MATHS

BOOKS - CENGAGE PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Single Correct Answer Type

1. If $f(x)=\left\{\left(\frac{\sin(2x^2)}{a}+\cos\left(\frac{3x}{b}\right)\right)^{ab/x^2}, x\neq 0\&e^3atx=0\right\}$ is continuous at $x=0\forall b\in R$ then minimum value of $a$is $-1/8$ b. $-1/4$ c. $-1/2$ d. 0

A. $-1/8$

B. $-1/4$

C. $-1/2$

D. 0

Answer: B

View Text Solution

2. Let $f: R \to R$ be any function. Also $g: R \to R$ is defined by $g(x) = |f(x)|$ for all x. Then g is

a. Onto if f is onto b. One-one if f is one-one c. Continuous if f is continuous d. None of these

A. onto if f is onto

B. one-one if f is one-one

C. continuous if f is continuous

D. None of these

Answer: C

Watch Video Solution

3. Let

$$f(x) = \left[\frac{1 - \sin \pi x}{1 + \cos 2\pi x}, x < \frac{1}{2} \text{ and } p, x = \frac{1}{2} \text{ and } \frac{\sqrt{2x-1}}{\sqrt{4 + \sqrt{2x-1}} - 2}\right.$$

.Determine the value of p, if possible, so that the function is continuous at $x = \frac{1}{2}$.

A. 1

B. $1/4$

C. 4

D. none of these

Answer: D

Watch Video Solution

4. For which of the following functions $f(0)$ exists such that $f(x)$ is continuous at $f(x) = \frac{1}{(\log)_e |x|}$ b. $f(x) = \frac{1}{(\log)_e |x|}$ c. f(x)=x sinpi/x d. $f(x) = \frac{1}{1 + 2^{\cot x}}$

A. $f(x) = \frac{1}{\log_e |x|}$

B. $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

C. $f(x) = x\frac{\sin(\pi)}{x}$

D. $f(x) = (1) = \frac{1}{1 + 2^{\cot x}}$

Answer: C

Watch Video Solution

5. Let $f(x) = x^3 - x^2 - 3x - 1, g(x) = (x+1)a$ and $h(x) = \frac{f(x)}{g(x)}$

where h is a rational function such that

(i) It is continuous everywhere except when $x = -1$,

(ii) $\lim_{x \to -1} h(x) = \frac{1}{2}$.

The value of h(1) is

A. $1/2$

B. $1/4$

C. $-1/2$

D. 1

Answer: C

Watch Video Solution

6. If the function $f(x) = \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ is continuous at $x = -2$, then the value of $f(-2)$ is

A. 0

B. -1

C. 1

D. 2

Answer: B

View Text Solution

7. Let $f(x)=\begin{cases}8^{\frac{1}{x}}, & x<0\\ a[x], & a\in R-\{0\},\quad x\geq 0\end{cases}$ (where [.] denotes the greatest integer function).

Then f(x) is

A. continuous only at a finite number of points.

B. discontinuous at a finite number of points.

C. discontinuous at an infinite number of points.

D. discontinuous at x = 0.

Answer: C

View Text Solution

8. Let $f(x)=(1-x)^2\sin^2 x+x^2$ for all x ∈ R, and let $g(x)=\int\left(\frac{2(t-1)}{t+1}-\ln t\right)f(t)dt$ for t ∈ [1, x] for all x ∈ (1, ∞).Which of the following is true ?

A. f is continuous at $x=\pi/2$

B. f has an irremovable discontinuity at $x = \pi/2$

C. f has a removable discontinuity at $x = \pi/2$

D. none of these

Answer: B

Watch Video Solution

9. If

$$f(x) = \left\{ \sin\left(\frac{\pi}{2}\right)(x - [x]), x < 55(b-1), x = 5 \frac{ab^2|x^2 - 11x + 24|}{x - 3}, x >\right.$$

is continuous at $x = 5, a, b \in R$ then ([.] denotes the greatest integer function) $a = \frac{25}{108}, b = \frac{6}{5}$ b. $a = \frac{6}{13}, b = \frac{17}{29}$ c. $a = \frac{1}{2}, b = \frac{25}{36}$ d. $a = \frac{23}{100}, b = \frac{6}{5}$

A. $a = \frac{25}{108}, b = \frac{6}{5}$

B. $a = \frac{6}{13}, b = \frac{17}{29}$

C. $a = \frac{1}{2}, b = \frac{25}{36}$

D. $a = (23), (100), b = \frac{6}{5}$

Answer: A

Watch Video Solution

10. The function $f(x)$ is discontinuous only at $x=0$ such that $f^2(x)=1 \forall x \in R$. The total number of such functions is 2 b. 3 c. 6 d. none of these

A. 2

B. 3

C. 6

D. none of these

Answer: C

Watch Video Solution

11. $f(x) = \left\{ \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1} k,\ x = 2,\ x \neq 2 \right.$ is continuous from right at the point $x = 2$, then k equals

a. 0

b. $1/4$

c. $-1/4$

d. none of these

A. 0

B. 1\4

C. $-1/4$

D. none of these

Answer: B

Watch Video Solution

12. Let $g(x) = f(f(x))$ where $f(x) = \{1 + x; 0 \leq x \leq 2\}$ and $f(x) = \{3 - x; 2 < x \leq 3\}$ then the number of points of discontinuity of $g(x)$ in [0,3] is :

A. 0

B. 1

C. 2

D. 3

Answer: C

View Text Solution

13. If the function $f(x)=\frac{(128a+ax)^{1/8}-2}{(32+bx)^{1/5}-2}$ is continuous at $x=0$, then the value of a/b is $\frac{3}{5}f(0)$ b. $2^{8/5}f(0)$ c. $\frac{64}{5}f(0)$ d. none of these

A. $\frac{3}{5}f(0)$

B. $2^{8/5}f(0)$

C. $\frac{64}{5}f(0)$

D. none of these

Answer: C

Watch Video Solution

14. If $f(x) = \begin{cases} \dfrac{1-\cos\left(1-\frac{\cos x}{2}\right)}{2^m x^n} 1 x = 0, x \neq 0 \end{cases}$ is continous at $x = 0$

then the value of $m + n$ is a. 2 b. 3 c. -3 d. 7

A. 2

B. 3

C. -3

D. 7

Answer: C

Watch Video Solution

15. Let $f(x) = \begin{cases} \frac{\alpha \cot x}{x} + \frac{\beta}{x^2} & 0 < |x| \leq 1 \\ \frac{1}{3} & x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$ then the value of $\alpha^2 + \beta^2$ is

A. 1

B. 2

C. 5

D. 9

Answer: B

Watch Video Solution

16. Let $f(x) = \left\{ \frac{2}{1+x^2}, \xi srationalb, \xi srational \right.$ has exactly two points of continuity then the value of b are $(0, 3]$ b. $[0, 1]$ c. $(0, 2]$ d.φ

A. (0,3]

B. [0,1]

C. (0,2]

D. ϕ

Answer: C

Watch Video Solution

17. If $f(x)$ ' $\begin{cases} \sin\left(\frac{a-x}{2}\right)\tan\left[\frac{\pi x}{2a}\right] & \text{for} \quad x > a \\ \frac{\left[\cos\left(\frac{\pi x}{2a}\right)\right]}{a-x} & \text{for} \quad x < a \end{cases}$

(where [x] is the greatest integer function of x) and a gt 0, then

A. $f(a^{-}) < 0$

B. f has a removable discontinuity at x = a

C. f has an irremovable discontinuity at x = a

D. $f(a^{+}) < 0$

Answer: B

View Text Solution

18. Let $f(x) = [tanx[\cot x]], x\left[\frac{\pi}{12}, \frac{\pi}{12}\right]$, (where [.] denotes the greatest integer less than or equal to$x$). Then the number of points, where $f(x)$ is discontinuous is a. one b. zero c. three d. infinite

A. one

B. zero

C. three

D. infinite

Answer: C

Watch Video Solution

19. Let $f: [a, b] \to R$ be any function which is such that $f(x)$ is rational for irrational x and that $f(x)$ is iirrational for rational x, then in [a,b]

A. f is discontinuous everywhere

B. f is discontinuous only at x = 0 and discontinuous everywhere

C. f is continuous for all irrational x and discontinuous for rational x

D. f is continuous for rational x and discontinuous for irrational x

Answer: A

View Text Solution

20. If $f(x) = [x](\sin kx)^p$ is continuous for real x, then (where [.] represents the greatest integer function)

A. $k \in [n\pi, n \in I], p > 0$

B. $k \in \{2n\pi, n \in I\}, p > 0$

C. $k \in \{n\pi, n \in I\}, p \in R - \{0\}$

D. $k \in \{n\pi, nI, n \neq 0\}, p \in R - \{0\}$

Answer: A

View Text Solution

21. Statement 1: Minimum number of points of discontinuity of the function $f(x) = (g(x)[2x-1]\ \forall x \in (-3, -1)$, where [.] denotes the greatest integer function and $g(x) = ax^3 = x^2 + 1$ is zero. Statement 2: $f(x)$ can be continuous at a point of discontinuity, say $x = c_1 of[2x-1]$ if $g(c_1) = 0.$ Statement 1 is True, Statement 2 is True, Statement 2 isa correct explanation for Statement 1. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for statement 1. Statement 1 is True, Statement 2 is False Statement 1 is False, Statement 2 is True.

A. Statement 1 is True, Statement 2 is True, Statement 2 is a correct explaination for Statement 1.

B. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for Statement 1/

C. Statement 1 is True, Statement 2 is False.

D. Statement 1 is False, Statement 2 is True.

Answer: D

View Text Solution

22. Number of points of discontinuity of $f(x) = \left[\sin^{-1} x\right] - [x]$ in its domain is equal to (where [.] denotes the greatest integer function) a. 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: D

Watch Video Solution

23. If $g(x) = (\lim)_{m \overrightarrow{\infty}} \frac{x^m f(x) + h(x) + 3}{2x^m + 4x + 1}$ when $x \neq 1 and g(1) = e^3$ such that $f(x), g(x) and h(x)$ are continuous functions at $x = 1$ then the value of $5f(1) - 2h(1)$ is 7 b. 6 c. 9 d. 8

A. 7

B. 6

C. 9

D. 8

Answer: B

Watch Video Solution

24. The number of points of discontinuity of $f x\Big) = \left[2x^2\right] - \{2x2\}^2$ (where [] denotes the greatest integer function and {} is fractional part of x) in the interval $(\,-2, 2),$ is 1 b. 6 c. 2 d. 4

A. 1

B. 6

C. 2

D. 5

Answer: B

Watch Video Solution

25. If $f(x) = \{(|x| - 3 \text{ when } x < 1)$, and $(|x - 2| + a, \text{ when } x \geq 1)$ &

$g(x) = \{2 - |x|$when $x < 2$ and $sgn(x) - b$, when $x \geq 2$.

if $h(x) = f(x) + g(x)$ is discontinuous at exactly one point, then -

(a). $a = -3, b = 0$

(b). $a = -3, b = -1$

(c) $a = 2, b = 1$

(d) $a = 0, b = 1$

A. $a = -3, b = 0$

B. $a = 0, b = 1$

C. $a = 2, b = 1$

D. $a = -3, b = 1$

Answer: D

Watch Video Solution

Watch Video Solution

26. The function $f(x)=\frac{x^3}{8}-s\in\pi x+4\in[-4,4]$ does not take the value -4 b. 10 c. 18 d. 12

A. -4

B. 10

C. 18

D. 12

Answer: C

Watch Video Solution

27. Let $f(x)$ be continuous functions $f\colon \overrightarrow{RR}$ satisfying $f(0)=1andf(2x)-f(x)=x\cdot$ Then the value of $f(3)$ is 2 b. 3 c. 4 d. 5

A. 2

B. 3

C. 4

D. 5

Answer: C

Watch Video Solution

28. about to only mathematics

A. $a = b = 4$

B. $a = b = -4$

C. $a = 4 \text{ and } b = -4$

D. $a = -4 \text{ and } b = 4$

Answer: C

View Text Solution

29. If $f(x)=\begin{cases}[x]+\sqrt{\{x\}}, & x<1\\ \frac{1}{[x]+\{x\}^2}, & x\geq 1\end{cases}$, then

[where [.] and {.} represent the greatest integer and fractional part functions respectively]

A. f(x) is continuous at x = 1 but not differentiable

B. f(x) is not continuous at x = 1

C. f(x) is differentiable at x = 1

D. $\lim_{x\to 1} f(x)$ does not exist

Answer: A

View Text Solution

30. If is an even function such that $\lim_{h\to 0}\frac{f(h)-f(0)}{h}$ has some fininte non-zero value, then

A. f is continuous and derivable at x =0

B. f is continuous but not differentiable at x = 0

C. f may be discontinuous at x = 0

D. None of these

Answer: B

Watch Video Solution

31. Let $f(x)$ be differentiable for real x such that $f'(x) > 0 on(-\infty, -4)$, $f'(x) < 0 on(-4, 6)$, $f'(x) > 0 on(6, \infty)$, If $g(x) = f(10 - 2x)$, then the value of $g'(2)$ is a. 1 b. 2 c. 0 d. 4

A. 1

B. 2

C. 0

D. 4

Answer: C

View Text Solution

View Text Solution

32. Number of points where $f(x) = x^2 - \left|x^2 - 1\right| + 2||x| - 1| + 2|x| - 7$ is non-differentiable is a. 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: A

View Text Solution

33. If $f(x) = |x - 1|. ([x] = [-x]),$ then (where [.] represents greatest integer function)

A. f(a) is continuous and differentiable at x = 1

B. f(x) is discontinuous at x = 1

C. f(x) is continuous at x = 2

D. f(x) is continuous but non-differentiable at x= 1

Answer: D

View Text Solution

34. Number of point where function f(x) defined as $f: [0, 2\pi] \rightarrow R,\ f(x) = \begin{cases} 3 - \left|\cos x - \frac{1}{\sqrt{2}}\right|, & |\sin x| < \frac{1}{\sqrt{2}} \\ 2 + \left|\cos x + \frac{1}{\sqrt{2}}\right|, & |\sin x| \geq \frac{1}{\sqrt{2}} \end{cases}$ is non differentiable is

A. 2

B. 4

C. 6

D. 0

Answer: B

Watch Video Solution

35. If $y = e^x \sin x$, then find $\frac{dy}{dx}$

Watch Video Solution

36. If $y = \cos\left(\sin^{-1} x\right)$, then find $\frac{dy}{dx}$

Watch Video Solution

37. If $y = \sin a^x$, then find $\frac{dy}{dx}$

Watch Video Solution

38. If $y = \frac{1}{1 + \frac{1}{x}}$, then find $\frac{dy}{dx}$

Watch Video Solution

39. If $y = \cos(x^3)$, then find $\frac{dy}{dx}$

Watch Video Solution

40. If $y = (1-x)\tan\left(\frac{x}{2}\right)$, then find $\frac{dy}{dx}$

Watch Video Solution

Multiple Correct Answer Type

1. Which of the following functions is/are discontinuous at $x = 1$?

$f(x) = \frac{1}{1+2^{tanx}}$ $\quad g(x) = (\lim)_{x \vec{\infty}} \frac{1}{1+n \in s^2(\pi x)}$

$h(x) = 2^{-2}\ \hat{}\ \left(\left(\left(\frac{1}{1-x}\right)\right)\right), x \neq 1andh(1) = 1$

$\varphi(x) = \frac{x-1}{|x-1|+2(x-1)^2}, x = 1and\varphi(1) = 1$

A. $f(x)=\frac{1}{1+2^{\tan x}}$

B. $g(x)=\lim_{n\to\infty}\frac{1}{1+n\sin^2(\pi x)}$

C. $h(x)=2^{-2^{\left(\frac{1}{1-X}\right)}}, x\neq 1 \text{ and } h(1)=1$

D. $\phi(x)=\frac{x-1}{|x-1|+2(x-1)^2}, x\neq 1 \text{ and } \phi(1)=1$

Answer: A

View Text Solution

2. f is a continous function in $[a, b]$; g is a continuous function in [b,c]. A function h(x) is defined as $h(x)=f(x)f \text{ or } x\in[a,b), g(x)f \text{ or } x\in(b,c]$ if f(b) =g(b) then

A. h(x) may or may not be continuous in [a, c]

B. $h(b^+)=g(b^-) \text{ and } h(b^-)=f(b^+)$

C. $h(b^-)=g(b^+) \text{ and } h(b^+)=f(b^-)$

D. h(x) has a removable discontinuity at x = b

Answer: C::D

Watch Video Solution

3. If the function $f(x)$ defined as $f(x)$ defined as $f(x) = \left\{3, x = 0\left(1 + \frac{ax + bx^3}{x^2}\right), x > 0\right.$ is continuous at $x = 0$, then $a = 0$ b. $b = e^3$ c. $a = 1$ d. $b = (\log)_e 3$

A. a = 0

B. $b = e^3$

C. $a = 1$

D. $b = \log_e 3$

Answer: A::D

Watch Video Solution

4. Given $f(x) = \left\{3 - \left[\cot^{-1}\left(\frac{2x^3 - 3}{x^2}\right)\right]f \text{ or } x > 0 \text{ and } \{x^2\}\cos\left(e^{\frac{1}{x}}\right)f \text{ or } x <\right.$ (where {} and [] denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does not hold good?

A. $f(0^-) = 0$

B. $f(0^+) = 3$

C. If $f(0) = 0,$ then f(x) is continuous at x = 0

D. Irremovable discontinuity of f at x = 0

Answer: B::D

View Text Solution

5. Let $f(x) = \begin{cases} x\left[\frac{1}{x}\right] + x[x] & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$ (where [x] denotes the greatest integer function). Then the correct statement is/are

A. Limit exists for $x = -1$.

B. f(x) has a removable discontinuity at x = 1.

C. f(x) has a non removable discontinuity at x = 2.

D. f(x) is discontinuous at all positive integers.

Answer: A::B::C::D

Watch Video Solution

6. A function $f: R\vec{R}$ is defined as $f(x) = (\lim)_{n\vec{\infty}} \frac{ax^2 + bx + c + e^{nx}}{1 + \cdot e^{nx}}$ is continuous on then Point lies on the space Point represents the 2-dimensional Cartesian plane Locus of point $(a, c) and (c, b)$ intersect at one point Point (a, b, c) lies on the plane in space

A. point (a, b, c) lies on line in space

B. point (a, b) represents the 2-dimensional Cartesian plane

C. Locus of point (a, c) and (c, b) intersect at one point

D. point (a, b, c) lies on the plane in space

Answer: A::B::C

Watch Video Solution

7. If $y = \frac{1}{1 + x^3}$, then find $\frac{dy}{dx}$

Watch Video Solution

8. If $y = \sin\left(x^2 - 3x\right)$, then find $\frac{dy}{dx}$

Watch Video Solution

9. If $y = (x - 2)^{\frac{2}{3}}$, then find $\frac{dy}{dx}$

Watch Video Solution

Comprehension Type

1. If $y=(4x+2)(x-1)$, then find $\frac{dy}{dx}$

Watch Video Solution

2. If $y=(x-1)e^{x}$, then find $\frac{dy}{dx}$

Watch Video Solution

Illustration

1. If $y=xe^{x}$, then find $\frac{dy}{dx}$

Watch Video Solution

2. If $y=\frac{1}{2\sin x-1}$, then find $\frac{dy}{dx}$

Watch Video Solution

3. If $y = \log(1 + x^2)$, then find $\dfrac{dy}{dx}$

Watch Video Solution

4. $x^2 + \sqrt{3}. f(x) + 2\sqrt{3} - 3 = 0$, then the value of $f(\sqrt{3})$

Watch Video Solution

5. If $y = \dfrac{x}{x+3}$, then find $\dfrac{dy}{dx}$

Watch Video Solution

6. If $y = \sqrt{x + bx^2}$, then find $\dfrac{dy}{dx}$

Watch Video Solution

7. If $y = \log(1 + 4x) + x$, then find $\frac{dy}{dx}$

Watch Video Solution

8. If $y = \frac{1}{x^4 + x^2 + 1}$, then find $\frac{dy}{dx}$

Watch Video Solution

9. If $y = x \sin x$, then find $\frac{dy}{dx}$

Watch Video Solution

10. If $f(x) = \log_e(x^2 - 4)$ then find $\frac{df}{dx}$

Watch Video Solution

11. If $y = \cos^{-1}\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)$ then find $\frac{dy}{dx}$?

Watch Video Solution

12. If $y = x^2 \tan x$, then find $\frac{dy}{dx}$

Watch Video Solution

13. Find $\frac{dy}{dx}$ if:- $x = -\sin t, y = \cos t$

Watch Video Solution

14. Find $\frac{dy}{dx}$ if:- $x = \cos t, y = \tan t$

Watch Video Solution

15. Find $\frac{dy}{dx}$ if:- $x = \sec t, y = \tan t$

Watch Video Solution

16. Find $\frac{dy}{dx}$ if:- $x = 2t, y = t^3$

Watch Video Solution

17. Find $\frac{dy}{dx}$ if:- $x = t, y = t^2 + t$

Watch Video Solution

18. Find $\frac{dy}{dx}$ if:- $x = \sin t, y = \cos t$

Watch Video Solution

19. Find $\frac{dy}{dx}$ if:- $x = \sin t, y = \tan t$

Watch Video Solution

20. Draw the graph and find the points of discontinuity $f(x) = [2\cos x]$, $x \in [0, 2\pi]$. ([.] represents the greatest integer function.)

Watch Video Solution

21. Draw the graph and discuss the continuity of $f(x) = [\sin x + \cos x], x \in [0, 2\pi]$, where [.] represents the greatest integer function.

Watch Video Solution

22. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2), [.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

Watch Video Solution

23. Discuss continuity of

(i) $f(x) = sgn(x^3 - x)$ (ii) $f(x) = sgn(2\cos x - 1)$

(iii) $f(x) = sgn(x^2 - 2x + 3)$

Watch Video Solution

24. If $f(x) = sgn(2\sin x + a)$ is continuous for all x, then find the possible values of a.

Watch Video Solution

25. Discuss the continuity of $f(x) = |x| sgn(x^3 - x)$

Watch Video Solution

26. if $f(x)\begin{cases} sgn(x-2) \times [\log_e x] & 1 \leq x \leq 3 \\ \{x^2\} & 3 < x \leq 3.5 \end{cases}$

where [.] dentes inteatest function ans {.} repesents fractional part function find the points where the continuity of f(x) shoukd be checked ,

Hence find he points of discontinuity .

Watch Video Solution

27. Discuss the continuity of $f(x) = (\lim)_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$

Watch Video Solution

28. Find the values of a if $f(x) = (\lim)_{n \overrightarrow{\infty}} \frac{ax^{2n} + 2}{x^{2n} + a + 1}$ is continuous at $x = 1.$

Watch Video Solution

29. Let f(x) be given that $f(x) = \begin{cases} x & \text{if x is rational} \\ 1 - x & \text{if x is irrational} \end{cases}$

The number of points at which f(x) is continuous, is

Watch Video Solution

30. If $f(x) = \frac{x+1}{x-1} and g(x) = \frac{1}{x-2}, then$ discuss the continuity of $f(x), g(x), and fog(x)\cdot$

Watch Video Solution

31. If $f(x) = \begin{cases} x-2 & x \leq 0 \\ 4-x^2 & x > 0 \end{cases}, discusscont \in uityofy$=f(f(x))

Watch Video Solution

32. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]\cdot$

Watch Video Solution

33. Using intermediate value theorem, prove that there exists a number x such that $x^{2005} + \frac{1}{1+\sin^2 x} = 2005.$

Watch Video Solution

34. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$ then the value of $f(1.5)$ is :

Watch Video Solution

35. Let $f: [0, 1]\overrightarrow{0, 1}$ be a continuous function. Then prove that $f(x) = x$ for at least one $0 \leq x \leq 1$.

Watch Video Solution

36. Discuss the differentiability of f(x)=$\begin{cases} \frac{\sin x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at x=0

Watch Video Solution

37. $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ then at $x = 0$,value of f(x) is

Watch Video Solution

38. If

$f(x) = \{x, x \leq 1, x^2 + bx + c, x > 1''f \in db$ and c if $functioniscont$

x=1`

Watch Video Solution

39. Find the values of a and b if

$f(x) = \{a + \sin^{-1}(x+b), x \geq 1$ and $x, x < 1$ is differentiable at

$x = 1.$

Watch Video Solution

40.

$f(x) = \{ax(x-1) + b, x < 1x - 1, 1 \leq x \leq 3. px^2 + qx + 2, x > 3$

Find the values of the constants $a, b, pandq$ so that all the following

conditions are satisfied $f(x)$ is continuous for all $x \cdot f(1)$ does not exist.

$f'(x)$ is continuous at $x = 3$

Watch Video Solution

41. Discuss the differentiability of $f(x) = \sin|x|$

Watch Video Solution

42. Test the continuity and differentiability of the function $f(x) = \left|\left(x + \frac{1}{2}\right)[x]\right|$ by drawing the graph of the function when $-2 \leq x < 2,$ where $[.]$ represents the greatest integer function.

Watch Video Solution

43. Discuss the differentiability of $f(x) =$ [x] $+|1 - x|, x \in (-1, 3), where[.]$ represents greatest integer function.

Watch Video Solution

44. Discuss the differentiability of $f(x) = \left(x^2 - 1\right)\left|x^2 - x - 2\right| + \sin(|x|)$.

Watch Video Solution

45. Discuss the differentiability of $f(x) = |x|\sin x + |x| - 2sgn(x - 2) + |x - 3|$.

Watch Video Solution

46. Prove that function $f(x) = (\sin \pi x)(x - 1)^{1/5}$ is continuous and differentiable at x=1

Also show that f'(x) is differentiable at x=1.

View Text Solution

Solved Examples

1. Let $f: R \to R$ satisfying $|f(x)| \le x^2, \forall x \in R$, then show that f(x) is differentiable at x = 0.

Watch Video Solution

2. If a function $f: [-2a, 2a] \to R$ is an odd function such that, $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left-hand derivative at $x = a$ is 0, then find the left-hand derivative at $x = -a$.

Watch Video Solution

3. Let $\alpha \in R$. Prove that a function $f: R \to R$ is differentiable at α if and only if there is a function $g: R \to R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha), \forall x \in R$.

Watch Video Solution

Concept Application Exercise 4 1

1. A function $f(x)$ satisfies the following property: $f\left(x\dot{y}\right) = f(x)f(y)\cdot$ Show that the function $f(x)$ is continuous for all values of x if it is continuous at $x = 1$.

Watch Video Solution

2. Find the value of $f(0)$ so that the function. $f(x) = \frac{\sqrt{1+x} - 1 + x3}{x} becomescont \in uousatx = 0$

Watch Video Solution

3. The function $f : R\text{~}\{0\}\overrightarrow{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at x = 0 by defining f(0) as (1) 2 (2) -1 (3) 0 (4) 1

Watch Video Solution

4. $Let\ f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right],$ If $f(x) is$ continuous in $\left[0, \frac{\pi}{4}\right],$ then find the value of $f\left(\frac{\pi}{4}\right)$.

Watch Video Solution

5. If $f(x) = \left(\tan\left(\frac{\pi}{4} + (\log)_e x\right)\right)^{(\log)_x e}$ is to be made continuous at $x = 1,$ then what is the value of $f(1)$?

Watch Video Solution

6. If the function $f(x) = \frac{x^2 - (A + 2)x + A}{x - 2}, f \text{ or } x \neq 2 and f(2) = 2,$ is continuous at $x = 2,$ then find the value of A.

Watch Video Solution

7. Let $f(x)=\begin{cases}\{1+|\sin x|\}^{a/|\sin x|}, & -\pi/6<x<0\\ b, & x=0\\ e^{\tan 2x/\tan 3x}, & 0<x<\pi/6\end{cases}$ Determine a and b such that f(x) is continuous at x = 0

Watch Video Solution

8. Which of the following functions is not continuous $\forall x \in R$?

A. $\sqrt{2\sin x+3}$

B. $\dfrac{e^x+1}{e^x+3}$

C. $\left(\dfrac{2^{2x}+1}{2^3+5}\right)^{5:7}$

D. $\sqrt{sgn x+1}$

Answer:

Watch Video Solution

9. Let $f(x) = \{(1+3x)^{\frac{1}{x}}, x \neq 0 \text{ and } e^3, x = 0$. Discuss the continuity of $f(x)$ at $(a) x = 0,$ (b) $x = 1$.

Watch Video Solution

10. Discuss the continuity of

$$f(x) = \left\{\frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)}, x \neq 1, 26, x = 112, x = 2\right.$$

Watch Video Solution

Concept Application Exercise 4 2

1. Find the value of x in $[1, 3]$ where the function $[x^2 + 1]([.]$ represents the greatest integer function) is discontinuous.

Watch Video Solution

2. Discuss the continuity of $f(x) = [\tan^{-1} x]$ ($[.]$ represents the greatest integer function).

Watch Video Solution

3. Discuss the continuity of $f(x) = \{\cot^{-1} x]$ ($\{.\}$ represents the fractional part function).

Watch Video Solution

4. Discuss the continuity of $f(x) = (\log|x|) sgn(x^2 - 1), x \neq 0.$

Watch Video Solution

5. $Let f(x) = \begin{cases} [\sin \pi x] & 0 \leq x \leq 1 \\ sgn\left(x - \frac{5}{4}\right) \times \left\{x - \frac{2}{3}\right\} & 1 \leq x \leq 2 \end{cases}$

when [.] denotes the greatest integer function and {.} represents the fractional part function ,At what points should the continuity be checked ? Hence , find the points of discontinuity .

Watch Video Solution

6. Consider $f(x) = \lim_{x - \infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$ for $x > 0, x \neq 1, f(1) = 0$ then

Watch Video Solution

7. Discuss the continuity of $f(x) \in [0, 2], where f(x) = (\lim)_{n \to \infty} \left(\sin\left(\pi \frac{x}{2}\right)\right)^{2n}$

Watch Video Solution

8. Find the value of a for which $f(x) = \{x^2, x \in Qx + a, x \notin Q$ is not continuous at any x.

Watch Video Solution

9. If $y = \frac{1}{t^2 + t - 2}, where t = \frac{1}{x - 1}$, then find the number of points where $f(x)$ is discontinuous.

Watch Video Solution

10. Find the points of discontinuity of the function: $f(x) = \frac{1}{1 - e^{\frac{x-1}{x-2}}}$

Watch Video Solution

Concept Application Exercise 4 3

1. Prove that $f(x) = \frac{x^3}{4} - \sin \pi x + 3$ takes the value of $\frac{7}{3}$ for $x \in [-2, 2]$.

Watch Video Solution

2. Leg f be continuous on the interval [0,1] to R such that $f(0) = f(1)$.

Prove that there exists a point c in $\left[\frac{0, 1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.

Watch Video Solution

3. Suppose f is a continuous map from R to R and $f(f(a)) = a$ for some

a. Show that there is some b such that $f(b) = b$.

Watch Video Solution

Concept Application Exercise 4 4

1. If f is an even function such that $(\lim)_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then prove that $f(x)$ is not differentiable at $x = 0$.

Watch Video Solution

Concept Application Exercise 4 5

1. Discuss the continuity and differentiability of $f(x)=|x+1|+|x|+|x-1|\,\forall x\in R;$ also draw the graph of $f(x)$

Watch Video Solution

2. Find x where $f(x)=\max\left\{\sqrt{x(2-x)},2-x\right\}$ is non-differentiable.

Watch Video Solution

3. Discuss the differentiability of $f(x)=mim.\{|x|,|x-2|,2-|x-1|\}.$

Watch Video Solution

4. Discuss the differentiability of function $f(x)=x-\left|x-x^2\right|$

Watch Video Solution

Watch Video Solution

5. Discuss the differentiability of $f(x) = \max\{\tan^{-1} x, \cot^{-1} x\}$.

Watch Video Solution

6. Find the values of $aandb$ if $f(x) = \{ax^2 + 1, x \leq 1x^2 + ax + b, x > 1isd \Leftrightarrow erentiableatx = 1$

Watch Video Solution

7. Discuss the differentiability of $f\left(x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)\right.$

Watch Video Solution

8. Discuss the differentiability of $f(x) = \left|\left|x^2 - 4\right| - 12\right|$.

Watch Video Solution

Single Correct Answer Type

1. If the function $f(x)=\dfrac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at each point of its domain, then the value of $f(0)$ (a) 2 (b) $\dfrac{1}{3}$ (c) $-\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

A. 2

B. 43468

C. 43499

D. $-\dfrac{1}{3}$

Answer: B

Watch Video Solution

2. if $f(x)=\begin{cases}\frac{8^x-4^x-2^x+1}{x^2} & x>0\\ x^2 & x\le 0\end{cases}$

is continuous at x=0 , then the value of λ is

A. $4\log_e 2$

B. $2 \log_e 2$

C. $\log_e 2$

D. none of these

Answer: C

Watch Video Solution

3. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4a + b, \frac{x-4}{|x-4|} + b, x > 4 \end{cases}$ Then $f(x)$ is continous at $x = 4$ when $a = 0, b = 0$ b. $a = 1, b = 1$ c. $a = -1, b = 1$ d. $a = -1, b = -1$

A. a=0,b=0

B. a=1,b=1

C. a=-1,b=1

D. a=1,b=-1

Answer: D

Watch Video Solution

4. Let $f(x)$ $\lim_{x \to \infty} \dfrac{\log, (2+x) - x^{2x} \sin x}{1 + x^{2n}}$ then:

A. f is continuous at x=1

B. $\lim_{x \to 1^{+}} f(x) = \log 3$

C. $\lim_{x \to 1^{+}} f(x) = -\sin 1$

D. $\lim_{x \to 1^{-}} f(x)$ does not exist

Answer: C

Watch Video Solution