MATHS

BOOKS - CENGAGE PUBLICATION

INTEGRALS

Others

1. If $\int \frac{dx}{x^2 + ax + 1} = f(g(x)) + c,$ then $f(x)$ is inverse trigonometric function for $|a| > 2$ $f(x)$ is logarithmic function for $|a| < 2$ $g(x)$ is quadratic function for $|a| > 2$ $g(x)$ is rational function for $|a| < 2$

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2. If $f(x) = \int \frac{x^8 + 4}{x^4 - 2x^2 + 2} dx$ and $f(0) = 0, then$ (a)$f(x)$ is an odd function (b)$f(x)$ has range R (c)$f(x)$ has at least one real root (d)$f(x)$ is a monotonic function.

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3. $\int \frac{dx}{(x+1)(x-2)} = A\log(x+1) + B\log(x-2) + c$ where

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4. Each question has four choices, a,b,c and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. (a) If both the statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1. (b) If both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. (c) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE. (d) If STATEMENT 1 is FALSE and STATEMENT 2 is TRUE. Statement 1: $\int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + c$ Statement 2: $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$

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5. If $\int \frac{x^2 - x + 1}{(x^2 + 1)^{\frac{3}{2}}} e^x dx = e^x f(x) + c$, then (a) $f(x)$ is an even function (b) $f(x)$ is a bounded function (c) the range of $f(x)$ is $(0, 1)$ (d) $f(x)$ has two points of extrema

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6. $If \int \frac{x^4 + 1}{x^6 + 1} dx = \tan^{-1} f(x) - \frac{2}{3} \tan^{-1} g(x) + C, then$

a) both $f(x) and g(x)$ are odd functions

$b)$ $f(x)$ is monotonic function

$c)$ $f(x) = g(x)$ has no real roots

$d) \int \frac{f(x)}{g(x)} dx = -\frac{1}{x} + \frac{3}{x^3} + c$

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7. If $\int \sin^{-1} x \cos^{-1} x dx = f^{-1}(x) \left[\frac{\pi}{2} x - x f^{-1}(x) - 2\sqrt{1 - x^2}\right] + 2x + \left(\frac{\pi}{2}\right.$

$(a) f(x) = \sin x$

$(b) f(x) = \cos x$

$(c) f(x) = \tan x$

(d) none of these

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8. If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A \cos 4x + B, then$ (a)$A = -\frac{1}{8}$ (b) $B = \frac{1}{2}$ (c) $A = -\frac{1}{4}$ (d) None of this

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9. Statement 1: $\int \frac{dx}{x^3\sqrt{1+x^4}} = -\frac{1}{2}\sqrt{1+\frac{1}{x^4}} + C$ Statement 2: For integration by parts, we have to follow ILATE rule. Which of the following Statements is/are correct ?

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10. Statement 1: $\int \frac{\sin x dx}{x}, (x > 0),$ cannot be evaluated.

Statement 2: Only differentiable functions can be integrated.

(a) statement 1 is true, statement 2 is true, Statement 2 is the correct explanation for statement 1.

(b) statement 1 is true, statement 2 is true, Statement 2 is not correct explanation for statement 1.

(c) statement 1 is true, statement 2 is not true.

(d) statement 2 is true, statement 1 is not true.

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11. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log_e (9e^{2x} - 4) + c$ then

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12. $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ equals

A. $\sin x - 6\tan^{-1}(\sin x) + C$

B. $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$

C. $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$

D. $\sin x - 2(\sin x)^{-1} + C$

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13. If

$$\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx = (x+1)\tan^{-1}\left(\frac{2x+2}{3}\right) + \lambda In\left(4x^2 + 8\right.$$

, then the value of -4λ must be

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14. Evaluate for $m \in N$,

$$\int (x^{3m} + x^{2m} + x^m)\left(2x^{2m} + 3x^m + 6\right)^{\frac{1}{m}} dx, \, x > 0$$

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15. Evaluate: $\int \frac{\cos^4 x}{\sin^3 x (\sin^5 x + \cos^5 x)^{\frac{3}{5}}} dx$

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16. Evaluate: $\int \sqrt{1 + \cos ec x dx}$

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17. $\int \frac{x^2 - 1}{(x^3)\sqrt{2x^4 - 2x^2 + 1}} dx$ is equal to

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18. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{\frac{9}{2}}} dx$ equals (for some arbitrary constant K). $(a) - \frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

$(b) \frac{1}{(\sec x + \tan x)^{\frac{1}{11}}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(c) $-\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(d) $\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

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19. Evaluate: $\int \frac{x^3}{\sqrt{x^2+1}}dx$

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20. Evaluate : $\int \frac{2x}{(1-x^2)\sqrt{x^4-1}}dx$

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21. Evaluate $\int \sqrt{\sec x-1}dx, 0 < x < \frac{\pi}{2}$

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22. Evaluate: $\int \frac{(1 - x \sin x) dx}{x(1 - x^3 e^{3 \cos x})}$

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23. Evaluate: If $\int \cos^n x \dot{d}x$ prove that $I_n = \frac{1}{n}(\cos^{n-1} x \sin x) + \left(\frac{n-1}{n}\right) I_{n-2}$

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24. Evaluate : $\int \frac{x dx}{\sqrt{1 + x^4}}$

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25. Evaluate: $\int \frac{e^{\tan^{-1} x}}{(1 + x^2)} \left[\left(\sec^{-1} \sqrt{1 + x^2} + \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) \right] dx, (x > 0).$

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26. $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}} dx is equal \rightarrow$ $\ln\left|x-\sqrt{x^2-1}\right|-\tan^{-1}x+c,$ $\ln\left|x+\sqrt{x^2-1}\right|-\tan^{-1}x+c,$ $\ln\left|x-\sqrt{x^2-1}\right|-\sec^{-1}x+c,$ $\ln\left|x+\sqrt{x^2-1}\right|-\sec^{-1}x+c$

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27. $If I=\int \frac{dx}{(2ax+x^2)^{\frac{3}{2}}}$,then I is equal to (a)$-\frac{x+a}{\sqrt{2ax+x^2}}+c$ (b) $-\frac{1}{a}\frac{x+a}{\sqrt{2ax+x^2}}+c$ (c)$-\frac{1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}}+c$ (d) $-\frac{1}{a^3}\frac{x+a}{\sqrt{2ax+x^3}}+c$

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28. If $f'(x)=\frac{1}{-x+\sqrt{x^2+1}}$ and $f(0)=\frac{1+\sqrt{2}}{2}$ then $f(1)$ is equal to- (a) $\log\left(\sqrt{2}+1\right)$ (b) 1 (c)$1+\sqrt{2}$ (d) none of these

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29. $\int e^x\left(\frac{2\tan x}{1+\tan x}+\cot^2\left(x+\frac{\pi}{4}\right)\right)dx$ is equal to (a) $e^x\tan\left(\frac{\pi}{4}-x\right)+c$ (b)$e^x\tan\left(x-\frac{\pi}{4}\right)+c$ (c)$e^x\tan\left(\frac{3\pi}{4}-x\right)+c$ (d) none of these

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30. The value of the integral $\int(x^2+x)\left(x^{-8}+2x^{-9}\right)^{\frac{1}{10}}dx$ is $\frac{5}{11}\left(x^2+2x\right)^{\frac{11}{10}}+c$ (b) $\frac{5}{6}(x+1x)^{\frac{11}{10}}+c$ $\frac{6}{7}(x+1)^{\frac{11}{10}}+c$ (d) none of these

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31. $If\int\frac{dx}{(x+2)(x^2+1)}=a\ln\left(1+x^2\right)+b\tan^{-1}x+\frac{1}{5}\ln|x+2|+C$ Then (a) $a=\frac{1}{10}, b=-\frac{2}{5}$ (b) $a=\frac{1}{10}, b=-\frac{2}{5}$ (c) $a=-\frac{1}{10}, b=\frac{2}{5}$ (d) $a=\frac{1}{10}, b=\frac{2}{5}$

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32. If $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx=ax+b\ln|2\sin x+3\cos x|+C$, then (a) $a=-\frac{12}{13}, b=\frac{15}{39}$ (b) $a=-\frac{7}{13}, b=\frac{6}{13}$ (c) $a=\frac{12}{13}, b=-\frac{15}{39}$ (d) $a=-\frac{7}{13}, b=-\frac{6}{13}$

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33. $If \int \frac{3e^x-5e^{-x}}{4e^x+5e^{-x}}dx=ax+b\ln(4e^x+5e^{-x})+C, then$ (a) $a=-\frac{1}{8}, b=\frac{7}{8}$ (b) $a=\frac{1}{8}, b=\frac{7}{8}$ (c) $a=-\frac{1}{8}, b=-\frac{7}{8}$ (d) $a=\frac{1}{8}, b=-\frac{7}{8}$

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34. If $\int\sqrt{\frac{\cos x-\cos^3 x}{(1-\cos^3 x)}}dx=f(x)+c$, then f(x) is equal to

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35. If $l^r(x)$ means $\log\log\log......x$, the $\log$ being repeated r times, then $\int\left[xl(x)l^2(x)l^3(x)......l^r(x)\right]^{-1}dx$ is equal to $l^{r+1}(x)+C$ (b) $\frac{l^{r+1}(x)}{r+1}+C$ (c)$l^r(x)+C$ (d) none of these

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36. $I=\int\frac{\cos 4x+1}{\cot x-\tan x}dx$ is equal to ?

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37. If $\int\frac{dx}{\sqrt{\sin^3 x\cos^5 x}}=a\sqrt{\cot x}+b\sqrt{\tan^3 x}+c,$ then (a) $a=-1, b=\frac{1}{3}$ (b) $a=-3, b=\frac{2}{3}$ (c)$a=-2, b=\frac{4}{3}$ (d) none of these

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38. $\int e^{\tan^{-1}x}(1+x+x^2)d(\cot^{-1}x)$ is equal to (a) $-e^{\tan^{-1}x}+c$ (b) $e^{\tan^{-1}x}+c$ (c) $-xe^{\tan^{-1}x}+c$ (d) $xe^{\tan^{-1}x}+c$

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39. $\int \frac{x^9 dx}{(4x^2+1)^6}$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+c$

(b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+c$

(c) $\frac{1}{10}(1+4x^2)^{-5}+c$

(d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+c$

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40. $\int \frac{2\sin x}{3+\sin 2x}dx$ is equal to

A. (a) $\frac{1}{2}\ln\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right|-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)+c$

B. (b) $\frac{1}{2}\ln\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right|-\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)+c$

C. (c) $\frac{1}{4}\ln\left|\frac{2+\sin x-\cos x}{2-\sin x+\cos x}\right|-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)+c$

D. (d) None of these

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41. $\int\frac{\ln(\tan x)}{\sin x\cos x}dx$ is equal to
(a) $\frac{1}{2}\ln(\tan x)+c$ (b) $\frac{1}{2}\ln\left(\tan^2 x\right)+c$ (c) $\frac{1}{2}(\ln(\tan x))^2+c$ (d) none of these

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42. $I=\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ is equal to

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43. If $\int\frac{\left(1-x^7\right)}{x\left(1+x^7\right)}dx=P\log|x|+Q\log\left|x^7+1\right|+c$

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44. if $\int \frac{dx}{x^2(x^n+1)^{\left(\frac{n-1}{n}\right)}} = -[f(x)]^{\frac{1}{n}} + c, then f(x) is$ $(1+x^n)$ (b) $1+x^{-1}$ x^n+x^{-n} (d) none of these

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45. If $\int \frac{1}{x\sqrt{1-x^3}}dx = a\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + b, then a is equal$ $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d0 $-\frac{2}{3}$

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46. Evaluate: $\int \frac{\sqrt{\cos 2x}}{\sin x}dx$

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47. Evaluate: $\int\left(\sqrt{\frac{1+x^2}{x^2-x^4}}\right)dx$

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48. Evaluate: $\int\frac{x^2}{(a+bx)^2}dx$

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49. Evaluate: $\int\sqrt{1+\sin\left(\frac{x}{2}\right)dx}$

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50. Evaluate: $\int\frac{x^2}{\sqrt{1-x}}dx$

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51. Evaluate: $\int (e^{\log x} + \sin x)\cos x dx$

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52. If $f(x) = \sqrt{x}, g(x) = e^x - 1$, and $\int fog(x)dx = Afog(x) + B\tan^{-1}(fog(x)) + C$, then $A + B$ is equal to

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53. If $\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2}dx = A\ln|\cos x + \sin x - 2| + Bx + C$, then the value of $A + B + |\lambda|$ is_____

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54. If $\int \left[\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right]\ln x dx = A\left(\frac{x}{e}\right)^x + B\left(\frac{e}{x}\right)^x + C$ then the value of $A + B$ is___

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55. $\int \frac{\sin x}{\sin x - \cos x} dx$

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56. Evaluate: $\int \frac{(x-1)e^x}{(x+1)^3} dx$

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57. $\int \frac{dx}{x^2(x^4+1)^{3/4}}$

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58. $If \int x \log\left(1 + \frac{1}{x}\right) dx = f(x)\log(x+1) + g(x)x^2 + Ax + C,$ then

(a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = \log x$ (c) $A = 1$ (d) none of these

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59. $IfI = \int \frac{dx}{x^3\sqrt{x^2-1}}$, the equals

A. a. $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^3}+\tan^{-1}\sqrt{x^2-1}\right)+C$

B. b. $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2}+x\tan^{-1}\sqrt{x^2-1}\right)+C$

C. c. $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x}+\tan^{-1}\sqrt{x^2-1}\right)+C$

D. d. $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2}+\tan^{-1}\sqrt{x^2-1}\right)+C$

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60. The value of $\int \frac{ax^2-b}{x\sqrt{c^2x^2-(ax^2+b)^2}}$ (a) $\frac{1}{c}\sin^{-1}\left(ax+\frac{b}{x}\right)+k$ (b) $c\sin^{-1}\left(a+\frac{b}{x}\right)+e$ (c) $\sin^{-1}\left(\frac{ax+\frac{b}{x}}{c}\right)+k$ (d) none of these

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61. $If \int \frac{dx}{\cos^3 x \sqrt{\sin 2x}} = a\left(\tan^2 x + b\right)\sqrt{\tan x} + c, then$ (a) $a = \frac{\sqrt{2}}{5}, b = \frac{1}{\sqrt{5}}$ (b)$a = \frac{\sqrt{2}}{5}, b = 5$ (c)$a = \frac{\sqrt{2}}{5}, b = -\frac{1}{\sqrt{5}}$ (d) $a = \frac{\sqrt{2}}{5}, b = \sqrt{5}$

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62. $4\int \frac{\sqrt{a^6 + x^8}}{x} dx$ is equal to (a) $\sqrt{a^6 + x^8} + \frac{a^3}{2} \ln \left| \frac{\sqrt{a^6 + x^8} + a^3}{\sqrt{a^6 + x^8} - a^3} \right| + c$ (b) $a^6 \ln \left| \frac{\sqrt{a^6 + x^8} - a^3}{\sqrt{a^6 + x^8} + a^3} \right| + c$ (c)$\sqrt{a^6 + x^8} + \frac{a^3}{2} \ln \left| \frac{\sqrt{a^6 + x^8} - a^3}{\sqrt{a^6 + x^8} + a^3} \right| + c$ (d) $a^6 \ln \left| \frac{\sqrt{a^6 + x^8} + a^3}{\sqrt{a^6 + x^8} - a^3} \right| + c$

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63. If $I = \int e^{-x} \log(e^x + 1) dx,$ then I equals

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64. $If I_{m,n} = \int \cos^m x \sin nx dx, then 7I_{4,3} - 4I_{3,2}$ is equal to (a) constant (b) $-\cos^2 x + C$ (c) $-\cos^4 x \cos 3x + C$ (d) $\cos 7x - \cos 4x + C$

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65. if $\int \frac{dx}{x^2(x^n+1)^{\left(\frac{n-1}{n}\right)}} = -[f(x)]^{\frac{1}{n}} + c, then f(x) is$ (a)$(1+x^n)$ (b) $1+x^{-1}$ (c)$x^n + x^{-n}$ (d) none of these

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66. Evaluate: $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

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67. Evaluate: $\int \frac{\sqrt{\cos 2x}}{\sin x} dx$

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68. $If \int xe^{x} \cos x dx = ae^{x}(b(1-x)\sin x + cx \cos x) + d, then$ (a) $a = 1, b = 1, c = -1$ (b) $a = \frac{1}{2}, b = -1, c = 1$ (c) $a = 1, b = -1, c = 1$ (d) $a = \frac{1}{2}, b = 1, c = -1$

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69. If $I = \int \sqrt{\frac{5-x}{2+x}} dx$, then equal

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70. Evaluate: $\int \frac{x+1}{x(1+xe^{x})^{2}} dx$

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71. Evaluate $\int \cos 2\theta \log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) d\theta$

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72. Evaluate: $\int\left(\frac{1}{x^3+x^4}+\left(\frac{\ln\left(1+x^{\frac{1}{6}}\right)}{x^{\frac{1}{3}}+\sqrt{x}}\right)\right)dx$

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73. Evaluate: $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

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74. $IfI = \int\frac{dx}{\left(a^2-b^2x^2\right)^{\frac{3}{2}}}, thenIequal$ (a) $\frac{x}{\sqrt{a^2-b^2x^2}}+C$ (b) $\frac{x}{2\sqrt{a^2-b^2x^2}}+C$ (c) $\frac{ax}{\sqrt{a^2-b^2x^2}}+C$ (d) none of these

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75. If $I=\int\left(\sqrt{\cot x}-\sqrt{\tan x}\right)dx$, is equal to

(a) $\sqrt{2}\log\left(\sqrt{\tan x}-\sqrt{\cot x}\right)+C$

(b) $\sqrt{2}\log\left|\sin x+\cos x+\sqrt{\sin 2x}\right|+C$

(c) $\sqrt{2}\log\left|\sin x-\cos x+\sqrt{2}\sin x\cos x\right|+C$

(d) $\sqrt{2}\log\left|\sin\left(x+\frac{\pi}{4}\right)+\sqrt{2}\sin x\cos x\right|+C$

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76. $\int x\left(\frac{\ln a^{a^{\frac{x}{2}}}}{3a^{\frac{5x}{2}}b^{3x}}+\frac{\ln b^{b^x}}{2a^{2x}b^{4x}}\right)dx\,(where a, b\in R^+)is$ (a) $\frac{1}{6\ln a^2b^3}a^{2x}b^{3x}\frac{\ln\left(a^{2x}b^{3x}\right)}{e}+k$ (b) $\frac{1}{6\ln a^2b^3}\frac{1}{a^{2x}b^{3x}}\frac{\ln 1}{ea^{2x}b^{3x}}+k$ (c) $\frac{1}{6\ln a^2b^3}\frac{1}{a^{2x}b^{3x}}\ln\left(a^{2x}b^{3x}\right)+k$ (d) $-\frac{1}{6\ln a^2b^3}\frac{1}{a^{2x}b^{3x}}\ln\left(e\left(a^{2x}b^{3x}\right)\right)+k$

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77. $\left(\int e^{x^4}\left(x+x^3+2x^5\right)e^{x^2}dx\right)$ is equal to

(a) $\frac{1}{2}xe^{x^2}e^{x^4}+c$ (b) $\frac{1}{2}x^2e^{x^4}+c$ (c) $\frac{1}{2}e^{x^2}e^{x^4}+c$ (d) $\frac{1}{2}x^2e^{x^2}e^{x}\ \hat{}\ 4+c$

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78. $\int \frac{\cos ec^2 x - 2005}{\cos^{2005} x} dx$ is equal to (a) $-\frac{\cot x}{(\cos x)^{2005}} + c$ (b) $\frac{\tan x}{(\cos x)^{2005}} + c$ (c) $-\frac{\tan x}{(\cos x)^{2005} + c}$ (d) none of these

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79. If $\int \frac{x \ln\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx = a\sqrt{1+x^2} \ln\left(x + \sqrt{1+x^2}\right) + bx + c$, then (A) $a = 1, b = -1$ (B) $a = 1, b = 1$ (C) $a = -1, b = 1$ (D) $a = -1, b = -1$

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80. If $\int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \ln f(x) + c$,then f(x) is equal to

A. (a) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$

B. (b) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$

C. (c) $\frac{1}{a^2 \cos^2 x + b^2 \cos^2 x}$

D. (d) $\dfrac{1}{a^2 \cos^2 x - b^2 \cos^2 x}$

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81. $If x f(x) = 3f^2(x) + 2, then \int \frac{2x^2 - 12x f(x) + f(x)}{(6f(x) - x)(x^2 - f(x))^2} dx$ equal. (A) $\dfrac{1}{x^2 - f(x)} + c$ (B) $\dfrac{1}{x^2 + f(x)} + c$ (C) $\dfrac{1}{x - f(x)} + c$ (D) $\dfrac{1}{x + f(x)} + c$

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82. $\displaystyle\int \frac{dx}{(1+\sqrt{x})\sqrt{x - x^2}}$ is equal to ?

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83. The value of integral $\displaystyle\int e^x \left(\frac{1}{\sqrt{1+x^2}} + \frac{1 - 2x^2}{\sqrt{(1+x^2)^5}} \right) dx$ is equal to

$(a) e^x \left(\dfrac{1}{\sqrt{1+x^2}} + \dfrac{x}{\sqrt{(1+x^2)^3}} \right) + c$

$(b) e^x\left(\frac{1}{\sqrt{1+x^2}}-\frac{x}{\sqrt{(1+x^2)^3}}\right)+c$

$(c) e^x\left(\frac{1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{(1+x^2)^5}}\right)+c$

(d)none of these

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84. Evaluate: $\int\sqrt{1+\cos ecxdx}$

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85. If $I=\int\sec^2 x\cos ec^4xdx=A\cot^3 x+B\tan x+C\cot x+D,$ then find the values of A,B,C and D.

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86. A curve $g(x) = \int x^{27}(1 + x + x^2)^6(6x^2 + 5x + 4)dx$ is passing through origin. Then (a)$g(1) = \frac{3^7}{7}$ (b) $g(1) = \frac{2^7}{7}$ (c)$g(-1) = \frac{1}{7}$ (d) $g(-1) = \frac{3^7}{14}$

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87. If $\int \sqrt{\cos ecx + 1}dx = kfog(x) + c, wherek$ is a real constant, then

(a)$k = -2, f(x) = \cot^{-1}x, g(x) = \sqrt{\cos ecx - 1}$

(b) $k = -2, f(x) = \tan^{-1}x, g(x) = \sqrt{\cos ecx - 1}$

(c) $k = 2, f(x) = \tan^{-1}xg(x) = \frac{\cot x}{\sqrt{\cos ecx - 1}}$

(d) $k = 2, f(x) = \cot^{-1}xg(x) = \frac{\cot x}{\sqrt{\cos ecx - 1}}$

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88. $\int \frac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}}dx =$ (a) $\sqrt{x^2 + \frac{1}{x^2} + 1} + C$ (b) $\frac{\sqrt{x^4 + x^2 + 1}}{x^2} + C$ (c) $\frac{\sqrt{x^4 + x^2 + 1}}{x} + C$ (d) none of these

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89. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to

(a) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+C$

(b) $\frac{\sqrt{2x^4-2x^2+1}}{x}+C$

(c) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+C$

(d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+C$

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90. $\int \sqrt{e^x-1}dx$ is equal to

(a) $2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}\right]+c$

(b) $\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}+c$

(c) $\sqrt{e^x-1}+\tan^{-1}\sqrt{e^x-1}+c$

(d) $2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}\right]+c$

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91. If $\int \sin x d(\sec x) = f(x) - g(x) + c, then$ $f(x) = \sec x$ (b) $f(x) = \tan x$ $g(x) = 2x$ (d) $g(x) = x$

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92. If $I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx = P \cos x + Q \log|f(x)| + R,$ then (a) $P = \frac{1}{2}, Q = -\frac{3}{4\sqrt{2}}$ (b) $P = \frac{1}{4}, Q = \frac{1}{\sqrt{2}}$ (c) $f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$ (d) $f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

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93. If $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x dx = AF(x-1) + BF(x-4) + C$ and $F(x) = \int \frac{e^x}{x} dx, then$

A. (a) $A = -\frac{2}{3}$

B. (b) $B = \left(\frac{4}{3}\right)e^3$

C. (c) $A = \frac{2}{3}$

D. (b) $A = \left(\frac{4}{3}\right)e^3$

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94. Evaluate: $\int \frac{1}{x}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$

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95. $\int \frac{x^3+3x+2}{\left(x^2+1\right)^2(x+1)}dx$

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96. $\int \frac{3+2\cos x}{\left(2+3\cos x\right)^2}dx$ is equal to

(a) $\left(\frac{\sin x}{3\cos x+2}\right)+c$

(b) $\left(\frac{2\cos x}{3\sin x+2}\right)+c$

(c) $\left(\frac{2\cos x}{3\cos x+2}\right)+c$

(d) $\left(\frac{2\sin x}{3\sin x+2}\right)+c$

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97. $\int\left(\frac{x+2}{x+4}\right)^2 e^x dx$ is equal to

A. (a) $e^x\left(\frac{x}{x+4}\right)+c$

B. (b) $e^x\left(\frac{x+2}{x+4}\right)+c$

C. (c) $e^x\left(\frac{x-2}{x+4}\right)+c$

D. (d) $\left(\frac{2xe^2}{x+4}\right)+c$

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98. $IfI=\int\frac{\sin 2x}{(3+4\cos x)^3}dx, thenIequals$ (a) $\frac{3\cos x+8}{(3+4\cos x)^2}+C$ (b) $\frac{3+8\cos x}{16(3+4\cos x)^2}+C$ (c) $\frac{3\cos x}{(3+4\cos x)^2}+C$ (d) $\frac{3-8\cos x}{16(3+4\cos x)^2}+C$

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99. If $I = \int \frac{dx}{\sec x + \cos ecx}$, then equals

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100. $\int \frac{x^3 dx}{\sqrt{1+x^2}}$ is equal to (A) $\frac{1}{3}\sqrt{1+x^2}\left(2+x^2\right)+C$ (B) $\frac{1}{3}\sqrt{1+x^2}\left(x^2-1\right)+C$ (C) $\frac{1}{3}\left(1+x^2\right)^{\frac{3}{2}}+C$ (D) $\frac{1}{3}\sqrt{1+x^2}\left(x^2-2\right)+C$

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101. $\int e^{\tan x}(\sec x - \sin x)dx \text{is equal to}$

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102. $\int e^x \frac{(x^2+1)}{(x+1)^2} dx$ is equal to

(a) $\left(\frac{x-1}{x+1}\right)e^x + c$

(b) $e^x\left(\frac{x+1}{x-1}\right) + c$

$e^x(x+1)(x-1) + c$

(d) none of these

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103. $\int x \sin x \sec^3 x dx$ is equal to

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104. $IfI = \int \frac{dx}{x^3\sqrt{x^2-1}}, thenIequals$

$\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^3} + \tan^{-1}\sqrt{x^2-1}\right) + C,$

$\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2} + x\tan^{-1}\sqrt{x^2-1}\right) + C,$

$\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x}+\tan^{-1}\sqrt{x^2-1}\right)+C,$
$\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2}+\tan^{-1}\sqrt{x^2-1}\right)+C$

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105. $\int\frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2-1}dx$ is $equal$ to (a) $\frac{1}{2}\left(\ln\left(\frac{x-1}{x+1}\right)\right)^2+C$ (b) $\frac{1}{2}\left(\ln\left(\frac{x+1}{x-1}\right)\right)^2+C$ (c) $\frac{1}{4}\left(\ln\left(\frac{x-1}{x+1}\right)\right)^2+C$ (d) $\frac{1}{4}\left(\ln\left(\frac{x+1}{x-1}\right)\right)^2+C$

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106. Let $g(x)=\int\frac{1+2\cos x}{(\cos x+2)^2}dx and g(0)=0.$ then the value of $8g\left(\frac{\pi}{2}\right)$ is ________

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107.

Let $k(x)=\int\frac{(x^2+1)dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1)=\frac{1}{\sqrt[3]{2}}$. Then the value of k

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108. Column I a) $\int\frac{x^2-x+1}{x^3-4x^2+4x}dx$ b) $\int\frac{x^2-1}{x(x-2)^3}dx$ c) $\int\frac{x^3+1}{x(x-2)^2}dx$ d) $\int\frac{x^5+1}{x(x-2)^3}dx$ COLUMN II (which of the following functions appear in integration of functions in columnI) p) $\log|x|$ q) $\log|x-2|$ r.) $\frac{1}{(x-2)}$ s) x

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109. Let $f(x)=\int x^{\sin x}\left(1+x\cos x\ln x+\sin x\right)dx and f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}$.

Then the value of $|\cos(f(\pi))|$ is___

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110. Each question contains statements given in two columns which have to be matched. Statements (a,b,c,d) in column I have to be matched with statements (p,q,r,s) in column II. If the correct match are a-p, s, b-r c-p, q. and d-s, then the correctly bubbled $4x4$ matrix should be as follows: figure Column I, a) If $\int \frac{2^x}{\sqrt{1-4^x}} dx = k\sin^{-1}(f(x)) + C$, then $k$is greater than, b) If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx = a\frac{\ln(x^k)}{x^k+1} + c$, then $ak$is less than, c) If $\int \frac{x^4+1}{x(x^2+1)^2} dx = k\ln|x| + \frac{m}{1+x^2} + n$, where $n$is the constant of integration, then $mk$is greater than, d) If $\int \frac{dx}{5+4\cos x} = k\tan^{-1}\left(mtan\frac{x}{2}\right) + C$, then $k/m$is greater than, COLUMN II p) 0 q) 1 r) 3 s)4

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111. The value of $\int e^2 x(\frac{1}{x}$ - $\frac{1}{2x^2}$)dx is

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112. Statement 1: If the primitive of $f(x) = \pi \sin \pi x + 2x - 4$ has the value -2 for $x = 1$, then there are exactly two values of x for which primitive of $f(x)$ vanishes. Statement 2: $\cos \pi x$ has period 2.

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113. Let f(x)=$2^{2x-1}$and ϕ(x) =$-2^x + 2x \log 2$. If f'(x)>ϕ'(x), then value of x is-

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114. Evaluate: $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

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115. If $\int x^2 \dot{e}^{-2x} dx = e^{-2x}(ax^2 + bx + c) + d,$ then the value of $\left|\frac{a}{bc}\right|$ is______

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116. If $f(x) = \int \frac{3x^2 + 1}{(x^2 - 1)^3} dx$ and $f(0) = 0$, then the value of $\left| \frac{2}{f(2)} \right|$ is___

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117. Evaluate: $\int \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} dx$

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118.

If $\int x^5 (1 + x^3)^{2/3} dx = A(1 + x^3)^{8/3} + B(1 + x^3)^{5/3} + c$, then find A and B

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119. The value of the integral $\int \frac{(1-\cos\theta)^{\frac{2}{7}}}{(1+\cos\theta)^{\frac{9}{7}}} d\theta$ is

A. (a) $\frac{7}{11}\left(tan\frac{\theta}{2}\right)^{\frac{11}{7}}+C$ (b) $\frac{7}{11}\left(\frac{\cos\theta}{2}\right)^{\frac{11}{7}}+C$ (c) $\frac{7}{11}\left(s\int h\frac{\eta}{2}\right)^{\frac{11}{7}}+C$ (d) none of these

B. null

C. null

D. null

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120. Let $f(x)=\int \frac{x^2dx}{(1+x^2)\left(1+\sqrt{x^2+1}\right)} and f(0)=0.$ Then value of $f(1)$ will be

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121. If $y = \int \sqrt{x}\left(1 + x^{\frac{1}{3}}\right)^4 dx$ is equal to

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122. If $\int \sqrt{1 + \sin x} f(x) dx = \frac{2}{3}(1 + \sin x)^{\frac{3}{2}} + c, then f(x) equal \ \cos x$

(b) $\sin x$ (c) $\tan x$ (d) 1

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123. Let $\int e^x \{f(x) - f'(x)\} dx = \varphi(x)$. Then $\int e^x f(x) dx$ is

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124. Evaluate: $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

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125. $\int \frac{\sec x}{\sqrt{\cos 2x}} dx$ is equal to

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126. Evaluate : $\int \frac{x^2 + 20}{(x \sin x + 5 \cos x)^2} dx$

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127. $\int \frac{x+2}{(x^2+3x+3)\left(\sqrt{x+1}\right)} . dx$

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128. Evaluate: $\int \frac{1}{(x^2)\sqrt{1-x^2}} dx$

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129. Evaluate: $\int x \cdot \sqrt{1-x^4}dx$

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130. Evaluate: $\int \frac{1}{(x-3)\sqrt{x+1}}dx$

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131. Evaluate: $\int (x-5)\sqrt{x^2+x}dx$

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132. Evaluate: $\int \frac{x+1}{(x-1)\sqrt{x+2}}dx$

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133. Evaluate: $\int \sec^3 x dx$

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134. Evaluate: $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx$

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135. Evaluate: $\int \frac{1}{(x+1)\sqrt{x^2-1}}dx$

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136. Evaluate: $\int \frac{1}{(x^2-4)\sqrt{x+1}}dx$

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137. Evaluate: $\int \frac{1}{(x+1)\sqrt{x^2+x+1}}dx$

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138. Evaluate: $\int \frac{x}{(x^2+4)\sqrt{x^2+1}}dx$

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139. Evaluate: $\int \frac{x}{(x-1)(x^2+4)}dx$

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140. Evaluate: $\int e^{\sin^{-1}x}dx.$

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141. Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

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142. $\int \sin^3 x \cos^5 x$ dx

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143. Find: $\int \frac{dx}{\sin x \cos^3 x}$

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144. $\int \frac{e^x(1+x)}{\cos^2(xe^x)}$ dx is

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145. Evaluate: $\int \frac{\sin^3 x dx}{(\cos^4 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)}$

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146. Evaluate: $\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx$

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147. Evaluate: $\int \frac{\tan x}{a + b \tan^2 x} dx$

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148. Evaluate: $\int \frac{1}{\left(\sqrt{e^{5x}}\left(\left(e^{2x} + e^{-2x}\right)^3\right)^{\frac{1}{4}}\right)}$

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149. Find: $\int \sin^5 x dx$

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150. Evaluate: $\int \frac{1}{(x^2 + 2x + 2)} dx$

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151. Evaluate: $\int \left(\left(\frac{e}{x} \right)^x + \left(\frac{x}{e} \right)^x \right) \ln x \, dx.$

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152. Evaluate: $\int \frac{x}{x^4 + x^2 + 1} dx$

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153. Evaluate: $\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1) \tan^{-1} \left(x + \frac{1}{x} \right)} dx$

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154. Evaluate: $\int \frac{x^2 - 1}{(x^2 + 1)\sqrt{1 + x^4}} dx$

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155. Evaluate: $\int \frac{1}{2x^2 + x - 1} dx$

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156. Evaluate: $\int \frac{x^2 + 4}{x^4 + 16} dx$

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157. Evaluate: $\int \sqrt{\tan \theta} d\theta$

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158. If $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx = ax + b\ln|2\sin x + 3\cos x| + C$, then (a) $a = -\frac{12}{13}, b = \frac{15}{39}$ (b) $a = -\frac{7}{13}, b = \frac{6}{13}$ (c)$a = \frac{12}{13}, b = \frac{15}{39}$ (d) $a = -\frac{7}{13}, b = -\frac{6}{13}$

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159. Evaluate: $\int \frac{x^2 + 1}{x^4 + 1} dx$

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160. Evaluate: $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx\cdot$

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161. Evaluate: $\int \frac{1}{x^4 + 1} dx$

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162. Evaluate:

$$\int \frac{dx}{\sin^4 x + \cos^4 x}$$

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163. Evaluate: $\int [f(x)g''(x) - f''(x)g(x)]dx$

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164. Evaluate: $\int \left(\log(\log x) + \frac{1}{(\log x)^2}\right)dx$

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165. Evaluate: $\int x^3 d\left(\tan^{-1} x\right)$

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166. Evaluate: $\int \sin^2(\log x)dx$

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167. Evaluate: $\int \left(\frac{\cos x}{x} - \log x^{\sin x}\right)dx$.

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168. Evaluate: $\int \frac{dx}{x^2(1+x^5)^{\frac{4}{5}}}$

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169. Evaluate: $\int \frac{1+x^4}{(1-x^4)^{\frac{3}{2}}}dx$

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170. Evaluate: $\int \frac{dx}{\sqrt[3]{\sin^{11} x \cos x}}$

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171. Evaluate: $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

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172. Evaluate $\int \frac{\log_e\left(x + \sqrt{x^2+1}\right)}{\sqrt{x^2+1}} dx.$

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173. Evaluate: $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

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174. Evaluate: $\int \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$

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175. Evaluate: $\int \frac{1}{\left[(x-1)^3(x+2)^5\right]^{\frac{1}{4}}}dx$

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176. Evaluate: $\int \frac{x^3-1}{x^2}dx$

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177. Evaluate: Evaluate: $\int x^{-11}\left(1+x^4\right)^{-\frac{1}{2}}dx$

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178. Evaluate: $\int \frac{2x-\sqrt{\sin^{-1}x}}{\sqrt{1-x^2}}dx\cdot$

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179. Evaluate: $\int \frac{e^{2x}-1}{e^{2x}+1}dx\cdot$

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180. Evaluate: $\int \frac{\sin x}{\sin(x+a)}dx$

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181. Evaluate: $\int \frac{dx}{\cos x - \sin x}$

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182. Evaluate: $\int \sin(e^x)d(e^x)$

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183. Evaluate: $\int \tan^3 x dx$

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184. Evaluate : $\int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$

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185. Evaluate: $\int \cos^3 x \sqrt{\sin x} dx$

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186. $\int \frac{dx}{x^{1/2} + x^{1/3}}$

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187. Evaluate: $\int \left(2^{2^{2^x}} \left(2^{2^x}\right)(2^x)\right) dx$

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188. Evaluate: $\int \frac{e^x}{e^{2x} + 6e^x + 5} dx$

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189. The number of irrational roots of the equation $\frac{4x}{x^2 + x + 3} + \frac{5x}{x^2 - 5x + 3} = -\frac{3}{2}$ is

A. $a. 4$

B. $b. 0$

C. $c. 1$

D. $d. 2$

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190. Evaluate: $\int \sqrt{\frac{1 + x}{x}} dx.$

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191. Evaluate: $\int \frac{4x+1}{x^2+3x+2}dx$

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192. Evaluate: $\int \frac{dx}{\left(a^2+x^2\right)^{\frac{3}{2}}}$

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193. Evaluate: $\int \frac{1}{x^2\sqrt{1+x^2}}dx$

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194. Evaluate: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$

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195. Evaluate: $\int \frac{dx}{\sqrt{(x-1)(x-2)}}$

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196. Evaluate: $\int \sqrt{\frac{x}{a^3 - x^3}} dx$

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197. Evaluate: $\int \frac{x^2}{\sqrt{1 - x^6}} dx$

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198. Evaluate: $\int \frac{\left[\sqrt{1 + x^2} + x\right]^n}{\sqrt{1 + x^2}} dx$

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199. Evaluate: $\int \frac{dx}{(x-p)\sqrt{(x-p)(x-q)}}$

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200. Evaluate: $\int \sec^5 x \cos ec^3 x dx$

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201. Evaluate: $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$

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202. $\int \left(\left(x^4 - x\right)^{\frac{1}{4}} \right) \frac{dx}{x^5}$

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203. $\int \frac{dx}{x^2(x^4+1)^{3/4}}$

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204. Evaluate: $\int x^x 1n(ex)dx$

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205. Evaluate: $\int \frac{x+1}{(x-1)\sqrt{x+2}}dx$

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206. Evaluate: $\int \{1+2\tan x(\tan x+\sec x))^{\frac{1}{2}}dx$

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207. Evaluate: $\int \frac{dx}{(1+\sin x)^{\frac{1}{2}}}$

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208. Find $\int \frac{\sin^6 x}{\cos^8 x} dx$.

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209. Evaluate: $\int \sin 2x d(\tan x)$

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210. Evaluate: $\int \frac{1}{\sqrt{3}\sin x + \cos x} dx$

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211. Evaluate: $\int \frac{1}{\sin(x-a)\sin(x-b)}dx$

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212. Evaluate: $\int (1+2x+3x^2+4x^3+\ldots.)dx, (|x|<1)$

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213. Evaluate: $\int \frac{\ln(\ln x)}{x \ln x}dx, (x>0)$

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214. Evaluate: $\int \frac{dx}{x+x\log x}$

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215. Evaluate: $\int \sec^p x \tan x dx$

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216. Evaluate: $\int \cos ec^4 x dx$

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217. Evaluate: $\int \frac{\cos x - \sin x}{\cos x + \sin x}(2 + 2\sin 2x)dx$

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218. Evaluate: $\int \tan^2 x \sin^2 x dx$

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219. Evaluate: $\int \frac{e^{3x} + e^{5x}}{e^x + e^{-x}}dx$

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220. Evaluate: $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$

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221. Evaluate: $\int \frac{(1 + 1nx)^5}{x} dx$

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222. Evaluate: $\int \sin x \cos x \cos 2x \cos 4x \cos 8x dx$

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223. Evaluate: $\int \frac{\sin 2x}{(a + b \cos x)^2} dx$

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224. Evaluate: $\frac{1}{\sqrt{1-e^{2x}}}dx$

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225. Evaluate: $\int \frac{2x+3}{\sqrt{x^2+4x+1}}dx$

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226. Evaluate: $\int \frac{dx}{\sqrt{x}+\sqrt{x-2}}$

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227. Evaluate: $\int \frac{x^3}{x+1}dx$

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228. Evaluate: $\int x \sin 3x dx$.

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229. Evaluate: $\int x \log x dx$.

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230. Evaluate: $\int \sin^{-1} x dx$.

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231. Evaluate: $\int \frac{x - \sin x}{1 - \cos x} dx$

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232. Evaluate: $\int \sqrt{x^2 + 2x + 5}\, dx$

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233. Evaluate: $\int \sqrt{1+3x-x^2}\,dx$

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234. Evaluate: $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$

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235. Evaluate: $\int \frac{\cot x}{\sqrt{\sin x}}\,dx$

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236. Evaluate: $\int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}}\,dx$

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237. Evaluate: $\int\left(\frac{x+1}{x}\right)(x+\log x)^2 dx$

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238. Evaluate: $\int \tan^4 x dx$

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239. Evaluate: $\int(\tan x - x)\tan^2 x dx$

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240. Evaluate: $\int\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{\sin x}dx$

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241. Evaluate: $\int \frac{\sqrt{2+\log x}}{x} dx$

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242. Evaluate: $\int \frac{\log x}{(1+\log x)^2} dx$

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243. $\int [\sin(\log x) + \cos(\log x)] dx$

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244. Evaluate: $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$

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245. Evaluate :

$$\int \frac{xe^x dx}{(x+1)^2}$$

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246. If $f(x)$ is polynomaial function of degree n, prove that

$$\int e^x f(x) dx = e^x \left[f(x) - f'(x) + f''(x) - f'''(x) + \ldots\ldots + (-1)^n f^n(x\right.$$

where $f^n(x) = \frac{d^n f}{dx^n}$

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247. Show that $\int e^x \left(f(x) + f'(x) \right) dx = e^x f(x) + C$

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248. Evaluate: $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$ (A) $-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+c$ (B) $\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+c$ (C) $-\left(1-\frac{1}{x^4}\right)^{\frac{1}{4}}+c$ (D) $-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+c$

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249. Evaluate: $\int \frac{dx}{\sqrt{2ax-x^2}}$

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250. Evaluate: $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx\cdot$

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251. Evaluate: $\int \frac{x^{\frac{5}{2}}}{\sqrt{1+x^7}}dx$

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252. Evaluate: $\int \frac{\sec^2 x}{3 + \tan x} dx$

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253. Evaluate: $\int \frac{x}{\sqrt{x+2}} dx$

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254. Evaluate: $\int \frac{1 - \tan x}{1 + \tan x} dx$

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255. Evaluate: $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

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256. Evaluate: $\int \frac{1}{\sqrt{3x+4} - \sqrt{3x+1}} dx.$

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257. Evaluate: $\int \frac{dx}{(2x-7)\sqrt{(x-3)(x-4)}}$

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258. Evaluate: $\int \left(x+\frac{1}{x}\right)^{\frac{3}{2}}\left(\frac{x^2-1}{x^2}\right)dx$

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259. Find the values of a and b such that $\int \frac{dx}{1+\sin x} = \tan\left(\frac{x}{2}+a\right)+b$

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260. Evaluate: $\int \frac{\sin 2x}{a^2\sin^2 x + b^2\cos^2 x} dx\cdot$

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261. Evaluate: $\int \frac{1}{1+e^{-x}} dx \cdot$

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262. Evaluate: $\int \frac{e^x (2-x^2) dx}{(1-x)\sqrt{1-x^2}}$

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263. Evaluate: $\int e^x (1+\tan x+\tan^2 x) dx$

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264. If $\int g(x) dx = g(x)$, then evaluate $\int g(x)\{f(x)+f'(x)\} dx$

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265. Evaluate: $\int \cos \sqrt{x} dx$

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266. Evaluate: $\int x \sin^2 x dx$

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267. Evaluate: If $\int f(x)dx = g(x), then \int f^{-1}(x)dx$

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268. Evaluate: $\int \cos x \log\left(tan\frac{x}{2}\right)dx$

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269. Evalaute: $\int \left(\frac{\log x - 1}{1 + (\log x)^2}\right)^2 dx$

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270. Evaluate: $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

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271. Evaluate: $\int tan^{-1} \sqrt{x} dx$

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272. Evaluate $\int \sin^3 x dx$

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273. Evaluate $\int \sin 2x \sin 3x dx$.

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274. Evaluate: $\int a^{mx} b^{nx} dx$

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275. Evaluate $\int \frac{\tan x}{\sec x + \tan x} dx$

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276. If $\int \frac{1}{x + x^5} dx = f(x) + c$,then the value of $\int \frac{x^4}{x + x^5} dx$.

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277. find the value of $\int \frac{(x^3 + 8)(x - 1)}{x^2 - 2x + 4} dx$

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278. Integrate the functions $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

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279. Evaluate the following integrals : $\int \frac{\tan x}{\sec x + \tan x} dx$

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280. Evaluate: $\int \frac{x+2}{(x+1)^2} dx.$

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281. Evaluate $\int \left(\frac{8x+13}{\sqrt{4x+7}}\right) dx$

A. (A) $\frac{1}{3}(4x+7)^{\frac{3}{2}} - \frac{1}{2}(4x+7)^{\frac{1}{2}} + c$

B. (B) $\frac{1}{6}(4x+7)^{\frac{5}{2}} - \frac{2}{3}(4x+7)^{\frac{3}{2}} + c$

C. (C) $\frac{1}{3}(4x+7)^{\frac{5}{2}} - \frac{1}{2}(4x+7)^{\frac{3}{2}} + c$

D. (D) $(4x+7)^{\frac{3}{2}} - \frac{1}{2}(4x+7)^{\frac{1}{2}} + c$

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282. Evaluate: $\int \frac{\sec x dx}{\sqrt{\cos 2x}}$

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283. Evaluate: $\int \sin^3 x \cos^2 x dx$

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284. Evaluate: $\int \frac{x dx}{\sqrt{(1+x^2)+\sqrt{(1+x^2)^3}}}$

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285. Evaluate: $\int \frac{2x+1}{x^4+2x^3+x^2-1} dx$

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286. Evaluate: $\int \frac{1}{x^2 - x + 1} dx$

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287. Evaluate: $\int \frac{1}{2x^2 + x - 1} dx$

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288. Evaluate: $\int \frac{\cos x}{\sin\left(x - \frac{\pi}{6}\right)\sin\left(x + \frac{\pi}{6}\right)} dx$

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289. Evaluate: $\int \frac{\sin x}{\sin 3x} dx.$

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290. Evaluate: $\int \frac{1}{3+\sin 2x} dx$

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291. Evaluate: $\int \frac{1}{1+\sin x+\cos x} dx$

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292. Evaluate: $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

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293. Evaluate: $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}} dx$

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294. Evaluate: $\int \frac{1}{\sin x-\sin 2x} dx$

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295. Evaluate: $\int \frac{2x}{(x^2+1)(x^2+2)} dx$

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296. Evaluate: $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx$

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297. Evaluate: $\int \frac{1-\cos x}{\cos x(1+\cos x)} dx$

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298. Evaluate : $\int (1-\cos x)\cos ec^2 x dx$

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299. Evaluate: $\int(\sec x+\tan x)^2 dx$

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300. Evaluate: $\int\frac{\sec x}{\sec x+\tan x}dx\cdot$

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301. Evaluate: $\int\tan^{-1}\left\{\sqrt{\left(\frac{1-\cos 2x}{1+\cos 2x}\right)}\right\}$dx

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302. Evaluate: $\int\frac{1}{1+\sin x}dx$

Watch Video Solution

303. Evaluate $\int \frac{1 - \cos x}{1 + \cos x} dx.$

Watch Video Solution

304. Evaluate: $\int \sec^2 x \cos ec^2 x dx \cdot$

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305. Evaluate $\int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$

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306. Evaluate: $\int \frac{(1 + x)^3}{\sqrt{x}} dx \cdot$

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307. Evaluate $\int \frac{\cos 2x \sin 4x dx}{\cos^4 x(1 + \cos^2 2x)}$

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308. Evaluate: $\int \frac{ax^3 + bx}{x^4 + c^2} dx$

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309. Evaluate $\int \frac{e^{2x} - 2e^x}{e^{2x} + 1} dx$

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310. Evaluate: $\int \frac{dx}{9 + 16 \sin^2 x}$

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311. Find $\int \frac{dx}{\sqrt{x}+x}$

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312. Evaluate: $\int \frac{\cot x}{\sqrt{\sin x}}dx$

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313. Evaluate: $\int \frac{\sqrt{x}dx}{1+x}$

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314. Evaluate: $\int \frac{x^2 \tan^{-1} x^3}{1+x^6}dx$

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315. Evaluate: $\int \sin^2 x \cos^2 x dx$

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316. Evaluate: $\int \frac{1}{x^4 - 1} dx$

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317. If the product of n positive numbers is n^n. Then their sum is

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318. Evaluate: $\int \frac{dx}{\sin x(3 + \cos^2 x)}$

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319. Evaluate: $\int \frac{x^2 + 1}{x(x^2 - 1)} dx$

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320. Evaluate: $\int \frac{\sin x}{\sin 4x} dx$

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321. Evaluate $\int \frac{\log_{ex} e \cdot \log_{e^2 x} e \cdot \log_{e^3 x} e}{x} dx.$

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322. Evaluate: $\int e^{3 \log x} \left(x^4 + 1\right)^{-1} dx$

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323. Evaluate: $\int \frac{dx}{x^{\frac{2}{3}}\left(1 + x^{\frac{2}{3}}\right)}$

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324. $\lim_{x \to 0} \frac{1}{x}\left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt\right]$ is equal to

A. (a) $(0, 1]$

B. (b) $[1, \infty)$

C. (c) $(0, \infty)$

D. (d) none of these

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325. A function f is continuous for all x (and not everywhere zero) such that $f^2(x) = \int_0^x f(t) \frac{\cos t}{2 + \sin t} dt$. Then $f(x)$ is (a) $\frac{1}{2} 1n\left(\frac{x + \cos x}{2}\right); x \neq 0$ (b) $\frac{1}{2} 1n\left(\frac{3}{x + \cos x}\right); x \neq 0$ (c) $\frac{1}{2} 1n\left(\frac{2 + \sin x}{2}\right); x \neq n\pi, n \in I$ (d) $\frac{\cos x + \sin x}{2 + \sin x}; x \neq n\pi + \frac{3\pi}{4}, n \in I$

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326. If $\int_{\cos x}^{1}\left(t^2 f(t)dt = 1-\cos x \,\forall x \in \left(0, \frac{\pi}{2}\right)\right.$, then the value of $\left[f\left(\frac{\sqrt{3}}{4}\right)\right]$ is [.] denotes the greatest integer function) 4 (b) 5 (c) 6 (d) -7

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327. If $\int_0^y \cos t^2 dt = \int_0^{x^2} \frac{\sin t}{t} dt, theprovethat \frac{dy}{dx} = \frac{2\sin x^2}{x\cos y^2}$

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328. $If \int_0^x f(t)dt = x + \int_x^1 tf(t)dt,$ then the value of $f(1)$

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329. Evaluate: $\lim_{x \to \infty} \frac{\left(\int_0^x e^{x^2} dx\right)^2}{\int_0^x e^{2x^2} dx}$

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330. If $f(x) = 1 + \frac{1}{x}\int_1^x f(t)dt,$ then the value of $f\left(e^{-1}\right)$ is

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331. If $\int_0^1 e^{-x^2} dx = a,$ then find the value of $\int_0^1 x^2 e^{-x^2} dx$ in terms of a

.

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332. $If \int_0^{f(x)} t^2 dt = x \cos \pi x, then f'(9) is -\frac{1}{9}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) non-existent

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333. $If I_n = \int_0^{\pi} x^n \sin x$dx ,t h e n fin d t h e v a l u eof $I_5 + 20 I_3$.

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334. The value of $\int_0^1 \lim_{n \to \infty} \sum_{k=0}^{n} \frac{x^{k+2}2^k}{k!} dx$ is:

(a) $e^2 - 1$

(b) 2

(c) $\frac{e^2 - 1}{2}$

(d) $\frac{e^2 - 1}{4}$

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335. If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$, then show that $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$.

Hence prove that

$$I_n = \begin{cases} \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\dots\left(\frac{1}{2}\right)\frac{\pi}{2} & \text{if} \quad n \text{is even} \\ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\dots\left(\frac{2}{3}\right)1 & \text{if} \quad n \text{is odd} \end{cases}$$

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336. If $A = \int_0^1 x^{50}(2-2x)^{50}dx,\ B = \int_0^1 x^{50}(1-x)^{50}dx$, which of the following is true? (A) $A = 2^{50}B$ (B) $A = 2^{-50}B$ (C) $A = 2^{100}B$ (D) $A = 2^{-100}B$

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337. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n};\ n \in N,$ then prove that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

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338. Given that f satisfies $|f(u) - f(v)| \leq |u - v| f \text{ or } u \text{ and } v$ in $[a, b]$. Then $\left|\int_a^b f(x)dx - (b-a)f(a)\right| \leq$ (a) $\frac{(b-a)}{2}$ (b) $\frac{(b-a)^2}{2}$ $(b-a)^2$ (d) none of these

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339. $If I_n = \int_0^1 x^n (\tan^{-1} x) dx, then prove that$

$(n+1)I_n + (n-1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$

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340. $If I = \int_{-20\pi}^{20\pi} |\sin x|[\sin x] dx (where [.]$ denotes the greatest integer function), then the value of I is -40 (b) 40 (c) 20 (d) -20

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341. If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $(I_m) = k - lI_{m-1} then$

$A) k = e$

$B) l = m$

$C) k = \frac{1}{e}$

$D) l \neq m$

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342. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ i s (a)$\frac{\pi}{2}+1$ (b) $\frac{\pi}{2}-1$ (c) -1 (d) 1

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343. $If I_{m,n}=\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx,$ Then show that $I_{m,n}=\frac{m-1}{m+n}I_{m-2,n}(m,n \in N)$ Hence, prove that $I_{m,n}=f(x)=\left\{\frac{(n-1)(n-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)}\frac{\pi}{4}\right.$ when both m and n are even $\left.\frac{(m-1)(m-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)}\right\}$

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344. Evaluate: $\int_0^\pi \frac{x\sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}dx$

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345. If $I_K = \int_1^e (\ln x)^k dx \left(k \in I^+\right) dx \left(k \in I^+\right)$, then find the value of I_4.

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346. If $\int_0^x \left\{\int_0^u f(t)dt\right\} du$ is equal to

(a) $\int_0^x (x-u)f(u)du$

(b) $\int_0^x uf(x-u)du$

(c) $x\int_0^x f(u)du$

(d) $x\int_0^x uf(u-x)du$

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347. Prove that: $I_n = \int_0^\infty x^{2n+1}e^{-x^2}dx = \frac{n!}{2}, n \in N$.

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348. Solve the following $2 \leq |x - 2| \leq 5$

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349. The value of the integral

$$\int_{-1}^{3} \left(\tan^{1} \frac{x}{x^2 + 1} + \tan^{-1} \frac{x^2 + 1}{x} \right) dx$$ is equal to

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350. It is known that $f(x)$ is an odd function in the interval $\left[-\frac{p}{2}, \frac{p}{2} \right]$ and has a period p, Prove that $\int_a^x f(t) dt$ is also periodic function with the same period.

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351. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4} [3 - f(x)]dx = 7$ then the value of $\int_{-1}^{2} f(x)dx$ is

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352. If $\int_{0}^{\frac{\pi}{2}} \log \sin \theta d(\theta) = k,$ then find the value of $\int_{0}^{\frac{\pi}{2}} \left(\frac{\theta}{\sin \theta}\right)^{2} d(\theta)$ in terms of k

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353. Evaluate: $\int_{0}^{\frac{\pi}{2}} \sin x \cos x dx$

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354. Let $f(x)$ is continuous and positive for $x \in [a, b]$, $g(x)$ is continuous for $x \in [a, b] and \int_{a}^{b} |g(x)|dx > \left|\int_{a}^{b} g(x)dx\right|$ STATEMENT 1 : The value of

$\int_a^b f(x)g(x)dx$ can be zero. STATEMENT 2 : Equation $g(x) = 0$ has at least one root for $x \in (a, b)\cdot$

(a) statement 1 is true, statement 2 is true, Statement 2 is the correct explanation for statement 1.

(b) statement 1 is true, statement 2 is true, Statement 2 is not correct explanation for statement 1.

(c) statement 1 is true, statement 2 is not true.

(d) statement 2 is true, statement 1 is not true.

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355. Evaluate: $\int_{-5}^{5} x^2 \left[x + \frac{1}{2}\right] dx (where[.]$ denotes the greatest integer function).

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356. STATEMENT 1 : The value of $\int_{-4}^{-5} \sin(x^2 - 3)dx + \int_{-2}^{-1} \sin(x^2 + 12x + 33)$ is zero. STATEMENT 2 : $\int_{-a}^{a} f(x)dx = 0 \Leftrightarrow f(x)$ is an odd function.

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357. Evaluate: $\int_0^{2\pi} [\sin x]dx$,where$[.]$ denotes the greatest integer function.

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358. STATEMENT 1 : The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx = 0$

STATEMENT 2 : $\int_a^b f(x)dx = \int_0^b f(a+b-x)dx$

then Which of the following statement is correct ?

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359. Evaluate: $\int_0^{\infty}\left[2e^{-x}\right]dx, where[x]$ represents greatest integer function.

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360. STATEMENT 1 : On the interval $\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$. the least value of the function $f(x)=\int_{\frac{5x}{4}}^{x}(3\sin t+4\cos t)dt$ is 0 STATEMENT 2 : If $f(x)$ is a decreasing function on the interval $[a, b]$, then the least value of $f(x)$ is $f(b)$.

(a) statement 1 is true, statement 2 is true, Statement 2 is the correct explanation for statement 1.

(b) statement 1 is true, statement 2 is true, Statement 2 is not correct explanation for statement 1.

(c) statement 1 is true, statement 2 is not true.

(d) statement 2 is true, statement 1 is not true.

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361. Evaluate: $\int_0^{10\pi}\left[\tan^{-1}x\right]dx, where[x]$ represents greatest integer function.

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362. Consider the function $f(x)$ satisfying the relation $f(x+1)+f(x+7)=0\,\forall x\in R\cdot$ STATEMENT 1 : The possible least value of t for which $\int_a^{a+t}f(x)dx$ is independent of $ais12.$ STATEMENT 2 : $f(x)$ is a periodic function.

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363. Evaluate: $\int_0^{\frac{5\pi}{12}}[\tan x]dx,$ where $[.]$ denotes the greatest integer function.

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364. Consider

$I_1 = \int_0^{\pi/4} e^{x^2}\,dx,\ I_2 = \int_0^{\pi/4} e^{x}\,dx,\ I_3 = \int_0^{\pi/4} e^{x^2}\cos x\,dx$,

$I_4 = \int_0^{\pi/4} e^{x^2}\sin x\,dx.$

STATEMENT 1 : $I_2 > I_1 > I_3 > I_4$

STATEMENT 2 : For $x \in (0, 1),\ x > x^2 and \sin x > \cos x.$

(a) statement 1 is true, statement 2 is true, Statement 2 is the correct explanation for statement 1.

(b) statement 1 is true, statement 2 is true, Statement 2 is not correct explanation for statement 1.

(c) statement 1 is true, statement 2 is not true.

(d) statement 2 is true, statement 1 is not true.

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365. Evaluate: $\int_0^2 \left[x^2 - x + 1\right] dx, where[.]$ denotos the greatest integer function.

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366. $If f(x) = A\sin\left(\frac{\pi x}{2}\right) + b, f'\left(\frac{1}{2}\right) = \sqrt{2} and \int_0^1 f(x)dx = \frac{2A}{\pi}$ the constants A and B are (a) $\frac{\pi}{2} and \frac{\pi}{2}$ (b) $\frac{2}{\pi} and \frac{3}{\pi}$ (c) $0 and -\frac{4}{\pi}$ (d) $\frac{4}{\pi} and 0$

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367. if [x] denotes the greatest integer less than or equal to x then integral $\int_0^2 x^2[x]dx$ equals

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368. The value of $\int_0^{2\pi}[2\sin x]dx, where [.]$ represents the greatest integral function, is (a) $\frac{-5\pi}{3}$ (b) $-\pi$ (c) $\frac{5\pi}{3}$ (d) -2π

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369. $f(x) = \int_1^x \frac{\tan^{-1}(t)}{t} dt \ \forall x \in R^+,$ then find the value of $f(e^2) - f\left(\frac{1}{e^2}\right)$

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370. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} x f([x(1-x)]) dx,$ $I_2 = \int_{1-k}^{k} f[x(1-x)] dx, where 2k - 1 > 0. Then \frac{I_1}{I_2} is$ 2 (b) k (c) $\frac{1}{2}$ (d) 1

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371. Find the points of minima for $f(x) = \int_0^x t(t-1)(t-2) dt$

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372. $If f(x) = e^{g(x)} and g(x) = \int_2^x \frac{tdt}{1+t^4},$ then find the value of $f'(2)$

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373. If $g(x)=\int_0^x \cos^4 t dt$, then $g(x+\pi)$ equals (a)$g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ (c)$g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

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374. Evaluate $\lim_{x \to 4} \int_4^x \frac{(4t-f(t))}{(x-4)} dt$

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375. Evaluate: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$

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376. If $I_n=\int_0^1 (1-x^5)^n dx, then \frac{55}{7}\frac{I_{10}}{I_{11}}$ is equal to ___

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377. Let f be a one-to-one continuous function such that $f(2) = 3$ and $f(5) = 7$. Given $\int_2^5 f(x)dx = 17$, then find the value of $\int_3^7 f^{-1}(x)dx$.

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378. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + 1n\left(\frac{1+x}{1-x}\right)\right)dx$ is equal to (where [.] represents the greatest integer function)

A. (a) $-\frac{1}{2}$

B. (b) 0

C. (c) 1

D. (d) $2\ln\left(\frac{1}{2}\right)$

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379. Evaluate: $\int_0^{e-1} \frac{e^{\frac{x^2+2x-1}{2}}}{x+1} dx + \int_1^e x \log x e^{\frac{x^2-2}{2}} dx$

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380. Evaluate $\lim_{x \to 0} \frac{x - \int_0^x \cos t^2 dt}{x^3 - 6x}$

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381. The value of $2^{2010} \frac{\int_0^1 x^{1004}(1-x)^{1004} dx}{\int_0^1 x^{1004}(1-x^{2010})^{1004} dx}$

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382. If $\int_0^1 \frac{e^t}{1+t} dt = a$ then find the value of $\int_0^1 \frac{e^t}{(1+t)^2} dt$ in terms of a.

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383. If $f(x) = x + \int_0^1 t(x+t)f(t)\text{dt}$,then the value of $\frac{23}{2}f(0)$ is equal to ________

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384. Let $f:(0,\infty) \to (0,\infty)$ be a differentiable function satisfying, $\int_0^x (1-t)f(t)dt = \int_0^x tf(t)dt$; $x \in R^+$ and $f(1)=1$. Determine $f(x)$.

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385. If $f(x) and g(x)$ are continuous functions, then $\int_{1n\lambda}^{1n\frac{1}{\lambda}} \frac{f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x) + g(-x)]} dx$ is

(a)dependent on λ (b) a none-zero constant (c)zero (d) none of these

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386. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0 is

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387. If $b = \int_0^1 \frac{e^t}{t+1} dt$, then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}$ is

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388. If $f(x) = x + \sin x$, then find the value of $\int_{\pi}^{2\pi} f^{-1}(x) dx$.

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389. Let $F(x) = \int_{x}^{x^2+\frac{\pi}{6}} \left(2\cos^2 t\right) dt$ for all $x \in R$ and $f: \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function.For $a \in \left[0, \frac{1}{2}\right]$, if F'(a)+2 is the area of the region bounded by x=0,y=0,y=f(x) and x=a, then f(0) is

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390. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2 f(x)=0$ for all nonzero x, then evaluate $\int_{\sin\theta}^{\cos ec\theta} f(x)dx$

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391.

$I_1=\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x\cos x}dx,\ I_2=\int_0^{2\pi}\cos^6 x dx,\ I_3=\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^3 x dx,\ I_4=\int_{(}$

$I_2=I_3=I_4=0,\ I_1\neq 0$ $I_1=I_2=I_3=0,\ I_4\neq 0$

$I_1=I_2=I_3=0,\ I_4\neq 0\ I_1=I_4=I_3=0,\ I_2\neq 0$

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392. The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{(\log)_e x}{x}\right|dx$ is (A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) 5

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393. $f(x)=\int_1^x \frac{\log t}{1+t+t^2}dt(x\geq 1)$ then prove that f(x) =f(1/x)

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394. If $f(x)=\int_0^{\pi}\frac{t\sin t dt}{\sqrt{1+\tan^2 x\sin^2 t}}$ for $0<x<\frac{\pi}{2}$, then (a) $f(0^+)=-\pi$ (b) $f\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8}$ (c) f is continuous and differentiable in $\left(0,\frac{\pi}{2}\right)$ (d) f is continuous but not differentiable in $\left(0,\frac{\pi}{2}\right)$

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395. Evaluate: $\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{(x^2-1)}{(x^2+1)^2}dx$

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396. $\int_0^x\frac{2^t}{2^{[t]}}dt, where[.]$ denotes the greatest integer function and $x\in R^+$, is equal to

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397.

$Ify = \int_0^x f(t)\sin\{k(x-t)\}dt, thenprovethat \frac{d^2y}{dx^2} + k^2y = kf(x)\cdot$

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398. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \frac{n^2}{2} \forall n \in I\cdot$ Then $\int_{-3}^5 f(|x|)dx$ is equal to (a) $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) none of these

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399. If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$, then find a

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400. The value of $\int_{\frac{1}{e}}^{\tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e}}^{\cot x} \frac{dt}{t(1+t^2)}$ is

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401. Prove that: $y = \int_{\frac{1}{8}}^{\sin^2 x} \sin^{-1}\sqrt{t}dt + \int_{\frac{1}{8}}^{\cos^2 x} \cos^{-1}\sqrt{t}dt$, where $0 \le x \le \frac{\pi}{2}$, is the equation of a straight line parallel to the x-axis. Find the equation.

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402. $f(x) > 0 \forall x \in R$ and is bounded. If

$$\lim_{n \to \infty}\left[\int_0^a \frac{f(x)dx}{f(x)+f(a-x)} + a\int_a^{2a} \frac{f(x)dx}{f(x)+f(3a-x)}\right.$$

$$\left. + a^2\int_{2a}^{3a} \frac{f(x)dx}{f(x)+f(5a-x)} + \dots + a^{n-1}\int_{(n-1)a}^{na} \frac{f(x)dx}{f(x)+f[(2n-1)a-x]}\right.$$

$= 7/5$ (where $a < 1$), then a is equal to

A. (a) $\frac{2}{7}$

B. (b) $\frac{1}{7}$

C. (c) $\frac{14}{19}$

D. (d) $\frac{9}{14}$

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403. Evaluate: $\int_1^a x\dot{a}^{-[(\log)_a x]}dx, (a > 1)\cdot$

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404. $\int_0^\infty \frac{\sin^2 x}{x^2} dx$ must be same as:

A. (a) $\int_0^\infty \frac{\sin x}{x} dx$

B. (b) $\left(\int_0^\infty \frac{\sin x}{x}\right)^2$

C. (c) $\int_0^\infty \frac{\cos^2 x}{x^2}$

D. (d) none of these

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405. Evaluate: $\int_1^{e^6}\left[\frac{\log x}{3}\right]dx, where[.]$ denotes the greatest integer function.

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406. If $\int_0^{\infty}\frac{\sin x}{x}dx=\frac{\pi}{2}$, then $\int_0^{\infty}\frac{\sin^3 x}{x}dx$ is equal to

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407. Find the value of $\int_{-1}^{1}\left[x^2+\{x\}\right]dx$, where [.] and $\{.\}$ denote the greatest function and fractional parts of x, respectiely.

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408. $\int_0^x[\cos t]dt, wherex\in\left(2n\pi, 2n\pi+\frac{\pi}{2}\right), n\in N$,and $[.]$ denotes the greatest integer function .

then the value of $f\left(\frac{1}{\pi}\right)is$

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409. Prove that $\int_0^x [\cot^{-1} x] dx, where[.]$ denotes the greatest integer function.

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410. Evaluate: $\int_{-\frac{\pi}{2}}^{2\pi} [\cot^{-1} x] dx$, where $[.]$ denotes the greatest integer function

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411. $f(x)$ is continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a\cdot$ Then the value of a is equal to: (a)$-\frac{1}{24}$ (b) $\frac{17}{168}$ (c) $\frac{1}{7}$ (d) $-\frac{167}{840}$

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412. Prove that $\int_0^1 xe^x dx$=1

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413. $I_1 = \int_0^{\frac{\pi}{2}} \ln(\sin x) dx,\ I_2 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin x + \cos x) dx \cdot$ Then (a) $I_1 = 2I_2$ (b) $I_2 = 2I_1$ $I_1 = 4I_2$ (d) $I_2 = 4I_1$

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414. Prove that $\int_0^{\infty} [ne^{-x}] dx = \ln\left(\frac{n^n}{n!}\right)$, $wheren$ is a natural number greater than 1 and [.] denotes the greatest integer function..

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415. For $x \in R$ and a continuous function f, let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x f\{x(2-x)\} dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\} dx$.Then $\frac{I_1}{I_2}$ is (a) -1 (b) 1 (c) 2 (d) 3

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416. Evaluate of each of the following integrals $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

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417. Given a function $f: [0, 4] \to R$ is differentiable ,then prove that for some $\alpha, \beta \in (0, 2)$, $\int_0^4 f(t)dt = 2\alpha f(\alpha^2) + 2\beta f(\beta^2)$.

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418. Evaluate: $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$

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419. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$ is

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420. Evaluate: $\int_{-\pi}^{3\pi}\log(sec\theta - tan\theta)d\theta$

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421. The value of the integral $\int_0^1 e^{x^2}dx$ lies in the integral

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422. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1, -1), (1.1)$and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of 60°. Then find the value of $\int_0^1 p(x)dx$.

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423. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}; n \in N,$ then prove that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

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424. If $f(0) = 1, f(2) = 3, f'(2) = 5$,then find the value of $\int_0^1 xf''(2x)dx$

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425. $If \int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = 10, then$ (a) $b = 22, a = 2$ (b) $b = 15, 1 = -5$ $b = 10, a = -10$ (d) $b = 10, a = -2$

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426. Evaluate: $\int_0^{\pi} \frac{dx}{1+\sin x}$

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427. if $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta . d\theta, then n \in N, n(I_{n-1} + I_{n+1})$ equals

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428. Consider the integral $\int_0^{2\pi} \frac{dx}{5 - 2\cos x}$ making the substitution $\tan\left(\frac{x}{2}\right) = t$, we have $I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x}$ $= \int_0^0 \frac{2dt}{(1+t^2)\left[5 - 2\frac{1-t^2}{1+t^2}\right]} = 0$ The result is obviously wrong, since the integrand is positive and consequently the integral of this function cannot be equal to zero. Find the mistake.

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429. If $f(x)=\int_0^x |t-1|dt$, where $0 \le x \le 2$ then

(a) range of $f(x)$ is $[0, 1]$ (b) $f(x)$ is differentiable at $x=1$ (c) $f(x)=\cos^{-1}x$ has two real roots (d) $f'\left(\frac{1}{2}\right)=\frac{1}{2}$

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430. If $\int_0^1 \frac{e^{-x}dx}{1+e^x}=(\log)_e(1+e)+k$, then find the value of k.

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431. The value of $\int_0^\infty \frac{dx}{1+x^4}$ is

A. (a) same as that of $\int_0^\infty \frac{x^2+1}{1+x}dx$

B. (b) $\frac{\pi}{2\sqrt{2}}$

C. (c) same as that of $\int_0^\infty \frac{x^2+1}{1+x^4}dx$

D. (d) $\frac{\pi}{\sqrt{2}}$

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432. Let f be a continuous function on $[a, b]$. Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dt = \int_x^b f(t)dt$.

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433. If $f(x)$ is continuous for all real values of x, then $\sum_{r=1}^{n} \int_0^1 f(r-1+x)dx$is equal to (a) $\int_0^n f(x)dx$ (b) $\int_0^1 f(x)dx$ (c) $\int_0^1 f(x)dx$ (d) $(n-1)\int_0^1 f(x)dx$

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434. F ind the value of $\int_0^1 \log x dx$.

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435. If $A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx$, $B_n = \int_0^{\frac{\pi}{2}} \left(\frac{\sin nx}{\sin x}\right)^2 dx$ for $n \in N$,

Then

(A) $A_{n+1} = A_n$ (B) $B_{n+1} = B_n$ (C) $A_{n+1} - A_n = B_{n+1}$ (D) $B_{n+1} - B_n = A_{n+1}$

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436. A continuous real function f satisfies $f(2x) = 3f(x) \forall x \in R \dot{I} f \int_0^1 f(x) dx = 1$, then find the value of $\int_1^2 f(x) dx$

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437. The value of $\int_0^1 \frac{2x^2 + 3x + 3}{(x+1)(x^2 + 2x + 2)} dx$ is

A. (a) $\frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$

B. (b) $\frac{\pi}{4} + 2\log 2 - \tan^{-1} \frac{1}{3}$

C. (c)$2\log 2 - \cot^{-1} 3$

D. (d) $-\frac{\pi}{4}+\log 4+\cot^{-1} 2$

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438. Evaluate: $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

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439. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ equals (a) $\frac{2}{5}$ (b) $-\frac{5}{2}$ (c) 1 (d) $\frac{5}{2}$

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440. Evaluate: $\int_1^{\infty} \left(e^{x+1}+e^{3-x}\right)^{-1} dx$

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441. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in

A. (a) $(0, 2)$

B. (b) no value of x

C. (c) $(0, \infty)$

D. (d) $(-\infty, 0)$

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442. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

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443. If $I(m, n) = \int_0^1 t^m (1+t)^n . dt$, then the expression for $I(m, n)$ in terms of $I(m+1, n-1)$ is:

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444. Prove that $4 \leq \int_1^3 \sqrt{3+x^2}dx \leq 4\sqrt{3}$

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445. If $I_1 = \int_0^{\pi/2} \frac{\cos^2 x}{1+\cos^2 x}dx$, $I_2 = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin^2 x}dx$ and $I_3 = \int_0^{\pi/2} \frac{1+2\cos^2 x\sin^2 x}{4+2\cos^2 x\sin^2 x}dx$, then (a) $I_1 = I_2 > I_3$ (b) $I_3 > I_1 = I_2$ (c) $I_1 = I_2 = I_3$ (d) none of these

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446. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}\, dx$ cannot exceed $\sqrt{\frac{15}{8}}$.

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447. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)\cdot$ If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is (a)$\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

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448. Evaluate: $\int_0^1 \frac{1}{\sqrt{1-x^2}}\sin^{-1}\left(2x\sqrt{1-x^2}\right)dx\cdot$

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449. Let $f(x) = \int_1^x \sqrt{2-t^2}dt\cdot$ Then the real roots of the equation $x^2 - f'(x) = 0$ are (a)± 1 (b) $\pm\frac{1}{\sqrt{2}}$ (c)$\pm\frac{1}{2}$ (d) 0 and 1

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450. Evaluate: $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}dx$

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451. If $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)dx=5$, then $\int_0^{50} f(x)dx$ is equal to

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452. $\int_{-1}^{2} \left|x^3-x\right|dx$.

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453. If $f(x)$ is integrable over $[1, 2]$, then $\int_1^2 f(x)dx$ is equal to

(a) $(\lim)_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (b) $(\lim)_{n\to\infty}\frac{1}{n}\sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$ (c) $(\lim)_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (d) $(\lim)_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

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454. Evaluate: $\int_{-\frac{\pi}{2}}^{\pi} \sin^{-1}(\sin x)dx$

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455. The value of $\int_0^{\frac{\pi}{4}} \sin 2x dx$ is

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456. Let: $a_n = \int_0^{\frac{\pi}{2}} (1 - \sin t)^n \sin 2t dt$ Then find the value of $\lim_{n \to \infty} n a_n$

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457. Q. $\int_0^{\pi} e^{\cos^2 x} (\cos^3 (2n + 1) x dx, n \in I$

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458. Prove that $\int_0^{102}(x-1)(x-2)..(x-100)\times\left(\frac{1}{x-1}+\frac{1}{x-2}\right.$ $+\ldots.+\left.\frac{1}{x-100}\right)dx=101!-100!$

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459. Let f : [0,2] $\rightarrow$ R be a function which is continuous on [0 , 2] and is differentiable on (0 , 2) with f(0) = 1. Let

F(x) = $\int_0^{x^2} f(\sqrt{t})dt$

for $x\in[0,2]$. If F'(x) = f'(x) for all $x\in(0,2)$, then F(2) equals

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460. Evaluate: $\int_0^1 \frac{1}{(1+x^2)\sqrt{1-x^2}}dx$

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461. If $f(x)=\begin{cases} e^{\cos x}\sin x & |x|\leq 2 \\ 2 & otherwise \end{cases}$. Then $\int_{-2}^{3} f(x)dx =$ _____

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462. Evaluate: $\int_0^{\pi/2} \frac{1}{\left(a^2\cos^2 x + b^2\sin^2 x\right)^2}dx$

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463. Prove that: $\int_0^{2\pi} \frac{x\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx = \pi^2$

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464. Prove that $1 < \int_0^2 \left(\frac{5-x}{9-x^2}\right)dx < \frac{6}{5}$

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465. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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466. If m is the least value (global minimum) and M is the greatest value (global maximum) of the function $f(x)$ on the interval $[a, b]$ (estimation of an integral), then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)\cdot$

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467. Let $f: R \rightarrow R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt\cdot$ Then the value of $f(1n5)$ is_____

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468. Show that $\int_a^b \frac{|x|}{x} dx = |b| - |a| \cdot$

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469. The value of $\frac{\pi^2}{1n3} \int_{\frac{7}{6}}^{\frac{5}{6}} \sec(\pi x) dx is\_\_$

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470. Let: $f(x) = \int_0^x |2t - 3| dt \cdot$ Then discuss continuity and differentiability of $f(x) at x = \frac{3}{2}$

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471. If $\int_a^b (f(x) - 3x) dx = a^2 - b^2$ then the value of $f\left(\frac{\pi}{6}\right)$ is ___

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472.

$$Let I_1=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x}{x}dx,\ I_2=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin(\sin x)}{\sin x}dx,\ I_3=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin(\tan x)}{\tan x}dx$$

Then arrange in the decreasing order in which values I_1, I_2, I_3 lie.

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473. The value of $\int_0^1 e^{x^2-x}dx$ is (a) <1 (b) >1 (c) $>e^{-\frac{1}{4}}$ (d) `

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474. Evaluate: $\int_0^1 x\frac{dx}{\sqrt{1-x^2}}$

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475. The value of the definite integral $\int_0^1\left(1+e^{-x^2}\right)dx$ is

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476. Prove that $0 < \int_0^1 \frac{x^7 dx}{(1+x^8)^{\frac{1}{3}}} < \frac{1}{8}$

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477. Let a, b, c be nonzero real numbers such that $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx$ $= \int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx = 0$ Then show that the equation $ax^2+bx+c=0$ will have one root between 0 and 1 and other root between 1 and 2.

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478. Evaluate: $5050 \frac{\int_0^1 (1-x^{50})^{100} dx}{\int_0^1 (1-x^{50})^{101} dx}$

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479. $If \int_{\frac{\pi}{3}}^{x} \sqrt{(3 - \sin^2 t)} dt + \int_0^y \cos t dt = 0, thenevaluate \frac{dy}{dx}$

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480. If $f(x) = \int_0^x (\cos(\sin t) + \cos(\cos t) dt, thenf(x + \pi) is$ (a) $f(x) + f(\pi)$ (b) $f(x) + 2(\pi)$ (c)$f(x) + f\left(\frac{\pi}{2}\right)$ (d) $f(x) + 2f\left(\frac{\pi}{2}\right)$

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481. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{a - \sin\theta}{a + \sin\theta}\right) d\theta, a > 0$

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482. Evaluate: $\int_0^1 \frac{dx}{1 + x^2}$

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483. If $f(t)$ is an odd function, then prove that $\varphi(x) = \int_a^x f(t)dt$ is an even function.

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484. Evaluate: $\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)} dx$

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485. Find the value of $\int_0^1 \{(\sin^{-1} x)/x\} dx.$

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486. Show that $\int_0^{n\pi + v} |\sin x| dx = 2n + 1 - \cos v,$ where n is a positive integer and ,$0 \leq v < \pi$

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487. Find $I = \int_0^{\pi} \ln(1 + \cos x) dx$

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488. The value of $\int_a^b (x-a)^3 (b-x)^4 dx$ is

A. (a) $\frac{(b-a)^4}{6^4}$

B. (b) $\frac{(b-a)^8}{280}$

C. (c) $\frac{(b-a)^7}{7^3}$

D. (d) none of these

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489. The value of the integral

$\int_{-1/2}^{1/2} \left\{ \left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2 \right\}^{1/2} dx$ is equal to

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490. The value of $\int_{-a}^{a}\left(\cos^{-1}x-\sin^{-1}\sqrt{1-x^2}\right)dx$ is $(a>0)$ where $\left(\int_0^a \cos^{-1}x\,dx=A\right)$ is

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491. If f is an odd function, then evaluate $I=\int_{-a}^{a}\frac{f(\sin x)dx}{f(\cos x)+f\left(\sin^2 x\right)}$

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492. Evaluate : $\int_0^{\frac{\pi}{4}}\log(1+\tan x)dx.$

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493. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x^9-3x^5+7x^3-x+1}{\cos^2 x}dx$

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494. Let $a + b = 4, wherea < 2, andletg(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)dx$ increrases as $(b - a)$ increrases.

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495. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left\{\frac{ax^2 + bx + c}{ax^2 - bx + c}(a + b)|\sin x|\right\}dx$

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496. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x}dx$, where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$. Hence, deduce that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} = \frac{1}{2}n\pi$.

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497. Evaluate: $\int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

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498. Find the value of $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

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499. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x (\sin x + \cos x) dx$

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500. $\int_{-1}^{2} \left[\frac{[x]}{1 + x^2}\right] dx, where[.]$ denotes the greater integer function, is equal to (a)-2 (b) -1 (c)0 (d) none of these

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501. Evaluate: $\int_a^b \cos x dx$

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502. Let $f: R \to R$ be a continuous function and $f(x) = f(2x)$ is true $\forall x \in R$. If $f(1) = 3$, then the value of $\int_{-1}^{1} f(f(x))dx$ is equal to

A. (a)6

B. (b) 0

C. (c) $3f(3)$

D. (d) $2f(0)$

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503. Evaluate:

$$(\lim)_{n \to \infty} \frac{(1^2 + 2^2 + 3^3 + \; + n^2)(1^3 + 2^3 + 3^3 + \; + n^3)}{1^6 + 2^6 + 3^6 + \; + n^6}$$

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504. if $\int \frac{dx}{x^2(x^n+1)^{\left(\frac{n-1}{n}\right)} = -[f(x)]^{\frac{1}{n}} + c, then f(x) is} (1+x^n)$

(b) $1+x^{-1}$ $x^n + x^{-n}$ (d) none of these

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505. Evaluate the definite integrals as limit of sums $\int_1^2 x^2 dx$

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506. Evaluate: $\int_a^b \sin x dx$ using limit of sum

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507. $\int_{-3}^{3} x^8 \{x^{11}\} dx$

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508. Evaluate: $\int_a^b e^x dx$ using limit of sum

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509. The value of $\int_{-2}^{1}\left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right]+1\right]dx,$ where [.] denotes the greatest integer function, is (a)1 (b) $1/2$ (c) 2 (d) none of these

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510. Evaluate: $\lim_{n\to\infty}\frac{((n+1)(n+2)\dots(n+n))^{\frac{1}{n}}}{n}$

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511. If $g(x)=\int_0^x(|\sin t|+|\cos t|)dt$,then $g\left(x+\frac{\pi n}{2}\right)$ is equal to, where $n\in N,$ (a) $g(x)+g(\pi)$ (b) $g(x)+g\left(\frac{n\pi}{2}\right)$ (c) $g(x)+g\left(\frac{\pi}{2}\right)$ (d) none of these

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512. Evaluate: $\lim_{n \to \infty}\left[\frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + + \frac{1}{nb}\right]$

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513. $\int_0^x |\sin t| dt$, where $x \in (2n\pi, (2n+1)\pi)$, $n \in N$, is equal to (A) $4n - \cos x$ (B) $4n - \sin x$ (C) $4n + 1 - \cos x$ (D) $4n - 1 - \cos x$

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514. Evaluate

$$\lim_{n \to \infty} n\left[\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + ... + \frac{1}{6n^2}\right]$$

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515. Let f be an odd continous function which is periodic with priod 2 if

g(x) = $\int_0^x f(t)$ dt then

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516. Evaluate $\int_a^b \frac{dx}{\sqrt{x}}, wherea, b > 0.$

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517. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) - g(-x)]dxis$ (a)π (b) 1 (c) -1 (d) 0

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518. Evaluate: $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right] dx$

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519. Evaluate: $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

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520. Evaluate: $\int_0^{100} x - [x] dx$ where $[.]$ represents the greatest integer function).

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521. If $S_n = \left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + ... + \frac{1}{n+\sqrt{n^2}}\right]$ then $(\lim)_{n \to \infty} S_n$ is equal to (A) $\log 2$ (B) $\log 4$ (C) $\log 8$ (D) none of these

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522. The value of $(\lim)_{n\overrightarrow{\infty}}\sum_{r=1}^{4n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+\sqrt{n}\right)^{2}}$ is equal to $\frac{1}{35}$ (b) $\frac{1}{4}$

(c) $\frac{1}{10}$ (d) $\frac{1}{5}$

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523. The value of $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ is (where [x] and {x} denotes the integral part and fractional part functions of x and $x \in N$)

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524. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi-\frac{\pi}{4}}|\sin x+\cos x|dx$

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525. The value of $\int_0^{\frac{\pi}{2}}\frac{dx}{1+\tan^3 x}$ is

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526. Evaluate: $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x dx$

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527. Let f be a contiuous function on $[a, b]$. If $F(x)=\left(\int_a^x f(t)dt-\int_x^b f(t)dt\right)(2x-(a+b))$, then prove that there exist some $c\varepsilon(a, b)$ such that $\int_a^c f(t)dt-\int_c^b f(t)dt=f(c)(a+b-2c)$

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528. Evaluate the following: $\int_{-1}^{1}\frac{\sin x-x^2}{3-|x|}dx$

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529. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$, then find the value of $\int_a^b x \sin x dx =$

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530. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^{2n-1} x - \cos^{2n+1} x} dx, where n \in N$

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531. $f(x)$ is a continuous and bijective function on R. If $\forall t \in R$, then the area bounded by $y = f(x), x = a - t, x = a$, and the x-axis is equal to the area bounded by $y = f(x), x = a + t, x = a$, and the x-axis. Then prove that $\int_{-\lambda}^{\lambda} f^{-1}(x) dx = 2a\lambda (given that f(a) = 0)$.

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532. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1-x}{1+x}\right) dx$

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533. $If \int_0^x f(t)dt = x + \int_x^1 tf(t)dt,$ then the value of $f(1)$

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534. Evaluate: $\int_0^x [\cos t]dtwherex \in \left(2n\pi, \left(4n+1\frac{\pi}{2}\right), n \in N, and[.]\right.$ denotes the greatest integer function.

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535. Suppose f is a real-valued differentiable function defined on $[1, \infty]$ with $f(1) = 1.$ Moreover, suppose that f satisfies $f'(x) = \frac{1}{x^2 + f^2(x)} Showthatf(x) < 1 + \frac{\pi}{4} \forall x \geq 1.$

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536. Let f be a real valued function satisfying

$f(x)+f(x+4)=f(x+2)+f(x+6)$

Prove that $\int_{x}^{x+8} f(t)dt$ is a constant function.

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537. If $x\int_{0}^{x}\sin(f(t))dx=(x+2)\int_{0}^{1}t\sin(f(t))dt$, where $x>0$, then show that $f'(x)\cot f(x)+\frac{3}{1+x}=0$

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538. Evaluate: $\int_{0}^{2}\frac{dx}{(17+8x-4x^2)\left[e^{6(1-x)}+1\right]}$

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539. $If f(x) = \int_{\frac{\pi^2}{16}}^{x^2} \frac{\sin x \sin\sqrt{\theta}}{1+\cos^2\sqrt{\theta}} d\theta,$ then find_the value of $f'\left(\frac{\pi}{2}\right)\cdot$

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540. Find the equation of tangent to the $y = F(x)$ at $x = 1$, where

$$F(x) = \int_x^{x^3} \frac{dt}{\sqrt{1+t^2}}$$

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541. If $\int_0^x [x]dx = \int_0^{[x]} xdx,$ then prove that either x is purely fractional or x is such that $\{x\} = \frac{1}{2}(where[.]and\{.\}$ denote the greatest integer and fractional part, respectively).

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542. Show that: $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx\cdot$

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543. If $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx, I_4 = \int_1^2 2^{x^3} dx$ then

A. (a) $I_1 > I_2$

B. (b) $I_2 > I_1$

C. (c) $I_3 > I_4$

D. (d) $I_3 < I_4$

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544. If a$f(x)$+b$f\left(\frac{1}{x}\right) = \frac{1}{x} - 5, x \neq 0, a \neq b$, then $\int_1^2 f(x)dx$ equals

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545. Find the mistake in the following evaluation of the integral $I = \int_0^{\pi} \frac{dx}{1 + 2\sin^2 x}$, then : $I = \int_0^{\pi} \frac{dx}{\cos^2 x + 3\sin^2 x}$

$= \int_0^{\pi} \frac{\sec^2 x dx}{1 + 3\tan^2 x} = \frac{1}{\sqrt{3}} \left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\pi} = 0$

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546. Find the value of $\int_{-1}^{1} \frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right) dx$

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547. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$ is ______

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548. The value of $\int_{-2}^{2} \left|1 - x^2\right| dx$ is______

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549. If

$$I_I = \int_0^{\pi/2} \cos(\sin x)dx,\ I_2 = \int_0^{\frac{\pi}{2}} \sin(\cos x)dx\ ,\ and\ I_3 = \int_0^{\frac{\pi}{2}} \cos x dx,$$

then find the order in which the values $I_1, I_2, I_3,$ exist.

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550. The integral $\int_0^{1.5} \left[x^2\right] dx, where[.]$ denotoes the greatest integer function, equals ....

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551. Evaluate the following limit:

$$\lim_{n \to \infty} \frac{\sum_{r=1}^{n} \sqrt{r} \sum_{r=1}^{n} \frac{1}{\sqrt{r}}}{\sum_{r=1}^{n} r}$$

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552. value of $\lim_{n \to \infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}}, wheren \in N$ is equal to

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553. If the value of the definite integral $\int_0^1 {}^{207}C_7 x^{200}(1-x)^7 dx$ is equal to $\frac{1}{k}, wherek \in N,$ then the value of $\frac{k}{26}$ is____

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554. Evaluate: $\int_{-1}^{0} \frac{dx}{x^2+2x+2}$

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555. If $fandg$ are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)andg(x) + g(a-x) = 2,$ then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx.$

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556. If $f(x) = \min\left(|x|, 1-|x|, \frac{1}{4}\right) \forall x \in R,$ then find the value of $\int_{-1}^{1} f(x)dx$.

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557. Evaluate: $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x dx$.

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558. Evaluate: $\int_a^b x^3 dx$

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559. Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx$

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560. Evaluate: $(\lim)_{n \to \infty}\left(\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-2^2}}++\frac{1}{\sqrt{3n^2}}\right)$

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561. If $y(x)=\int_{\frac{\pi^2}{16}}^{x^2}\frac{\cos x\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$, find $\frac{dy}{dx}$ at $x=\pi$.

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562. Evaluate:

$$\lim_{n\to\infty}\left[\left(\frac{1}{n^2}\sec^2\left(\frac{1}{n^2}\right)+\frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right)\ldots.+\frac{1}{n}\sec^2(1)\right]$$

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563. For $x>0, let f(x)=\int_1^x\frac{\log t}{1+t}dt$. Find the function $f(x)+f\left(\frac{1}{x}\right)$ and find the value of $f(e)+f\left(\frac{1}{e}\right)$.

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564. Evaluate $(\lim)_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n^2 + k^2}$

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565. If" f, is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x \mapsto \infty$then show that every line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$

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566. Show that : $\frac{\pi}{6} \leq \int_0^1 \frac{dx}{\sqrt{4 - x^2 - x^3}} \leq \frac{\pi}{4\sqrt{2}}$

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567. Let $f\colon[-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1-x) f \text{ or } all x \in [-1, 2]\cdot$ Let $R_1 = \int_{-1}^{2} x f(x) dx,$ and R_2

be the area of the region bounded by $y = f(x), x = -1, x = 2,$ and the x-axis . Then (a)$R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c)$2R_1$ (d) $3R_1 = R_2$

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568. Prove that $\int_0^x e^{xt - t^2}\, dt = e^{\frac{x^2}{4}} \int_0^x e^{-\left(\frac{t^2}{4}\right)}\, dt$

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569. If $\alpha = \int_0^1 \left(e^{9x + 3\tan^{-1} x}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right) dx$

where $\tan^{-1} x$ takes only principal value, then the value of $\left(\log_e |1 + \alpha| - \frac{3\pi}{4}\right)$ is

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570. Evaluate $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{1/3}^{2/3} e^{9\left(x - \frac{2}{3}\right)^2} dx.$

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571. Prove that for any positive integer k, $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + + \cos(2k-1)x]$. Hence, prove that $\int_0^{\frac{\pi}{2}} \sin 2xk \cot x dx = \frac{\pi}{2}$.

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572. Compute the integrals: $\int_0^{\infty} f\left(x^n + x^{-n}\right) \log x \frac{dx}{x}$

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573. If for a real number y, $[y]$ is the greatest integral function less, then or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x] dx$ is $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

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574. Compute the integrals: $\int_0^{\infty} f\left(x^n + x^{-n}\right) \log x \frac{dx}{1+x^2}$

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575. If $\int_0^x \left\{\int_0^u f(t)dt\right\} du$ is equal to (a) $\int_0^x (x-u)f(u)$ (b) $\int_0^x uf(x-u)du$ (c) $x\int_0^x f(u)du$ (d) $x\int_0^x uf(u-x)du$

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576. Compute the integrals: $\int_{\frac{1}{e}}^{e} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx$

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577. STATEMENT 1: $\int_0^{\pi} x \sin x \cos^2 x dx = \frac{\pi}{2}\int_0^{\pi} \sin x \cos^2 x dx.$

STATEMENT 2: $\int_a^b x f(x) dx = \frac{a+b}{2}\int_a^b f(x) dx.$

a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.

b) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1.

c) Statement 1 is true, Statement 2 is false.

d) Statement 2 is true, Statement 1 is false.

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578. Evaluate: $\int_0^{\frac{\pi}{2}} x \cos x dx$

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579. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

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580. Evaluate: $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) dx$

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581. If $\alpha, \beta(\beta > \alpha)$, are the roots of $g(x) = ax^2 + bx + c = 0$ and $f(x)$ is an even function, then $\int_{\alpha}^{\beta} \frac{e^{f\left(\frac{g(x)}{x-\alpha}\right)}}{e^{f\left(\frac{g(x)}{x-\alpha}\right)} + e^{f\left(\frac{g(x)}{x-\beta}\right)}} =$ (a) $\left|\frac{b}{2a}\right|$ (b) $\frac{\sqrt{b^2 - 4ac}}{|2a|}$ (c) $\left|\frac{b}{a}\right|$ (d) none of these

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582. Consider the function $f(x)$ satisfying the relation $f(x+1) + f(x+7) = 0 \forall x \in R\cdot$ STATEMENT 1 : The possible least value of t for which $\int_{a}^{a+t} f(x)dx$ is independent of $ais12.$ STATEMENT 2 : $f(x)$ is a periodic function.

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583. Let $f\colon R\vec{R}$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}\cdot$ Then evaluate $\lim_{x \to 2} \int_{6}^{f(x)} \frac{4t^3}{x-2} dt$

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584. $If\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x, where. x \in \left(0, \frac{\pi}{2}\right)$, then find the value of $f\left(\frac{1}{\sqrt{3}}\right)$.

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585. Prove that $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$, and when $f(x)$ is odd function, $\int_{-a}^{a} f(x)dx = 0$

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586. Evaluate: $\int_0^{100\pi} \sqrt{(1-\cos 2x)}dx$.

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587. Evaluate: $\int_0^{\pi} \frac{xdx}{1+\cos\alpha\sin x}, where 0 < \alpha < \pi$.

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588. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

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589. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers a,b and two unequal non-zero positive integers m and n $\int_a^{a+n} f(x)dx = \int_b^{b+m} f(x)$ calculate the value of $\int_m^n f(x)dx$

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590. Evaluate: $\int \cos^3 x e^{\log \sin x} dx$

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591. The value of the integral $\int_{-\frac{3\pi}{4}}^{\frac{5\pi}{4}}\left(\frac{\sin x+\cos x}{e^{x-\frac{\pi}{4}}+1}\right)dx$ (A) 0 (B) 1 (C) 2

(D) none of these

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592. $If\int_0^{n\pi} f(\cos^2 x)dx = k\int_0^{\pi} f(\cos^2 x)dx, thenf \in dthevaluek.$

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593. $\int_{2-a}^{2+a} f(x)dx$ is equal to [where $f(2-\alpha)=f(2+\alpha)\,\forall\alpha\in R$

(a) $2\int_2^{2+a} f(x)dx$ (b) $2\int_0^{a} f(x)dx$ (c) $2\int_2^{2} f(x)dx$ (d) none of these

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594. Evaluate: $\int_0^a \frac{dx}{x+\sqrt{(a^2-x^2)}}$ or $\int_0^{\frac{\pi}{2}}\frac{d\theta}{1+\tan\theta}$

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595. Suppose that $F(x)$ is an anti derivative of $f(x)=\frac{\sin x}{x}$, where $x>0$. Then $\int_{1}^{3}\frac{\sin 2x}{x}dx$ can be expressed as

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596. Evaluate: $\int_{-\pi}^{\pi}\frac{x\sin x dx}{e^{x}+1}$

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597. $If\int_{0}^{1}\cot^{-1}(1-x+x^{2})dx=\lambda\int_{0}^{1}\tan^{-1}xdx, then \lambda is equal \rightarrow$

1 (b) 2 (c) 3 (d) 4

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598. Show that $\int_{0}^{\frac{\pi}{2}}\sqrt{(\sin 2\theta)}\sin\theta d\theta=\frac{\pi}{4}$

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599. The number of positive continuous $f(x)$ defined in $[0, 1]$ for with

$$I_1 = \int_0^1 f(x)dx = 1, I_2 = \int_0^1 xf(x)dx = a,$$

$I_3 = \int_0^1 x^2 f(x)dx = a^2$ is /are

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600. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$

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601. The value of the definite integral $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} dx$ is

(a) $\sqrt{2}\pi$ (b) $\frac{\pi}{\sqrt{2}}$ (c) $2\sqrt{2}\pi$ (d) $\frac{\pi}{2\sqrt{2}}$

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602. For $\theta \in \left(0, \frac{\pi}{2}\right)$, prove that $\int_0^{\theta} \log(1 + \tan\theta \tan x) dx = \theta \log(\sec\theta)$

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603. A function f is defined by $f(x) = \frac{1}{2^{r-1}}, \frac{1}{2^r} < x \leq 1\left(2^{r-1}\right), r = 1, 2, 3$ then the value of $\int_0^1 f(x) dx$

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604. Evaluate $\int_0^1 \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})} dx$

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605. If $P(x)$ is a polynomial of the least degree that has a maximum equal to 6 at $x = 1$, and a minimum equal to 2 at $x = 3$, then

$\int_0^1 P(x)dx$ equals

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606. Evaluate: $\int_0^{\pi} x \log \sin x dx$

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607. The value of the definite integral $\int_0^{\sqrt{1n\left(\frac{\pi}{2}\right)}} \cos e^{x^2} 2xe^{x^2} dxis$ 1 (b) $1+(\sin 1)$ $1-(\sin 1)$ (d) $(\sin 1)-1$

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608. Evaluate: $\int_0^{\frac{\pi}{2}} \cos 2x + 4 \sin 2x \cos 2x dx$

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609. $Thevalueof \int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1} \log\left(1+x-\frac{1}{x}\right) dx is$ (a) $\frac{\pi}{8}(\log)_e 2$ (b) $\frac{\pi}{2}(\log)_e 2$ (c) $-\frac{\pi}{2}(\log)_e 2$ (d) none of these

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610. If $f(x)$ is a function satisfying $f(x+a)+f(x)=0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x) dx$ is independent of b, then find the least positive value of c

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611. If $f(x)$ is monotonic differentiable function on $[a, b]$, then $\int_a^b f(x) dx + \int_{f(a)}^{f(b)} f^{-1}(x) dx =$ (a) $bf(a) - af(b)$ (b) $bf(b) - af(a)$ (c) $f(a) + f(b)$ (d) cannot be found

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612. $Let A = \int_0^{\infty} \frac{\log x}{1+x^3} dx$ Then find the value of $\int_0^{\infty} \frac{x \log x}{1+x^3} dx$ in terms of A

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613. Show that : $\int_0^1 \frac{\log x}{(1+x)} dx = -\int_0^1 \frac{\log(1+x)}{x} dx$

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614. The value of $\lim_{n \to \infty} \left[\tan\left(\frac{\pi}{2n}\right)\tan\left(\frac{2\pi}{2n}\right)\ldots\ldots.\tan\left(\frac{n\pi}{2n}\right)\right]^{\frac{1}{n}}$ is

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615. answer any one question : (ii) evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

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616. Match the statements/expression given in Column I with the values given in Column II. Column I, Column II (p) The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0)=0$ and $\int_0^1 f(x)dx = 1, is$, 8 (q) The number of points in the interval $\left[-\sqrt{13}, \sqrt{13}\right]$ at which $f(x)=\sin\left(x^2\right)+\cos\left(x^2\right)$ attains its maximum value, is, 2 (r) $\int_{-2}^{2} \frac{3x^2}{1+e^x} dx$ equals, 4 $(s)\frac{\int_{-2}^{2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx}{\int_0^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx}$, 0

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617. If $I_1 = \int_0^{\pi} x f\left(\sin^3 x + \cos^2 x\right) dx and$ $I_2 = \int_0^{\frac{\pi}{2}} f\left(\sin^3 x + \cos^2 x\right) dx, then relate I_1 and I_2$

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618. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

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619. Find the value of $\int_0^1 2x^3 - 3x^2 - x + 13dx$.

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620. If $\int_0^{\infty} x^{2n+1}\dot{e}^{-x^2} dx = 360,$ then the value of n is___

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621. Evaluate: $\int_0^{\infty} \frac{dx}{1+x^2}$

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622. $If I_1 = \int_0^{\frac{\pi}{2}} \cos\theta f\left(\sin\theta + \cos^2\theta\right) d(\theta)$

$\text{and } I_2 = \int_0^{\pi/2} \sin 2\theta f\left(\sin\theta + \cos^2\theta\right) d(\theta)$, then

(a) $I_1 = -2I_2$ (b) $I_1 = I_2$ (c) $2I_1 = I_2$ (d) $I_1 = -I_2$

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623. If $f(a+b-x)=f(x)$, then prove that $\int_a^b x f(x)dx = \frac{a+b}{2}\int_a^b f(x)dx$.

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624. Let $I_1=\int_{-2}^{2}\frac{x^6+3x^5+7x^4}{x^4+2}dx$ and $I_2=\int_{-3}^{1}\frac{2(x+1)^2+11(x+1)+14}{(x+1)^4+2}dt$ Then the value of I_1+I_2 is (A) 8 (B) $\frac{200}{3}$ (C) $\frac{100}{3}$ (D) none of these

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625. Find the value of $\int_0^{\frac{\pi}{2}}\sin 2x dx$

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626. Let $f: [a, b] \rightarrow [1, \infty)$ be a continuous function and lt $g: R \rightarrow R$ be defined as $g(x) = \begin{cases} 0 & \text{if x} < a \\ \int_a^x f(t)dt & \text{if a} \leq x \leq b \\ \int_a^b f(t)dt & \text{if x} > b \end{cases}$ Then

a) $g(x)$ is continuous but not differentiable at a

b) $g(x)$ is differentiable on R

c) $g(x)$ is continuous but nut differentiable at b

d) $g(x)$ is continuous and differentiable at either a or b but not both.

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627. The value of $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

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628. prove that $|\sin t|$ is non-differentiable at $t = \pi$.

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629. Find the value of $\int_0^1 x(1-x)dx$

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630. Let $S_n = \sum_{k=0}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$ for n=1,2,3... then(correct options may be more than one) (a)$S_n < \frac{\pi}{3\sqrt{3}}$ b) $S_n > \frac{\pi}{3\sqrt{3}}$(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

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631. The value of the definite integral $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{x^4+x^2+2}{(x^2+1)^2} dx equals\_\_$

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632. If a continuous function f on $[0, a]$ satisfies $f(x)f(a-x)=1, a>0,$ then find the value of $\int_0^a \frac{dx}{1+f(x)}$

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633.

$Let J = \int_{-5}^{-4} (3 - x^2)\tan(3 - x^2)dx$ and $K = \int_{-2}^{-1} (6 - 6x + x^2)$ $\tan(6x - x^2 - 6)dx$. Then (J+K) equals ____

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634. Let $g(x)$ be differentiable on R and $\int_{\sin t}^{1} x^2 g(x)dx = (1 - \sin t)$, where $t \in \left(0, \frac{\pi}{2}\right)$. Then the value of $g\left(\frac{1}{\sqrt{2}}\right)$ is___

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635. Show that $(\lim)_{n \to \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + + \frac{1}{6n}\right) = \log 6$

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636. $\int_{-\frac{\pi}{3}}^{0}\left[\cot^{-1}\left(\frac{2}{2\cos x-1}\right)+\cot^{-1}\left(\cos x-\frac{1}{2}\right)\right]dx$ i s equal to $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{3\pi^2}{8}$

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637. $If\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{e^{\frac{\pi}{4}}dx}{\left(e^x+e^{\frac{\pi}{4}}\right)(\sin x+\cos x)}=k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sec x dx$, then the value of k is (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

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638. If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)}xg(x(1-x))dx,$ and $I_2=\int_{f(-a)}^{f(a)}g(x(1-x))dx,$ then the value of $\frac{I_2}{I_1}$ is

A. (a) -1

B. (b) -2

C. (c) 2

D. (d) 1

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639. If $f(y) = e^y, g(y) = y > 0, and\ F(t) = \int_0^t f(t-y)g(y)dy$,then

A. (a) $F(t) = e^t - (1+t)$

B. (b) $F(t) = te^t$

C. (c) $F(t) = te^{-1}$

D. (d) $F(t) = 1 - e^t(1+t)$

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640. $Given \int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x+\cos x} = \log 2.$ Then the value of integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\sin x+\cos x} dx$ is equal to

(a) $\frac{1}{2}\log 2$ (b) $is\frac{\pi}{2}-\log 2$ (c)$\frac{\pi}{4}-\frac{1}{2}\log 2$ (d) $\frac{\pi}{2}+\log 2$

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641. $\mathrm{I\,f}I_1=\int_{-100}^{101}\frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ $\mathrm{and}I_2=\int_{-100}^{101}\frac{dx}{5+2x-2x^2}, then\frac{I_1}{I_2}\mathrm{i\;s}$ (a)2 (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{1}{2}$

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642. $\int_0^{\infty}\frac{xdx}{(1+x)(1+x^2)}$ is equal to (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) none of these"

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643. $\int_0^{\infty}\frac{dx}{\left[x+\sqrt{x^2+1}\right]^3}$is equal to (a)$\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) none of these

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644. The value of $\int_{1}^{e} \frac{1+x^2 Inx}{x+x^2 Inx} dx$ is equal (a) $In(1+e)e + In(1+e)$ (b) $e - In(1+e)$

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645. The value of integral $\int_{0}^{\frac{1}{\sqrt{3}}} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ must be (a) $3+2\pi$ (b) $4-\pi^2$ (c) π (c) non of these

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646. $If \int_{0}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then the value of $\int_{0}^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$ (a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

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647. The value of the integral $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$ is equal to (a)$\sin\alpha$ (b) $\alpha\sin\alpha$ (c)$\frac{\alpha}{2\sin\alpha}$ (d) $\frac{\alpha}{2}\sin\alpha$

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648. The natural number $n \leq 5$ for which $I_n = \int_0^1 e^x (x-1)^n dx = 16 - 6e$ is

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649. Let f be a continuous function on $[a, b]$. Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dt = \int_x^b f(t)dt$.

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650. If $f(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$ and $g(x) = f(x-1) + f(x+1)$, then find the value of $\int_{-3}^{5} g(x)dx$.

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651. Evaluate: $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

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652. Evaluate: $\int_0^{\frac{\pi}{4}} \left(\tan^{-1}\left(\frac{2\cos^2\theta}{2-\sin 2\theta}\right)\right)\sec^2\theta d\theta.$

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653. $Let\ f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}.$ Then show that $f(n) = \int_0^{\frac{\pi}{2}} \cot\left(\frac{\theta}{2}\right)(1-\cos^n\theta)d\theta$

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654. Let $f(x)$ be a continuous function $\forall x \in R$, except at $x = 0$, such that $\int_0^a f(x)dx$, $a \in R^+$ exists. If $g(x) = \int_x^a \frac{f(t)}{t}dt$, prove that $\int_0^a f(x)dx = \int_0^a g(x)dx$

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655. If $f(x) = \frac{\sin x}{x} \forall x \in (0, \pi]$, prove that $\frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(x)f\left(\frac{\pi}{2} - x\right)dx = \int_0^{\pi} f(x)dx$

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656. Determine a positive integer n such that $\int_0^{\frac{\pi}{2}} x^n \sin x dx = \frac{3}{4}\left(\pi^2 - 8\right)$

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657. f, g, h are continuous in $[0, a], f(a-x) = f(x), g(a-x) = -g(x), 3h(x) - 4h(a-x) = 5.$ Then prove that $\int_0^a f(x)g(x)h(x)dx = 0.$

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658. If $f(x + f(y)) = f(x) + y \forall x, y \in R$ and $f(0) = 1,$ then prove that $\int_0^2 f(2-x)dx = 2\int_0^1 f(x)dx.$

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659. Let $\int_x^{x+p} f(t)dt$ be independent of $x and I_1 = \int_0^p f(t)dt, I_2 = \int_{10}^{p^n + 10} f(z)dz$ for some $p,$ where $n \in N$. Then evaluate $\frac{l_2}{l_1}$.

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660. Evaluate: $\int_3^{10} [\log[x]]dx$

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661. If the function $f:[0,8] \to R$ is differentiable, then for $0 < \alpha < 1 < \beta < 2, \int_0^8 f(t)dt$ is equal to

A. (a) $3\left[\alpha^3 f(\alpha^2) + \beta^2 f(\beta^2)\right]$

B. (b) $3\left[\alpha^3 f(\alpha) + \beta^2 f(\beta)\right]$

C. (c) $3\left[\alpha^3 f(\alpha^2) + \beta^2 f(\beta^3)\right]$

D. (d) $3\left[\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3)\right]$

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662. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is/are (a) $\frac{22}{7} - \pi$ (b) $\frac{2}{105}$ (c)0 (d) $\frac{71}{15} - \frac{3\pi}{2}$

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663. Let $\int_{x}^{x+p} f(t)dt$ be independent of $x and I_1 = \int_0^p f(t)dt,\ I_2 = \int_{10}^{p^n+10} f(z)dz$ for some p, where $n \in N$. Then evaluate $\frac{l_2}{l_1}$.

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664. Evaluate: $\int_{-\pi}^{\pi}\left((2x)\frac{1+\sin x}{1+\cos^2 x}\right)dx$

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665. Q. if $\int_0^{100}(f(x)dx = a$, then $\sum_{r=1}^{100}\left(\int_0^1 (f(r-1+x)dx)\right)$ =

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666. Let $f(x)$ be a derivable function satisfying $f(x) = \int_0^x e^t \sin(x-t)dt$ and $g(x) = f''(x) - f(x)$ Then the possible integers in the range of $g(x)$ is______

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667. If $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, then (a)$\alpha < 2$ (b)$\alpha < 0$ $(c)0 < \alpha < 1$ (d) $\alpha = 0$

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668. $\int_{\frac{5}{2}}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4}dx$ is equal to

A. (a) $\frac{\pi}{6}$

B. (b) $\frac{2\pi}{3}$

C. (c) $\frac{5\pi}{6}$

D. (d) $\frac{\pi}{3}$

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669. The value of $\int_0^{\frac{3\pi}{2}} \frac{\left|\tan^{-1}(\tan x)\right| - \left|\sin^{-1}(\sin x)\right|}{\left|\tan^{-1}(\tan x)\right| + \left|\sin^{-1}(\sin x)\right|} dx$ is equal to

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670. Let $f(0) = 0 and \int_0^2 f'(2t) e^{f(2t)} dt = 5. thenvalueof f(4) is$ log 2 (b) log 7 (c) log 11 (d) log 13

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671. The value of the integral $\int_0^\infty \frac{x \log x}{\left(1 + x^2\right)^2} dx, is$ (a)0 (b) log 7 (c) 5 log 13 (d) none of these

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672. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx is \frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{3\pi^2}{2}$ (d) $\frac{\pi^2}{3}$

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673. $If \int_{\log 2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}, \text{ then } x \text{ is equalto}$ 4 (b) 1n 8 (c) 1n 4 (d) none of these

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674. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is π (b) $\frac{\sqrt{3}}{2}$ (c) $2\sqrt{2}$ (d) none of these

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675. Which of the following is incorrect? $\int_{a+c}^{b+c} f(x)dx = \int_a^b f(x+c)dx$ $\int_{ac}^{bc} f(x)dx = c\int_a^b f(cx)dx$ $\int_{-a}^{a} f(x)dx = \frac{1}{2}\int_{-a}^{a} (f(x) + f(-x)dx$

None of these

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676. The value of the integral $\int_{-\pi}^{\pi} \sin mx \sin nx dx,$ for $m \neq n(m, n \in I), is$

A. (a) 0

B. (b) π

C. (c) $\pi/2$

D. (d) 2π

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677. Evaluate: $\int \frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$

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678. $\int_0^{\infty}\left(\frac{\pi}{1+\pi^2x^2}-\frac{1}{1+x^2}\right)\log x dx$ is equal to (a) $-\frac{\pi}{2}\log\pi$ (b) 0 (c) $\frac{\pi}{2}\log 2$ (d) none of these

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679. If $f(x)=\cos\left(\tan^{-1}x\right)$, then the value of the integral $\int_0^1 xf''(x)dx$ is (a) $\frac{3-\sqrt{2}}{2}$ (b) $\frac{3+\sqrt{2}}{2}$ (c) 1 (d) $1-\frac{3}{2\sqrt{2}}$

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680. The equation of the curve is $y=f(x)\cdot$ The tangents at $[1, f(1), [2, f(2)], and[3, f(3)]$ make angles $\frac{\pi}{6}, \frac{\pi}{3}, and\frac{\pi}{4}$, respectively, with the positive direction of x-axis. Then the value of $\int_2^3 f'(x)f''(x)dx+\int_1^3 f''(x)dx$ is equal to

A. (a)$-\frac{1}{\sqrt{3}}$

B. (b) $\frac{1}{\sqrt{3}}$

C. (c)0

D. (d) none of these

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681. The value of $\int_1^e \left(\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right) dx, is$ (a) $\tan e$ (b) $\tan^{-1} e$ (c)$\tan^{-1}\left(\frac{1}{e}\right)$ (d) none of these

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682. If $f(\pi) = 2$ and $\int_0^\pi \left(f(x) + f''(x)\right)\sin x dx = 5,$ then $f(0)$ is equal to (it is given that $f(x)$ is continuous in $[0, \pi]$). (a) 7 (b) 3 (c) 5 (d) 1

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683. If $\int_1^2 e^{x^2} dx = a, then \int_e^{e^4} \sqrt{1nx} dx$ is equal to (a)$2e^4 - 2e - a$ (b) $2e^4 - e - a$ (c)$2e^4 - e - 2a$ (d) $e^4 - e - a$

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684. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{|\sin x|} \cos x}{1 + e^{\tan x}} dx$ is equal to (a) $e + 1$ (b) $1 - e$ (c) $e - 1$ (d) none of these

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685. The value of the expression $\frac{\int_0^a x^4 \left(\sqrt{a^2 - x^2}\right) dx}{\int_0^a x^2 \left(\sqrt{a^2 - x^2}\right) dx}$ is

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686. $IfA = \int_0^\pi \frac{\cos x}{(x+2)^2} dx, then \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{x+1} dx$ is equal to (a) $\frac{1}{2} + \frac{1}{\pi + 2} - A$ (b) $\frac{1}{\pi + 2} - A$ (c)$1 + \frac{1}{\pi + 2} - A$ (d)

$A-\frac{1}{2}-\frac{1}{\pi+2}$

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687. $\int_0^4 \frac{(y^2-4y+5)\sin(y-2)dy}{[2y^2-8y+11]}$ is equal to (a) 0 (b) 2 (c) -2 (d) none of these

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688. The values of a for which the integral $\int_0^2 |x-a|dx \geq 1$ is satisfied are (a)$(2, \infty)$ (b) $(-\infty, 0)$ (c)$(0, 2)$ (d) none of these

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689. evaluate: $I=\int \frac{2x}{(x^2+1)(x^2+3)} dxis$

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690. Let $I = \int_1^3 \sqrt{3 + x^3} dx$, then the value of I will lie in the interval

(a)$[4, 6]$ (b) $[1, 3]$ (c)$\left[4, 2\sqrt{30}\right]$ (d) $\left[\sqrt{15}, \sqrt{30}\right]$

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691. about to only mathematics

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692. If $f(x) = \{0$,where $x = \frac{n}{n+1}, n = 1, 2, 3....$ and 1 else where then the value of $\int_0^2 f(x) dx$ is

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693. The function f and g are positive and continuous. If f is increasing and g is decreasing, then $\int_0^1 f(x)[g(x) - g(1-x)]dx$

(a)is always non-positive

(b)is always non-negative

(c)can take positive and negative values

(d)none of these

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694. A Function $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 e^x f(t) dt$. Then (a) $f(0) < 0$ (b) $f(x)$ is a decreasing function. (c) $f(x)$ is an increasing function. (d) $\int_0^1 f(x) dx > 0$

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695. Let $f(x)$ be positive, continuous, and differentiable on the interval $(a, b) and \lim_{x \to a^+} f(x) = 1, \lim_{x \to b^-} f(x) = 3^{\frac{1}{4}}$. If $f'(x) \geq f^3(x) + \frac{1}{f(x)}$, then the greatest value of $b - a$ is (a) $\frac{\pi}{48}$ (b) $\frac{\pi}{36}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{12}$

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696. Let $f:(0,\infty)\rightarrow R$ and $F(x^3)=\int_0^{x^3} f(t)dt,\ \text{ if }\ F(x^3)=x^3(x^2+x+1)$.Then f(27)=

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697. If $g(x)=\int_0^x 2|t|dt$,then (a)$g(x)=x|x|$ (b)$g(x)$ is monotonic (c) g(x) is differentiable at x=0 (d)$g'(x)$is differentiable at x=0

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698. Prove that the value of the integral, $\int_0^{2a}\frac{f(x)}{f(x)+f(2a-x)}dx$ is equal to a.

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699. Let $f:\left[\frac{1}{2},1\right]\rightarrow R$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x)<2f(x)$ and

$f\left(\frac{1}{2}\right) = 1$. Then the value of $\int f(x)dx$ lies in the interval for x:[1/2,1] (a) $(2e - 1, 2e)$ (b) $(3 - 1, 2e - 1)$ (c) $\left(\frac{e-1}{2}, e - 1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

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700. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1}dt$, for all, $x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to (a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

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701. $\int_{-\frac{1}{\sqrt{3}}}^{-\frac{1}{\sqrt{3}}} \frac{x^4}{1 - x^4} \cos^{-1}\left(\frac{2x}{1 + x^2}\right)dx$

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702. The value of $\int_{\in 2}^{\sqrt{\in 3}} \frac{x \sin x^2}{\sin x^2 + \sin(In6 - x^2)dx}$ is

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703. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1, and\ f(0) = 0$, then

A. (A) $f\left(\frac{1}{2}\right) < \frac{1}{2}\ and\ f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}\ and\ f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. (C) $f\left(\frac{1}{2}\right) < \frac{1}{2}\ and\ f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. (D) $f\left(\frac{1}{2}\right) > \frac{1}{2}\ and\ f\left(\frac{1}{3}\right) < \frac{1}{3}$

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704. $(\lim)_{n \to \infty} \frac{n}{2^n} \int_0^2 x^n dx$ equals___

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705. Let $f:[0,\infty)\to R$ be a continuous strictly increasing function, such that $f^3(x)=\int_0^x t\dot{f}^2(t)dt$ for every $x\geq 0$. Then value of $f(6)$ is______

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706. A continuous real function f satisfies $f(2x)=3f(x)\,\forall x\in R\dot{I}f\int_0^1 f(x)dx=1$, then find the value of $\int_1^2 f(x)dx$

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707. $Letf(x)=x^3-\dfrac{3x^2}{2}+x+\dfrac{1}{4}$ Then the value of $\left(\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(x))dx\right)^{-1}$ is___

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708. Evaluate: $\int_0^2 [x^2 - x + 1] dx, where [.]$ denotos the greatest integer function.

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709. Consider the polynomial $f(x) = ax^2 + bx + c \cdot$ If $f(0) = 0, f(2) = 2,$ then the minimum value of $\int_0^2 |f'(x)| dx is\_\_$

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710. Find the points at which the function f given by $f(x) = x^3 - 3x$ has local minima and local maxima.

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711. STATEMENT 1: If $f(x)$ is continuous on $[a, b]$, then there exists a point $c \in (a, b)$ such that $\int_a^b f(x) dx = f(c)(b - a)$

STATEMENT 2: For $a < b$, if $m and M$ are, respectively, the smallest and greatest values of $f(x) on [a, b]$, then $m(b - a) \leq \int_a^b f(x) dx \leq (b - a) M$.

(a) statement 1 is true, statement 2 is true, Statement 2 is the correct explanation for statement 1.

(b) statement 1 is true, statement 2 is true, Statement 2 is not correct explanation for statement 1.

(c) statement 1 is true, statement 2 is not true.

(d) statement 2 is true, statement 1 is not true.

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712. Each question contains statements given in two columns which have to be matched. Statements (a,b,c,d) in column I have to be matched with statements (p,q,r,s) in column II. If the correct match are a-p, s, b-r c-p, q. and d-s, then the correctly bubbled $4x4$ matrix should be as follows: figure Column I, a) If $\int \frac{2^x}{\sqrt{1 - 4^x}} dx = k \sin^{-1}(f(x)) + C$, then $k$is greater than, b) If $\int \frac{\left(\sqrt{x}\right)^5}{\left(\sqrt{x}\right)^7 + x^6} dx = a \frac{\ln(x^k)}{x^k + 1} + c$, then $ak$is less than, c) If $\int \frac{x^4 + 1}{x(x^2 + 1)^2} dx = k \ln|x| + \frac{m}{1 + x^2} + n$, where $n$is the constant of integration, then $mk$is greater than, d) If $\int \frac{dx}{5 + 4\cos x} = k \tan^{-1}\left(m \tan\frac{x}{2}\right) + C$, then $k/m$is greater than, COLUMN II p) 0 q) 1 r) 3 s)4

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713. The value of the definite integral, $\int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx$ is

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714. The value of $\int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx$ is, $n \in I$ (a) $\frac{\pi}{2}$ (b) 0 (c) π (d) 2π

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715. $If I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, (m,n\in I, m,n\geq 0), then$

(a) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}}dx$ (b) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$

(c) $I(m,n)=\int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$ (d) $I(m,n)=\int_0^\infty \frac{x^{n}}{(1+x)^{m+n}}dx$

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716. If $I_n=\int_0^{\sqrt{3}} \frac{dx}{1+x^n}, (n=1,2,3..)$, then find the value of $(\lim)_{n\to\infty} I_n$. (a)0 (b) 1 (c) 2 (d) $\frac{1}{2}$

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717. $\int_0^1 \frac{\tan^{-1}x}{x}dx$ is equals to (a) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x}dx$ (b) $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x}dx$ (c) $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x}{x}dx$ (d) $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{x}{\sin x}dx$

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718. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha,$ then the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\sin\left(\frac{t}{2}\right)}{4\pi+2-t} dt$ is (1)2α (2) -2α (3) α (4) $-\alpha$

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719. $Let I_1 = \int_0^1 \frac{e^x dx}{1+x} and I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)} \dot{t}ehn \frac{I_1}{I_2} is equal \rightarrow$

(a)3/e(b)e/3(c)3e(d)1/(3e)`

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720. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx \left(where \theta \neq \frac{n\pi}{2}, n \in I\right)\Big)$ is equal to (a) $-\cos\theta \int_1^{\tan\theta} f(x\sin\theta)dx$ (b)$-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x)dx$ (c) $\sin\theta\theta \int_1^{\tan\theta} f(x\cos\theta)dx$ (d)$\frac{1}{\tan\theta}\theta \int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

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721. Let $g(x) = \int_0^x f(t).\, dt$,where f is such that $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1, 2]$.Then g(2) satisfies the inequality

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722. The value of $\int_0^{\frac{\pi}{2}} \sin|2x - \alpha| dx,$ where $\alpha \in [0, \pi],$ is (a) $1 - \cos\alpha$ (b) $1 + \cos\alpha$ (c) 1 (d) $\cos\alpha$

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723. $If I_n = \int_0^{\pi} e^x (\sin \mathrm{x})^{\mathrm{n}} dx, then \frac{I_3}{I_1}$ is equal to (a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

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724. $\int_{-2}^{0} \{(x+1)^3 + 2 + (x+1)\cos(x+1) dx$ is equal to (A) -4 (B) 0 (C) 4 (D) 6

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725. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[-10, 10]$ be $f(x) = \{x - [x], \quad \text{if} \quad [x]$ is odd, $1 + [x] - x, \quad \text{if} \quad [x]$ is even Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx$ is___

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726. evaluate: $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx is$

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727. If f is continuous function and $F(x) = \int_0^x \left((2t+3)\cdot\int_t^2 f(u)du\right)dt$, then $\left|\frac{F''(2)}{f(2)}\right|$ is equal to____

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728. Let $f'(x) = \dfrac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with f(1/2)=0.If $m \leq \int_{1/2}^{1} f(x)dx \leq M$, then the possible values of m and M are

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729. If the value of $\lim_{n \to \infty} \left(n^{-\frac{3}{2}}\right) \sum_{j=1}^{6n} \sqrt{j}$ is equal to $\sqrt{N}$, then the value of $\dfrac{N}{12}$ is __________

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730. $If \int_0^x f(t)dt = x + \int_x^1 tf(t)dt,$ then the value of $f(1)$

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731. If $I = \int_0^{\frac{3\pi}{4}} [(1+x)\sin x + (1-x)\cos x]dx,$ then value of $(\sqrt{2}-1)I$ is_____

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732. The value of $\int_1^a [x] f'(x) dx$,where $a > 1$, and $[x]$ denotes the greatest integer not exceeding x, is

(A) $af(a) - \{f(1)f(2) + \ldots . + f([a])\}$

(B) $[a]f(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$

(C) $[a]f(a) - \{f(1) + f(2) + \ldots\ldots. + fA\}$

(D) $af([a]) - \{f(1) + f(2) + \ldots\ldots + fA\}$

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733. The value of $\int_0^x [\cos t] dt,\ x\varepsilon\left[(4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2}\right]$ and $n\varepsilon N$ is equal to (where [.] represents greatest integer function.

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734. Let $f(x) = \min(\{x\}, \{-x\})\ \forall\varepsilon R$, where {.} denotes the fractional part of x. Then $\int_{-100}^{100} f(x) dx$ is equal to

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735. $\int_1^4 \{x - 0.4\} dx$ equals (where $\{x\}$ is a fractional part of x

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736. The value of the definite integral $\int_2^4 (x(3-x)(4+x)(6-x)(10-x) + \sin x) dx$ equal (a)$\cos 2 + \cos 4$ (b) $\cos 2 - \cos 4$ (c)$\sin 2 + \sin 4$ (d) $\sin 2 - \sin 4$

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737. $If x = \int_c^{\sin t} \sin^{-1} z dz, y = \int_k^{\sqrt{t}} \frac{\sin z^2}{z} dz, then \frac{dy}{dx} is\ equals$

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738. If $f(x)=\int_0^1 \frac{dt}{1+|x-t|}$,then $f'\left(\frac{1}{2}\right)$ is equal to (a)0 (b) $\frac{1}{2}$ (c) 1

(d) none of these

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739. Let $f(x)=\int_2^x \frac{dt}{\sqrt{1+t^4}} andg(x)$ be the inverse of $f(x)$. Then the value of $g'(0)$

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740. STATEMENT 1: $\int_0^{\frac{\pi}{4}} \log(1+tan\theta)d\theta=\frac{\pi}{8}\log 2.$

STATEMENT 2: $\int_0^{\frac{\pi}{2}} \log\sin\theta d\theta=-\pi\log 2.$

(a) statement 1 is true, statement 2 is true, Statement 2 is the correct explanation for statement 1.

(b) statement 1 is true, statement 2 is true, Statement 2 is not correct explanation for statement 1.

(c) statement 1 is true, statement 2 is not true.

(d) statement 2 is true, statement 1 is not true.

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741. STATEMENT 1: $f(x)$ is symmetrical about $x = 2$. Then $\int_{2-a}^{2+a} f(x)dx$ is equal to $2\int_{2}^{2+a} f(x)dx\cdot$ STATEMENT 2: If $f(x)$ is symmetrical about $x = b$, then $f(b-\alpha) = f(b+\alpha)\,\forall\alpha \in R\cdot$

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742. If $f(x) = \cos x - \int_0^x (x-t)f(t)dt, then f''(x) + f(x)$ is equal to (a)$-\cos x$ (b) $-\sin x$ (c) $\int_0^x (x-t)f(t)dt$ (d) 0

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743. If $f'(x) = f(x) + \int_0^1 f(x)dx, given f(0) = 1,$ then the value of $f\left((\log)_e 2\right)$ is (a) $\frac{1}{3+e}$ (b) $\frac{5-e}{3-e}$ (c) $\frac{2+e}{e-2}$ (d) none of these

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744. STATEMENT 1: $\int_0^{\pi} \sqrt{1-\sin^2 x}\,dx = 0$, STATEMENT 2: $\int_0^{\pi} \cos x\,dx = 0$

A. (a) statement 1 is correct but 2 is false

B. (b) statement 2 is correct but 1 is false

C. (c) both the statements are correct

D. (d) both the statements are false

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745. STATEMENT 1: Let m be any integer. Then the value of $I_m = \int_0^{\pi} \frac{\sin 2mx}{\sin x} dx$ is zero.

STATEMENT 2 : $I_1 = I_2 = I_3 = I_m$

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746. STATEMENT 1: $\int_a^x f(t)dt$ is an even function if $f(x)$ is an odd function. STATEMENT 2: $\int_a^x f(t)dt$ is an odd function if $f(x)$ is an even function.

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747. STATEMENT 1: The value of $\int_0^{2\pi} \cos^{99} x dx$ is 0 STATEMENT 2: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$,if $f(2a - x) = f(x)$ which of the statement is correct?Is statement 2 correct explanation of statement 1?

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748. For $a \varepsilon R$ (the set of all real numbers) $a \neq -1, \lim_{n \to \infty} \frac{(1^a + 2^a + \dots\dots\dots + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + \dots\dots\dots\dots + (na+n$

Then $a =$

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749. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$. Then

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750. Let f : $(0, \infty) \rightarrow R$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t + \frac{1}{t}\right)} \frac{dt}{t}$$

then

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751. Show that $\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$.

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752. Let $f(x)$ be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x) and f'\left(\frac{1}{4}\right) = 0$. Then

(a) $f'(x)$ vanishes at least twice on $[0, 1]$

(b) $f'\left(\frac{1}{2}\right) = 0$

(c) $\int_{-\frac{1}{2}}^{\frac{1}{2}} f\left(x + \frac{1}{2}\right) \sin x dx = 0$

(d) $\int_{-\frac{1}{2}}^{\frac{1}{2}} f(t) e^{\sin \pi t} dt = \int_{\frac{1}{2}}^{1} f(1-t) e^{\sin \pi t} dt$

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753. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}} and g(x)$ be the inverse of $f(x)$. Then the value of $g'(0)$

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754. Let f be a real-valued function defined on interval $(0, \infty)$,by $f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}. dt.$ Then which of the following statement(s) is (are) true? (A). f"(x) exists for all $\in (0, \infty)$. (B). f'(x) exists for all x $\in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$. (C). there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all x $\in (\alpha, \infty)$. (D). there exists $\beta > 1$ such that $|f(x)| + |f'(x)| \leq \beta$ for all x $\in (0, \infty)$.

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755. Let $f: R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t|f(f(t))|dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, Then the value of $f\left(\frac{1}{2}\right)$ is

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756. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2 - x + 1}dx$ is $\frac{\pi^2}{\sqrt{n}}$ (where $n \in N$), then the value of $\frac{n}{27}$ is

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757. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. The correct expression(s) is (are)

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758. The option(s) with the values of a and L that satisfy the following equation is (are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)\,dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\,dt} = L$$

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759. Let $f: R \to R$ be a function defined by $f(x) = \{[x], (x \leq 2)\ (0, x > 2)$ where $[x]$ is the greatest integer less than or equal to $x\cdot$ If

$$I = \int_{-1}^{2} \frac{x f(x^2)}{2 + f(x+1)} dx, then\ the\ value\ of\ (4I - 1) is$$

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760. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = 2\int_0^1 \tan^{-1} x dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}(1-x+x^2)dx$

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761. Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cos ecx \\ \cos^2 x & \cos^2 x & \cos ec^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$ then, $\int_{-\pi}^{\pi} f(x) dx equals$

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762. If $f(x) = \int_a^x [f(x)]^{-1} dx$ and $\int_a^1 [f(x)]^{-1} dx = \sqrt{2}$, then

A. (a) $f(2) = 2$

B. (b) $f'(2) = \frac{1}{2}$

C. (c) $f^{-1}(2) = 2$

D. (d) $\int_0^1 f(x) dx = \sqrt{2}$

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