MATHS

BOOKS - CENGAGE PUBLICATION

DEFINITE INTEGRATION

ILLUSTRATION_TYPE

1. Evaluate the definite integrals as limit of sums $\int_{z}^{1} x^2 dx$

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2. Evaluate: $\int_{a}^{b} e^{x} dx$ using limit of sum

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3. Evaluate: $\int_a^b \sin x dx$ using limit of sum

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4. Evaluate $\int_a^b \frac{dx}{\sqrt{x}}, wherea, b > 0.$

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5. Column I, Column II $Atx = 1, f(x) = \{\log x, x < 12x - x^2, x \geq 1$, p. is increasing At $x = 2, f(x) = \{x - 1, x < 20, x = 2\sin x, x > 2$, q. is decreasing At $x = 0, f(x) = \{2x + 3, x < 05, x = 0x^2 + 7, x > 0$, r. has point of maxima At $x = 0, f(x) = \{e^{-x}x < 00, x = 0 - \cos x, x > 0$, s. has point of minima

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6. Evaluate: $\int_{-\frac{\pi}{2}}^{\pi} \sin^{-1}(\sin x)dx$

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7. Evaluate: $\int_0^1 \frac{1}{\sqrt{1-x^2}} \sin^{-1}\left(2x\sqrt{1-x^2}\right)dx\cdot$

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8. Evaluate: $\int_0^{2\pi} [\sin x]dx, where[.]$ denotes the greatest integer function.

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9. Prove that $\frac{1+\sqrt{2}}{2} < \int_0^{\pi/2} \frac{\sin x}{x} dx < \frac{\pi + 2\sqrt{2}}{4}$

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10. Evaluate: $\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$

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11. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1, -1), (1, 1)$ and a point with abscrissa 0 at which the curve is inclined to the axis of abscissa at an angle of 60^0. Then find the value of $\int_0^1 P(x)dx$

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12. Let f be a continuous function on $[a, b]$. Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dt = \int_x^b f(t)dt$.

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13. $\int_0^1 \frac{dx}{e^x + e^{-x}}$

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14. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\tan x dx}{1 + m^2 \tan^2 x}$

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15. Find the mistake in the following evaluation of the integral $I = \int_0^{\pi} \frac{dx}{1 + 2\sin^2 x}$, then : $I = \int_0^{\pi} \frac{dx}{\cos^2 x + 3\sin^2 x}$
$= \int_0^{\pi} \frac{\sec^2 x dx}{1 + 3\tan^2 x} = \frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_{\pi}^{0} = 0$

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16. Let $\frac{d}{dx}(F(x)) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_1^4 \frac{2e^{\sin\left(x^2\right)}}{x} dx = F(k) - F(1)$, then possible value of k is:

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17. If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$ then the value of $f\left(\frac{\pi}{6}\right)$ is ___

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18. If $f(0) = 1,\ f(2) = 3,\ f(2) = 5,\ thenf \in dthevalueof \int_0^1 x f^{2x} dx$

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19. F ind the value of $\int_0^1 \log x dx \cdot$

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20. Evaluate: $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$

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21. If $\lambda = \int_0^1 \frac{e^t}{1+t}$, then $\int_0^1 e^t \log_e(1+t)dt$ is equal to

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22. If $\int_0^1 e^{-x^2} dx = a$, then find the value of $\int_0^1 x^2 e^{-x^2} dx$ in terms of a.

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23. If $f(x) = x + \sin x$, then find the value of $\int_\pi^{2\pi} f^{-1}(x)dx$.

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24. Find the value of $\int_0^{\pi/2} \cos^5 x \sin^7 x dx$

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25. Evaluate

$$\lim_{n \to \infty} n\left[\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + \dots + \frac{1}{6n^2}\right]$$

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26. Evaluate: $\left(\lim\right)_{n \overrightarrow{\infty}} n\left[\frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \quad + \frac{1}{nb}\right]$

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27. Evaluate: $\left(\lim\right)_{n \overrightarrow{\infty}} \left(\frac{(n+1)(n+2)(n+n)^{\frac{1}{n}}}{n}\right.$

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28. Evaluate: $(\lim)_{n \to \infty} \frac{\left(1^2 + 2^2 + 3^3 + \quad + n^2\right)\left(1^3 + 2^3 + 3^3 + \quad + n^3\right)}{1^6 + 2^6 + 3^6 + \quad + n^6}$

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29. `P rov et h a t0

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30. Evaluate: $\int_0^1 x \frac{dx}{\sqrt{1-x^2}}$

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31.

$$Let I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx,\ I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(\sin x)}{\sin x} dx,\ I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(\tan x)}{\tan x} dx$$

Then arrange in the decreasing order in which values I_1, I_2, I_3 lie.

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32. find I= $\int_0^2 \left(\frac{5-x}{9-x^2}\right) dx$

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33. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

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34. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}\, dx$ cannot exceed $\sqrt{\frac{15}{8}}$.

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35. If $f(a+b-x)=f(x)$, then prove that $\int_a^b x f(x) dx \frac{a+b}{2} \int_a^b f(x) dx$.

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36. $\int_{-1}^{2} |x^3 - x| dx$.

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37. Find the value of $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

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38. Show that $\int_a^b \frac{|x|}{x} dx = |b| = |a| \cdot$

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39. If $f(n) = \int_0^{2015} \frac{e^x}{1 + x^n} dx$, then find the value of $\lim_{n \to \infty} f(n)$

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40. Let: $f(x) = \int_0^x |2t - 3| dt \cdot$ Then discuss continuity and differentiability of $f(x) at x = \frac{3}{2}$

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41. A continuous real function f satisfies $f(2x) = 3f(x) \, \forall x \in R\dot{I}f \int_0^1 f(x)dx = 1$, then find the value of $\int_1^2 f(x)dx$

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42. Let $g(x) = \int_0^x f(t)dt$ where f is such that $1/2 \leq f(t) \leq 1$ for $t \in [0, 1]$ and $0 \leq f(t) \leq 1/2$ for `tin[1, 2]

Then the interval in which g(2) lies.

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43. if [x] denotes the greatest integer less than or equal to x then integral $\int_0^2 x^2[x]dx$ equals

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44. Evaluate: $\int_0^{\frac{5\pi}{12}} [\tan x]dx, where[.]$ deontoes the greatest integer function.

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45. Evaluate: $\int_0^{10\pi} [\tan^{-1} x]dx, where[x]$ represents greatest integer function.

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46. Evaluate: $\int_0^2 [x^2 - x + 1]dx, where[.]$ denotos the greatest integer function.

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47. Prove that $\int_0^\infty [ne^{-x}]dx = 1n\left(\frac{n^n}{n!}\right), wheren$ is a natural number greater than 1 and [.] denotes the greatest integer function..

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48. Evaluate $\int_0^{\sqrt{3}}\left(\frac{1}{1+x^2}\right)\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

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49. Evaluate of each of the following integrals $\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

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50. Evaluate: $\int_0^a\frac{dx}{x+\sqrt{(a^2-x^2)}}$ or $\int_0^{\frac{\pi}{2}}\frac{d\theta}{1+\tan\theta}$

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51. Evaluate $\int_0^{\pi}\frac{\sin 6x}{\sin x}dx.$

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52. Evaluate: $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

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53. Evaluate: $\int_{-\pi}^{3\pi} \log(sec\theta - tan\theta)d\theta$

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54. Prove that $\int_0^{2a} f(x)dx = \int_0^{a} [f(a-x)+f(a+x)]dx$

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55. Evaluate : $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx.$

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56. Evaluate: $\int_{-5}^{5} x^2 \left[x + \frac{1}{2}\right] dx (where[.]$ denotes the greatest integer function).

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57. Evaluate: $\int_{-\pi}^{\pi} \frac{x \sin x dx}{e^x + 1}$

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59. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1 - x + x^2}\right) dx = 2\int_0^1 \tan^{-1} x dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}(1 - x + x^2) dx$

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60. Show that $\int_0^{\frac{\pi}{2}} \sqrt{(\sin 2\theta)} \sin \theta d\theta = \frac{\pi}{4}$

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61. For $\theta \in \left(0, \frac{\pi}{2}\right)$, prove that $\int_0^{\theta} \log(1 + \tan\theta \tan x) dx = \theta \log(\sec\theta)$

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62. Evaluate the definite integral: $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1 - x^4}\right) \cos^{01}\left(\frac{2x}{1 + x^2}\right) dx \cdot$

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63. How many terms of the A.P. : 24, 21, 18, ... must be taken so that their sum is 78?

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64. Evaluate $\int_0^{2\pi} \frac{dx}{1 + 3\cos^2 x}$

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65. Prove that: $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} + \cos^{2n} x} dx$

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66. Evaluate: $\int_0^{\pi} e^{|\cos x|} \left(2s \in \left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x dx.$

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67. Find the value of the integral is $\int_0^{\pi} x \log \sin x dx$

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68. Evaluate: $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) dx$

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69. Evaluate: $\int_0^{\frac{\pi}{2}} x \cot x dx$

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70. Evaluate: $\int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{dx}{1 + x^2}$

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71. Evaluate: $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$

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72. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta,\ a>0$

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73. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left\{\frac{ax^2+bx+c}{ax^2-bx+c}(a+b)|\sin x|\right\}dx$

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74. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^9-3x^5+7x^3-x+1}{\cos^2 x}dx$

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75. If f is an odd function, then evaluate $I=\int_{-a}^{a}\frac{f(\sin x)dx}{f(\cos x)+f(\sin^2 x)}$

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76. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left[\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right]^{\frac{1}{2}}dx$

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77. Find the value of $\int_{-2}^{2}\frac{\sin^{-1}(\sin x)+\cos^{-1}(\cos x)}{(1+x^2)\left(1+\left[\frac{x^2}{5}\right]\right)}dx$, where [.] represents the greatest integer function.

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79. Evaluate $\int_0^{16\pi/3}|\sin x|dx.$

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80. The value of $\dfrac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ is (where [x] and {x} denotes the integral part and fractional part functions of x and $x \in N$)

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81. Let $f(x)$ be a continuous and periodic function such that $f(x) = f(x+T)$ for all $x \varepsilon R, T > 0$.If $\int_{-2T}^{a+5T} f(x)dx = 19(a > 0)$ and $\int_0^T f(x)dx = 2$, then find the value of $\int_0^a f(x)dx$.

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82. If $g(x) = \int_0^x \cos^4 t dt$, then prove that $g(x+\pi) = g(x) + g(\pi)$.

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83. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi - \frac{\pi}{4}} |\sin x + \cos x| dx$

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84. Evaluate: $\int_0^x [\cos t] dtwheren \in \left(2n\pi, \left(4n+1\frac{\pi}{2}\right), n \in N, and[.]\right.$ denotes the greatest integer function.

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85. Let f be a real-valued function satisfying $f(x)+f(x+4)=f(x+2)+f(x+6)$ Prove that $\int_x^{x+8} f(t)dt$ is constant function.

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86. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers a, b and two unequal non-zero positive integers $mandn$ $\int_a^{a+n} f(x)dx = \int_b^{b+m} f(x)dx\dot{C}alc\underline{a}tethevalueof\int_m^n f(x)dx$

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87. If $y = \int_{x^2}^{x^3} \frac{1}{\log t} dt (x > 0)$, then find $\frac{dy}{dx}$

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88. If $\int_0^y \cos t^2 \, dt = \int_0^{x^2} \frac{\sin t}{t} \, dt$, then $\frac{dy}{dx}$ is equal to

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89. $If x = \int_0^y \frac{dt}{\sqrt{1+9t^2}} and \frac{d^2y}{dx^2} = ay, then f \in da$

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90. $If \int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x, where. x \in \left(0, \frac{\pi}{2}\right)$, then find the value of $f\left(\frac{1}{\sqrt{3}}\right)$.

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91. Let $f: \overrightarrow{RR}$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}$.

Then evaluate $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x - 2} dt$

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92. Evaluate: $(\lim)_{x \overrightarrow{\infty}} \frac{\left(\int 0 x e^x \ \hat{} \ 2 dx\right)^2}{\int 0 x e^{2x} \ \hat{} \ 2 dx}$

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93. Prove that: $y = \int_{\frac{1}{8}}^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_{\frac{1}{8}}^{\cos^2 x} \cos^{-1} \sqrt{t} dt,$ where $0 \leq x \leq \frac{\pi}{2}$, is the equation of a straight line parallel to the x-axis. Find the equation.

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94. If $f(x)=\int_{x^2}^{x^2+1} e^{-t^2}dt$, then $f(x)$ increases in

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95. Let $f:(0,\infty)\overrightarrow{0,\infty}$ be a differentiable function satisfying, $\xi nt_0^x(1-t)f(t)dt=\int_0^x tf(t)dtx\in R^+ andf(1)=1.$ Determine $f(x)\cdot$

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96. Let $f:R\to(0,\infty)$ be a real valued function satisfying $\int_0^x tf(x-t)dt=e^{2x}-1$ then find f(x) ?

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97. Let $f:R\to R$ be a differntiable function satisfying $f(x)=x^2+3\int_0^{x^{\frac{1}{3}}} e^{-t^3}.\,f(x-t^3)dt.$ Then find $f'(x)$.

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98. $If y = \int_0^x f(t)\sin\{k(x-t)\}dt, then prove that \frac{d^2y}{dx^2} + k^2 y = kf(x).$

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99. Prove that $\int_0^x e^{xt - t^2}\, dt = e^{\frac{x^2}{4}} \int_0^x e^{-\left(\frac{t^2}{4}\right)} dt$

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100. Evaluate: $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{\frac{1}{3}}^{\frac{2}{3}} e^{9\left(x - \frac{2}{3}\right)^2} dx$

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101. Compute the integrals: $\int_0^{\infty} f\left(x^n + x^{-n}\right)\log x \frac{dx}{x}$

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102. Compute the integrals: $\int_0^{\infty} f(x^n + x^{-n}) \log x \frac{dx}{1+x^2}$

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103. Compute the integrals: $\int_{\frac{1}{e}}^{e} \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$

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104. $Let A = \int_0^{\infty} \frac{\log x}{1+x^3} dx \dot{T} hen f \in d the value of \int_0^{\infty} \frac{x \log x}{1+x^3} dx$ in terms of $A \cdot$

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105. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha,$ then the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\sin\left(\frac{t}{2}\right)}{4\pi+2-t} dt$ is (1)2α (2) -2α (3) α (4) $-\alpha$

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106. $\int_0^1 \frac{\tan^{-1}x}{x}dx$ is equals to (a) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x}dx$ (b) $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x}dx$ (c) $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x}{x}dx$ (d) $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{x}{\sin x}dx$

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107. For $x > 0, let f(x) = \int_1^x \frac{\log t}{1+t}dt.$ Find the function $f(x) + f\left(\frac{1}{x}\right)$ and find the value of $f(e) + f\left(\frac{1}{e}\right).$

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108. Determine a positive integer n such that $\int_0^{\frac{\pi}{2}} x^n \sin x dx = \frac{3}{4}(\pi^2 - 8)$

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109. The natural number $n \leq 5$ for which $I_n = \int_0^1 e^x (x-1)^n dx = 16 - 6e$ is

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110. Prove that: $I_n = \int_0^{\infty} x^{2n+1} e^{-x^2} dx = \frac{n!}{2}, n \in N.$

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111. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}; n \in N,$ then prove that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

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112. If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx,$ then show that $I_n = ((n-1)n)I_{n-2}.$

Hence prove that

$$I_n = \begin{cases} \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\left(\frac{1}{2}\right)\frac{\pi}{2} & \text{if } n \text{is even} \\ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\left(\frac{2}{3}\right)1 & \text{if } n \text{is odd} \end{cases}$$

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SOLVED EXAMPLE_TYPE

1. f, g, h are continuous in $[0, a], f(a-x) = f(x), g(a-x) = -g(x), 3h(x) - 4h(a-x) = 5.$ Then prove that $\int_0^a f(x)g(x)h(x)dx = 0.$

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2. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin 3x}{\sin x + \cos x} dx.$

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3. Let $f:[0, 4] \rightarrow R$ be a differentiable function then for some α, β in $(0, 2)$ $\int_0^4 f(t)dt$=

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4. Prove that $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \int_0^{\infty} \frac{\sin x}{x} dx$

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5. If $\int_0^{\frac{\pi}{2}} \log \sin \theta d th\eta = k$, then find the value of $\int_{\pi}^{\frac{\pi}{2}} \left(\frac{\theta}{s \int h\eta}\right)^2 d th\eta$ in terms of k

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6. Evaluate: $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2} \cos x\right)}{2x - \pi} dx$

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7. Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos\left(|x| \frac{\pi}{3}\right)} dx$

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8. It is known that $f(x)$ is an odd function and has a period p. Prove that $\int_a^x f(t)dt$ is also periodic function with the same period.

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9. Evaluate: $\int_0^{\frac{\pi}{4}}\left(\tan^{-1}\left(\frac{2\cos^2\theta}{2-\sin 2\theta}\right)\right)\sec^2\theta d\theta.$

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10. If $f(x)=\frac{\sin x}{x}\forall x\in(0,\pi],$ prove that $\frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(x)f\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi} f(x)dx$

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11. Let $f(x)$ be a continuous function $\forall x\in R,$ except at $x=0,$ such that $\int_0^a f(x)dx,\ a\in R^+$ exists. If $g(x)=\int_x^a \frac{f(t)}{t}dt,$ prove that $\int_0^a f(x)dx=\int_0^a g(x)dx$

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12. If $\xi nt_0^x \sin(f(t))dt = (x+2)\int_0^x t\sin(f(t))dt, wherex > 0,$ then show that $f'(x)\cot f(x) + \frac{3}{1+x}0.$

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13. Show that: $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx\cdot$

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14. Let $a+b=4, wherea < 2, andletg(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)dx$ increrases as $(b-a)$ increrases$\cdot$

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15. about to only mathematics

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16. If $f(x+f(y))=f(x)+y \forall x, y \in R$ and $f(0)=1,$ then prove that $\int_0^2 f(2-x)dx=2\int_0^1 f(x)dx.$

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17. Suppose f is a real-valued differentiable function defined on $[1, \infty]$ with $f(x1)=1.$ Moreover, suppose that f satisfies $f'(x)=\frac{1}{x^2+f^2(x)} Showthat f(x) < 1+\frac{\pi}{4} \forall x \geq 1.$

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18. Let f be a contiuous function on $[a, b].$ If $F(x)=\left(\int_a^x f(t)dt-\int_x^b f(t)dt\right)(2x-(a+b)),$

then prove that there exist some $c\varepsilon(a, b)$ such that $\int_a^c f(t)dt - \int_c^b f(t)dt = f(c)(a+b-2c)$

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19. $f(x)$ is a continuous and bijective function on $R\cdot$ If $\forall t \in R,$ then the area bounded by $y = f(x), x = a - t, x = a,$ and the x-axis is equal to the area bounded by $y = f(x), x = a + t, x = a,$ and the x-axis. Then prove that $\int_{-\lambda}^{\lambda} f^{-1}(x)dx = 2a\lambda(giventhat f(a) = 0)\cdot$

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20. If $f(x) = x + \int_0^1 t(x+t)f(t)dt,$ then find the value of the definite integral $\int_0^1 f(x)dx.$

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CAE_TYPE

1. Evaluate the following integrals using limit of sum.

$$\int_a^b \cos x dx$$

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2. Evaluate the following integrals .

$$\int_a^b x^3 dx$$

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3. Find the value of $\int_0^4 [x]dx$, where $[.]$ represents the gretest integer function.

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4. If $f(x) = \{1 - |x|, |x| \leq 1$ and 0 , |x|>1 and g(x)=f(x-1)+f(x+1), find the value of $\int_{-3}^5 g(x)dx\cdot$

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5. Consider the integral $\int_0^{2\pi} \frac{dx}{5 - 2\cos x}$ making the substitution $\frac{\tan 1}{x} = t$, we have $I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x}$ $= \int_0^0 \frac{2dt}{(1 + t^2)\left[\frac{5 - 2(1 - t^2)}{(1 + t^2)}\right]} = 0$ The result is obviously wrong, since the integrand is positive and consequently the integral of this function cannot be equal to zero. Find the mistake.

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6. Evaluate the following : $\int_0^{\pi} \frac{dx}{1 + \sin x}$

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7. Evaluate: $\int_1^{\infty} \left(e^{x+1} + e^{3-x}\right)^{-1} dx$

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8. Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)\sqrt{1-x^2}} dx$

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9. Evaluate: $\int_0^1 \frac{2-x^2}{(1+x)\sqrt{1-x^2}} dx$

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10. Evaluate the following : $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$

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11. Evaluate: $\int_{\pi/6}^{\pi/4} \frac{1+\cot x}{e^x \sin x} dx$

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12. Evaluate $\int_0^1 \frac{e^{-x}dx}{1+e^x}$

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13. Prove that $\int_0^{102} (x-1)(x-2)..(x-100) \times \left(\frac{1}{x-1} + \frac{1}{x-2} + ...+ \frac{1}{x-100}\right)dx = 101! - 100!$

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14. Show that : $\int_0^1 \frac{\log x}{(1+x)}dx = -\int_0^1 \frac{\log(1+x)}{x}dx$

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15. If $\int_0^1 \frac{e^t}{1+t}dt = a$, then find the value of $\int_0^1 \frac{e^t}{(1+t)^2}dt$ in terms of a .

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16. Let f be a one-to-one continuous function such that $f(2)=3$ and $f(5)=7$. Given $\int_2^5 f(x)dx=17$, then find the value of $\int_3^7 f^{-1}(x)dx$.

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17. Evaluate:

$$(\lim)_{n\to\infty}\left(\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-2^2}}+\ldots\ldots.+\frac{1}{\sqrt{3n^2}}\right)$$

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18. Evaluate: $\lim_{n\to\infty}\left[\left(\frac{1}{n^2}\sec^2\left(\frac{1}{n^2}\right)+\frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right)\ldots.+\frac{1}{n}\sec^2(1)\right]$

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19. Evaluate $(\lim)_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}$

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20. Evaluate the following limit:

$$\lim_{n \to \infty} \frac{\sum_{r=1}^{n} \sqrt{r} \sum_{r=1}^{n} \frac{1}{\sqrt{r}}}{\sum_{r=1}^{n} r}$$

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21. Evaluate the following limit:

$$\lim_{n \to \infty} \left[\frac{n!}{n^n}\right]^{1/n}$$

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22. Prove that $4 \leq \int_1^3 \sqrt{3 + x^2} dx \leq 4\sqrt{3}$

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23. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$, $I_4 = \int_1^2 2^{x^3} dx$ then

A. $I_1 > I_2$

B. $I_2 > I_1$

C. $I_3 > I_4$

D. $I_3 < I_4$

Answer: A::D

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24. If

$$I_I = \int_0^{\pi/2} \cos(\sin x) dx,\ I_2 = \int_0^{\frac{\pi}{2}} \sin(\cos x) dx \text{ , } and\ I_3 = \int_0^{\frac{\pi}{2}} \cos x dx,$$

then find the order in which the values $I_1, I_2, I_3,$ exist.

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25. Show that : $\frac{\pi}{6} \leq \int_0^1 \frac{dx}{\sqrt{4 - x^2 - x^3}} \leq \frac{\pi}{4\sqrt{2}}$

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26. Evaluate $\int_0^{\pi/2} |\sin x - \cos x| dx.$

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27. Evaluate: $\int_{-1}^{4} f(x)dx = 4 and \int_2^4 (3 - f(x))dx = 7,$ then find the value of $\int_2^{-1} f(x)dx.$

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28. Evaluate $\int_1^5 \sqrt{(x - 2)\sqrt{x - 1}} dx.$

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29. Evaluate: $\int_{-1}^{3}\left(\frac{\tan^{-1}x}{x^2+1}+\frac{\tan^{-1}(x^2+1)}{x}\right)dx$

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30. Evaluate $\int_{1}^{a} x.\,a^{-[\log_a x]}dx,\ (a>1)$.Here $[.]$ represents the greatest integer function.

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31. Evaluate: $\int_{1}^{e^6}\left[\frac{\log x}{3}\right]dx, where[.]$ denotes the greatest integer function.

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32. Find the value of $\int_{-1}^{1}[x^2+\{x\}]dx, where[.]and\{.\}$ denote the greatest function and fractional parts of x, respectively.

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33. Prove that $\int_0^x [\cot^{-1} x] dx, where[.]$ denotes the greatest integer function.

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34. Prove that $\int_0^x f[t] dt = \frac{[x]([x]-1)}{2} + [x](x-[x]),$ where [.] denotes the greatest integer function.

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35. Evaluate: $\int_0^\infty [2e^{-e}] dx, where[x]$ represents greatest integer function.

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36. If $f(a+b-x)=f(x)$, then prove that $\int_a^b x f(x) dx = \frac{a+b}{2}\int_a^b f(x) dx$.

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37. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is

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38. Find the value of $\int_0^1 \sqrt[3]{2x^3-3x^2-x+1} dx$.

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39. Show that $\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x) dx$.

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40. Find the value of $\int_0^1 x(1-x)^n dx$

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41. If a continuous function f on $[0, a]$ satisfies $f(x)f(a-x)=1, a>0,$ then find the value of $\int_0^a \frac{dx}{1+f(x)}$

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42. If $fandg$ are continuous function on $[0, a]$ satisfying $f(x)=f(a-x)andg(x)(a-x)=2,$ then show that $\int_0^a f(x)g(x)dx=\int_0^a f(x)dx.$

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43. Find the value of $\int_0^{\pi/2} \sin 2x \log \tan x dx.$

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44. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0 is

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45. answer any one question : (ii) evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

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46. Evaluate $\int_0^{\pi} \frac{x dx}{1+\cos\alpha \sin x}$,where $0 < \alpha < \pi$.

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47. Find the value of $\int_0^{2\pi} \frac{1}{1+\tan^4 x} dx$

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48. $\int_0^{2\pi} \sin^{100} x \cos^{99} x dx$ equals to ?

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49. For $U_n = \int_0^1 x^n (2-x)^n dx; V_n = \int_0^1 x^n (1-x)^n dx n \in N,$ which of the following statement(s) is/are true?

(a)$U_n = 2^n V_n$ (b) $U_n = 2^{-n} V_n$ $U_n = 2^{2n} V_n$ (d) $V_n = 2^{-2n} U_n$

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50. Evaluate: $\int_0^{\pi} \log(1+\cos x) dx$

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51. Find the value of $\int_0^1 (\sin^{-1} x) dx$

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52. Evaluate $\int_{-\infty}^{0} \frac{te^t}{\sqrt{1-e^{2t}}} dt$

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53. If $I_1 = \int_0^{\pi} x f(\sin^3 x + \cos^2 x) dx$ and $I_2 = \int_0^{\frac{\pi}{2}} f(\sin^3 x + \cos^2 x) dx, then relate I_1$ and I_2

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54. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x (\sin x + \cos x) dx$

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55. Evaluate: $\int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

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56. Evaluate: $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x dx$

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57. Evaluate: $\int_{-1}^{1}\frac{\sin x - x^2}{3-|x|}dx$

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58. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos x\frac{\log(1-x)}{1+x}dx$

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59. Evaluate: $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$

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60. Evaluate: $\int_0^{100} (x - [x]dx(where[.]$ represents the greatest integer function).

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61. Evaluate: $\int_0^{100\pi} \sqrt{(1 - \cos 2x)}dx.$

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62. $If \int_0^{n\pi} f(\cos^2 x)dx = k\int_0^{\pi} f(\cos^2 x)dx, thenf \in dthevaluek.$

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63. Evaluate $\int_0^{n\pi+t} (|\cos x| + |\sin x|)dx,$ where $n\varepsilon N$ and $t\varepsilon[0, \pi/2]$.

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64. Find the value of : $\int_0^{10} e^{2x-[2x]} d(x-[x]) where [.]$ denotes the greatest integer function).

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65. If $f(x)$ is a function satisfying $f(x+a)+f(x)=0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x)dx$ is independent of b, then find the least positive value of c

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66. Show that $\int_0^{n\pi+v} |\sin x| dx = 2n+1-\cos v$, where n is a positive integer and `,lt=v

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67. $If \int_{\frac{\pi}{3}}^{x} \sqrt{(3-\sin^2 t)} dt + \int_0^y \cos t dt = 0, thenevaluate \frac{dy}{dx}$

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68. $If f(x) = e^{g(x)} and g(x) = \int_2^x \frac{tdt}{1+t^4}$, then find the value of $f'(2)$

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69. Evaluate $(\lim)_{x \to 4} \int_4^x \frac{(4t - f(t))}{(x-4)} dt$

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70. Evaluate: $\lim_{x \to 0} \frac{\int_0^{x^2} \cos^2 t dt}{x \sin x}$

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71. Find the points of minima for $f(x) = \int_0^x t(t-1)(t-2)dt$

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72. Find the equation of tangent to $y = \int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}} at x = 1.$

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73. $If f(x) = \int_{\frac{\pi^2}{16}}^{x^2} \frac{\sin x \sin \sqrt{\theta}}{1+\cos^2 \sqrt{\theta}} d\theta,$ then find_the value of $f'\left(\frac{\pi}{2}\right)$.

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74. If $f(x)$ is a function satisfying $f(x+a)+f(x)=0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x)dx$ is independent of b, then find the least positive value of c

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75. Let $f(x)$ be a differentiable function satisfying $f(x) = \int_0^x e^{(2tx - t^2)} \cos(x-t)dt$, then find the value of $f''(0)$.

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76. $If \int_0^1 \frac{e^t dt}{t+1} = a$,then find the value of $\int_{b-1}^{b} \frac{e^{-t} dt}{t-b-1}$

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77. $f(x) = \int_1^x \frac{\log t}{1+t+t^2} dt (x \geq 1)$ then prove that f(x) =f(1/x)

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78. $f(x) = \int_1^x \frac{\tan^{-1}(t)}{t} dt \ \forall x \in R^+,$ then find the value of $f(e^2) - f\left(\frac{1}{e^2}\right)$

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79. Evaluate: $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{(x^2-1)}{(x^2+1)^2} dx$

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80. Evaluate: $\int_0^{e-1} \frac{\frac{x^2+2x-1}{2}}{x+1} dx + \int_1^e x \log x e^{\frac{x^2-2}{2}} dx$

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81. Find the value of $\int_{\frac{1}{2}}^{2} e^{\left|x-\frac{1}{x}\right|} dx.$

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82. If $I_1 = \int_0^1 \frac{dx}{e^x(1+x)}$ and $I_2 = \int_0^{\pi/4} \frac{e^{\tan^2\theta}\sin\theta}{(2-\tan^2\theta)\cos^3\theta} d\theta$,then find the value of $\frac{l_1}{l_2}$.

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83. If $I_K = \int_1^e (1nx)^k dx\left(k \in I^+\right)dx\left(k \in I^+\right),$ then find the value of I_4.

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84. Given $I_m = \int_1^e (\log x)^m dx$,then prove that $\frac{I_m}{1-m} + mI_{m-2} = e$

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85. If $I_n = \int_0^\pi x^n \sin x dx$, then find the value of $I_5 + 20I_3$.

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86. If $L(m, n) = \int_0^1 t^m (1+t)^n, dt$, then prove that

$L(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} L(m+1, n-1)$

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87. $If I_n = \int_0^1 x^n (\tan^{-1} x) dx, then prove that$

$(n+1)I_n + (n-1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$

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88. $If I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx,$ Then show that $I_{m,n} = \frac{m-1}{m+n} I_{m-2,n} (m, n \in N)$ Hence, prove that $I_{m,n} = f(x) = \left\{ \frac{(n-1)(n-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} \frac{\pi}{4} \right.$

when both m and n are even

$$\left. \frac{(m-1)(m-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} \right\}$$

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SCQ_TYPE

1. Let

$$f(x) = \lim_{n \to \infty} \frac{1}{n} \left(\left(x + \frac{1}{n}\right)^2 + \left(x + \frac{2}{n}\right)^2 + \dots\dots\dots + \left(x + \frac{n-1}{n}\right.\right.$$

Then the minimum value of $f(x)$ is

A. $1/4$

B. $1/6$

C. $1/9$

D. $1/12$

Answer: D

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2. $IfS_n = \left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + + \frac{1}{n+\sqrt{n^2}}\right], then(\lim)_{n\overrightarrow{\infty}} S_n$

is equal to log 2 (b) log 4 log 8 (d) none of these

A. $\log 2$

B. $\log 4$

C. $\log 8$

D. none of these

Answer: B

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3. The value of $(\lim)_{n \overrightarrow{\infty}} \sum_{r=1}^{4n} \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+\sqrt{n})^2}$ is equal to $\frac{1}{35}$ (b) $\frac{1}{4}$ (c) $\frac{1}{10}$ (d) $\frac{1}{5}$

A. $\frac{1}{35}$

B. $\frac{1}{14}$

C. $\frac{1}{10}$

D. $\frac{1}{5}$

Answer: C

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4. The value of

$$\lim_{n \to \infty} \frac{(1^2 + 2^2 + \dots\dots\dots + n^2)(1^3 + 2^3 + \dots\dots\dots + n^3)(1^4 + 2^4 + \dots}{(1^5 + 2^5 + \dots\dots\dots\dots + n^5)^2}$$

is equal to

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$

Answer: A

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5. $The value of (\lim)_{n \to \infty} \left[tan\frac{\pi}{2n}\frac{\tan(2\pi)}{2n}\frac{\tan(n\pi)}{2n}\right]^{1/n} is\ e$ (b) e^2 (c) 1 (d) e^3

A. e

B. e^2

C. 1

D. e^3

Answer: C

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6. $\int_{2-a}^{2+a} f(x)dx$ is equal to [where $f(2-\alpha) = f(2+\alpha)\,\forall \alpha \in R$

(a) $2\int_{2}^{2+a} f(x)dx$ (b) $2\int_{0}^{a} f(x)dx$ (c) $2\int_{2}^{2} f(x)dx$ (d) none of these

A. $2\int_{2}^{2+a} f(x)dx$

B. $2\int_{0}^{a} f(x)dx$

C. $2\int_{2}^{2} f(x)dx$

D. none of these

Answer: A

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7. Let $f(x) = \min(\{x\}, \{-x\})\,\forall \varepsilon R$, where {.} denotes the fractional part of x. Then $\int_{-100}^{100} f(x)dx$ is equal to

A. (a) 50

B. (b) 100

C. (c) 200

D. (d) none of these

Answer: A

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8. Which of the following is incorrect? $\int_{ac}^{b+c} f(x)dx = \int_{a}^{b} f(x+c)dx$ $\int_{ac}^{bc} f(x)dx = \int_{a}^{b} f(cx)dx$ $\int_{-a}^{a} f(x)dx = \frac{1}{2}\int_{-a}^{a} (f(x) + f(-x)dx$

None of these

A. (a) $\int_{a+c}^{b+c} f(x)dx = \int_{a}^{b} f(x+c)dx$

B. (b) $\int_{ac}^{bc} f(x)dx = c\int_{a}^{b} f(cx)dx$

C. (c) $\int_{-a}^{a} f(x)dx = \frac{1}{2}\int_{-a}^{a} f(x) + f(-x)dx$

D. (d) none of these

Answer: D

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9. Evaluate $\int_0^1 \frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$

A. $\frac{\sqrt{e}}{2}(\sqrt{3}+1)$

B. $\frac{\sqrt{3e}}{2}$

C. $2e-1$

D. $\sqrt{\frac{e}{3}}$

Answer: C

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10. $If\int_{\log 2}^{x} \frac{dy}{\sqrt{e^y-1}} = \frac{\pi}{6}, \text{ then } \ x \ \text{ is equal to}$

(a)4 (b) 1n 8 (c) 1n 4 (d) none of these

A. 4

B. $In8$

C. $In4$

D. none of these

Answer: C

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11. $\int_{\frac{5}{2}}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4} dx$ is equal to

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{3}$

Answer: D

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12. If $f(x)$ satisfies the condition of Rolle's theorem in $[1, 2]$, then $\int_1^2 f'(x)dx$ is equal to (a) 1 (b) 3 (c) 0 (d) none of these

A. 1

B. 3

C. 0

D. none of these

Answer: C

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13. The value of the integral $\int_0^{\log 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3}dx$

A. $3 + 2\pi$

B. $4 - \pi$

C. $2+\pi$

D. none of these

Answer: B

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14. The value of the integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}, 0<\alpha<\pi$ is

A. $\sin\alpha$

B. $\alpha\sin\alpha$

C. $\frac{\alpha}{\sin\alpha}$

D. $\frac{\alpha}{2}\sin\alpha$

Answer: C

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15. $\int_0^{\infty} \frac{dx}{\left[x+\sqrt{x^2+1}\right]^3}$ is equal to (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) none of these

A. $\frac{3}{8}$

B. $\frac{1}{8}$

C. $-\frac{3}{8}$

D. none of these

Answer: A

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16. If $f(y) = e^y, g(y) = y > 0, \text{and } F(t) = \int_0^t f(t-y)g(y)dy$,then

A. $F(t) = e^t - (1+t)$

B. $F(t) = te^t$

C. $F(t) = te^{-t}$

D. $F(t) = 1 - e^{t}(1 + t)$

Answer: A

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17. about to only mathematics

A. $\frac{17}{4}$

B. $\frac{13}{4}$

C. $\frac{19}{4}$

D. $\frac{5}{4}$

Answer: C

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18. The numbers of possible continuous $f(x)$ defined in $[0, 1]$ for which $I_1 = \int_0^1 f(x)dx = 1, I_2 = \int_0^1 xf(x)dx - a, I_3 = \int_0^1 x^2 f(x)dx = a^2 is / a$

1 (b) ∞ (c) 2 (d) 0

A. 1

B. ∞

C. 2

D. 0

Answer: D

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19. Suppose that $F(x)$ is an anti-derivative of $f(x) = \frac{\sin x}{x}, where x > 0$. Then $\int_1^3 \tan^{-1} dx$ can be expressed as $F(6) - F(2)$ (b) $\frac{1}{2}(F(6) - f(2))$ $\frac{1}{2}(F(3) - f(1))$ (d) $2(F(6)) - F(2))$

A. $F(6) - F(2)$

B. $\frac{1}{2}(F(6) - F(2))$

C. $\frac{1}{2}(F(3) - F(1))$

D. $2(F(6) - F(2))$

Answer: A

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20. $\int_{-\frac{\pi}{3}}^{0} \left[\cot^{-1}\left(\frac{2}{2\cos x - 1}\right) + \cot^{-1}\left(\cos x - \frac{1}{2}\right)\right] dxisequa < o$

$\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{3\pi^2}{8}$

A. $\frac{\pi^2}{6}$

B. $\frac{\pi^2}{3}$

C. $\frac{\pi^2}{8}$

D. $\frac{3\pi^2}{8}$

Answer: A

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21. Evaluate the definite integrals $\int_0^{\pi/4} \frac{\sin x + \cos x}{25 - 16(\sin x - \cos x)^2} dx$

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22. $\int_{-1}^{1} \frac{e^{-\frac{1}{x}}}{x^2\left(1 + e^{-\frac{2}{x}}\right)} dx$ is equal to :

A. $\frac{\pi}{2} - 2\tan^{-1} e$

B. $\frac{\pi}{2} - 2\cot^{-1} e$

C. $2\tan^{-1} e$

D. $\pi - 2\tan^{-1} e$

Answer: D

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23. If $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$, then $\int_0^{\infty} \frac{\sin^3 x}{x} dx$ is equal to

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/2$

Answer: B

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24. The range of the function $f(x) = \int_{-1}^{1} \frac{\sin x dt}{1 - 2t \cos x + t^2}$ is

A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $[0, \pi]$

C. $\{0, \pi\}$

D. $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

Answer: D

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25. If the function $f:[0,8]\to R$ is differentiable, then for $0<\alpha<1<\beta<2, \int_0^8 f(t)dt$ is equal to

A. $3\left[\alpha^3 f\left(\alpha^2\right)+\beta^2 f\left(\beta^2\right)\right]$

B. $3\left[\alpha^3 f(\alpha)+\beta^3 f(\beta)\right]$

C. $3\left[\alpha^2 f\left(\alpha^3\right)+\beta^2 f\left(\beta^3\right)\right]$

D. $3\left[\alpha^2 f\left(\alpha^2\right)+\beta^2 f\left(\beta^2\right)\right]$

Answer: C

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26. If $f(x)=x^5+5x-1$ then $\int_5^{41}\frac{dx}{\left(f^{-1}(x)\right)^5+5f^{-1}(x)}$ equals

A. 0

B. $\log_e 3$

C. $\log_e 4$

D. $\log_e 7$

Answer: D

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27. Let $f(0) = 0 and \int_0^2 f'(2t)e^{f(2t)}\,dt = 5. then value of f(4) is$ log 2 (b) log 7 (c) log 11 (d) log 13

A. $\log 2$

B. $\log 7$

C. $\log 11$

D. $\log 13$

Answer: C

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28. If $f(x) = \cos\left(\tan^{-1}x\right)$, then the value of the integral $\int_0^1 xf''(x)dx$ is (a) $\frac{3-\sqrt{2}}{2}$ (b) $\frac{3+\sqrt{2}}{2}$ (c) 1 (d) $1 - \frac{3}{2\sqrt{2}}$

A. $\frac{3-\sqrt{2}}{2}$

B. $\frac{3+\sqrt{2}}{2}$

C. 1

D. $1 - \frac{3}{2\sqrt{2}}$

Answer: D

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29. The equation of the curve is $y = f(x)\cdot$ The tangents at $[1, f(1), [2, f(2)], and[3, f(3)]$ make angles $\frac{\pi}{6}, \frac{\pi}{3}, and\frac{\pi}{4}$, respectively,

with the positive direction of x-axis. Then the value of $\int_2^3 f'(x)f''(x)dx + \int_1^3 f''(x)dx$ is equal to

A. $-1/\sqrt{3}$

B. $1/\sqrt{3}$

C. 0

D. none of these

Answer: A

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30. The value of $\int_1^e \left(\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right)dx, is$ (a) $\tan e$ (b) $\tan^{-1} e$ (c) $\tan^{-1}\left(\frac{1}{e}\right)$ (d) none of these

A. $\tan e$

B. $\tan^{-1} e$

C. $\tan^{-1}(1/e)$

D. none of these

Answer: B

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31. If $f(\pi) = 2$ and $\int_0^{\pi} (f(x) + f''(x)) \sin x dx = 5$, then $f(0)$ is equal to (it is given that $f(x)$ is continuous in $[0, \pi]$). (a) 7 (b) 3 (c) 5 (d) 1

A. 7

B. 3

C. 5

D. 1

Answer: B

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32. If $\int_1^2 e^x \hat{} 2dx = a, then \int_e^{e^4} \sqrt{1nx} dxisequa < o\ 2e^4 - 2e - a$ (b) $2e^4 - e - a\ 2e^4 - e - 2a$ (d) $e^4 - e - a$

A. $2e^4 - 2e - a$

B. $2e^4 - e - a$

C. $2e^4 - e - 2a$

D. $e^4 - e - a$

Answer: B

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33. If $f(x)$ is continuous for all real values of x, then $\sum_{r=1}^{n} f(r - 1 + x) dxisequa < o \int_0^n f(x)dx$ (b) $\int_0^1 f(x)dx\ n \int_0^1 f(x)dx$ (d) $(n-1)\int_0^1 f(x)dx$

A. $\int_0^n f(x)dx$

B. $\int_0^1 f(x)dx$

C. $n\int_0^1 f(x)dx$

D. $(n-1)\int_0^1 f(x)dx$

Answer: A

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34. $Thevalueof\int_0^{\frac{\pi}{2}} \sin|2x-\alpha|dx, where\alpha \in [0, \pi], is \quad 1-\cos\alpha$ (b) $1+\cos\alpha$ 1 (d) $\cos\alpha$

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35. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \frac{n^2}{2} \forall n \in I\cdot$ Then $\int_{-3}^5 f(|x|)dx$ is equal to (a) $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) none of these

A. $19/2$

B. $35/2$

C. $17/2$

D. none of these

Answer: B

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36. If $f(x) = \int_{-1}^{x} |t| dt$, then for any $x \geq 0$, $f(x)$ equals

A. $\frac{1}{2}(1 - x^2)$

B. $\frac{1}{2}x^2$

C. $\frac{1}{2}(1 + x^2)$

D. none of these

Answer: C

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37. The value of $\int_{-a}^{a}\left(\cos^{-1}x - \sin^{-1}\sqrt{1-x^2}\right)dx$ is $(a > 0)$ where $\left(\int_0^a \cos^{-1}xdx = A\right)$ is

A. 0

B. 2

C. 3

D. none of these

Answer: B

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38. The value of $\int_1^a [x]f'(x)dx$,where $a > 1$, and $[x]$ denotes the greatest integer not exceeding x, is

(A) $af(a) - \{f(1)f(2) + \ldots.. + f([a])\}$

(B) $[a]f(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$

(C) $[a]f(a) - \{f(1) + f(2) + \ldots\ldots. + fA\}$

(D) $af([a]) - \{f(1) + f(2) + \ldots\ldots + fA\}$

A. $af(a) - (f(1) + f(2) + \dots\dots\dots + f([a]))$

B. $[a]f(a) - (f(1) + f(2)) + \dots\dots\dots + f([a]))$

C. $[a]f([a]) - (f(1) + f(2) + \dots\dots\dots + f(a))$

D. $af([a]) - (f(1) + f(2) + \dots\dots\dots + f(a))$

Answer: B

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39. $\int_3^{10} [\log[x]]dx$ is equal to (where [.] represents the greatest integer function)

A. 9

B. $16 - e$

C. 10

D. $10 + e$

Answer: A

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40. $\int_{-1}^{2}\left[\frac{[x]}{1+x^2}\right]dx, where[.]$ denotes the greater integer function, is equal to (a)-2 (b) -1 (c)0 (d) none of these

A. -2

B. -1

C. zero

D. none of these

Answer: B

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41. The value of $\int_{-2}^{1}\left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right]+1\right]dx,$ where [.] denotes the greatest integer function, is (a)1 (b) $1/2$ (c) 2 (d) none of these

A. 1

B. $1/2$

C. 2

D. none of these

Answer: C

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42. The value of $\int_0^{2\pi} [2\sin x]dx, where[.]$ represents the greatest integral function, is $\frac{-5\pi}{3}$ (b) $-\pi$ $\frac{5\pi}{3}$ (d) -2π

A. $\frac{-5\pi}{3}$

B. $-\pi$

C. $\frac{5\pi}{3}$

D. -2π

Answer: B

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43.

$I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx,\ I_2 = \int_0^{2\pi} \cos^6 x dx,\ I_3 = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x dx,\ I_4 = \int_0$

$I_2 = I_3 = I_4 = 0, I_1 \neq 0$ $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

$I_1 = I_2 = I_3 = 0, I_4 \neq 0$ $I_1 = I_4 = I_3 = 0, I_2 \neq 0$

A. $I_2 = I_3 = I_4 = 0, I_1 \neq 0$

B. $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

C. $I_1 = I_3 = I_4 = 0, I_2 \neq 0$

D. $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

Answer: C

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44. Given $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} = A$. Then the value of the definite integral $\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x} dx$ is equal to

A. $\frac{1}{2}A$

B. $\frac{\pi}{2} - A$

C. $\frac{\pi}{4} - \frac{1}{2}A$

D. $\frac{\pi}{2} + A$

Answer: C

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45. $\text{I f}I_1 = \int_{-100}^{101} \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})}$ $\text{and}I_2 = \int_{-100}^{101} \frac{dx}{5 + 2x - 2x^2}, then\frac{I_1}{I_2}$i s (a)2 (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{1}{2}$

A. 2

B. $\frac{1}{2}$

C. 1

D. $-\frac{1}{2}$

Answer: B

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46. Find the value of $\int_0^{\infty}\frac{xdx}{(1+x)(1+x^2)}$ equals to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none of these

Answer: A

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47. Q. $\int_0^{\pi} e^{\cos^2 x}(\cos^3(2n+1)xdx, n \in I$

A. π

B. 1

C. 0

D. none of these

Answer: C

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48. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} x f([x(1-x)]dx,$ $I_2 = \int_{1-k}^{k} f[x(1-x)]dx, where2k - 1 > 0. Then \frac{I_1}{I_2} is$ 2 (b) k (c) $\frac{1}{2}$ (d) 1

A. 2

B. k

C. $\frac{1}{2}$

D. 1

Answer: C

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49. If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)} xg(x(1-x))dx,$ and $I_2=\int_{f(-a)}^{f(a)} g(x(1-x))dx,$ then the value of $\frac{I_2}{I_1}$ is

A. -1

B. -2

C. 2

D. 1

Answer: C

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50. $Thevalueof\int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1}\log\left(1+x-\frac{1}{x}\right)dxis$ (a)$\frac{\pi}{8}(\log)_e 2$ (b) $\frac{\pi}{2}(\log)_e 2$ (c)$-\frac{\pi}{2}(\log)_e 2$ (d) none of these

A. $\frac{\pi}{8}\log_e 2$

B. $\frac{\pi}{2}\log_e 2$

C. $-\frac{\pi}{2}\log_e 2$

D. none of these

Answer: A

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51. The value of the definite integral $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} dx$ is

(a) $\sqrt{2}\pi$ (b) $\frac{\pi}{\sqrt{2}}$ (c) $2\sqrt{2}\pi$ (d) $\frac{\pi}{2\sqrt{2}}$

A. $\sqrt{2}\pi$

B. $\frac{\pi}{\sqrt{2}}$

C. $2\sqrt{2}\pi$

D. $\frac{\pi}{2\sqrt{2}}$

Answer: B

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52. $f(x) > 0 \forall x \in R$ and is bounded. If

$$\lim_{n \to \infty} \left[\int_0^a \frac{f(x)dx}{f(x) + f(a-x)} + a\int_a^{2a} \frac{f(x)dx}{f(x) + f(3a-x)} + a^2 \int_{2a}^{3a} \frac{f(x)dx}{f(x) + f(5a-x)} + \ldots + a^{n-1} \int_{(n-1)a}^{na} \frac{f(x)dx}{f(x) + f[(2n-1)a - x]} \right.$$

$= 7/5$ (where $a < 1$), then a is equal to

A. $\frac{2}{7}$

B. $\frac{1}{7}$

C. $\frac{14}{19}$

D. $\frac{9}{14}$

Answer: C

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53. $If \int_0^1 \cot^{-1}(1 - x + x^2) dx = \lambda \int_0^1 \tan^{-1} x dx, then \lambda is equal \to$ 1

(b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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54. The value of the definite integral $\int_{-1}^{1}(1+x)^{1/2}(1-x)^{3/2}dx$ equals

A. π

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: D

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55. $Thevalueofthe \in tegral \int_{-\frac{3\pi}{4}}^{\frac{5\pi}{4}} \left(\frac{\sin x + \cos x}{e^{x - \frac{\pi}{4}} + 1} dx$ 0 (b) 1 (c) 2 (d) none of these

A. 0

B. 1

C. 2

D. none of these

Answer: A

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56. $I_1 = \int_0^{\frac{\pi}{2}} \ln(\sin x) dx, I_2 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin x + \cos x) dx \cdot$ Then (a) $I_1 = 2I_2$ (b) $I_2 = 2I_1$ $I_1 = 4I_2$ (d) $I_2 = 4I_1$

A. $I_1 = 2I_2$

B. $I_2 = 2I_1$

C. $I_1 = 4I_2$

D. $I_2 = 4I_1$

Answer: A

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57. If $I_1 = \int_0^{\pi/2} \frac{\cos^2 x}{1 + \cos^2 x} dx$, $I_2 = \int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin^2 x} dx$ and $I_3 = \int_0^{\pi/2} \frac{1 + 2\cos^2 x \sin^2 x}{4 + 2\cos^2 x \sin^2 x} dx$, then (a) $I_1 = I_2 > I_3$ (b) $I_3 > I_1 = I_2$ (c) $I_1 = I_2 = I_3$ (d) none of these

A. $I_1 = I_2 > I_3$

B. $I_3 > I_1 = I_2$

C. $I_1 = I_2 = I_3$

D. none of these

Answer: C

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58. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{8}$

D. $\frac{\pi^2}{16}$

Answer: D

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59. For $x \in R$ and a continuous function f, let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x f\{x(2-x)\} dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\} dx$.Then $\frac{I_1}{I_2}$ is (a) -1 (b) 1 (c) 2 (d) 3

A. -1

B. 1

C. 2

D. 3

Answer: B

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60. $If \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{e^{\frac{\pi}{4}} dx}{\left(e^x + e^{\frac{\pi}{4}}\right)(\sin x + \cos x)} = k \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec x dx$, then the value of k is (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$

D. $-\frac{1}{\sqrt{2}}$

Answer: C

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61. The value of the definite integral $\int_2^4 x(3-x)(4+x)(6-x)(10-x) + \sin x \Big) dx equals$ $\cos 2 + \cos 4$

(b) $\cos 2 - \cos 4 \sin 2 + \sin 4$ (d) $\sin 2 - \sin 4$

A. $\cos 2 + \cos 4$

B. $\cos 2 - \cos 4$

C. $\sin 2 + \sin 4$

D. $\sin 2 - \sin 4$

Answer: B

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62. $IfI = \int_{-20\pi}^{20\pi} |\sin x|[\sin x]dx(where[.]$ denotes the greatest integer function), then the value of I is -40 (b) 40 (c) 20 (d) -20

A. -40

B. 40

C. 20

D. -20

Answer: A

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63. The function f and g are positive and continuous. If f is increasing and g is decreasing, then $\int_0^1 f(x)[g(x) - g(1-x)]dx$

(a)is always non-positive

(b)is always non-negative

(c)can take positive and negative values

(d)none of these

A. is always non-positive

B. is always non-negative

C. can take positive and negative values

D. none of these

Answer: A

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64. Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

A. $\frac{\pi^2}{4}$

B. $\frac{\pi^2}{2}$

C. $\frac{3\pi^2}{2}$

D. $\frac{\pi^2}{3}$

Answer: A

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65. If $f(x)=\int_0^{\pi}\frac{t\sin t dt}{\sqrt{1+\tan^2 x\sin^2 t}}$ for $0<x<\frac{\pi}{2}$, then (a) $f(0^+)=-\pi$ (b) $f\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8}$ (c) f is continuous and differentiable in $\left(0,\frac{\pi}{2}\right)$ (d) f is continuous but not differentiable in $\left(0,\frac{\pi}{2}\right)$

A. $f(0^+)=-\pi$

B. $f\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8}$

C. f is continuous and differntiable in $\left(0,\frac{\pi}{2}\right)$

D. f is continuous but not differentiable in $\left(0,\frac{\pi}{2}\right)$

Answer: C

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66. about to only mathematics

A. 3^8

B. 3^7

C. 3^9

D. none of these

Answer: B

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67. The value of $\int_0^{4\pi} \log_e |3\sin x + 3\sqrt{3}\cos x| dx$ then the value of I is equal to

A. $\pi \log_e 3$

B. $2\pi \log_e 3$

C. $4\pi \log_e 3$

D. $8\pi \log_e 3$

Answer: C

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68. The value of $\int_0^{\pi} \frac{|x|\sin^2 x}{1+2|\cos x|\sin x} dx$ is equal to

A. a. $\pi/4$

B. b. $\pi/2$

C. c. π

D. d. 2π

Answer: B

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69. The value of the integral $\int_{-\pi}^{\pi} \sin mx \sin nx dx$, for $m \neq n (m, n \in I), is$

A. 0

B. π

C. $\pi/2$

D. 2π

Answer: A

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70. If $f(x) and g(x)$ are continuous functions, then $\int_{1n\lambda}^{1n\frac{1}{\lambda}} \frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x)+g(-x)]} dx$ is

(a)dependent on λ (b) a none-zero constant (c)zero (d) none of these

A. depenent on λ

B. a non zero constant

C. zero

D. none of these

Answer: C

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71. $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is equal to

A. $1/4$

B. $1/2$

C. 1

D. 2

Answer: B

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72. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{|\sin x|}\cos x}{(1+e^{\tan x})dxisequa < o}$ $e+1$ (b) $1-e$ $e-1$ (d) none of these

A. $e+1$

B. $2e$

C. $e-1$

D. $e-2$

Answer: C

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73. about to only mathematics

A. π

B. π^2

C. $2\pi^2$

D. $\pi^2/2$

Answer: B

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74. The value of $\int_{-\pi}^{\pi}\sum_{r=0}^{999}\cos rx\left(1+\sum_{r=1}^{999}\sin rx\right)$dx, is

A. 2π

B. 999π

C. 0

D. π

Answer: A

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75. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}I$

B. $2I$

C. $3I$

D. $6I$

Answer: C

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76. $\int_{1}^{4}\{x-0.4\}dx$ equals (where $\{x\}$ is a fractional part of x

A. 13

B. 6.3

C. 1.5

D. 7.5

Answer: C

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77. The value of $\int_{0}^{x}[\cos t]dt, x\in\left[(4n+1)\frac{\pi}{2},(4n+3)\frac{\pi}{2}\right] and n\in N,$ is equal to where [.] represents greatest integer function. (a)

$\frac{\pi}{2}(2n-1)-2x$ (b) $\frac{\pi}{2}(2n-1)+x$ (c) $\frac{\pi}{2}(2n+1)-x$ (d) $\frac{\pi}{2}(2n+1)+x$

A. $\frac{\pi}{2}(2n-1)-2x$

B. $\frac{\pi}{2}(2n-1)+x$

C. $\frac{\pi}{2}(2n+1)-x$

D. $\frac{\pi}{2}(2n+1)+x$

Answer: C

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78. Evaluate: $\int_0^{2\pi}[\sin x]dx, where[.]$ denotes the greatest integer function.

A. $4n-\cos x$

B. $4n-\sin x$

C. $4n+1-\cos x$

D. $4n - 1 - \cos x$

Answer: C

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79. $\int_0^x \frac{2^t}{2^{[t]}} dt, where[.]$ denotes the greatest integer function and $x \in R^+$, is equal to

A. $\frac{1}{1n2}\left([x] + 2^{\{x\}} - 1\right)$

B. $\frac{1}{1n2}\left([x] + 2^{\{x\}}\right)$

C. $\frac{1}{1n2}\left([x] - 2^{\{x\}}\right)$

D. $\frac{1}{1n2}\left([x] + 2^{\{x\}} + 1\right)$

Answer: A

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80. Let f be an odd continous function which is periodic with priod 2 if g(x) = $\int_0^x f(t)$ dt then

A. $g(x)$ is odd

B. $2(n) = 0, n\varepsilon N$

C. $g(2n) = 0, n\varepsilon N$

D. $g(x)$ is non-periodic

Answer: C

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81. If $g(x) = \int_0^x (|\sin t| + |\cos t|)dt$,then $g\left(x + \frac{\pi n}{2}\right)$ is equal to, where $n \in N$, (a) $g(x) + g(\pi)$ (b) $g(x) + g\left(\frac{n\pi}{2}\right)$ (c) $g(x) + g\left(\frac{\pi}{2}\right)$ (d) none of these

A. $g(x) + g(\pi)$

B. $g(x) + ng\left(\frac{\pi}{2}\right)$

C. $g(x)+g\left(\frac{\pi}{2}\right)$

D. none of these

Answer: B

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82. $If x=\int_{c}^{\sin t}\sin^{-1}z dz, y=\int_{k}^{\sqrt{t}}\frac{\sin z^2}{z}dz, then \frac{dy}{dx} is equals$

A. $\frac{\tan t}{2t}$

B. $\frac{\tan t}{t^2}$

C. $\frac{\tan t}{2t^2}$

D. $\frac{\tan t^2}{2t^2}$

Answer: C

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83. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}} and g(x)$ be the inverse of $f(x)$. Then the value of $g'(0)$

A. 1

B. 17

C. $\sqrt{17}$

D. none of these

Answer: C

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84. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals (a) $\frac{2}{5}$ (b) $-\frac{5}{2}$ (c)1 (d) $\frac{5}{2}$

A. $2/5$

B. $-5/2$

C. 1

D. $5/2$

Answer: A

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85. If $f(x) = \cos x - \int_0^x (x-t) f(t) dt, then f'(x) + f(x)$ is equal to $-\cos x$ (b) $-\sin x \int_0^x (x-t) f(t) dt$ (d) 0

A. $-\cos x$

B. $-\sin x$

C. $\int_0^x (x-t) f(t) dt$

D. 0

Answer: A

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86. A function f is continuous for all x (and not everywhere zero) such that $f^2(x)=\int_0^x f(t)\frac{\cos t}{2+\sin t}dt$. Then $f(x)$ is (a) $\frac{1}{2}\ln\left(\frac{x+\cos x}{2}\right); x\neq 0$ (b) $\frac{1}{2}\ln\left(\frac{3}{x+\cos x}\right); x\neq 0$ (c) $\frac{1}{2}\ln\left(\frac{2+\sin x}{2}\right); x\neq n\pi, n\in I$ (d) $\frac{\cos x+\sin x}{2+\sin x}; x\neq n\pi+\frac{3\pi}{4}, n\in I$

A. $\frac{1}{2}In\left(\frac{x+\cos x}{2}\right)$

B. $\frac{1}{2}In\left(\frac{3}{2+\cos x}\right)$

C. $\frac{1}{2}In\left(\frac{2+\sin x}{2}\right)$

D. $\frac{\cos x+\sin x}{2+\sin x}$

Answer: C

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87. $\lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}dt-\int_{x+y}^a e^{\sin^2 t}dt\right]$ is equal to

A. $e^{\sin^2 y}$

B. $\sin 2y e^{\sin^2 y}$

C. 0

D. none of these

Answer: A

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88. Let $f(x) = \int_1^x \frac{e^t}{t} dt, x \in R^+$. Then complete set of valuesof x for which $f(x) \leq In$ x is

A. $(0, 1]$

B. $[1, \infty)$

C. $(0, \infty)$

D. none of these

Answer: A

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89. $If\int_0^x f(t)dt = x + \int_x^1 tf(t)dt,$ then the value of $f(1)$

A. $1/2$

B. 0

C. 1

D. $-1/2$

Answer: A

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90. If $f(x) = 1 + \frac{1}{x}\int_1^x f(t)dt,$ then the value of $f(e^{-1})$ is

A. 1

B. 0

C. -1

D. none of these

Answer: B

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91. If $\left[f\left(\frac{\sqrt{3}}{2}\right)\right]$ is [.] denotes the greatest integer function)

A. 4

B. 5

C. 6

D. -7

Answer: B

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92. $f(x)$ is continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$. Then the value of a is equal to: (a)$-\frac{1}{24}$ (b) $\frac{17}{168}$ (c) $\frac{1}{7}$ (d) $-\frac{167}{840}$

A. (a) $-\frac{1}{24}$

B. $\frac{17}{168}$

C. $\frac{1}{7}$

D. $-\frac{167}{840}$

Answer: D

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93. The value of $\int_{\frac{1}{e}}^{\tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e}}^{\cot x} \frac{dt}{t(1+t^2)}$ is

A. (a) 0

B. (b) 2

C. (c) 1

D. (c) none of these

Answer: C

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94. $\lim_{x \to \infty} \frac{\int_0^x \tan^{-1} t \, dt}{\sqrt{x^2+1}}$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 1

D. π

Answer: A

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95. A function is defined by $f(x) = \int_0^{\pi} \cos t \cos(x - t) dt, 0 \leq x \leq 2\pi$ then which of the following equals?

A. $\frac{\pi}{4} \cos x$

B. $\frac{\pi}{2} \cos x$

C. $\frac{-\pi}{2} \cos x$

D. $\frac{-\pi}{4} \cos x$

Answer: C

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96.

If f' is a differentiable function satisfying $f(x) = \int_0^x \sqrt{1 - f^2(t)} dt + \frac{1}{2}$ then the value of $f(\pi)$ is equal to

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

Answer: B

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97. If $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, then (a)$\alpha < 2$ (b)$\alpha < 0$ $(c)0 < \alpha < 1$ (d) $\alpha = 0$

A. $1 < \alpha < 2$

B. $\alpha < 0$

C. $0 < \alpha < 1$

D. $\alpha = 0$

Answer: C

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98.

The value of the integral $\int_0^1 e^{x^2} dx$ lies in the interval (a) $(0, 1)$ (b) $(-1, 0)$ (c) $(1, e)$ (d) none of these

A. $(0, 1)$

B. $(-1, 0)$

C. $(1, e)$

D. none of these

Answer: C

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99. Given that f satisfies $|f(u) - f(v)| \leq |u - v| f \text{ or } u \text{ and } v$ in $[a, b]$.

Then $\left|\int_a^b f(x)dx - (b-a)f(a)\right| \leq$ (a) $\frac{(b-a)}{2}$ (b) $\frac{(b-a)^2}{2}$ $(b-a)^2$

(d) none of these

A. $\frac{(b-a)}{2}$

B. $\frac{(b-a)^2}{2}$

C. $(b-a)^2$

D. none of these

Answer: B

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100. The value of the integral $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx, is$ (a)0 (b) log 7 (c) 5 log 13 (d) none of these

A. 0

B. $\log 7$

C. $5 \log 13$

D. none of these

Answer: A

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101. $\int_0^{\infty}\left(\frac{\pi}{1+\pi^2x^2}-\frac{1}{1+x^2}\right)\log x dx is equa < o \quad -\frac{\pi}{21}n\pi$ (b) 0 $\frac{\pi}{21}n2$ (d) none of these

A. $-\frac{\pi}{2}In\pi$

B. 0

C. $\frac{\pi}{2}In2$

D. none of these

Answer: A

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102. $If A=\int_0^{\pi}\frac{\cos x}{(x+2)^2}dx, then \int_0^{\frac{\pi}{2}}\frac{\sin 2x}{x+1}dx is equa < o$ $\frac{1}{2}+\frac{1}{\pi+2}-A$ (b) $\frac{1}{\pi+2}-A\ 1+\frac{1}{\pi+2}-A$ (d) $A-\frac{1}{2}-\frac{1}{\pi+2}$

A. $\frac{1}{2}+\frac{1}{\pi+2}-A$

B. $\frac{1}{\pi+2} - A$

C. $1 + \frac{1}{\pi+2} - A$

D. $A - \frac{1}{2} - \frac{1}{\pi+2}$

Answer: A

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103. $\int_0^4 \frac{(y^2-4y+5)\sin(y-2)dy}{[2y^2-8y+11]}$ is equal to (a) 0 (b) 2 (c) -2 (d) none of these

A. 0

B. 2

C. -2

D. none of these

Answer: A

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104. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx$ (where $\theta \neq \frac{n\pi}{2}, n \in I$) is equal to

A. a. $-\cos\theta \int_{1}^{\tan\theta} f(x\sin\theta)dx$

B. b. $-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x)dx$

C. c. $\sin\theta \int_{1}^{\tan\theta} f(x\cos\theta)dx$

D. d. $-\frac{1}{\tan\theta} \int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

Answer: A

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105. $Let I_1 = \int_0^1 \frac{e^x dx}{1+x} and I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)} \dot{t}ehn \frac{I_1}{I_2} isequal \rightarrow$

(a)3/e(b)e/3(c)3e(d)1/(3e)`

A. $3/e$

B. $e/3$

C. $3e$

D. $1/3e$

Answer: C

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106. $Let I_1 = \int_{-2}^{2} \frac{x^6 + 3x^5 + 7x^4}{x^4 + 2} dx and$
$I_2 = \int_{-3}^{1} \frac{2(x+1)^2 + 11(x+1) + 14}{(x+1)^4 + 2} dt$. Then the value of $I_1 + I_2 is$ 8
(b) $\frac{200}{3}$ (c) $\frac{100}{3}$ (d) $none of these$

A. 8

B. $200/3$

C. $100/3$

D. noe

Answer: C

Watch Video Solution

107. Let f beintegrable over $[0, a]$ for any real value of a.

If $I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta)\,d\theta$ and $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta)\,d\theta$, then

A. $I_1 = -2I_2$

B. $I_1 = I_2$

C. $2I_1 = I_2$

D. $I_1 = -I_2$

Answer: B

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108. The value of $\int_a^b (x-a)^3(b-x)^4 dx$ is

A. $\frac{(b-a)^4}{6^4}$

B. $\frac{(b-a)^8}{280}$

C. $\frac{(b-a)^7}{7^3}$

D. none of these

Answer: B

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109. $If I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx, (m, n \in I, m, n \geq 0), then$

(a) $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}} dx$ (b) $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

(c) $I(m, n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$ (d) $I(m, n) = \int_0^\infty \frac{x^n}{(1+x)^{m+n}} dx$

A. $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}} dx$

B. $I(m, n) = \int_0^\infty \frac{x^m}{(1+x)^{m+n}} dx$

C. $I(m, n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$

D. $I(m, n) = \int_0^\infty \frac{x^n}{(1+x)^{m+n}} dx$

Answer: C

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110. $The value of the def \in ite \in tegral \int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx is$ 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

A. 0

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: A

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111. $If I_n = \int_0^{\pi} e^x (\sin x)^n dx, then \frac{I_3}{I_1}$ is equal to (a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

A. $3/5$

B. $1/5$

C. 1

D. $2/5$

Answer: A

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112. If $f'(x) = f(x) + \int_0^1 f(x)dx$, given $f(0) = 1$, then find the value of $f(\log_e 2)$ is

A. $\dfrac{1}{3+e}$

B. $\dfrac{5-e}{3-e}$

C. $\dfrac{2+e}{e-2}$

D. none of these

Answer: B

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113. Let $f(x)$ be positive, continuous, and differentiable on the interval $(a, b) and (\lim)_{x \overrightarrow{a}^{+}} f(x) = 1, (\lim)_{x \overrightarrow{b}^{-}} f(x) = 3^{\frac{1}{4}} \dot{I} f f'(x) \geq f^3(x) + \frac{1}{f(x)}$ then the greatest value of $b - a$ is $\frac{\pi}{48}$ (b) $\frac{\pi}{36}$ $\frac{\pi}{24}$ (d) $\frac{\pi}{12}$

A. $\frac{\pi}{48}$

B. $\frac{\pi}{36}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{12}$

Answer: C

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MCQ_TYPE

1. If $f(x)$ is integrable over $[1, 2]$, then $\int_1^2 f(x)dx$ is equal to

(a) $(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (b) $(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$ (c)

$(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (d) $(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

A. (a) $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

B. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

C. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$

D. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

Answer: B::C

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2. If $L = \lim_{n \to \infty} \frac{n^3\left(e^{1/n} + e^{2/n} + \dots\dots\dots + e\right)}{(n+1)^m\left(1^m + 4^m + \dots. + n^{2m}\right)}$ is non zero finite real, then

A. $L = 3(e-1)$

B. $L = 2(e-1)$

C. $m = 1$

D. $m = 1/3$

Answer: A::C

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3. Let $p = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{120}}$ and $q = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{121}}$ then

A. $p > 20$

B. $q < 20$

C. $p + q < 40$

D. $p + q > 40$

Answer: A::B::D

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4. Let $S_n = \sum_{k=0}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for n=1,2,3... then(correct options may be more than one) (a)$S_n < \frac{\pi}{3\sqrt{3}}$ b) $S_n > \frac{\pi}{3\sqrt{3}}$(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

A. $S_n < \frac{\pi}{3\sqrt{3}}$

B. $S_n > \frac{\pi}{3\sqrt{3}}$

C. $T_n < \frac{\pi}{3\sqrt{3}}$

D. $T_n > \frac{\pi}{3\sqrt{3}}$

Answer: A::D

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5. The value of $\int_0^1 \frac{2x^2 + 3x + 3}{(x+1)(x^2 + 2x + 2)} dx$ is

A. $\frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$

B. $\frac{\pi}{4} + 2\log 2 - \tan^{-1}\frac{1}{3}$

C. $2\log 2 - \cot^{-1} 3$

D. $-\frac{\pi}{4} + \log 4 + \cot^{-1} 2$

Answer: A::C::D

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6. Let $f(x) = \int_1^x \frac{3^t}{1+t^2} dt$, where $x > 0$, Then

A. for $0 < \alpha < \beta$, $f(\alpha) < f(\beta)$

B. for $0 < \alpha < \beta$, $f(\alpha) > f(\beta)$

C. $f(x) + \pi/4 < \tan^{-1} x \, \forall x \geq 1$

D. $f(x) + \pi/4 > \tan^{-1} x \, \forall x \geq 1$

Answer: A::D

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7. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$, then find the value of $\int_a^b x \sin x dx =$

A. $a + b = \frac{9\pi}{2}$

B. $|a = b| = 4\pi$

C. $\frac{a}{b} = 15$

D. $\int_a^b \sec^2 x dx = 0$

Answer: A::B

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8. $Iff(x) = \int_0^x 2|t| dt, then\ g(x) = x|x|$ $g(x)$ is monotonic $g(x)$ is differentiable at $x = 0$ $g'(x)$ is differentiable at $x = 0$

A. $g(x) = x|x|$

B. g(x) is monotonic

C. g(x) is differentiable at x = 0

D. g'(x) is differentiable at x = 0

Answer: A::B::C

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9. If $A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx, B_n = \int_0^{\frac{\pi}{2}} \left(\frac{\sin nx}{\sin x}\right)^2 dx$ for $n \in N$,

Then

(A) $A_{n+1} = A_n$ (B) $B_{n+1} = B_n$ (C) $A_{n+1} - A_n = B_{n+1}$ (D) $B_{n+1} - B_n = A_{n+1}$

A. $A_{n+1} = A_n$

B. $B_{n+1} = B_n$

C. $A_{n+1} - A_n = B_{n+1}$

D. $B_{n+1} - B_n = A_{n+1}$

Answer: A::D

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10. The value of $\int_0^{\infty} \frac{dx}{1+x^4}$ is

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11. The value of $\int_0^1 e^{x^2-x}$dx$is(a)$<1(b)>1(c)> $e^{-\frac{1}{4}}$ (d) $< e^{-\frac{1}{4}}$

A. <1

B. >1

C. $> e^{-\frac{1}{4}}$

D. $< e^{-\frac{1}{4}}$

Answer: A::C

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12. $If \int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = 10, then$ (a) $b = 22, a = 2$ (b) $b = 15, 1 = -5$ $b = 10, a = -10$ (d) $b = 10, a = -2$

A. $b = 22, a = 2$

B. $b = 15, a = -5$

C. $b = 10, a = -10$

D. $b = 10, a = -2$

Answer: A::B::C

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13. The values of a for which the integral $\int_0^2 |x-a|dx \geq 1$ is satisfied are $(2, \infty)$ (b) $(-\infty, 0)$ $(0, 2)$ (d) none of these

A. $[2, \infty)$

B. $(-\infty, 0]$

C. $(0, 2)$

D. none of these

Answer: A::B::C

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14. If $f(x) = \int_0^x |t-1| dt$, where $0 \leq x \leq 2$ then

(a) range of $f(x)$ is $[0, 1]$ (b) $f(x)$ is differentiable at $x = 1$ (c) $f(x) = \cos^{-1} x$ has two real roots (d) $f'\left(\frac{1}{2}\right) = \frac{1}{2}$

A. range of $f(x)$ is $[0, 1]$

B. $f(x)$ is differentiable at $x = 1$

C. $f(x) = \cos^{-1} x$ has two real roots

D. $f'(1/2) = 1/2$

Answer: B

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15. If $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)dx=5$, then $\int_0^{50} f(x)dx$ is equal to

A. 125

B. $\int_{-4}^{46} f(x)dt$

C. $\int_1^{51} f(x)dx$

D. $\int_2^{52} f(x)dx$

Answer: A::B::D

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16. If $f(x)=\int_0^x (\cos(\sin t)+\cos(\cos t)dt, then f(x+\pi) is$ (a) $f(x)+f(\pi)$ (b) $f(x)+2(\pi)$ (c)$f(x)+f\left(\frac{\pi}{2}\right)$ (d) $f(x)+2f\left(\frac{\pi}{2}\right)$

A. $f(x)+f(\pi)$

B. $f(x)+2f(\pi)$

C. $f(x) + f\left(\frac{\pi}{2}\right)$

D. $f(x) + 2f\left(\frac{\pi}{2}\right)$

Answer: A::D

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17. If $I_n = \int_0^{\pi/4} \tan^n x dx, (n > 1$ is an integer), then (a) $I_n + I_{n-2} = \frac{1}{n+1}$ (b) $I_n + I_{n-2} = \frac{1}{n-1}$ (c) $I_2 + I_4, I_6, \ldots\ldots$ are in H.P. (d) $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

A. $I_n + I_{n-2} = \frac{1}{n+1}$

B. $I_n + I_{n-2} = \frac{1}{n-1}$

C. $I_2 + I_4, I_6, \ldots\ldots$ are in H.P.

D. $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

Answer: B::C::D

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18. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}; n \in N,$ then prove that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

A. $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

B. $I_2 = \frac{\pi}{8} + \frac{1}{4}$

C. $I_2 = \frac{\pi}{8} - \frac{1}{4}$

D. $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

Answer: A::B::D

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19. $Let f : [1, \infty) \rightarrow R and f(x) = \int_1^x \frac{e^t}{t} dt - e^x \dot{T}hen$

A. $f(x)$ is an increasing function

B. $\lim_{x \to \infty} f(x) \to \infty$

C. $f'(x)$ has a maxima at $x = e$

D. $f(x)$ is a decreasing function

Answer: A::B

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20. If $f(x) = \int_a^x [f(x)]^{-1} dx$ and $\int_a^1 [f(x)]^{-1} dx = \sqrt{2}$, then

A. $f(2) = 2$

B. $f'(2) = 1/2$

C. $f^{-1}(2) = 2$

D. $\int_0^1 f(x) dx = \sqrt{2}$

Answer: A::B::C

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21. A Function $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 e^x f(t) dt \dot{T}hen$

$f(0) < 0$ $f(x)$ is a decreasing function. $f(x)$ is an increasing function.

$\int_0^1 f(x) dx > 0$

A. $f(0) < 0$

B. $f(x)$ is a decreasing function

C. $f(x)$ is increasing function

D. $\int_0^1 f(x) dx > 0$

Answer: A::B

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22. If $\int_0^x \left\{ \int_0^u f(t) dt \right\} du$ is equal to

(a) $\int_0^x (x - u) f(u)$

(b) $\int_0^x u f(x - u) du$

(c) $x\int_0^x f(u)du$

(d) $x\int_0^x uf(u-x)du$

A. $\int_0^x (x-u)f(u)du$

B. $\int_0^x uf(x-u)du$

C. $x\int_0^x f(u)du$

D. $x\int_0^x uf(u-x)du$

Answer: A::B

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23. Which of the following statement(s) is/are TRUE?

A. If function $y=f(x)$ is continuous at $x=c$ such that $f(c)\neq 0$, then $f(x)f(c)>0\forall x\varepsilon(c-h, c+h)$, where h is sufficiently small positive quantity.

B. $\lim_{n\to\infty}\frac{1}{n}\left(n\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\ldots\left(1+\frac{n}{n}\right)\right)=1+2In2.$

C. Let f be a continuous and non-negative function defined on $[a, b]$ If $\int_a^b f(x)dx = 0$, then $f(x) = \forall x \varepsilon [a, b]$

D. Let f be continuous function defined on $[a, b]$ such that $\int_a^b f(x)dx = 0$.Then there exists at least one $c\varepsilon(a, b)$ for which $f(c) = 0$

Answer: A::C::D

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24. If $\int_0^x [x]dx = \int_0^{[x]} xdx, x \notin$ integer (where, [.] and {.} denotes the greatest integer and fractional parts respectively, then the value of 4{x} is equal to ...

A. $x\varepsilon[0, 1)$

B. $\{x\} = 2$

C. $\{x\} = 1/3$

D. $x > 0$

Answer: A::B

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25. Consider the function $f(\theta) = \int_0^1 \frac{\left|\sqrt{1-x^2} - \sin\theta\right|}{\sqrt{1-x^2}} dx$, where $0 \le \theta \le \frac{\pi}{2}$, then

A. $f_{\min} = \sqrt{2} - 1$

B. $f_{\min} = \sqrt{2} + 1$

C. $f_{\max} = 1$

D. $f_{\max} = \frac{\pi}{2} - 1$

Answer: A::B::C::D

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26. $f:[0,1) \rightarrow R$ be a non increasing function then for $\alpha \varepsilon (0,1)$

A. $\alpha \int_0^1 f(x)dx \leq \int_0^\alpha f(x)dx$

B. $\alpha \int_0^1 f(x)dx \geq \int_0^\alpha f(x)dx$

C. $\alpha^2 \int_0^1 f(x)dx \leq \int_0^\alpha f(x)dx$

D. $\sqrt{\alpha} \int_0^1 f(x)dx \geq \int_0^\alpha f(x)dx$

Answer: A::C

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27.

Let f(x) be a non-constant twice differentiable function defined on (∞, ∞) such that $f(x) = f(1-x)$ and $f''\left(\frac{1}{4}\right) = 0$. Then

(a) $f'(x)$vanishes at least twice on $[0,1]$ (b) $f'\left(\frac{1}{2}\right) = 0$

(c) $\int_{-\frac{1}{2}}^{\frac{1}{2}} f\left(x+\frac{1}{2}\right)\sin x dx = 0$

(d) $\int_0^{\frac{1}{2}} f(t)e^{\sin x} t dt = \int_{\frac{1}{2}}^1 f(1-t)e^{\sin \pi} t dt$

A. $f'(x)$ vanishes at least twice on $[0, 1]$

B. $f'\left(\frac{1}{2}\right) = 0$

C. $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x dx = 0$

D. $\int_0^{1/2} f(t)e^{\sin xt}dt = \int_{t/2}^{1} f(1-t)e^{\sin \pi t}dt$

Answer: A::B::C::D

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LC_TYPE

1.

$y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The range of $y = f(x)$ is (a) $[0, \infty)$ (b)R (c)$(-\infty, 0]$ (d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

A. $[0, \infty)$

B. R

C. $(-\infty, 0]$

D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer: D

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2. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The value of $\int_{-2}^2 f(x)dx$ is

A. 0

B. -2

C. $2\log_e 2$

D. none of these

Answer: A

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3. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The range of $y = f(x)$ is

A. $(-\infty, 1]$

B. $[-1, \infty)$

C. $[-1, 1]$

D. none of these

Answer: C

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4. Let $f: R \to R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt$. $f(x)$ increases for

A. $x > 1$

B. $x < -2$

C. $x > 2$

D. none of these

Answer: B

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5. Let $f: R \to R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$

$y = f(x)$ is

A. (a) injective but not surjective

B. (b) surjective but not injective

C. (c) bijective

D. (d) neither injective nor surjective

Answer: B

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6. Let $f(x)$ be a differentiable function such that $f(x)=x^2+\int_0^x e^{-t}f(x-t)dt$ then $\int_0^1 f(x)dx=$

A. $\frac{1}{4}$

B. $-\frac{1}{12}$

C. $\frac{5}{12}$

D. $\frac{12}{7}$

Answer: C

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7. $f(x)$ satisfies the relation $f(x)-\lambda\int_0^{\pi/2}\sin x\cdot\cos t f(t)dt=\sin x$ If $\lambda>2$ then $f(x)$ decreases in

A. $(0,\pi)$

B. $\left(\frac{\pi}{2},3\pi/2\right)$

C. $(-\pi/2,\pi/2)$

D. none of these

Answer: C

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8. $f(x)$ satisfies the relation $f(x) - \lambda \int_0^{\pi/2} \sin x \cos t f(t) dt = \sin x$

If $f(x) = 2$ has the least one real root, then

A. $\lambda \varepsilon [1, 4]$

B. $\lambda \varepsilon [-1, 2]$

C. $\lambda \varepsilon [0, 1]$

D. $\lambda \varepsilon [1, 3]$

Answer: D

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9. $f(x)$ satisfies the relation $f(x)-\lambda\int_0^{\pi/2}\sin x\cdot\cos t f(t)dt=\sin x$ If $\lambda>2$ then `f(x) increases in

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10. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x)=\int_a^x f(t)dt, a\ne 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha)+g(x)=0\forall x\varepsilon R, \alpha>0,$ and $\int_b^{2k} g(t)dt$ is independent of b

If $f(x)$ is an odd function, then

A. (a) $\phi(x)$ is also an odd function

B. (b) $\phi(x)$ is an even function

C. (c) $\phi(x)$ is neither an even nor an odd function

D. (d) for $\phi(x)$ to be an even function, it must satisfy $\int_0^a f(x)dx=0$

Answer: B

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11. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x) = \int_a^x f(t)dt, a \neq 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha)+g(x)=0 \forall x \varepsilon R, \alpha > 0,$ and $\int_b^{2k} g(t)dt$ is independent of b

If $f(x)$ is an even function, then

A. $\phi(x)$ is also an even function

B. $\phi(x)$ is an odd function

C. $\phi(x)$ is an neither even nor odd function

D. None of these

Answer: D

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12. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x)=\int_a^x f(t)dt, a\ne 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha)+g(x)=0 \forall x \varepsilon R, \alpha>0,$ and $\int_b^{2k} g(t)dt$ is independent of b

Least positive value fo c if c, k, b are n A.P. is

A. 0

B. 1

C. α

D. 2α

Answer: D

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13. Find the area of a parallelogram whose adjacent sides are given by the vectors $\overrightarrow{a}=3\hat{i}+\hat{j}+4\hat{k}$and $\overrightarrow{b}=\hat{i}-\hat{j}+\hat{k}.$

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14. The value of $\int_0^1 \frac{x^a - 1}{\log x} dx$ is

A. $\log(a - 1)$

B. $\log(a + 1)$

C. $a \log(a + 1)$

D. none of these

Answer: B

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15. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

The value $\int_0^{\pi/2} \log(\sin^2\theta + k^2\cos^2\theta)\,d\theta$, where $k \geq 0,$ is

A. $\pi\log(1+k) + \pi\log 2$

B. $\pi\log(1+k)$

C. $\pi\log(1+k) - \pi\log 2$

D. $\log(1+k) - \log 2$

Answer: C

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16.

The value of $\frac{dI}{da}$ when $I = \int_0^{\pi/2} \log\left(\frac{1+a\sin x}{1-a\sin x}\right)\frac{dx}{\sin x}$ (where $|a| < 1$) is

A. $\frac{\pi}{\sqrt{1-a^2}}$

B. $-\pi\sqrt{1-a^2}$

C. $\sqrt{1-a^2}$

D. $\frac{\sqrt{1-a^2}}{\pi}$

Answer: A

Watch Video Solution

17. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

If $\int_0^{\pi} \frac{dx}{(a-\cos x)} = \frac{\pi}{\sqrt{a^2-1}}$, then the value of $\int_0^{\pi} \frac{dx}{\left(\sqrt{10}-\cos x\right)^3}$ is

A. (a) $\frac{\pi}{81}$

B. (b) $\frac{7\pi}{162}$

C. (c) $\frac{7\pi}{81}$

D. (d) none of these

Answer: C

Watch Video Solution

18. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t \cos x) f(t) dt$

The range of $f(x)$ is

A. A. $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

B. B. $\left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$

C. C. $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

D. D. none of these

Answer: B

Watch Video Solution

19. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t) dt$

$f(x)$ is not invertible for

A. A. $x \varepsilon \left[-\frac{\pi}{2} - \tan^{-1} 2, \frac{\pi}{2} - \tan^{-1} 2\right]$

B. B. $x \varepsilon \left[\tan^{-1}\left(\frac{1}{2}\right), \pi + \tan^{-1}\frac{1}{2}\right]$

C. C. $x \varepsilon \left[\pi + \cot^{-1} 2, 2\pi + \cot^{-1} 2\right]$

D. D. none of these

Answer: D

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20. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t) dt$

The value of $\int_0^{\pi/2} f(x) dx$ is

A. A. 1

B. B. -2

C. C. -1

D. D. 2

Answer: C

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21. Let $u=\int_0^\infty \frac{dx}{x^4+7x^2+1}$ and $v=\int_0^\infty \frac{x^2dx}{x^4+7x^2+1}$ then find the value of u+v

A. $\pi/3$

B. $\pi/6$

C. $\pi/12$

D. $\pi/9$

Answer: B

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22. Let $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$ and $v = \int_0^{\infty} \frac{x^2 dx}{x^4 + 7x^2 + 1}$ then find the value of u+v

A. $\pi/3$

B. $\pi/6$

C. $\pi/12$

D. $\pi/9$

Answer: B

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23. If $f(x) = \int_0^1 \frac{dt}{1 + |x - t|}, x \in R$. The value of $f'(1/2)$ is equal to

A. $1/2$

B. 0

C. 1

D. 2

Answer: B

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24. If $f(x) = \int_0^1 \frac{dt}{1+|x-t|}, x \in R$. The value of $f'(1/2)$ is equal to

A. $f(x)$ is decreasing for $x > 1$

B. $f(x)$ is increasing for $x < 1$

C. $f(1) = \log_e 2$

D. $f(1/2) = \log_e(3/2)$

Answer: D

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25. Let f be a differentiable function satisfying $\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t)dt = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$

The value of $\int_0^{\pi/2} f(x)dx$ lies in the interval

A. $\left(\frac{2}{\pi}, 1\right)$

B. $\left(1, \frac{\pi}{2}\right)$

C. $\left(\frac{3}{2}, \frac{\pi}{2}\right)$

D. $\left(0, \frac{2}{\pi}\right)$

Answer: B

Watch Video Solution

26. Let f be a differentiable function satisfying $\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t)dt = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$

The value of $\lim_{x \to 0} \frac{\cos x}{f(x)}$ is equal to where [.] denotes greatest integer function

A. 0

B. 1

C. $1/2$

D. 2

Answer: B

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27. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx$, where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$. Hence, deduce that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} = \frac{1}{2} n\pi$.

A. $\pi / 2$

B. π

C. $n\pi / 2$

D. $n\pi$

Answer: D

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28. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx$, where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$. Hence, deduce that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} = \frac{1}{2} n\pi$.

A. $\pi / 2$

B. π

C. $n\pi / 2$

D. $n\pi$

Answer: C

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29. Assertion : Millikan's experiment established that electric charge is quantised.

Reason : From this experiment mass of the electron could not be determined.

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30. If p_1 and p_2 are the lengths of the perpendiculars from the point (2,3,4) and (1,1,4) respectively from the plane 3x-6y+2z+11=0, then for which equation p_1 and p_2 will be the roots?

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31. Let the definite integral be defined by the formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$. For more accurate result, for $c\varepsilon(a,b)$, we can use $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$ so that for $c = \frac{a+b}{2}$ we get $\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$. If $f''(x) < 0 \forall x\varepsilon(a,b)$ and c is a point such that $a < c < b$, and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum then $f'(c)$ is equal to

A. a. $\frac{f(b)-f(a)}{b-a}$

B. b. $\frac{2(f(b)-f(a))}{b-a}$

C. c. $\dfrac{2f(b)-f(a)}{2b-a}$

D. d. 0

Answer: B

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MATRIX MATCH_TYPE

1. If [.] denotes the greatest integer function, then match the following lists:

List I	List II
a. $\int_{-1}^{1}[x+[x+[x]]]\,dx$	p. 3
b. $\int_{2}^{5}([x]+[-x])\,dx$	q. 5
c. $\int_{-1}^{3}\operatorname{sgn}(x-[x])\,dx$	r. 4
d. $25\int_{0}^{\pi/4}\left(\tan^6(x-[x])+\tan^4(x-[x])\right)dx$	s. -3

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2. Match the following lists:

List I	List II
a. $\int \frac{e^{2x}-1}{e^{2x}+1} dx$ is equal to	p. $x - \log\left[1+\sqrt{1-e^{2x}}\right]+c$
b. $\int \frac{1}{(e^{x}+e^{-x})^2} dx$ is equal to	q. $\log(e^{x}+1)-x-e^{-x}+c$
c. $\int \frac{e^{-x}}{1+e^{x}} dx$ is equal to	r. $\log(e^{2x}+1)-x+c$
d. $\int \frac{1}{\sqrt{1-e^{2x}}} dx$ is equal to	s. $-\frac{1}{2(e^{2x}+1)}+c$

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3. Factorise the following : $25a^2 - 4b^2 + 28bc - 49c^2$

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4. Choose the correct answer:

The hybridisation of the carbon atom (underlined) present in

`(PAT_CHE_0XI_B02_C03_E01_022_Q01.png" width="80%">

is

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5. Let $\int_0^{\infty} \frac{\sin x}{x} dx = \alpha$ Then match the following lists and choose the correct code. :

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6. If $I(m) = \int_0^{\pi} \log_e \left(1 - 2m \cos x + m^2\right) dx$, Then find the value of I(81/9)

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NUMERICAL VALUE_TYPE

1. If the value of $(\lim)_{n\overrightarrow{\infty}}\left(n^{-\frac{3}{2}}\right)\sum_{j=1}^{\dot{6n}}\sqrt{j}$ is equal to $\sqrt{N}$ then the value of $\frac{N}{8}$ is ________

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2. $(\lim)_{n\to\infty}\frac{n}{2^n}\int_0^2 x^n dx$ equals___

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3. A continuous real function f satisfies $f(2x)=3f(x)\,\forall x\in R\dot{I}f\int_0^1 f(x)dx=1$, then find the value of $\int_1^2 f(x)dx$

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4. Consider the polynomial $f(x)=ax^2+bx+c\dot{}$ If $f(0)=0,\ f(2)=2,$ then the minimum value of $\int_0^2 |f'(x)|dxis_-$

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5. If $I = \int_0^{\frac{3\pi}{4}} [(1+x)\sin x + (1-x)\cos x]dx,$ then value of $(\sqrt{2}-1)I$ is_____

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6. about to only mathematics

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7. The value of $\int_0^{\frac{3\pi}{2}} \frac{|\tan^{-1}(\tan x)| - |\sin^{-1}(\sin x)|}{|\tan^{-1}(\tan x)| + |\sin^{-1}(\sin x)|} dx$ is equal to

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8. $Let f(x) = x^3 = \frac{3x^2}{2} + x + \frac{1}{4}$ Then the value of $\left(\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(x))dx\right)^{-1}$ os____

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9. The value of $\int_0^1 \frac{\tan^{-1} x}{\cot^{-1}(1 - x + x^2)} dx$ is____.

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10. Let $f(x)$ be a differentiable function symmetric about $x = 2$, then the value of $\int_0^4 \cos(\pi x) f'(x) dx$ is equal to________.

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11. Let $f: [0, \infty) \to R$ be a continuous strictly increasing function, such that $f^3(x) = \int_0^x t f^{.2}(t) dt$ for every $x \geq 0$. Then value of $f(6)$ is_______

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12. If f is continuous function and $F(x)=\int_0^x\left((2t+3)\cdot\int_t^2 f(u)du\right)dt$, then $\left|\frac{F''(2)}{f(2)}\right|$ is equal to____

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13. If the value of the definite integral $\int_0^1\frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$ is $\frac{\pi^2}{\sqrt{n}}$ (where $n\in N)$, then the value of $\frac{n}{27}$ is

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14. Let $f(x)=\int_0^x\frac{dt}{\sqrt{1+t^3}}$ and $g(x)$ be the inverse of $f(x)$. Then the value of $4\frac{g''(x)}{g(x)^2}$ is_______.

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15. Let $g(x)$ be differentiable on R and $\int_{\sin t}^{1} x^2 g(x)dx = (1 - \sin t)$, where $t \in \left(0, \frac{\pi}{2}\right)$. Then the value of $g\left(\frac{1}{\sqrt{2}}\right)$ is____

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16. If $\int_0^{\infty} x^{2n+1} \dot{e}^{-x dx} = 360,$ then the value of n is___

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17. Let $f(x)$ be a derivable function satisfying $f(x) = \int_0^x e^t \sin(x - t)dt$ and $g(x) = f''(x) - f(x)$ Then the possible integers in the range of $g(x)$ is______

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18. Let $f(x) = \frac{1}{x^2} \int_4^x \left(4t^2 - 2f'(t)\right)dt$ then find the value of $9f'(4)$

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19. If the value of the definite integral $\int_0^1 \hat{} \, (2007) C_7 x^{2000} 1 \dot{-} x^{7} dx$ is equal to $\frac{1}{k}, \, where k \in N,$ then the value of $\frac{k}{26}$ is____

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20. If $I_n = \int_0^1 (1 - x^5)^n dx, \, then \frac{55}{7} \frac{I_{10}}{I_{11}}$ is equal to ___

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21. Evaluate: $5050 \frac{\int_0^1 (1 - x^{50})^{100} dx}{\int_0^1 (1 - x^{50})^{101} dx}$

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22.

$Let J = \int_{-5}^{-4} (3 - x^2) \tan(3 - x^2) dx \text{ and } K = \int_{-2}^{-1} (6 - 6x + x^2)$

$\tan\left(6x - x^2 - 6\right)dx$. Then (J+K) equals ____

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23. The value of the definite integral $\int_{2-1}^{\sqrt{2}+1} \frac{x^4 + x^2 + 2}{(x^2+1)^2} dx equals\_\_$

Watch Video Solution

24. Consider a real valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t(f(t))dt$. If M and m are maximum and minimum values of function f, then the value of M/m is__________.

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25. If $f(x) = x + \int_0^1 t(x+t)f(t)dt$, then the value of $\frac{23}{2} f(0)$ is equal to ________

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26. Let $y = f(x) = 4x^3 + 2x - 6,$ then the value of $\int_0^2 f(x)dx + \int_0^{30} f^{-1}(y)dy$ is equal to ________.

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27.

The value of $\int_1^3 \left(\sqrt{1 + (x - 1)^3} + (x^2 - 1)^{\frac{1}{3}} + 1\right) dx$ is ________.

Watch Video Solution

28. The value of $\int_0^1 \cos^{-1}(x - x^2) - \sqrt{(1 - x^2)(2x - x^2)} dx$ is equal to ________.

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JEE MAIN

1. $\int_0^{\pi} [\cot x] dx$, where [.] denotes the greatest integer function, is equal to

A. $\frac{\pi}{2}$

B. 1

C. -1

D. $-\frac{\pi}{2}$

Answer: D

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2.

Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$ for all $x\varepsilon[0, 1], p(0) = 1$ and $p(1) = 41.$

Then $\int_0^1 p(x)dx$ is equals to (a)42 (b)$\sqrt{41}$ (c)21 (d)41

A. 42

B. $\sqrt{41}$

C. 21

D. 41

Answer: C

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3. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

A. $\log 2$

B. $\pi \log 2$

C. $\frac{\pi}{8}\log 2$

D. $\frac{\pi}{2}\log 2$

Answer: B

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4. For $x\varepsilon\left(0, \frac{5\pi}{2}\right)$, definite $f(x)=\int_0^x \sqrt{t}\sin t dt$. Then f has

A. local maximum at π and local minima at 2π

B. local maximum at π and 2π

C. local minimum at π and 2π

D. local minimum at π and local maximum at `2pi

Answer: A

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5. If $g(x)=\int_0^x \cos^4 t dt$, then $g(x+\pi)$ equals $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

A. $\frac{g(x)}{g(\pi)}$

B. $g(x)+g(\pi)$

C. $g(x) - g(\pi)$

D. $g(x).\ g(\pi)$

Answer: B

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6. Statement - 1 : The value of the integral

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} \text{ is equal to } \frac{\pi}{6}$$

Statement-2 : $\int_a^b f(x) = \int_a^b f(a+b-x)dx$

A. Statement I is true, statement II is true, statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is a not a correct explanation for statement I

C. Statement I is true, statement II is false

D. Statement I is false, statement II is true

Answer: D

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7. The intercepts on x-axis made by tangents to the curve, $\int_0^x |t| dt, x \in R$ which are parallel to the line y = 2x, are equal to :

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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8. The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2\left(\frac{x}{2}\right) - 4\sin\left(\frac{x}{2}\right)}dx$ equals :

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - \sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \frac{\pi}{3}$

Answer: D

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9. The integral $\int_2^4 \frac{\log x^2}{\left(\log x^2\right) + \log(36 - 12x + x^2)} dx$ is equal to:

A. 2

B. 4

C. 1

D. 6

Answer: C

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10. $\lim_{n \to \infty} \left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

A. $\frac{27}{e^2 0}$

B. $\frac{9}{e^2}$

C. $3\log 3 - 2$

D. $\frac{18}{e^4}$

Answer: A

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11. Evaluate: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$

A. -1

B. -2

C. 2

D. 4

Answer: C

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12. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ is

A. $\pi/4$

B. $\pi/8$

C. $\pi/2$

D. 4π

Answer: A

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JEE ADVANCED

1. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt, 0 \le x \le 1, and\ f(0) = 0$, then

A. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer: C

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2. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dxis/are$ $\frac{22}{7} - \pi$ (b) $\frac{2}{105}$ 0 (d) $\frac{71}{15} - \frac{3\pi}{2}$

A. $\frac{22}{7} - \pi$

B. $\frac{2}{105}$

C. 0

D. $\frac{71}{15} - \frac{3\pi}{2}$

Answer: A

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3. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}dt,$ for all, $x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $\left(f^{-1}\right)'(2)$ is equal to (a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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4. The value of $\int_{\in 2}^{\sqrt{\in 3}} \frac{x \sin x^2}{\sin x^2 + \sin(In6 - x^2) dx}$ is

A. $\frac{1}{4} In \frac{3}{2}$

B. $\frac{1}{2} In \frac{3}{2}$

C. $In \frac{3}{2}$

D. $\frac{1}{6} In \frac{3}{2}$

Answer: A

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5. Let $f : [-1, 2]\overrightarrow{0, \infty}$ be a continuous function such that $f(x) = f(1 - x) f\ or\ all x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x) dx,$ and R_2 be the area of the region bounded by $y = f(x), x = -1, x = 2,$ and the $x - a\xi s$. Then $R_1 = 2R_2$ (b) $R_1 = 3R_2$ $2R_1$ (d) $3R_1 = R_2$

A. $R_1 = 2R_2$

B. $R_1 = 3R_2$

C. $2R_1 = R_2$

D. $3R_1 = R_2$

Answer: C

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6. Let $f: \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int f(x)dx$ lies in the interval for x:[1/2,1] (a) $(2e-1, 2e)$ (b) $(3-1, 2e-1)$ (c) $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

A. $(2e-1, 2e)$

B. $(e-1, 2e-1)$

C. $\left(\frac{e-1}{2}, e-1\right)$

D. $\left(0, \frac{e-1}{2}\right)$

Answer: D

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7. Let f : [0,2] $\rightarrow$ R be a function which is continuous on [0 , 2] and is differentiable on (0 , 2) with f(0) = 1. Let

F(x) = $\int_0^{x^2} f\left(\sqrt{t}\right)dt$

for $x \in [0, 2]$. If F'(x) = f'(x) for all $x \in (0, 2)$, then F(2) equals

A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. e^4

Answer: B

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8. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\cos ecx)^{17}$ dx

A. (a) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

B. (b) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{17} du$

C. (c) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{17} du$

D. (d) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

Answer: A

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9. Let $f'(x) = \frac{192x^3}{2 + \sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0$. If $m \leq \int_{\frac{1}{2}}^{1} f(x)dx \leq M$ then for $x \in \left[\left(\frac{1}{2}\right), 1\right]$ the possible values of m and M are

A. a. $m = 13, M = 24$

B. b. $m = \frac{1}{4}, M = \frac{1}{2}$

C. c. $m = -11, M = 0$

D. d. $m = 1, M = 12$

Answer: D

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10. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$ is equal to

A. $\frac{\pi^2}{4} - 2$

B. $\frac{\pi^2}{4} + 2$

C. $\pi^2 - e^{\frac{\pi}{2}}$

D. $\pi^2 + \frac{e^{\pi}}{2}$

Answer: A

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11. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots$ then which one of the following is not true ?

A. $I_n = I_{n+2}$

B. $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{10} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: A::B::C

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12. about to only mathematics

A. $f''(x)$ exists for al $x\varepsilon(0, \infty)$

B. $f'(x)$ exists for all $x\varepsilon(0, \infty)$ and f' is continuous on $(0, \infty)$ but not differentiable on $(0, \infty)$.

C. There exists $\alpha > 1$ suchthat $|f'(x)| < \ | f(x) +$ for all $x\varepsilon(\alpha, \infty)$

D. There exists $\beta > 0$ such that $|f(x)| + \ | f'(x) + \ \leq \beta$ for all $x\varepsilon(0, \infty)$

Answer: B::C

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13. Let S be the area of the region enclosed by $y = e^{-x^2}, y = 0, x = 0$ and $x = 1$. Then

A. $S \geq \frac{1}{e}$

B. $S \geq 1 - \frac{1}{e}$

C. $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

D. $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Answer: A::B::D

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14. For $a \varepsilon R$ (the set of all real numbers) $a \neq -1, \lim_{n \to \infty} \frac{(1^a + 2^a + \ldots\ldots\ldots + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + \ldots\ldots\ldots\ldots + (na + n$

Then $a =$

A. 5

B. 7

C. $\frac{-15}{2}$

D. $\frac{-17}{2}$

Answer: B::D

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15. Let $f: [a, b] \rightarrow [1, \infty)$ be a continuous function and lt $g: R \rightarrow R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if } x \leq a \\ \int_a^x f(t)dt & \text{if } a \leq x \leq b \\ \int_a^b f(t)dt & \text{if } x > b \end{cases}$$

Then

A. (a) $g(x)$ is continuous but not differentiable at a

B. (b) $g(x)$ is differentiable on R

C. (c) $g(x)$ is continuous but not differentiable at b

D. (d) $g(x)$ is continuous and differentiable at either a or b but not both

Answer: A::C

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16. Let $f: (0, \infty) \to \mathbb{R}$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{\left(t + \frac{1}{t}\right)} \frac{dt}{t}$$

Then

A. $f(x)$ is monotonically increasing on $[1, \infty)$

B. $f(x)$ is monotonocally decreasing on $(0, 1)$

C. $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \varepsilon (0, \infty)$

D. $f(2^x)$ is an odd function of x on R

Answer: A::C::D

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17. The option(s) with the values of a and L that satisfy the following equation is (are)

$$\frac{\int_0^{4\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt}{\int_0^{\pi} e^t\left(\sin^6 at + \cos^4 at\right)dt} = L$$

A. $a = 2, L = \dfrac{e^{4\pi} - 1}{e^{\pi} - 1}$

B. $a = 2, L = \dfrac{e^{4\pi + 1}}{e^{\pi} + 1}$

C. $a = 4, L = \dfrac{e^{4\pi} - 1}{e^{\pi} - 1}$

D. $a = 4, L = \dfrac{e^{4\pi} + 1}{e^{\pi} + 1}$

Answer: A::C

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18. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. The correct expression(s) is (are)

A. $\int_0^{\pi/4} x f(x) dx = \frac{1}{12}$

B. $\int_0^{\pi/4} f(x) dx = 0$

C. $\int_0^{\pi/4} x f(x) = \frac{1}{6}$

D. $\int_0^{\pi/4} f(x) dx = 1$

Answer: A::B

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19. find the period of $\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{3}\right)$ is

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20. Let $f: R \to (0, 1)$ be a continuous function. Then, which of the following function (s) has (have) the value zero at some point in the interval (0,1)?

A. a. $e^x - \int_0^x f(t) \sin t dt$

B. b. $x^9 - f(x)$

C. c. $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t dt$

D. d. $x - \int_0^{\frac{\pi}{2} - x} f(t) \cos t dt$

Answer: B::D

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21. If $I \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx, then:$

A. $I > \log_e 99$

B. $I < \log_e 99$

C. $I < \frac{49}{50}$

D. $I > \frac{49}{50}$

Answer: B::D

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22. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt, then:$
(a) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (b) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ (c) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (d) $g'\left(\frac{\pi}{2}\right) = 2\pi$

A. $g'\left(\frac{\pi}{2}\right) = -2\pi$

B. $g'\left(-\frac{\pi}{2}\right) = 2\pi$

C. $g'\left(\frac{\pi}{2}\right) = 2\pi$

D. $g'\left(-\frac{\pi}{2}\right) = -2\pi$

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23. Given that for each $a \varepsilon (0, 1)$, $\lim \left(h \to 0^{+}\right) \int_{h}^{1-h} t^{-a}(1-t)^{a-1} dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is differentiable on $(0, 1)$.

The value of $g\left(\frac{1}{2}\right)$ is

A. a. π

B. b. 2π

C. c. $\frac{\pi}{2}$

D. d. $\frac{\pi}{4}$

Answer: A

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24. Given that for each $a \varepsilon (0, 1)$, $\lim \left(h \to 0^{+}\right) \int_{h}^{1-h} t^{-a}(1-t)^{a-1} dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is differentiable on $(0, 1)$.

The value of $g\left(\frac{1}{2}\right)$ is

A. $\frac{\pi}{2}$

B. π

C. $-\frac{\pi}{2}$

D. 0

Answer: D

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25. Let $F: R \to R$ be a thrice differntiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x\varepsilon(1/2, 3)$. Let $f(x) = xF(x)$ for all $x \in R$. Then the correct statement(s) is (are)

A. a. $f'(1) < 0$

B. b. $f(2) < 0$

C. c. $f'(x) \neq 0$ for an $x \in (1, 3)$

D. d. $f'(x) = 0$ for some $x \in (1, 3)$

Answer: A::B::C

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26. Let $F : R \to R$ be a thrice differentiable function. Suppose that F(1)=0, F(3)=-4 and $F'(x) < 0$ for all $x \in (1/2, 3)$. Let f(x)=xF(x) for all $x \in R$. If $\int_1^3 x^2 F'(x) dx = -12$ and $\int_1^3 x^3 F''(x) dx = 40$, then the correct expression(s) is (are)

A. $9f'(3) + f'(1) - 32 = 0$

B. $\int_1^3 f(x) dx = 12$

C. $9f'(3) - f'(1) + 32 = 0$

D. $\int_1^3 f(x) dx = -12$

Answer: C::D

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27. Match the terms given in Column I with the compound given in Column II.

	Column I		Column II
A.	Acid rain	1.	$CHCl_2$-CHF_2
B.	Photochemical smog	2.	CO
C.	Combination with haemoglobin	3.	CO_2
D.	Depletion of ozone layer	4.	SO_2
		5.	Unsaturated hydrocarbons

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28. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad \text{if} \quad [x]$ is odd, $1 + [x] - x, \quad \text{if} \quad [x]$ is even Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx$ is___

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29. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, x \in R,$ where $f'(x)$ denotes $\frac{df'(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is_____

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30. The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} \left(1 - x^2\right)^5 \right\} dx$ is

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31. Let $f : R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t|f(f(t))|dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, Then the value of $f\left(\frac{1}{2}\right)$ is

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32. If $\alpha = \int_0^1 \left(e^{9x + 3\tan^{-1} x}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right)dx$

where $\tan^{-1} x$ takes only principal value, then the value of $\left(\log_e |1 + \alpha| - \frac{3\pi}{4}\right)$ is

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34. Let $f: R \to R$ be a function defined by $f(x) = \{[x], (x \le 2)\ (0, x > 2)$ where $[x]$ is the greatest integer less than or equal to x. If $I = \int_{-1}^{2} \frac{x f(x^2)}{2 + f(x+1)} dx, then the value of (4I - 1) is$

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35. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is

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36. Let $f: R \to R$ be a differentiable function such that $f(0) = 0, f\left(\frac{\pi}{2}\right) = 3 and f'(0) = 1.$ If $g(x) = \int_x^{\frac{\pi}{2}} \left[f'(t)\cos ect - \cot t \cos ect f(t)\right] dt$ for $x\left(0, \frac{\pi}{2}\right]$, then $(\lim)_{x \to 0} g(x) =$

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37. For each positive integer n, let $y_n = \frac{1}{n}((n+1)(n+2)\ldots.(n+n))^{\frac{1}{n}}$ For $x \in R$ let $[x]$ be the greatest integer less than or equal to x. If $(\lim)_{n \to \infty} y_n = L$, then the value of $[L]$ is _____.

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38. The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}} dx$ is ______.

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Single Correct Answer Type

1. $\lim_{n \to \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n}\right]$ is equal to

A. $\frac{3}{8}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. None of these

Answer: A

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2. The value of $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{\sin\left\{\frac{(n+r)\pi}{4n}\right\}} \cdot \frac{\pi}{n}$ is equal to

A. $2\ln(\sqrt{2}-1)$

B. $4\ln(\sqrt{2}-1)$

C. $4\ln(\sqrt{2}+1)$

D. $\ln\sqrt{2}$

Answer: C

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3. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k^{1/a}\left\{n^{a-\frac{1}{a}} + k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$ is equal to

A. 1

B. 2

C. 43467

D. 4

Answer: A

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4. If $\int_0^3 (3ax^2 + 2bx + c)\,dx = \int_1^3 (3ax^2 + 2bx + c)\,dx$ where a, b, c are constants then $a + b + c =$

A. $a + b + c = 3$

B. $a + b + c = 1$

C. $a + b + c = 0$

D. $a + b + c = 2$

Answer: C

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5. The number of solution of the equation $\int_{-2}^{x}|\cos x|dx = 0, 0 < x < \frac{\pi}{2}$, is

A. 0

B. 1

C. 2

D. 4

Answer: A

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6. $\int_{0}^{1} e^{2x} e^{e^x}$ dx

A. $e^e(2e - 1)$

B. $e^e(e - 1)$

C. $e^{2e}(e - 1)$

D. none of these

Answer: B

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7. Let $f(x) = \lim_{n \to \infty} \frac{\cos x}{1 + (\tan^{-1} x)^n}$. Then the value of $\int_o^\infty f(x)dx$ is equal to

A. $\cos(\tan 1)$

B. $\sin(\tan 1)$

C. $\tan(\tan 1)$

D. none of these

Answer: B

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8. The value of difinite integral $\int_0^1 = \dfrac{dx}{\sqrt{(x+1)^3(3x+1)}}$ equals

A. $\sqrt{2}-1$

B. $\tan. \dfrac{\pi}{12}$

C. $\tan. \dfrac{5\pi}{12}$

D. none of these

Answer: A

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9. If f(x) is continuous and $\int_0^9 f(x)dx = 4$, then the value of the integral $\int_0^3 x.\, f(x^2)dx$ is

A. 2

B. 18

C. 16

D. 4

Answer: A

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10. $\lim_{t \to 0} \int_0^{2\pi} \frac{|\sin(x+t) - \sin x|}{|t|} dx$ equals

A. 2

B. 4

C. 43469

D. 1

Answer: B

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11. The value of $I = \int_{-1}^{1} (1+x)^{1/2}(1-x)^{3/2} dx$ is

A. π

B. $\frac{\pi}{2}$

C. 2π

D. none of these

Answer: A

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12. The value of $I = \int_0^{\pi} x\left(\sin^2(\sin x) + \cos^2(\cos x)\right)dx$ is

A. π^2

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{4}$

D. none of these

Answer: B

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13. $\int_0^a \log(\cot a + \tan x)dx$ where $a \in \left(0, \frac{\pi}{2}\right)$ is

A. (a) $a \ln(\sin a)$

B. (b) $-a \ln(\sin a)$

C. (c) $-a \ln(\cos a)$

D. (d) none of these

Answer: B

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14. IF $f(x + f(y)) = f(x) + y \forall x, y \in R$ and $f(0) = 1$, then $\int_0^{10} f(10 - x)dx$ is equal to

A. 1

B. 10

C. $\int_0^1 f(x)dx$

D. $10\int_0^1 f(x)dx$

Answer: D

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15. $u = \int_0^{\frac{\pi}{2}} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx$ and $v = \int_0^{\frac{\pi}{2}} \cos\left(\frac{\pi}{3}\sin x\right)dx$

A. 2u = v

B. 2u = 3v

C. u = v

D. u = 2v

Answer: A

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16. $\int_0^{100\pi}\left(\sum_{r=1}^{10}\tan rx\right)dx$ is equal to

A. 0

B. 100π

C. -50π

D. 50π

Answer: A

View Text Solution

17. $\int_0^{\pi/2}\sin x\sin 2x\sin 3x\sin 4x dx =$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{16}$

D. $\frac{\pi}{32}$

Answer: C

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18. $\int_1^{2013} [(x-1)(x-2)...(x-2013)]dx$

A. $(2013)^2$

B. $(2012)(2013)(2014)$

C. $2013!$

D. 0

Answer: D

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19. $f:[0,5] \to R, y=f(x)$ such that $f''(x)=f''(5-x)\ \forall x \in [0,5]\ f'(0)=1$ and $f'(5)=7$, then the value of $\int_1^4 f'(x)dx$ is

A. 4

B. 6

C. 8

D. 10

Answer: C

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20. $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 dx}{e^{2x} - 1}$ is equal to (i)0 (ii)2 (iii)e (iv)none of these

A. 0

B. 2

C. e

D. 2e

Answer: A

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21. If f and g are two continuous functions being even and and odd, respectively, then $\int_{-a}^{a} \frac{f(x)}{b^{g(x)}+1} dx$ is equal to (a being any non-zero number and b is positive real number, $b \neq 1$)

A. independent of f

B. independent of g

C. independent of both f and g

D. none of these

Answer: B

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22. If $\int_{0}^{4\pi} \ln\left|13\sin x + 3\sqrt{3}\cos x\right| dx = k\pi \ln 7$, then the value of k is

A. 2

B. 4

C. 8

D. 16

Answer: B

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23. f is a real valued function from R to R such that $f(x) + f(-x) = 2$, then $\int_{1-x}^{1+X} f^{-1}(t)dt =$

A. -1

B. 0

C. 1

D. none of these

Answer: B

View Text Solution

24. if $\int_{\log 2}^{x} \frac{du}{(e^u - 1)^{\frac{1}{2}}} = \frac{\pi}{6}$ then $e^x =$

A. 1

B. 2

C. 4

D. -1

Answer: C

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25. The value of $\int_{e}^{\pi^2} [\log_\pi x] d(\log_e x)$ (where [.] denotes greatest integer function) is

A. $2 \log_e \pi$

B. $\log_e \pi$

C. 1

D. 0

Answer: B

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26. The value of $\dfrac{\int_0^1 \frac{dt}{\sqrt{1-t^4}}}{\int_0^1 \frac{1}{\sqrt{1+t^4}} dt}$ is

A. 1

B. 2

C. $2\sqrt{3}$

D. $\sqrt{2}$

Answer: D

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27. Let a and b be two positive real numbers. Then the value of $\int_a^b \frac{e^{x/a} - e^{b/x}}{x} dx$ is

A. 0

B. ab

C. 1/ab

D. e^{ab}

Answer: A

View Text Solution

28. The value of $\int_0^\infty \frac{\log x}{a^2 + x^2} dx$ is

A. $\frac{2\pi \log a}{a}$

B. $\frac{\pi \log a}{2a}$

C. $\pi \log a$

D. 0

Answer: B

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29. $\int_{1/3}^{3} \frac{1}{x} \log_e\left(\left|\frac{x+x^2-1}{x-x^2+1}\right|\right) dx$ is equal to

A. $\frac{8}{3}$

B. $-\frac{8}{3}$

C. 0

D. 3

Answer: C

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30. If

$$I_n = \int_0^1 \left(1+x+x^2+\ldots.+x^{n-1}\right)\left(1+3x+5x^2+\ldots.+(2n-3)x^{n-2}\right.$$

then the value of $\sqrt{I_9}$ is (a) 3 (b) 6 (c) 9 (d) 12

A. 3

B. 6

C. 9

D. 12

Answer: C

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31. A function f(x) satisfie $f(x) = f\left(\frac{c}{x}\right)$ for some real number c(gt 1) and all positive number 'x'. If $\int_1^{\sqrt{c}} \frac{f(x)}{x} dx = 3$, then $\int_1^{c} \frac{f(x)}{x} dx$ is

A. 4

B. 6

C. 8

D. 9

Answer: B

View Text Solution

32. Let $I_1 = \int_0^{\infty} \frac{x^2\sqrt{x}}{(1+x)^6} dx,\ I_2 = \int_0^{\infty} \frac{x\sqrt{x}}{(1+x)^6} dx$, then

A. $I_1 = 2I_2$

B. $I_2 = 2I_1$

C. $I_1 = I_2$

D. $I_1 = -I_2$

Answer: D

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33. If $\int_0^{x^2(1+x)} f(t)dt = x$, then the value of f(2) is.

A. $1/2$

B. $1/3$

C. $1/4$

D. $1/5$

Answer: D

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34. If $f(x) = \int_0^x \log_{0.5}\left(\frac{2t-8}{t-2}\right)dt$, then the interval in which f(x) is increasing is (a) $(-\infty, 2) \cup (6, \infty)$ (b) $(4, 6)$ (c) $(-\infty, 2) \cup (4, \infty)$ (d) (2,6)

A. $(-\infty, 2) \cup (6, \infty)$

B. $(4, 6)$

C. $(-\infty, 2) \cup (4, \infty)$

D. (2,6)

Answer: B

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35. If a, b and c are real numbers, then the value of $\lim_{t \to 0} \left(\frac{1}{t} \int_0^t (1 + a \sin bx)^{c/x} dx \right)$ equals

A. abc

B. $\frac{ab}{c}$

C. $\frac{bc}{a}$

D. $\frac{ca}{b}$

Answer: A

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36. If $f(x) = \int_2^x \frac{dt}{1+t^4}$, then

A. $f(3) < \frac{1}{17}$

B. $f(3) > \frac{1}{17}$

C. $f(3) = \frac{1}{17}$

D. $f(3) > 1$

Answer: A

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37. If $\int_0^x f(x) \sin t dt = \text{constant}, \ 0 < x < 2\pi$ and $f(\pi) = 2$, then the value of $f(\pi/2)$ is

A. 3

B. 2

C. 4

D. 8

Answer: C

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38. A function f, continuous on the positive real axis, has the property that for all choices of x gt 0 and y gt 0, the integral $\int_{x}^{xy} f(t)dt$ is independent of x (and therefore depends only on y). If f(2) = 2, then $\int_{1}^{e} f(t)dt$ is equal to

A. e

B. 4e

C. 4

D. none of these

Answer: C

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39. The maximum value of the integral $\int_{a-1}^{a+1} \frac{1}{1+x^4} dx$ is attained

A. exactly at two values of a

B. only at one value of a which is positive

C. only a one value of a which is negative

D. only at a = 0

Answer: D

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40. $\lim_{x \to 0} \frac{\int_0^x \frac{t^2}{\sqrt{a+t}} dt}{x - \sin x} = 1 (a > 0)$. Then the value of a is

A. $1/2$

B. $1/4$

C. 2

D. 4

Answer: D

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41. Let f(x) be a differentiable non-decreasing function such that $\int_0^x (f(t))^3 dt = \frac{1}{x^2}\left(\int_0^x f(x)dt\right)^3 \forall x \in R - \{0\}$ and $f(1) = 1$. If $\int_0^x f($

is

A. always equal to 1

B. always equal to -2

C. may be 1 or -2

D. not independent of x

Answer: A

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42. Let f be continuous and the function g is defined as $g(x) = \int_0^x \left(t^2 \int_1^t f(u)du\right)dt$ where $f(1) = 3$. then the value of $g'(1) + g''(1)$ is

A. 1

B. 2

C. 3

D. 4

Answer: C

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43. Let $I_n = \int_0^{\pi/2} (\sin x + \cos x)^n dx (n \geq 2)$. Then the value of n. $I_n - 2(n-1)I_{n-1}$ is

A. 5

B. 9

C. 2

D. 7

Answer: C

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44. Let $I_n = \int_0^1 x^n \sqrt{1-x^2}dx$. Then $\lim_{n \to \infty} \frac{I_n}{I_{n-2}} =$

A. 2

B. 1

C. -1

D. -2

Answer: B

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45. If $\int_0^\infty e^{-ax}dx = \frac{1}{a}$, then $\int_0^\infty (x^n)e^{-ax}dx$ is

A. $\frac{(-1)^n n!}{a^{n+1}}$

B. $\frac{(-1)^n (n-1)!}{a^n}$

C. $\frac{n!}{a^{n+1}}$

D. none of these

Answer: C

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46. Let $n \geq 1, n \in Z$. The real number $a \in 0, 1$ that minimizes the integral $\int_0^1 |x^n - a^n| dx$ is

A. $\frac{1}{2}$

B. 2

C. 1

D. $\frac{1}{3}$

Answer: A

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47. Let f be a continuous function satisfying $f'(lnx) = [1$ for $0 < x \le 1, x$ for $x > 1$ and $f(0) = 0$ then $f(x)$ can be defined as

A. $f(x) = \begin{cases} 1 & \text{if} \quad x \le 1 \\ 1 - e^x & \text{if} \quad x > 1 \end{cases}$

B. $f(x) = \begin{cases} 1 & \text{if} \quad x \le 1 \\ e^x - 1 & \text{if} \quad x > 1 \end{cases}$

C. $f(x) = \begin{cases} 1 & \text{if} \quad x < 1 \\ e^x & \text{if} \quad x > 1 \end{cases}$

D. $f(x) = \begin{cases} x & \text{if} \quad x \le 1 \\ e^x - 1 & \text{if} \quad x > 1 \end{cases}$

Answer: D

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48. $\dfrac{626\int_0^{\infty} e^{-x} \sin^{25} x dx}{\int_0^{\infty} e^{-x} \sin^{23} x dx}$ is equal to

A. 300

B. 625

C. 600

D. 1200

Answer: C

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49. If $g(x)$ is the inverse of $f(x)$ and $f(x)$ has domain $x \in [1, 5]$, where $f(1) = 2$ and $f(5) = 10$ then the values of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$ equals

A. 72

B. 56

C. 36

D. 48

Answer: D

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50. If $f(x) = x + \sin x$, then $\int_{\pi}^{2\pi} f^{-1}(x)dx$ is equal to

A. $\frac{3\pi^2}{2} - 2$

B. $\frac{3\pi^2}{2} + 2$

C. $3\pi^2$

D. none of these

Answer: B

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51. Given a real-valued function f which is monotonic and differentiable.

Then $\int_{f(a)}^{f(b)} 2x\left(b - f^{-1}(x)\right)dx =$

A. $\int_a^b \left(f^2(x) - 2f^2(a)\right)dx$

B. $\int_a^b \left(2f^2(x) - f^2(a)\right)dx$

C. $\int_a^b \left(f^2(x)\right)dx - f^2(a)(b-a)$

D. none of these

Answer: C

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52. Let $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$ then

A. $\frac{1}{3} < I < \frac{1}{\sqrt{8}}$

B. $\frac{1}{4} < I < \frac{1}{3}$

C. $\frac{1}{4} < I < 0$

D. none of these

Answer: A

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53. Consider the function $h(x) = \frac{g^2(x)}{2} + 3x^3 - 5$, where g(x) is a continuous and differentiable function. It is given that h(x) is a monotonically increasing function and g(0) = 4. Then which of the following is not true ?

A. $g^2(1) > 10$

B. $h(5) > 3$

C. $h\left(\frac{5}{2}\right) < 2$

D. $g^{-1} < 22$

Answer: C

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Multiple Correct Answer Type

1. If $A(x+y) = A(x)A(y)$ and $A(0) \neq 0$ and $B(x) = \dfrac{A(x)}{1+(A(x))^2}$, then

A. $\int_{-2010}^{2010} B(x)dx = \int_0^{2011} B(x)dx$

B. $\int_{-2010}^{2011} B(x)dx = \int_{0}^{2010} B(x)dx + \int_{0}^{2011} B(x)dx$

C. $\int_{-2010}^{2011} B(x)dx = 0$

D. $\int_{-2010}^{2010} (2B(-x) - B(x))dx = 2\int_{0}^{2010} B(x)dx$

Answer: B:D

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2. A function f is defined by $f(x) = \int_{0}^{\pi} \cos t \cos(x - t)dt, 0 \leq x \leq 2\pi.$ Which of the following hold(s) good? (A) $f(x)$ is continuous but not differentiable in $(0, 2\pi)$ (B) There exists at least one $c \in (0, 2\pi)$ such that $f'(c) = 0$ (C) Maximum value of f is $\frac{\pi}{2}$ (D) Minimum value of f is $-\frac{\pi}{2}$

A. f(x) is continuous but not differentiable in $(0, 2\pi)$.

B. Maximum value of f is $\pi/2$

C. There exists atleast one $c \in (0, 2\pi)$ such that $f'(c) = 0$

D. Minimum value of f is $-\frac{\pi}{2}$.

Answer: B::C::D

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3. $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ then

A. $\int_{+0}^{\infty} e^{-2x^2} dx = \frac{\sqrt{\pi}}{2\sqrt{2}}$

B. $\int_0^{\infty} x e^{-x^2} dx = \frac{1}{2}$

C. $\int_0^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$

D. $\int_0^{\infty} x^2 e^{-x^2} dx = \frac{\pi}{4}$

Answer: A::B::C

View Text Solution

4. Let $f(x) = \int_0^{x} \frac{e^t}{t} dt (x > 0)$,

then $e^{-a}[f(x+1) - f(1+a)] =$

A. $\int_0^x = \frac{e^t}{(t+a)}dt$

B. $\int_1^x \frac{e^t}{t+a}dt$

C. $e^{-a}\int_{1+a}^{x+a}\frac{e^t}{t}dt$

D. $\int_0^x \frac{e^{t-a}}{(t+a)}dt$

Answer: B::C

View Text Solution

5. Let $f(x) = \int_2^x f(t^2 - 3t + 4)dt$. Then

A. f(2) = 0

B. $f(-2) = 0$

C. $f'(2) = 0$

D. $f'(2) = 2$

Answer: A::C

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6. If $\int_0^x f(t)dt = e^x - ae^{2x}\int_0^1 f(t)e^{-t}dt$, then

A. $a = \dfrac{1}{3-2e}$

B. $f(x) = e^x - 2e^{2x}$

C. $a = \dfrac{1}{e}$

D. $f(x) = e^x - e^{-x}$

Answer: A::B

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7. A function $f(x)$ satisfies $f(x) = \sin x + \int_0^x f'(t)\left(2\sin t - \sin^2 t\right)dt$ is

A. $f\left(\frac{\pi}{6}\right) = 1$

B. $g(x) = \int_0^x f(t)$ dt is increasing on $(0, \pi)$

C. $f(0) = 0$

D. f(x) is increasing on $(0, \pi)$

Answer: A::B::C

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Comprehension Type

1. Consider the function $f(x) = \int_0^x \left(5\ln\left(1+t^2\right) - 10t\tan^{-1}t + 16\sin t\right)dt.\ f(x)$ is

A. negative for all $x \in (0, 1)$

B. increasing for all $x \in (0, 1)$

C. decreasing for all $x \in (0, 1)$

D. non-monotonic function for $x \in (0, 1)$

Answer: B

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2. Consider the function $f(x) = \int_0^x \left(5\ln\left(1+t^2\right) - 10t\tan^{-1}t + 16\sin t\right)dt$. Which is not true for $\int_0^x f(t)dt$ gt?

A. positive for all $x \in (0, 1)$

B. increasing for all $x \in (0, 1)$

C. non-monotonic for all $x \in (0, 1)$

D. none of these

Answer: C

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3. $\int\left(\frac{2a+x}{a+x}\right)\sqrt{\frac{a-x}{a+x}}dx$

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4. $f(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$

A. $\dfrac{f(n+1)}{(m+1)^n}$

B. $\dfrac{f(n)}{(m+1)^{n+1}}$

C. $\dfrac{f(n+1)}{(m+1)^{n+1}}$

D. $g(m+1), n+1)$

Answer: C

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5. $\int \dfrac{dx}{x^5(1+x^5)^{\frac{1}{5}}}$

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Subjective Type

1. Prove that $\int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n}$ for n $\in N$

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