MATHS

BOOKS - CENGAGE PUBLICATION

DIFFERENTIATION

ILLUSTRATION

1. If $y=\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}, x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then find $\frac{dy}{dx}$.

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2. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x using the first principle.

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3. If $f(x) = x\tan^{-1}x,$ find $f'(\sqrt{3})$ using the first principle.

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4. $f(x) = [2x]\sin 3\pi x and f'(k') = \lambda k\pi(-1)^k$ (where [.] denotes the greatest integer function and $k \in N),$ then find the value of λ .

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5. Let $f: R^{\rightarrow}$ satisfying $|f(x)| \leq x^2 \forall x \in R$ be differentiable at $x = 0.$ The find $f'(0)$.

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6. A function $f: R \rightarrow R$ satisfies the equation f(x+y)=f(x)f(y) for all $x, y \in R,$ $f(x) \neq 0.$ Suppose that the function is differentiable at x=0 and f'(0)=2. Prove that f'(x)=2f(x).

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7. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$, then find $\frac{dy}{dx}$.

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8. If $f(x) = x|x|,$ then prove that $f'(x) = 2|x|$

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9. If $y = 1 = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + + \frac{x^n}{n!},$ show that $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0.$

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10. Find $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x)$

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11. Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to x, if $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

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12. $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, where $-\frac{\pi}{2} < x < \pi$ and $\frac{a}{b}\tan x > -1$. Find $\frac{dy}{dx}$

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13. $y = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$. find $\frac{dy}{dx}$.

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14. Find $\frac{dy}{dx}$ for $y = \sin\left(x^2+1\right)\cdot$

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15. If $y = \sqrt{\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}, then f \in d\frac{dy}{dx}\cdot$

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16. Differentiate the function $f(x) = \sec(\tan(\sqrt{x}))$ with respect to $x\cdot$

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17. Find $\frac{dy}{dx}$ for $y = \log\left(x + \sqrt{a^2 + x^2}\right).$

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18. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$, where $-1 < x < 1$, find $\frac{dy}{dx}$

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19. Find $\frac{dy}{dx}$ for $y = \tan^{-1}\sqrt{\frac{a-x}{a+x}}, \; -a<x<a$

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20. The value of $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ is equal to $\sin^{-1}x + \sin^{-1}\sqrt{x}$ $\sin^{-1}x - \sin^{-1}\sqrt{x}$ $\sin^{-1}\sqrt{x} - \sin^{-1}x$ none of these

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21. If $y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right)$ upto n terms, then find the value of $y'(0)$

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22. Let f: $R \rightarrow R$ be a one-one onto differentiable function, such that $f(2) = 1$ and $f'(2) = 3$. Then, find the value of $\left(\frac{d}{dx}\left(f^{-1}(x)\right)\right)_{x=1}$

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23.

If $f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x$, then find $f'\left(\frac{\pi}{4}\right)$.

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24. if cos y=x cos (a+y), $(a \neq 0)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

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25. Find $\frac{dy}{dx} f$ or $y = x \sin x \log x$.

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26. Evaluate $\lim_{h \to 0} \frac{(a+h)^2 \sin^{-1}(a+h) - a^2 \sin^{-1} a}{h}$

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27. Differentiate $y = \frac{e^x}{1 + \sin x}$

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28. If

$$f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x, \quad \text{then find} \quad f'\left(\frac{\pi}{4}\right).$$

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29. if cos y=x cos (a+y), $(a \neq 0)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

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30. If $y=\sqrt{\frac{1-x}{1+x}}$ find $\frac{dy}{dx}$ and prove that $\left(1-x^2\right)\frac{dy}{dx}+y=0$

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31. Find the sum of the series $1+2x+3x^2+\ldots.(n-1)x^{n-2}$ using differentiation.

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32. If $\sqrt{x}+\sqrt{y}=4, \quad \text{then find } \frac{dy}{dx}.$

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33. If $xy+y^2$=$\tan x+y$,then find $\frac{dy}{dx}$

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34. If $y = x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \ldots}}}$, prove that $\frac{dy}{dx} = \frac{y}{2y - x}$.

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35. If $\sec(x + y) = xy$ find dy//dx

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36. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \to \infty}}}$,prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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37. If $\log(x^2 + y^2) = \tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$

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38. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to_____

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39. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

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40. If $x = a\sec^3\theta andy = a\tan^3\theta, f \in d\frac{dy}{dx}a\text{h}\eta = \frac{\pi}{3}$.

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41. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then move that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.

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42. Let

$y=x^3-8x+7$ and $x=f(t)$. If $\frac{dy}{dt}=2$ and $x=3att=0$, then $\frac{dx}{dt}$ at t = 0 is given by`

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43. Find the derivative of $\frac{\sqrt{x}(x+4)^{\frac{3}{2}}}{(4x-3)^{\frac{4}{3}}}$

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44. If $x^m y^n=(x+y)^{m+n}, provethat \frac{dy}{dx}=\frac{y}{x}.$

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45. Differentiate $(\log x)^{\cos x}$ with respect to $x\cdot$

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46. If $f(x) = |x|^{|\sin x|}$, then find $f'\left(-\frac{\pi}{4}\right)$

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47. If $y = x^{x} \hat{} x \wedge ((((\infty))))$, find $\frac{dy}{dx}$.

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48. Let $f(x) = (\lim)_{h \overset{\rightarrow}{0}} \frac{(\sin(x+h))^{1n(x+h)} - (\sin x)^{1nx}}{h}$. Then $f\left(\frac{\pi}{2}\right)$ equal to 0 (b) equal to 1 In $\frac{\pi}{2}$ (d) non-existent

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49. If $x < 1, provethat \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \infty = \frac{1}{1-x}$

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50. Differentiate $\log \sin x$ w.r.t. $\sqrt{\cos x}$.

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51. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left((2x)\cdot\sqrt{1-x^2}\right)$,

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52. Find the derivative of $f(\tan x) w \dot{r} t g(\sec x) a t x = \frac{\pi}{4}$, where $f'(1) = 2 a n d g'(\sqrt{2}) = 4$.

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53. Let f(x) $= \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the values of f'$(\pi/2)$.

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54. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$. Then value of $\lim_{x \to 0} \frac{f'(x)}{x}$ is equal to

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55. If $y = \tan^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

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56. Multiply the polynomials: $(2pq + 3q^2)$ and $(3pq - 2q^2)$

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57. If $y = x \log\{\frac{x}{(a + bx)}\}$, then show that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

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58. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

is a constant independent of a and b.

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59. If $y=e^{a\cos^{-1}x}, \ -1\le x<1, showthat$

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$$

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60. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{d^2y}{dx^2}$

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61. If g is inverse of f then prove that $f''(g(x)) = -g''(x)(f'(g(x)))^3$.

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62. Let f(x) and g(x) be real valued functions such that f(x)g(x)=1, $\forall x \in R$. If $f''(x)$ and $g''(x)$ exists $\forall x \in R$ and $f'(x)$ and $g'(x)$ are never zero, then prove that $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)} = \frac{2f'(x)}{f(x)}$

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63. If $f(x)$, $g(x)$ and $h(x)$ are three polynomials of degree 2, then prove that $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

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64. If $f\left(\frac{x+y}{3}\right)=\frac{2+f(x)+f(y)}{3}$ for all real $x and y$ and $f'(2)=2$, then determine $y=f(x)$.

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65.

If $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R, (xy \neq 1)$, and $\lim_{x \to 0} \frac{f(}{x}$

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66. Let $f: R-\{0\} \to R$ be a function which is differentiable in its domain and satisfying the equation $f(x+y)=f(x)+f(y)+\frac{x+y}{xy}-\frac{1}{x+y}$, also f'(1)=2. Then find the function.

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67. A real valued function satisfies the relation $f(x+y)=f(x)+f(y)+(e^{x}-1)(e^{y}-1),\ \forall x, y \in R$. If f'(0) = 2, find f(x).

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68. If $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$,$\forall y$, $f(y)\neq 0$ and $f'(1)=2$, find f(x) .

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69. Let $f: R\vec{R}$ be a function satisfying condition $f(x+y^{3})=f(x)+[f(y)]^{3} f \text{ or } all x, y \in R\cdot$ If $f'(0)\geq 0,$ find $f(10)\cdot$

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70. Let $f(x+y)=f(x)+f(y)+2xy-1$ for all real $x and y$ and $f(x)$ be a differentiable function. If $f'(0)=\sin\alpha,$ the prove that

$f(x) > 0 \forall x \in R.$

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71. If $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}, x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then find $\frac{dy}{dx}$.

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72. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x.

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73. If $f(x) = x \tan^{-1} x$, find $f'(\sqrt{5})$.

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74. $f(x) = [2x] \sin 3\pi x and f'(k') = \lambda k\pi(-1)^k$ (where [.] denotes the greatest integer function and $k \in N$), then find the value of λ.

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75. Let $f: R \rightarrow$ satisfying $|f(x)| \leq x^2 \forall x \in R$ be differentiable at $x = 0$. The find $f'(0)$.

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76. A function $f: R\vec{R}$ satisfies the equation $f(x+y) = f(x)f(y)$ for all $x, y \in R and f(x) \neq 0 f$ or $all x \in R$. If $f(x)$ is differentiable at $x = 0 and f'(0) = 2,$ then prove that $f'(x) = 2f(x)$.

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77. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)(1 + x)\left(1 - x^{\frac{1}{4}}\right)$, then find $\frac{dy}{dx}$.

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78. If $f(x) = x|x|$, then prove that $f'(x) = 2|x|$

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79. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + + \frac{x^n}{n!}$, show that $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0.$

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80. Find $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x), x \in (0, \pi) \cup (\pi, 2\pi)$.

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81. Differentiate $\sin^{-1}\left(2x\sqrt{1 - x^2}\right)$ with respect to x if $\frac{1}{\sqrt{2}} < x < 1$

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82. If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, $\left(\frac{b}{a}\right)\tan x < 1$where $-\frac{\pi}{2} < x < \frac{\pi}{2}$ then find $\frac{dy}{dx}$.

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83. $y = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$. find $\frac{dy}{dx}$.

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84. Find $\frac{dy}{dx}$ for $y = \sin(2x^2 + 3x + 1)$.

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85. If $y = \sqrt{\log\left\{\cos\left(\frac{x^3}{3} - 1\right)\right\}}$, then find $\frac{dy}{dx}$.

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86. Differentiate the function f(x) = sin $\left(\cos\sqrt{x}\right)$ with respect to x,

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87. Find $\dfrac{dy}{dx}$ for $y = \log\left(x - \sqrt{a^2 + x^2}\right)$.

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88. $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, where $-1 < x < 1$, find $\dfrac{dy}{dx}$

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89. Find $\dfrac{dy}{dx}$ for $y = \tan^{-1}\sqrt{\dfrac{a-x}{a+x}}, \ -a < x < a.$

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90. if $y=\sin^{-1}\left[\sqrt{x}\sqrt{1-x^2}-x\sqrt{1-x}\right]$ and $0<x<1$, then find $\frac{dy}{dx}$.

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91. If $y=\tan^{-1}\left(\frac{1}{1+x+x^2}\right)+\tan^{-1}\left(\frac{1}{x^2+3x+3}\right)+\tan^{-1}\left(\frac{1}{x^2+5x+7\right.}$

upto n terms, then find the value of $y'(0)$

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94.

If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a +}{\sin a}$

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95. Find $\frac{dy}{dx}$ for $y = x\cos x \log x$.

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96. Evaluate $\lim_{h \to 0} \frac{(a+h)^2 \sin^{-1}(a+h) - a^2 \sin^{-1} a}{h}$

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97. Differentiate $y = \dfrac{e^x}{1 + \tan x}$

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98.

If $f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x, \quad \text{then find} \quad f'\left(\frac{\pi}{4}\right).$

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99.

If $\cos y = x \cos(a + y), \quad \text{with } \cos a \neq \pm 1, \quad \text{prove that} \quad \dfrac{dy}{dx} = \dfrac{\cos^2(a +}{\sin a}$

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100. If $y = \sqrt{\dfrac{1 - x}{1 + x}}$ find $\dfrac{dy}{dx}$ and prove that $\left(1 - x^2\right)\dfrac{dy}{dx} + y = 0$

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101. Find the sum of the series $1 + 2x + 3x^2 + \ldots. (n-1)x^{n-2}$ using differentiation.

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102. If $\sqrt{x} + \sqrt{y} = 4, \quad \text{then find } \dfrac{dy}{dx}.$

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103. If $xy + y^2 = \tan x + y$,then find $\dfrac{dy}{dx}$

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104. If $y = x + \dfrac{1}{x + \dfrac{1}{x + \dfrac{1}{x + \ldots.}}}$ " prove that "$\dfrac{dy}{dx} = \dfrac{y}{2y - x}$

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105. If tan (x+y) = xy, then find $\frac{dy}{dx}$

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106. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\rightarrow\infty}}}$,prove that $\frac{dy}{dx}=\frac{\cos x}{2y-1}$

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107. If $\log(x^2+y^2)=\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

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108. If $y=y(x)$ and it follows the relation $4xe^{xy}=y+5\sin^2 x$, then $y'(0)$ is equal to_____

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109. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)\cdot$

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110. If $x = a\sec^3\theta andy = a\tan^3\theta, f \in d\frac{dy}{dx}ah\eta = \frac{\pi}{3}\cdot$

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111. If $e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = \frac{-y\log x}{x\log y}$

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112. Let

$y = x^3 - 8x + 7$ and $x = f(t)$. If $\frac{dy}{dt} = 2$ and $x = 3att = 0$, then

$\frac{dx}{dt}$ at t = 0 is given by`

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113. Find the derivative of $\frac{(x)^{\frac{5}{2}}(x-3)^{\frac{3}{2}}}{(7x-5)^{\frac{4}{3}}}$

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114. If $x^m y^n = (x+y)^{m+n}, provethat \frac{dy}{dx} = \frac{y}{x}.$

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115. Differentiate $(\log x)^{\cos x}$ with respect to $x.$

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116. If $f(x) = |x|^{|\sin x|}, \quad \text{then find} \quad f'\left(-\frac{\pi}{4}\right).$

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117. If $y = x^x \hat{} x \wedge ((((\infty))))$, find $\frac{dy}{dx}.$

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118. Let $f(x) = (\lim)_{h \to 0} \frac{(\sin(x+h))^{1n(x+h)} - (\sin x)^{1nx}}{h}.$ Then $f\left(\frac{\pi}{2}\right)$ equal to 0 (b) equal to 1 In $\frac{\pi}{2}$ (d) non-existent

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119. If $x < 1, provethat \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \infty = \frac{1}{1-x}$

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120. Differentiate log sin x w.r.t. $\sqrt{x}$.

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121. Find the derivative of $f(\tan x) w\dot{r}tg(\sec x) at x = \frac{\pi}{4},$ where $f'(1) = 2 and g'\left(\sqrt{2}\right) = 4.$

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122. Let f(x) $= \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the values of f'$(\pi/2)$.

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123. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$ then find the value of

$$\lim_{x \to 0} \frac{f(x)}{x^2}$$

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124. If $y = \sin^{-1} x$, find $\frac{d^2y}{dx^2}$.

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125. about to only mathematics

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126. If $y = x \log\left\{\frac{x}{(a+bx)}\right\}$, then show that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

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127. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

is a constant independent of a and b.

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128. If $y = e^{a\cos^{-1}x}, \ -1 \leq x < 1, showthat$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

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129. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$

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130. If g is inverse of f then prove that $f''(g(x)) = -g''(x)(f'(g(x)))^3$.

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131. Let f(x) and g(x) be real valued functions such that f(x)g(x)=1, $\forall x \in R$. If $f''(x)$ and $g''(x)$ exists $\forall x \in R$ and $f'(x)$ and $g'(x)$ are never zero, then prove that $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)} = \frac{2f'(x)}{f(x)}$

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132. If $f(x), g(x) and h(x)$ are three polynomial of degree 2, then prove that

$\phi(x) = |[f(x)g(x)h(x)]; [f'(x)g'(x)h'(x)]; [f''(x)g''(x)h''(x)]|$ is: constant

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133. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ for all real x and y. If $f'(0)$ exits and equals -1 and $f(0) = 1$, then find $f(2)$.

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134. $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$,for all$x, y \in R.(xy \neq 1)$,and $\lim_{x \to 0} \frac{f(x)}{x} = 2$.Find$f(\sqrt{3})$and $f'(-2)$.

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135. A real valued function satisfies the relation $f(x+y)=f(x)+f(y)+(e^x-1)(e^y-1), \forall x, y \in R$. If f'(0) = 2, find f(x).

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136. Find function f(x) which satisfy the relation $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)} \forall x, y \in R, y \neq 0, f(y) \neq 0 \text{ and } f'(1)=2$

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137. Let $f: R\overrightarrow{R}$ be a function satisfying condition $f(x+y^3)=f(x)+[f(y)]^3 f \text{ or } all x, y \in R\cdot$ If $f'(0) \geq 0,$ find $f(10)\cdot$

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138. Let $f(x+y)=f(x)+f(y)+2xy-1$ for all real $x and y$ and $f(x)$ be a differentiable function. If $f'(0)=\sin\alpha,$ the prove that $f(x)>0 \forall x \in R.$

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Solved Examples

1. If $f(x)=(\log)_x(\log x), then f'(x)$ at $x=e$ is equal to (a) $\frac{1}{e}$ (b) e (c) 1 (d) zero

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2. If $\frac{\cos x}{2}\frac{\cos x}{2^2}\frac{\cos x}{2^3}\infty=\frac{\sin x}{x},$ then find the value of $\frac{1}{2^2}\frac{\sec^2 x}{2}+\frac{1}{2^4}\frac{\sec^2 x}{2^2}+\frac{1}{2^6}\frac{\sec^2 x}{2^3}+\infty$

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3. If $y = f(a^x) and f'(\sin x) = (\log)_e x, then find \left(\frac{dy}{dx}\right)$, if it exists, where pi/2 less than x less than pi

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4. If P_n is the sum of a $G\dot{P}\cdot$ upto n terms $(n \geq 3)$, then prove that $(1-r)\frac{dP_n}{dr} = (1-n)P_n + nP_{n-1}$, where r is the common ratio of $G\dot{P}\cdot$

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5. If $g(x) = \frac{f(x)}{(x-a)(x-b)(x-c)}, where f(x)$ is a polynomial of degree < 3, then prove that $\frac{dg(x)}{dx} = \frac{\left|1af(a)(x-a)^{-2}1bf(b)(x-b)^{-2}1cf(c)(x-c)^{-2}\right|}{|a^2a1b^2b1c^2c1|}$

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6. If $x = \cos ec\theta - \sin\theta$ and $y = \cos ec^n\theta - \sin^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$.

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7. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then prove that $\frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$

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8. Find the differential equation of the curves given by $y = Ae^{2x} + Be^{-2x}$ where A and B are parameters.

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9. If $y = \left(\frac{1}{2}\right)^{n-1}\cos\left(n\cos^{-1}x\right)$, then prove that y satisfies the differential equation $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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10. Let $f(x) and g(x)$ be two function having finite nonzero third-order derivatives $f'''(x)$ and $g'''(x)$ for all $x \in R$. If $f(x).\,g(x) = 1$ for all $x \in R$, then prove that $\frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} - \frac{g''}{g}\right)$.

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11. If a curve is represented parametrically by the equation $x = f(t)$ and $y = g(t)$ then prove that $\frac{d^2y}{dx^2} = -\left[\frac{g'(t)}{f'(t)}\right]^3\left(\frac{d^2x}{dy^2}\right)$

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12. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real $x and y$ and $f'(2) = 2$, then determine $y = f(x)\cdot$

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13. If $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than 0 and $f(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}$, then find $f(x)$

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14. If $|a_1 \sin x + a_2 \sin 2x + \quad + a_n \sin nx| \leq |\sin x|$ for $x \in R$, then prove that $|a_1 + 2a_2 + 3a_3 + na_n| \leq 1$

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15. Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$. If $|p(x)| \leq |e^{x-1} - 1| \leq 1$ then prove |a1 + 2a2 +...... + n an| ≤ 1.

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16. If $f(x) = \log_x(\log x)$, then find $f'(x) at x = e$

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17. If $\frac{\cos x}{2}\frac{\cos x}{2^2}\frac{\cos x}{2^3}\infty=\frac{\sin x}{x}$, then find the value of $\frac{1}{2^2}\frac{\sec^2 x}{2}+\frac{1}{2^4}\frac{\sec^2 x}{2^2}+\frac{1}{2^6}\frac{\sec^2 x}{2^3}+\infty$

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18. If $y=f(a^x) and f'(\sin x)=(\log)_e x, then find\left(\frac{dy}{dx}\right)$, if it exists, where pi/2 less than x less than pi

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19. If P_n is the sum of a $G\dot{P}$ upto n terms $(n\geq 3)$, then prove that $(1-r)\frac{dP_n}{dr}=(1-n)P_n+nP_{n-1}$, where r is the common ratio of $G\dot{P}$

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20. If $x = \cos ec\theta - \sin\theta$ and $y = \cos ec^n\theta - \sin^n\theta$, then show that $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$.

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21.

If $= \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, prove tha

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22.

If $y = \frac{2}{\sqrt{a^2 - b^2}}\left\{\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)\right\}$, then show that $\frac{d^2y}{dx^2} =$

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23. If $y = \left(\frac{1}{2}\right)^{n-1}\cos(n\cos^{-1}x)$, then prove that y satisfies the differential equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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24. If a curve is represented parametrically by the equation $x = f(t)$ and $y = g(t)$ then prove that $\frac{d^2y}{dx^2} = -\left[\frac{g'(t)}{f'(t)}\right]^3\left(\frac{d^2x}{dy^2}\right)$

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25. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real $x and y$ and $f'(2) = 2$, then determine $y = f(x)$.

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26. If $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than 0 and $f(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}$, then find $f(x)$

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27. If $|a_1 \sin x + a_2 \sin 2x + \quad + a_n \sin nx| \leq |\sin x|$ for $x \in R,$ then prove that $|a_1 + 2a_2 + 3a_3 + na_n| \leq 1$

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28. Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n.$ If $|p(x)| \leq \left|e^{x-1} - 1\right| \leq 1$ then prove |a1 + 2a2 +...... + n an| ≤ 1.

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Concept Application 3.1

1. Using the definition of derivative find the derivative of $\sqrt{\sin x}$

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Concept Application 3.2

1. Find the derivative of $\sqrt{4-x}$ w.r.t. x using the first principle.

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2. If $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then find $\frac{dy}{dx}$

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3. find $\frac{dy}{dx}$of $xy=e^{x-y}$

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4. If $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right); 0<x<(\sqrt{2})$, then find= $\frac{dy}{dx}$

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5. Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left(\frac{4x}{1+5x^2}\right)+\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$

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6. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, where $x \neq 0$

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7. Simplify $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

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8. Find $\frac{dy}{dx}$

when $y = \sin^{-1}x + \cos^{-1}x$

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9. if $y = x^x$, $then\text{Find}\frac{dy}{dx}$

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10. If $y = e^x \sin x$, find $\frac{dy}{dx}$

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11. Find $\frac{dy}{dx}$ for the function: $y = \log \sqrt{\sin \sqrt{e^x}}$

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12. Find $\frac{dy}{dx}$ for the function: $y = a^{\left(\sin^{-1} x\right)^2}$

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13. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x \left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

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14. $y = \sin^{-1}\left[\sqrt{x - ax} - \sqrt{a - ax}\right]$ then find $\frac{dy}{dx}$

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15. Find $\frac{dy}{dx}$ if y=sin(2x)

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16. Show that $\frac{dy}{dx} = \sec x$. If $y = (\log)\sqrt{\frac{1 + \sin x}{1 - \sin x}}$

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17. Find $\frac{dy}{dx}$ for the functions: $y = \frac{x + \sin x}{x + \cos x}$

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18. If $y = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\ldots\left(1 + x^{2^n}\right)$ then $\frac{dy}{dx}$ at $x = 0$ is

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19. $x\sqrt{1+y}+y\sqrt{1+x}=0$ then $\frac{dy}{dx}=$

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20. If g(x) is the inverse function and $f'(x)=\sin x$ then prove that $g'(x)=\cos ec[g(x)]$

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21. Find the derivative of $\sqrt{5-x}$ w.r.t. x.

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22. $y=\sin^{-1}\frac{2x}{1+x^2}, \ -1\le x\le 1$

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23. If $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \ -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then find $\frac{dy}{dx}$

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24. $y = \cos^{-1}(2x^2 - 1), 0 < x < \frac{1}{\sqrt{2}}$

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25. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{4x}{1 + 5x^2}\right) + \tan^{-1}\left(\frac{2 + 3x}{3 - 2x}\right)$

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26. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$, where $x \neq 0$

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27. $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ find $\frac{dy}{dx}$

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28. Find $\frac{dy}{dx}$ for the function: $y = \sin^{-1}\sqrt{(1-x)} + \cos^{-1}\sqrt{x}$

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29. $y = \sqrt{\tan\sqrt{x}}$ find $\frac{dy}{dx}$

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30. $y = e^{\sin x^3}$ find $\frac{dy}{dx}$

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31. Find $\frac{dy}{dx}$ for the function: $y = \log\sqrt{\sin\sqrt{e^x}}$

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32. Find $\frac{dy}{dx}$ for the function: $y = a^{(\sin^{-1} x)^2}$

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33. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

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34. $y = \sin^{-1}\left[\sqrt{x - ax} - \sqrt{a - ax}\right]$ then find $\frac{dy}{dx}$

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35. Find $\frac{dy}{dx}$ for the functions: $y = x^3 e^x \sin x$

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36. Find $\frac{dy}{dx}$ for the function: $y = (\log)_e \sqrt{\frac{1 + \sin x}{1 - \sin x}}, where x = \frac{\pi}{3}$

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37. $y = \frac{x + \cos x}{x + \sin x}$, find $\frac{dy}{dx}$

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38.

If $y = (1 + x)(1 + x^2)(1 + x^4)\dots\left(1 + x^{2^n}\right), \quad \text{then find } \frac{dy}{dx} at x = 0.$

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39. If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$

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40. If g(x) is the inverse function and $f'(x) = \sin x$ then prove that $g'(x) = \cos ec[g(x)]$

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Concept Application 3.3

1. Solve : tan − 1 (x − 1 / x − 2) + tan − 1 (x + 1 / x + 2) = π / 4 Then find the value of x.

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2. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$

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3. If $y = b\tan^{-1}\left(\frac{x}{a} + \frac{\tan^{-1} y}{x}\right), f \in d\frac{dy}{dx}.$

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4.

If $\log_e(\log_e x - \log_e y) = e^{x^2 y}(1 - \log_e x)$, then find the value of $y'(e)$.

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5.

If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots\infty}}}}$, then prove that $\frac{dy}{dx} = \frac{y^2 -}{2y^3 - 2}$

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6. Let $f(x) = \frac{(2^x + 2^{-x})\sin x\sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$. Then find the value of f'(0).

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7. If $x^3+y^3=3axy$, find $\frac{dy}{dx}$.

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8. If $y=b\tan^{-1}\left(\frac{x}{a}+\frac{\tan^{-1}y}{x}\right)$, $f\in d\frac{dy}{dx}$.

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9.

If $\log_e(\log_e x-\log_e y)=e^{x^2y}(1-\log_e x)$, then find the value of $y'(e)$.

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10.

If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\ldots\infty}}}}$, then prove that $\frac{dy}{dx}=\frac{y^2-}{2y^3-2}$

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Concept Application 3.4

1. Let IR be the set of real numbers andf : IR be such that for all $x, y \in IR$

$|f(x) - f(y) \leq |x - y|^3$ Prove that f is a constant function.

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2. If $x = \frac{2t}{1+t^2}$,$y = \frac{1-t^2}{1+t^2}$,$then$ find $\frac{dy}{dx}$.

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3. If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$ show that $\frac{dy}{dx} = -\frac{y}{x}$.

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4. Find $\frac{dy}{dx}$ if x= 2 cosθ- $\cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$.

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5. If $x = 3\cos t - 2\cos^3 t, y = 3\sin t - 2\sin^3 t, then$dy/dx is

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6. about to only mathematics

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7. Statement 1: Let $f: R \to R$ be a real-valued function $\forall x, y \in R$ such that $|f(x) - f(y)| \leq |x - y|^3$. Then $f(x)$ is a constant function. Statement 2: If the derivative of the function w.r.t. x is zero, then function is constant.

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8. If $x = \frac{2t}{1+t^2}, y = \frac{1-t^2}{1+t^2}, then$ find $\frac{dy}{dx}$.

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9. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$ show that $\frac{dy}{dx} = -\frac{y}{x}$.

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10. Find $\frac{dy}{dx}$ if $x = \cos\theta - \cos 2\theta$

and $y = \sin\theta - \sin 2\theta$

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11. Find $\frac{dy}{dx}$ if $x = 3\cos\theta - 2\cos^3\theta, y = 3\sin\theta - 2\sin^3\theta.$

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12. Find $\frac{dy}{dx}$ when $x = a\log t$ and $y = b\sin t.$

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Concept Application 3.5

1. Derivative of the function f(x)= (x-1)(x-2) is

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2. If $x^y = e^{x-y}$, Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

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3. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

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4. If $y^x = x^y, then f \in d\frac{dy}{dx}$.

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5. If $x = e^y + e^{(y + \to \infty)}$, where $x > 0, then f \in d\frac{dy}{dx}$

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6. Find $\frac{dy}{dx}$ if $x^y = y^x$

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7. Differentiate $\left(x^{\sin x}\right), x > 0$ with respect to $x\cdot$

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8. If $y = (\tan x)^{(\tan x)}$, then find $\frac{dy}{dx}$. at $x = \frac{\pi}{4}$

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9. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x

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10. If $y \log x = x - y$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

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11. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

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12. If $y^x = x^y, then f \in d\frac{dy}{dx}$.

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13. If $x = e^y + e^{(y + \to \infty)}$, where $x > 0, then f \in d\frac{dy}{dx}$

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14. Find $\frac{dy}{dx}$ for $y = x^x$.

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15. Differentiate $(x \sin x)^x$ with respect to x.

Watch Video Solution

16. If $y = (\tan x)^{(\tan x)^{\tan x}}$, then find $\frac{dy}{dx}$.

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Concept Application 3.6

1. If $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$,then find $\frac{dy}{dx}$

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2. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1-x^2}$ at $x=\frac{1}{2}$

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3. Differentiate $\frac{x}{\sin x}$ w.r.t . sinx.

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4. If $y=f(x^3), z=g(x^5), f'(x)=\tan x, andg'(x)=\sec x,$ then find the value of $\lim_{x\to 0}\frac{\left(\frac{dy}{dz}\right)}{x}$

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5. Find the derivative of $\cos^{-1}\frac{1-x^2}{1+x^2}$ $\text{w.r.t.}\ \sin^{-1}\frac{2x}{1+x^2}.$

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6. Find the derivative of $\cos^{-1}(2x^2-1)$ w.r.t. $\sqrt{1-x^2}$ at $x=\frac{1}{2}$.

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7. Differentiate $\frac{x}{\cos x}$ w.r.t . cosx.

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8. If $y=f(x^3), z=g(x^5), f'(x)=\tan x, and g'(x)=\sec x,$ then find the value of $\lim_{x\to 0}\frac{\left(\frac{dy}{dz}\right)}{x}$

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Concept Application 3.7

1. If $f(x)=\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, then prove that $f'(x)=3x^2+2x(a^2+b^2+c^2)$.

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2. Let $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin(2x^2) \end{vmatrix}$. Find the value of $f'(0)$.

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3. Let $g(x) = \begin{vmatrix} f(x+c) & f(x+2c) & f(x+3c) \\ f(c) & f(2c) & f(3c) \\ f'(c) & f'(2c) & f'(3c) \end{vmatrix}$,

where c is constant, then find $\lim_{x \to 0} \frac{g(x)}{x}$.

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4. If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, then prove that

$f'(x) = 3x^2 + 2x(a^2+b^2+c^2)$.

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5. Let $f(x)=\begin{vmatrix}\cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2)\\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2)\\ \sin 2x & 0 & \sin(2x^2)\end{vmatrix}$. Find the value of $f'(0)$.

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6. Let $g(x)=\begin{vmatrix}f(x+c) & f(x+2c) & f(x+3c)\\ f(c) & f(2c) & f(3c)\\ f'(c) & f'(2c) & f'(3c)\end{vmatrix}$,

where c is constant, then find $\lim_{x\to 0}\frac{g(x)}{x}$.

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Concept Application 3.8

1. If $f(x)=(1+x)^n$, then the value of

$f(0)+f'(0)+\frac{f''(0)}{2!}+\frac{f'''(0)}{3!}+\ldots\ldots\frac{f^n(0)}{n!}$.

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2. If $e^{y}(x+1)=1$, prove that $\frac{d^{2}y}{dx^{2}}=\left(\frac{dy}{dx}\right)^{2}$

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3. Prove that $\frac{d^{n}}{dx^{n}}\left(e^{2x}+e^{-2x}\right)=2^{n}\left[e^{2x}+(-1)^{n}e^{-2x}\right]$

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4. If $y=\sin(\sin x)$ and $\frac{d^{2}y}{dx^{2}}+\frac{dy}{dx}\tan x+f(x)=0$, then find $f(x)$.

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5. If $y=\log(1+\sin x)$, prove that $y_4+y_3y_1+y_2^{2}=0$.

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6. if $f(x)=\begin{vmatrix} x^n & n! & 2 \\ \cos x & \cos.\frac{n\pi}{2} & 4 \\ \sin x & \sin.\frac{n\pi}{2} & 8 \end{vmatrix}$, then find the value of $\frac{d^n}{dx^n}[f(x)]_{x=0}.\ (n \in z)$.

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7. If $x=a\cos\theta,\ y=b\sin\theta,$ then prove that $\frac{d^3y}{dx^3}=-\frac{3b}{a^3}\cos ec^4\theta\cot\theta$.

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8. If $x=a\cos^3\theta,\ y=b\sin^3\theta$,fin d $\frac{d^3y}{dx^3}$ at $\theta=0$.

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9. If $f(x)=(1+x)^n$, then the value of $f(0)+f'(0)+\frac{f''(0)}{2!}+\frac{f'''(0)}{3!}+\ldots\ldots\frac{f^n(0)}{n!}$.

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10. If $e^y(x+1)=1$, find $\frac{d^2y}{dx^2}$

Watch Video Solution

11. Prove that $\frac{d^n}{dx^n}\left(e^{2x}+e^{-2x}\right)=2^n\left[e^{2x}+(-1)^n e^{-2x}\right]$

Watch Video Solution

12. If $y=\sin(\sin x)$ and $\frac{d^2y}{dx^2}+\frac{dy}{dx}\tan x+f(x)=0$, then find $f(x)$.

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13. If $y=\log(1+\sin x)$, prove that $y_4+y_3y_1+y_2^2=0$.

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14. If $x = a\cos\theta, y = b\sin\theta,$ then prove that $\frac{d^3y}{dx^3} = -\frac{3b}{a^3}\cos ec^4\theta\cot\theta$.

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15. If $x = a\cos^3\theta, y = b\sin^3\theta$,fin d $\frac{d^3y}{dx^3}$ at $\theta = 0$.

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Concept Application 3.9

1. Let $f(x+y) = f(x)f(y)$ for all $x and y$. Suppose $f(5) = 2 and f'(0) = 3$. Find $f'(5)$.

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2. Let $f(xy) = f(x)f(y)\,\forall x, y \in R$ and f is differentiable at $x = 1$ such that $f'(1) = 1.$ Also, $f(1) \neq 0$, $f(2) = 3.$ Then find $f'(2)$

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3. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all $x and y and f(x) = (2x^2 + 3x)g(x)$ for all$x$, where $g(x)$ is continuous and $g(0) = 3.$ Then find $f'(x)$.

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4. Let $g: R\overrightarrow{R}$ be a differentiable function satisfying $g(x) = g(y)g(x-y)\,\forall x, y \in R$ and $g'(0) = a and g'(3) = b.$ Then find the value of $g'(-3)$.

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5. Let $f(x^m y^n) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$. If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $\lim_{x \to 0} \frac{f(1+x)}{x}$

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6. If $f\left(\frac{x+2y}{3}\right) = \frac{f(x) + 2f(y)}{3} \forall x, y \in R and f'(0) = 1, f(0) = 2$, then find $f(x)$.

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7. Prove that $\lim_{x \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$

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8. Let $f(x+y) = f(x)f(\dot{y})$ for all $x and y$. Suppose $f(5) = 2 and f'(0) = 3$. Find $f'(5)$.

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9. Let $f(xy) = f(x)f(y) \forall x, y \in R$ and f is differentiable at $x = 1$ such that $f'(1) = 1$. Also, $f(1) \neq 0$, $f(2) = 3$. Then find $f'(2)$

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10. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all $x and y and f(x) = (2x^2 + 3x)g(x)$ for all$x$, where $g(x)$ is continuous and $g(0) = 3$. Then find $f'(x)$.

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11. Let $g: R\overrightarrow{R}$ be a differentiable function satisfying $g(x) = g(y)g(x-y) \forall x, y \in R$ and $g'(0) = a and g'(3) = b$. Then find the value of $g'(-3)$.

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12. Let $f(x^m y^n) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$. If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $\lim_{x \to 0} \frac{f(1+x)}{x}$

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13. If $f\left(\frac{x+2y}{3}\right) = \frac{f(x) + 2f(y)}{3}\ \forall x, y \in R and f'(0) = 1, f(0) = 2$, then find $f(x)$.

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14. Prove that $\lim_{x \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$

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Execrises

1. If $y = a \sin x + b \cos x, then y^2 + \left(\frac{dy}{dx}\right)^2$ is a function of x (b) function of y function of $x and y$ (d) constant

A. function of x

B. function of y

C. function of x and y

D. constant

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2. find $\frac{dy}{dx}$, $\left(\sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}\right)$ is equal to, $0 < x < \frac{\pi}{2}$

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3. If $y = |\cos x| + |\sin x|, then \left(\frac{dy}{dx}\right)$ at $x = \frac{2\pi}{3}$ is

A. $\frac{1-\sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}(\sqrt{3}-1)$

D. none of these

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4. If f(x) $= |\log_e |x||$, then f'(x) equals

A. A. $\frac{1}{|x|}$, where $x \neq 0$

B. B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

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5. If $f(x)=\sqrt{1-\sin 2x}$, then $f'(x)$ is equal to

(a)$-(\cos x+\sin x)$,for $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

(b)$\cos x+\sin x$,for $x\in\left(0,\frac{\pi}{4}\right)$

(c)$-(\cos x+\sin x)$,for $x\in\left(0,\frac{\pi}{4}\right)$

(d)$\cos x-\sin x$,for $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

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6. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D^{\cdot}F(x)$ by the formula $D\cdot(x)=(\lim)_{h\overset{\to}{0}}\frac{f^2(x+h)-f^2(x)}{h}, where f^2(x)$ mean $[f(x)]^2 and \Leftrightarrow(x)=x\log x, then D^{\cdot}f(x)(\mid)_{x=e}$ has the value e (B) 2e

(c) 4e (d) none of these

A. e

B. 2e

C. 4e

D. none of these

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7. If `y=cot^(-1)[(sqrt(1+sinx)+sqrt(1-sinx))/(sqrt(1+sinx)-sqrt(1-sinx))](0

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. 3

D. 1

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8. If $y=\frac{(a-x)\sqrt{a-x}-(b-x)\sqrt{x-b}}{a}, then \frac{dy}{dx}$ wherever it is defined is $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$ (b) $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$ $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

(d) $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

A. A. $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$

B. B. $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$

C. C. $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

D. D. $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

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9. The derivative of $y=(1-x)(2-x)....(n-x)$ at $x=1$ is

(a)0 (b) $(-1)(n-1)!$ (c)$n!-1$ (d) $(-1)^{n-1}(n-1)!$

A. 0

B. $(-1)(n-1)!$

C. $n!-1$

D. $(-1)^{n-1}(n-1)!$

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10. If $y=\sqrt{\frac{1-x}{1+x}}$, then $(1-x^2)\frac{dy}{dx}$ is equal to

A. A. y^2

B. B. $1/y$

C. C. $-y$

D. D. $-y/x$

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11. If $y=\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}, then\frac{dy}{dx}isequa < o \quad \frac{ay}{x\sqrt{a^2-x^2}}$ (b) $\frac{ay}{\sqrt{a^2-x^2}}$ $\frac{ay}{x\sqrt{a^2-x^2}}$ (d) none of these

A. $\frac{ay}{x\sqrt{a^2-x^2}}$

B. $\frac{ay}{\sqrt{a^2-x^2}}$

C. $\dfrac{ay}{x\sqrt{x^2-a^2}}$

D. none of these

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12. Let $u(x)$ and $v(x)$ be differentiable functions such that $\dfrac{u(x)}{v(x)}=7$. If $\dfrac{u'(x)}{v'(x)}=p$ and $\left(\dfrac{u'(x)}{v'(x)}\right)=q$, then $\dfrac{p+q}{p-q}$ has the value of (a) 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

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13. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log a, then\frac{dy}{dx}$ is equal to (a) $\frac{x}{y}$ (b) $\frac{y}{x^2}$ (C) $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2-y^2}{x^2+y^2}$

D. $\frac{y}{x}$

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14. Let $h(x)$ be differentiable for all x and let $f(x)=(kx+e^x)h(x)$, where k is some constant. If $h(0)=5, h'(0)=-2, and f'(0)=18,$ then the value of k is 5 (b) 4 (c) 3 (d) 2.2.

A. 5

B. 4

C. 3

D. 2.2

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15. If $\lim_{t \to x} \frac{e^t f(x) - e^x f(t)}{(t-x)(f(x))^2} = 2$ and $f(0) = \frac{1}{2}$, then find the value of $f'(0)$.

A. 4

B. 2

C. 0

D. 1

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16. If $f(0)=0, f'(0)=2,$ then the derivative of $y=f(f(f(x)))$ at $x=0$ is 2 (b) 8 (c) 16 (d) 4

A. 2

B. 8

C. 16

D. 4

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17. If $f(x)=\sqrt{1+\cos^2(x^2)}, thenf'\left(\frac{\sqrt{\pi}}{2}\right)$ is (a) $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\pi/6}$ (c) $1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

A. $\sqrt{\pi/6}$

B. $-\sqrt{\pi/6}$

C. $1/\sqrt{6}$

D. $\pi / \sqrt{6}$

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18. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x} isequa < o \quad \frac{1}{2}\sqrt{1 + secx}$ (b) $\sqrt{1 + \sec x}$ $-\frac{1}{2}\sqrt{1 + \sec x}$ (d) $-\sqrt{1 + \sec x}$

A. $\frac{1}{2}\sqrt{1 + \sec x}$

B. $\sqrt{1 + \sec x}$

C. $-\frac{1}{2}\sqrt{1 + \sec x}$

D. $-\sqrt{1 + \sec x}$

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19. if $y = \tan^{-1}\left(\frac{2^x}{1 + 2^{2x+1}}\right)$ then $\frac{dy}{dx} atx = 0$ is

A. 1

B. 2

C. In 2

D. $-\frac{1}{10}\ln 2$

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20. If $y=\left(x+\sqrt{x^2+a^2}\right)^n, then \frac{dy}{dx}$ is $\frac{ny}{\sqrt{x^2+a^2}}$ (b) $-\frac{ny}{\sqrt{x^2+a^2}}$ $\frac{nx}{\sqrt{x^2+a^2}}$ (d) $-\frac{nx}{\sqrt{x^2+a^2}}$

A. $\frac{ny}{\sqrt{x^2+a^2}}$

B. $-\frac{ny}{\sqrt{x^2+a^2}}$

C. $\frac{nx}{\sqrt{x^2+a^2}}$

D. $-\frac{nx}{\sqrt{x^2+a^2}}$

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21. If $y = e^{\log\left(x^5\right)}$, find $\frac{dy}{dx}$

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22. Evaluate, $\frac{d}{dx}\left[\sin^2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ is

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

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23. The differential coefficient of $f\left((\log)_e x\right)$ with respect to x, where $f(x) = (\log)_e x$, is

(a) $\frac{x}{(\log)_e x}$ (b) $\frac{1}{x}(\log)_e x$ (c) $\frac{1}{x(\log)_e x}$ (d) none of these

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x \log_e x}$

D. none of these

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24. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2), then \frac{dy}{dx} at x = 1$ is (a)2 (b) 1 (c) -2 (d) none of these

A. 2

B. 1

C. -2

D. none of these

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25. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2}x\cos x^3 \cos ecx^2$ $\frac{2}{3}\sin x^3 \sec x^2$ $\tan x$ (d) none of these

A. $\frac{3}{2}x \quad \cos \quad x^2 \quad \text{cosec} \quad x^2$

B. $\frac{3}{2} \quad \sin \quad x^3 \quad \sec \quad x^2$

C. $\tan x$

D. none of these

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26. A function f, defined for all positive real numbers, satisfies the equation $f(x^2) = x^3$ for every $x > 0$. Then the value of $f'(4)$ is 12 (b) 3 (c) $3/2$ (d) cannot be determined

A. 12

B. 3

C. $3/2$

D. cannot be determined

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27. Let $f:(-5,5)\to R$ be a differentiable function of with $f(4)=1, f'(4)=1, f(0)=-1$ and $f'(0)=1 If f, g(x)=\left(f\left(2f^2(x)+2\right.\right.$

then g'(0) equals

A. 4

B. -4

C. 8

D. -8

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28. The function $f(x) = e^x + x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}(f^{-1})$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{In2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. none of these

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29. If $f(x) = x + \tan x$ and f is the inverse of g, then $g'(x)$ is equal to

A. $\frac{1}{1 + [g(x) - x]^2}$

B. $\frac{1}{2 - [g(x) - x]^2}$

C. $\frac{1}{2 + [x - g(x)]^2}$

D. none of these

Answer: C

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30. If g is the inverse of a function f and f ' (x) = $\frac{1}{1+x^n}$, Then g ' (x) i

equal to

A. $\frac{-1}{6}$

B. 6

C. $\frac{-1}{3}$

D. non-existent

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31. If $y = $ ((sin^ (−1)x)) `, prove that (1 − x^ 2) d 2 y d x 2 − x d y d x = 0

A. x+y

B. 1+xy

C. 1-xy

D. xy-2

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32. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\infty}}}, then \frac{dy}{dx} is$ $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

A. $\frac{x}{2y-1}$

B. $\frac{x}{2y+1}$

C. $\frac{1}{x(2y-1)}$

D. $\frac{1}{x(1-2y)}$

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33. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right] is$

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$ (c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

A. $\frac{1}{2(1+x)\sqrt{x}}$

B. $\frac{3}{(1+x)\sqrt{x}}$

C. $\frac{2}{(1+x)\sqrt{x}}$

D. $\frac{3}{2(1+x)\sqrt{x}}$

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34. Suppose the function $f(x)-f(2x)$ has the derivative 5 at $x=1$ and derivative 7 at $x=2$. The derivative of the function $f(x)-f(4x)$ at x=1 has the value equal to

(a) 19 (b) 9 (c) 17 (d) 14

A. 19

B. 9

C. 17

D. 14

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35. answeer the following : (i) if $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that ,$(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

A. 0

B. 1

C. 2

D. none of these

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36.

Let $e^y = \dfrac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}$ and $\tan\dfrac{x}{2} = \sqrt{\dfrac{1-\alpha}{1+\alpha}}, \alpha \in [-1, 0] \cup (0, 1$

A. $1/2$

B. 1

C. 2

D. $1/3$

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37. or,find the derivatives of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x=0.

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

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38. If $\ln\left((e-1)e^{xy}+x^2\right)=x^2+y^2$ then $\left(\frac{dy}{dx}\right)_{1,0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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39. If $y=x^{(x^x)}$ then $\frac{dy}{dx}$ is

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40. The first derivative of the function $\left[\cos^{-1}\left(s \in \sqrt{\frac{1+x}{2}}\right) + x^x\right]$ with respect to x at $x = 1$ is $3/4$ (b) 0 (c) $1/2$ (d) $-1/2$

A. $3/4$

B. 0

C. $1/2$

D. $-1/2$

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41. Let $f(x) = x^x, x \in (0, \infty)$ and let g(x) be inverse of f(x), then g'(x) must be

A. $x(1 + \log x)$

B. $x(1 + \log(x))$

C. $\dfrac{1}{x(1 + \log g(x))}$

D. non-existent

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42. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to (a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

A. n(n-1) y

B. n(n+1)y

C. ny

D. n^2y

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43. If $y = ax^{n+1} + bx^{-n}, thenx^2 \frac{d^2y}{dx^2}$ is equal to (a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

A. $m^2(ae^{mx} - be^{-mx})$

B. 1

C. 0

D. none of these

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44. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b,$ and that $f'' (x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of $\frac{|ab|}{3}$ is ___

A. 25

B. 9

C. -15

D. -9

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45. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x) isequa < o$ $2^{20}\left(\cos 2x - 2^{20} os 3x\right)$ $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$ $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$ $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

A. $2^{20}\left(\cos 2x - 2^{20}\cos 3x\right)$

B. $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$

C. $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$

D. $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

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46. $\frac{d^n}{dx^n}(\log x) = ?$ (a) $\frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ (c) $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

A. $\frac{(n-1)!}{x^n}$

B. $\frac{n!}{x^n}$

C. $\frac{(n-2)!}{x^n}$

D. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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47. The nth derivative of the function $f(x) = \frac{1}{1-x^2}[where x \in (-1, 1)]$ at the point $x = 0$ where n is even is (a) 0 (b) $n!$ (c) n^nC_2 (d) 2^nC_2

A. 0

B. $n!$

C. $b^n C_2$

D. $2^n C_2$

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48. If $y = x \log\left\{\frac{x}{(a+bx)}\right\}$, then show that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

A. $x\frac{dy}{dx} - y$

B. $\left(x\frac{dy}{dx} - y\right)^2$

C. $y\frac{dy}{dx} - x$

D. $\left(y\frac{dy}{dx} - x\right)^2$

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49. If $ax^2+2hxy+by^2=1, then \frac{d^2y}{dx^2}$ is $\frac{h^2-ab}{(hx+by)^2}$ (b) $\frac{ab-h^2}{(hx+by)^2}$ $\frac{h^2+ab}{(hx+by)^2}$ (d) none of these

A. $\frac{h^2-ab}{(hx+by)^3}$

B. $\frac{ab-h^2}{(hx+by)^2}$

C. $\frac{h^2+ab}{(hx+by)^2}$

D. none of these

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50. If $y^{\frac{1}{m}}=\left(x+\sqrt{1+x^2}\right), then\left(1+x^2\right)y_2+xy_1$ is (where y_r represents the rth derivative of y w.r.t. $x\cdot m^2y$ (b) my^2 (c) m^2y^2 (d) none of these

A. m^2y

B. my^2

C. m^2y^2

D. none of these

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51. If $(\sin x)(\cos y) = \frac{1}{2}, then \frac{d^2y}{dx^2} at \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -4 (b) -2 (c) -6 (d) 0

A. -4

B. -2

C. -6

D. 0

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52. A function f satisfies the condition $f(x) = f'(x) + f''(x) + f'''(x)....$, where $f(x)$ is an indefinitely differentiable function and dash denotes the order of derivatives. If $f(0) = 1$,then $f(x)$ is

(a)$e^{\frac{x}{2}}$ (b) e^{x} (c) e^{2x} (d) e^{4x}

A. $e^{x/2}$

B. e^{x}

C. e^{2x}

D. e^{4x}

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53. Let $f(x)$ be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f''(3) = 0, and f'''(3) = 12.$ Then the value of $f'(1)$ is

(a)12 (b)23 (c) -13 (d) none of these

A. 12

B. 23

C. -13

D. none of these

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54. If $y^2 = ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is (a) a constant (b) a function of x only (c) a function of y only (d) a function of x and y

A. a constant

B. a function of x only

C. a function of y only

D. a function of x and y

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55. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is

(a) e^x

(b) $-\frac{e^x}{(1+e^x)^3}$

(c) $-\frac{e^x}{(1+e^x)^2}$

(d) $\frac{-1}{(1+e^x)^3}$

A. $(-\sin x + e^x)^{-1}$

B. $\frac{\sin x - e^x}{(\cos x + e^x)^2}$

C. $\frac{\sin x - e^x}{(\cos x + e^x)^3}$

D. $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

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56. if y= sin mx, them the value of the determinant

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} \text{ Where } y_n = \frac{d^n y}{dx^n} \text{ is}$$

A. 1

B. 0

C. -1

D. none of these

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57. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5$, then $F(10)$ is (a)5 (b) 10 (c) 0 (d) 15

A. 5

B. 10

C. 0

D. 15

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58. Let $y = 1n(1 + \cos x)^2$. Then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$ equal (a) 0 (b) $\frac{2}{1 + \cos x}$ (c) $\frac{4}{1 + \cos x}$ (d) $\frac{-4}{(1 + \cos \mathrm{x})^2}$

A. 0

B. $\frac{2}{1 + \cos x}$

C. $\frac{4}{1 + \cos x}$

D. $\frac{-4}{(1 + \cos x)^2}$

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59. If $x = t\cos t, y = t + \sin t$. Then $\frac{d^2x}{dy^2}$ at $t = \frac{\pi}{2}$ is

(a) $\frac{\pi + 4}{2}$ (b) $-\frac{\pi + 4}{2}$ (c) -2 (d) none of these

A. $\frac{\pi + 4}{2}$

B. $-\frac{\pi + 4}{2}$

C. -2

D. none of these

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60. If $f(x) = (x-1)^4(x-2)^3(x-3)^2(x-4)$, then the value of $f'''(1) + f''(2) + f'(3) + f'(4)$ equals

A. 0

B. 50

C. 324

D. 648

Answer: 648

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61. let $y = t^{10} + 1$, and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is

A. $\frac{5}{2}t$

B. $20t^8$

C. $\frac{5}{16t^6}$

D. none of these

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62. If x = log p and $y = \frac{1}{p}$, then

A. $\frac{d^2y}{dx^2} - 2p = 0$

B. $\frac{d^2y}{dx^2} + y = 0$

C. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

D. $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

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63. If $x = \sin t, y = \tan t, \quad \text{then } \frac{dy}{dx}$ is

A. $\cos^3 t$

B. $\frac{1}{\cos^3 t}$

C. $\sin^2 t$

D. $\frac{1}{\sin^2 t}$

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64. If y=$x - x^2 + x^3 - x^4 + \ldots.$. then x is equal to` is

A. $\dfrac{1-y}{y}$

B. $\dfrac{1+y}{y}$

C. $\dfrac{y}{1+y}$

D. $\dfrac{y}{1-y}$

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65. If graph of $y = f(x)$ is symmetrical about the y-axis and that of $y = g(x)$ is symmetrical about the origin and if $h(x) = f(x)g(\dot{x}), then \dfrac{d^3h(x)}{dx^3}$ at x=0 is (a)cannot be determined (b) $f(0)g(\dot{0})$ (c)0 (d) none of these

A. cannot be determined

B. $f(0) \cdot g(0)$

C. 0

D. none of these

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66. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g^{f(x)}$ equals. $-\frac{f^x}{(f'(x))^3}$ (b) $\frac{f'(x)f^x-(f'(x))^3}{f'(x)}$ $\frac{f'(x)f^x-(f'(x))^2}{(f'(x))^2}$ (d) none of these

A. $-\frac{f''(x)}{(f'(x))^3}$

B. $\frac{f'(x)f''(x)-(f(x))^3}{f'(x)}$

C. $\frac{f'(x)f''(x)-(f'(x))^2}{(f'(x))^2}$

D. none of these

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67. $f(x) = e^{x} - e^{-x} - 2\sin x - \frac{2}{3}x^{3}$. Then the least value of n for which $\frac{d^n}{dx^n} f(x) \Big|_{x=0}$ is nonzero is a. 5 b. 6 c. 7 d. 8

A. 5

B. 6

C. 7

D. 8

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68. Let $y = f(x)$ and $x = \frac{1}{z}$. If $\frac{d^2y}{dx^2} = \lambda(z^3)\frac{dy}{dz} + z^4\frac{d^2y}{dz^2}$, then the value of λ is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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69. Let x=f(t) and y=g(t), where x and y are twice differentiable function. If f'(0)= g'(0) =f''(0) = 2. g''(0) = 6, then the value of $\left(\frac{d^2y}{dx^2}\right)_{t=0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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70. If $f(x)$ satisfies the relation $f\left(\frac{5x-3y}{2}\right)=\frac{5f(x)-3f(y)}{2}\forall x, y \in R,$ and $f(0)=3andf'(0)=2,$ then the period of $\sin(f(x))$ is 2π (b) π (c) 3π

(d) 4π

A. 2π

B. π

C. 3π

D. 4π

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71. If $y^2+b^2=2xy, then \frac{dy}{dx} equals$

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72. If $y = a\sin x + b\cos x, then y^2 + \left(\frac{dy}{dx}\right)^2$ is a function of x (b) function of$y$ function of $x and y$ (d) constant

A. function of x

B. function of y

C. function of x and y

D. constant

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73. find $\frac{dy}{dx}$, $\left(\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\right)$ is equal to, $0 < x < \frac{\pi}{2}$

A. $\sec^2 x$

B. $-\sec^2\left(\frac{\pi}{4} - x\right)$

C. $\sec^2\left(\frac{\pi}{4} + x\right)$

D. $\sec^2\left(\frac{\pi}{4} - x\right)$

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74. If $y = |\cos x| + |\sin x|, then\left(\frac{dy}{dx}\right) \text{at} x = \frac{2\pi}{3}$ is

A. $\frac{1 - \sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}\left(\sqrt{3} - 1\right)$

D. none of these

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75. If f(x) $= |\log_e |x||,$ then f'(x) equals

A. $\frac{1}{|x|}$, where $x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

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76. if f(x) = $\sqrt{1 - \sin 2x}$, then f'(x) is equal to

A. $-(\cos x + \sin x)$, for x $\in (\pi/4, \pi/2)$

B. cosx + sin x for x $\in (0, \pi/4)$

C. $-(\cos x + \sin x)$, for $x \in (0, \pi/4)$

D. $\cos x - \sin x$, for $x \in (\pi/4, \pi/2)$

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77. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D^{\cdot}F(x)$ by the formula $D\cdot(x) = (\lim)_{h \vec{0}} \frac{f^2(x+h) - f^2(x)}{h}, where f^2(x)$ mean $[f(x)]^2 and \Leftrightarrow (x) = x \log x, then D^{\cdot} f(x)(\mid)_{x=e}$ has the value e (B) 2e (c) 4e (d) none of these

A. e

B. 2e

C. 4e

D. none of these

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78. if $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{\sqrt{a-x} + \sqrt{x-b}}$, then $\frac{dy}{dx}$ wherever it is defined is

A. $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$

B. $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$

C. $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

D. $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

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79. The derivative of $y=(1-x)(2-x)\ldots.(n-x)$ at $x=1$ is

(a)0 (b) $(-1)(n-1)!$ (c)$n!-1$ (d) $(-1)^{n-1}(n-1)!$

A. 0

B. $(-1)(n-1)!$

C. $n!-1$

D. $(-1)^{n-1}(n-1)!$

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80. If $y=\sqrt{\frac{1-x}{1+x}}$, then $(1-x^2)\frac{dy}{dx}$ is equal to

A. y^2

B. $1/y$

C. $-y$

D. $-y/x$

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81. If $y=\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}, then \frac{dy}{dx} isequa < o$ $\frac{ay}{x\sqrt{a^2-x^2}}$ (b) $\frac{ay}{\sqrt{a^2-x^2}}$ $\frac{ay}{x\sqrt{a^2-x^2}}$ (d) none of these

A. $\frac{ay}{x\sqrt{a^2-x^2}}$

B. $\frac{ay}{\sqrt{a^2-x^2}}$

C. $\frac{ay}{x\sqrt{x^2-a^2}}$

D. none of these

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82. Let $u(x)$ and $v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7$. If $\frac{u'(x)}{v'(x)} = p$ and $\left(\frac{u'(x)}{v'(x)}\right) = q$, then $\frac{p+q}{p-q}$ has the value of (a) 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

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83. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log a, then \frac{dy}{dx}$ is equal to (a) $\frac{x}{y}$ (b) $\frac{y}{x^2}$ (C) $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2-y^2}{x^2+y^2}$

D. $\frac{y}{x}$

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84. Let $h(x)$ be differentiable for all x and let $f(x)=(kx+e^x)h(x)$, where k is some constant. If $h(0)=5, h'(0)=-2, and f'(0)=18,$ then the value of k is 5 (b) 4 (c) 3 (d) 2.2.

A. 5

B. 4

C. 3

D. 2.2

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85. $If \lim_{t \to x} \frac{e^t f(x) - e^x f(t)}{(t-x)(f(x))^2} = 2$ and $f(0) = \frac{1}{2}$, then find the value of $f'(0)$.

A. 4

B. 2

C. 0

D. 1

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86. If $f(0)=0, f'(0)=2,$ then the derivative of $y=f(f(f(x)))$ at $x=0$ is 2 (b) 8 (c) 16 (d) 4

A. 2

B. 8

C. 16

D. 4

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87. If $f(x)=\sqrt{1+\cos^2(x^2)}, then f'\left(\frac{\sqrt{\pi}}{2}\right)$ is (a) $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\pi/6}$ (c) $1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

A. $\sqrt{\pi/6}$

B. $-\sqrt{\pi/6}$

C. $1/\sqrt{6}$

D. $\pi / \sqrt{6}$

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88. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x} is equa < o$ $\frac{1}{2}\sqrt{1+secx}$ (b) $\sqrt{1+\sec x}$ $-\frac{1}{2}\sqrt{1+\sec x}$ (d) $-\sqrt{1+\sec x}$

A. $\frac{1}{2}\sqrt{1+\sec x}$

B. $\sqrt{1+\sec x}$

C. $-\frac{1}{2}\sqrt{1+\sec x}$

D. $-\sqrt{1+\sec x}$

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89. If $tany = \frac{2^x}{1+2^{2x+1}}, then \frac{dy}{dx} at x = 0 is$

A. 1

B. 2

C. In 2

D. $-\frac{1}{10} In 2$

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90. If $y = \left(x + \sqrt{x^2 + a^2}\right)^n, then \frac{dy}{dx}$ is $\frac{ny}{\sqrt{x^2 + a^2}}$ (b) $-\frac{ny}{\sqrt{x^2 + a^2}}$ $\frac{nx}{\sqrt{x^2 + a^2}}$ (d) $-\frac{nx}{\sqrt{x^2 + a^2}}$

A. $\frac{ny}{\sqrt{x^2 + a^2}}$

B. $-\frac{ny}{\sqrt{x^2 + a^2}}$

C. $\frac{nx}{\sqrt{x^2 + a^2}}$

D. $-\frac{nx}{\sqrt{x^2 + a^2}}$

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91. If $y=(\log)_{\sin x}(\tan x), then\left(\left(\frac{dy}{dx}\right)\right)_{\frac{\pi}{4}} isequa < o$ $\frac{4}{\log 2}$ (b) $-4\log 2$ $\frac{-4}{\log 2}$ (d) none of these

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. none of these

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92. Evaluate, $\frac{d}{dx}\left[\sin^2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ is

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

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93. The differential coefficient of $f(\log_e x)$ with respect to x, where $f(x) = \log_e x$, is :

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x\log_e x}$

D. none of these

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94. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$, $then \frac{dy}{dx} at x = 1$ is (a)2 (b) 1 (c) -2 (d) none of these

A. 2

B. 1

C. -2

D. none of these

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95. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2} x \cos x^3 \cos ecx^2$ $\frac{2}{3} \sin x^3 \sec x^2$ $\tan x$ (d) none of these

A. $\frac{3}{2} x \quad \cos \quad x^2 \quad \text{cosec} \quad x^2$

B. $\frac{3}{2} \quad \sin \quad x^3 \quad \sec \quad x^2$

C. $\tan x$

D. none of these

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96. A function f, defined for all positive real numbers, satisfies the equation $f\left(x^2\right) = x^3$ for every $x > 0$. Then the value of $f'(4)$ is 12 (b) 3 (c) $3/2$ (d) cannot be determined

A. 12

B. 3

C. $3/2$

D. cannot be determined

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97. Let $f:(-5, 5) \to R$ be a differentiable function of with $f(4) = 1, f'(4) = 1, f(0) = -1$ and $f'(0) = 1 If f, g(x) = \left(f\left(2f^2(x) + 2\right.\right.$

then g'(0) equals

A. 4

B. -4

C. 8

D. -8

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98. The function $f(x) = e^x + x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}(f^{-1})$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{In2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. none of these

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99. If $f(x) = x + \tan x$ and f is the inverse of g, then $g'(x)$ is equal to

A. $\dfrac{1}{1 + [g(x) - x]^2}$

B. $\dfrac{1}{2 - [g(x) - x]^2}$

C. $\dfrac{1}{2 + [g(x) - x]^2}$

D. none of these

Answer: C

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100. If $f(x) = x^3 + 3x + 4$ and g is the inverse function of f(x), then the value of $\dfrac{d}{dx}\left(\dfrac{g(x)}{g(g(x))}\right)$ at x = 4 equals

A. $\dfrac{-1}{6}$

B. 6

C. $\dfrac{-1}{3}$

D. non-existent

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101. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}, then\left(1-x^2\right)\frac{dy}{dx}$ is equal to (a)$x+y$ (b) $1+xy$ (c) $1-xy$ (d) $xy-2$

A. x+y

B. 1+xy

C. 1-xy

D. xy-2

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102. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\infty}}}, then \frac{dy}{dx} is$ $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

A. $\frac{x}{2y-1}$

B. $\frac{x}{2y+1}$

C. $\frac{1}{x(2y-1)}$

D. $\frac{1}{x(1-2y)}$

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103. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right] is$

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$ (c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

A. $\frac{1}{2(1+x)\sqrt{x}}$

B. $\frac{3}{(1+x)\sqrt{x}}$

C. $\frac{2}{(1+x)\sqrt{x}}$

D. $\frac{3}{2(1+x)\sqrt{x}}$

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104. Suppose the function $f(x)-f(2x)$ has the derivative 5 at $x=1$ and derivative 7 at $x=2$. The derivative of the function $f(x)-f(4x)$ at x=1 has the value equal to

(a) 19 (b) 9 (c) 17 (d) 14

A. 19

B. 9

C. 17

D. 14

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105. answeer the following : (i) if $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$, show that ,$\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$

A. 0

B. 1

C. 2

D. none of these

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106.

Let $e^y=\frac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}$ and $\tan\frac{x}{2}=\sqrt{\frac{1-\alpha}{1+\alpha}}, \alpha\in[-1,0]\cup(0,1$

A. $1/2$

B. 1

C. 2

D. $1/3$

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107. or,find the derivatives of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x=0.

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

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108. If $\log_e((e-1)e^{xy}+x^2)=x^2+y^2, then\frac{dy}{dx}$ at (1, 0) is equal to

A. 0

B. 1

C. 2

D. 3

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109. If $y = x^{(x^x)}$ then $\frac{dy}{dx}$ is

A. $y[x^x(\log ex)\log x + x^x]$

B. $y[x^x(\log ex)\log x + x]$

C. $y\left[x^x(\log ex)\log x + x^{-1}\right]$

D. $y\left[x^x(\log_e x)\log x + x^{-1}\right]$

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110. The first derivative of the function $\left[\cos^{-1}\left(s \in \sqrt{\frac{1+x}{2}}\right)+x^{x}\right]$

with respect to x at $x=1$ is $3/4$ (b) 0 (c) $1/2$ (d) $-1/2$

A. $3/4$

B. 0

C. $1/2$

D. $-1/2$

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111. Let $f(x)=x^{x}, x \in (0, \infty)$ and let g(x) be inverse of f(x), then g'(x) must be

A. $x(1+\log x)$

B. $x(1+\log(x))$

C. $\dfrac{1}{x(1+\log g(x))}$

D. non-existent

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112. If $y = ax^n + bx^{-n}$,then

$x^2 \frac{d^2y}{dx^2}$ is equal to

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113. If $y = ax^{n+1} + bx^{-n}, thenx^2 \frac{d^2y}{dx^2}$ is equal to (a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

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114. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b,$ and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of $\frac{|ab|}{3}$ is ___

A. 25

B. 9

C. -15

D. -9

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115. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x) is equa < o$ $2^{20}(\cos 2x - 2^{20} os 3x)$
$2^{20}(\cos 2x + 2^{20}\cos 4x)$ $2^{20}(\sin 2x + 2^{20}\sin 4x)$
$2^{20}(\sin 2x - 2^{20}\sin 4x)$

A. $2^{20}(\cos 2x - 2^{20}\cos 3x)$

B. $2^{20}(\cos 2x + 2^{20}\cos 4x)$

C. $2^{20}(\sin 2x + 2^{20}\sin 4x)$

D. $2^{20}(\sin 2x - 2^{20}\sin 4x)$

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116. $\frac{d^n}{dx^n}(\log x) = ?$ (a) $\frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ (c) $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

A. $\frac{(n-1)!}{x^n}$

B. $\frac{n!}{x^n}$

C. $\frac{(n-2)!}{x^n}$

D. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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117. If $y = x\log\left(\frac{x}{a+bx}\right)$, then $x^3\frac{d^2y}{dx^2} =$

A. $x\frac{dy}{dx} - y$

B. $\left(x\frac{dy}{dx} - y\right)^2$

C. $y\frac{dy}{dx}-x$

D. $\left(y\frac{dy}{dx}-x\right)^2$

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118. If $ax^2+2hxy+by^2=1, then \frac{d^{2y}}{dx^2}$ is $\frac{h^2-ab}{(hx+by)^2}$ (b) $\frac{ab-h^2}{(hx+by)^2}$ $\frac{h^2+ab}{(hx+by)^2}$ (d) none of these

A. $\frac{h^2-ab}{(hx+by)^3}$

B. $\frac{ab-h^2}{(hx+by)^2}$

C. $\frac{h^2+ab}{(hx+by)^2}$

D. none of these

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119. If $y^{1/m} = \left(x + \sqrt{1+x^2}\right)$, then $(1+x^2)y_2 + xy_1$ is (where y_r represents the rth derivative of y w.r.t. x)

A. m^2y

B. my^2

C. m^2y^2

D. none of these

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120. If (sin x) (cos y) = $1/2$, then d^2y/dx^2 at $(\pi/4, \pi/4)$ is

A. -4

B. -2

C. -6

D. 0

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121. A function f satisfies the condition $f(x) = f'(x) + f''(x) + f'''(x)....$, where $f(x)$ is an indefinitely differentiable function and dash denotes the order of derivatives. If $f(0) = 1$,then $f(x)$ is

(a)$e^{\frac{x}{2}}$ (b) e^x (c) e^{2x} (d) e^{4x}

A. $e^{x/2}$

B. e^x

C. e^{2x}

D. e^{4x}

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122. Let $f(x)$ be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f''(3) = 0, and f'''(3) = 12$. Then the value of $f'(1)$ is

(a)12 (b)23 (c) -13 (d) none of these

A. 12

B. 23

C. -13

D. none of these

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123. If $y^2 = ax^2 + bx + c, \quad \text{then} \quad y^3 \frac{d^2y}{dx^2}$ is

A. a constant

B. a function of x only

C. a function of y only

D. a function of x and y

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124. If $y = \cos x + e^{4x}$, then $\frac{d^2y}{dx^2} =$

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125. If $''(x) = -f(x)$ and $g(x) = -f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)$

and given that f(5)=5, then f(10) is equal to

A. 5

B. 10

C. 0

D. 15

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126. Let $y=\ln(1+\cos x)^2$. Then the value of $\frac{d^2y}{dx^2}+\frac{2}{e^{\frac{y}{2}}}$ equal

(a)0

(b) $\frac{2}{1+\cos x}$

(c) $\frac{4}{1+\cos x}$

(d) $\frac{-4}{(1+\cos \mathrm{x})^2}$

A. 0

B. $\frac{2}{1+\cos x}$

C. $\frac{4}{1+\cos x}$

D. $\dfrac{-4}{(1+\cos x)^2}$

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127. If $x = t\cos t, y = t + \sin t$. Then $\dfrac{d^2x}{dy^2}$ at $t = \dfrac{\pi}{2}$ is

(a) $\dfrac{\pi+4}{2}$ (b) $-\dfrac{\pi+4}{2}$ (c) -2 (d) none of these

A. $\dfrac{\pi+4}{2}$

B. $-\dfrac{\pi+4}{2}$

C. -2

D. none of these

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128. Let $y = t^{12} + 1$ and $x = t^6 + 1$. Then $\dfrac{d^2y}{dx^2}$ is

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129. If x = log p and $y = \frac{1}{p}$, then

A. $\frac{d^2y}{dx^2} - 2p = 0$

B. $\frac{d^2y}{dx^2} + y = 0$

C. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

D. $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

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130. If $x = \varphi(t), y = \psi(t), then \frac{d^2y}{dx^2}$ is (a) $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^2}$ (b) $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^3}$ (c) $\frac{\varphi''}{\psi''}$ (d) $\frac{\psi''}{\varphi''}$

A. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$

B. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$

C. $\dfrac{\phi''}{\psi''}$

D. $\dfrac{\psi''}{\phi''}$

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131. If $f(x) = x^4 \tan(x^3) - x 1n(1 + x^2)$, then the value of $\dfrac{d^4(f(x))}{dx^4}$ at $x = 0$ is 0 (b) 6 (c) 12 (d) 24

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132. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g^{f(x)}$ equals. $-\dfrac{f^x}{(f'(x))^3}$ (b) $\dfrac{f'(x)f^x - (f'(x))^3}{f'(x)}$ $\dfrac{f'(x)f^x - (f'(x))^2}{(f'(x))^2}$ (d) none of these

A. $-\dfrac{f''(x)}{(f'(x))^3}$

B. $\dfrac{f'(x)f''(x) - (f(x))^3}{f'(x)}$

C. $\frac{f'(x)f''(x)-(f'(x))^2}{(f'(x))^2}$

D. none of these

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133. $f(x)=e^x-e^{-x}-2\sin x-\frac{2}{3}x^3$. Then the least value of n for which $\frac{d^n}{dx^n}f(x)\,|_{x=0}$ is nonzero is a. 5 b. 6 c. 7 d. 8

A. 5

B. 6

C. 7

D. 8

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134. Let $y = f(x)$ and $x = \frac{1}{z}$. If $\frac{d^2y}{dx^2} = \lambda_{z^3}\frac{dy}{dz} + z^4\frac{d^2y}{dz^2}$, then the value of λ is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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135. Let x=f(t) and y=g(t), where x and y are twice differentiable function. If f'(0)= g'(0) =f''(0) = 2. g''(0) = 6, then the value of $\left(\frac{d^2y}{dx^2}\right)_{t=0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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136. If $f(x)$ satisfies the relation $f\left(\frac{5x-3y}{2}\right)=\frac{5f(x)-3f(y)}{2}\forall x, y \in R,$ and $f(0)=3 and f'(0)=2,$ then the period of $\sin(f(x))$ is 2π (b) π (c) 3π (d) 4π

A. 2π

B. π

C. 3π

D. 4π

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137. A function $f: R \to R$ satisfies $\sin x \cos y(f(2x+2y) - f(2x-2y)) = \cos x \sin y(f(2x+2y) + f(2x-2y)$

If $f'(0) = \frac{1}{2}, then$ (a)$f''(x) = f(x) = 0$ (b)$4f''(x) + f(x) = 0$ (c) $f''(x) + f(x) = 0$ (d)$4f''(x) - f(x) = 0$

A. $f''(x) = f(x) = 0$

B. $4f''(x) + f(x) = 0$

C. $f''(x) + f(x) = 0$

D. $4f''(x) - f(x) = 0$

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Multiple Correct Answers Type

1. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to (a)$\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$ (c) $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$ (d) $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

A. $\frac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2\sqrt{x}}$

B. $\frac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2x}$

C. $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$

D. $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

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2. Let $y=\sqrt{x+\sqrt{x+\sqrt{x+\infty}}}$, $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2y-1}$

(b) $\frac{x}{x+2y}$

(c) $\frac{1}{\sqrt{1+4x}}$

(d) $\frac{y}{2x+y}$

A. $\frac{1}{2y-1}$

B. $\frac{x}{x+2y}$

C. $\frac{1}{\sqrt{1+4x}}$

D. $\frac{y}{2x+y}$

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3. $If \sin\theta$ = $n\sin(\theta+2\alpha)$ then $\tan(\theta+\alpha)$=

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4. $f(x)=\left|x^2-3|x|+2\right|$. Then which of the following is/are true

$f'(x)=2x-3$ for x in $(0,1)\cup(2,\infty)$

$f'(x)=2x+3$ for x in $(-\infty,-2)\cup(-1,0)$

$f'(x)=-2x-3$ for x in (-2,-1)`

None of these

A. $f'(x) = 2x - 3 \text{ for } x \in (0, 1) \cup (2, \infty)$

B. $f'(x) = 2x + 3f \text{ or } x \in (-\infty, -2) \cup (-1, 0)$

C. $f'(x) = -2x - 3 \text{ for } x \in (-2, -1)$

D. None of these

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5. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1} x$. Then $f'(10 = 1 \quad f'\left(\frac{3}{2}\right) = -1$

$doma \in off(x)isx \geq 1$ range of $f(x)is(-2-1) \cup (2, \infty)$

A. $f(10) = 1$

B. $f'(3/2) = -1$

C. domain of $f(x)isx \geq 1$

D. range of $f(x)is(-2, -1)] \cup (2, 00)$

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6. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0 and y(1) = 1, then \; y'(1) = \frac{4}{3}$ (b) $y^1 = -\frac{4}{3}$ $y^1 = -8\frac{22}{27}$ (d) $y'(1) = \frac{2}{3}$

A. $y'(1) = 4/3$

B. $y''(1) = -4/3$

C. $y''(1) = -8\frac{22}{27}$

D. $y'(1) = 2/3$

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7. If $y = x^{(\log x)}$ then $\frac{dy}{dx} =$

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8. If $f(x-y), f(x)f(y)$ and $f(x+y)$ are in A.P. for all $x, y,$ and $f(0) \neq 0,$ then (a) $f(4) = f(-4)$ (b) $f(2) + f(-2) = 0$ (c) $f'(4) + f'(-4) = 0$ (d) $f'(2) = f'(-2)$

A. $f(4) = f(-4)$

B. $f(2) + f(-2) = 0$

C. $f'(4) + f'(-4) = 0$

D. $f'(2) = f'(-2)$

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9. If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), then \frac{dy}{dx} is$

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10.

$f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is

A. $f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$

B. $f_n(x)f_{n-1}(x)$

C. $f_n(x)f_{n-1}(x)...f_2(x).\ f_1(x)$

D. None of these

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11. Let $f(x) and g(x)$ be two function having finite nonzero third-order derivatives $f'''(x)$ and $g'''(x)$ for all $x \in R$. If $f(x).\ g(x) = 1$ for all $x \in R$, then prove that $\frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} - \frac{g''}{g}\right)$.

A. $\frac{-2f'(x)}{f(x)}$

B. $\frac{-2g'(x)}{g(x)}$

C. $\frac{-f'(x)}{f(x)}$

D. $\frac{2f'(x)}{f(x)}$

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12. If $y = e^{-x} \cos x and y_n + k_n y = 0, where y_n = \frac{d^n y}{dx^n} and k_n$ are constants $\forall n \in N$, then $k_4 = 4$ (b) $k_8 = -16$ $k_{12} = 20$ (d) $k_{16} = -24$

A. $k_4 = 4$

B. $k_8 = -16$

C. $k_{12} = 20$

D. $k_{16} = -24$

Answer: B

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13. If a function is represented parametrically be the equations $x = \frac{1 + (\log)_e t}{t^2}; y = \frac{3 + 2(\log)_e t}{t}$, then which of the following statements are true?

A. $y''(x - 2xy') = y$

B. $yy' = 2x(y')^2 + 1$

C. $xy' = 2y(y')^2 + 2$

D. $y''(y - 4xy') = (y')^2$

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14. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2+1} + (\log)_e\sqrt{x + \sqrt{x^2+1}}$, prove that $2y = xy' + (\log)_e y', where y'$ denotes the derivative w.r.t x.

A. $y' = x + \sqrt{x^2+1}$

B. $y' = \frac{1}{x + \sqrt{x^2+1}}$

C. $2y = xy' + \log_e y'$

D. $2y = xy' - \log_e y'$

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15. A curve parametrically given by $x = t + t^3$ and $y = t^2$, where $t \in R$. For what vlaue(s) of t is $\frac{dy}{dx} = \frac{1}{2}$?

A. $1/3$

B. 2

C. 3

D. 1

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16. If y=$(\sin x)^{x}$,then $\frac{dy}{dx}$=

A. $\frac{1}{3\sqrt{\pi}}$

B. $-\frac{1}{3\sqrt{\pi}}$

C. $-\frac{1}{5\sqrt{\pi}}$

D. $-\frac{1}{3\sqrt{5\pi}}$

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17. If g is the inverse of a function $f$and $f(x)=\frac{1}{1+x^5}$, Then $g'(x)$ i

equal to :

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

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18. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\dot{g}(x)$, then $g(x)$ equals:

A. $\lim_{x \to 0^+} f'\left(\frac{1}{x}\right) = 1$

B. $\lim_{x \to 0^+} x f\left(\frac{1}{x}\right) = 2$

C. $\lim_{x \to 0^+} x^2 f'(x) = 0$

D. $|f(x)| \leq 2 \quad \text{for all} \quad x \in (0, 2)$

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19. Let $f(x) = x\sin \pi x,\ x > 0$. Then for all natural numbers $n,\ f'(x)$ vanishes at (a) A unique point in the interval $\left(n,\ n+\frac{1}{2}\right)$ (b) a unique point in the interval $\left(n+\frac{1}{2},\ n+1\right)$ (c) a unique point in the interval $(n,\ n+1)$ (d) two points in the interval $(n,\ n+1)$

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

Answer: A and C

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20. Let $f:(0, \infty) \rightarrow R$ be a differentiable function such that $f'(x)=2-\frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1)=1$, then

A. $\lim_{x \rightarrow 0^+} f'\left(\frac{1}{x}\right)=1$

B. $\lim_{x \rightarrow 0^+} x f\left(\frac{1}{x}\right)=2$

C. $\lim_{x \rightarrow 0^+} x^2 f'(x)=0$

D. $|f(x)| \leq 2$ for all $x \in (0, 2)$

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21. Let $f: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then, find the value of h'(1).

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22. If $y = e^{\sqrt{x}} - e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{e^{\sqrt{x}} + e^{-\sqrt{x}}}{2\sqrt{x}}$

B. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$

C. $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$

D. $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

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23. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots \infty}}}}$ then $\frac{dy}{dx}$ equal to

A. $\frac{1}{2y - 1}$

B. $\frac{x}{x + 2y}$

C. $\frac{1}{\sqrt{1 + 4x}}$

D. $\frac{y}{2x + y}$

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24. If $f(x - y), f(x)f(y)$ and $f(x + y)$ are in A.P. for all $x, y,$ and $f(0) \neq 0,$ then (a) $f(4) = f(-4)$ (b) $f(2) + f(-2) = 0$ (c) $f'(4) + f'(-4) = 0$ (d) $f'(2) = f'(-2)$

A. $f(4) = f(-4)$

B. $f(2) + f(-2) = 0$

C. $f'(4) + f'(-4) = 0$

D. $f'(2) = f'(-2)$

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25. If a function is represented parametrically be the equations $x = \frac{1 + (\log)_e t}{t^2}; y = \frac{3 + 2(\log)_e t}{t}$, then which of the following statements are true?

A. $y''(x - 2xy') = y$

B. $yy' = 2x(y')^2 + 1$

C. $xy' = 2y(y')^2 + 2$

D. $y''(y - 4xy') = (y')^2$

Watch Video Solution

26. If $y=\frac{x^2}{2}+\frac{1}{2}x\sqrt{x^2+1}+(\log)_e\sqrt{x+\sqrt{x^2+1}}$, prove that $2y=xy'+(\log)_e y', wherey'$ denotes the derivative w.r.t $x\cdot$

A. $y' = x+\sqrt{x^2+1}$

B. $y' = \frac{1}{x+\sqrt{x^2+1}}$

C. $2y = xy' + \log_e y'$

D. $2y = xy' - \log_e y'$

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27. A curve parametrically given by $x=t+t^3$ and $y=t^2$, where $t\in R$. For what vlaue(s) of t is $\frac{dy}{dx}=\frac{1}{2}$?

A. $1/3$

B. 2

C. 3

D. 1

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28. Let $f(x) = x\sin\pi x,\ x > 0$. Then for all natural numbers $n,\ f'(x)$ vanishes at (a) A unique point in the interval $\left(n,\ n+\frac{1}{2}\right)$ (b) a unique point in the interval $\left(n+\frac{1}{2},\ n+1\right)$ (c) a unique point in the interval $(n,\ n+1)$ (d) two points in the interval $(n,\ n+1)$

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

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29. Let $f:(0,\infty)\to R$ be a differentiable function such that $f'(x)=2-\frac{f(x)}{x}$ for all $x\in(0,\infty)$ and $f(1)=1$, then

A. $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right)=1$

B. $\lim_{x\to 0^+} x f\left(\frac{1}{x}\right)=2$

C. $\lim_{x\to 0^+} x^2 f'(x)=0$

D. $|f(x)|\leq 2 \text{ for all } x\in(0,2)$

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Linked Comprehension Type

1. f(x) is a polynomial function, $f:R\to R$, such that $f(2x)=f'(x)f''(x)$.

$f(3)$ is

A. 4

B. 12

C. 15

D. none of these

Answer: B

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2. f(x) is a polynomial function, $f: R \rightarrow R$, such that $f(2x) = f'(x)f''(x)$. f(x) is (A) one-one and onto (B) one-one and into (C) many-one and onto (D) many-one and into

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

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3. f(x) is a polynomial function, $f: R \to R$, such that $f(2x) = f'(x)f''(x)$. Equation f(x) = x has (A) three real and positive roots (B) three real and negative roots (C) one real root (D) three real roots such that sum of roots is zero

A. three real and positive roots

B. three real and negative roots

C. one real root

D. three real roots such that sum of roots is zero

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4.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$.

The value of f(1) is

A. 2

B. 3

C. -1

D. 4

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5. Let $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in R$. Then f(2) equals

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

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6. Let $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in R$. Then f(2) equals

A. 0

B. -1

C. 2

D. 3

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7. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x-\alpha)^2(x-\beta)\ldots=0$, from which we can conclude that

$f(x) = 0$ or $2(x-\alpha)[(x-\beta)\ldots] + (x-\alpha)^2[(x-\beta)\ldots]' = 0$ or $(x-\alpha)$

has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f''(x)=0. If x-c is a factor of order m of the polynomial f(x) of degree n $(1 < m < n)$, then x=c is a root of the polynomial [where $f^r(x)$ represent rth derivative of f(x) w.r.t. x]

A. $f^m(x)$

B. $f^{m-1}(x)$

C. $f^n(x)$

D. none of these

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8. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x-\alpha)^2(x-\beta)\ldots = 0,$ from which we can conclude that

$f(x) = 0$ or $2(x - \alpha)[(x - \beta)...] + (x - \alpha)^2[(x - \beta)...]' = 0$ or $(x - \alpha)$

has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f'''(x)=0.

If $a_1x^3 + b_1x^2 + c_1x + d_1 = 0$ and $a_2x^3 + b_2x^2 + c_2x + d_2 = 0$ have a pair of repeated roots common, then

$$\begin{vmatrix} 3a_1 & 2b_1 & c_1 \\ 3a_2 & 2b_2 & c_2 \\ a_2b_1 - a_1b_2 & c_1a_2 - c_2a_1 & d_1a_2 - d_2a_1 \end{vmatrix} =$$

A. 0

B. 1

C. -1

D. 2

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9. If roots of an equation $x^n - 1 = 0are1, a_{1,2}, a_{n-1},$ then the value of $(1 - a_1)(1 - a_2)(1 - a_3)(1 - a_{n-1})$ will be n b. n^2 c. n^n d. 0

A. $n^2/2$

B. n

C. $(-1)^n n$

D. none of these

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10. Equation

$x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_{n-1}.$

The value of $\sum_{n-1}^{r=1} \frac{1}{1-a_r}$ is

A. $\dfrac{2^{n-1}(n-2)+1}{2^n - 1}$

B. $\dfrac{2^n(n-2)+1}{2^n - 1}$

C. $\dfrac{2^{n-1}(n-1)-1}{2^n - 1}$

D. none of these

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11. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_{n-1}$.

The value of $\sum_{r=1}^{n-1} \frac{1}{2 - a_r}$ is

A. $\frac{n}{4}$

B. $\frac{n(n-1)}{2}$

C. $\frac{n-1}{2}$

D. none of these

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12.

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The value of f(3) is

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13.

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f'(x)$.

The value of g(0) is

A. 0

B. -3

C. 2

D. none of these

Answer: C

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14.

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f'(x)$.

Then find the function $f(x)$ and $g(x)$.

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15.

$g(x+y) = g(x) + g(y) + 3xy(x+y)\, \forall x, y \in R$ and $g'(0) = -4.$

Number of real roots of the equation g(x) = 0 is

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16. If f(x) =2x-1 and g(x)=3x+2 then find (fog) (x) a)2(3x+1) b)2(3x+2) c)(2x+1) d)3(3x+1)

A. 2(3x+1)

B. 2(3x+2)

C. 3(2x+1)

D. 3(3x+1)

Answer: [-1,1]

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17. If x+y=3 and xy=2,then the value of $x^3 - y^3$ is equal to

A. 6

B. 7

C. 8

D. 0

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18. `If p(x)=x+3,then p(x) +p(-x) is equal to:

A. 3

B. 2x

C. 0

D. 6

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19. If $x^3 + y^3 - 3axy$=0,then $\frac{dy}{dx}$ equals

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20. A curve is represented parametrically by the equations $x = f(t) = a^{In(b')}$ and $y = g(t) = b^{-In(a^t)} a, b > 0$ and $a \neq 1, b \neq 1$ $Where t \in R.$

The value of $\frac{f(t)}{f'(t)} \cdot \frac{f''(-t)}{f'(-t)} + \frac{f(-t)}{f'(-t)} \cdot \frac{f''(t)}{f'(t)} \forall t \in R$ is

A. -2

B. 2

C. -4

D. 4

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21. Let $f: R \rightarrow R$ be a differentiable function satisfying $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, then

The value of f(9) is

A. 8

B. 10

C. 12

D. 18

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22. Let $f: R \rightarrow R$ be a differentiable function satisfying $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, then

The value of f(9) is

A. 240

B. 356

C. 252

D. 730

Answer: C

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23. If roots of an equation $x^n - 1 = 0 are 1, a_{1,2}, a_{n-1},$ then the value of $(1 - a_1)(1 - a_2)(1 - a_3)(1 - a_{n-1})$ will be n b. n^2 c. n^n d. 0

A. $n^2/2$

B. n

C. $(-1)^n n$

D. none of these

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Matrix Match Type

1. Multiply the polynomials: (a + 3b) and (x + 5)

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2. Multiply the polynomials: (y – 8) and (3y – 4)

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3. Differentiate with respect to x, $y = e^{\tan x}$

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4. Match Column-I to II

Column-I	Column-II
(A) Tetragonal and Hexagonal	(P) are two crystal systems
(B) Cubic and Rhombohedral	(Q) $\alpha = \beta = \gamma$
(C) Monoclinic and Triclinic	(R) $a \neq b \neq c$
(D) Cubic and Orthorhombic	(S) $a = b = c$

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5. Match List I with List II and choose the correct answer

List I		List II	
A	Hypothalamus	1.	Sperm lysins
B	Acrosome	2.	Estrogen
C	Graafian follicle	3.	Relaxin
D	Leydig cells	4.	GnRH
E	Parturition	5.	Testosterone

A. $\begin{matrix} a & b & c & d \\ q & p & s & r \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ r & q & s & p \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ q & p & r & s \end{matrix}$

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6. Match the statements/expressions given in List (with the value given in List II.

A. $\begin{array}{cccc} a & b & c & d \\ s & r & q & p \end{array}$

B. $\begin{array}{cccc} a & b & c & d \\ q & s & r & p \end{array}$

C. $\begin{array}{cccc} a & b & c & d \\ s & r & p & q \end{array}$

D. $\begin{array}{cccc} a & b & c & d \\ q & s & p & r \end{array}$

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Numerical Value Type

1. $f'(x) = \phi(x)$ and $\phi'(x) = f(x)$ for all x. Also,$f(3) = 5$ and $f'(3) = 4$. Then the value of $[f(10)]^2 - [\phi(10)]^2$ is ____

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2. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then|f'(-3)|$ equals________.

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3. If $x^3 + 3x^2 - 9x = c$ is of the form $(x - \alpha)^2(x - \beta)$, then c is equal to 27 b. -27 c. 5 d. -5

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4. If graph of $y = f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7) = 3,$ then the value of $f'(3)$ is________

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5. Let $g(x) = f(x)\sin x, where f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. The value of $|g''(-\pi)|$ equals

____

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6. Let

$f(x) = (x-1)(x-2)(x-3)...(x-n), n \in N, and f'(n) = 5040.$

Then the value of n is________

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7. y=f(x), where f satisfies the relation

$f(x+y) = 2f(x) + xf(y) + y\sqrt{f(x)}, \forall x, y \varepsilon R$ and f'(0)=0.Then f(6)is

equal ______

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8. If function f satisfies the relation $f(x)xf'(-x) = f(-x)xf'(x)f$ or $allx, andf(0) = 3, and \Leftrightarrow (3) = 3,$ then the value of $f(-3)$ is ____________

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9. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}} andy' = 0atx = 5,$ then the value of $\frac{a^2}{b^2}$ is______

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10. Prove that $\frac{2^{(\log)_{2\frac{1}{4}x}} - 3^{(\log)_{27}(x^2+1)^3} - 2x}{7^{4(\log)_{49}x} - x - 1} > 0$

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11. $(\lim)_{h\vec{0}} \frac{(e+h)^{1n(e+h)} - e}{h} is_{-}-$

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12. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3} and g(x) = f^{-1}(x),$ then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is________

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13. Suppose that $f(0) = 0 and f'(0) = 2,$ and let $g(x) = f(-x + f(f(x)))$. The value of g' (0) is equal to _____

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14. f(x) is a polynomial of degree 4 with real coefficients such that f(x)=0 is satisfied by x=1,2,3 only then f'(1).f'(2).f'(3) is equal to :`

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15. A nonzero polynomial with real coefficient has the property that $f(x) = f'(x)\dot{f}'(x)$. If a is the leading coefficient of $f(x)$, then the value of $1/2a)$ is____

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16. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}$ then the value of $\left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is__________

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17. Let $z = (\cos x)^2$ and $y = \sin x$. Then the value of $\frac{d^2z}{dy^2} at x = \frac{\pi}{3}$ is____________.

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18. Let $g(x) = \begin{cases} \dfrac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x}, x \neq 00, x = 0 \end{cases}$ If $g'(0)$ exists and is equal to nonzero value b, then $52\dfrac{b}{a}$ is equal to ____

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19. Let $f(x) = x + \dfrac{1}{2x + \dfrac{1}{2x + \dfrac{1}{2x + \ldots\ldots\infty}}}$.

Then find the value of $f(50)\cdot f'(50)$

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20. Let $F(x) = f(x)g(x)h(x)$ for all real x, where $f(x)$, $g(x)$, and $h(x)$ are differentiable functions. At some point x_0, $F'(x_0) = 21F(x_0)$, $f'(x_0) = 4f(x_0)$, $g'(x_0) = -7g(x_0)$, and $h'(x_0) = kh(x_0)$. Then k is_______

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21. If $y = \frac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$, then y'(0) is equal to -

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22. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ then the value of $\frac{d}{d(\tan\theta)}f(\theta)$ is

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23. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1,3) is

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24. Let $f : R \to R$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x + y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in R$. Then, the value of $(\log)_e(f(4))$ is ______

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25. $f'(x) = \phi(x)$ and $\phi'(x) = f(x)$ for all x. Also, $f(3) = 5$ and $f'(3) = 4$. Then the value of $[f(10)]^2 - [\phi(10)]^2$ is ____

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27. If $x^3 + 3x^2 - 9x + \lambda$ is of the form $(x - \alpha)^2(x - \beta)$ then λ is equal to

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$f(x) = (x - 1)(x - 2)(x - 3)...(x - n), n \in N, and f'(n) = 5040.$

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38. Let $z = (\cos x)^3$ and $y = \sin x\cdot$ Then the value of $\frac{d^2z}{dy^2}atx = \frac{\pi}{6}$ is__________.

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44. The slope of the tangent to the curve $\left(y-x^5\right)^2 = x\left(1+x^2\right)^2$ at the point (1,3) is

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Archives

1. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals: a.1 b. $\log 2$ c. $-\log 2$ d. -1

A. -1

B. 1

C. log 2

D. -log 2

Answer: A

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2. Let $f: (-5, 5) \rightarrow R$ be a differentiable function of with $f(4) = 1, f'(4) = 1, f(0) = -1$ and $f'(0) = 1 If f, g(x) = \left(f\left(2f^2(x) + 2\right.\right.$

then g'(0) equals

A. -2

B. 4

C. -4

D. 0

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3. $\frac{d^2x}{dy^2}$ equals: (1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (3) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

A. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

B. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

C. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

D. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

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4. If $y = \sec(\cos^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) -1 (d) $\sqrt{2}$

A. $1/2$

B. 1

C. $\sqrt{2}$

D. $1\sqrt{2}$

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5. If g is the inverse of a function $f$and $f(x) = \dfrac{1}{1 + x^5}$, Then $g'(x)$ i

equal to :

A. $1 + x^5$

B. $5x^4$

C. $\dfrac{1}{1 + \{g(x)\}^5}$

D. $1 + \{g(x)\}^5$

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6. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g\dot{(}x)$, then $g(x)$ equals:

A. $\frac{3}{1+9x^3}$

B. $\frac{9}{1+9x^3}$

C. $\frac{3x\sqrt{x}}{1-9x^3}$

D. $\frac{3x}{1-9x^3}$

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7. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals: a.1 b. $\log 2$ c. $-\log 2$ d. -1

A. -1

B. 1

C. log 2

D. `-log 2

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8. Let $f:(-5, 5) \rightarrow R$ be a differentiable function of with $f(4) = 1, f'(4) = 1, f(0) = -1$ and $f'(0) = 1 If f, g(x) = \left(f\left(2f^2(x) + 2\right.\right.$

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A. -2

B. 4

C. -4

D. 0

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9. $\frac{d^2x}{dy^2}$ equals: (1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (3) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

A. -1

B. 1

C. log 2

D. `-log 2

Answer: A

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10. If $y = \operatorname{cosec}\left(\cot^{-1} x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

Watch Video Solution

11. If g is the inverse of a function $f$and $f(x) = \frac{1}{1+x^5}$, Then $g'(x)$ i

equal to :

A. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

B. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

C. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

D. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

Watch Video Solution

12. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g(x)$, then

$g(x)$ equals:

A. $1/2$

B. 1

C. $\sqrt{2}$

D. $1\sqrt{2}$

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13. Let $f(x)=x\sin\pi x,\ x>0$. Then for all natural numbers $n,\ f'(x)$ vanishes at (a) A unique point in the interval $\left(n,\ n+\frac{1}{2}\right)$ (b) a unique point in the interval $\left(n+\frac{1}{2},\ n+1\right)$ (c) a unique point in the interval $(n,\ n+1)$ (d) two points in the interval $(n,\ n+1)$

A. $1+x^5$

B. $5x^4$

C. $\dfrac{1}{1+\{g(x)\}^5}$

D. $1+\{g(x)\}^5$

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14. Let $f:(0,\infty)\to R$ be a differentiable function such that $f'(x)=2-\dfrac{f(x)}{x}$ for all $x\in(0,\infty)$ and $f(1)=1$, then

A. $\dfrac{3}{1+9x^3}$

B. $\dfrac{9}{1+9x^3}$

C. $\dfrac{3x\sqrt{x}}{1-9x^3}$

D. $\dfrac{3x}{1-9x^3}$

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