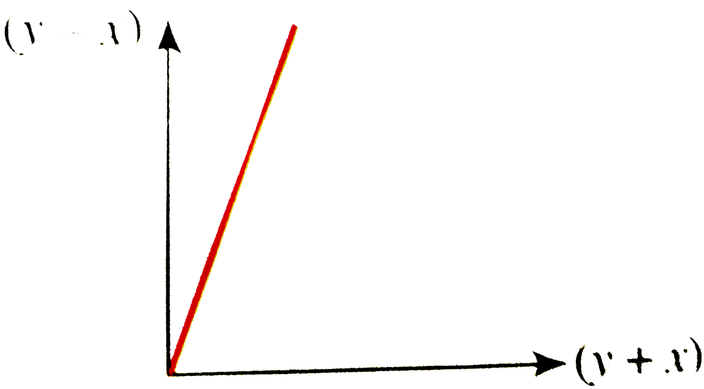
MATHS

BOOKS - CENGAGE PUBLICATION

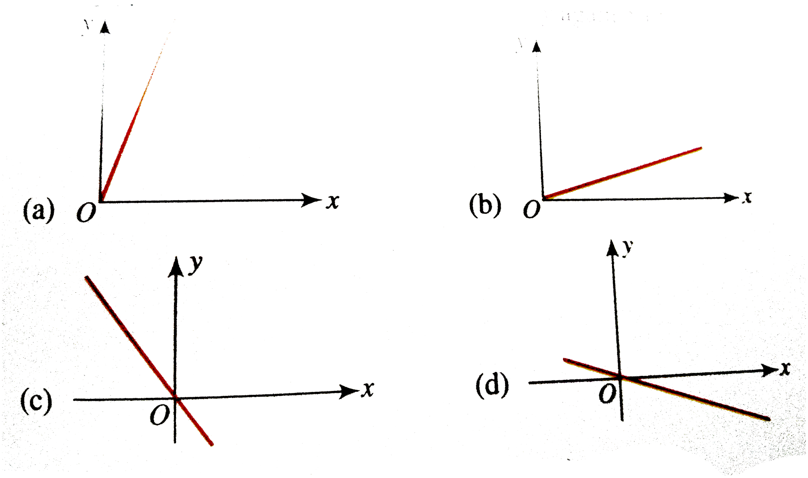
GRAPHS OF ELEMENTARY FUNCTIONS

Illustrations

1. The graph of $(y - x)$ against $(y + x)$ is shown below.



Which one of the following shows the graph of y against x ?



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2. Draw the graph of $f(x) = \dfrac{x^3 - x}{x^2 - 1}$.

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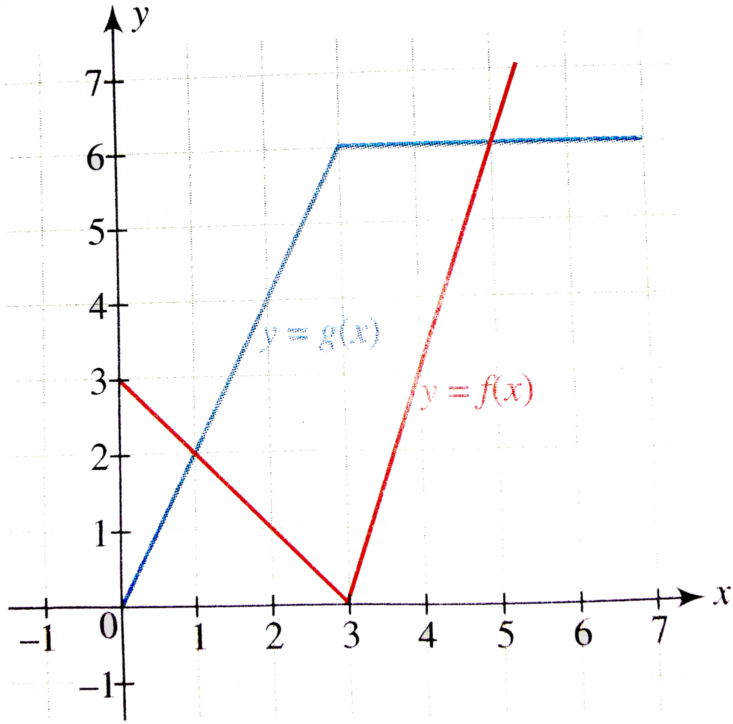
3. Graph of $y = f(x)$ and $y = g(x)$ is given in the following figure. If $h(x) = f(g(x))$, then find the value of $h'(2)$.



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4. Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all real x and y. If f'(0) exists and equals-1 and f(0)=1, find f(2)

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5. Sketch the regions satisfying the following inequalities:

(a) $x > 2$

(b) $|y| \geq 1$

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6. Shade the regions where points satisfy $|x - y| < 1$.

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7. Plot the region satisfying $|x| + |y| \leq 2 \text{ and } |x| + |y| > 2$.

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8. If $x < 2$, then find the values of x^2 graphically.

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9. If $x < -1$, then find the values of x^2 graphically.

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10. Draw the graph of $f(x) = \begin{cases} x^3, x^2 < 1 \\ x, x^2 \geq 1 \end{cases}$

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11. If $x > 2$, then find the values of $1/x$ graphically.

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12. If $x < -1$, then find the values of $1/x$ graphically.

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13. When $x > -2$, find the values of $1/x$.

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14. When $x < 3$, find the values of $1/x$.

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15. Draw the graph of $\frac{1}{x} + \frac{1}{y} = 1$.

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16. Draw the graph of $y = \frac{1}{x^2}$.

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17. Draw the graphs of following quadratic functions.

(i) $y = x^2 + x + 1$

(ii) $y = x^2 - 2x - 3$

(iii) $y = 2 + x - x^2$

(iv) $y = x - 1 - x^2$

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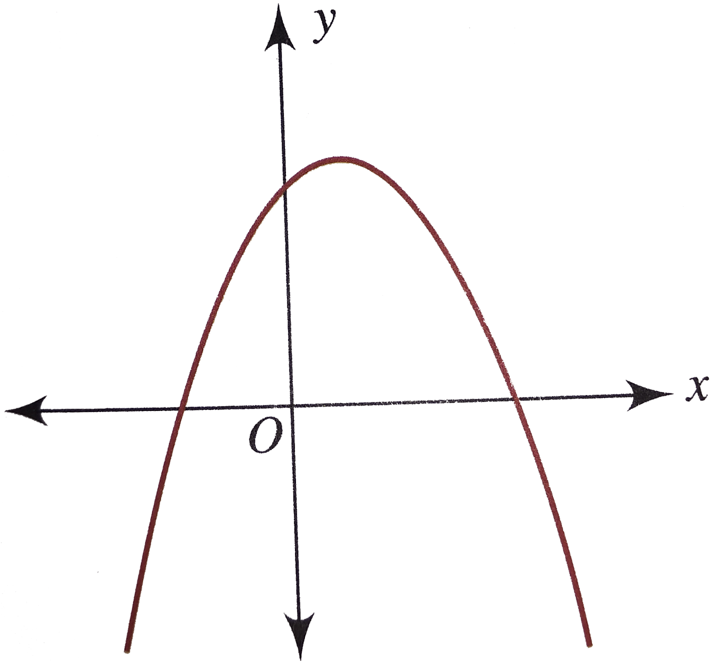
18. The following figure shows the graph of $f(x) = ax^2 + bx + c$, find the signs of a, b and c.



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19. Let $f(x) = 2x(2 - x), 0 \leq x \leq 2$. Then find the number of solutions of $f(f(f(x))) = \frac{x}{2}$.

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20. $f: R \to R$ is defined as $f(x) = \begin{cases} x^2 + kx + 3, & \text{for } x \geq 0 \\ 2kx + 3, & \text{for } x < 0 \end{cases}$. If $f(x)$ is injective, then find the values of k.

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21. If $f(x) = x^3 + 4x^2 + \lambda x + 1$ is a monotonically decreasing function of x in the largest possible interval $\left(-2, -\frac{2}{3}\right)$. Then

(a)$\lambda = 4$ (b) $\lambda = 2$ $\lambda = -1$ (d) λ has no real value

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22. For what real value of 'a' do the roots of $x^2 - 2x - (a^2 - 1) = 0$ lie between the-root $x^2 - 2(a+1)x + a(a-1) = 0$

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23. Value (s) of 'a' for which $ax^2 + (a-3)x + 1 < 0$ for at least one positive x.

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24. Consider the inequation $9^x - a3^x - a + 3 \leq 0$, where a is real parameter.

The given inequality has at least one negative solution for $a \in$

(a) $(-\infty, 2)$ (b) $(3, \infty)$ (c) $(-2, \infty)$ (d) $(2, 3)$

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25. Let a,b,c be real. If $ax^2 + bx + c = 0$ has two real roots α, β where $\alpha < -1$ and $\beta > 1$,then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0.$

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26. If $b > a$, then the equation $(x-a)(x-b)-1=0$ has

(a) Both roots in (a, b)

(b) Both roots in $(-\infty, a)$

(c) Both roots in $(b, +\infty)$

(d) One root in $(-\infty, a)$ and the other in $(b, +\infty)$

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27. When $x > -2$, find the values of $|x|$ graphically.

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28. When $x < 3$, find the values of $|x|$ graphically.

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29. If $2 \leq |x| \leq 5$, then find the values of x from the graph of $y = |x|$.

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30. Draw the graph of $f(x) = \frac{|x-1|}{x-1}$.

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31. Draw the graph of $x + |y| = 2y$ and check the differentiability.

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32. Draw the graph of $f(x) = (x+2)|x-1|$.

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33. Draw the graph of the function $f(x) = x - |x - x^2|, \ -1 \leq x \leq 1$ and find the points of non-differentiability.

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34. Solve : $x^2 - |x + 2| + x > 0$

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35. Draw the graph of $f(x) = |2x - 1| + |2x - 3|$. Find the range of the function.

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36. Draw the graph of $f(x) = |x| - |2x - 3|$. Find the range of the function.

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37. Let $f(x) = x + 2|x + 1| + 2|x - 1|$. Find the values of k if $f(x) = k$

(i) has exactly one real solution,

(ii) has two negative solutions,

(iii) has two solutions of opposite sign.

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38. $f(x) = |ax - b| + c|x| \forall x \in (-\infty, \infty)$, where $a > 0, b > 0, c > 0$. Find the condition if $f(x)$ attains the minimum value only at one point.

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39. about to only mathematics The tangent to the curve y=e^x drawn at the point (c,e^c) intersects the line joining the points (c−1,e^(c−1)) and (c+1,e^(c+1))

A. On the left of $x = c$

B. On the right of $x = c$

C. At no point

D. At all points

Answer:

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40. If a continuous function f defined on the real line R assume positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$, for all real x where k is a real constant.

For k > 0, the set of all values of k for which $y = ke^x - x = 0$has only one root is

A. No point

B. One point

C. Two points

D. More than two points

Answer:

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41. If a continuous function f defined on the real line R assume positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$, for all real x where k is a real constant.

For k > 0, the set of all values of k for which $y = ke^x - x = 0$has only one root is

A. a. $\frac{1}{e}$

B. b. 1

C. c. e

D. d. $\log_e 2$

Answer:

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42. If a continuous function f defined on the real line R assume positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R. Consider $f(x) = ke^x - x$, for all real x where k is a real constant.

For k > 0, the set of all values of k for which $y = ke^x - x = 0$has two distinct roots is

A. $\left(0, \frac{1}{e}\right)$

B. $\left(\frac{1}{e}, 1\right)$

C. $\left(\frac{1}{e}, \infty\right)$

D. $(0, 1)$

Answer:

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43. Find the number of solution of $2^x + 3^x + 4^x - 5^x = 0$

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44. Draw the graph of $y = \log_x \sqrt{x}$

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45. Find the number of roots of the equation $x \log_e x = 1$.

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46. If the graphs of the functions $y = \log_e x$ and $y = ax$ intersect at exactly two points,then find the value of a.

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47. draw the graph of $f(x) = x + [x]$, [.] denotes greatest integer function.

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48. Draw the graph of the function $f(x) = x - |x^2 - x| - 1 \leq x \leq 1$, where $[\cdot]$ denotes the greatest integer function. Find the points of discontinuity and non-differentiability.

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49. Draw the graph of $f(x) = [x^2], x \in [0, 2)$, where $[\cdot]$ denotes the greatest integer function.

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50. Draw the graph of $f(x) = [\sqrt{x}], x \in [0, 16)$, where $[\cdot]$ denotes the greatest ineger function.

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51. Draw the graph of $y = [x] + \sqrt{x - [x]}$, where$[\cdot]$ denotes the greatest ineger function.

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52. Draw the graph of $f(x) = [\log_e x], e^{-2} < x < 10$, where $[\cdot]$ represents the greatest integer function.

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53. Solve $x^2 - 4 - [x] = 0$ (where $[]$ denotes the greatest integer function).

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54. Sketch the region of relation $[x] + [y] = 5, x, y \geq 0$, where $[\cdot]$ denots the greatest integer function.

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55. Draw the graph of $f(x) = \{2x\}$, where $\{\cdot\}$ represents the fractional part function.

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56. Find the domain of $f(x) = \sqrt{|x| - \{x\}}$ (where $\{\cdot\}$ denots the fractional part of x.

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57. Solve : $x^2 = \{x\}$, where $\{x\}$ represents the fractional part function.

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58. Draw the graph of $y^2 = \{x\}$, where $\{\cdot\}$ represents the fractional part function.

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59. Draw the graph of $y = \frac{1}{\{x\}}$, where $\{\cdot\}$ denotes the fractional part function.

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60. Solve : $4\{x\} = x + [x]$(where $[\cdot]$ denotes the greatest integer function and $\{\cdot\}$ denotes the fractional part function.

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61. In the following graph, state the absolute and the local maximum and minimum values of the function.

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62. Draw the graph of $f(x) = \operatorname{sgn}(x^3 - x)$.

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63. Draw the graph of $f(x) = \operatorname{sgn}(\log_e x)$.

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64. Let a function $f(x)$ be defined in $[-2, 2]$ as

$$f(x) = \begin{cases} \{x\}, & -2 \leq x < -1 \\ |\text{sgn } x|, & -1 \leq x \leq 1 \\ \{-x\}, & 1 < x \leq 2 \end{cases}$$ where $\{x\}$ and sgn x denote fractional part and signum functions, respectively. Then find the area bounded by the graph of $f(x)$ an the x-axis.

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65. Let $f: R \to R$ be defined as $f(x) = e^{\text{sgn } x} + e^{x^2}$. Then find the range of the function, and also indentify the type of the function : one-one or many-one.

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66. Draw the graph of the function $f(x) = \max\{x, x^2\}$ and write its equivalent definition.

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67. Let $f: R \rightarrow R$ be a function defined by $f(x) = \max . \{x, x^3\}$. The set of all points where $f(x)$ is NOT differenctiable is

(a) $\{-1, 1\}$

(b) $\{-1, 0\}$

(c) $\{0, 1\}$

(d) $\{-1, 0, 1\}$

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68. Find the equivalent definition of $f(x) = maxx^2, (-x)^{2^{\cdot}}, 2x(1-x) whre 0 \leq x \leq 1$

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69. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: R \rightarrow R$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \leq 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

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70. Sketch the region of the points satisfying $\max\{|x|, |y|\} \leq 4$

.

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71. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \leq 100\}$ and $B = \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then the area of the region $A \cap B$ is

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72. Draw the graphs of the following parabolas :

(i) $x = y^2 - 2y - 3$

(ii) $x = 6 + y - y^2$

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73. Find the number of roots of the equation $e^x = \sqrt{-x}$.

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74. Let $g(x) = \sqrt{x - 2k}, \forall 2k \leq x < 2(k+1)$, where $k \in$ integer. Check whether $g(x)$ is periodic or not.

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75. Plot the region in the first quadrant in which points are nearer to the origin than to the line $x = 3$.

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76. Draw the graph of $y = \sqrt{x^2 - 1}$

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77. Draw the graph of $y = -\sqrt{6 - 3x^2}$

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78. The eccentricity of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$ is a. $\frac{25}{16}$ b. $\frac{4}{5}$ c. $\frac{16}{25}$ d. $\frac{5}{4}$

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79. Find the area enclosed by the curves $y = \sqrt{x}$ and $x = -\sqrt{y}$ and the circle $x^2 + y^2 = 2$ above the x-axis.

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80. Consider a square with vertices at $(1, 1), (-1, 1), (-1, -1), and (1, -1)$. Set S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

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Exercises

1. Draw the graph of $y = \frac{1}{(1/x)}$.

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2. (a) Draw the graph of

$$f(x) = = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, & \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2,3,\ldots \\ 0, & x = 0 \end{cases}$$

(b) Sketch the region $y \leq -1$.

(c) Sketch the region $|x| < 3$.

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3. Sketch the regions which points satisfy $|x + y| \geq 2$.

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4. Sketch the region satisfying $|x| < |y|$.

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5. For a point P in the plane, let $d_1(P) and d_2(P)$ be the distances of the point P from the lines $x - y = 0 and x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4,$ is

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6. Draw the graph of $y = \frac{x-1}{x-2}$.

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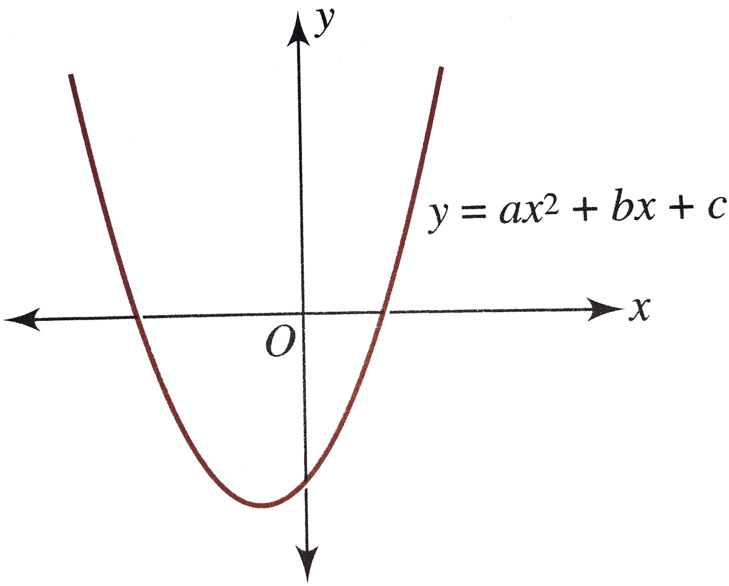
7. The following figure shows the graph of $f(x) = ax^2 + bx + c$, then find the sign of values of a, b and c.



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8. The entire graphs of the equation $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

A. $k < 7$

B. $-5 < k < 7$

C. $k > \; -5$

D. None of these

Answer:

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9. If $x^2 + 2ax + a < 0 \forall x \in [1, 2]$, the find the values of a.

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10. Draw the graph of $f(x) = x|x|$.

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11. Draw the graph of the function: Solve $\left|\frac{x^2}{x-1}\right| \leq 1$ using the graphical method.

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12. Draw the graph of $y = \left|x^2 - 2x\right| - x$.

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13. Draw the graph of $y = \begin{cases} 2^x, & x^2 - 2x \leq 0 \\ 1 + 3.5x - x^2 & x^2 - 2x > 0 \end{cases}$

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14. Draw the graph of $f(x) = |2x - 1| + |2x - 3|$. Find the range of the function.

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15. Draw the graph of $f(x) = y = |x - 1| + 3|x - 2| - 5|x - 4|$ and find the values of λ for which the equation $f(x) = \lambda$ has roots of opposite sign.

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16. Find the set of real value(s) of a for which the equation $|2x + 3| + 2x - 3| = ax + 6$ has more than two solutions.

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17. Draw the graph of $y = 2^{\frac{(|x| + x)}{x}}$.

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18. Draw the graph of $y = x^{\frac{1}{\log_e x}}$.

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19. Find the number of solutions to the equation $x + \log_e x = 0$.

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20. draw the graph of $f(x) = x + [x]$, [.] denotes greatest integer function.

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21. Given $f(x)$ is a periodic function with period 2 and it is defined as

$$f(x) = \begin{cases} \left[\cos\frac{\pi x}{2}\right] + 1, & 0 < x < 1 \\ 2 - x, & 1 \le x < 2 \end{cases}$$

Here $[\cdot]$ represents the greatest integer $\le x$. If $f(0) = 1$, then draw the graph of the function for $x \in [-2, 2]$.

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22. Draw the region of relation $[x][y] = 6, x, y \ge 0$. Here $[\cdot]$ denotes the greatest integer function.

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23. $\lim_{x \to c} f(x)$ does not exist for

wher $[.]$ represent greatest integer function $\{.\}$ represent fractional part function

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24. Let $f(x) = \dfrac{[x]+1}{\{x\}+1}$ for $f : \left[0, \dfrac{5}{2}\right) \rightarrow \left(\dfrac{1}{2}, 3\right]$, where $[\cdot]$ represents the greatest integer function and $\{\cdot\}$ represents the fractional part of x.

Draw the graph of $y = f(x)$. Prove that $y = f(x)$ is bijective.

Also find the range of the function.

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25. Draw the graph of $y = 2^{\{x\}}$, where $\{\cdot\}$ represents the fractional part function.

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26. The area of the region containing the points (x, y) satisfying $4 \leq x^2 + y^2 \leq 2(|x| + |y|)$ is (a)$8squnits$ (b) $2squnits$ (c)$4\pi squnits$ (d) $2\pi squnits$

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27. Draw the graph of $y = -\sqrt{x^2+2}$

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28. Draw the graph of $y = |x|^{\frac{1}{2}}$ for $-1 \leq x < 1$.

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29. Draw the graph of $f(x) = \text{sgn}(\log_{0.5} x)$.

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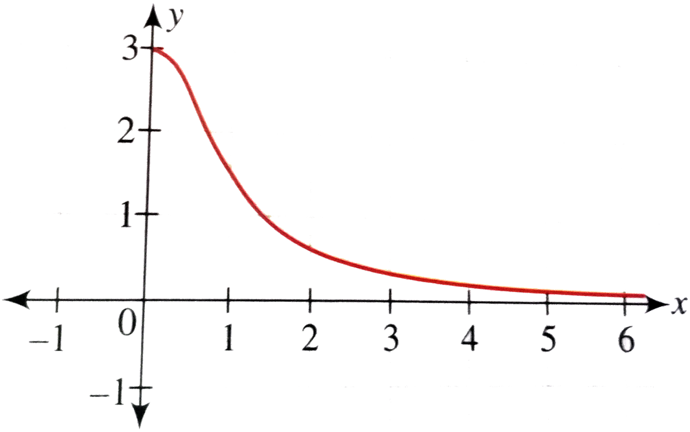
30. The graph of $y = f(x)$ is as shown in the following figure.

Draw the graph of $y = [f(x)]$.



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31. Discuss the continuity of $f(x) = (\lim)_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$

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32. An even periodic function $f: R \to R$ with period 4 is such that

$$f(x) = \begin{cases} \max.\left(|x|, x^2\right), & 0 \le x < 1 \\ x, & 1 \le x \le 2 \end{cases}$$. Then draw the graph of $y = f(x)$ for $x \in R$

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33. The function $f(x) = \max\{(1-x), (1+x), 2\}, x \in (-\infty, \infty)$ is

A. Continuous at all points

B. Differentiable at all points

C. Differentiable at all points except at $x = 1$ and $x = -1$

D. Continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous

Answer:

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34. If $f(x) = \min$ { 1, x^2 , x^3}` then

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