



## MATHS

### BOOKS - CENGAGE PUBLICATION

# GRAPHS OF TRIGONOMETRIC FUNCTIONS

#### Illustration

1. Plot  $y = \sin x$  and  $y = \sin 2x$ .



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2. Plot  $y = \sin x$  and  $y = \frac{\sin x}{2}$



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3. Draw the graph of  $y = \tan(3x)$



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4. Draw the graph of  $y = \sec^2 x - \tan^2 x$ . Is  $f(x)$  periodic? If yes, what is its fundamental period?



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5. Draw the graph of  $y = \sec^2 x - \tan^2 x$ . Is  $f(x)$  periodic? If yes, what is its fundamental period?

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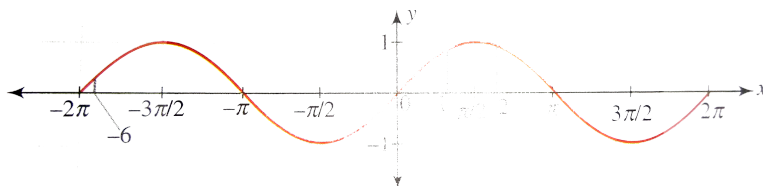
6. Which of the following is highest?

(a) cosec 1

(b) cosec 2

(c) cosec 4

(d) cosec (-6)



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7. Draw the graph of the function

$$y = f(x) = \lim_{n \rightarrow \infty} \cos^{2n} x \text{ and find its period.}$$



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8. Find the number of solution to the equation

$$x^2 \tan x = 1, x \in [0, 2\pi].$$



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9. Solve  $\tan x > \cot x$ , where  $x \in [0, 2\pi]$ .



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10. Let  $f(x) = x \sin \pi x$ ,  $x > 0$ . Then for all natural numbers  $n$ ,  $f'(x)$  vanishes at (a) A unique point in the interval  $\left(n, n + \frac{1}{2}\right)$  (b) a unique point in the interval  $\left(n + \frac{1}{2}, n + 1\right)$  (c) a unique point in the interval  $(n, n + 1)$  (d) two points in the interval  $(n, n + 1)$

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11. If  $0 < \alpha < \frac{\pi}{3}$ , then prove that  $\alpha(\sec \alpha) < \frac{2\pi}{3}$ .

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12. Draw the graph of  $y = [\sin x]$ ,  $x \in [0, 2\pi]$ , where  $[\cdot]$  represents the greatest integer function.

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13. Draw the graph of  $f(x) = [\tan x]$ ,  $0 \leq x \leq 5\pi/12$ , where  $[\cdot]$  represents the greatest integer function.

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14. Draw the graph of  $f(x) = e^{\sin x}$ .

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15. Draw the graph  $y = \sin^2 x$ .



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16. Draw the graph of  $y = (\sin 2x) \sqrt{1 + \tan^2 x}$ , find its domain and range.



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17. Draw the graph of  $y = \sin^3 x$ .



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18. Draw the graph of  $y = \sin^3 x$ .



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19. Draw the graph of

$$f(x) = |\sin x| + |\cos x|, x \in \mathbb{R}.$$



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20. Draw the graph of  $f(x) = \sqrt{\sin x}$ .



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21. Draw the graph of  $y = \frac{\cos\left(|x| + \frac{\pi}{2}\right)}{\sin x}$ . Is the function periodic?

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22. Draw the graph of  $f(x) = \cos \pi[x]$ , where  $[\cdot]$  represents the greatest integer function. Find the period of the function.

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23. Draw the graph of

$$f(x) = \sec x + \operatorname{cosec} x, x \in (0, 2\pi) - \{\pi/2, \pi, 3\pi/2\}$$

Also find the values of 'a' for which the equation  $\sec x + \operatorname{cosec} x = a$  has two distinct root and four distinct roots.

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24. if:  $f(x) = \frac{\sin x}{\sqrt{1 + \tan^2 x}} - \frac{\cos x}{\sqrt{1 + \cot^2 x}}$ , then  
find the range of  $f(x)$

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25. Find the area bounded by the following curve :

(i)  $f(x) = \sin x, g(x) = \sin^2 x, 0 \leq x \leq 2\pi$

(ii)  $f(x) = \sin x, g(x) = \sin^4 x, 0 \leq x \leq 2\pi$



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26. Write the equivalent (piecewise) definition of

$$f(x) = \operatorname{sgn}(\sin x).$$



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27. Draw the graph of  $f(x) = \{\sin x\}$ , where  $\{\cdot\}$  represents the fractional part function.



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28.

Draw a graph of  $f(x) = \sin\{x\}$ , where  $\{x\}$  represents the greatest integer function.



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29. Draw the graph of

$f(x) = \max\{2 \sin x, 1 - \cos x\}$ ,  $x \in (0, \pi)$ .

Also find the range of

$g(x) = \min\{2 \sin x, 1 - \cos x\}$ ,  $x \in (0, \pi)$



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30. Draw the graph of  $y = \log_e(\sin x)$ .



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31. Draw the graph of  $[y] = \sin x, x \in [0, 2\pi]$  where

$[\cdot]$  denotes the greatest integer function



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32. Draw the graph of  $y = x \sin x$ .



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33. Draw the graph of  $y = e^x \sin 2\pi x$ .



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34. Let  $[x]$  denotes the greatest integer less than or equal to  $x$ . If  $f(x) = [x \sin \pi x]$ , then  $f(x)$  is

- A. a. continuous at  $x=0$
- B. b. continuous in  $(-1,0)$
- C. c. differentiable at  $x=1$
- D. d. differentiable in  $(-1,1)$

**Answer:**



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35. Evaluate :  $\left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} \right]$ , where  $[\cdot]$  represents the greatest integer function.



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36. Discuss the extrema of

$$f(x) = \frac{x}{1 + x \tan x}, x \in \left(0, \frac{\pi}{2}\right)$$



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37. Find the values of  $a$  if equation

$$1 - \cos x = \frac{\sqrt{3}}{2}|x|xa, x \in (0, \pi), \text{ has exactly one}$$

solution.



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38. Find the number of solutions of the equation

$$\sin x = x^2 + x + 1.$$



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39.

Prove

that

$$\sin x + 2x \geq \frac{3x(x+1)}{\pi}, \forall x \in \left[0, \frac{\pi}{2}\right] \text{ (Justify the$$



inequality, if any used).



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**40.** Find the ratio of the areas of two regions of the curve  $C_1 \equiv 4x^2 + \pi^2 y^2 = 4\pi^2$  divided by the curve  $C_2 \equiv y = -\left(\operatorname{sgn}\left(x - \frac{\pi}{2}\right)\right)\cos x$  (where  $\operatorname{sgn}(x) = \operatorname{signum}(x)$ ).



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**41.** Solve  $\tan x < 2$ .



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42. Solve  $\sin x > \frac{1}{2}$  or find the domain of

$$f(x) = \frac{1}{\sqrt{1 + 2\sin x}}$$



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43. Solve :  $2 \cos^2 \theta + \sin \theta \leq 2$ , where  $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$ .



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44. Solve  $\sin \theta + \sqrt{3} \cos \theta \geq 1$ ,  $\pi$



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45. Let  $f(x) = \begin{cases} x^2 + 3x, & -1 \leq x < 0 \\ -\sin x, & 0 \leq x < \pi/2 \\ -1 - \cos x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$ .

Draw the graph of the function and find the following

(a) Range of the function

(b) Point of inflection

(c) Point of local minima



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46. If  $0 < x_1 < x_2 < x_3 < \pi$ , then prove that

$$\sin\left(\frac{x_1 + x_2 + x_3}{3}\right) > \frac{\sin x_1 + \sin x_2 + \sin x_3}{3}.$$

Hence or otherwise prove that if  $A, B, C$  are angles

of a triangle, then the maximum value of  $\sin A + \sin B + \sin C$  is  $\frac{3\sqrt{3}}{2}$



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## Exercises

1. Draw the graph of  $y = \operatorname{cosec}^2 x - \cot^2 x$ . Is  $f(x)$  periodic? If yes, what is its fundamental period?



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2. Draw the graph of  $y = \cos \pi x$ .

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3. Draw the graph of  $y = \cos^2 x$ .

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4. Find the value of  $x$  for which

$f(x) = \sqrt{\sin x - \cos x}$  is defined,  $x \in [0, 2\pi)$ .

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5. Draw the graph of  $y = \tan^2 x$ .

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6. Draw the graph of  $y = \sin x + \cos x$ ,  $x \in [0, 2\pi]$ .



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7. Draw the graph of  $y = [\cos x]$ ,  $x \in [0, 2\pi]$ , where  $[\cdot]$  represents the greatest integer function.



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8. Draw the graph of  $y = \sin \pi \sqrt{x}$ .



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9. The number of roots of the equation  $x \sin x = 1$ ,  $x \in [-2\pi, 0) \cup (0, 2\pi]$  is (a) 2 (b) 3 (c) 4 (d) 0

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10. Evaluate :  $\left[ \lim_{x \rightarrow 0} \frac{\tan x}{x} \right]$ , where  $[\cdot]$  represents the greatest integer function.

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11. For  $f(x) = \sin x - x^2 + 1$ , check whether the function is increasing, decreasing or has a point of extremum ?



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12. Draw the graph of the function  $f(x) = \max\{\sin x, \cos 2x\}$ ,  $x \in [0, 2\pi]$ . Write the equivalent definition of  $f(x)$  and find the range of the function.



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13. Draw the graph of  $[y] = \sin x, x \in [0, 2\pi]$  where  $[\cdot]$  denotes the greatest integer function

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14. The total number of solution of  $\sin\{x\} = \cos\{x\}$  (where  $\{ \}$  denotes the fractional part) in  $[0, 2\pi]$  is equal to 5 (b) 6 (c) 8 (d) none of these

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15. Draw the graph of  $f(x) = |\sin x| + |\cos x|, x \in R.$



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16. Find the number of solutions to

$$\cos x = \frac{x}{10}, x > 0.$$



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17. The number of solutions of

$$\tan x - mx = 0, m > 1, \text{ in } \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \text{ is 1 (b) 2 (c)}$$

3 (d)  $m$



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18. Find the number of solutions to

$$\log_e |\sin x| = -x^2 + 2x \text{ in } \left[ -\frac{\pi}{2}, \frac{3\pi}{2} \right].$$



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19. Solve :  $\cos x \leq -\frac{1}{2}$ .



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20. Prove that the least positive value of  $x$ , satisfying

$$\tan x = x + 1, \text{ lies in the interval } \left( \frac{\pi}{4}, \frac{\pi}{2} \right).$$



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21. Draw the graph of  $y = \frac{x^2}{10} \sin x$ .



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22. Draw the graph of  $y = \frac{\sin x}{x}$ .



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