MATHS

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HYPERBOLA

Illustration

1. If the base of a triangle and the ratio of tangent of half of base angles are given, then identify the locus of the opposite vertex.

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2. The locus of the center of a variable circle which always touches two given circles externally is

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3. The equation of one of the directrices of a hyperboda is $2x + y = 1$, the corresponding focus is (1, 2) and $e = \sqrt{3}$. Find the equation of the hyperbola.

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4. The eccentricity of the conic represented by $2x^2 + 5xy + 2y^2 + 11x - 7y - 4 = 0$ is

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5. If equation $\left|\sqrt{(x - \tan\theta)^2 + (y - \sqrt{3}\tan\theta)^2} - \sqrt{(x - 2\tan\theta)^2 + y^2}\right| = 2, \theta \in [0, \pi]$ represents hyperbola, then find the value of θ.

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6. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$ then equation of the hyperbola is

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7. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then find the value

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8. If hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ passes through the focus of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then find the eccentricity of hyperbola.

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9. Find the eccentricity of the hyperbola given by equations $x = \frac{e^t + e^{-t}}{2} and y = \frac{e^t - e^{-t}}{3}, t \in R$.

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10. An ellipse and a hyperbola have their principal axes along the coordinate axes and have a common foci separated by distance $2\sqrt{3}$. The difference of their focal semi-axes is equal to 4. If the ratio of their eccentricities is 3/7 , find the equation of these curves.

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11. If the latus rectum subtends a right angle at the center of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then find its eccentricity.

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12. Find the equation of hyperbola if centre is (1, 0), one focus is (6, 0) and transverse axis 6 .

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13. Two straight lines rotate about two fixed points. If they start from their position of coincidence such that one rotates at the rate double that of the other. Then find the locus of their point of intersection of two straight lines

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14. Find the coordinates of the foci, the eccentricity, the latus rectum, and the equations of directrices for the hyperbola $9x^2 - 16y^2 - 72x + 96y - 144 = 0$

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15. Find the coordinates of the foci and the center of the hyperbola, $x^2 - 3y^2 - 4x = 8$

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16. about to only mathematics

A. x^2 +2y^2 <_1

B. max{(|x|,|y|)}<_1

C. x^2 -y^2<_1

D. y^2 -x<_0

Answer:

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17. Find the locus of the midpoints of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to y = 2x.

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18. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the center of the hyperbola, then find the range of the eccentricity e of the hyperbola.

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19. If $(a\sec\theta; b\tan\theta)$ and $(a\sec\phi; b\tan\phi)$ are the ends of the focal chord of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then prove that $\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\phi}{2}\right)=\frac{1-e}{1+e}$

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20. Find the point on the hyperbola $x^2-9y^2=9$ where the line $5x+12y=9$ touches it.

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21. Find the value of m for which $y=mx+6$ is a tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{49}=1$

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22. Find the equation of tangents to the curve $4x^2 - 9y^2 = 1$ which are parallel to $4y = 5x + 7$.

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23. If it is possible to draw the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having slope 2,then find the range of eccentricity

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24. Find the equation of tangents to hyperbola $x^2 - y^2 - 4x - 2y = 0$ having slope 2.

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25. Find the minimum value of $(2 - a - 4\sec\theta)^2 + (a - 3\tan\theta)^2$, $a \in R$.

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26. Find the locus of the-mid points of the chords of the circle $x^2 + y^2 = 16$, which are tangent to the hyperbola $9x^2 - 16y^2 = 144$

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27. Find the equation of tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at $(2, 1)$.

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28. A tangent to the hyperbola $x^2 - 2y^2 = 4$ meets x-axis at P and y-aixs at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is origin).Find the locus of R.

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29. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2).

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30. Find the equation of pair of tangents drawn from point (4, 3) to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Also, find the angle between the tangents.

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31. Tangents drawn from the point (c, d) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ make angles α and β with the x-axis.

If $\tan\alpha\tan\beta = 1$, then find the value of $c^2 - d^2$.

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32. On which curve does the perpendicular tangents drawn to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ intersect?

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33. Find the equation of hyperbola having foci S(2, 1) and S'(10, 1) and a straight line $x + y - 9 = 0$ as its tangent.

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34. Find the eccentricity of the hyperbola with asymptotes $3x + 4y = 2$ and $4x - 3y = 2$.

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35. Find the equation of the hyperbola which has $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ as its asymptotes and which passes through the origin.

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36. Find the equation of the asymptotes of the hyperbola $3x^2 + 10xy + 9y^2 + 14x + 22y + 7 = 0$

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37. If a hyperbola passing through the origin has $3x - 4y - 1 = 0$ and $4x - 3y - 6 = 0$ as its asymptotes, then find the equation of its transvers and conjugate axes.

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38. Show that the locus represented by $x = \frac{1}{2}a\left(t + \frac{1}{t}\right), y = \frac{1}{2}a\left(t - \frac{1}{t}\right)$ is a rectangular hyperbola.

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39. If two distinct tangents can be drawn from the point $(\alpha, \alpha+1)$ on different branches of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then find the values of α.

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40. From a point $P(1, 2)$, two tangents are drawn to a hyperbola H in which one tangent is drawn to each arm of the hyperbola. If the equations of the asymptotes of hyperbola H are $\sqrt{3}x - y + 5 = 0$ and $\sqrt{3}x + y - 1 = 0$, then the eccentricity of H is 2 (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

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41. Find the equation of normal to the hyperbola $x^2 - 9y^2 = 7$ at point (4, 1).

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42. Find the equation of normal to the hyperbola $3x^2 - y^2 = 1$ having slope $\frac{1}{3}$.

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43. If the normal at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2a^2} = 1$ meets the transvers axis at G, then prove that $AG\dot{A}'G = a^2\left(e^4\sec^2\theta - 1\right)$, where $AandA'$ are the vertices of the hyperbola.

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44. Normal are drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at point θ_1 and θ_2 meeting the conjugate axis at $G_1 and G_2$, respectively. If $\theta_1 + \theta_2 = \frac{\pi}{2}$, prove that $CG_1 \cdot CG_2 = \frac{a^2e^4}{e^2 - 1}$, where C is the center of the hyperbola and e is the eccentricity.

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45. Let P(6,3) be a point on the hyperbola parabola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$If the normal at the point intersects the x-axis at (9,0), then the eccentricity of the hyperbola is

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46. Prove that any hyperbola and its conjugate hyperbola cannot have common normal.

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47. A ray emerging from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8. Find the equation of the reflected ray after the first reflection if point P lies in the first quadrant.

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48. Normal to a rectangular hyperbola at P meets the transverse axis at N. If foci of hyperbola are S and S', then find the value of $\frac{SN}{SP}$.

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49. Consider hyperbola xy = 16 to find the equation of tangent at point (2, 8) .

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50. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

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51. If $A, B, and C$ are three points on the hyperbola $xy = c^2$ such that AB subtends a right angle at C, then prove that AB is parallel to the normal to the hyperbola at point C.

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52. Prove that product of parameters of four concyclic points on the hyperbola $xy = c^2$ is 1. Also, prove that the mean of these four concyclic points bisects the distance between the centres of the hyperbola and the circle.

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Solved Examples

1. A variable line $y = mx - 1$ cuts the lines $x = 2y$ and $y = -2x$ at points $AandB$. Prove that the locus of the centroid of triangle $OAB(O$ being the origin) is a hyperbola passing through the origin.

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2. Let P be a point on the hyperbola $x^2 - y^2 = a^2$, where a is a parameter, such that P is nearest to the line $y = 2x$. Find the locus of P.

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3. Show that the midpoints of focal chords of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ lie on another similar hyperbola.

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4. From the center C of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, perpendicular CN is drawn on any tangent to it at the point $P(a \sec \theta, b \tan \theta)$ in the first quadrant. Find the value of θ so that the area of CPN is maximum.

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5. Semi transverse axis of hyperbola is 5. Tangent at point P and normal to this tangent meet conjugate axis at A and B, respectively. The circle on AB as diameter passes through tow fixed points, the distance between which is 20. Find the eccentricity of hyperbola.

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6. The exhaustive set of values of α^2 such that there exists a tangent to the ellipse $x^2 + \alpha^2 y^2 = \alpha^2$ and the portion of the tangent intercepted by the hyperbola $\alpha^2 x^2 - y^2 = 1$ subtends a right angle at the center of the curves is:

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7. Prove that the part of the tangent at any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ intercepted between the point of contact and the transvers axis is a harmonic mean between the lengths of the perpendiculars drawn from the foci on the normal at the same point.

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8. If one of varying central conic (hyperbola) is fixed in magnitude and position, prove that the locus of the point of contact of a tangent drawn to it from a fixed point on the other axis is a parabola.

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9. If normal at P to a hyperbola of eccentricity e intersects its transverse and conjugate axes at L and M, respectively, then prove that the locus of midpoint of LM is a hyperbola. Find the eccentricity of this hyperbola

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10. If the normal at a point P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the x-axis at G, show that the $SG = eSP$. S being the focus of the hyperbola.

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11. $(x-1)(y-2)=5$ and $(x-1)^2+(y+2)^2=r^2$ intersect at four points A, B, C, D and if centroid of $\triangle ABC$ lies on line $y=3x-4$, then locus of D is

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Concept Application Exercise 7 1

1. The equation $\sqrt{(x-4)^2+(y-2)^2}+\sqrt{(x+4)^2+(y-2)^2}=8$ represents a

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2. If cot $\theta=\frac{3}{4}$ find the value of :

$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

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3. The equation of the transvers axis of the hyperbola $(x-3)^2+(y+1)^2=(4x+3y)^2$ is (a)$x+3y=0$ (b) $4x+3y=9$ $3x-4y=13$ (d) $4x+3y=0$

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Concept Application Exercise 7 2

1. Write the length of the latus rectum of the hyperbola $16x^2-9y^2=144.$

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2. If the latus rectum of a hyperbola forms an equilateral triangle with the vertex at the center of the hyperbola ,then find the eccentricity of the hyperbola.

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3. The distance between two directrices of a rectangular hyperbola is 10 units. Find the distance between its foci.

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4. An ellipse and a hyperbola are confocal (have the same focus) and the conjugate axis of the hyperbola is equal to the minor axis of the ellipse. If $e_1 and e_2$ are the eccentricities of the ellipse and the hyperbola, respectively, then prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 2$.

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5. If S ans S' are the foci, C is the center , and P is point on the rectangular hyperbola, show that $SP \times S'P = (CP)^2$

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6. Find the equation of the hyperbola whose foci are $(8, 3) and (0, 3)$ and eccentricity is $\frac{4}{3}$.

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7. Find all the aspects of hyperbola $16x^2 - 3y^2 - 32x + 12y - 44 = 0$.

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8. Show that the locus represented by $x = \frac{1}{2}a\left(t + \frac{1}{t}\right), y = \frac{1}{2}a\left(t - \frac{1}{t}\right)$ is a rectangular hyperbola.

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9. Two straight lines pass through the fixed points $(\pm a, 0)$ and have slopes whose products is $p > 0$ Show that the locus of the points of intersection of the lines is a hyperbola.

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10. If $AOBandCOD$ are two straight lines which bisect one another at right angles, show that the locus of a points P which moves so that $PA \cdot PB = PC \cdot PD$ is a hyperbola. Find its eccentricity.

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11. Find the equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point (5, 3).

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12. PN is the ordinate of any point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and A' is its transvers axis. If Q divides AP in the ratio $a^2 : b^2$, then prove that NQ is perpendicular to $A'P$.

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Concept Application Exercise 7 3

1. The tangents from $(1, 2\sqrt{2})$ to the hyperbola $16x^2 - 25y^2 = 400$ include between them an angle equal to:

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2. Tangents are drawn to the hyperbola $3x^2 - 2y^2 = 25$ from the point $\left(0, \frac{5}{2}\right)$. Find their equations.

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3. A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$, is

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4. The locus a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is (A) a parabola (B) an ellipse (C) a hyperbola (D) a circle

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5. A normal to the hyperbola, $4x^2 \_ 9y^2 = 36$ meets the co-ordinate axes x and y at A and $B.$ respectively. If the parallelogram $OABP(O$ being the origin) is formed, then the locus of P is :-

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6. A point P moves such that the chord of contact of the pair of tangents from P on the parabola $y^2 = 4ax$ touches the rectangular hyperbola $x^2 - y^2 = c^2$. Show that the locus of P is the ellipse $\frac{x^2}{c^2} + \frac{y^2}{(2a)^2} = 1.$

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7. If a tangent to the parabola $y^2 = 4ax$ intersects the $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $A$and B, then the locus of the point of intersection of tangents at A and B to the ellipse is

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8. If the chords of contact of tangents from two points $(-4, 2)$ and $(2, 1)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are at right angle, then find then find the eccentricity of the hyperbola.

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9. Statement 1 : If from any point $P(x_1, y_1)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the corresponding chord of contact lies on an other branch of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Statement 2 : From any point outside the hyperbola, two tangents can be drawn to the hyperbola.

(a) Statement 1 and Statement 2 are correct and Statement 2 is the correct explanation for Statement 1.

(b) Statement 1 and Statement 2 are correct and Statement 2 is not the correct explanation for Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 2 is true but Statement 1 is false.

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10. Let 'p' be the perpendicular distance from the centre C of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to the tangent drawn at a point R on the hyperbola. If $S \& S'$ are the two foci of the hyperbola, then show that $(RS + RS')^2 = 4a^2\left(1 + \frac{b^2}{p^2}\right)$.

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Concept Application Exercise 7 4

1. Find the angle between the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

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2. Find the asymptotes of the curve $xy - 3y - 2x = 0$.

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3. If asymptotes of hyperbola bisect the angles between the transverse axis and conjugate axis of hyperbola, then what is eccentricity of hyperbola?

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4. The asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ form with an tangent to the hyperbola triangle whose area is $a^2 \tan \lambda$ in magnitude then its

eccentricity is: (a) $\sec\lambda$ (b) $\cos ec\lambda$ (c) $\sec^2\lambda$ (d) $\cos ec^2\lambda$

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5. If the foci of a hyperbola lie on $y = x$ and one of the asymptotes is $y = 2x$, then the equation of the hyperbola, given that it passes through (3, 4), is (a)$x^2 - y^2 - \frac{5}{2}xy + 5 = 0$ (b) $2x^2 - 2y^2 + 5xy + 5 = 0$ (c) $2x^2 + 2y^2 - 5xy + 10 = 0$ (d) none of these

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Concept Application Exercise 7 5

1. If any line perpendicular to the transverse axis cuts the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the conjugate hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ at points $PandQ$, respectively, then prove that normal at $PandQ$ meet on the x-axis.

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2. A normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the axes at $M and N$ and lines MP and NP are drawn perpendicular to the axes meeting at P. Prove that the locus of P is the hyperbola $a^2x^2 - b^2y^2 = (a^2 + b^2)$.

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3. Prove that the locus of the point of intersection of the tangents at the ends of the normal chords of the hyperbola $x^2 - y^2 = a^2$ is $a^2(y^2 - x^2) = 4x^2y^2$.

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4. Find the value of m, for which the line $y = mx + 25\frac{\sqrt{3}}{3}$ is a normal to the conic $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

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5. Normal is drawn at one of the extremities of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which meets the axes at points $AandB$. Then find the area of triangle $OAB(O$ being the origin).

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Concept Application Exercise 7 6

1. Find the asymptotes and axes of hyperbola having equation $xy - 3y - 4x + 7 = 0.$

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2. The chord PQ of the rectangular hyperbola $xy = a^2$ meets the axis of x at $A; C$ is the midpoint of $PQ;$ and O is the origin. Then ACO is (a) equilateral (b) isosceles (c) right-angled (d) right isosceles

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3. If $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ and $S(x_4, y_4)$ are four concyclic points on the rectangular hyperbola and $xy = c^2$, then find coordinates of the orthocentre of the triangle PQR

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4. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda(\lambda \in R^+)$, then the locus of point P is (a) $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$ (c) $xy = \lambda c^2$ (d) none of these

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Exercises

1. If the distance between the foci and the distance between the two directricies of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3:2, then $b : a$ is (a)$1 : \sqrt{2}$ (b) $\sqrt{3} : \sqrt{2}$ (c)$1 : 2$ (d) $2 : 1$

A. $1 : \sqrt{2}$

B. $\sqrt{3} : \sqrt{2}$

C. $1 : 2$

D. $2 : 1$

Answer: A

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2. The is a point P on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ such that its distance from the right directrix is the average of its distance from the two foci. Then the x-coordinate of P is (a)$-\frac{64}{5}$ (b)$-\frac{32}{9}$ (c)$-\frac{64}{9}$ (d)none of these

A. $-64/5$

B. $-32/9$

C. $-64/9$

D. none of these

Answer: A

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3. The equation, $2x^2 + 3y^2 - 8x - 18y + 35 = K$ represents (a) no locus if $k > 0$ (b) an ellipse if $k < 0$ (c) a point if $k = 0$ (d) a hyperbola if $k > 0$

A. no locus if k gt 0

B. an ellipse if k lt 0

C. a point if k = 0

D. a hyperbola if k gt0

Answer: C

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4. Let 'a' and 'b' be non-zero real numbers. Then, the equation $\left(ax^2+by^2+c\right)\left(x^2-5xy+6y^2\right)$ represents :

A. four staright lines, when c = 0 and a, b are of the same sign

B. two straight lines and a circle, when a = b and c is of sign opposite to that of a

C. two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

D. a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Answer: B

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5. For the hyperbola $\dfrac{x^2}{\cos^2\alpha}-\dfrac{y^2}{\sin^2\alpha}=1;\left(0<\alpha<\dfrac{\pi}{4}\right)$. Which of the following remains constant when alpha varies?

A. Eccentricity

B. Abscissa of foci

C. Directrix

D. Vertex

Answer: B

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6. Which of the following pairs may represent the eccentricities of two conjugate hyperbolas, for $\alpha \in (0, \pi/2)$?

a.$\sin\theta, \cos\theta$ b.$\tan\theta, \cot\theta$ c.$\sec\theta, \text{cosec}\theta$ d.$1 + \sin\theta, 1 + \cos\theta$

A. $\sin\theta, \cos\theta$

B. $\tan\theta, \cot\theta$

C. $\sec\theta, \text{cosec}\theta$

D. $1 + \sin\theta, 1 + \cos\theta$

Answer: C

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7. If a variable line has its intercepts on the coordinate axes e and e', where $\frac{e}{2} and \frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2 + y^2 = r^2$, where $r =$

A. 1

B. 2

C. 3

D. cannot be decided

Answer: C

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8. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

A. (a) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

B. (b) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$

C. (c) $x^2\sin^2\theta - y^2\cos^2\theta = 1$

D. (d) $x^2\cos^2\theta - y^2\cos^2\theta = 1$

Answer: A

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9. If the distances of one focus of hyperbola from its directrices are 5 and 3, then its eccentricity is

A. $\sqrt{2}$

B. 2

C. 4

D. 8

Answer: B

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10. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ be confocal $(a > A \text{ and } a > b)$ having the foci at s_1 and S_2, respectively. If P is their point of intersection, then S_1P and S_2P are the roots of quadratic equation

A. (a) $x^2+2Ax+\left(a^2-A^2\right)=0$

B. (b) $x^2+2ax+\left(a^2-A^2\right)=0$

C. (c) $x^2-2Ax+\left(a^2+A^2\right)=0$

D. (d) $x^2-2ax+\left(a^2-A^2\right)=0$

Answer: D

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11. Two tangents are drawn from a point on hyperbola $x^2 - y^2 = 5$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If they make angle α and β with x-axis, then

A. $\alpha - \beta = \pm \frac{\pi}{2}$

B. $\alpha + \beta = \frac{\pi}{2}$

C. $\alpha + \beta = \pi$

D. $\alpha + \beta = 0$

Answer: B

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12. Equation of the rectangular hyperbola whose focus is $(1, -1)$ and the corresponding directrix is $x - y + 1 = 0$.

A. $x^2 - y^2 = 1$

B. $xy = 1$

C. $2xy - 4x + 4y + 1 = 0$

D. $2xy + 4x - 4y - 1 = 0$

Answer: C

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13. If two circles $(x+4)^2 + y^2 = 1$ and $(x-4)^2 + y^2 = 9$ are touched extermally by a circle, then locus of centre of variable circle is

A. (a) $\frac{x^2}{15} - \frac{y^2}{1} = 1$

B. (b) $\frac{x^2}{4} - \frac{y^2}{12} = 1$

C. (c) $\frac{x^2}{1} - \frac{y^2}{15} = 1$

D. (d) $\frac{x^2}{12} - \frac{y^2}{4} = 1$

Answer: C

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14. If the vertex of a hyperbola bisects the distance between its center and the correspoinding focus, then the ratio of the square of its conjugate axis to the square of its transverse axis is (a) 2 (b) 4 (c) 6 (d) 3

A. 2

B. 4

C. 6

D. 3

Answer: C

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15. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : (1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\sqrt{3}$

A. $3/4$

B. $4/\sqrt{3}$

C. $2/\sqrt{3}$

D. none of these

Answer: C

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16. Let LL' be the latus rectum through the focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and A' be the farther vertex. If $A'LL'$ is equilateral, then the eccentricity of the hyperbola is (axes are coordinate axes).

A. $\sqrt{3}$

B. $\sqrt{3}+1$

C. $(\sqrt{3}+1)/\sqrt{2}$

D. $(\sqrt{3}+1)/\sqrt{3}$

Answer: D

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17. The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is (a) 2 (b) $2\sqrt{3}$ (c) 4 (d) $\frac{4}{5}$

A. 2

B. $2/\sqrt{3}$

C. 4

D. $4/5$

Answer: A

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18. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}t = 0 \& \sqrt{3}tx + ty - 4\sqrt{3} = 0$ (where t is a parameter) is a hyperbola whose eccentricity is: (a)$\sqrt{3}$ (b) 2 (c)$\frac{2}{\sqrt{3}}$ (d) $\frac{4}{3}$

A. $\sqrt{3}$

B. 2

C. $2/\sqrt{3}$

D. $4/3$

Answer: B

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19. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2 \alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2(\sec^2 \alpha) + y^2 = 25,$ then a value of α is : (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

A. $\pi/6$

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: B

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20. The equation of the transvers and conjugate axes of a hyperbola are, respectively, $x+2y-3=0$ and $2x-y+4=0$, and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is

a) $\frac{2}{5}(x+2y-3)^2-\frac{3}{5}(2x-y+4)^2=1$

b) $\frac{2}{5}(x-y-4)^2-\frac{3}{5}(x+2y-3)^2=1$

c) $\frac{2}{5}(2x-y+4)^2-\frac{3}{5}(x+2y-3)^2=1$

d) $2(x+2y-3)^2-3(2x-y+4)^2=1$

A. $\frac{2}{5}(x+2y-3)^2-\frac{3}{5}(2x-y+4)^2=1$

B. $\frac{2}{5}(2x-y+4)^2-\frac{3}{5}(x+2y-3)^2=1$

C. $2(2x-y+4)^2-3(x+2y-3)^2=1$

D. $2(x+2y-3)^2-3(2x-y+4)^2=1$

Answer: B

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21. Factorise the expression: $(x^2 - 2xy + y^2) - z^2$

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22. If two points $P \& Q$ on the hyperbola , $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose centre is C be such that CP is perpendicularal to CQ and $a < b1$,then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}.$

A. $\frac{b^2 - a^2}{2ab}$

B. $\frac{1}{a^2} + \frac{1}{b^2}$

C. $\frac{2ab}{b^2 - a^2}$

D. $\frac{1}{a^2} - \frac{1}{b^2}$

Answer: D

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23. The angle between the lines joining the origin to the points of intersection of the line $\sqrt{3}x + y = 2$ and the curve $y^2 - x^2 = 4$ is

A. $\tan^{-1}(2/\sqrt{3})$

B. $\pi/6$

C. $\tan^{-1}(\sqrt{3}/2)$

D. $\pi/2$

Answer: C

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24. A variable chord of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1, (b > a),$ subtends a right angle at the center of the hyperbola if this chord touches. a fixed circle concentric with the hyperbola a fixed ellipse concentric with the hyperbola a fixed hyperbola concentric with the hyperbola a fixed parabola having vertex at (0, 0).

A. a fixed circle concentric with the hyperbola

B. a fixed ellipse concentric with the hyperbola

C. a fixed hyperbola concentric with the hyperbola

D. a fixed parabola having vertex at (0, 0)

Answer: A

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25. If the distance between two parallel tangents having slope m drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{49} = 1$ is 2, then the value of $2|m|$ is____

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26. If $ax + by = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a^2 - b^2$ is equal to (a) $\frac{1}{a^2e^2}$ (b) a^2e^2 (c)b^2e^2 (d) none of these

A. $1/a^2e^2$

B. a^2e^2

C. b^2e^2 none of these

D. none of these

Answer: A

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27. A tangent drawn to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P\left(\frac{\pi}{6}\right)$ froms a triangle of area a^2 square units, with the coordinate axes, then the square of its eccentricity is (A) 15 (B) 24 (C) 17 (D) 14

A. 15

B. 24

C. 17

D. 14

Answer: C

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28. The number of roots of the equation x^2+5x+6=0 is

A. 2

B. 4

C. zero

D. none of these

Answer: C

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29. The locus of a point whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is a/an ellipse (b) circle hyperbola (d) parabola

A. ellipse

B. circle

C. hyperbola

D. parabola

Answer: C

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30. The sides $AC and AB$ of a ABC touch the conjugate hyperbola of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the vertex A lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the side BC must touch parabola (b) circle hyperbola (d) ellipse

A. parabola

B. circle

C. hyperbola

D. ellipse

Answer: D

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31. The number of possible tangents which can be drawn to the curve $4x^2 - 9y^2 = 36,$ which are perpendicular to the straight line $5x + 2y - 10 = 0$, is zero (b) 1 (c) 2 (d) 4

A. a. zero

B. b. 1

C. c. 2

D. d. 4

Answer: A

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32. The tangent at a point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point $(0, -b)$ and the normal at P passes through the point $(2a\sqrt{2}, 0)$. Then the eccentricity of the hyperbola is

A. a. 2

B. b. $\sqrt{2}$

C. c. 3

D. d. $\sqrt{3}$

Answer: B

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33. Locus of the feet of the perpendiculars drawn from either foci on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$ is

A. $x^2 + y^2 = 9$

B. $x^2 + y^2 = 1/9$

C. $x^2 + y^2 = 7/144$

D. $x^2 + y^2 = 1/16$

Answer: D

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34. P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then OT.ON is equal to

A. e^2

B. a^2

C. b^2

D. b^2/a^2

Answer: B

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35. The coordinates of a point on the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$ which is nearest to the line $3x + 2y + 1 = 0$ are

A. (6, 3)

B. $(-6, -3)$

C. $(-6, 3)$

D. $(6, -3)$

Answer: C

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36. The tangent at a point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets one of the directrix at $F\cdot$ If PF subtends an angle θ at the corresponding focus, then $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) π

A. $\pi/4$

B. $\pi/2$

C. $3\pi/4$

D. π

Answer: B

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37. The locus of a point, from where the tangents to the rectangular hyperbola $x^2-y^2=a^2$ contain an angle of 45^0, is (a) $(x^2+y^2)^2+a^2(x^2-y^2)=4a^2$ (b) $2(x^2+y^2)^2+4a^2(x^2-y^2)=4a^2$ (c) $(x^2+y^2)^2+4a^2(x^2-y^2)=4a^2$ (d) $(x^2+y^2)+a^2(x^2-y^2)=a^4$

A. $(x^2+y^2)^2+a^2(x^2-y^2)=4a^2$

B. $2(x^2+y^2)^2+4a^2(x^2-y^2)=4a^2$

C. $(x^2+y^2)^2+4a^2(x^2-y^2)=4a^4$

D. $(x^2+y^2)^2+a^2(x^2-y^2)=a^4$

Answer: C

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38. If tangents $PQandPR$ are drawn from a variable point P to thehyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, (a>b)$, so that the fourth vertex S of parallelogram $PQSR$ lies on the circumcircle of triangle PQR , then the locus of P is (a) $x^2+y^2=b^2$ (b) $x^2+y^2=a^2$ (c) $x^2+y^2=a^2-b^2$ (d) none of these

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39. The number of points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from which mutually perpendicular tangents can be drawn to the circle $x^2+y^2=a^2$ is/are (a) 0 (b) 2 (c) 3 (d) 4

A. 0

B. 2

C. 3

D. 4

Answer: A

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40. If a ray of light incident along the line $3x + (5 - 4\sqrt{2})y = 15$ gets reflected from the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, then its reflected ray goes along the line. $x\sqrt{2} - y + 5 = 0$ (b) $\sqrt{2}y - x + 5 = 0$ $\sqrt{2}y - x - 5 = 0$

(d) none of these

A. $x\sqrt{2} - y + 5 = 0$

B. $\sqrt{2}y - x + 5 = 0$

C. $\sqrt{2}y - x - 5 = 0$

D. none of these

Answer: D

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41. The chord of contact of a point P w.r.t a hyperbola and its auxiliary circle are at right angle. Then the point P lies (a) on conjugate hyperbola (b) one of the directrix (c) one of the asymptotes (d) none of these

A. conjugate hyperbola

B. one of the directrix

C. asymptotes

D. none of these

Answer: C

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42. The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $a^2x^2 - y^2 = 4$ intersect at right angles. Then the equation of the circle through the points of intersection of two conics is (a) $x^2 + y^2 = 5$ (b) $\sqrt{5}(x^2 + y^2) - 3x - 4y = 0$ (c) $\sqrt{5}(x^2 + y^2) + 3x + 4y = 0$ (d) $x^2 + y^2 = 25$

A. $x^2 + y^2 = 5$

B. $\sqrt{5}(x^2 + y^2) - 3x - 4y = 0$

C. $\sqrt{5}(x^2 + y^2) + 3x + 4y = 0$

D. $x^2 + y^2 = 25$

Answer: A

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43. The locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ forms a triangle of constant area with the coordinate axes is (a) straight line (b) a hyperbola (c) an ellipse (d) a circle

A. a straight line

B. a hyperbola

C. an ellipse

D. a circle

Answer: B

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44. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$ then the equation of the corresponding pair of tangents is (A) $9x^2 - 8y^2 + 18x - 9 = 0$ (B) $9x^2 - 8y^2 - 18x + 9 = 0$ (C) $9x^2 - 8y^2 - 18x - 9 = 0$ (D) $9x^2 - 8y^2 + 18x + 9 = 0$`

A. $9x^2 - 8y^2 + 18x - 9 = 0$

B. $9x^2 - 8y^2 - 18x = 0$

C. $9x^2 - 8y^2 - 9 = 0$

D. $9x^2 - 8y^2 + 18x + 9 = 0$

Answer: B

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45. If the tangent at point P(h, k) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the circle $x^2 + y^2 = a^2$ at points $Q(x_1, y_1)$ and $R(x_2, y_2)$, then the vlaue of $\frac{1}{y_1} + \frac{1}{y_2}$ is

A. $\frac{1}{k}$

B. $\frac{2}{k}$

C. $\frac{ab}{k}$

D. $\frac{a+b}{k}$

Answer: B

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46. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ (where $\theta+\phi=\frac{\pi}{2}$) be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ If (h,k) is the point of intersection of the normals at P and Q then k is equal to

(A) $\frac{a^2+b^2}{a}$ (B) $-\left(\frac{a^2+b^2}{a}\right)$ (C) $\frac{a^2+b^2}{b}$ (D) $-\left(\frac{a^2+b^2}{b}\right)$

A. $\frac{a^2+b^2}{a}$

B. $-\left(\frac{a^2+b^2}{a}\right)$

C. $\frac{a^2+b^2}{b}$

D. $-\left(\frac{a^2+b^2}{b}\right)$

Answer: D

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47. A normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ has equal intercepts on the positive x- and y-axis. If this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then a^2+b^2 is equal to (a) 5 (b) 25 (c) 16 (d) none of these

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48. Portion of asymptote of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (between centre and the tangent at vertex) in the first quadrant is cut by the line $y + \lambda(x - a) = 0$ (lambda is a parameter) then (A) $\lambda \in R$ (B) $\lambda \in (0, \infty)$ (C) $\lambda \in (-\infty, 0)$ (D) $\lambda \in R - \{0\}$

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49. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 120^0 and the product of perpendiculars drawn from the foci upon its any tangent is 9, then the locus of the point of intersection of perpendicular tangents of the hyperbola can be (a) $x^2 + y^2 = 6$ (b) $x^2 + y^2 = 9$ $x^2 + y^2 = 3$ (d) $x^2 + y^2 = 18$

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50. Let any double ordinate PNP' of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ be produced on both sides to meet the asymptotes in $QandQ'$. Then $PQ\dot{P}^{'}Q$ is equal to 25 (b) 16 (c) 41 (d) none of these

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51. For hyperbola whose center is at (1, 2) and the asymptotes are parallel to lines $2x + 3y = 0$ and $x + 2y = 1$, the equation of the hyperbola passing through (2, 4) is (a) $(2x + 3y - 5)(x + 2y - 8) = 40$ (b) $(2x + 3y - 8)(x + 2y - 5) = 40$ (c) $(2x + 3y - 8)(x + 2y - 5) = 30$ (d) none of these

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52. The asymptotes of the hyperbola $\frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} = 1$ and $\frac{x^2}{a_2^2} - \frac{y^2}{b_2^2} = 1$ are perpendicular to each other. Then, (a) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (b) $a_1 a_2 = b_1 b_2$ (c) $a_1 a_2 + b_1 b_2 = 0$ (d) $a_1 - a_2 = b_1 - b_2$

A. $a_1 / a_2 = b_1 / b_2$

B. $a_1 a_2 = b_1 b_2$

C. $a_1 a_2 + b_1 b_2 = 0$

D. $a_1 - a_2 = b_1 - b_2$

Answer: C

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53. If $S = 0$ is the equation of the hyperbola $x^2 + 4xy + 3y^2 - 4x + 2y + 1 = 0$, then the value of k for which $S + K = 0$ represents its asymptotes is

A. 20

B. -16

C. -22

D. 18

Answer: C

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54. If two distinct tangents can be drawn from the point $(\alpha, \alpha + 1)$ on different branches of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then find the values of α.

A. $|\alpha| < 3/2$

B. $|\alpha| > 2/3$

C. $|\alpha| > 3$

D. none of these

Answer: A

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55. A hyperbola passes through (2,3) and has asymptotes $3x-4y+5=0$ and $12x+5y-40=0$. Then, the equation of its transverse axis is:

A. $77x-21y-265=0$

B. $21x-77y+265=0$

C. $21x-77y-265=0$

D. $21x+77y-265=0$

Answer: B

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56. From any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$. The area cut-off by the chord of contact on the asymptotes is equal to: (a) $\frac{a}{2}$ (b) ab (c) $2ab$ (d) $4ab$

A. $a/2$

B. ab

C. $2ab$

D. $4ab$

Answer: D

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57. The combined equation of the asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ is -

A. $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$

B. $2x^2 + 5xy + 2y^2 + 4x + 5y - 2 = 0$

C. $2x^2 + 5xy + 2y^2 = 0$

D. none of these

Answer: A

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58. The asymptotes of the hyperbola $xy = hx + ky$ are (1)$x - k = 0$ and $y - h = 0$ $(2)x + h = 0$ and $y + k = 0$ (3)$x - k = 0$ and $y + h = 0$ $(4)x + k = 0$ and $y - h = 0$

A. x - k = 0 and y - h = 0

B. x + h = 0 and y + k = 0

C. x - k = 0 and y + h = 0

D. x + k = 0 and y - h = 0

Answer: A

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59. The center of a rectangular hyperbola lies on the line $y = 2x$. If one of the asymptotes is $x + y + c = 0$, then the other asymptote is: (a) $6x + 3y - 4c = 0$ (b) $3x + 6y - 5c = 0$ (c) $3x - 6y - c = 0$ (d) none of these

A. $6x + 3y - 4c = 0$

B. $3x + 6y - 5c = 0$

C. $3x - 6y - c = 0$

D. none of these

Answer: D

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60. The equation of a rectangular hyperbola whose asymptotes are $x = 3$ and $y = 5$ and passing through (7,8) is

A. $xy - 3y + 5x + 3 = 0$

B. $xy + 3y + 4x + 3 = 0$

C. $xy - 3y + 5x - 3 = 0$

D. $xy - 3y - 5x + 3 = 0$

Answer: D

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61. If tangents OQ and OR are dawn to variable circles having radius r and the center lying on the rectangular hyperbola $xy = 1$, then the locus of the circumcenter of triangle OQR is $(O$ being the origin). (a) $xy = 4$ (b) $xy = \frac{1}{4}$ $xy = 1$ (d) none of these

A. xy = 4

B. xy = 1/4

C. xy = 1

D. none of these

Answer: B

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62. Four points are such that the line joining any two points is perpendicular to the line joining other two points. If three point out of

these lie on a rectangular hyperbola, then the fourth point will lie on

A. the same hyperbola

B. the conjugate hyperbola

C. one of the directrix

D. one of the asymptotes

Answer: A

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63. If $S_1 and S_2$ are the foci of the hyperbola whose length of the transverse axis is 4 and that of the conjugate axis is 6, and $S_3 and S_4$ are the foci of the conjugate hyperbola, then the area of quadrilateral $S_1S_3S_2S_4$ is 24 (b) 26 (c) 22 (d) none of these

A. 24

B. 26

C. 22

D. none of these

Answer: B

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64. Suppose the circle having equation $x^2 + y^2 = 3$ intersects the rectangular hyperbola $xy = 1$ at points $A, B, C, and D$. The equation $x^2 + y^2 - 3 + \lambda(xy - 1) = 0, \lambda \in R,$ represents.

A. a pair of lines through the origin for $\lambda = -3$

B. an ellipse through A, B, C and D for $\lambda = -3$

C. a parabola through A, B, C and D for $\lambda = -3$

D. a circle for any $\lambda \in R$

Answer: A

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65. The equation to the chord joining two points $(x_1, y_1) and (x_2, y_2)$ on the rectangular hyperbola $xy = c^2$ is:

A. $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$

B. $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

C. $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$

D. $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

Answer: A

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66. The locus of the foot of the perpendicular from the center of the hyperbola $xy = 1$ on a variable tangent is

A. $(x^2 - y^2)^2 = 4xy$

B. $(x^2 + y^2)^2 = 2xy$

C. $(x^2 + y^2) = 4xy$

D. $\left(x^2+y^2\right)^2=4xy$

Answer: D

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67. The curve $xy=c(c>0)$ and the circle $x^2+y^2=1$ touch at two points, then distance between the points of contact is

A. (a) 1

B. (b) 2

C. (c) $2\sqrt{2}$

D. (d) none of these

Answer: B

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68. Let C be a curve which is the locus of the point of intersection of lines $x = 2 + m$ and $my = 4 - m$. A circle $s: (x - 2)^2 + (y + 1)^2 = 25$ intersects the curve C at four points: $P, Q, R, andS$. If O is center of the curve C, then $OP^2 + OQ^2 + OR^2 + OS^2$ is (a) 50 (b) 100 (c) 25 (d) $\frac{25}{2}$

A. 50

B. 100

C. 25

D. 25/5

Answer: B

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Multiple Correct Answers Type

1. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ at four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$, and $S(x_4, y_4)$, then

A. $x_1 + x_2 + x_3 + x_4 = 0$

B. $y_1 + y_2 + y_3 + y_4 = 0$

C. $x_1 x_2 x_3 x_4 = c^4$

D. $y_1 y_2 y_3 y_4 = c^4$

Answer: A::B::C::D

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2. The equation $(x - \alpha)^2 + (y - \beta)^2 = k(lx + my + n)^2$ represents

A. a parabola for $k < (l^2 + m^2)^{-1}$

B. an ellipse for $0 < k < (l^2 + m^2)^{-1}$

C. a hyperbola for $k > (l^2 + m^2)^{-1}$

D. a point circle for k = 0

Answer: B::C::D

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3. If $(5, 12) and (24, 7)$ are the foci of a hyperbola passing through the origin, then (a)$e = \frac{\sqrt{386}}{12}$ (b) $e = \frac{\sqrt{386}}{13}$ (c)$LR = \frac{121}{6}$ (d) $LR = \frac{121}{3}$

A. $e = \frac{\sqrt{386}}{12}$

B. $e = \frac{\sqrt{386}}{13}$

C. $LR = 121/6$

D. $LR = 121/3$

Answer: A::C

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4. Show that the equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola.

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5. If a hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. Its transverse and conjugate axes coincide respectively with the major and minor axes of the ellipse and if the product of eccentricities of hyperbola and ellipse is 1 then the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$ b. the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$ c. focus of hyperbola is (5, 0) d. focus of hyperbola is $\left(5\sqrt{3}, 0\right)$

A. the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$

B. the equation of the hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

C. the vertex of the hyperbola is (5, 0)

D. the vertex of the hyperbola is $\left(5\sqrt{3}, 0\right)$

Answer: A::C

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6. If the foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ coincide with the foci of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and the eccentricity of the hyperbola is 2, then

A. $a^2 + b^2 = 16$

B. there is no director circle to the hyperbola

C. the centre of the director circle is (0, 0)

D. the length of latus rectum of the hyperbola is 12

Answer: A::B::D

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7. The differential equation $\frac{dy}{dx} = \frac{2x}{3y}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity. (a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{5}{3}}$ (c) $\sqrt{\frac{2}{5}}$ (d) $\sqrt{\frac{5}{2}}$

A. $\sqrt{3/5}$

B. $\sqrt{5/3}$

C. $\sqrt{2/5}$

D. $\sqrt{5/2}$

Answer: B::D

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8. find the equation of parabola which is symmetric about y-axis,and passes through point (2,-3).

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9. If the ellipse $x^2 + 2y^2 = 4$ and the hyperbola S = 0 have same end points of the latus rectum, then the eccentricity of the hyperbola can be

A. A. $\operatorname{cosec}\frac{\pi}{4}$

B. B. $\operatorname{cosec}\frac{\pi}{3}$

C. C. $2\sin.\ \frac{\pi}{3} + \sin.\ \frac{\pi}{4}$

D. D. $\sqrt{2}\sin.\ \frac{\pi}{3} + \sin.\ \frac{\pi}{4}$

Answer: A::D

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10. For which of the hyperbolas, can we have more than one pair of perpendicular tangents?

A. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

B. $\frac{x^2}{4} - \frac{y^2}{9} = -1$

C. $x^2 - y^2 = 4$

D. $xy = 44$

Answer: B

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11. The lines parallel to the normal to the curve $xy = 1$ is/are (a) $3x + 4y + 5 = 0$ (b) $3x - 4y + 5 = 0$ (c)$4x + 3y + 5 = 0$ (d) $3y - 4x + 5 = 0$

A. $3x + 4y + 5 = 0$

B. $3x - 4y + 5 = 0$

C. $4x + 3y + 5 = 0$

D. $3y - 4x + 5 = 0$

Answer: B::D

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12. From the point (2, 2) tangent are drawn to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1.$ Then the point of contact lies in the (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant

A. first quadrant

B. second quadrant

C. third quadrant

D. forth quadrant

Answer: C::D

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13. For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, let n be the number of points on the plane through which perpendicular tangents are drawn.

A. If n = 1, then $e = \sqrt{2}$

B. If n gt 1, then $0 < e < \sqrt{2}$.

C. If n = 0, then $e > \sqrt{2}$.

D. none of these

Answer: A::B::C

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14. Ifthe normal at P to the rectangular hyperbola $x^2 - y^2 = 4$ meets the axes in G and g and C is the centre of the hyperbola, then

A. $PG = PC$

B. $Pg = PC$

C. $PG = Pg$

D. $Gg = 2PC$

Answer: A::B::C::D

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15. Find the equation of tangent to the hyperbola $y = \frac{x+9}{x+5}$ which passes through $(0, 0)$ origin

A. $x + 25y = 0$

B. $x + y = 0$

C. $5x - y = 0$

D. $x - 25y = 0$

Answer: A::B

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16. Tangents which are parrallel to the line $2x + y + 8 = 0$ are drawn to hyperbola $x^2 - y^2 = 3$. The points of contact of these tangents is/are

A. (2,1)

B. $(2, \ -1)$

C. $(-2, \ -1)$

D. $(-2, 1)$

Answer: B::D

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17. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2).

A. equation of one of the tangents is x = 3

B. equation of one of the tangents is $5x - 12y + 9 = 0$

C. the area of triangle that these tangents form with their chord of contact is 12 sq. untis

D. the area of triangle that these tangents form with their chord of contact is 8 sq. units

Answer: A::B::D

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18. Find the equation of the circle which passes through the point (2, -2) and (3, 4) and whose centre lies on the x + y = 2

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19. If $P(\alpha, \beta)$, the point of intersection of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2}(1 - e^2) = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{a^2(E^2 - 1)} = \frac{1}{4}$ is equidistant from the foci of the curves all lying in the right of y-axis then

A. $2\alpha = a(2e + E)$

B. $a - e\alpha = E\alpha - \alpha/2$

C. $E = \frac{\sqrt{e^2 + 24} - 3e}{2}$

D. $E = \frac{\sqrt{e^2 + 12} - 3e}{2}$

Answer: B::C

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20. A hyperbola having the transverse axis of length $\frac{1}{2}$ unit is confocal with the ellipse $3x^2 + 4y^2 = 12$, then

A. Equation of the hyperbola is $\frac{x^2}{15} - \frac{y^2}{1} = \frac{1}{16}$

B. Eccentricity of the hyperbola is 4

C. Distance between the directries of the hyperbola is $\frac{1}{8}$ units

D. Length of latus rectum of the hyperbola is $\frac{15}{2}$ units

Answer: B::C::D

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21. In X-Y plane, the path defined by the equation $\frac{1}{x^m}+\frac{1}{y^m}+\frac{k}{(x+y)^n}=0$, is (a) a parabola if $m=\frac{1}{2}, k=-1, n=0$ (b) a hyperbola if $m=1, k=-1, n=0$ (c) a pair of lines if $m=k=n=1$ (d) a pair of lines if $m=k=-1, n=1$

A. a parabola if $m=\frac{1}{2}, k=-1, n=0$

B. a hyperbola if $m=1, k=-1, n=0$

C. a pair of lines if $m=k=n=1$

D. a pair of lines if $m=k=-1, n=1$

Answer: A::B::C::D

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22. A point moves such that the sum of the squares of its distances from the two sides of length 'a' of a rectangle is twice the sum of the squares of its distances from the other two sides of length b. The locus of the point can be:

A. a circle

B. an ellipse

C. a hyperbola

D. a pair of lines

Answer: C::D

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23. The equation of a hyperbola with co-ordinate axes as principal axes, and the distances of one of its vertices from the foci are 3 and 1 can be

A. $3x^2 - y^2 = 3$

B. $x^2 - 3y^2 + 3 = 0$

C. $x^2 - 3y^2 - 3 = 0$

D. none of these

Answer: A::B

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24. Three points A,B and C taken on rectangular hyperbola $xy = 4$ where $B(-2, -2)$ and $C(6, 2/3)$. The normal at A is parallel to BC, then

A. circumcentre of ΔABC is $(2, -2/3)$

B. equation of circumcircle of ΔABC is $3x^2 + 3y^2 - 12x + 4y - 40 = 0$

C. orthocenter of ΔABC is $\left(\frac{2}{\sqrt{3}}, 2\sqrt{3}\right)$

D. none of these

Answer: A::B::C

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25. A tangent is drawn at any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this tangent is intersected by the tangents at the vertices at points P and Q, then which of the following is/are true

A. S,S',P and Q are concyclic

B. PQ is diameter of the circle

C. S,S', P and Q forms rhombus

D. PQ is diagonal of acute angle of the rhombus formed by S,S',P and Q

Answer: A::B

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26. If two tangents can be drawn the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from (α, α^2), then

A. $\alpha \in (-2, 0)$

B. $\alpha \in (0, 2)$

C. $\alpha \in (-\infty, -2)$

D. $\alpha \in (2, \infty)$

Answer: C::D

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27. The director circle of a hyperbola is $x^2 + y^2 - 4y = 0$. One end of the major axis is (2,0) then a focus is (a) $(\sqrt{3}, 2 - \sqrt{3})$ (b) $(-\sqrt{3}, 2 + \sqrt{3})$ (c) $(\sqrt{6}, 2 - \sqrt{6})$ (d) $(-\sqrt{6}, 2 + \sqrt{6})$

A. $(\sqrt{3}, 2 - \sqrt{3})$

B. $(-\sqrt{3}, 2 + \sqrt{3})$

C. $\left(\sqrt{6}, 2-\sqrt{6}\right)$

D. $\left(-\sqrt{6}, 2+\sqrt{6}\right)$

Answer: C::D

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28. The points on the ellipse $\frac{x^2}{2}+\frac{y^2}{10}=1$ from which perpendicular tangents can be drawn to the hyperbola $\frac{x^2}{5}-\frac{y^2}{1}=1$ is/are

A. $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{5}{2}}\right)$

B. $\left(\sqrt{\frac{3}{2}}, -\sqrt{\frac{5}{2}}\right)$

C. $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{5}{2}}\right)$

D. $\left(\sqrt{\frac{5}{2}}, \sqrt{\frac{3}{2}}\right)$

Answer: A::B::C

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Comorehension Type

1. Consider an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Let a hyperbola is having its vertices at the extremities of minor axis of an ellipse and length of major axis of an ellipse is equal to the distance between the foci of hyperbola. Let e_1 and e_2 be the eccentricities of an ellipse and hyperbola respectively. Again let A be the area of the quadrilateral formed by joining all the foci and A, be the area of the quadrilateral formed by all the directrices. The relation between e_1 and e_2 is given by

A. $e_1 e_2 = 1$

B. $e_2^2(1 - e_1^2) = 1$

C. $e_1^2(e_1^2 - 1) = 1$

D. $e_1 e_2(1 - e_1^2) = 1$

Answer: B

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2. Consider an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$. A hyperbola has its vertices at the extremities of minor axis of the ellipse and the length of major axis of the ellipse is equal to the distance between the foci of hyperbola. Let e_1 and e_2 be the eccentricities of ellipse and hyperbola, respectively. Also, let A_1 be the area of the quadrilateral fored by joining all the foci and A_2 be the area of the quadrilateral formed by all the directries.

The relation between e_1 and e_2 is given by

A. $\tan^{-1}\left(\frac{1}{\sqrt{1-e_1^2}}\right)$

B. $\tan^{-1}\left(\frac{e_1}{\sqrt{1-e_1^2}}\right)$

C. $\tan^{-1}\left(\frac{1}{\sqrt{e_2^2-1}}\right)$

D. $\tan^{-1}\sqrt{1=e_1^2}$

Answer: C

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3. Consider the ellipse E_1, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$. An ellipse E_2 passes through the extremities of the major axis of E_1 and has its foci at the ends of its minor axis.Consider the following property:Sum of focal distances of any point on an ellipse is equal to its major axis. Equation of E_2 is

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4. Consider the hyperbola $\frac{X^2}{9} - \frac{y^2}{a^2} = 1$ and the circle $x^2 + (y - 3) = 9$.

Also, the given hyperbola and the ellipse $\frac{x^2}{41} + \frac{y^2}{16} = 1$ are orthogonal to each other.

The number of points on the hyperbola and the circle from which tangents drawn to the circle and the hyperbola, respectively, are perpendicular to each other is

A. $x^2 - y^2 = 0$

B. $x^2 - 9 = 0$

C. $9y^2 - 19x^2 = 0$

D. No common tangent.

Answer: B

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5. Consider the hyperbola $\frac{x^2}{9} - \frac{y^2}{a^2} = 1$ and the circle $x^2 + (y-3) = 9$. Also, the given hyperbola and the ellipse $\frac{x^2}{41} + \frac{y^2}{16} = 1$ are orthogonal to each other.

Combined equation of pair of common tangents between the hyperbola and the circle is given be: (1).$x^2 - y^2 = 0$ (2).$x^2 - 9 = 0$ (3). $9y^2 - 19x^2 = 0$ (4).No common tangent.

A. 0

B. 2

C. 4

D. 6

Answer: C

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6. Consider the hyperbola $\frac{x^2}{9} - \frac{y^2}{a^2} = 1$ and the circle $x^2 + (y-3) = 9$. Also, the given hyperbola and the ellipse $\frac{x^2}{41} + \frac{y^2}{16} = 1$ are orthogonal to each other.

Combined equation of pair of common tangents between the hyperbola and the circle is given be:

A. $x^2 + y^2 \pm 5x - 3y = 0$

B. $x^2 + y^2 + 5x \pm 3y = 0$

C. $x^2 - y^2 \pm 5x - 3y = 0$

D. $x^2 - y^2 + 3x \pm 3y = 0$

Answer: A

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7. The locus of the foot of perpendicular from my focus of a hyperbola upon any tangent to the hyperbola is the auxiliary circle of the hyperbola. Consider the foci of a hyperbola as $(-3, -2)$ and (5,6) and the foot of perpendicular from the focus (5, 6) upon a tangent to the hyperbola as (2, 5).

The conjugate axis of the hyperbola is

A. $4\sqrt{11}$

B. $2\sqrt{11}$

C. $4\sqrt{22}$

D. $2\sqrt{22}$

Answer: D

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8. The locus of the foot of perpendicular from my focus of a hyperbola upon any tangent to the hyperbola is the auxiliary circle of the hyperbola.

Consider the foci of a hyperbola as $(-3, -2)$ and (5,6) and the foot of perpendicular from the focus (5, 6) upon a tangent to the hyperbola as (2, 5).

The directrix of the hyperbola corresponding to the focus (5, 6) is

A. $2x + 2y - 1 = 0$

B. $2x + 2y - 11 = 0$

C. $2x + 2y - 7 = 0$

D. $2x + 2y - 9 = 0$

Answer: B

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9. The locus of the foot of perpendicular from my focus of a hyperbola upon any tangent to the hyperbola is the auxiliary circle of the hyperbola. Consider the foci of a hyperbola as $(-3, -2)$ and (5,6) and the foot of perpendicular from the focus (5, 6) upon a tangent to the hyperbola as (2, 5).

The directrix of the hyperbola corresponding to the focus (5, 6) is

A. $(2/9, 31/3)$

B. $(7/4, 23/4)$

C. $(2/3, 9)$

D. $(7/9, 7)$

Answer: C

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10. Let P(x, y) be a variable point such that

$$\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$$

which represents a hyperbola.

IF the origin is shifted to the point (3, 7/2) and the axes are rotated through an angle θ in clockwise sense so that the equation of the given hyperbola changes to the standard form $\dfrac{X^2}{a^2}-\dfrac{y^2}{b^2}=1$, then θ is

A. $5/3$

B. $4/3$

C. $5/4$

D. $3/\sqrt{7}$

Answer: C

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11. Let P(x, y) be a variable point such that

$$\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$$

which represents a hyperbola.

IF the origin is shifted to the point (3, 7/2) and the axes are rotated through an angle θ in clockwise sense so that the equation of the given hyperbola changes to the standard form $\frac{X^2}{a^2}-\frac{y^2}{b^2}=1$, then θ is

A. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{55}{4}$

B. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{25}{4}$

C. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{7}{4}$

D. none of these

Answer: D

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12. Let $P(x,y)$ is a variable point such that $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$, which represents hyperbola. The eccentricity e' of the corresponding conjugate hyperbola is (A) $\frac{5}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{4}$ (D) $\frac{3}{\sqrt{7}}$

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13. In a hyperbola, the portion of the tangent intercepted between the asymptotes is bisected at the point of contact.

Consider a hyperbola whose center is at the origin. A line $x+y=2$ touches this hyperbola at P(1,1) and intersects the asymptotes at A and B such that AB = $6\sqrt{2}$ units.

The equation of the tangent to the hyperbola at (-1, 7/2) is

A. $5xy + 2x^2 + 2y^2 = 0$

B. $3x + 2y = 4$

C. $2x^2 + 2y^2 - 5xy = 0$

D. none of these

Answer: A

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14. In a hyperbola, the portion of the tangent intercepted between the asymptotes is bisected at the point of contact.

Consider a hyperbola whose center is at the origin. A line $x + y = 2$ touches this hyperbola at P(1,1) and intersects the asymptotes at A and B such that AB = $6\sqrt{2}$ units.

The equation of the pair of asymptotes is

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15. In a hyperbola, the portion of the tangent intercepted between the asymptotes is bisected at the point of contact.

Consider a hyperbola whose center is at the origin. A line $x + y = 2$ touches this hyperbola at P(1,1) and intersects the asymptotes at A and B such that AB = $6\sqrt{2}$ units.

The equation of the tangent to the hyperbola at $(-1, 7/2)$ is

A. (a) $5x + 2y = 2$

B. (b) $3x + 2y = 4$

C. (c) $3x + 4y = 11$

D. (d) none of these

Answer: B

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16. A point P moves such that sum of the slopes of the normals drawn from it to the hyperbola xy=16 is equal to the sum of ordinates of feet of normals. The locus of P is a curve C

A. (A) $x^2 = 4y$

B. (B) $x^2 = 16y$

C. (C)$x^2 = 12y$

D. (D) $y^2 = 8x$

Answer: B

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17. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola xy = 16 is equal to the sum of ordinates of feet of normals . The locus of P is a curve C.

The equation of the curve C is

A. $x^2 = 16y$

B. $x^2 = 2y$

C. $x^2 + 2y = 0$

D. $x^2 + 4y = 0$

Answer: C

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18. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola xy = 16 is equal to the sum of ordinates of feet of normals . The locus of P is a curve C.

the equation of the curve C is

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19. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

A. $xy - 1 = x - y$

B. $xy + 1 = x + y$

C. $2xy = x + y$

D. none of these

Answer: B

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20. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

A. $xy = 2x + y - 2$

B. $2xy = x + 2y + 5$

C. $xy = x + y + 1$

D. none of these

Answer: C

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21. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

A. 4

B. 0

C. 3

D. 2

Answer: D

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Matrix Mathc Type

1. Let the foci of the hyperbola $\frac{X^2}{A^2} - \frac{y^2}{B^2} = 1$ be the vertices of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the foci of the ellipse be the vertices of the hyperbola. Let the eccentricities of the ellipse and hyperbola be e_E and e_H, respectively. Then match the following lists.

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2. Factorise the following: $p^2 + 6p - 16$

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3. Consider the lines represented by equation $(x^2 + xy - x) \times (x - y) = 0$ forming a triangle. Then match the following lists:

List I	List II
a. Orthocenter of triangle	**p.** $(1/6, 1/2)$
b. Circumcenter	**q.** $(1/(2+2\sqrt{2}), 1/2)$
c. Centroid	**r.** $(0, 1/2)$
d. Incenter	**s.** $(1/2, 1/2)$

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4. Factorise the following: $p^2 + 6p + 8$

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5. Consider the lines represented by equation $(x^2 + xy - x) \times (x - y) = 0$ forming a triangle. Then match the following lists:

List I	List II
a. Orthocenter of triangle	**p.** $(1/6,\ 1/2)$
b. Circumcenter	**q.** $(1/(2 + 2\sqrt{2}),\ 1/2)$
c. Centroid	**r.** $(0,\ 1/2)$
d. Incenter	**s.** $(1/2,\ 1/2)$

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6. Consider all possible permutations of the letters of the word ENDEANOEL. Match the statements/ expressions in column I with the statement/expressions in

	Column I		Column II
(A)	The number of permutations containing the word ENDEA is	(p)	5!
(B)	The number of permutations in which the letters E occurs in the first and the last positions, is	(q)	$2 \times 5!$
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions, is	(r)	$7 \times 5!$
(D)	The number of permutations in which the letters A, E, O occur only in odd positions, is	(s)	$21 \times 5!$

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7. Match the following and mark the correct options

Animal	Respiratory Organ
A Earthworm	i. Moist cuticle
B. Aquatic Arthropods	ii. Gills
C. Fishes	iii. Lungs
D. Birds/Reptiles	iv. Trachea

A. (II) (iii) (R)

B. (IV) (iv) (S)

C. (IV) (iii) (S)

D. (II) (iv) (R)

Answer: D

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8. Match the items of colums I and II.

Column-I		Column-II	
(P)	Z for ideal gas behaviour	(1)	3/8
(Q)	Z for real gas at low pressure	(2)	$\left(1+\frac{Pb}{RT}\right)$
(R)	Z for real gas at high pressure	(3)	1
(S)	Z for critical state	(4)	$\left(1-\frac{a}{RTV}\right)$

A. (II) (i) (P)

B. (III) (i0) (Q)

C. (II) (iv) (R)

D. (I) (ii) (Q)

Answer: A

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9. Lists I, II and III contains conics, equation of tangents to the conics and points of contact, respectively.

For $a = \sqrt{2}$ if a tangent is drawn to a suitable conic (List I) at the point of contact $(-1, 1)$, which of the following options is the only CORRECT combination for obtaining its equation?

A. (II) (ii) (Q)

B. (III) (i) (P)

C. (I) (i) (P)

D. (I) (ii) (Q)

Answer: D

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10. The equivalent capacitance between points A and B will be

A. $P \rightarrow IV, Q \rightarrow II, R \rightarrow I, S \rightarrow III$

B. $P \rightarrow IV, Q \rightarrow III, R \rightarrow I, S \rightarrow II$

C. $P \rightarrow IV, Q \rightarrow I, R \rightarrow III, S \rightarrow II$

D. $P \rightarrow III, Q \rightarrow IV, R \rightarrow II, S \rightarrow I$

Answer: B

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Numerical Value Type

1. The eccentricity of the hyperbola $\left|\sqrt{(x-3)^2+(y-2)^2}-\sqrt{(x+1)^2+(y+1)^2}\right|=1$ is _____

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2. If $y = mx + c$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, having eccentricity 5, then the least positive integral value of m is____

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3. Consider the graphs of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and $x, y \in R$. Number of points in which the two graphs intersect, is

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4. If $4\left(x - \sqrt{2}\right)^2 + \lambda\left(y - \sqrt{3}\right)^2 = 45$ and $\left(x - \sqrt{2}\right)^2 - 4\left(y - \sqrt{3}\right)^2 = 5$ cut orthogonally, then integral value of λ is ________.

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5. If the hyperbola $x^2 - y^2 = 4$ is rotated by 45^0 in the anticlockwise direction about its center keeping the axis intact, then the equation of the hyperbola is $xy = a^2$, where a^2 is equal to__________

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6. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined atv angle θ and ϕ to the x-axis.If $\tan\theta. \tan\phi = 2$, prove that $\beta^2 = 2\alpha^2 - 7$.

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7. The area of triangle formed by the tangents from the point (3, 2) to the hyperbola $x^2 - 9y^2 = 9$ and the chord of contact w.r.t. the point (3, 2) is____________

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8. Find the value of m for which $y = mx + 6$ is a tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{49} = 1.$

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9. If tangents drawn from the point $(a, 2)$ to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ are perpendicular, then the value of a^2 is ____

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10. If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $2r$ and r respectively, let e_E and e_H are the eccentricities of ellipse and hyperbola respectively, then

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11. If L is the length of the latus rectum of the hyperbola for which $x = 3andy = 2$ are the equations of asymptotes and which passes through the point (4, 6), then the value of $\frac{L}{\sqrt{2}}$ is_____

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12. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{\pi}{3}$, then the eccentricity of conjugate hyperbola is ________.

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13. If the chord $x\cos\alpha + y\sin\alpha = p$ of the hyperbola $\frac{x^2}{16} - \frac{y^2}{18} = 1$ subtends a right angle at the center, and the diameter of the circle, concentric with the hyperbola, to which the given chord is a tangent is d, then the value of $\frac{d}{4}$ is__________

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14. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

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Jee Main

1. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

A. (a) $4/\sqrt{3}$

B. (b) $2/\sqrt{3}$

C. (c) $\sqrt{3}$

D. (d) $4/3$

Answer: B

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2. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point:

A. (a) $(-\sqrt{2}, -\sqrt{3})$

B. (b) $(3\sqrt{2}, 2\sqrt{3})$

C. (c) $(2\sqrt{2}, 3\sqrt{3})$

D. (d) $(\sqrt{3}, \sqrt{2})$

Answer: C

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3. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0,3) then the area (in sq units) of $\triangle PTQ$ is

A. $36\sqrt{5}$

B. $45\sqrt{5}$

C. $54\sqrt{3}$

D. $60\sqrt{3}$

Answer: B

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Jee Advanced

1. Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at point P intersects the x-axis at (9, 0), then find the eccentricity of the hyperbola.

A. $\sqrt{5/2}$

B. $\sqrt{3/2}$

C. $\sqrt{2}$

D. $\sqrt{3}$

Answer: B

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2. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal to that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then (a) the foci of ellipse are $(\pm 1, 0)$ (b) equation of ellipse is $x^2 + 2y^2 = 2$ (c) the foci of ellipse are $(\pm \sqrt{2}, 0)$ (d) equation of ellipse is $x^2 + 2y^2 = 1$

A. the equation of the ellipse is $x^2 + 2y^2 = 1$

B. the foci of the ellipse are $(\pm 1, 0)$

C. the equation of the ellipse is $x^2 + 2y^2 = 4$

D. the foci of the ellipse are $(\pm \sqrt{2}, 0)$

Answer: A::B

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3. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

A. the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

B. a focus of the hyperbola is (2,0)

C. the eccentricity of the hyperbola is $\frac{2}{\sqrt{3}}$

D. the equation of the hyperbola is $x^2 - 3y^2 = 3$

Answer: B::D

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4. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the sraight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are (A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (C) $(3\sqrt{3}, -2\sqrt{2})$ (D) $(-3\sqrt{3}, 2\sqrt{2})$

A. $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

B. $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

C. $(3\sqrt{3}, -2\sqrt{2})$

D. $(3\sqrt{3}, -2\sqrt{2})$

Answer: A::B

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5. Find the equations of the tangent and the normal to $16x^2 + 9y^2 = 144$ at (x_1, y_1) where $x_1 = 2$ and $y_1 > 0$.

A. $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2} \quad \text{for} \quad x_1 > 1$

B. $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}} \quad \text{for} \quad x_1 > 1$

C. $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2} \quad \text{for} \quad x_1 > 1$

D. $\frac{dm}{dy_1} = \frac{1}{3} \quad \text{for} \quad x_1 > 0$

Answer: A::B::D

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6. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ then which of the following CANNOT be sides of a right angled triangle?

A. (a) 2a, 4, 1

B. (b) 2a, 8, 1

C. (c) a, 4, 1

D. (d) a, 4, 2

Answer: B::C::D

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Comprehension Type

1. The circle $x^2+y^2-8x=0$ and hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ I intersect at the points A and B. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is (A) $2x-\sqrt{5}y-20=0$ (B) $2x-\sqrt{5}y+4=0$ (C) $3x-4y+8=0$ (D) $4x-3y+4=0$

A. $2x-\sqrt{5}y-20=0$

B. $2x-\sqrt{5}y+4=0$

C. $3x-4y+8=0$

D. $4x-3y+4=0$

Answer: B

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2. The circle $x^2+y^2-8x=0$ and hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ intersect at the points A and B. Then the equation of the circle with AB as its diameter is

A. a. $x^2 + y^2 - 12x + 24 = 0$

B. b. $x^2 + y^2 + 12x + 24 = 0$

C. c. $x^2 + y^2 + 24x - 12 = 0$

D. d. $x^2 + y^2 - 24x - 12 = 0$

Answer: A

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3. Consider a hyperbola $xy = 4$ and a line $y = 2x = 4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B.

Locus of circumcentre of triangle OAB is

A. an ellipse with eccentricity $\frac{1}{\sqrt{2}}$

B. a parabola

C. a hyperbola with eccnetricity $\sqrt{2}$

D. a circle

Answer: C

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4. Consider a hyperbola $xy = 4$ and a line $y = 2x = 4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B.

Shortest distance between the line and hyperbola is

A. $\frac{8\sqrt{2}}{\sqrt{5}}$

B. $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$

C. $\frac{2\sqrt{2}}{\sqrt{5}}$

D. $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$

Answer: B

Watch Video Solution

5. Consider a hyperbola $xy = 4$ and a line $y = 2x = 4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B.

Let the given line intersects the x-axis at R. if a line through R. intersect the hyperbolas at S and T, then minimum value of $RS \times RT$ is

A. 2

B. 4

C. 6

D. 8

Answer: D

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6. Consider a hyperbola: $\frac{(x-7)^2}{a} - \frac{(y+3)^2}{b^2} = 1$. The line $3x - 2y - 25 = 0$, which is not a tangent, intersect the hyperbola at $H\left(\frac{11}{3}, -7\right)$ only. A variable point $P(\alpha + 7, \alpha^2 - 4)\ \forall \alpha \in R$ exists in the plane of the given hyperbola.

The eccentricity of the hyperbola is

A. $\sqrt{\frac{7}{5}}$

B. $\sqrt{2}$

C. $\frac{\sqrt{13}}{2}$

D. $\frac{3}{2}$

Answer: C

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7. Consider a hyperbola: $\frac{(x-7)^2}{a} - \frac{(y+3)^2}{b^2} = 1$. The line $3x - 2y - 25 = 0$, which is not a tangent, intersect the hyperbola at $H\left(\frac{11}{3}, -7\right)$ only. A variable point $P(\alpha + 7, \alpha^2 - 4)\ \forall \alpha \in R$ exists in the plane of the given hyperbola.

The eccentricity of the hyperbola is

A. $(2, \infty)$

B. $(-\infty, -2)$

C. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

D. None of these

Answer: D

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Single Correct Answer Type

1. The locus of $P(x, y)$ such that $\sqrt{x^2 + y^2 + 8y + 16} - \sqrt{x^2 + y^2 - 6x + 9} = 5,$ is

A. A. hyperbola

B. B. circle

C. C. finite line segment

D. D. infinite ray

Answer: D

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2. The distance of the focus of $x^2 - y^2 = 4$, from the directrix, which is nearer to it, is

A. $2\sqrt{2}$

B. $\sqrt{2}$

C. $4\sqrt{2}$

D. $8\sqrt{2}$

Answer: B

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3. If $\frac{x^2}{36} - \frac{y^2}{k^2} = 1$ is a hyperbola, then which of the following points lie on hyperbola?

A. A. $(3, 1)$

B. B. $(-3, 1)$

C. C. $(5, 2)$

D. D. $(10, 4)$

Answer: D

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4. The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common

A. A. centre and vertices only

B. B. centre, foci and vertices

C. C. centre, foci and directrices

D. D. centre only

Answer: A

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5. The equation to the hyperbola having its eccentricity 2 and the distance between its foci is 8 is

A. A. $\frac{x^2}{12} - \frac{y^2}{4} = 1$

B. B. $\frac{x^2}{4} - \frac{y^2}{12} = 1$

C. C. $\frac{x^2}{8} - \frac{y^2}{2} = 1$

D. D. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Answer: B

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6. If the centre, vertex and focus of a hyperbola be (0,0), (4,0) and (6,0) respectively, then the equation of the hyperbola is

A. $4x^2 - 5y^2 = 8$

B. $4x^2 - 5y^2 = 80$

C. $5x^2 - 4y^2 = 80$

D. $5x^2 - 4y^2 = 8$

Answer: C

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7. The equation $\frac{x^2}{9-\lambda} + \frac{y^2}{4-\lambda} = 1$ represents a hyperbola when $a < \lambda < b$ then $(b-a) =$

A. A. 3

B. B. 4

C. C. 5

D. D. 6

Answer: C

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8. If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1,$ then the point $\left(\frac{1}{e}, \frac{1}{e'}\right)$ lies on the circle (A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$ (C) $x^2 + y^2 = 3$ (D) $x^2 + y^2 = 4$

A. $x^2 + y^2 = 1$

B. $x^2 + y^2 = 2$

C. $x^2 + y^2 = 3$

D. $x^2 + y^2 = 4$

Answer: A

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9. the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$ is

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10. The length of the transverse axis of the hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ is

A. 8

B. 4

C. 6

D. 2

Answer: A

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11. A hyperbola has centre 'C" and one focus at $P(6, 8)$. If its two directrixes are $3x + 4y + 10 = 0$ and $3x + 4y - 10 = 0$ then $CP =$

A. 14

B. 8

C. 10

D. 6

Answer: C

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12. If the foci of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ coincide, the value of a is

A. 3

B. 2

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Answer: A

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13. A rectangular hyperbola of latus rectum 4 units passes through (0,0) and has (2,0) as its one focus. The equation of locus of the other focus is

A. $x^2 + y^2 = 36$

B. $x^2 + y^2 = 4$

C. $x^2 - y^2 = 4$

D. $x^2 + y^2 = 9$

Answer: A

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14. If the curves $x^2 - y^2 = 4$ and $xy = \sqrt{5}$ intersect at points A and B, then the possible number of points (s) C on the curve $x^2 - y^2 = 4$ such that triangle ABC is equilateral is

A. 0

B. 1

C. 2

D. 4

Answer: A

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15. The point $(3\tan(\theta + 60^{\circ}), 2\tan(\theta + 30^{\circ}))$ lies on the conic, then its centre is (θ is the parameter)

A. $(-3\sqrt{3}, 2\sqrt{3})$

B. $(3\sqrt{3}, -2\sqrt{3})$

C. $(-3\sqrt{3}, -2\sqrt{3})$

D. (0,0)

Answer: A

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16. The equation of a tangent to the hyperbola $3x^2 - y^2 = 3$, parallel to the line $y = 2x + 4$ is

A. $y = 2x + 3$

B. $y = 2x + 1$

C. $y = 2x + 4$

D. $y = 2x + 2$

Answer: B

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17. A tangent to the hyperbola $y = \frac{x+9}{x+5}$ passing through the origin in

A. $x + 25y = 0$

B. $5x + y = 0$

C. $5x - y = 0$

D. $x - 25y = 0$

Answer: A

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18. The absolute value of slope of common tangents to parabola $y^2 = 8x$ and hyperbola $3x^2 - y^2 = 3$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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19. For the hyperbola $xy = 8$ any tangent of it at P meets co-ordinates at Q and R then area of triangle CQR where 'C' is centre of the hyperbola is

A. 16 sq. units

B. 12 sq. units

C. 24 sq. units

D. 18 sq. units

Answer: A

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20. The tangents and normal at a point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the y-axis A and B. Then the circle on AB as diameter passes through the focii of the hyperbola

A. one of the vertex of the hyperbola

B. one of the foot of directrix on x-axis of the hyperbola

C. foci of the hyperbola

D. none of these

Answer: C

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21. If $4x^2 + py^2 = 45$ and $x^2 - 4y^2 = 5$ cut orthogonally, then the value of p is

A. $1/9$

B. $1/3$

C. 3

D. 9

Answer: D

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22. A tangent drawn to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes, then the squae of its eccentricity is equal to

A. 15

B. 16

C. 17

D. 18

Answer: C

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23. If m is the slope of a tangent to the hyperbola $\frac{x^2}{a^2-b^2}-\frac{y^2}{a^3-b^3}=1$ where $a>b>1$ when

A. $(a+b)m^2+ab\geq(a+b)^2$

B. $(a+b)^2m+ab\geq(a+b)$

C. $abm^2+(a+b)\geq(a+b)^2$

D. $(a+b)m^2+a^2b^2\geq(a+b)^2$

Answer: A

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24. Two tangents to the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$, having slopes 2 and m where $(m \neq 2)$ cuts the axes at four concyclic points then the slope m is/are

A. $-\frac{1}{2}$

B. -2

C. $\frac{1}{2}$

D. 2

Answer: C

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25. The equation of that chord of hyperbola $25x^2 - 16y = 400$, whose mid point is (5,3) is

A. $115x - 117y = 17$

B. $125x - 48y = 481$

C. $127x + 33y = 341$

D. $15x - 121y = 105$

Answer: B

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26. If a chord joining $P(a\sec\theta, a\tan\theta), Q(a\sec\alpha, a\tan\alpha)$ on the hyperbola $x^2 - y^2 = a^2$ is the normal at P, then $\tan\alpha$ is (a) $\tan\theta(4\sec^2\theta + 1)$ (b) $\tan\theta(4\sec^2\theta - 1)$ (c) $\tan\theta(2\sec^2\theta - 1)$ (d) $\tan\theta(1 - 2\sec^2\theta)$

A. $\tan\theta(4\sec^2\theta + 1)$

B. $\tan\theta\left(4\sec^2\theta - 1\right)$

C. $\tan\theta\left(2\sec^2\theta - 1\right)$

D. $\tan\theta\left(1 - 2\sec^2\theta\right)$

Answer: B

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27. The number of normal (s) of a rectangular hyperbola which can touch its conjugate is equal to

A. 0

B. 2

C. 4

D. 8

Answer: C

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28. If the normal at a point P to the hyperbola meets the transverse axis at G, and the value of SG/SP is 6, then the eccentricity of the hyperbola is (where S is focus of the hyperbola)

A. 2

B. 4

C. 6

D. 8

Answer: C

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29. If the normal at $P(a\sec\theta, b\tan\theta)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the transverse axis in G then minimum length of PG is

A. $\frac{b^2}{a}$

B. $\left|\frac{a}{b}(a+b)\right|$

C. $\left|\frac{a}{b}(a-b)\right|$

D. $\left|\frac{a}{b}(a-b)\right|$

Answer: A

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30. If normal to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ drawn at an extremity of its latus-rectum has slope equal to the slope of line which meets hyperbola only once, then the eccentricity of hyperbola is

A. $e=\sqrt{\frac{1+\sqrt{5}}{2}}$

B. $e=\sqrt{\frac{\sqrt{5}+3}{2}}$

C. $e=\sqrt{\frac{2}{\sqrt{5}-1}}$

D. None of these

Answer: A

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31. At the point of intersection of the rectangular hyperbola $xy = c^2$ and the parabola $y^2 = 4ax$ tangents to the rectangular hyperbola and the parabola make angles θ and ϕ , respectively with x-axis, then

A. $\theta = \tan^{-1}(-2\tan\phi)$

B. $\theta = \frac{1}{2}\tan^{-1}(-\tan\phi)$

C. $\phi = \tan^{-1}(-2\tan\theta)$

D. $\phi = \frac{1}{2}\tan^{-1}(-\tan\theta)$

Answer: A

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32. The number of points from where a pair of perpendiculartangents can be drawn to the hyperbola, $x^2\sec^2\alpha - y^2\cos ec^2\alpha = 1, \alpha \in \left(0, \frac{\pi}{4}\right)$, is

(A) 0 (B) 1 (C) 2 (D) infinite

A. 0

B. 1

C. 2

D. infinite

Answer: D

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33. If e is the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and θ is the angle between the asymptotes, then $\cos.\ \frac{\theta}{2}$ is equal to

A. $\frac{1-e}{e}$

B. $\frac{2}{e} - e$

C. $\frac{1}{e}$

D. $\frac{2}{e}$

Answer: C

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34. The equation of a hyperbola whose asymptotes are $3x \pm 5y = 0$ and vertices are $(\pm 5, 0)$ is

A. $9x^2 - 25y^2 = 225$

B. $25x^2 - 9y^2 = 225$

C. $5x^2 - 3y^2 = 225$

D. $3x^2 - 5y^2 = 25$

Answer: A

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35. The tangent at P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets one of the asymptote in Q. Then the locus of the mid-point of PQ is

A. $3\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = 4$

B. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$

C. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{1}{2}$

D. $4\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = 3$

Answer: D

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36. Find the locus of the foot of perpendicular from the centre upon any normal to line hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

A. $\left(x^2 - + y^2\right)^2\left(\frac{a^2}{x^2} + \frac{b^2}{y^2}\right) = \left(a^2 - b^2\right)^2$

B. $\left(x^2 + y^2\right)^2\left(\frac{a^2}{x^2} - \frac{b^2}{y^2}\right) = \left(a^2 + b^2\right)^2$

C. $\left(x^2 + y^2\right)^2\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = \left(a^2 + b^2\right)^2$

D. None of these

Answer: B

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37. Let the transverse axis ofa varying hyperbola be fixed with length of transverse axis being 2a. Then the locus of the point of contact of any tangent drawn to it from a fixed point on conjugate axis is

A. a parabola

B. a circle

C. an ellipse

D. a hyperbola

Answer: A

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38. The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is (A) $\left(x^2 - y^2\right) = 4c^2xy$ (B) $\left(x^2 + y^2\right)^2 = 2c^2xy$ (C) $\left(x^2 + y^2\right) = 4c^2xy$ (D) $\left(x^2 + y^2\right)^2 = 4c^2xy$

A. $\left(x^2 - y^2\right)^2 = 4c^2xy$

B. $\left(x^2 + y^2\right)^2 = 2c^2xy$

C. $\left(x^2 - y^2\right)^2 = 2c^2xy$

D. $\left(x^2 + y^2\right)^2 = 4c^2xy$

Answer: D

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