i\sin\left(\frac{\pi}{2^2}\right)\right) \ldots \ldots \infty$$

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51. Find the principal argument of the complex number

$$\frac{(1+i)^5\left(1+\sqrt{3i}\right)^2}{-1i\left(-\sqrt{3}+i\right)}$$

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52. If $z = \frac{\left(\sqrt{3} + i\right)^{17}}{(1 - i)^{50}}$, then find $amp(z)$.

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53. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$, show that $|w| = 1z$ is purely real.

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54. It is given the complex numbers z_1 and z_2, $\left|z_1\right| = 2$ and $\left|z_2\right| = 3$. If the included angle of their corresponding vectors is $60\ °$, then find value of $\left|\dfrac{z_1 + z_2}{z_1 - z_2}\right|$

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55. Solve the equation $z^3 = \bar{z}(z \neq 0)$

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56. If $2z_1 / 3z_2$ is a purely imaginary number, then find the value of $|\left((z_1 - z_2) / \left(z_1 + z_2\right)\right|$

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57. Find the complex number satisfying the system of equations $z^3 + \omega^7 = 0 and z^5 \omega^{11} = 1$.

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58. Express the following in $a + ib$ form:

(i) $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4$

(ii) $\frac{(\cos 2\theta - i\sin 2\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2} (\cos 3\theta - i\sin 3\theta)^{-9}}$

(iii) $\frac{(\sin\pi/8 + i\cos\pi/8)^8}{(\sin\pi/8 - i\cos\pi/8)^8}$

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59. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If R(z) and I(z), respectively, denote the real and imaginary parts of z, then

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60. Prove that the roots of the equation $x^4 - 2x^2 + 4 = 0$ forms a rectangle.

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61. If $z +$ 1/z $= 2\cos\theta$, prove that $\left|\left(z^{2n} - 1\right)/\left(z^{2n} + 1\right)\right| = |\tan n\theta|$

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62. If $z = x + iy$ is a complex number with $x, y \in Qand|z| = 1$, then show that $\left|z^{2n} - 1\right|$ is a rational number for every $n \in N$.

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63. If $z = \cos\theta + i\sin\theta$ is a root of the equation $a_0z^n + a_2z^{n-2} + \ldots . + a_{n-1}z + a_n = 0$, then prove that

$a_0 + a_1\cos\theta + a_2^{\cos 2}\theta + \quad + a_n\cos n\theta = 0$

$a_1\sin\theta + a_2^{\sin 2}\theta + \quad + a_n\sin n\theta = 0$

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64. If $\left|z_1\right| = 1, \left|z_2\right| = 2, \left|z_3\right| = 3, and \left|9z_1z_2 + 4z_1z_3 + z_2z_3\right| = 12,$ then find the value of $\left|z_1 + z_2 + z_3\right|$.

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65. If $\alpha\, and\, \beta$ are different complex numbers with $|\beta| = 1,\ f \in d\left|\frac{\beta - \alpha}{1 - \alpha\beta}\right|$

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66. Given that $\left|z_1 + z_2\right|^2 = \left|z_1\right|^2 + \left|z_2\right|^2$, prove that $\frac{z_1}{z_2}$ is purely imaginary.

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67. Let $\left|\left(z_1 - 2z_2\right)/\left(2 - z_1z_2\right)\right| = 1$ and $\left|z_2\right| \neq 1$, where z_1 and z_2 are complex numbers. shown that $\left|z_1\right| = 2$

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68. If $z_1 and z_2$ are two complex numbers and $c > 0$, then prove that $\left|z_1 + z_2\right|^2 \leq (1 + c)\left|z_1\right|^2 + \left(1 + c^{-1}\right)\left|z_2\right|^2$.

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69. If z_1, z_2, z_3, z_4 are the affixes of four point in the Argand plane, z is the affix of a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then prove that z_1, z_2, z_3, z_4 are concyclic.

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70. if $|z_1 + z_2| = |z_1| + |z_2|$, then prove that $arg(z_1) = arg(z_2)$ if $|z_1 - z_2| = |z_1| + |z_2|$, then prove that $arg(z_1) = arg(z_2) = \pi$

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71. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz + iz$ is $\frac{1}{2}|z|^2$

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72. Find the minimum value of $|z - 1|$ if $||z - 3| - |z + 1||= 2.$

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73. Find the greatest and the least value of $\left|z_1 + z_2\right|$ if $z_1 = 24 + 7iand\left|z_2\right| = 6.$

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74. If z is a complex number, then find the minimum value of $|z| + |z - 1| + |2z - 3|^{.}$

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75. If $\left|z_1 - 1\right| \leq , \left|z_2 - 2\right| \leq 2, \left|z_{33}\right| \leq 3,$ then find the greatest value of $\left|z_1 + z_2 + z_3\right|^{.}$

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76. If sec α and α are the roots of $x^2 - px + q = 0,$ then (a) $p^2 = q(q - 2)$ (b) $p^2 = q(q + 2)$ (c)$p^2q^2 = 2q$ (d) none of these

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77. Identify the locus of z if $z = a + \frac{r^2}{z - a}, \; > 0.$

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78. If z is any complex number such that $|3z - 2| + |3z + 2| = 4$, then identify the locus of z.

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79. If $|z| = 1$ and let $\omega = \dfrac{(1 - z)^2}{1 - z^2}$, then prove that the locus of ω is equivalent to $|z - 2| = |z + 2$

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80. Let z be a complex number having the argument θ, $0 < \theta < \frac{\pi}{2}$, and satisfying the equation $|z - 3i| = 3$. Then find the value of $\cot\theta - \frac{6}{z}$

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81. How many solutions the system of equations $||z + 4| - |z - 3i||$ = 5 and |z| = 4 ` has ?

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82. Prove that $|Z - Z_1|^2 + |Z - Z_2|^2 = a$ will represent a real circle [with center $\left(|Z_1 + Z_2|^{/}2 + \right)$] on the Argand plane if $2a \geq |Z_1 - Z_1|^2$

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83. If $|z - 2 - 3i|^2 + |z - 5 - 7i|^2 = \lambda$ respresents the equation of circle with least radius, then find the value of λ.

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84. If $\frac{|2z - 3|}{|z - i|} = k$ is the equation of circle with complex number 'I' lying inside the circle, find the values of K.

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85. Find the point of intersection of the curves $arg(z - 3i) = \frac{3\pi}{4} andarg(2z + 1 - 2i) = \pi/4.$

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86. If complex numbers $z_1 z_2$ and z_3 are such that $|z_1| = |z_2| = |z_3|$, then prove that $arg\left(\frac{z_2}{z_1}\right) = arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)^2.$

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87. If the triangle fromed by complex numbers z_1, z_2 and z_3 is equilateral then prove that $\frac{z_2 + z_3 - 2z_1}{z_3 - z_2}$ is purely imaginary number

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88. Show that the equation of a circle passings through the origin and having intercepts a and b on real and imaginary axis, respectively, on the argand plane is $Re\left(\frac{z - a}{z - ib}\right) = 0$

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89. The triangle formed by $A(z_1), B(z_2)$ and $C(z_3)$ has its circumcentre at origin .If the perpendicular form A to BC intersect the circumference at z_4 then the value of $z_1z_4 + z_2z_3$ is

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90. Let vertices of an acute-angled triangle are $A(z_1), B(z_2), andC(z_3)^{\cdot}$ If the origin O is he orthocentre of the triangle, then prove that $z_1(z)_2 + (z)_1z_2 =_2(z)_3 + (z)_2z_3 = z_3(z)_1 + (z)_3z_1^{\cdot}$

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91. If z_1, z_2, z_3 are three complex numbers such that $5z_1 - 13z_2 + 8z_3 = 0,$ then prove that $\left|z_1(z)_1 1 z_2(z)_2 1 z_3(z)_3 1\right| = 0$

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92. If $z = z_0 + A\left(z - (z)_0\right), whereA$ is a constant, then prove that locus of z is a straight line.

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93. $z_1 and z_2$ are the roots of $3z^2 + 3z + b = 0.$ if $O(0), \left(z_1\right), \left(z_2\right)$ form an equilateral triangle, then find the value of $b^{.}$

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94. Let z_1, z_2 and z_3 be three complex number such that

$$\left|z_1 - 1\right| = \left|z_2 - 1\right| = \left|z_3 - 1\right| \text{ and } arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \frac{\pi}{6}$$

then prove that $z_2^3 + z_3^3 + 1 = z_2 + z_3 + z_2 z_3$.

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95. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equailateral triangle. If z_0 is the circumcentre of the triangle , then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$.

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96. In the Argands plane what is the locus of $z(\neq 1)$ such that $arg\left\{\frac{3}{2}\left(\frac{2z^2 - 5z + 3}{2z^2 - z - 2}\right)\right\} = \frac{2\pi}{3}\cdot$

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97. If $\left(\frac{3 - z_1}{2 - z_1}\right)\left(\frac{2 - z_2}{3 - z_2}\right) = k(k > 0)$, then prove that points $A(z_1), B(z_2), C(3), andD(2)$ (taken in clockwise sense) are concyclic.

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98. If z_1, z_2, z_3 are complex numbers such that $\left(2/z_1\right) = \left(1/z_2\right) + \left(1/z_3\right)$, then show that the points represented by z_1, z_2, z_3 lie one a circle passing through the origin.

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99. $A\left(z_1\right), B\left(z_2\right), C\left(z_3\right)$ are the vertices of he triangle ABC (in anticlockwise). If $\angle ABC = \pi/4$ and $AB = \sqrt{2}(BC)$, then prove that $z_2 = z_3 + i\left(z_1 - z_3\right)$.

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100. If sec α and α are the roots of $x^2 - px + q = 0$, then

(a) $p^2 = q(q - 2)$ (b) $p^2 = q(q + 2)$ (c)$p^2q^2 = 2q$ (d) none of these

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101. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\frac{(z - z_1)}{(z - z_2)}$ is $\frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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102. Complex numbers z_1, z_2 and z_3 are the vertices A,B,C respectivelt of an isosceles right angled triangle with right angle at C. show that $\left(z_1 - z_2\right)^2 = 2\left(z_1 - z_3\right)\left(z_3 - z_2\right)$.

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103. Let $z_1, z_2 and z_3$ represent the vertices $A, B, and C$ of the triangle ABC, respectively, in the Argand plane, such that $\left|z_1\right| = \left|z_2\right| = 5.$ Prove that $z_1 \sin 2A + z_2 \sin 2B + z_3 \sin 2C = 0.$

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104. F $a = \cos(2\pi/7) + i\sin(2\pi/7)$, then find the quadratic equation whose roots are $\alpha = a + a^2 + a^4 and \beta = a^3 + a^5 + a^7$.

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105. If ω is an imaginary fifth root of unity, then find the value of $loe_2\left|1 + \omega + \omega^2 + \omega^3 - 1/\omega\right|$.

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106. If 1, $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_s$ are ninth roots of unity (taken in counter -clockwise sequence in the Argard plane). Then find the value of

$|(2-\alpha_1)(2-\alpha_3), (2-\alpha_5)(2-\alpha_7)|$.

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107. find the sum of squares of all roots of the equation.

$x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$

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108. Find roots of the equation $(z+1)^5 = (z-1)^5$.

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109. If the roots of $(z-1)^n = i(z+1)^n$ are plotted in ten Arg and plane, then prove that they are collinear.

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110. Let 1, $z_1, z_2, z_3, \ldots, z_{n-1}$ be the nth roots of unity. Then prove that $\left(1-z_1\right)\left(1-z_2\right)\ldots\left(1-z_{n-1}\right) = n$.

Also,deduce that $\sin.\frac{\pi}{n}\sin.\frac{2\pi}{\pi}\sin.\frac{3\pi}{n}\ldots\sin.\frac{(n-1)\pi}{n} = \frac{\pi}{2^{n-1}}$

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Sloved Examples

1. if $\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)]+[1/(b+\omega)]+[1/(c+\omega)]=2\omega^2$ and $\left[1/(a+\omega)^2\right]+\left[1/(b+\omega)^2\right]+\left[1/(c+\omega)^2\right]=2\omega$, then find the value of $[1/(a+1)]+[1/(b+1)]+[1/(c+1)]$.

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2. If $z_1 and z_2$ are complex numbers and $u=\sqrt{z_1z_2}$, then prove that $|z_1|+|z_2|=\left|\frac{z_1+z_2}{2}+u\right|+\left|\frac{z_1+z_2}{2}-u\right|$

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3. If a is a complex number such that $|a| = 1$, then find the value of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root.

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4. Let z, z_0 be two complex numbers. It is given that $|z| = 1$ and the numbers $z, z_0, z\bar{z}_0, 1$ and 0 are represented in an Argand diagram by the points P,P_0,Q,A and the origin, respectively. Show that $\triangle POP_0$ and $\triangle AOQ$ are congruent. Hence, or otherwise, prove that

$$|z - z_0| = |z\bar{z}_0 - 1| = |\bar{z}z_0 - 1|.$$

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5. Let a, b and c be any three nonzero complex number. If $|z| = 1$ and $'z'$ satisfies the equation $az^2 + bz + c = 0$, prove that $a.\bar{a} = c.\bar{c}$ and |a||b|=$\sqrt{ac\left(\bar{b}\right)^2}$

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6. Let x_1, x_2 are the roots of the quadratic equation $x^2 + ax + b = 0$, where a,b, are complex numbers and y_1, y_2 are the roots of the quadratic equation $y^2 + |a|yy + |b| = 0$. If $\left|x_1\right| = \left|x_2\right| = 1$, then prove that $\left|y_1\right| = \left|y_2\right| = 1$

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7. If $\alpha = (z - i)/(z + i)$ show that, when z lies above the real axis, αwill lie within the unit circle which has centre at the origin. Find the locus of α as z travels on the real axis form $- \infty \text{to} + \infty$

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8. If $|z| \leq 1, |w| \leq 1,$ then show that $|z - w|^2 \leq (|z| - |w|)^2 + (argz - argw)^2$

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9. Prove that the distance of the roots of the equation $\left|\sin\theta_1\right|z^3 + \left|\sin\theta_2\right|z^2 + \left|\sin\theta_3|z + |\sin\theta_4| = |3\right|$ from z=0 is greater than $2/3$.

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10. If $|z - (4 + 3i)| = 1$, then find the complex number z for each of the following cases:

(i) $|z|$ is least

(ii) $|z|$ is greatest

(iii) $arg(z)$ is least

(iv) arg(z) is greatest

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11. If a ,b,c, and u,v,w are complex numbers representing the vertices of two triangle such that they are similar, then prove that $\frac{a - c}{a - b} = \frac{u - w}{u - v}$

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12. Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane, respectively. If $\angle AOB = \theta \neq 0$ and $OA = OB$, *where* O is the origin, prove that $p^2 = 4q\cos^2(\theta/2)$.

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13. The altitude form the vertices A, B and C of the triangle ABC meet its circumcircle at D,E and F, respectively . The complex number representing the points D,E, and F are z_1, z_2 and z_3, respectively. If $(z_3 - z_1)/(z_2 - z_1)$ is purely real, then show that triangle ABC is right-angled at A.

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14. Let A, B, C, D be four concyclic points in order in which $AD:AB = CD:CB$ If A, B, C are repreented by complex numbers a, b, c representively, find the complex number associated with point D

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15. If $n \geq 3$ and $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are the n,nth roots of unity, then find value of $\sum \sum_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$

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Exercise 3 1

1. Is the following computation correct? If not give the correct computation: $\left[\sqrt{(-2)}\sqrt{(-3)}\right] = \sqrt{(-2) \cdot 3} = \sqrt{6}$

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2. Find the value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$

$(1 + i)^6 + (1 - i)^6$

A. -2

B. 0

C. 2

D. -1

Answer: A

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3. If $(a + b) + i(a - b) = 2 - 3i$ find a and b

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4. Express the following complex number in $a + ib$ form:

$$\frac{3 - 2i}{(2 - i)}$$

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Exercise 3 2

1. प्रश्न 11 से 13 तक कि सम्मिश्र संख्याओं में प्रत्येक का गुणात्मक प्रतिलोम ज्ञात कीजिए ।

$4 - 3i$

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2. Express the following complex number in $a + ib$ form:

$$\frac{i}{1 - i}$$

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3. Express the following complex number in $a + ib$ form:

$$\frac{i}{1 + i}$$

Watch Video Solution

4. Express the following complex number in $a + ib$ form:

$$(1 - i)^3$$

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5. If ω is the complex cube root of unity, then find $\left(1 - \omega + \omega^2\right)^3$.

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6. Express the following complex number in $a + ib$ form: $(1 - i)^2$

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7. Find the real numbers x and y, if $(x - iy)i$ is the conjugate of $5 - 3i$

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8. If z_1, z_2, z_3 are three nonzero complex numbers such that $z_3 = (1 - \lambda)z_1 + \lambda z_2 where\lambda \in R - \{0\},$ then prove that points corresponding to $z_1, z_2 and z_3$ are collinear .

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9. Find the square root of $16 + 30i$

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Exercise 3 3

1. Find the square root of $7 + 24i$

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2. Find the square root of i.

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3. Find the square root of $1 + 2\sqrt{6}i$.

Watch Video Solution

4. Find the square root of $8 + 6i$.

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5. If ω is the complex cube root of unity, then find $\omega^{99} + \omega^{100} + \omega^{101}$

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6. Simplify: $\dfrac{i + i^2 + i^3 + i^4}{1 + i}$

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7. Find the square root of $3 + 4i$

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Exercise 3 4

1. if α and β are imaginary cube root of unity then prove

$(\alpha)^4 + (\beta)^4 + (\alpha)^{-1}.(\beta)^{-1} = 0$

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2. If ω is a complex cube roots of unity, then find the value of the $(1 + \omega)\left(1 + \omega^2\right)\left(1 + \omega^4\right)\left(1 + \omega^8\right)$... to 2n factors.

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3. Write the complex number in a + ib form using cube roots of unity: $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{1000}$

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4. If z is a complex number, then find the minimum value of $|z| + |z - 1| + |2z - 3|$.

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5. Find the common roots of $x^{12} - 1 = 0$ and $x^4 + x^2 + 1 = 0$

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6. If α, β and γ are the roots of $X^3 - 3X^2 + 3X + 7 + 0,$ find the value of $\frac{\alpha = 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$.

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7. Prove that $t^2 + 3t + 3$ is a factor of $(t+1)^{n+1} + (t+2)^{2n-1}$ for all intergral values of n $\in$ N.

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Exercise 3 5

1. If sec α and α are the roots of $x^2 - px + q = 0$, then (a) $p^2 = q(q-2)$ (b) $p^2 = q(q+2)$ (c)$p^2q^2 = 2q$ (d) none of these`

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2. Find the modulus and argument of the following complex number: $\frac{1 + i}{1 - i}$

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3. If $\frac{3\pi}{2} < \alpha < 2\pi$ then the modulus argument of $(1 + \cos 2\alpha) + i\sin 2\alpha$

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4. Find the principal argument of the complex number $\frac{\sin(6\pi)}{5} + i\left(1 + \frac{\cos(6\pi)}{5}\right)$.

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5. If $z = re^{i\theta}$, then prove that $\left|e^{iz}\right| = e^{-rs\int h\eta}$.

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6. Find the complex number z satisfying $Re\left(z^2\right) = 0, |z| = \sqrt{3}.$

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7. If $|z - iRe(z)| = |z - Im(z)|$, then prove that z, lies on the bisectors of the quadrants.

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8. Find the locus of the points representing the complex number z for which $|z+5|^2 = |z-5|^2 = 10$.

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9. Solve that equation $z^2 + |z| = 0$, where z is a complex number.

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10. Let $z = x + iy$ be a complex number, where $x and y$ are real numbers. Let $A and B$ be the sets defined by $A = \{z : |z| \le 2\} and B = \{z : (1-i)z + (1+i)\dot{z} \ge 4\}$. Find the area of region $A \cup B$

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11. Real part of $\left(e^{e}\right)^{\iota\theta}$ is

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12. Prove that $z = i^{i}, where i = \sqrt{-1}$, is purely real.

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Exercise 3 6

1. For $z_1 = {}^6\sqrt{(1-i)/\left(1+i\sqrt{3}\right)}, z_2 = {}^6\sqrt{(1-i)/\left(\sqrt{3}+i\right)}$, $z_3 = {}^6\sqrt{(1+i)/\left(\sqrt{3}-i\right)}$, prove that $|z_1| = |z_2| = |z_3|$

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2. If sec α and α are the roots of $x^2 - px + q = 0$, then (a) $p^2 = q(q-2)$ (b) $p^2 = q(q+2)$ (c)$p^2q^2 = 2q$ (d) none of these

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3. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers then $arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right) =$

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4. Find the modulus, argument and the principal agrument of the complex number $(\tan 1 - i)^2$

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5. If $(1 + i)(1 + 2i)(1 + 3i)1 + m) = (x + iy)$, then show that $2 \times 5 \times 10 \times \times \left(1 + n^2\right) = x^2 + y^2$.

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6. If $a + ib = \frac{(x + i)^2}{2x + 1}$, prove that $a^2 + b^2 = \frac{(x + i)^2}{(2x + 1)^2}$

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7. Let z be a complex number satisfying the equation $\left(z^3 + 3\right)^2 = -16$, then find the value of $|z|$

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8. If θ is real and z_1, z_2 are connected by $z12 + z22 + 2z_1z_2\cos\theta = 0$, then prove that the triangle formed by vertices $O, z_1 and z_2$ is isosceles.

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9. If $\left|z_1 - z_0\right| = z_2 - z_1 = \frac{\pi}{2}$, then find z_0.

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10. If $\sec\alpha$ and α are the roots of $x^2 - px + q = 0$, then (a) $p^2 = q(q - 2)$ (b) $p^2 = q(q + 2)$ (c) $p^2q^2 = 2q$ (d) none of these

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Exercise 3 7

1. Simplify the following and express in the form (a+ib): $(5 + 7i)i^4$

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2. Simplify the following and express in the form (a+ib):

$(6 - 5i)(2 + 3i)$

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3. Simplify the following and express in the form (a+ib):

$(7 + i)i^3$

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4. Simplify the following and express in the form (a+ib):

$7i(3 - 4i)$

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5. Simplify the following and express in the form (a+ib):

$(9 - 5i)(1 + i)$

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6. Simplify the following and express in the form (a+ib):

$(5i + 4)8i$

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Exercise 3 8

1. Simplify the following and express in the form (a+ib):

$$\left(1 - 2i + 7i^2\right)i$$

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2. Simplify the following and express in the form (a+ib):

$$(1 - 2i)(1 + i)$$

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3. Simplify the following and express in the form (a+ib):

$$24+\sqrt{-24}$$

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4. Simplify the following and express in the form (a+ib): 2-

$\sqrt{-9}+\sqrt{-49}$

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5. Simplify the following and express in the form (a+ib): 5-

$\sqrt{-125}$

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6. Simplify the following and express in the form (a+ib):

$7+\sqrt{-8}$

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7. Simplify the following and express in the form (a+ib):

$5+\sqrt{-64}$

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8. Evaluate : $3\sqrt{-49} - 8\sqrt{-9}$

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9. Evaluate : $5\sqrt{-25} + 2\sqrt{-625}$

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10. Evaluate : $3\sqrt{-81} - \sqrt{-9}$

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11. Evaluate : $3\sqrt{-16} - 9\sqrt{-9}$

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Exercise 3 9

1. Evaluate : $8\sqrt{-8} - 2\sqrt{-1}$

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2. Evaluate : $5\sqrt{-9} + 2\sqrt{-4}$

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3. Evaluate : $\sqrt{-18} + 3\sqrt{-16}$

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4. If ω is the complex cube root of unity, then find ω^{-97}

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5. Evaluate : $3\sqrt{-12} + 2\sqrt{-9}$

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6. Evaluate : $3\sqrt{-4} + 2\sqrt{-9}$

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7. Evaluate : $\sqrt{-25} + 3\sqrt{-4}$

Watch Video Solution

8. Evaluate : $\sqrt{-25} + \sqrt{-24}$

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Exercise 3 10

1. If ω is the complex cube root of unity, then find $\frac{1}{\omega^{77}}$

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2. If ω is the complex cube root of unity, then find $\omega^{22} + \frac{1}{\omega^{22}}$

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3. If ω is the complex cube root of unity, then find $\frac{1}{\omega^{405}}$

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4. If ω is the complex cube root of unity, then find ω^{-73}

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5. If ω is the complex cube root of unity, then find ω^{-303}

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6. If ω is the complex cube root of unity, then find

$\omega + \frac{1}{\omega} + \frac{1}{\omega^2}$

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7. If ω is the complex cube root of unity, then find $\omega + \frac{1}{\omega}$

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8. If a-5b+3ai=7-3i, then find a and b.

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9. If 7a-b+5ai=7+5i, then find a and b.

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10. If 2b+(a-b)i=2+3i, then find a and b..

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Exercise 3 11

1. If a+(b+2a)i=1-4i, then find a and b.

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2. If a-b+2ai=3+6i, then find a and b.

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3. Simplify: $\left(4i^9 - 3i^9\right)$

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4. Simplify: $\left(4i^8 - 3i^9 + 3\right)$

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5. Simplify: $\left(i^9 + i^{19} + i^8\right)$

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6. Simplify: $\left(i^{50} + i^{51} + i^{49}\right)$

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Single Correct Answer Type

1. Simplify: $\left(i^{30}+i^{20}+i^{4}\right)$

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2. Simplify: $\left(i^{29}+i^{39}+i^{49}\right)$

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3. Find the amplitude of the complex number $z = 6$.

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4. Find the amplitude of the complex number $z = -i$.

Watch Video Solution

5. Find the amplitude of the complex number $z = 7 + 7i$.

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6. Find the amplitude of the complex number $z = 2\sqrt{3} + 6i$

.

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7. Find the amplitude of the complex number $z = 3 + \sqrt{3}i$.

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8. Find the amplitude of the complex number $z = -3 - 3i$.

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9. Find the amplitude of the complex number $z = 7$.

Watch Video Solution

10. Find the amplitude of the complex number $z = 5$.

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11. Find the amplitude of the complex number $z = 3i$.

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12. Find the amplitude of the complex number $z = a - ib$.

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13. Find the amplitude of the complex number $z = \sqrt{3}i$.

Watch Video Solution

14. Find the amplitude of the complex number $z = 2i$.

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15. Find the amplitude of the complex number $z = \sqrt{3} + i$.

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16. Find the amplitude of the complex number $z = 1 + i$.

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17. Find the amplitude of the complex number $z = 1 + i\sqrt{3}$

.

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18. Find the conjugate of the complex number $\sqrt{-1}$

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19. Find the conjugate of the complex number $5 + 2\sqrt{-1}$

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20. Find the conjugate of the complex number $3 - \sqrt{-1}$

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21. Find the modulus of the complex number z=13-5i

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22. Find the modulus of the complex number z=8+6i

Watch Video Solution

23. Find the modulus of the complex number z=11-i

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24. Find the modulus of the complex number z=7i

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25. Find the modulus of the complex number z=8+i

Watch Video Solution

26. Find the modulus of the complex number z=5+2i

Watch Video Solution

27. Find the modulus of the complex number z=4-i

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28. Find the modulus of the complex number z=9-2i

Watch Video Solution

29. Find the modulus of the complex number z=5+i `

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30. Evaluate : i^{341}

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31. Evaluate : i^{503}

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32. Find the modulus of the complex number $z = 3 + 7i$

Watch Video Solution

33. Find the modulus of the complex number z=4+3i

Watch Video Solution

34. Evaluate : i^{50}

Watch Video Solution

35. Evaluate : i^{97}

Watch Video Solution

36. Evaluate : i^{43}

Watch Video Solution

37. Evaluate : i^{13}

Watch Video Solution

38. Evaluate : i^{29}

Watch Video Solution

39. Evaluate : i^{-23}

Watch Video Solution

40. Evaluate : i^{-22}

Watch Video Solution

41. Evaluate : i^{28}

Watch Video Solution

42. Evaluate : i^{-78}

Watch Video Solution

43. Evaluate : i^{-47}

Watch Video Solution

44. Evaluate : i^{-35}

Watch Video Solution

45. Evaluate : i^{-52}

Watch Video Solution

46. Evaluate : i^{-66}

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47. Find the modulus of the complex number $z = \cos\theta + i\sin\theta$

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48. If $z = (i)^{(i)^i}$ w h e r e $i = \sqrt{-1}, then|z|$ is equal to a.1 b. $e^{-\pi/2}$ c. $e^{-\pi}$ d. none of these

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49. If $z = i\log\left(2 - \sqrt{3}\right)$, then $\cos z =$

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50. Evaluate : i^{45}

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51. Evaluate : i^{30}

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52. Evaluate : i^{-41}

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53. Evaluate : i^{31}

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54. Find the modulus of the complex number z=2+3i

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55. Find the modulus of the complex number z=1+i

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56. Let C_1 and C_2 are concentric circles of radius 1 and $\frac{8}{3}$ respectively having centre at $(3, 0)$ on the argand plane. If the complex number z satisfies the inequality $\log_{\frac{1}{3}}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$, then (a) z lies outside C_1 but inside C_2 (b) z line inside of both C_1 and C_2 (c) z line outside both C_1 and C_2 (d) none of these

A. z lies outside C_1 but inside C_2

B. z line inside of both C_1 and C_2

C. z line outside both C_1 and C_2

D. none of these

Answer: A

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57. If $z_1 = 3 + 2i, z_2 = 2 - i$, then find $\bar{z}_1 . \bar{z}_2$

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58. If $z_1 = 3 + i, z_2 = 1 - 3i$, then find $\bar{z}_1 . \bar{z}_2$

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59. If $z_1 = 1 - 3i, z_2 = 1 + i$, then find $\bar{z}_1 . \bar{z}_2$

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60. If $z_1 = 1 - i, z_2 = 2 + i$, then find $\bar{z}_1 . \bar{z}_2$

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61. Find the conjugate of the complex number $z = 13 - i$

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62. Find the conjugate of the complex number $z = 11 + 13i$

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63. Find the conjugate of the complex number $z = \sqrt{7} - \sqrt{5}i$

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64. Find the conjugate of the complex number $z = 5 + 7i$

Watch Video Solution

65. Find the conjugate of the complex number $z = \sqrt{6} + 7i$

Watch Video Solution

66. Find the conjugate of the complex number $z = \sqrt{5} - 3i$

Watch Video Solution

67. Find the conjugate of the complex number $z = \sqrt{5} - 2i$

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68. If $\left|z_1\right| + \left|z_2\right| = 1 and z_1 + z_2 + z_3 = 0$ then the area of the triangle whose vertices are z_1, z_2, z_3 is $3\sqrt{3}/4$ b. $\sqrt{3}/4$ c. 1 d. 2

A. $3\sqrt{3/4}$

B. $\sqrt{3/4}$

C. 1

D. 2

Answer: A

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69. Let $z and \omega$ be two complex numbers such that $|z| \leq 1, |\omega| \leq 1 and |z - i\omega| = |z - i\omega| = 2, then z$ equals 1 or i b. i or $-i$ c. 1 or -1 d. i or -1

A. $\frac{2}{3}$

B. $\frac{\sqrt{5}}{3}$

C. $\frac{3}{2}$

D. $\frac{2\sqrt{5}}{3}$

Answer: C

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70. Let z_1, z_2, z_3, z_4 are distinct complex numbers satisfying $|z| = 1$ and $4z_3 = 3(z_1 + z_2)$, then $|z_1 - z_2|$ is equal to

A. 1 or i

B. i or $-i$

C. 1 or i

D. i or -1

Answer: D

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71. z_1, z_2, z_3, z_4 are distinct complex numbers representing the vertices of a quadrilateral $ABCD$ taken in order. If $z_1 - z_4 = z_2 - z_3$ and $\arg\left[\left(z_4 - z_1\right)/\left(z_2 - z_1\right)\right] = \pi/2$, the quadrilateral is

A. rectangle

B. rhombus

C. square

D. trapezium

Answer: A

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72. If $k + |k + z^2| = |z|^2 \left(k \in R^-\right)$, then possible argument of z is

A. 0

B. π

C. $\pi/2$

D. none of these

Answer: C

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73. If z_1, z_2, z_3 are the vertices of an equilational triangle ABC such that $|z_1 - i| = |z_2 - i| = |z_3 - i|$, then $|z_1 + z_2 + z_3|$ equals to

A. $3\sqrt{3}$

B. $\sqrt{3}$

C. 3

D. $\frac{1}{3\sqrt{3}}$

Answer: C

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74. If z=6-i, then find $z - \bar{z}$

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75. If z=6+i, then find $z + \bar{z}$

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76. If z=5-3i, then find $z - \bar{z}$

Watch Video Solution

77. If z=5-3i, then find $z + \bar{z}$

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78. about to only mathematics

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: B

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79. $z_1 and z_2$ lie on a circle with center at the origin. The point of intersection z_3 of he tangents at $z_1 and z_2$ is given by $\frac{1}{2}\left(z_1 + (z)_2\right)$ b. $\frac{2z_1z_2}{z_1 + z_2}$ c. $\frac{1}{2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)$ d. $\frac{z_1 + z_2}{(z)_1(z)_2}$

A. $\frac{1}{2}\left(\bar{z}_1 + \bar{z}_2\right)$

B. $\frac{2z_1z_2}{z_1 + z_2}$

C.

D.

Answer: B

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80. If $\arg\left(\frac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \frac{\pi}{2}$ and $\left|\frac{z}{|z|} - z_1\right| = 3$, then $|z_1|$

equals to a. $\sqrt{3}$ b. $2\sqrt{2}$ c. $\sqrt{10}$ d. $\sqrt{26}$

A. $\sqrt{26}$

B. $\sqrt{10}$

C. $\sqrt{3}$

D. $2\sqrt{2}$

Answer: B

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81. about to only mathematics

A. $\frac{1}{2}\left|z_1 - z_2\right|^2$

B. $\frac{1}{2}\left|z_1 - z_2\right|r$

C. $\frac{1}{2}\left|z_1 - z_2\right|^2 r^2$

D. $\frac{1}{2}\left|z_1 - z_2\right|^2$

Answer: B

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82. If $\sec\alpha$ and α are the roots of $x^2 - px + q = 0$, then (a) $p^2 = q(q - 2)$ (b) $p^2 = q(q + 2)$ (c)$p^2q^2 = 2q$ (d) none of these

A. $\pi + 8$

B. $\pi + 4$

C. $2\pi + 4$

D. $\pi + 6$

Answer: A

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83. about to only mathematics

A. $e^{i\theta}$

B. $e^{-i\theta}$

C. $\omega, \bar{\omega}$

D. $\omega + \bar{\omega}$

Answer: D

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84. If $p and q$ are distinct prime numbers, then the number of distinct imaginary numbers which are pth as well as qth roots of unity are. $\min(p, q)$ b. $\min(p, q)$ c. 1 d. $zero$

A. min(p,q)

B. max(p,q)

C. 1

D. zero

Answer: D

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85. Given z is a complex number with modulus 1. Then the equation $[(1+ia)/(1-ia)]^4 = z$ has all roots real and distinct two real and two imaginary three roots two imaginary one root real and three imaginary

A. all roots real and distinct

B. two real and tw imaginary

C. three roots real and one imaginary

D. one root real and three imaginary

Answer: A

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86. The value of z satisfying the equation

$\log z + \log z^2 + \dot{} + \log z^n = 0 is$

A. $\cos. \frac{4m\pi}{n(n+1)} + i\sin. \frac{4m\pi}{n(n+1)}, m = 0, 1, 2, ...$

B. $\cos. \frac{4m\pi}{n(n+1)} - i\sin. \frac{4m\pi}{n(n+1)}, m = 0, 1, 2, ...$

C. $\sin. \frac{4m\pi}{n} + i\cos. \frac{4m\pi}{n}, m = 0, 1, 2, ...$

D. 0

Answer: A

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87. If $n \in N > 1$, then the sum of real part of roots of $z^n = (z+1)^n$ is equal to

A. $\frac{n}{2}$

B. $\frac{(n - 1)}{2}$

C. $-\frac{n}{2}$

D. $\frac{(1 - n)}{2}$

Answer: D

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88. Which of the following represents a points in an Argand pane, equidistant from the roots of the equation $(z+1)^4 = 16z^4$? (0, 0) b. $\left(-\frac{1}{3}, 0\right)$ c. $\left(\frac{1}{3}, 0\right)$ d. $\left(0, \frac{2}{\sqrt{5}}\right)$

A. $(0, 0)$

B. $\left(-\frac{1}{3}, 0\right)$

C. $\left(\frac{1}{3}, 0\right)$

D. $\left(0, \frac{2}{\sqrt{5}}\right)$

Answer: C

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89. Let a be a complex number such that $|a| < 1 and z_1, z_2, z_3, \ldots$ be the vertices of a polygon such that $z_k = 1 + a + a^2 + \ldots + a^{k-1}$ for all $k = 1, 2, 3, Then z_1, z_2$ lie within the circle (a) $\left|z - \frac{1}{1 - a}\right| = \frac{1}{|a - 1|}$ (b) $\left|z + \frac{1}{a + 1}\right| = \frac{1}{|a + 1|}$ (c) $\left|z - \frac{1}{1 - a}\right| = |a - 1|$ (d) $\left|z + \frac{1}{a + 1}\right| = |a + 1|$

A. $\left|z - \frac{1}{1 - a}\right| = \frac{1}{|a - 1|}$

B. $\left|z + \frac{1}{a + 1}\right| = \frac{1}{|a + 1|}$

C. $\left|z - \frac{1}{1 - a}\right| = |a - 1|$

D. $\left|z + \frac{1}{1 - a}\right| = |a - 1|$

Answer: A

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90. Let $z = x + iy$ be a complex number where $xandy$ are integers. Then, the area of the rectangle whose vertices are the roots of the equation $zz^3 + zz^3 = 350$ is 48 (b) 32 (c) 40 (d) 80

A. 48

B. 32

C. 40

D. 80

Answer: A

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91. Let z be a complex number such that the imaginary part of z is nonzero and a = z2 + z + 1 is real. Then a cannot take the value (A) –1 (B) 1 3 (C) 1 2 (D) 3 4

A. -1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: D

Watch Video Solution

92. Let complex numbers α and $\frac{1}{\alpha}$ lies on circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$ then $|\alpha|$ is equal to

A. $1/\sqrt{2}$

B. $1/2$

C. $1/\sqrt{7}$

D. $1/3$

Answer: C

Multiple Correct Answers Type

1. If $z = \omega, \omega^2 where \omega$ is a non-real complex cube root of unity, are two vertices of an equilateral triangle in the Argand plane, then the third vertex may be represented by $z = 1$ b. $z = 0$ c. $z = -2$ d. $z = -1$

A. $z = 1$

B. $z = 0$

C. $z = -2$

D. $z = -1$

Answer: A::C

Watch Video Solution

2. If $amp(z_1z_2) = 0 and |z_1| = |z_2| = 1, then \ z_1 + z_2 = 0$ b. $z_1z_2 = 1$ c. $z_1 = z_2$ d. none of these

A. $z_1 + z_2 = 0$

B. $z_1z_2 = 1$

C. $z_1 = \bar{z}_2$

D. none of these

Answer: B::C

Watch Video Solution

3. If sec α and α are the roots of $x^2 - px + q = 0,$ then (a) $p^2 = q(q - 2)$ (b) $p^2 = q(q + 2)$ (c)$p^2q^2 = 2q$ (d) none of these

A. $-\dfrac{\pi}{4}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{3\pi}{4}$

D. $-\dfrac{3\pi}{4}$

Answer: A::B::C::D

Watch Video Solution

4. Values $(s)(-i)^{1/3}$ is/are $\dfrac{\sqrt{3}-i}{2}$ b. $\dfrac{\sqrt{3}+i}{2}$ c. $\dfrac{-\sqrt{3}-i}{2}$ d. $\dfrac{-\sqrt{3}+i}{2}$

A. $s\frac{\sqrt{3}-i}{2}$

B. $\frac{\sqrt{3}+i}{2}$

C. $\frac{-\sqrt{3}-i}{2}$

D. $\frac{-\sqrt{3}+i}{2}$

Answer: A::C

Watch Video Solution

5. If $a^3 + b^3 + 6abc = 8c^3 \& \omega$ is a cube root of unity then: (a)a, b, c are in $A.P.$ (b) $a, b, c,$ are in $H.P.$ (c) $a + b\omega - 2c\omega^2 = 0$ (d) $a + b\omega^2 - 2c\omega = 0$

A. a, c, b are in A.P

B. a,c,b are in H.P

C. $a + b\omega - 2c\omega^2 = 0$

D. $a + b\omega^2 - 2c\omega = 0$

Answer: A::C::D

Watch Video Solution

6. Let z_1 and z_2 be two non -zero complex number such that $|z_1 + z_2| = |z_1| = |z_2|$. Then $\frac{z_1}{z_2}$ can be equal to (ω is imaginary cube root of unity).

A. $1 + \omega$

B. $1 + \omega^2$

C. ω

D. ω^2

Answer: C::D

Watch Video Solution

7. If $p = a + b\omega + c\omega^2$, $q = b + c\omega + a\omega^2$, and $r = c + a\omega + b\omega^2$, where $a, b, c \neq 0$ and ω is the complex cube root of unity, then (a) $p + q + r = a + b + c$ (b) $p^2 + z^2 + r^2 = a^2 + b^2 + c^2$ (c) $p^2 + z^2 + r^2 = -2(pq + qr + rp)$ (d) none of these

A. If p,q,r lie on the circle |z|=2, the trinagle formed by these point is equilateral.

B. $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$

C. $p^2 + q^2 + r^2 = 2(pq + qr + rp)$

D. none of these

Answer: A::C

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8. Find the square root of the complex number z=2i .

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9. Find the square root of the complex number z=8i .

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10. Evaluate: $1 + i^{100} + i^{101}$

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11. Evaluate: $1 + i^{24} + i^{25}$

Watch Video Solution

12. If z=4+3i, then find $z - \bar{z}$

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13. Evaluate: $1 + i^{49} + i^{50}$

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14. If z=4-3i, then find $z - \bar{z}$

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15. If $z_1 = 1 + 2i, z_2 = -i$, then find $\bar{z}_1 + \bar{z}_2$.

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16. If $z_1 = 1 - i, z_2 = 1 + i$, then find $\bar{z}_1 + \bar{z}_2$.

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17. If z=1+i, then find $z\bar{z}$.

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18. If z=1-i, then find $z\bar{z}$.

Watch Video Solution

19. If z=2-i, then find $z\bar{z}$.

Watch Video Solution

20. If z=2+i, then find $z\bar{z}$.

Watch Video Solution

21. If $z = 2 - \sqrt{3}i$, then find $z\bar{z}$.

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22. If $z = 2 + \sqrt{3}i$, then find $z\bar{z}$.

Watch Video Solution

23. If $z = 3 + \sqrt{7}i$, then find $z\bar{z}$.

Watch Video Solution

24. If $z = 3 - \sqrt{7}i$, then find $z\bar{z}$.

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25. If z=5-3i, then find $\bar{\bar{z}}$.

Watch Video Solution

26. Evaluate : i^{555}

Watch Video Solution

27. If z=2+7i, then find $\overset{=.}{z}$

Watch Video Solution

28. If z=2-7i, then find $\overset{=.}{z}$

Watch Video Solution

29. If z=5+3i, then find $\overset{=.}{z}$

Watch Video Solution

30. If z=$2+\sqrt{3}i$, then find $\overset{=.}{z}$

Watch Video Solution

31. If z=2 - $\sqrt{3}i$, then find $\overline{\overline{z}}$.

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32. Evaluate: $i^2 + i^3$

Watch Video Solution

33. Evaluate: $i^3 + i^{12}$

Watch Video Solution

34. Evaluate: $i^4 + i^{13}$

Watch Video Solution

35. Evaluate: $i^{18} + i^{20}$

Watch Video Solution

36. Evaluate: $i^9 + i^{19}$

Watch Video Solution

37. Evaluate: $1 + i^{32} + i^3$

Watch Video Solution

38. Evaluate: $1 + i^{15} + i^{16}$

Watch Video Solution

39. Evaluate: $1 + i^{99} + i^{100}$

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40. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $3\sqrt{3} + 3i$

Watch Video Solution

41. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $1 + \sqrt{3}i$

Watch Video Solution

42. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $(1 - i)^2$

Watch Video Solution

43. If ω is the complex cube root of unity, then find $(1 + \omega)\left(1 + \omega^2\right)^{.}$

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44. If ω is the complex cube root of unity, then find $(1-\omega)\left(1-\omega^2\right)$.

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45. If ω is the complex cube root of unity, then find $\left(1+\omega^2\right)\left(1+\omega^4\right)$

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46. If ω is the complex cube root of unity, then find $\left(1-\omega^2\right)\left(1-\omega^4\right)$

Watch Video Solution

47. If ω is the complex cube root of unity, then find $(1-\omega)\left(1-\omega^5\right)$

Watch Video Solution

48. If ω is the complex cube root of unity, then find $(1+\omega)\left(1+\omega^5\right)$

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49. If ω is the complex cube root of unity, then find $\left(1-\omega-\omega^2\right)^{3}$.

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Linked Comprehension Type

1. If ω is the complex cube root of unity, then find $\left(1-\omega+\omega^2\right)^4$

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2. If ω is the complex cube root of unity, then find $\left(1+\omega-\omega^2\right)^3$

Watch Video Solution

3. If ω is the complex cube root of unity, then find $\left(1 - 3\omega + \omega^2\right)^3$

Watch Video Solution

4. If ω is the complex cube root of unity, then find $\left(2 + \omega + \omega^2\right)^3$

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5. If ω is the complex cube root of unity, then find $\left(2 - \omega + 2\omega^2\right)^4$

Watch Video Solution

6. If ω is the complex cube root of unity, then find

$\left(2 + 3\omega + 3\omega^2\right)^2$

Watch Video Solution

7. If ω is the complex cube root of unity, then find

$\left(1 - 2\omega + \omega^2\right)^5$

Watch Video Solution

8. If ω is the complex cube root of unity, then find

$\left(2 + \omega + \omega^2\right)^5$

Watch Video Solution

9. If ω is the complex cube root of unity, then find $\left(1 - 3\omega + \omega^2\right)^5$

Watch Video Solution

10. If $z = \sqrt{23} + \sqrt{5}i$, then find $z\bar{z}$.

Watch Video Solution

11. If $z = \sqrt{11} + \sqrt{7}i$, then find $z\bar{z}$.

Watch Video Solution

12. If $z = \sqrt{12} - \sqrt{7}i$, then find $z\bar{z}$.

Watch Video Solution

13. If $z = \sqrt{13} - \sqrt{11}i$, then find $z\bar{z}$.

Watch Video Solution

14. If $z = \sqrt{7} - \sqrt{5}i$, then find $z\bar{z}$.

Watch Video Solution

15. If $z = \sqrt{3} - \sqrt{2}i$, then find $z\bar{z}$.

Watch Video Solution

16. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $2 + 2i$

Watch Video Solution

17. Express the following complex number in polar form $\left(re^{i}\theta\right)$: 2

Watch Video Solution

18. Express the following complex number in polar form

$\left(re^{i}\theta\right)$: 10

Watch Video Solution

19. Express the following complex number in polar form

$\left(re^{i}\theta\right)$: $-\dfrac{\sqrt{3}}{2}-i\dfrac{1}{2}$

Watch Video Solution

20. Express the following complex number in polar form

$\left(re^{i}\theta\right)$: $-\sqrt{6}-\sqrt{2}i$

Watch Video Solution

21. Express the following complex number in polar form

$\left(re^{i}\theta\right)$: $1 - i + i^2 - i^3$

Watch Video Solution

22. Express the following complex number in polar form

$\left(re^{i}\theta\right)$: $-3i$

Watch Video Solution

23. Consider the equation $az + b\bar{z} + c = 0$, where a,b,c $\in$

Z

If $|a| \neq |b|$, then z represents

A. circle

B. straight line

C. one point

D. ellispe

Answer: C

View Text Solution

24. Consider the equation $az + b\bar{z} + c = 0$, where a,b,c $\in$

Z

If $|a| = |b|$ and $\bar{a}c \neq b\bar{c}$, then z has

A. infnite solutions

B. no solutions

C. finite solutions

D. cannot say anything

Answer: B

View Text Solution

25. Consider the equation $az + b\bar{z} + c = 0$, where a,b,c $\in$

Z

If $|a| \neq |b|$, then z represents

A. an ellipse

B. a circle

C. a point

D. a straight line

Answer: D

View Text Solution

26. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $2 + 2\sqrt{3}i$

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27. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $(2 - 3i)^2$

Watch Video Solution

28. Express the following complex number in polar form $\left(re^{i}\theta\right)$: i^3

Watch Video Solution

29. Express the following complex number in polar form $\left(re^{i}\theta\right)$: i^2

Watch Video Solution

30. Express the following complex number in polar form $\left(re^{i}\theta\right)$: $(2+i)^2$

Watch Video Solution

31. Express the following complex number in polar form $\left(re^{i}\theta\right)$: -6 - 4$i$

Watch Video Solution

32. Express the following complex number in polar form $\left(re^{i}\theta\right)$: -6 + 8$i$

Watch Video Solution

33. Express the following complex number in polar form

$\left(re^{i}\theta\right)$:$3 + 4i$

Watch Video Solution

34. Express the following complex number in polar form

$\left(re^{i}\theta\right)$:$-5 - 5i$

Watch Video Solution

35. Let $S = S_1 \cap S_2 \cap S_3$, where

$s_1 = \{z \in C : |z| < 4\}, S_2 = \left\{z \in C : \ln\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{31}}\right] > 0\right\}$ and

$S_3 = \{z \in C : Rez > 0\}$ Area of S=

A. $\frac{10\pi}{3}$

B. $\frac{20\pi}{3}$

C. $\frac{16\pi}{3}$

D. $\frac{32\pi}{3}$

Answer: B

Watch Video Solution

36. If ω is the complex cube root of unity, then find $\omega^2 + \omega^3 + \omega^4$.

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Matrix Match Type

1. Express the following complex number in polar form $\left(re^{i}\theta\right)$:$1 + i\sqrt{3}$

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2. Express the following complex number in polar form $\left(re^{i}\theta\right)$:$1 + i$

Watch Video Solution

3. Express the following complex number in polar form

$\left(re^{i}\theta\right):\left(-\frac{1}{2}\right)-i\left(\frac{\sqrt{3}}{2}\right)$

Watch Video Solution

4. Express the following complex number in polar form

$\left(re^{i}\theta\right):-3i$

Watch Video Solution

5. Express the following complex number in polar form

$\left(re^{i}\theta\right):9i$

Watch Video Solution

6. Express the following complex number in polar form $\left(re^{i}\theta\right)$:5

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7. Find the complex number $z = e^{-i\left(\frac{\pi}{4}\right)}$ in the form x+iy .

Watch Video Solution

8. Find the complex number $z = e^{-i(2\pi)}$ in the form x+iy .

Watch Video Solution

9. Find the complex number $z = e^{i(\pi)}$ in the form x+iy .

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Numerical Value Types

1. Write the following in $z = a + ib$ form: $z = \dfrac{1}{3 - 2i}$

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2. Write the following in $z = a + ib$ form: $z = \dfrac{e + \text{if}}{c + id}$

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3. Write the following in $z = a + ib$ form: $z = \dfrac{1}{2 - 2\sqrt{3}i}$

Watch Video Solution

4. Write the following in $z = a + ib$ form: $z = \dfrac{1}{c + id}$

Watch Video Solution

5. Write the following in $z = a + ib$ form: $z = \dfrac{1}{5 + i}$

Watch Video Solution

6. Write the following in $z = a + ib$ form: $z = \dfrac{1}{6 + 8i}$

Watch Video Solution

7. Write the following in $z = a + ib$ form: $z = \frac{1}{3 + 4i}$

Watch Video Solution

8. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{i\frac{7\pi}{2}}$

Watch Video Solution

9. Write the following in $z = a + ib$ form: $z = \sqrt{3}e^{i\frac{7\pi}{6}}$

Watch Video Solution

10. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{i\frac{9\pi}{4}}$

Watch Video Solution

11. Write the following in $z = a + ib$ form: $z = \frac{e^{-ia}}{e^{-ib}}$

Watch Video Solution

12. Write the following in $z = a + ib$ form: $z = \frac{e^{ia}}{e^{ib}}$

Watch Video Solution

13. Write the following in $z = a + ib$ form: $z = e^{a + i\theta}$

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14. If $|z + 2 - i| = 5$ and maxium value of $|3z + 9 - 7i|$ is M, then the value of M is ______.

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15. Write the following in $z = a + ib$ form: $z = e^{-i\theta}$

Watch Video Solution

16. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{i\frac{5\pi}{4}}$

Watch Video Solution

17. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{i3\frac{\pi}{4}}$

Watch Video Solution

18. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{-i\frac{\pi}{4}}$

Watch Video Solution

19. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{-i\frac{\pi}{4}}$

Watch Video Solution

20. Write the following in $z = a + ib$ form: $z = \sqrt{2}e^{i\frac{\pi}{4}}$

Watch Video Solution

21. Write the following in $z = a + ib$ form: $z = 5e^{i\frac{\pi}{6}}$

Watch Video Solution

22. Write the following in $z = a + ib$ form: $z = 5e^{i\frac{\pi}{3}}$

Watch Video Solution

23. Write the following in $z = a + ib$ form: $z = 5e^{i\frac{\pi}{4}}$

Watch Video Solution

24. Write the following in $z = a + ib$ form: $z = 5e^{i\frac{\pi}{2}}$

Watch Video Solution

25. Write the following in $z = a + ib$ form: $z = 5e^{i\pi}$

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26. Let $A(z_1)$ and $B(z_2)$ be lying on the curve $|z - 3 - 4i| = 5$, where $|z_1|$ is maximum. Now, $A(z_1)$ is rotated about the origin in anticlockwise direction through 90° reaching at $P(z_0)$. If A, B and P are collinear then the value of $\left(|z_0 - z_1| \cdot |z_0 - z_2|\right)$ is ______.

Watch Video Solution

27. If z_1, z_2, z_3 are three points lying on the circle $|z| = 2$ then the minimum value of the expression $\left|z_1 + z_2\right|^2 + \left|z_2 + z_3\right|^2 + \left|z_3 + z_1\right|^2 =$

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28. Minimum value of $\left|z_1 + 1\right| + \left|z_2 + 1\right| + \left|z_1 z_2 + 1\right|$ if $\left[z_1\right| = 1$ and $|z_2\ | = 1$ is ________.

Watch Video Solution

29. If $\left|z_1\right| = 2$ and $(1 - i)z_2 + (1 + i)\bar{z}_2 = 8\sqrt{2}$, then the minimum value of $\left|z_1 - z_2\right|$ is ______.

Watch Video Solution

30. Given that $1 + 2|z|^2 = \left|z^2 + 1\right|^2 + 2|z + 1|^2$, then the value of $|z(z + 1)|$ is ______.

Watch Video Solution

31. Find the complex number $z = e^{i\frac{\pi}{2}}$ in the form x+iy .

Watch Video Solution

32. Find the complex number $z = e^{i\frac{\pi}{3}}$ in the form x+iy .

Watch Video Solution

33. Find the complex number $z = e^{i\frac{4\pi}{3}}$ in the form x+iy .

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Archives Single Correct Answer Type

1. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|Z|$ is equal to

(1) $\sqrt{3} + 1$ (2) $\sqrt{5} + 1$ (3) 2 (4) $2 + \sqrt{2}$

A. $\sqrt{3} + 1$

B. $\sqrt{5} + 1$

C. 2

D. $2 + \sqrt{2}$

Answer: B

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2. If z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$ then

A. ∞

B. 0

C. 1

D. 2

Answer: C

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3. Let α and β be real and z be a complex number. If $z^2 + az + \beta = 0$ has two distinct roots on the line Re(z)=1, then it is necessary that

A. $\beta \in (1, \infty)$

B. $\beta \in (0, 1)$

C. $\beta \in (-1, 0)$

D. $|\beta| = 1$

Answer: A

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4. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^3 = A + B\omega$. Then (A, B) equals to ?

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5. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis

A. either on the real axis or on a circle passing thorugh the origin.

B. on a circle with centre at the origin.

C. either on the real axis or an a circle not possing through the origin .

D. on the imaginary axis .

Answer: A

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6. If z is a complex number of unit modulus and argument q, then $arg\left(\frac{1+z}{1+\bar{z}}\right)$ equal (1) $\frac{\pi}{2}$ - θ (2) θ (3) π - θ (4) -θ

A. $-\theta$

B. $\frac{\pi}{2} - \theta$

C. θ

D. $\pi - \theta$

Answer: C

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7. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$ (1) is equal to $\frac{5}{2}$ (2) lies in the interval (1, 2) (3) is strictly greater than $\frac{5}{2}$ (4) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

A. is equal to $\frac{5}{2}$

B. lies in the interval (1,2)

C. is strictly gerater than $\frac{5}{2}$

D. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

Answer: B

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8. If z_1 and z_2 are two complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular whereas z_1 is not unimodular then $|z_1|$=

A. Straight line parallel to x-axis

B. sraight line parallel to y-axis

C. circle of radius 2

D. circle of radius $\sqrt{2}$

Answer: C

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9. A value of for which $\dfrac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ purely imaginary, is : (1) $\dfrac{\pi}{3}$ (2) $\dfrac{\pi}{6}$ (3) $\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$ (4) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

A. $\dfrac{\pi}{6}$

B. $\sin^{-1}\left(\dfrac{Sqrt(3)}{4}\right)$

C. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D. $\frac{\pi}{3}$

Answer: C

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10. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. $If \left|1111 - \omega^2 - 1\omega^2 1\omega^2\omega^7\right| = 3k$, thenk is equal to : -1 (2) 1 (3) $-z$ (4) z

A. 1

B. z

C. $-z$

D. -1

Answer: B

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11. If $\alpha, \beta \in C$ are distinct roots of the equation $x^2 + 1 = 0$ then $\alpha^{101} + \beta^{107}$ is equal to

A. 2

B. -1

C. 0

D. 1

Answer: D

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Multiple Correct Answer Type

1. Let z_1 and z_2 be two distinct complex numbers and $z = (1 - t)z_1 + tz_2$, for some real number t with $0 < t < 1$ and $i = \sqrt{-1}$. If arg(w) denotes the principal argument of a non-zero compolex number w, then

A. $\left|z - z_1\right| + \left|z - z_2\right| = \left|z_1 - z_2\right|$

B. $\left(z - z_1\right) = \left(z - z_2\right)$

C. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D. $arg\left(z - z_1\right) = arg\left(z_2 - z_1\right)$

Answer: A::C::D

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2. about to only mathematics

A. $\pi/2$

B. $\pi/6$

C. $2\pi/3$

D. $5\pi/6$

Answer: C::D

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3. Let $a, b \in R$ and $a^2 + b^2 \neq 0$.

Suppose $S = \left\{ z \in C : z = \frac{1}{a + ibt}, t \in R, t \neq 0 \right\}$, where $i = \sqrt{-1}$. If z=x+iy and $z \in S$, then (x,y) lies on

A. the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0 \right)$ for

$a > 0 be \neq 0$

B. the circle with radius $-\frac{1}{2a}$ and centre

$\left(-\frac{1}{2}, 0 \right) a < 0, b \neq 0$

C. the axis for $a \neq 0, b = 0$

D. the y-axis for $a = 0, b \neq 0$

Answer: A::C::D

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4. Let $a, b, x and y$ be real numbers such that $a - b = 1 and y \neq 0.$ If the complex number $z = x + iy$ satisfies $Im\left(\frac{az + b}{z + 1}\right) = y$, then which of the following is (are) possible value9s) of x?| $-1 - \sqrt{1 - y^2}$ (b) $1 + \sqrt{1 + y^2}$ $-1 + \sqrt{1 - y^2}$ (d) $-1 - \sqrt{1 + y^2}$

A. $-1 - \sqrt{1 - y^2}$

B. $1 + \sqrt{1 + y^2}$

C. $1 - \sqrt{1 + y^2}$

D. $-1 + \sqrt{1 - y^2}$

Answer: A::D

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5. For a non-zero complex number z, let $arg(z)$ denote the principal argument with $-\pi < arg(z) \leq \pi$ Then, which of the following statement(s) is (are) FALSE?

$arg(-1, -i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$ (b) The function $f: R \to (-\pi, \pi]$, defined by $f(t) = arg(-1 + it)$ for all $t \in R$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$ (c) For any two non-zero complex numbers z_1 and z_2,

$arg\left(\frac{z_1}{z_2}\right) - arg\left(z_1\right) + arg\left(z_2\right)$ is an integer multiple of 2π

(d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition

$arg\left(\frac{\left(z - z_1\right)\left(z_2 - z_3\right)}{\left(z - z_3\right)\left(z_2 - z_1\right)}\right) = \pi$, lies on a straight line

A. $arg(-1-i)=\frac{\pi}{4}$, where $i=\sqrt{-1}$

B. The function $f: R \to (-\pi, \pi]$, defined by $f(t)=arg(-1+it)$ for all $t \in R$, is continous at all points of R, where $i=\sqrt{-1}$

C. For any tow non-zero complex number z_1 and z_2, $arg\left(\frac{z_1}{z_2}\right)-arg(z_1)+arg(z_2)$ is an integer multiple of 2π

D. For any three given distinct complex numbers z_1, z_2 and z_3 the locus of the point z satisfying the condition $\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right)=\pi$, lies on a straight line.

Answer: A::B::D

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6. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $\left(x, y \in \mathbb{R}, \ i = \sqrt{-1}\right)$ of the equation $sz + tz + r = 0$, where $z = x - iy$. Then, which of the following statement(s) is (are) TRUE? If L has exactly one element, then $|s| \neq |t|$ (b) If $|s| = |t|$, then L has infinitely many elements (c) The number of elements in $\ln n\{z : |z - 1 + i| = 5\}$ is at most 2 (d) If L has more than one element, then L has infinitely many elements

A. If L has exactly one element, then $|s| \neq |t|$

B. If $|s| = |t|$ then L has infinitely many elements

C. The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

D. If L has most than one elements, then L has infinitely many elements.

Answer: A::C::D

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