MATHS

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ELLIPSE

Single Correct Answer Type

1. The second degree equation $x^2 + 4y^2 - 2x - 4y + 2 = 0$ represents

A. a parabola

B. a pair of straight line

C. an ellipse

D. a hyperbola

Answer: C

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2. In the standard ellipse, the lines joining the ends of the minor axis to one focus are at right angles. The distance between the focus and the nearer vertex is $\sqrt{10}-\sqrt{5}$. The equation of the ellipse (a) $\frac{x^2}{36}+\frac{y^2}{18}=1$ (b) $\frac{x^2}{40}+\frac{y^2}{20}=1$ (c) $\frac{x^2}{20}+\frac{y^2}{10}=1$ (d) $\frac{x^2}{10}+\frac{y^2}{5}=1$

A. $\frac{x^2}{36}+\frac{y^2}{18}=1$

B. $\frac{x^2}{40}+\frac{y^2}{20}=1$

C. $\frac{x^2}{20}+\frac{y^2}{10}=1$

D. $\frac{x^2}{10}+\frac{y^2}{5}=1$

Answer: D

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3. The foci of an ellipse are $(-2,4)$ and (2,1). The point $\left(1,\frac{23}{6}\right)$ is an extremity of the minor axis. What is the value of the eccentricity?

A. $\frac{9}{13}$

B. $\frac{3}{\sqrt{13}}$

C. $\frac{2}{\sqrt{13}}$

D. $\frac{4}{13}$

Answer: B

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4. Let $Q = (3, \sqrt{5}), R = (7, 3\sqrt{5})$. A point P in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A. 6

B. 12

C. 18

D. 9

Answer: B

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5. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{3\sqrt{2}}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

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6. Area bounded by the circle which is concentric with the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ and which passes through $\left(4,\ -\frac{9}{5}\right)$, the vertical chord common to both circle and ellipse on the positive side of x-axis is

A. $\frac{481}{25}\tan^{-1}\left(\frac{9}{20}\right)-\frac{36}{5}$

B. $2\tan^{-1}\left(\frac{9}{20}\right)$

C. $\frac{481}{25}\tan^{-1}\left(\frac{9}{20}\right)$

D. none of these

Answer: A

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7. If A and B are foci of ellipse $(x-2y+3)^2+(8x+4y+4)^2=20$ and P is any point on it, then $PA+PB=$ (a) 2 (b) 4 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

A. 2

B. 4

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: B

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8. The distance between directrix of the ellipse $(4x-8)^2+16y^2=\left(x+\sqrt{3}y+10\right)^2$ is

A. 12

B. 16

C. 20

D. 24

Answer: B

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9. A chord is drawn passing through $P(2, 2)$ on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ such that it intersects the ellipse at A and B. Then maximum value of $PA.\ PB$ is (a) $\frac{61}{4}$ (b) $\frac{59}{4}$ (c) $\frac{71}{4}$ (d) $\frac{63}{4}$

A. $\frac{61}{4}$

B. $\frac{59}{4}$

C. $\frac{71}{4}$

D. $\frac{63}{4}$

Answer: B

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10. If (x, y) lies on the ellipse $x^2 + 2y^3 = 2$, then maximum value of $x^2 + y^2 + \sqrt{2}xy - 1$ is

A. $\frac{\sqrt{5}+1}{2}$

B. $\frac{\sqrt{5}-1}{2}$

C. $\frac{\sqrt{5}+1}{4}$

D. $\frac{\sqrt{5}-1}{4}$

Answer: A

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11. If the eccentric angles of two points P and Q on the ellipse $\frac{x^2}{28}+\frac{y^2}{7}=1$ whose centre is C differ by a right angle then the area of ΔCPQ is

A. 5

B. 6

C. 7

D. 8

Answer: C

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12. P and Q are points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose center is C. The eccentric angles of P and Q differ by a right angle. If $\angle PCQ$ minimum, the eccentric angle of P can be (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{12}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{12}$

Answer: B

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13. If eccentric angle of a point lying in the first quadrant on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be θ and the line joining the centre to that point makes an angle ϕ with the x-axis, then $\theta-\phi$ will be maximum when θ is equal to

A. $\tan^{-1}\sqrt{\frac{a}{b}}$

B. $\tan^{-1}\sqrt{\frac{b}{a}}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: A

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14. Let P and Q be points of the ellipse $16x^2 + 25y^2 = 400$ so that $PQ = 96/25$ and P and Q lie above major axis. Circle drawn with PQ as diameter touch major axis at positive focus, then the value of slope m of PQ is

A. -1

B. $1/2$

C. 2

D. $1/3$

Answer: A

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15. If the reflection of the ellipse $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$ in the mirror line $x-y-2=0$ is $k_1x^2+k_2y^2-160x-36y+292=0,$ then $\frac{k_1+k_2}{5}$ is equal to

A. A. 4

B. B. 5

C. C. 6

D. D. 7

Answer: B

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16. A point P moves on x-y plane such that $PS+PS'=4$ where $S(K,0)$ and $S'(-K,0),$ then which of the following is not true about the locus of P?

A. ellipse if $K\in(-2,2)$

B. pair of coincidence lines if $K = \pm 2$

C. empty if $K \in (-\infty, -2) \cup (2, \infty)$

D. none of these

Answer: D

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17. The ratio of the area enclosed by the locus of the midpoint of PS and area of the ellipse is (P-be any point on the ellipse and S, its focus)

A. A. $\frac{1}{2}$

B. B. $\frac{1}{3}$

C. C. $\frac{1}{5}$

D. D. $\frac{1}{4}$

Answer: D

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18. Find the set of those value(s) of α for which $\left(7-\frac{5\alpha}{4}, \alpha\right)$ lies inside the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$

A. 0

B. 1

C. 2

D. 3

Answer: B

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19. The coordinates of the vertices $BandC$ of a triangle ABC are (2, 0) and (8, 0), respectively. Vertex A is moving in such a way that $4\frac{\tan B}{2}\frac{\tan C}{2}=1.$ Then find the locus of A

A. $\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$

B. $\frac{(x-5)^2}{16}+\frac{y^2}{25}=1$

C. $\frac{(x-5)^2}{25}+\frac{y^2}{9}=1$

D. $\frac{(x-5)^2}{9}+\frac{y^2}{25}=1$

Answer: A

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20. PQ and QR are two focal chords of an ellipse and the eccentric angles of P,Q,R are $2\alpha, 2\beta, 2\gamma$, respectively then $\tan\beta\gamma$ is equal to

A. $\cot\alpha$

B. $\cot^2\alpha$

C. $2\cot\alpha$

D. None of these

Answer: B

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21. the value of λ for which the line $2x-\frac{8}{3}\lambda y=-3$ is a normal to the conic $x^2+\frac{y^2}{4}=1$ is:

A. $\frac{3}{8}$

B. $\frac{1}{2}$

C. $-\frac{\sqrt{3}}{2}$

D. $\frac{\sqrt{3}}{2}$

Answer: D

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22. If the length of the major axis intercepted between the tangent and normal at a point $P(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the length of semi-major axis, then eccentricity of the ellipse is

A. $\frac{\cos\theta}{\sqrt{1-\cos\theta}}$

B. $\frac{\sqrt{1-\cos\theta}}{\cos\theta}$

C. $\frac{\sqrt{1-\cos\theta}}{\sin\theta}$

D. $\frac{\sin\theta}{\sqrt{1-\sin\theta}}$

Answer: B

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23. How many tangents to the circle $x^2+y^2=3$ are normal to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$?

A. A. 3

B. B. 2

C. C. 1

D. D. 0

Answer: D

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24. An ellipse passes through the point (2,3) and its axes along the coordinate axes, $3x+2y-1=0$ is a tangent to the ellipse, then the equation of the ellipse is

A. $\frac{x^2}{4}+4y^2=1$

B. $\frac{x^2}{8}+\frac{y^2}{1}=1$

C. $4x^2+\frac{y^2}{4}=1$

D. No such ellipse exists

Answer: D

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25. If $x\cos\alpha+y\sin\alpha=4$ is tangent to $\frac{x^2}{25}+\frac{y^2}{9}=1$, then the value of α is

A. $\tan^{-1}(3/\sqrt{7})$

B. $\tan^{-1}(7/3)$

C. $\tan^{-1}(\sqrt{3}/7)$

D. $\tan^{-1}(3/7)$

Answer: A

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26. If the normal at any point P of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ meets the coordinate axes at M and N respectively, then $|PM|:|PN|$ equals

A. $4:3$

B. $16:9$

C. $9:16$

D. $3:4$

Answer: C

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27. If the normal at any point P on the ellipse cuts the major and minor axes in G and g respectively and C be the centre of the ellipse, then

A. $a^2(CG)^2 + b^2(Cg)^2 = (a^2 - b^2)^2$

B. $a^2(CG)^2 - b^2(Cg)^2 = (a^2 - b^2)^2$

C. $a^2(CG)^2 - b^2(Cg)^2 = (a^2 + b^2)^2$

D. None of these

Answer: A

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28. The area of the parallelogram formed by the tangents at the points whose eccentric angles are $\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

A. A. ab

B. B. 4ab

C. C. 3ab

D. D. 2ab

Answer: D

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29. The straight line $\frac{x}{4}+\frac{y}{3}=1$ intersects the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at two points A and B, there is a point P on this ellipse such that the area of ΔPAB is equal to $6(\sqrt{2}-1)$. Then the number of such points (P) is/are

A. 0

B. 1

C. 2

D. 3

Answer: D

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30. The tangent at any point on the ellipse $16x^2 + 25y^2 = 400$ meets the tangents at the ends of the major axis at T_1 and T_2. The circle on T_1T_2 as diameter passes through

A. $(3, 0)$

B. $(0, 0)$

C. $(0, 3)$

D. $(4, 0)$

Answer: A

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31. The minimum value of $\left\{(r+5-4|\cos\theta|)^2 + (r-3|\sin\theta|)^2\right\} \forall r, \theta \in R$ is

A. 0

B. 2

C. 3

D. None of these

Answer: A

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32. Let S_1 and S_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. The value of a for which the line $y = ax$ contains the centre of a circle which touches S_2 externally and S_1 internally is

A. $\pm \frac{3}{10}$

B. $\pm \frac{1}{5}$

C. $\pm \frac{\sqrt{13}}{10}$

D. $\pm \frac{10}{13}$

Answer: C

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33. If ω is one of the angles between the normals to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (b>a)$ at the point whose eccentric angles are θ and $\frac{\pi}{2}+\theta$, then prove that $\frac{2\cot\omega}{\sin 2\theta}=\frac{e^2}{\sqrt{1-e^2}}$

A. $\frac{e^2}{\sqrt{1-e^2}}$

B. $\frac{e^2}{\sqrt{1+e^2}}$

C. $\frac{e^2}{1-e^2}$

D. $\frac{e^2}{1+e^2}$

Answer: A

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34. From any point on the line $(t+2)(x+y)=1, t \neq -2$, tangents are drawn to the ellipse $4x^2+16y^2=1$. It is given that chord of contact passes through a fixed point. Then the number of integral values of 't' for which the fixed point always lies inside the ellipse is

A. 0

B. 1

C. 2

D. 3

Answer: C

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35. At a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ tangents PQ is drawn. If the point Q be at a distance $\frac{1}{p}$ from the point P, where 'p' is distance of the tangent from the origin, then the locus of the point Q is

A. (a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{1}{a^2b^2}$

B. (b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1-\frac{1}{a^2b^2}$

C. (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2b^2}$

D. (d) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{1}{a^2b^2}$

Answer: A

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36. From a point P perpendicular tangents PQ and PR are drawn to ellipse $x^2+4y^2=4$, then locus of circumcentre of triangle PQR is

A. $x^2+y^2=\frac{16}{5}\left(x^2+4y^2\right)^2$

B. $x^2+y^2=\frac{5}{16}\left(x^2+4y^2\right)^2$

C. $x^2+4y^2=\frac{16}{5}\left(x^2+y^2\right)^2$

D. $x^2+4y^2=\frac{16}{5}\left(x^2+y^2\right)^2$

Answer: B

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37. If the normal at any point P on ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle at Q and R such that $\angle QOR=90^{\circ}$ where O is centre of ellipse, then

A. (a) $a^4+2b^3 \geq 3a^2b^2$

B. (b) $a^4+2b^4 \geq 5a^2b^2+2a^3b$

C. (c) $a^4+2b^4 \geq 3a^2b^2+ab$

D. (d) None of these

Answer: B

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38. Tangents are drawn from any point on the circle $x^2+y^2=41$ to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ then the angle between the two tangents is

A. A. $\frac{\pi}{4}$

B. B. $\frac{\pi}{3}$

C. C. $\frac{\pi}{6}$

D. D. $\frac{\pi}{2}$

Answer: D

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39. If radius of the director circle of the ellipse $\frac{(3x+4y-2)^2}{100}+\frac{(4x-3y+5)^2}{625}=1$ is

A. 6

B. $\sqrt{34}$

C. $\sqrt{29}$

D. $\sqrt{26}$

Answer: C

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40. If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$ then a possible value of α^2 is

A. 1

B. 2

C. 3

D. 4

Answer: B

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41. An ellipse has the points $(1, -1)$ and $(2, -1)$ as its foci and $x + y = 5$ as one of its tangent then the value of $a^2 + b^2$ where a, b are the lenghta of semi major and minor axes of ellipse respectively is :

A. $\frac{41}{2}$

B. 10

C. 19

D. $\frac{81}{4}$

Answer: D

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42. An ellipse has foci at $F_1(9, 20)$ and $F_2(49, 55)$ in the xy-plane and is tangent to the x-axis. Find the length of its major axis.

A. 85

B. 75

C. 65

D. 55

Answer: A

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43. The maximum distance of the centre of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ from the chord of contact of mutually perpendicular tangents of the ellipse is

A. $\frac{144}{5}$

B. $\frac{9}{5}$

C. $\frac{16}{5}$

D. $\frac{8}{5}$

Answer: C

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44. P_1 and P_2 are the lengths of the perpendicular from the foci on the tangent of the ellipse and P_3 and P_4 are perpendiculars from extermities of major axis and P from the centre of the ellipse on the same tangent, then $\frac{P_1P_2 - P^2}{P_3P_4 - P^2}$ equals (where e is the eccentricity of the ellipse)

A. A. e

B. B. $\sqrt{e}$

C. C. e^2

D. D. none of these

Answer: C

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45. From the focus $(-5, 0)$ of the ellipse $\frac{x^2}{45} + \frac{y^2}{20} = 1$, a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A. A. $-\frac{3}{2}$

B. B. $-\frac{7}{3}$

C. C. $-\frac{5}{4}$

D. D. $-\frac{2}{11}$

Answer: D

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46. Let $5x - 3y = 8\sqrt{2}$ be normal at $P\left(\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. If m, m' are feet of perpendiculars from foci s, s' respectively. or tangents at p, then point of intersection of sm' and $s'm$ is

A. $\left(\frac{5}{2}, 0\right)$

B. $\left(0, \frac{5}{2}\right)$

C. $\left(\frac{41}{10\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$

D. $\left(\frac{3}{2\sqrt{2}}, \frac{41}{10\sqrt{2}}\right)$

Answer: C

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47. If the normals at α, β, γ and δ on an ellipse are concurrent then the value of $(\sigma \cos \alpha)(\sigma \sec \alpha)$ I

A. 2

B. 4

C. 6

D. none of these

Answer: B

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48. about to only mathematics

A. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{(2a^2+b^2)}$

B. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{2}{(a^2-b^2)}$

C. $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{(a^2+b^2)}$

D. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2}{(3a^2-b^2)}$

Answer: C

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49. Consider an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ with centre c and a point P on it with eccentric angle $\frac{\pi}{4}$. Nomal drawn at P intersects the major and minor axes in A and B respectively. N_1 and N_2 are the feet of the perpendiculars from the foci S_1 and S_2 respectively on the tangent at P and N is the foot of the perpendicular from the centre of the ellipse on the normal at P. Tangent at P intersects the axis of x at T.

A. $\begin{matrix} P & Q & R & S \\ 2 & 3 & 4 & 1 \end{matrix}$

B. $\begin{matrix} P & Q & R & S \\ 3 & 1 & 4 & 2 \end{matrix}$

C. $\begin{matrix} P & Q & R & S \\ 2 & 4 & 1 & 3 \end{matrix}$

D. $\begin{matrix} P & Q & R & S \\ 4 & 1 & 2 & 3 \end{matrix}$

Answer: C

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Multiple Correct Answers Type

1. In triangle ABC, $a = 4$ and $b = c = 2\sqrt{2}$. A point P moves within the triangle such that the square of its distance from BC is half the area of rectangle contained by its distance from the other two sides. If D be the centre of locus of P, then

A. locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

B. locus of P is a hyperbola with eccentricity $\sqrt{\frac{3}{2}}$

C. area of the quadr5ilateral $ABCD = \frac{16}{3}$ sq. units

D. area of the quadrilateral $ABCD = \frac{32}{3}$ sq. units

Answer: A::C

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2. Extremities of the latera recta of the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ having a given major axis 2a lies on

A. A. $x^2 = a(a - y)$

B. B. $x^2 = a(a + y)$

C. C. $y^2 = a(a + x)$

D. D. $y^2 = a(a - x)$

Answer: A::B

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3. Identify correct statement(s) about conic $\sqrt{(x-5)^2 + (y-7)^2} + \sqrt{(x+1)^2 + (y+1)^2} = 12$

A. A. centre of conic is (2,3)

B. B. conic is hyperbola with foci (5,7) and $(-1, -1)$

C. C. conic is ellipse with major axis $4x - 3y + 1 = 0$

D. D. eccentricity of conic is $\frac{5}{7}$

Answer: A::C

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4. P and Q are two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are differ by 90°, then

A. Locus of point of intersection of tangents at P and Q is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

B. Locus of mid-point (P, Q) is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$

C. Product of slopes of OP and OQ ehere O is the centre is $\frac{-b^2}{a^2}$

D. Max. area of ΔOPQ is $\frac{1}{2}ab$

Answer: A::B::C::D

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5. Find the equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

A. (a) The foci of each ellipse always lie within the other ellipse

B. (b) Their auxiliary circles are the same

C. (c) Their director circles are the same

D. (d) The ellipses encloses the same area

Answer: B::C::D

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6. AB and CD are two equal and parallel chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Tangents to the ellipse at A and B intersect at P and tangents at C and D at Q. The line PQ

A. passes through the origin

B. is bisected at the origin

C. cannot pass through the origin

D. is not bisected at the origin

Answer: A::B

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7. $\frac{x^2}{r^2+r-6}+\frac{y^2}{r^2-6r+5}=1$ will represent the ellipse if r lies in the interval

A. $(-\infty, -2)$

B. $(3, -\infty)$

C. $(5, \infty)$

D. $(1, \infty)$

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8. The equation $3x^2 + 4y^2 - 18 + 16y + 43 = k$ represents an empty set, if $k < 0$ represents an ellipse, if $k > 0$ represents a point, if $k = 0$ cannot represent a real pair of straight lines for any value of k

A. represent an empty set, if $k < 0$

B. represent an ellipse if $k > 0$

C. represent a poin, if k=0

D. cannot repreent a real paor of straight lines for any value of k

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9. If the equation of the ellipse is $3x^2 + 2y^2 + 6x - 8y + 5 = 0$, then which of the following is/are true? $e = \frac{1}{\sqrt{3}}$ Center is $(-1, 2)$. Foci are

$(-1, 1) and (-1, 3)$. Directrices are $y = 2 \pm \sqrt{3}$

A. $e = 1\sqrt{3}$

B. Center is (-1,2)

C. Foci are (-1,2)

D. Directrices are y=2 $\pm \sqrt{3}$

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10. about to only mathematics

A. $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

B. $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

C. $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

D. $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

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11. ABCD is a rhombus with AC=2BD. Diagonals AC and BD intersect at P. E_1, E_2, E_3 and E_4 are four ellipes passing through P and their foci are A and B, B and C, C and D and D and A, respectively . If for $i = 1, 2, 3, 4$, e_i are the eccentricities fo E_i, then

A. $e_i = e_3$

B. $e_2 = e_4$

C. $e_1 = 2e_2$

D. $e_1 = e_2$

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12. The major axis and minor axis of an ellipse are, respectively, $x - 2y - 5 = 0$ and $2x + y + 10 = 0$. If the end of the latus rectum is (3,4) find focii

A. (5,0)

B. (-7,-6)

C. (-11,-8)

D. (11,3)

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13. If the chord through the points whose eccentric angles are θ and φ on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ passes through a focus, then the value of $\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\varphi}{2}\right)$ is $\frac{1}{9}$ (b) -9 (c) $-\frac{1}{9}$ (d) 9

A. $1/9$

B. -9

C. $-1/9$

D. 9

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14. Which of the following is/are true about the ellipse $x^2+4y^2-2x-16y+13=0$? $(a)$the latus rectum of the ellipse is 1. (b)The distance between the foci of the ellipse is $4\sqrt{3}\cdot$ (c)The sum of the focal distances of a point $P(x, y)$ on the ellipse is 4. (d)Line $y=3$ meets the tangents drawn at the vertices of the ellipse at points P and Q . Then PQ subtends a right angle at any of its foci.

A. The latus rectum of the ellipse is 1

B. The distance between the foci of the ellipse is $4\sqrt{3}$

C. The sum of the focal distances of a point P(x,y) on the ellipse is 4

D. Liney=3 meets the tangents drawn at the vertices of the ellipse at points P and Q. Then PQ subtends a right angle at any of its foci.

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15. Which of the following statement(s) is/are TRUE?

A. Theren are infinite positive integral values of a for which $(13x-1)^2+(13y-2)^2=\left(\frac{5x+12-1}{a}\right)^2$ represents an ellipse

B. The minimum distance of a point (1,2) from the ellipse $4x^2+9y^2-36y+4=0$ is 1.

C. If from a point $P(0,\alpha)$ two normals other thn the axes are drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{16=1}$, hen $|\alpha|<9/4$

D. If the length of the latus rectum of the ellipse is one -third of its major axis, then its eccentrticity is equal to $1\sqrt{3}$

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16. Let $E_1 and E_2,$ respectively, be two ellipses $\frac{x^2}{a^2}+y^2=1, and x^2+\frac{y^2}{a^2}=1$ (where a is a parameter). Then the locus of the points of intersection of the ellipses $E_1 and E_2$ is a set of curves comprising two straight lines (b) one straight line one circle (d) one parabola

A. two straigths

B. one straiths line

C. one circle

D. one parabola

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17. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(k+11)} = 1$. If $f(x)$ is a positive decr4easing function, then the set of values of k for which the major axis is the x-axis is $(-3, 2)$. the set of values of k for which the major axis is the y-axis is $(-\infty, 2)$. the set of values of k for which the major axis is the y-axis is $(-\infty, -3) \cup (2, \infty)$ the set of values of k for which the major axis is the y-axis is $(-3, -\infty,)$

A. the set of values of k for which the major axis is the x-asix is (-3,2)

B. the set of values of k for which the major axis is the x-asix is

$(-\infty, 2)$

C. the set of values of k for which the major axis is the x-asix is

$(-\infty, -3) \cup (2, \infty)$

D. the set of values of k for which the major axis is the x-asix is

$(-3, \infty)$

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18. Two concentric ellipses are such that the foci of one are on the other and their major axes are equal. Let $eande'$ be their eccentricities. Then. the quadrilateral formed by joining the foci of the two ellipses is a parallelogram the angle θ between their axes is given by $\theta = \cos^{-1}\sqrt{\frac{1}{e^2} + \frac{1}{e'^2} = \frac{1}{e^2 e'^2}}$ If $e^2 + e'^2 = 1$, then the angle between the axes of the two ellipses is 90^0 none of these

A. the quadrilateral formed by joining the foci of the two ellipe is a parallelogram

B. the angle θ between their axes is give by $\theta = \cos^{-1}\sqrt{\frac{1}{e^2} + \frac{1}{e'^2} - \frac{1}{e^2 e'^2}}$

C. If $e^2 + e'^2 = 1$, then the angle between th axes of the two ellipse is 90°

D. none of these

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19. A point moves so that the sum of the squares of its distances from two intersecting straight lines is constant. Prove that its locus is an ellipse.

A. $\frac{\sqrt{\cos 2\alpha}}{\sin \alpha} \text{ if } \alpha < \frac{\pi}{4}$

B. $\frac{\sqrt{-\cos 2\alpha}}{\cos \alpha} \text{ if } \alpha < \frac{\pi}{4}$

C. $\dfrac{\sqrt{\cos 2\alpha}}{\cos\alpha}$ if $\alpha < \dfrac{\pi}{4}$

D. $\dfrac{\sqrt{-\cos 2\alpha}}{\sin\alpha}$ if $\alpha < \dfrac{\pi}{4}$

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20. The coordinates (2, 3) and (1, 5) are the foci of an ellipse which passes through the origin. Then the equation of the tangent at the origin is $(3\sqrt{2}-5)x+(1-2\sqrt{2})y=0$ tangent at the origin is $(3\sqrt{2}+5)x+(1+2\sqrt{2y})=0$ tangent at the origin is $(3\sqrt{2}+5)x-(2\sqrt{2+1})y=0$ tangent at the origin is $(3\sqrt{2}-5)-y(1-2\sqrt{2})=0$

A. tangent at the origin is $(3\sqrt{2}-5)x+(1-2\sqrt{2})y=0$

B. tangent at the origin is $(3\sqrt{2}-5)x+(1+2\sqrt{2})y=0$

C. tangent at the origin is $(3\sqrt{2}-5)x-(2\sqrt{2}+1)y=0$

D. tangent at the origin is $(3\sqrt{2}-5)-y-(1-2\sqrt{2})=0$

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21. about to only mathematics

A. $x - 2y = 0$

B. $2x - y = 0$

C. $x + 2y = 0$

D. $2x + y = 0$

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22. A point on the ellipse $x^2 + 3y^2 = 37$ where the normal is parallel to the line $6x - 5y = 2$ is $(5, -2)$ (b) (5, 2) (c) $(-5, 2)$ (d) $(-5, -2)$

A. (5,-2)

B. (5,2)

C. (-5,2)

D. (-5,-2)

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23. The locus of the image of the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} - 1, (a > b)$, with respect to any of the tangents to the ellipse is $(x+4)^2 + y^2 = 100$ (b) $(x+2)^2 + y^2 = 50$ $(x-4)^2 + y^2 = 100$ (d) $(x+2)^2 + y^2 = 50$

A. $(x+4)^2 + y^2 = 100$

B. $(x+2)^2 + y^2 = 50$

C. $(x-4)^2 + y^2 = 100$

D. $(x-2)^2 + y^2 = 50$

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24. If the tangent drawn at point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is the same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on the ellipse $4x^2 + 5y^2 = 20$, then $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ $t = -\frac{2}{\sqrt{5}}$ (d) $t = -\frac{1}{\sqrt{5}}$

A. $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$

B. $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

C. $t = -\frac{2}{\sqrt{5}}$

D. $t = -\frac{1}{\sqrt{5}}$

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25. If a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ u=is $4 - 3y = 7$ and its ecentricity is $\frac{\sqrt{7}}{4}$, then the volume of L.R can be

A. $\frac{9}{\sqrt{2}}$

B. $\frac{9}{2}$

C. $\frac{8\sqrt{337}}{19}$

D. $\frac{3\sqrt{337}}{8}$

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26. A triangle ABC with fixed base BC , the vertex A moves such that $\cos B + \cos C = 4\frac{\sin^2 A}{2}$. If $a, bandc,$ denote the length of the sides of the triangle opposite to the angles $A, B, andC$, respectively, then (a) $b+c=4a$ (b) $b+c=2a$ (c)the locus of point A is an ellipse (d)the locus of point A is a pair of straight lines

A. b+c=4a

B. b+c=2a

C. the locus of point A is an ellipse

D. the locus of point A is a pair of straight lines

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27. Let E_1 and E_2 be two ellipse whsoe centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis , and the y-axis, respectively. Let S be the circle $x^2+(y-1)^2=2$. The straigth line x+y=3 touches the curves, S, E_1 and E_2 at P,Q and R, respectively . Suppose that $PQ=PR=\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2 respectively, thent hecorrect expression (s) is (are)

A. $e_1^2+e_2^2=\frac{43}{40}$

B. $e_1e_2=\frac{\sqrt{7}}{2\sqrt{10}}$

C. $\left|e_1^2-e_2^2\right|=\frac{5}{8}$

D. $e_1e_2=\frac{\sqrt{3}}{7}$

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28. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q . Consider the ellipse whose center is at the origin $O(0,\ 0)$ and whose semi-major axis is OQ . If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

A. For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length the latcus rectum is 1

B. For the ellipse, the eccentricity is $\frac{1}{2}$ and the length the latcus rectue is $\frac{1}{2}$

C. The area of the reagion bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1 \text{is} \frac{1}{4\sqrt{4}}(\pi - 2)$

D. The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1 \text{is} \frac{1}{16}(\pi - 12)$

Answer: A:C

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ILLUSTRATION

1. Two circles are given such that one is completely lying inside the other without touching. Prove that the locus of the center of variable circle which touches the smaller circle from outside and the bigger circle from inside is an ellipse.

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2. Base BC of triangle ABC is fixed and opposite vertex A moves in such a way that $\tan.\frac{B}{2}\tan.\frac{C}{2}$ is constant. Prove that locus of vertex A is ellipse.

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3. If the equation $(5x-1)^2+(5y-2)^2=(\lambda^2-2\lambda+1)(3x+4y-1)^2$ represents an ellipse, then find values of λ.

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4. Find the equation of the ellipse whose focus is $S(-1, 1)$, the corresponding directrix is $x - y + 3 = 0$, and eccentricity is 1/2. Also find its center, the second focus, the equation of the second directrix, and the length of latus rectum.

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5. Find the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus rectum is half of its major axis.

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6. Find the equation of the ellipse (referred to its axes as the axes of $x and y$, respectively) whose foci are $(\pm 2, 0)$ and eccentricity is $\frac{1}{2}$

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7. If the focal distance of the one end of the minor axis of standard ellipse is k and distance between its foci is $2h(k > h)$, then find its equation.

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8. Find the equation of the ellipse having minor axis of length 1 and foci (0,1), (0,-1). Also find its latus rectum.

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9. If $P(x, y)$ is any point on the ellipse $16x^2 + 25y^2 = 400$ and $f_1 = (3, 0)F_2 = (- 3, 0)$, then find the value of $PF_1 + PF_2$.

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10. A rod of length l cm moves with its ends A (on x=-axis) and B (on y-axis) always touching the coordinae axes. Prove that the point P on the rod which divides AB in the ratio $\lambda(\neq 1)$ is ellipse. Alos, find the eccentricity of the ellipse.

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11. The first artificial satellite to orbit the earth was Sputnik I. Its highest point above earths surface was 947 km, and its lowest point was 228 km. The center of the earth was at one focus of the elliptical orbit. The radius of the earth is 6378 km. Find the eccentricity of the orbit.

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12. Find the normal to the ellipse $\frac{x^2}{18}+\frac{y^2}{8}=1$ at point (3, 2).

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13. Variable complex number z satisfies the equation $|z-1+2i|+|z+3-i|=10$. Prove that locus of complex number z is ellipse. Also, find the centre, foci and eccentricity of the ellipse.

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14. Find the center, foci, the length of the axes, and the eccentricity of the ellipse $2x^2+3y^2-4x-12y+13=0$

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15. Find the equation of the ellipse whose axes are of length 6 and $2\sqrt{6}$ and their equations are $x-3y+3=0$ and $3x+y-1=0$, respectively.

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16. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.

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17. If $S and S'$ are two foci of ellipse $16x^2 + 25y^2 = 400 and PSQ$ is a focal chord such that $SP = 16$, then find $S'Q$.

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19. The locus of mid-points of focal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

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20. Find the eccentric angle of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the center of the ellipse is $\sqrt{5}$

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21. Find the number of rational points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

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22. Find the equation of the curve whose parametric equation are $x = 1 + 4\cos\theta, y = 2 + 3\sin\theta, \theta \in R.$

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23. A line passing through the origin $O(0, 0)$ intersects two concentric circles of radii $a and b$ at $P and Q,$ If the lines parallel to the X-and Y-axes through $Q and P,$ respectively, meet at point $R,$ then find the locus of $R.$

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24. The auxiliary circle of a family of ellipses passes through the origin and makes intercepts of 8 units and 6 units on the x and y-axis, respectively. If the eccentricity of all such ellipses is $\frac{1}{2}$, then find the locus of the focus.

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25. If the line $lx + my + n = 0$ cuts the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ at points whose eccentric angles differ by $\frac{\pi}{2}$, then find the value of $\frac{a^2 l^2 + b^2 m^2}{n^2}$.

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26. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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27. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$ having its vertex coincident with one extremity of the major axis.

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28. Prove that the sum of eccentric angles of four concylic points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $2n\pi,$ where $n \in Z$

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29. The ratio of the area of triangle inscribed in ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5. Then, find the eccentricity of the ellipse.

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30. If α and β are the eccentric angles of the extremities of a focal chord of an ellipse, then prove that the eccentricity of the ellipse is $\frac{\sin\alpha + \sin\beta}{\sin(\alpha + \beta)}$

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31. Find the equation of chord of an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ joining two points $P\left(\frac{\pi}{3}\right)$ and $Q\left(\frac{\pi}{6}\right)$

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32. Find the points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ such that the tangent at each point makes equal angles with the axes.

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33. An ellipse passes through the point $(4, -1)$ and touches the line $x + 4y - 10 = 0$. Find its equation if its axes coincide with the coordinate axes.

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34. Find the maximum area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which touches the line $y = 3x + 2$.

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35. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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36. If the line $3x + 4y = \sqrt{7}$ touches the ellipse $3x^2 + 4y^2 = 1$, then find the point of contact.

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37. If $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then find the eccentric angle θ of point of contact.

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38. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ [where $\theta \in \left(0, \frac{\pi}{2}\right)$] Then the value of θ such that sum of intercepts on axes made by this tangent is minimum is (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

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39. The tangent at a point P on an ellipse intersects the major axis at $T, and N$ is the foot of the perpendicular from P to the same axis. Show that the circle drawn on NT as diameter intersects the auxiliary circle orthogonally.

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41. Find the point on the ellipse $16x^2 + 11y^2 = 256$ where the common tangent to ti and the circle $x^2 + y^2 - 2x = 15$ toch.

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42. Consider an ellipse $\frac{x^2}{4} + y^2 = \alpha(\alpha$ is parameter $> 0)$ and a parabola $y^2 = 8x$. If a common tangent to the ellipse and the parabola meets the coordinate axes at $A and B$, respectively, then find the locus of the midpoint of AB.

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43. Find the equations of the tangents drawn from the point (2, 3) to the ellipse $9x^2 + 16y^2 = 144.$

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44. Find the angle between the pair of tangents from the point (1,2) to the ellipse $3x^2 + 2y^2 = 5.$

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45. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

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46. If there exists exactly one point of the line $3x + 4y + 25 = 0$ from which perpendicular tangesnt can be brawn to ellipse $\frac{x^2}{a^2} + y^2 = 1 (a > 1)$,

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47. If from a point P, tangents $PQandPR$ are drawn to the ellipse $\frac{x^2}{2} + y^2 = 1$ so that the equation of QR is $x + 3y = 1$, then find the coordinates of P.

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48. Prove that the chord of contact of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with respect to any point on the directrix is a focal chord.

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49. Tangents are drawn from the points on the line $x - y - 5 = 0$ to $x^2 + 4y^2 = 4$. Then all the chords of contact pass through a fixed point. Find the coordinates.

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50. Find the locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ form a triangle of constant area with the coordinate axes.

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52. Tangent are drawn from the point (3, 2) to the ellipse $x^2 + 4y^2 = 9$. Find the equation to their chord of contact and the middle point of this chord of contact.

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53. If F_1 and F_2 are the feet of the perpendiculars from the foci $S_1 and S_2$ of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ on the tangent at any point P on the ellipse, then prove that $S_1F_1 + S_2F_2 \geq 8$.

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54. An ellipse has point $(1, -1)and(2, -1)$ as its foci and $x + y - 5 = 0$ as one of its tangents. Then the point where this line touches the ellipse is $\left(\frac{32}{9}, \frac{22}{9}\right)$ (b) $\left(\frac{23}{9}, \frac{2}{9}\right)$ $\left(\frac{34}{9}, \frac{11}{9}\right)$ (d) none of these

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55. Find the normal to the ellipse $\frac{x^2}{18} + \frac{y^2}{8} = 1$ at point (3, 2).

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56. Find the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ on which the normals are parallel to the line $2x - y = 1$.

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57. The value of λ, for which the line $2x - \frac{8}{3}\lambda y = -3$ is a normal to the conic $x^2 + \frac{y^2}{4} = 1$ is

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58. If ω is one of the angles between the normals to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (b > a)$ at the point whose eccentric angles are θ and $\frac{\pi}{2} + \theta$, then prove that $\frac{2\cot\omega}{\sin 2\theta} = \frac{e^2}{\sqrt{1 - e^2}}$

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59. If the normal at any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the axes at G and g respectively, then find the ratio $PG : Pg$.

(a) $a : b$ (b) $a^2 : b^2$ (c) $b : a$ (d) $b^2 : a^2$

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60. Find the maximum distance of any normal of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from its centre,

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62. Normal to the ellipse $\frac{x^2}{84}+\frac{y^2}{49}=1$ intersects the major and minor axes at $PandQ$, respectively. Find the locus of the point dividing segment PQ in the ratio 2:1.

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63. If the normal at $P\left(2\frac{3\sqrt{3}}{2}\right)$ meets the major axis of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at Q , and S and S' are the foci of the given ellipse, then

find the ratio $SQ : S'Q$.

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SOLVED EXAMPLE

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2. Find the values of a for which three distinct chords drawn from $(a, 0)$ to the ellipse $x^2 + 2y^2 = 1$ are bisected by the parabola $y^2 = 4x$.

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3. Prove that if any tangent to the ellipse is cut by the tangents at the endpoints of the major axis at $TandT'$, then the circle whose diameter is TT' will pass through the foci of the ellipse.

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5. about to only mathematics

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6. If a triangle is inscribed in an ellipse and two of its sides are parallel to the given straight lines, then prove that the third side touches the fixed ellipse.

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7. The tangent at a point $P(a\cos\varphi, b\sin\varphi)$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets its auxiliary circle at two points, the chord joining which subtends a right angle at the center. Find the eccentricity of the ellipse.

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8. Find the locus of point P such that the tangents drawn from it to the given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meet the coordinate axes at concyclic points.

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9. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P&Q.

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10. show that the area of the triangle inscribed in the circle $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meet the ellipse respictively at P,Q,R so that P,Q,R lie on the same side of the major axis as A,B,C respictively. Prove that the normal to the ellipse drawn at the points P,Q and R are concurrent.

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CONCEPT APPLICATION EXERCISE 6.1

1. Find the equation of ellipse having focus at (1,2) corresponding directirx $x - y = 2 = 0$ and eccentricity 0.5.

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2. Find the equation of parabola having focus at (0,-3) its directrix is y = 3.

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3. Find the eccentricity, one of the foci, the directrix, and the length of the latus rectum for the conic $(3x-12)^2+(3y+15)^2=\frac{(3x-4y+5)^2}{25}$.

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CONCEPT APPLICATION EXERCISE 6.2

1. Find the equation of an ellipse whose eccentricity is 2/3, the latus rectum is 5 and the centre is at the origin.

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2. An ellipse has OB as the semi-minor axis, $FandF'$ as its foci, and $\angle FBF'$ a right angle. Then, find the eccentricity of the ellipse.

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3. An ellipse is inscribed in a reactangle and the angle between the diagonals of the reactangle is $\tan^{-1}(2\sqrt{2})$, then find the ecentricity of the ellipse

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4. Find the equation of an ellipse whose axes are the x-and y-axis and whose one focus is at (4,0) and eccentricity is 4/5.

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5. An ellipse is described by using an endless string which is passed over two pins. If the axes are $6cm$ and $4cm$, the length of the string and distance between the pins are ....

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6. An arc of a bridge is semi-elliptical with the major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal, then prove that the best approximation of the height of the acr 2 m from the center of the base is $\frac{8}{3}m$.

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7. If the foci of an ellipse are $(0, \pm 1)$ and the minor axis is of unit length, then find the equation of the ellipse. The axes of ellipse are the coordinate axes.

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8. A point P lies on the ellipe $\frac{(y-1)^2}{64} + \frac{(x+2)^2}{49} = 1$. If the distance of P from one focus is 10 units, then find its distance from other focus.

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9. An ellipse circumscribes a quadrilateral whose sides are given by $x = - +2$ and $y = \pm 4$. If the distance between focis is $4\sqrt{6}$ and major axis is along yaxis then find the eqution of ellipse

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10. Find the eccentricity of an ellipse beween whose foci is 10 and that between focus and corresponding directirx is 15

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11. A circle whose diameter is major aixs of ellipe $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b > 0)$ meets minor axis at point P. Ifthe orthocentre of ΔPF_1F_2 lies on ellipse where F_1 and F_2 are foci of ellipse , then find the eccenricity of the ellipse

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12. What is the ratio of the greatest and least focal distances ofa point on the ellipse $4x^2 + 9y^2 = 36$?

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13. An ellipse is drawn with major and minor axis of length 10 and 8 respectively. Using one focus a centre, a circle is drawn that is tangent to ellipse, with no part of the circle being outside the ellipse. The radius of the circle is

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14. The moon travels an elliptical path with Earth as one focus. The maximum distance from the moon to the earth is 405, 500 km and the minimum distance is 363,300 km. What is the eccentricity of the orbit?

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15. Find the foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225.$

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16. Prove that the area bounded by the circle $x^2 + y^2 = a^2$ and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to the area of another ellipse having semi-axis $a - b$ and b, $a > b$.

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17. Find the lengths of the major and minor axis and the eccentricity of the ellipse $\frac{(3x - 4y + 2)^2}{16} + \frac{(4x + 3y - 5)^2}{9} = 1$

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18. Find the locus of the middle points of all chords of $\frac{x^2}{4} + \frac{y^2}{9} = 1$ which are at a distance of 2 units from the vertex of parabola $y^2 = -8ax\cdot$

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CONCEPT APPLICATION EXERCISE 6.3

1. Prove that any point on the ellipse whose foci are $(-1, 0)$ and $(7, 0)$ and eccentricity is $\frac{1}{2}$ is $\left(3 + 8\cos\theta, 4\sqrt{3}\sin\theta\right), \theta \in R$.

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2. Find the eccentric angles of the extremities of the latus recta of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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3. If the chord joining points $P(\alpha)$ and $Q(\beta)$ on the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ subtends a right angle at the vertex $A(a, 0)$, then prove that $\tan\left(\frac{a}{2}\right)\tan\left(\frac{\beta}{2}\right) = -\frac{b^2}{a^2}$.

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4. If the area of the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$ is 4π , then find the maximum area of rectangle inscribed in the ellipse.

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5. Find the point $(\alpha,)\beta$ on the ellipse $4x^2+3y^2=12,$ in the first quadrant, so that the area enclosed by the lines $y=x, y=\beta, x=\alpha$, and the x-axis is maximum.

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6. Find the maximum length of chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then find the locus of midpoint of PQ

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7. If $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ are two points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ then find the locus of midpoint of PQ

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8. Let P any point on ellipse $3x^2+4y^2=12$. If S and S'' are its foci then find the the locus of the centroid of trianle PSS''

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9. The line x+2y=1 cuts the ellipse $x^2+4y^2=1$ 1 at two distinct points A and B. Point C is on the ellipse such that area of triangle ABC is maximum, then find point C.

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CONCEPT APPLICATION EXERCISE 6.4

1. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

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2. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ intersects the major and minor axes at points A and B, respectively. If C is the center of the ellipse, then find area of triangle ABC.

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3. Find the slope of a common tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and a concentric circle of radius r.

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4. If any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intercepts equal lengths l on the axes, then find l.

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5. If two points are taken on the minor axis of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the same distance from the center as the foci, then prove that the sum of the squares of the perpendicular distances from these points on any tangent to the ellipse is $2a^2$.

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6. If the tangent at any point of the ellipse $\frac{x^2}{a^3} + \frac{y^2}{b^2} = 1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact, then show that the eccentricity of the ellipse is given by $e = \frac{\cos \beta}{\cos \alpha}$

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7. From any point on any directrix of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b,$ a pari of tangents is drawn to the auxiliary circle. Show that the chord of contact will pass through the correspoinding focus of the ellipse.

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8. If the tangent to the ellipse $x^2 + 2y^2 = 1$ at point $P\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ meets the auxiliary circle at point R and Q, then find the points of intersection of tangents to the circle at Q and $R\cdot$

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9. If the line $2px + y\sqrt{5 - 6p^2} = 1, p \in \left[-\frac{\sqrt{5}}{6}, \frac{\sqrt{5}}{6}\right]$, always touches the standard ellipse. Then find the eccentricity of the standard ellipse.

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10. If a quadrilateral is formed by four tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ then the area of the square is equal to

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11. A variable tangent to the circle $x^2 + y^2 = 1$ intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at point P and Q. The lous of the point of the intersection of tangents to the ellipse at P and Q is another ellipse. Then find its eccentricity.

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12. If the chords of contact of tangents from two poinst (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles, then find the value of $\frac{x_1 x_2}{y_1 y_2}$.

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13. From the point $A(4, 3)$, tangent are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ to touch the ellipse at B and $C\dot{E}F$ is a tangent to the ellipse parallel to line BC and towards point $A \cdot$ Then find the distance of A from $EF \cdot$

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14. Tangents $PQandPR$ are drawn at the extremities of the chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, which get bisected at point $P(1, 1) \cdot$ Then find the point of intersection of the tangents.

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15. Find the normal to the ellipse $\frac{x^2}{18} + \frac{y^2}{8} = 1$ at point (3, 2).

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CONCEPT APPLICATION EXERCISE 6.5

1. Let S(3,4) and S (9,12) be two foci of an ellipse. If foot of the perpendicular from focus S to a tangent of the ellipse is (1,-4), then find the eccentricity of the ellipse.

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2. Find the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the positive end of the latus rectum.

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3. The area of the rectangle formed by the perpendicular from the center of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\frac{\pi}{4}$, is

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4. If the straight line $4ax + 3by = 24$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$, then find the the coordinates of focii and the ellipse

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5. If normal at any poin P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b > 0)$ meets the major and minor axes at Q and R, respectively, so that 3PQ=2PR, then find the eccentricity of ellipse

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6. If the normal at one end of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through one end of the monor axis, then prove that eccentricity is constant.

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7. The foci of an ellipse are $S(3, 1)$ and $S'(11, 5)$The normal at P is $x + 2y - 15 = 0$ Then point P is

A. a) 17,1

B. b) -17,-1

C. c) -17,1

D. d) 17,-1

Answer: (17,1)

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CONCEPT APPLICATION EXERCISE 6.6

1. If the area of a square circumscribing an ellipse is 10 square units and maximum distance of a normal from the centre of the ellipse is 1 unit, than find the eccentricity of ellipse

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EXERCISES (SINGLE CORRECT ANSWER TYPE)

1. $PandQ$ are the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis. If PBQ is an equilateral triangle, then the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{3}$ (d) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

A. $1/\sqrt{2}$

B. $1/3$

C. $1/2$

D. $\sqrt{3}/2$

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2. $S_1 and S_2$ are the foci of an ellipse of major axis of length 10 units, and P is any point on the ellipse such that the perimeter of triangle PS_1 is 15.

Then the eccentricity of the ellipse is 0.5 (b) 0.25 (c) 0.28 (d) 0.75

A. 0.5

B. 0.25

C. 0.28

D. 0.75

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3. If the eccentricity of the ellipse, $\frac{x^2}{a^2+1}+\frac{y^2}{a^2+2}=1$ is $\frac{1}{\sqrt{6}}$ then latus rectum of ellipse is

A. $5/\sqrt{6}$

B. $10/\sqrt{6}$

C. $8/\sqrt{6}$

D. none of these

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4. If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ at four distinct points and $a = b^2 - 5b + 7$, then b does not lie in $[4, 5]$ (b) $(- \infty, 2) \cup (3, \infty)$ $(- \infty, 0)$ (d) $[2, 3]$

A. [4,5]

B. $(- \infty, 2) \cup (3, \infty)$

C. $(- \infty, 0)$

D. [2,3]

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5. A point moves so that its distance from the point (2,0) is always (1)/(3)` of its distances from the line x-18. If the locus of the points is a conic, then length of its latus rectum is

A. $\frac{16}{3}$

B. $\frac{32}{3}$

C. $\frac{8}{3}$

D. $\frac{15}{4}$

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6. if vertices of an ellipse are $(-4, 1), (6, 1)$ and $x - 2y = 2$ is focal chord then the eccentricity of the ellipse is

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$

Answer: A

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7. Let P be a pooint on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ in the first or second quadrants whowse foci are S_1 and S_2. Then the least possible value of circumradius of $\Delta P S_1 S_2$ will be

A. ae

B. be

C. $\frac{ae}{b}$

D. $\frac{ae^2}{b}$

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8. An equilateral triangle is inscribed in an ellipse whose equation is $x^2 + 4y^2 = 4$ If one vertex of the triangle is (0,1) then the length of each side is

A. $\frac{8\sqrt{3}}{13}$

B. $\frac{24\sqrt{3}}{13}$

C. $\frac{16\sqrt{3}}{13}$

D. $\frac{48\sqrt{3}}{13}$

Answer: C

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9. A circle has the same center as an ellipse and passes through the foci $F_1 and F_2$ of the ellipse, such that the two curves intersect at four points. Let P be any one of their point of intersection. If the major axis of the ellipse is 17 and the area of triangle PF_1F_2 is 30, then the distance between the foci is

A. 13

B. 10

C. 11

D. non of these

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10. There are exactly two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose distances from its center are the same and are equal to $\frac{\sqrt{a^2 + 2b^2}}{2}$. Then the eccentricity of the ellipse is $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{3\sqrt{2}}$

A. $1/2$

B. $1/\sqrt{2}$

C. $1/3$

D. $1/3\sqrt{2}$

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11. The eccentricity of the locus of point $(3h+2, k)$, where (h, k) lies on the circle $x^2+y^2=1$, is (a) $\frac{1}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{1}{\sqrt{3}}$

A. $1/3$

B. $\sqrt{2}/3$

C. $2\sqrt{2}/3$

D. $1/\sqrt{3}$

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12. Let $SandS'$ be two foci of the ellipse $\frac{x^2}{a^3}+\frac{y^2}{b^2}=1$. If a circle described on SS' as diameter intersects the ellipse at real and distinct points, then the eccentricity$e$ of the ellipse satisfies $c=\frac{1}{\sqrt{2}}$ (b) $e \in \left(\frac{1}{\sqrt{2}}, 1\right)$ $e \in \left(0, \frac{1}{\sqrt{2}}\right)$ (d) none of these

A. $e=1/\sqrt{2}$

B. $e \in (1\sqrt{2}, 1)$

C. $e \in (0, 1/\sqrt{2})$

D. none of these

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13. Point P represent the complex number z=x+iy and point Qrepresents the complex number z+1/z. If P moves on the circle |z|=2, then the eccentricity of locus of point Q is

A. $3/5$

B. $4/5$

C. $3/4$

D. $1/2$

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14. The locus of the point which divides the double ordinates of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the ratio $1:2$ internally is

A. $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$

B. $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=\frac{1}{9}$

C. $\frac{9x^2}{a^2}+\frac{9y^2}{b^2}=1$

D. none of these

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15. The equation of the line passing through the center and bisecting the chord $7x+y-1=0$ of the ellipse $\frac{x^2}{1}+\frac{y^2}{7}=1$ is

A. x=y

B. 2x=y

C. x=2y

D. x+y=0

Answer: A

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16. A parabola is drawn with focus at one of the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If the latus rectum of the ellipse and that of the parabola are same, then the eccentricity of the ellipse is (a) $1-\frac{1}{\sqrt{2}}$ (b) $2\sqrt{2}-2$ (c) $\sqrt{2}-1$ (d) none of these

A. $1-\frac{1}{\sqrt{2}}$

B. $2\sqrt{2}-2$

C. $\sqrt{2}-1$

D. none of these

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17. P is any point lying on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ whose foci are S and S'. If $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then the value of $\tan.\frac{\alpha}{2}\tan.\frac{\beta}{2}$ is

A. $\frac{1+e}{1-e}$

B. $\frac{1+e^2}{1-e^2}$

C. $\frac{1-e}{1+e}$

D. 1

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18. With a given point and line as focus and directrix, a series of ellipses are described. The locus of the extremities of their minor axis is an ellipse (b) a parabola a hyperbola (d) none of these

A. an ellipse

B. a parabola

C. a hyperbola

D. none of these

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19. The angle subtended by common tangents of two ellipses $4(x-4)^2+25y^2=100and4(x+1)^2+y^2$ at the origin is $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

A. $\pi/3$

B. $\pi/4$

C. $\pi/6$

D. $\pi/2$

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20. The length of major ofthe ellipse $(5x-10)^2+(5y+15)^2=\frac{1}{4}(3x-4y+7)^2$ is

A. 10

B. $20/3$

C. $20/7$

D. 4

Answer: B

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21. The foci of an ellipse are $S(-1,-1), S'(0,-2)$its $e=\frac{1}{2}$, then the equation of the directrix corresponding to the focus S is

A. x-y+3-=0

B. x-y+7=0

C. x-y+5=0

D. x-y+4=0

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22. If the curves $\frac{x^2}{4} + y^2 = 1$ and $\frac{x^2}{a^2} + y^2 = 1$ for a suitable value of a cut on four concyclic points, the equation of the circle passing through these four points is

A. $x^2 + y^2 = 2$

B. $x^2 + y^2 = 1$

C. $x^2 + y^2 = 4$

D. none of these

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23. If the maximum distance of any point on the ellipse $x^2+2y^2+2xy=1$ from its center is r, then r is equal to $3+\sqrt{3}$ (b) $2+\sqrt{2}$ $\dfrac{\sqrt{2}}{\sqrt{3-\sqrt{5}}}$ (d) $\sqrt{2-\sqrt{2}}$

A. $3+\sqrt{3}$

B. $2+\sqrt{2}$

C. $\sqrt{2}/\sqrt{3-\sqrt{5}}$

D. $\sqrt{2-\sqrt{2}}$

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24. If $SandS''$ are the foci of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, and P is any point on it, then the range of values of $SP\dot{S}'P$ is $9\leq f(\theta)\leq 16$ (b) $9\leq f(\theta)\leq 25$ $16\leq f(\theta)\leq 25$ (d) $1\leq f(\theta)\leq 16$

A. $9\leq f(\theta)\leq 16$

B. $9 \leq f(\theta) \leq 25$

C. $16 \leq f(\theta) \leq 25$

D. $1 \leq f(\theta) \leq 25$

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25. A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.

A. 15π

B. 20π

C. 27π

D. 30π

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26. The eccentric angle of a point on the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at a distance of 5/4 units from the focus on the positive x-axis is $\cos^{-1}\left(\frac{3}{4}\right)$ (b) $\pi-\cos^{-1}\left(\frac{3}{4}\right)$ $\pi+\cos^{-1}\left(\frac{3}{4}\right)$ (d) none of these

A. $\cos^{-1}(3/4)$

B. $\cos^{-1}(4/5)$

C. $\cos^{-1}(3/5)$

D. none of these

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27. If PQR is an equilateral triangle inscribed in the auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b),$ and $P'Q'R'$ is the correspoinding triangle inscribed within the ellipse, then the centroid of triangle $P'Q'R'$ lies at center of ellipse focus of ellipse between focus and center on major axis none of these

A. center of ellipse

B. focus of ellipse

C. between focus and center on major axis

D. none of these

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28. Let $d_1 and d_2$ be the length of the perpendiculars drawn from the foci $S and S'$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent at any point P on the ellipse. Then, $SP : S'P = \ d_1 : d_2$ (b) $d_2 : d_1$ $d12 : d22$ (d) $\sqrt{d_1} : \sqrt{d_2}$

A. $d_1 : d_2$

B. $d_2 : d_1$

C. $d_1^2 : d_2^2$

D. $\sqrt{d_1} : \sqrt{d_1}$

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29. The slopes of the common tangents of the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ and the circle $x^2+y^2=3$ are ± 1 (b) $\pm\sqrt{2}$ (c) $\pm\sqrt{3}$ (d) none of these

A. ± 1

B. $\pm\sqrt{2}$

C. $\pm\sqrt{3}$

D. none of these

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30. If the line y=mx+c is a tangent to the ellipse $x^2+2y^2=4$, then the minimum possible value of c is (a) $-\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) 1

A. $-\sqrt{2}$

B. $\sqrt{2}$

C. 2

D. 1

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31. If $(\sqrt{3})bx + ay = 2ab$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the eccentric angle of the point of contact is $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

A. $\pi/6$

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

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32. Length of the perpendicular from the centre of the ellipse $27x^2 + 9y^2 = 243$ on a tangent drawn to it which makes equqal intercepts on the coordiantes axes is

A. $3/2$

B. $3\sqrt{2}$

C. $3/\sqrt{3}$

D. 6

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33. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its corresponding point Q on the auxiliary circle meet on the line $x = \frac{a}{e}$ (b) $x = 0$ $y = 0$ (d) none of these

A. x=a/e

B. x=0

C. y=0

D. none of these

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34. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is (a) $\frac{27}{4}$ (b) 18 (c) $\frac{27}{2}$ (d) 27

A. $27/4$ sq units

B. 9 sq. units

C. $27/2$ sq .units

D. 27 sq.units

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35. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the midpoint of the intercept made by the tangents between the coordinate axes is (a) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (b) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (c) $\frac{x^2}{2} + y^2 = 1$ (d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

A. $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

B. $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

C. $\frac{x^2}{2} + \frac{y^2}{4} = 1$

D. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

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36. The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and coordinates axes is

A. ab sq. units

B. $\frac{a^2 + b^2}{2}$ sq. units

C. $\frac{(a+b)^2}{2}$ sq. units

D. $\frac{a^2+ab+b^2}{3}$ sq. units

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37. Tangents are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b),$ and the circle $x^2+y^2=a^2$ at the points where a common ordinate cuts them (on the same side of the x-axis). Then the greatest acute angle between these tangents is given by $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ (b) $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$ $\tan^{-1}\left(\frac{2ab}{\sqrt{a-b}}\right)$ (d) $\tan^{-1}\left(\frac{2ab}{\sqrt{a+b}}\right)$

A. $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

B. $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$

C. $\tan^{-1}\left(\frac{2ab}{2\sqrt{a-b}}\right)$

D. $\tan^{-1}\left(\frac{2ab}{2\sqrt{a+b}}\right)$

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38. Let P be any point on a directrix of an ellipse of eccentricity e, S be the corresponding focus, and C the center of the ellipse. The line PC meets the ellipse at A. The angle between PS and tangent a A is α . Then α is equal to $\tan^{-1} e$ (b) $\frac{\pi}{2}$ $\tan^{-1}(1-e^2)$ (d) none of these

A. $\tan^{-1} e$

B. $\pi/2$

C. $\tan^{-1}(1-e^2)$

D. none of these

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39. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is 4 (b) 2 (c)

1 (d) none of these

A. 4

B. 2

C. 1

D. none of these

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40. If the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ make angles $\alpha and \beta$ with the major axis such that $\tan\alpha + \tan\beta = \gamma,$ then the locus of their point of intersection is $x^2 + y^2 = a^2$ (b) $x^2 + y^2 = b^2$ $x^2 - a^2 = 2\lambda xy$ (d) $\lambda(x^2 - a^2) = 2xy$

A. $x^2 + y^2 = a^2$

B. $x^2 + y^2 = b^2$

C. $x^2 - y^2 = 2\lambda xy$

D. $\lambda\left(x^2 - b^2\right) = 2xy$

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41. If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(b > a)$ and the parabola $y^2 = 4ax$ cut at right angles, then eccentricity of the ellipse is (a) $\frac{3}{5}$ (b) $\frac{2}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

A. $\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

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42. If $\alpha - \beta =$ constant, then the locus of the point of intersection of tangents at $P(a\cos\alpha, b\sin\alpha)$ and $Q(a\cos\beta, b\sin\beta)$ to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a circle (b) a straight line an ellipse (d) a parabola

A. a cricle

B. a straigth line

C. an ellipse

D. a parabola

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43. Find the llocus of the points of the intersection of tangents to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which make an angle 0.

A. parabola

B. circle

C. ellipse

D. straight line

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44. The length of the tangent of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ intercepted between auxiliary circle such that the portionof the tangent intercepted between the auxiliary circle subtends equal angles at foci is

A. 6

B. 3

C. 4

D. 8

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45. A tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ at any point P meets the line $x=0$ at a point Q. Let R passes through a fixed point. The fixed point is (3, 0) (b) (5, 0) (c) (0, 0) (d) (4, 0)

A. (3,0)

B. (5,0)

C. (0,0)

D. (4,0)

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46. about to only mathematics

A. 9

B. 13

C. 4

D. 5

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47. If the ellipse $\frac{x^2}{a^2-7}+\frac{y^2}{13=5a}=1$ is inscribed in a square of side length $\sqrt{2}a$, then a is equal to $\frac{6}{5}$ $(-\infty,-\sqrt{7})\cup\left(\sqrt{7},\frac{13}{5}\right)$ $(-\infty,-\sqrt{7})\cup\left(\frac{13}{5},\sqrt{7},\right)$ no such a exists

A. $6/5$

B. $11/10$

C. $13/10$

D. no such a exists

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48. The locus of the point of intersection of the tangent at the endpoints of the focal chord of the ellipse `(x^2)/(a^2)+(y^2)/(b^2)=1(b

A. circle

B. ellipse

C. hyperbola

D. pair of straight lines

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49. The length of the sides of the square which can be made by four perpendicular tangents $PQandPR$ are drawn to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$. Then the angle subtended by QR at the origin is $\frac{\tan^{-1}\left(\sqrt{6}\right)}{65}$ (b) $\frac{\tan^{-1}\left(4\sqrt{6}\right)}{65}$ $\frac{\tan^{-1}\left(8\sqrt{6}\right)}{65}$ (d) $\frac{\tan^{-1}\left(48\sqrt{6}\right)}{455}$

A. $\tan^{-1}.\frac{\sqrt{6}}{65}$

B. $\tan^{-1}.\frac{4\sqrt{6}}{65}$

C. $\tan^{-1}.\frac{8\sqrt{2}}{65}$

D. $\tan^{-1}.\frac{48\sqrt{6}}{455}$

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50. If tangents $PQandPR$ are drawn from a point on the circle $x^2 + y^2 = 25$ to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1, (b < 4)$, so that the fourth vertex S of parallelogram $PQRS$ lies on the circumcircle of triangle PQR, then the eccentricity of the ellipse is $\frac{\sqrt{5}}{4}$ (b) $\frac{\sqrt{7}}{4}$ (c) $\frac{\sqrt{7}}{4}$ (d) $\frac{\sqrt{5}}{3}$

A. $\sqrt{5}/4$

B. $\sqrt{7}/3$

C. $\sqrt{7}/4$

D. $\sqrt{5}/3$

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51. If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is inscribed in a rectangle whose length to breadth ratio is $2:1$, then the area of the rectangle is $4.\ \frac{a^2 + b^2}{7}$ (b) $4.\ \frac{a^2 + b^2}{3}$ $12.\ \frac{a^2 + b^2}{5}$ (d) $8.\ \frac{a^2 + b^2}{5}$

A. $4 \cdot \dfrac{a^2+b^2}{7}$

B. $4 \cdot \dfrac{a^2+b^2}{3}$

C. $12 \cdot \dfrac{a^2+b^2}{5}$

D. $8 \cdot \dfrac{a^2+b^2}{5}$

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52. An ellipse hase semi-major of length 2 and semi-minor axis of length 1. It slides between the coordinates axes in the first quadrant while mantaining contact with both x-axis and y-axis. The locus of the centre of the ellipse is

A. $x^2+y^2=8$

B. $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

C. $y^2=4x$

D. $x^2+y^2=5$

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53. The number of points on the ellipse $\frac{x^2}{50} + \frac{y^2}{20} = 1$ from which a pair of perpendicular tangents is drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is 0 (b) 2 (c) 1 (d) 4

A. 0

B. 2

C. 1

D. 4

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54. Let P_i and P_i' be the feet of the perpendiculars drawn from the foci $SandS$' on a tangent T_i to an ellipse whose length of semi-major axis is

20. If $\sum_{i=0}^{10} (SP_i)(S'\Pi') = 2560$, then the value of eccentricity is (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

A. $1/5$

B. $2/5$

C. $3/5$

D. $4/5$

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55. The normal at a variable point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of eccentricity e meets the axes of the ellipse at $Q and R\cdot$ Then the locus of the midpoint of QR is a conic with eccentricity e' such that (a)e' is independent of e (b) $e' = 1$ (c)$e' = e$ (d) $e' = \frac{1}{e}$

A. e' is independent of e

B. e'=1

C. e'=e

D. $e' = 1/e$

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56. Any ordinate MP of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ meets the auxiliary circle at Q. Then locus of the point of intersection of normals at $PandQ$ to the respective curves is $x^2 + y^2 = 8$ (b) $x^2 + y^2 = 34$ $x^2 + y^2 = 64$ (d) $x^2 + y^2 = 15$

A. $x^2 + y^2 = 8$

B. $x^2 + y^2 = 34$

C. $x^2 + y^2 = 64$

D. $x^2 + y^2 = 15$

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57. The number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from the point $P(0, 6)$ is one (b) two (c) three (d) four

A. one

B. two

C. three

D. four

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58. The line $y = mx - \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$ is normal to the ellise $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for all values of m belonging to $(0, 1)$ (b) $(0, \infty)$ (c) R (d) none of these

A. (0,1)

B. $(0, \infty)$

C. R

D. none of these

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LINKED COMPREHENSION TYPE

1. Consider an ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focus of the ellipse, radius of the incircle of ΔOCS_1 be 1 unit and $OS_1 = 6$ units.

Q. If perimeter of $\Delta\, OCS_1$ is p units, then the value of p is

A. 20 units

B. 10 units

C. 15 units

D. 25 units

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2. Consider an ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focus of the ellipse, radius of the incircle of ΔOCS_1 be 1 unit and $OS_1 = 6$units.

Q. The equation of the director circle of (E) is

A. $x^2 + y^2 = 48.5$

B. $x^2 + y^2 = \sqrt{97}$

C. $x^2 + y^2 = 97$

D. $x^2 + y^2 = \sqrt{48.5}$

Watch Video Solution

3. Consider an ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focus of the ellipse, radius of the incircle of ΔOCS_1 be 1 unit and $OS_1 = 6$ units.

Q. If perimeter of $\Delta\, OCS_1$ is p units, then the value of p is

A. $65\pi/4$

B. $64\pi/5$

C. 64π

D. 65π

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4. Consider the ellipse whose major and minor axes are x-axis and y-axis, respectively. If ϕ is the angle between the CP and the normal at point P on the ellipse, and the greatest value $\tan\phi$ is $\frac{3}{4}$ (where C is the centre of the ellipse). Also semi-major axis is 10 units . The eccentricity of the ellipse is

A. $1/2$

B. $1/3$

C. $\sqrt{3}/2$

D. none of these

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5. Consider the ellipse whose major and minor axes are x-axis and y-axis, respectively ϕ is the angle between CP and the normal at point P on the ellipse, and the greatest value of $\tan\phi$ is 3/2 (where C is the center of he ellipse) . Also, the length of the semi-major axis is 10 units

The eccentricity of the ellipse is

A. 50 units

B. 100 units

C. 25 units

D. none of these

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6. Consider the ellipse whose major and minor axes are x-axis and y-axis, respectively ϕ is the angle between CP and the normal at point P on the ellipse, and the greatest value of $\tan\phi$ is 3/2 (where C is the center of he ellipse) . Also, the length of the semi-major axis is 10 units

The eccentricity of the ellipse is

A. $x^2 + y^2 = 125$

B. $x^2 + y^2 = 150$

C. $x^2 + y^2 = 200$

D. none of these

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7. A cruve is respresented by $C = 21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$

The eccentricity of the cruve is

A. $1/3$

B. $1/\sqrt{3}$

C. $2/3$

D. $2/\sqrt{5}$

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8. A cruve is respresented by $C = 21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$

The eccentricity of the cruve is

A. $6, 2\sqrt{6}$

B. $5, 2\sqrt{5}$

C. $4, 4\sqrt{5}$

D. none of these

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9. A cruve is respresented by $C = 21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$

The center of the conic C is

A. (1,0)

B. (0,0)

C. (0,1)

D. none of these

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10. Show that for all real values of 't' the line $2tx + y\sqrt{1-t^2} = 1$ touches the ellipse.Find the eccentricity of the ellipse.

A. $2/3$

B. $\sqrt{3}/2$

C. $1/\sqrt{3}$

D. $1/2$

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11. For all real p, the line $2px + y\sqrt{1-p^2} = 1$ touches a fixed ellipse whose axex are the coordinate axes

The foci of the ellipse are

A. $(0, \pm\sqrt{3})$

B. $(0, +2/3)$

C. $(\pm\sqrt{3}/2, 0)$

D. none of these

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12. For all real p, the line $2px + y\sqrt{1-p^2} = 1$ touches a fixed ellipse whose axex are the coordinate axes

The locus of the point of intersection of perpendicular tangent is

A. $x^2 + y^2 = 5/4$

B. $x^2 + y^2 = 3/2$

C. $x^2 + y^2 = 2$

D. none of these

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13. Let S and S'' be the fociof the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentricity is i.e. P is a variable point on the ellipse. Consider the locus the incenter of $\Delta PSS''$

The eccentricity of the locus oc the P is (a) ellipse (b) hyperbola (a) parabola (d) circle

A. ellipse

B. hyperbola

C. parabola

D. circle

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14. Let S and S'' be the fociof the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentricity is i.e. P is a variable point on the ellipse. Consider the locus the incenter of $\Delta PSS''$

The eccentricity of the locus oc the P is (a) ellipse (b) hyperbola (a) parabola (d) circle

A. $\sqrt{\frac{2e}{1-e}}$

B. $\sqrt{\frac{2e}{1+e}}$

C. 1

D. none of these

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15. Let S and S'' be the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentricity is i.e. P is a variable point on the ellipse. Consider the locus of the incenter of $\Delta PSS''$ The maximum area of recatangle inscribed in the locus is

A. $\frac{2abe^2}{1+e}$

B. $\frac{2abe}{1-e}$

C. $\frac{2abe}{1+e}$

D. none of these

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16. Curves $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ intersect at four distinct points A,B,C and D. Their common tangents from a parallelogram PQRS. Q. If ABCD is sqyare, then the value of $25r^2$ is

A. $\frac{12}{5}\sqrt{2}$

B. $\frac{12}{5}$

C. $\frac{12}{5\sqrt{5}}$

D. none of these

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17. Curves $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ intersect at four distinct points A,B,C and D. Their common tangents from a parallelogram PQRS. Q. If ABCD is sqyare, then the value of $25r^2$ is

A. $\sqrt{120}$

B. $\sqrt{12}$

C. $\sqrt{15}$

D. none of these

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18. $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ interset at four distinct points A,B,C, and D. Their common tangents form a peaallelogram A'B'C'D'. If A'B'C'D' is a square, then the ratio of the area of circle C_1 to the area of circumcircle of $\Delta A'B'C'$ is

A. $9/16$

B. $3/4$

C. $1/2$

D. none of these

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19. Comprehension- I A coplanar beam of light emerging from a point source have equation $\lambda x - y + 2(1 + \lambda) = 0, \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having equation $\mu x - y + 2(1 - \mu) = 0, \mu \in R$. Foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$ The eccentricity of the ellipse is equal to

A. $1/3$

B. $1/\sqrt{3}$

C. $2/3$

D. none of these

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20. A coplanar beam of ligth emerging from a point source has the equation $\lambda x - y + 2(1 + a\lambda) - 0, \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having eqution $\mu x - y + 2(1 - \mu) = 0, \mu \in R$. Further, it is found that the foot of the perpendicular from the point (2,2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$ The eccentricity of the ellipse of is equal to

A. $4\sqrt{5}$

B. `2sqrt(5)~

C. $\sqrt{5}$

D. none of these

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21. A coplanar beam of ligth emerging from a point source has the equation $\lambda x - y + 2(1 + a\lambda) - 0, \lambda \in R.$ The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having eqution $\mu x - y + 2(1 - \mu) = 0, \mu \in R.$ Further, it is found that the foot of the perpendicular from the point (2,2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$ The eccentricity of the ellipse of is equal to

A. 9

B. 8

C. 7

D. 6

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22. The tagents at any point P of the circle $x^2 + y^2 = 16$ meets the tangents at a fixed point A at T. Point is joined to B, the other end of the diameter, through, A.

The locus of the intersection of AO and BT is a conic whosee eccentricity is

A. $1/2$

B. $1/\sqrt{2}$

C. $1/3$

D. $1/\sqrt{3}$

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23. The tagents at any point P of the circle $x^2 + y^2 = 16$ meets the tangents at a fixed point A at T. Point is joined to B, the other end of the diameter, through, A.

The sum of focal distance of any point on the curce is

A. 12

B. 16

C. 20

D. 8

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24. The tagents at any point P of the circle $x^2 + y^2 = 16$ meets the tangents at a fixed point A at T. Point is joined to B, the other end of the diameter, through, A.

Which of the following does not change by changing the radius of the circle ?

A. Coordinartes of foci

B. Length of he major axis

C. Eccentricity

D. Length of the minor axis

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25. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is such that it has the least area but contains the circle $(x-1)^2 + y^2 = 1$

The eccentricity of the ellipse is

A. $(2\sqrt{2}/3)$

B. $1\sqrt{3}$

C. $1/2$

D. none of these

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26. Area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is equal to

A. $x^2 + y^2 = 6.5$

B. $x^2 + y^2 = 5$

C. $x^2 + y^2 = 4.5$

D. none of these

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27. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is such that its has the least area but contains the circel $(x - 1)^2 + y^2 = 1$

The length of latus of ellipse is

A. 2 units

B. $\sqrt{2}$units

C. 3 units

D. 2.5 units

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28. A particle just clears a wall of height b at distance a and strikes the ground at a distance c from the point of projection. The angle of projection is (1) $\frac{\tan^{-1} b}{ac}$ (2) 45^o (3) $\frac{\tan^{-1}(bc)}{a(c-a)}$ (4) $\frac{\tan^{-1}(bc)}{a}$

A. $(3, 0)$ and $(0, 2)$

B. $\left(-8/5, 2\sqrt{161}/15\right)$ and $(-9/5, 8/5)$

C. $(8/5, 2\sqrt{161}/15$ and $(0, 2)$

D. $(3, 0)$ and $(-9/5, 8/5)$

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29. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at point A and B. Q. The coordinates of A and B are

A. $(5, 8/7)$

B. $(7/5, 25/8)$

C. $(11/, 8/5)$

D. $(8/25, 7/5)$

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30. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at point A and B. Q. The equation of the locus of the points whose distance from the point P and the line AB are equal, is

A. $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

B. $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

C. $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

D. $x^2 + y^2 - 2xy + 27 + 31y - 120 = 0$

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31. let $A(x_1, 0)$ and $B(x_2, 0)$ be the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ suppose parabola having vertex at origin and focus at B intersect the hyperbola at P in first quadrant and at point Q in fourth quadrant.

A. $\left(-\frac{9}{10}, 0\right)$

B. $\left(\frac{2}{3}, 0\right)$

C. $\left(\frac{9}{10}, 0\right)$

D. $\left(\frac{2}{3}, \sqrt{6}\right)$

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32. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF1NF2 is

A. $3:4$

B. $4:5$

C. $5:8$

D. $2:3$

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MATRIX MATCH TYPE

1. Match the following lists:

List I	List II
a. The distance between the points on the curve $4x^2 + 9y^2 = 1$, where the tangent is parallel to the line $8x = 9y$, is less than	**p.** 1
b. The sum of the distance between the foci of the curve $25(x+1)^2 + 9(y+2)^2 = 225$ from (−1, 0) is more than	**q.** 4
c. The sum of distances from the x axis of the points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4}$, where the normal is parallel to the line $2x + y = 1$, is less than	**r.** 7
d. Tangents are drawn from the points on the line $x - y + 2 = 0$ to the ellipse $x^2 + 2y^2 = 2$. Then all the chords of contact pass through the point whose distance from (2, 1/2) is more than	**s.** 5

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2. The tangents drawn from a point P to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ make angles α and β with the major axis, Now, match the following lists :

List I	List II
a. If $\alpha+\beta=\frac{c\pi}{2}$, $(c \in N)$, then the locus of p can be	p. Circle
b. If $\tan\alpha \tan\beta = c$, $(c \in R)$, then the locus of P can be	q. Ellipse
c. If $\tan\alpha + \tan\beta = c$, (where $c \in R$), then the locus of P can be	r. Hyperbola
d. If $\cot\alpha + \cot\beta = c$, (where $c \in R$), then the locus of P can be	s. Pair of straight lines

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3. Match the following lists:

List I	List II (which of the following functio appear in integration of functi in List I)
a. $\int \frac{x^2 - x + 1}{x^3 - 4x^2 + 4x} dx$	p. $\log \lvert x \rvert$
b. $\int \frac{x^2 - 1}{x(x-2)^3} dx$	q. $\log \lvert x - 2 \rvert$
c. $\int \frac{x^3 + 1}{x(x-2)^2} dx$	r. $\frac{1}{(x-2)}$
d. $\int \frac{x^5 + 1}{x(x-2)^3} dx$	s. x

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4. Match the following lists:

List I	List II
a. An ellipse passing through the origin has its foci (3, 4) and (6, 8). Then the length of its minor axis is	**p.** 8
b. If PQ is a focal chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes through $S \equiv (3, 0)$ and $PS = 2$, then the length of chord PQ is	**q.** $10\sqrt{2}$
c. If the line $y = x + K$ touches the ellipse $9x^2 + 16y^2 = 144$, then the difference of values of K is	**r.** 10
d. The sum of distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the foci is	**s.** 12

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5. Match the following lists and then choosen the correct code.

List I	List II
a. A stick of length 10 m slides on the co-ordinate axes. Then locus of the point dividing this stick from the x-axis in the ratio 6 : 4 is a curve whose eccentricty is e. Then $3e$ is equal to	**p.** $\sqrt{6}$
b. AA' is the major axis of the ellipse $3x^2 + 2y^2 + 6x - 4y - 1 = 0$ and P is a variable point on it. Then the greatest area of triangle APA' is	**q.** $2\sqrt{7}$
c. The distance between the foci of the curve represented by the equation $x = 1 + 4\cos\theta$, $y = 2 + 3\sin\theta$ is	**r.** $\frac{128}{3}$
d. Tangents are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ at the endpoints of the latus rectum. The area of the quadrilateral so formed is	**s.** $\sqrt{5}$

A. $\begin{matrix} a & b & c & d \\ p & r & q & q \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & r & p & s \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ r & p & s & q \end{matrix}$

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6. Match the following lists and then choosen the corrct code.

List I	List II
a. If the vertices of a rectangle of maximum area inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are extremities of latus rectum, then the eccentricity of the ellipse is	**p.** $\frac{2}{\sqrt{5}}$
b. If the extremities of the diameter of the circle $x^2 + y^2 = 16$ are the foci of the ellipse, then the eccentricity of the ellipse, if its size is just sufficient to contain the circle, is	**q.** $\frac{1}{\sqrt{2}}$
c. If the normal at point (6, 2) to the ellipse passes through its nearest focus (5, 2), having center at (4, 2), then its eccentricity is	**r.** $\frac{1}{3}$
d. If the extremities of the latus rectum of the parabola $y^2 = 24x$ are the foci of ellipse, and if the ellipse passes through the vertex of the parabola, then its eccentricity is	**s.** $\frac{1}{2}$

A. $\begin{matrix} a & b & c & d \\ q & p & s & r \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & r & s & q \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ q & q & s & p \end{matrix}$

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7. Match the conics in List I with the statements/expressions in List II

List I	List II
a. Circle	**p.** The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
b. Parabola	**q.** Points z in the complex plane satisfying $\|z+2\|-\|z-2\|=\pm 3$
c. Ellipse	**r.** Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), \ y = \frac{2t}{1+t^2}$
d. Hyperbola	**s.** The eccentricity of the conic lies in the interval $1 \le x < \infty$
	t. Points z in the complex plane satisfying $\text{Re}\,(z+1)^2 = \|z\|^2 + 1$

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8. Match the following lists :

List I	List II
a. Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1], x \neq \pm \frac{\sqrt{3}}{2}$ Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals	**p.** 1
b. Let $A_1, A_2, \ldots, A_n (n>2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k = 1, 2, \ldots n$. If $\left\|\sum_{k=1}^{n-1}(\vec{a}_k \times \vec{a}_{k+1})\right\| = \left\|\sum_{k=1}^{n-1}(\vec{a}_k \cdot \vec{a}_{k+1})\right\|$, then the minimum value of n is	**q.** 2
c. If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h is	**r.** 8
d. Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is	**s.** 9

A. $\begin{matrix} a & b & c & d \\ s & r & q & p \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & s & p & r \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & r & p & q \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ q & s & p & r \end{matrix}$

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NUMERICAL VALUE TYPE

1. If $x, y \in R,$ satisfies the equation $\frac{(x-4)^2}{4} + \frac{y^2}{9} = 1$, then the difference between the largest and the smallest valus of the expression $\frac{x^2}{4} + \frac{y^2}{9}$ is_____

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2. The value of a for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b),$ if the extremities of the latus rectum of the ellipse having positive ordinates lie on the parabola $x^2 = 2(y-2)$ is ___

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3. If the variable line $y = kx + 2h$ is tangent to an ellipse $2x^2 + 3y^2 = 6$, then the locus of $P(h, k)$ is a conic C whose eccentricity is 3. Then the value of $3e^2$ is_________

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4. about to only mathematics

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5. Points P and D are taken on the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$. If a , b ,c and d are the lengths of the side of quadrilateral PADB, where A and B are foci of the ellipse, then maximum value of (abcd) is ________

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6. If $a\left(x^2+y^2+2y+1\right)=(x-2y+3)^2$ is an ellipse and $a\in(b,\infty)$, then the value of b is __________

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7. If the midpoint of the chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$is $(0,3)$

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8. Let the distance between a focus and the corresponding directrix of an ellipse be 8 and the eccentricity be $\frac{1}{2}$. If the length of the minor axis is $k,$ then $\frac{\sqrt{3}k}{2}$ is ___________

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9. find the radius of director circle of auxilliary circle of ellipse $(3x+4y-1)^2+5(4x-3y+2)^2=250$ is _________

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10. Suppose $x and y$ are real numbers and that $x^2+9y^2-4x+6y+4=0$. Then the maximum value of $\frac{(4x-9y)}{2}$ is__________

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11. Rectangle ABCD has area 200.An ellipse with area 200π passes through A and C and has foci at B and D. Find the perimeter of the rectangle.

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12. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q.Let the tangents to the ellipse at P and Q meet at R. If $\Delta(h)$ Area of triangle $\Delta PQR,$ and $\Delta_1=\max_{\frac{1}{2}\leq h\leq 1}\Delta(h)$ and $\Delta_2=\min_{\frac{1}{2}\leq h\leq 1}\Delta(h)$ Then $\frac{8}{\sqrt{5}}\Delta_1-8\Delta_2$

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13. Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are $(f_1, 0) and (f_2, 0)$ where $f_1 > 0 and f_2 < 0.$ Let $P_1 and P_2$ be two parabolas with a common vertex at (0, 0) and with foci at $(f_1 .0) and$ (2f_2 , 0), respectively. LetT_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangents to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of $T_2,$ then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is

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JEE MAIN (SINGLE CORRECT ANSWER TYPE)

1. The ellipse $x^2 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinates axes, whicj in turn is inscribed in another ellipse that passes through the point (0,0). Then, the equation of the ellipse is

A. $x^2 + 16y^2 = 16$

B. $x^2 + 12y^2 = 16$

C. $4x^2 + 48y^2 = 16$

D. $4x^2 + 64y^2 = 48$

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2. The ellipse $x^2 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinates axes, whicj in turn is inscribed in another ellipse that passes through the point (0,0). Then, the equation of the ellipse is

A. $5x^2 + 3y^2 - 32 = -0$

B. $3x^2 + 5y^2 - 32 = -0$

C. $5x^2 + 3y^2 - 48 = -0$

D. $3x^2 + 5y^2 - 15 = -0$

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3. Statement 1: An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4 is y = 2x + 2\sqrt{3}$. Statement 2: If the line $y = mx + \dfrac{4\sqrt{3}}{m}, (m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

A. Statement 1 is false 2 is true

B. Statement 1 is true, statement 2 true , statemens 2 is a correct explanation for statement 1

C. Statement 1 is true, statement 2 is true: statement 2 is not a correct explanation for statement 1

D. Statement 1 is true, statement 2 is false.

4. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ as its semi-major axis. If the centre of the ellipse is the origin and its axes are the coordinate axes, then the equation of the ellipse is (1) $4x^2+y^2=4$ (2) $x^2+4y^2=8$ (3) $4x^2+y^2=8$ (4) $x^2+4y^2=16$

A. $4x^2+y^2=4$

B. $x^2+4y^2=8$

C. $4x^2+y^2=8$

D. $x^2+4y^2=16$

5. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9}$ and having its center (0, 3) is 4 (b) 3 (c) $\sqrt{12}$ (d) $\frac{7}{2}$

A. $x^2 + y^2 - 6y - 7 = 0$

B. $x^2 + y^2 - 6y + 7 = 0$

C. $x^2 + y^2 - 6y - 5 = 0$

D. $x^2 + y^2 - 6y + 5 = 0$

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6. The locus of the foot of the perpendicular from the centre of the ellipse $x^2 + 3y^2 = 3$ on any tangent to it is

A. $(x^2 - y^2)^2 = 6x^2 + 2y^2$

B. $(x^2 - y^2)^2 = 6x^2 - 2y^2$

C. $(x^2 + y^2)^2 = 6x^2 + 2y^2$

D. $\left(x^2+y^2\right)^2=6x^2-2y^2$

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7. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is (a) $\frac{27}{4}$ (b) 18 (c) $\frac{27}{2}$ (d) 27

A. $27/4$

B. 18

C. $27/2$

D. 27

Answer: D

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8. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = - 4, then the equation of the normal to it at (1, 3/2) is :

A. $x + 2y = 4$

B. $2y - x = 0$

C. $4x - 2y = 1$

D. $4x + 2y = 7$

Answer: B

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9. Two sets A and B are as under $A = |(a, b) \in R \times R: |a - 5| < 1 \text{ and } |b - 5| < 1\Big\} B = \Big[(a, b) \in R \times F$

(1) $B \subset A$ (2) $A \subset B$ (3) $A \cap B = \phi(anemptyset)(4) \neq ither$A sub B

n or B sub A\`

A. neither $A \subset B \text{nor} B \subset A$

B. $B \subset A$

C. $A \subset B$

D. $A \subset B = \phi$ (an empty set)

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JEE ADVANCED (SINGLE CORRECT ANSWER TYPE)

1. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets is auxiliary circle at the point M. Then the area of the triangle with vertices at $A, M,$ and O (the origin) is (a) 31/10 (b) 29/10 (c) 21/10 (d) 27/10

A. $31/10$

B. $29/10$

C. $21/10$

D. $27/10$

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2. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at points. $\left(\pm\frac{(3\sqrt{5})}{2} \pm \frac{2}{7}\right)$ (b) $\left(\pm\frac{(3\sqrt{5})}{2} \pm \frac{\sqrt{19}}{7}\right)$ $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (d) $\left(\pm 2\sqrt{3} \pm \frac{4\sqrt{3}}{7}\right)$

A. $(\pm 3\sqrt{5}/2, \pm 2/7)$

B. $(\pm 3\sqrt{5}/2, \pm\sqrt{19}/7)$

C. $\pm 2\sqrt{3}, \pm 17)$

D. $(\pm 2\sqrt{3}, \pm 4\sqrt{3}/7)$

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3. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

A. $\sqrt{2}/2$

B. $\sqrt{3}/2$

C. $1/2$

D. $3/4$

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