MATHS

BOOKS - CENGAGE PUBLICATION

INEQUALITIES INVOLVING MEANS

Single Correct Answer

1. If a, b, c be three positive numbers in $A.P.$ and $E = \frac{a+8b}{2b-a} + \frac{8b+c}{2b-c}$, then a value of E can be

A. 16

B. 15

C. 17

D. 21

Answer: D

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2. Let $ab = 1$, then the minimum value of $\frac{1}{a^4} + \frac{1}{4b^4}$ is

A. 1

B. 2

C. $1/4$

D. $1/2$

Answer: A

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3. If $x > 0, y > 0, z > 0$, the least value of

$x^{\log_e y - \log_e z} + y^{\log_e z - \log_e x} + Z^{\log_e x - \log_e y}$ is

A. 3

B. 1

C. 5

D. 6

Answer: A

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4. Let $p, q,\ r \in R^{+}$ and $27pqr \geq (p+q+r)^{3}$ and $3p+4q+5r=12.$

Then the value of $8p+4q-7r=$

A. 2

B. 3

C. 4

D. 5

Answer: D

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5. Minimum value of $f(x) = \cos^2 x + \frac{\sec x}{4}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

A. $3/2$

B. $3/4$

C. $3/8$

D. none of these

Answer: B

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6. Let $x, y, z \in R^+$ and $2xy + 3yz + 4xz = 18$. If α, β and γ be the values of x, y and z respectively, for which xyz attains its maximum value, then the value of $2\alpha + \beta + \gamma =$

(a) 4 (b) 6 (c) 8 (d) 12

A. 4

B. 6

C. 8

D. 12

Answer: B

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7. The minimum value of $\frac{x^4+y^4+z^2}{xyz}$ for positive real numbers x, y, z is (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) $8\sqrt{2}$

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. $8\sqrt{2}$

Answer: B

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8. If $n_1,\ n_2,\ n_3,......,n_{100}$ are positive real numbers such that $n_1 + n_2 + n_3 + \ldots + n_{100} = 20$ and $k = n_1(n_2 + n_3 + n_4)(n_5 + n_6 + \ldots + n_9)(n_{10} + \ldots. + n_{16})\ldots(\ldots + n_{10}$

, then k belongs to

A. A. $(o, 100]$

B. B. $(o, 128]$

C. C. $(o, 144]$

D. D. $(o, 1024]$

Answer: D

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9. If $a,\ b,\ c$ are the sides of triangle , then the least value of $\dfrac{a}{c+a-b} + \dfrac{b}{a+b-c} + \dfrac{c}{b+c-a}$ is

A. $1/3$

B. 1

C. 3

D. 6

Answer: C

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10. If $x > 0, \dfrac{x^n}{1 + x + x^2 + \dots + x^{2n}}$ is

A. A. $\leq \dfrac{1}{2n+1}$

B. B. $< \dfrac{2}{2n+1}$

C. C. $\geq \dfrac{1}{2n+1}$

D. D. $> \dfrac{2}{2n+1}$

Answer: A

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11. If positive quantities a, b, c are in $H.P.$, then which of the following is not true ? a. $b > \frac{a+c}{2}$ b. $\frac{1}{a-b} - \frac{1}{b-c} > 0$ c. $ac > b^2$ d. none of these

A. $b > \frac{a+c}{2}$

B. $\frac{1}{a-b} - \frac{1}{b-c} > 0$

C. $ac > b^2$

D. none of these

Answer: D

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12. Given that x, y, z are positive real numbers such that $xyz = 32$, the minimum value of $\sqrt{(x+2y)^2 + 2z^2 - 15}$ is equal to

A. 6

B. 8

C. 9

D. 12

Answer: C

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13. If x, y, z are positive real numbers such that $x^2 + y^2 + Z^2 = 7$ and $xy + yz + xz = 4$ then the minimum value of xy is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: C

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14. If x lies between -5 and 11, then the greatest value of $(11-x)^3(x+5)^5$ is

A. $6^5 \cdot 10^3$

B. $6^3 \cdot 10^3$

C. $6^3 \cdot 10^4$

D. $6^3 \cdot 10^5$

Answer: D

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15. The least integral value of $f(x)=\dfrac{(x-1)^7+3(x-1)^6+(x-1)^5+1}{(x-1)^5}, \forall x>1$ is (a) 8 (b) 6 (c) 12 (d) 18

A. 8

B. 6

C. 12

D. 18

Answer: B

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16. If a, b, c are positive real numbers and $2a + b + 3c = 1$, then the maximum value of $a^4 b^2 c^2$ is equal to

A. $\frac{1}{3 \cdot 4^8}$

B. $\frac{1}{9 \cdot 4^7}$

C. $\frac{1}{9 \cdot 4^8}$

D. $\frac{1}{27 \cdot 4^8}$

Answer: C

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17. If x, y, z be three positive numbers such that xyz^2 has the greatest value $\frac{1}{64}$, then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is

A. 6

B. 8

C. 10

D. 12

Answer: C

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18. If x_1, x_2 and x_3 are the positive roots of the equation $x^3-6x^2+3px-2p=0$, $p\in R$, then the value of $\sin^{-1}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)+\cos^{-1}\left(\frac{1}{x_2}+\frac{1}{x_3}\right)-\tan^{-1}\left(\frac{1}{x_3}+\frac{1}{x_1}\right)$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. π

Answer: A

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Comprehension

1. If a, b, x, y are real number and $x, y > 0$, then $\frac{a^2}{x} + \frac{b^2}{y} \geq \frac{(a+b)^2}{x+y}$ so on solving it we have $(ay - bx)^2 \geq 0$.

Similarly, we can extend the inequality to three pairs of numbers, i.e,

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq \frac{(a+b+c)^2}{x+y+z}$$

Now use this result to solve the following questions.

If $abc = 1$, then the minimum value of

$\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)}$ is

A. $\geq (a+b+c)$

B. $\geq \frac{1}{2}(a+b+c)$

C. $\frac{3}{2} \leq (a+b+c)$

D. None of these

Answer: A

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2. If a, b, x, y are real number and $x, y > 0$, then $\frac{a^2}{x} + \frac{b^2}{y} \geq \frac{(a+b)^2}{x+y}$ so on solving it we have $(ay - bx)^2 \geq 0$.

Similarly, we can extend the inequality to three pairs of numbers, i.e,

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq \frac{(a+b+c)^2}{x+y+z}$$

Now use this result to solve the following questions.

The value of $\frac{a^2+b^2}{a+b} + \frac{b^2+c^2}{b+c} + \frac{a^2+c^2}{a+c}$ is

A. 3

B. $3/2$

C. 6

D. 9

Answer: B

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Illustration

1. Prove that $(ab+xy)(ax+by) > 4abxy(a, b, x, y > 0)$.

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2. Prove that $b^2c^2+c^2a^2+a^2b^2 > abc(a+b+c)$, where $a, b, c > 0$.

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3. Prove that $\frac{2}{b+c}+\frac{2}{c+a}+\frac{2}{a+b}\left\langle\frac{1}{a}+\frac{1}{b}+\frac{1}{c}, wherea, b, c\right\rangle 0.$

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4. If $a, b, andc$ are distinct positive real numbers such that $a+b+c=1,$ then prove that $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)} > 8.$

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5. Find the minimum value of $4^{\sin}\ \hat{}\ (2x) + 4^{\cos}\ \hat{}\ (2x)$.

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6. If $(\log)_2(a+b) + (\log)_2(c+d) \geq 4.$ Then find the minimum value of the expression $a+b+c+\cdot\cdot$

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7. Find all real solutions to $2^x + x^2 = 2 - \frac{1}{2^x}.$

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8. Find all positive real solutions to $4x + \frac{18}{y} = 14, 2y + \frac{9}{z} = 15, 9z + \frac{16}{x} = 17.$

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9. Let A;G;H be the arithmetic; geometric and harmonic means between three given no. a;b;c then the equation having a;b;c as its root is

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11. If $a+b+c=1$, then prove that $\frac{8}{27abc} > \left\{\frac{1}{a}-1\right\}\left\{\frac{1}{b}-1\right\}\left\{\frac{1}{c}-1\right\} > 8.$

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12. If $yz+zx+xy=12, where x, y, z$ are positive values, find the greatest value of $xyz\cdot$

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13. If a, b, c are positive, then prove that $a/(b+c)+b/(c+a)+c/(a+b) \geq 3/2.$

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14. Prove that $2^n > 1+n\sqrt{2^{n-1}}, \forall n > 2$ where n is a positive integer.

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15. If $S + a_1 + a_2 + a_3 + \quad + a_n, a_1 \in R^+$ for $i = 1 \rightarrow n,$ then prove that $\frac{S}{S - a_1} + \frac{S}{S - a_2} + \quad + \frac{S}{S - a_n} \geq \frac{n^2}{n - 1}, \forall n \geq 2$

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16. If $a_1 + a_2 + a_3 + \ldots\ldots + a_n = 1 \forall a_i > 0, i = 1, 2, 3, \ldots\ldots, n,$ then find the maximum value of $a_1 a_2 a_3 a_4 a_5 \ldots\ldots a_n$.

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17. If $a, b, c,$ are positive real numbers, then prove that (2004, 4M) $\{(1 + a)(1 + b)(1 + c)\}^7 > 7^7 a^4 b^4 c^4$

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18. Prove that $\frac{\sec^4 \alpha}{\tan^2 \beta} + \frac{\sec^4 \beta}{\tan^2 \alpha} \geq 8$. If each term in the expression is well defined.

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19. Prove that

$$\left[\frac{x^2 + y^2 + z^2}{x + y + z}\right]^{x+y+z} > x^x y^y z^z > \left[\frac{x + y + z}{3}\right]^{x+y+z} (x, y, z > 0)$$

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20. Prove that

$1^1 \times 2^2 \times 3^3 \times \quad \times un \leq [(2n + 1) / 3]n(n + 1) / 2, n \in N.$

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21. Find the greatest value of $x^2 y^3$, where x and$y$ lie in the first quadrant on the line $3x + 4y = 5$.

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22. Find the maximum value of $(7-x)^4(2+x)^5 when x$ lies between $-2 and 7.$

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23. Find the maximum value of xyz when $\frac{x}{1}+\frac{y^2}{4}+\frac{z^3}{27}=1,$ where $x, y, z > 0.$

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24. If $a, b > 0$ such that $a^3+b^3=2,$ then show that $a+b \leq 2.$

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25. If $m > 1, n \in N$ show that $1m + 2m + 2^{2m} + 2^{3m} + \quad + 2^{nm-m} > n^{i-m}(2^n - 1)^m$.

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26. Prove that in an acute angled triangle $ABC, \sec A + \sec B + \sec C \geq 6$.

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27. Prove that $\frac{b^2 + c^2}{b + c} + \frac{c^2 + a^2}{c + a} + \frac{a^2 + b^2}{a + b} > a + b + c$

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28. Prove that $\frac{a^8 + b^8 + c^8}{a^3 b^3 c^3} > \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

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29. If $a, b, andc$ are positive and $a + b + c = 6,$ show that $(a + 1/b)2 + (b + 1/c)2 + (c + 1/a)2 \geq 75/4.$

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Example 1

1. Let $x_1, x_2, , x_n$ be positive real numbers and we define $S = x_1 + x_2 + \; + x_n.$ Prove that $(1 + x_1)(1 + x_2)(1 + x_n) \leq 1 + S + \frac{S^2}{2!} + \frac{S^3}{3!} + \; + \frac{S^n}{n!}$

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Example 2

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Example 3

1. Calculate the greatest and least values of the function

$$f(x) = \frac{x^4}{x^8 + 2x^6 - 4x^4 + 8x^2 + 16}$$

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Example 4

1. If a,b,c are three distinct positive real numbers in G.P., than prove that

$c^2 + 2ab > 3ac.$

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Example 5

1. If x and y are real numbers such that $16^{x^2+y}+16^{x+y^2}=1$, then find the values of x and y.

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Example 6

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Example 7

1. If $x+y+z=1 and x, y, z$ are positive, then show that $\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2+\left(z+\frac{1}{z}\right)^2>\frac{100}{3}$

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Example 8

1. If a, b,c are three positive real numbers , then find minimum value of $\frac{a^2+1}{b+c}+\frac{b^2+1}{c+a}+\frac{c^2+1}{a+b}$

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Concept Application Eexercises 6 1

1. Prove that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{1}{\sqrt{(bc)}}+\frac{1}{\sqrt{(ca)}}+\frac{1}{\sqrt{(ab)}}$, where a,b,c $>$ 0

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2. If a, b, c are real numbers such that $0<a<1, 0<b<1, 0<c<1, a+b+c=2,$ then prove that $\frac{a}{1-a}\frac{b}{1-b}\frac{c}{1-c}\geq 8$

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3. If x,y are positive real numbers and m, n are positive integers, then prove that $\frac{x^n y^m}{(1+x^{2n})(1+y^{2m})} \leq \frac{1}{4}$

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4. If $a_i > 0\ (i = 1, 2, 3, \ldots .\ n)$ prove that

$$\sum_{1 \leq i \leq j \leq n} \sqrt{a_i a_j} \leq \frac{n-1}{2}(a_1 + a_2 + \ldots . + a_n)$$

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5. Find the minimum value of $2^{\sin x} + 2^{\cos x}$

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6. If $(\log)_{10}(x^3 + y^3) - (\log)_{10}(x^2 + y^2 - xy) \le 2,$ and x, y are positive real number, then find the maximum value of xy.

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7. Prove that the greatest value of xy is $c^3 / \sqrt{2ab}$. if $a^2x^4 + b^4y^4 = c^6$.

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8. If $x, y \in R^+$ such that x + y = 8, then find the minimum value of $\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right)$

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Concept Application Eexercises 6 2

1. Prove that $\left(\frac{n+1}{2}\right)^n > n!$

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2. If $a_1, a_2, \ldots\ldots, a_n > 0,$ then prove that $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_4} + \ldots\ldots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} > n$

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3. In ABC , prove that $tanA + tanB + tanC \geq 3\sqrt{3}, whereA, B, C$ are acute angles.

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4. If n $\geq$ 1 is a positive integer, then prove that $3^n \geq 2^n + n.\, 6^{\frac{n-1}{2}}$

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5. If $abcd = 1$ where a,b,c,d are positive reals then the minimum value of $a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + cd$ is

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6. If x,y,z $>$ 0 and x + y + z = 1, the prove that

$$\frac{2x}{1-x} + \frac{2y}{1-y} + \frac{2z}{1-z} \geq 3.$$

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Concept Application Eexercises 6 3

1. Prove that $\left[\frac{a^2 + b^2}{a+b}\right]^{a+b} > a^a b^b > \left\{\frac{a+b}{2}\right\}^{a+b}.$

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2. Prove that $a^p b\&q > \left(\frac{ap + bp}{p+q}\right)^{p+q}.$

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3. Prove that $px^{q-r}+qx^{r-p}+rx^{p-q}>p+q+r, wherep, q, r$ are distinct and $x\ne1$.

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4. Given are positive rational numbers a, b, c such that $a+b+c=1$, then prove that $a^ab^bc^c+\ \wedge cb^ab^ac^b\le1$.

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5. If a and b are positive numbers such that $a^2+b^2=4$, then find the maximum value of a^2b.

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6. Find the greatest value of $x^2y^3z^4$ if $x^2+y^2+z^2=1, wherex, y, z$ are positive.

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Concept Application Eexercises 6 4

1. Prove that $a^4+b^4+c^4>abc(a+b+c), wherea, b, c>0.$

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2. If $C_r=\frac{n!}{[r!(n-r)]}$, the prove that `sqrt(C_1)+sqrt(C_2)++sqrt(C_n)

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3. If a + b =1, a $>$ 0,b $>$ 0, prove that $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2\geq\frac{25}{2}$

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4. If A+B+C=π Prove that in triangle ABC, sin $A + \sin B + C \le \frac{3\sqrt{3}}{2}$

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Exercises Single Correct Answer Type

1. The minimum value of $\frac{x^4 + y^4 + z^2}{xyz}$ for positive real numbers x, y , z is

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. $8\sqrt{2}$

Answer: B

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2. A rod of fixed length k slides along the coordinates axes, If it meets the axes at $A(a, 0) and B(0, b)$, then the minimum value of $\left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2$ 0 8 $k^2 = 4 + \frac{4}{k^2}$ $k^2 + 4 + \frac{4}{k^2}$

A. 0

B. 8

C. $k^2 - 4 + \frac{4}{k^2}$

D. $k^2 + 4 + \frac{4}{k^2}$

Answer: D

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3. The least value of $6\tan^2\varphi + 54\cot^2\varphi + 18$ is 54 when A.M. $\geq$ GM. Is applicable for $6\tan^2\varphi, 54\cot^2\varphi, 18$ 54 when A.M. $\geq$ GM. Is applicable for $6\tan^2\varphi, 54\cot^2\varphi, 18$ is added further 78 when $\tan^2\varphi = \cot^2\varphi$ (I) is correct, (II) is false (I) and (II) are correct (III) is correct None of the above are correct

A. (I) is correct, (II) is false

B. (I) and (II) are correct

C. (III) is correct

D. (III) is correct

Answer: B

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4. If $ab^2c^3, a^2b^3c^4, a^3b^4c^5$ are in A.P. $(a, b, c > 0)$, then the minimum value of $a + b + c$ is (a) 1 (b) 3 (c) 5 (d) 9

A. 1

B. 3

C. 5

D. 9

Answer: B

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5. If $y = 3^{x-1} + 3^{-x-1}$, then the least value of y is 2 6 $2/3$ $3/2$

A. 2

B. 6

C. $2/3$

D. $3/2$

Answer: C

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6. Minimum value of $(b+c)/a + (c+a)/b + (a+b)/c$ (for real positive numbers a, b, c) is 1 2 4 6

A. 1

B. 2

C. 3

D. 6

Answer: D

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7. If the product of n positive numbers is n^n , then their sum is a positive integer b. divisible by n equal to $n + 1/n$ never less than n^2

A. a positive integer

B. divisible by n

C. equal to $n + l/n$

D. never less than n^2

Answer: D

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8. The minimum value of $P = bcx + cay + abz$, when $xyz = abc$, is $a. 3abc\ b. 6abc\ c. abc\ d. 4abc$

A. 3abc

B. 6abc

C. abc

D. 4abc

Answer: A

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9. If l, m, n are the three positive roots of the equation $x^3 - ax^2 + bx - 48 = 0,$ then the minimum value of $(1/l) + (2/m) + (3/n)$ equals a1 b2 c$\frac{3}{2}$ d$\frac{5}{2}$

A. 1

B. 2

C. $3/2$

D. $5/2$

Answer: C

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10. If positive numbers a, b, c are in H.P., then equation $x^2 - kx + 2b^{101} - a^{101} - c^{101} = 0 (k \in R)$ has both roots positive both roots negative one positive and one negative root both roots imaginary

A. both roots positive

B. both roots

C. one positive and one negative root

D. both roots imaginary

Answer: C

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11. For $x^2 - (a+3)|x| - 4 = 0$ to have real solutions, the range of a is

a)$(-\infty, -7] \cup [1, \infty)$

b)$(-3, \infty)$

c)$(-\infty, -7]$

d)$[1, \infty)$

A. $(-\infty, -7] \cup [1, \infty)$

B. $(-3, \infty)$

C. $(-\infty, -7]$

D. $[1, \infty)$

Answer: D

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12. If a, b, c are the sides of a triangle, then the minimum value of $\frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$ is equal to 3 6 9 12

A. 3

B. 6

C. 9

D. 12

Answer: A

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13. If $a, b, c, d \in R^{+} - \{1\}$, then the minimum value of $\log_d a + \log_c b + \log_a c + \log_b d$ is

A. 4

B. 2

C. 1

D. none of these

Answer: A

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14. If $a, b, c \in R^{+}, then \frac{bc}{b+c}+\frac{ac}{a+c}+\frac{ab}{a+b}$ is always $\leq \frac{1}{2}(a+b+c)$ $\geq \frac{1}{3}\sqrt{abc}$ $\leq \frac{1}{3}(a+b+c)$ $\geq \frac{1}{2}\sqrt{abc}$

A. $\leq \frac{1}{2}(a+b+c)$

B. $\geq \frac{1}{2}\sqrt{abc}$

C. $\leq \frac{1}{3}(a+b+c)$

D. $\geq \frac{1}{2}\sqrt{abc}$

Answer: A

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15. If $a, b, c \in R^{+} then (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is always ≥ 12 ≥ 9 ≤ 12 none of these

A. ≥ 12

B. ≥ 9

C. ≤ 12

D. none of these

Answer: B

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16. If $a, b, c \in R^{+}$, then the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is equal to abc $2abc$ $3abc$ $6abc$

A. abc

B. 2abc

C. 3abc

D. 6abc

Answer: D

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17. If $a, b, c, d \in R^{+} and a, b, c$ are in H.P., then $a + d > b + c$ $a + b > c + d$ $a + c > b + d$ none of these

A. $a + d > b + c$

B. $a + b > c + d$

C. $a + c > b + d$

D. none of these

Answer: A

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18. If $a, b, c, d \in R^{+}$ such that $a + b + c = 18$, then the maximum value of $a^2 b^3 c^4$ is equal to a. $2^{18} \times 3^2$ b. $2^{18} \times 3^3$ c. $2^{19} \times 3^2$ d. $2^{19} \times 3^3$

A. $2^{18} \times 3^2$

B. $2^{18} \times 3^3$

C. $2^{19} \times 3^{2}$

D. $2^{19} \times 3^{3}$

Answer: D

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19. $f(x) = \dfrac{(x-2)(x-1)}{x-3}, \forall x > 3.$ The minimum value of $f(x)$ is equal to

A. $3 + 2\sqrt{2}$

B. $3 + 2\sqrt{3}$

C. $3\sqrt{2} + 2$

D. $3\sqrt{2} - 2$

Answer: A

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20. If $a > 0$, then least value of $\left(a^3 + a^2 + a + 1\right)^2$ is (a)$64a^2$ (b)$16a^4$ (c) $16a^3$ (d)d. none of these

A. $64a^2$

B. $16a^4$

C. $16a^3$

D. none of these

Answer: C

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Multiple Correct Answers Type

1. If A is the area and $2s$ is the sum of the sides of a triangle, then $A \leq \frac{s^2}{4}$ (b) $A \leq \frac{s^2}{3\sqrt{3}}$ $2R \sin A \sin B \sin C$ (d) $noneofthese$

A. $A \leq \frac{s^2}{4}$

B. $A \leq \frac{s^2}{3\sqrt{3}}$

C. $A < \frac{s^2}{\sqrt{3}}$

D. none of these

Answer: A::B

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2. If x, y, z are positive numbers is $A\dot{P}; then\ y^2 \geq xz\ xy + yz \geq 2xz$ $\frac{x+y}{2y-x} + \frac{y+z}{2y-z}$ d. none of these

A. $y^2 \geq xz$

B. $xy + yz \geq 2xz$

C. $\frac{x+y}{2y-x} + \frac{y+z}{2y-z} \geq 4$

D. none of these

Answer: A::C

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3. For positive real numbers a, bc such that $a + b + c = p,$ which one holds? $(p-a)(p-b)(p-c) \leq \frac{8}{27}p^3$ $(p-a)(p-b)(p-c) \geq 8abc$ $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \leq p\, noneofthese$

A. $(p-a)(p-b)(p-c) \leq \frac{8}{27}p^3$

B. $(p-a)(p-b)(p-c) > 8abc$

C. $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \leq p$

D. none of these

Answer: A::B

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4. The corresponding first and the (2n-1)th terms of an A.P a G.P and a H.P are equal ,If their nth terms are a, b and c , respectively , then

A. $a = b = c$

B. $a + c = b$

C. $a > b > c$

D. $ac - b^2 = 0$

Answer: C::D

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5. If $a > 0, b > 0, c > 0$ and $2a + b + 3c = 1$, then

A. $a^4b^2c^2$ is greatest then $a = \frac{1}{4}$

B. $a^4b^2c^2$ is greatest then $b = \frac{1}{4}$

C. $a^4b^2c^2$ is greatest then $c = \frac{1}{12}$

D. greatest value of $a^4b^2c^2 is \frac{1}{9.4^8}$

Answer: A::B::C::D

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Linked Comprehension Type

1. If roots of the equation $f(x) = x^6 - 12x^5 + bx^4 + cx^3 + dx^2 + ex + 64 = 0$are positive, then

Which has the greatest absolute value ? (a) b (b) c (c) d (d) e

A. b

B. c

C. d

D. e

Answer: C

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2. If roots of the equation $f(x) = x^6 - 12x^5 + bx^4 + cx^3 + dx^2 + ex + 64 = 0$are positive, then

Which has the greatest absolute value ? (a) b (b) c (c) d (d) e

A. b

B. c

C. d

D. e

Answer: A

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3. If roots of the equation $f(x) = x^6 - 12x^5 + bx^4 + cx^3 + dx^2 + ex + 64 = 0$are positive, then remainder when $f(x)$ is divided by $x - 1$ is (a) 2 (b) 1 (c) 3 (d) 10

A. 2

B. 1

C. 3

D. 10

Answer: B

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4. Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a,b,c are non-negative).

Minimum non-negative real value of a is (a) 10 (b) 9 (c) 6 (d) 4

A. 10

B. 9

C. 6

D. 4

Answer: D

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5. Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a,b,c are non-negative).

Minimum non-negative real value of b is (a) 12 (b) 15 (c) 6 (d) 10

A. 12

B. 15

C. 6

D. 10

Answer: C

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6. Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a,b,c are non-negative). Minimum non-negative real value of c is (a) 10 (b) 9 (c) 6 (d) 4

A. 10

B. 9

C. 6

D. 4

Answer: D

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Numerical Value Type

1. For $x \geq 0$, the smallest value of the function $f(x) = \frac{4x^2 + 8x + 13}{6(1+x)}$, is _______.

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2. Let $x^2 - 3x + p = 0$ has two positive roots $a and b$, then minimum value if $\left(\frac{4}{a} + \frac{1}{b}\right)$ is,

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3. If x, y and z are posirive real umbers and $x = \dfrac{12 - yz}{y + z}$. The maximum value of xyz equals.

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4. If a,b, and c are positive and $9a + 3b + c = 90$, then the maximum value of $(\log a + \log b + \log c)$ is (base of the logarithm is 10)_______.

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5. Given that x, y, z are positive reals such that $xyz = 32$. The minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is _________.

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6. If $x, y \in R^+$ satisfying $x + y = 3,$ then the maximum value of $x^2 y$ is.

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7. For any$x, y, \in R^+, xy > 0$. Then the minimum value of $\frac{2x}{y^3} + \frac{x^3 y}{3} + \frac{4y^2}{9x^4}$ is.

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8. Let a,b,c,d and e be positive real numbers such that $a + b + c + d + e = 15$ and $ab^2c^3d^4e^5 = (120)^3 \times 50$. Then the value of $a^2 + b^2 + c^2 + d^2 + e^2$ is ________.

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9. Consider the system of equations $x_1 + x_2^2 + x_3^3 + x_4^4 + x_5^5 = 5$ and $x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 15$ where x_1, x_2, x_3, x_4, x_5 are positive real numbers. Then numbers of $(x_1, x_2, x_3, x_4, x_5)$ is ________.

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10. The minimum value of the sum of real number $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8, and a^{10} with a > 0$ is

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Jee Advanced Single

1. The least value of $a \in \mathbb{R}$ for which $4ax^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{27}$

D. $\frac{1}{25}$

Answer: C

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