MATHS

BOOKS - CENGAGE PUBLICATION

LIMITS

ILLUSTRATION

1. Find the value of $\lim_{x \to 3^-} \frac{x-2}{x-3}$.

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2. Prove that $\lim_{x \to 2} [x]$ does not exists, where [.] represents the greatest integer function.

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3. $Let f(x) = \begin{cases} x+1 & , \quad \text{if} \quad x \geq 0 \\ x-1 & , \quad \text{if} \quad x < 0 \end{cases}$ .Then prove that $\lim_{x \to 0} f(x)$ does not exist.

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4. IF $f(x) = \begin{cases} x & \text{if} \quad x \quad \text{is rational} \\ 1-x & \text{if} \quad x \quad \text{is irrational} \end{cases}$, then find $\lim_{x \to 1/2} f(x)$ if exists.

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5. Evaluate $\lim_{x \to 1^+} 2^{-2^{\frac{1}{1-x}}}$.

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6. Evaluate $\lim_{x \to 0^-} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$.

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7. Evaluate $(\lim)_{x \to 0} \frac{\sin x - 2}{\cos x - 1}$

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8. If a and b are positive and [x] denotes greatest integer less than or equal to x, then find $\lim_{x \to 0^+} \frac{x}{a}\left[\frac{b}{x}\right]$.

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9. $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)} & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$ check limit at $x = 4$ is.

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10. Evaluate the left-and right-hand limits of the function defined by $f(x) = \{1 + x^2, \text{ if } 0 \leq x < 1 2 - x, \text{ if } x > 1$ at $x = 1$. Also, show that $(\lim)_{x \to 1} f(x)$ does not exist

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11. Let $f(x)=\begin{cases}\cos[x], & x\leq 0\\ |x|+a, & x>0\end{cases}$. Then find the value of a, so that $\lim_{x\to 0} f(x)$ exists, where [x] denotes the greatest integer function less than or equal to x.

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12. Evaluate: $\lim_{x\to\frac{5\pi}{4}}[\sin x+\cos x]$, [.] denotes the greatest integer function.

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13. Let $f(x)=\begin{cases}x+1, & x>0\\ 2-x, & x\leq 0\end{cases}$ and $g(x)=\begin{cases}x+3, & x<1\\ x^2-2x-2, & 1\leq x<2\\ x-5, & x\geq 2\end{cases}$

Find the LHL and RHL of g(f(x)) at x=0 and, hence, find $\lim_{x\to 0} g(f(x))$.

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14. If $\lim_{x \to a} [f(x) + g(x)] = 2$ and $\lim_{x \to a} [f(x) - g(x)] = 1$, then find the value of $\lim_{x \to a} f(x)g(x)$.

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15. Find the following limits.

(i) $\lim_{x \to 2} \frac{4x}{x^3 - 3}$ (ii) $\lim_{x \to 1} \frac{\log_{10} x - 3}{3x - 2}$ (iii) $\lim_{x \to \pi} \frac{3 + \cos}{2 - \sin}$

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16. Evaluate $\lim_{x \to 0} \left(\frac{x + 4}{2 - x} \right)^{\frac{x^2 + 2x - 3}{x - 1}}$

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17. If $a_1 = 1$ and $a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n}, n \geq 1$and if $\lim_{n \to \infty} a_n = a$, then find the value of a.

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18. Evaluate $\lim_{x \to \infty} \frac{\sin x}{x}$.

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19. Find $\lim_{x \to 0} [x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$, (where $[.]$ represents the greatest integer funciton).

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20. In the neighbourhood of $x = 0$ it is known that $1 + |x| < \frac{e^x - 1}{x} < 1 - |x|$ then find $\lim_{x \to 0} \frac{e^x - 1}{x}$.

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21. Evaluate $\lim_{x \to \infty} \frac{\log_e x}{x}$

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22. If $3-\left(\frac{x^2}{12}\right) \leq f(x) \leq 3+\left(\frac{x^3}{9}\right)$ in the neighborhood of x=0, then find the value of $\lim_{x \to 0} f(x)$.

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23. Evaluate $\lim_{x \to 2} \frac{x^2-5x+6}{x^2-4}$.

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24. Evaluate $\lim_{x \to 1} \left(\frac{2}{1-x^2}-\frac{1}{1-x}\right)$.

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25. Evaluate $\lim_{x \to 1} \frac{x^2+x\log_e x-\log_e x-1}{(x^2)-1}$

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26. Evaluate: $\left(\lim\right)_{x \to \frac{3\pi}{4}} \dfrac{1 + (\tan x)^{\frac{1}{3}}}{1 - 2\cos^2 x}$

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27. Evaluate $\lim\limits_{x \to \infty} \dfrac{\log_e x}{x}$

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28. Evaluate: $\lim\limits_{n \to \infty} (4^n + 5^n)^{\frac{1}{n}}$

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29. Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$

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30. Evaluate: $(\lim)_{x \to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}, (a \neq 0)$.

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31. Evaluate

$$\lim_{x \to \frac{\pi}{2}} \tan^2 x \left[\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right]$$

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32. Evaluate $\lim_{n \to \infty} \dfrac{1^3 + 2^3 + 3^3 + \ldots + n^3}{\sqrt{4n^8 + 1}}$.

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33. If $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 7x + 12}$, then which of the following limits exists?

(i) $\lim_{x \to \infty} \sin^{-1} f(x)$ (ii) $\lim_{x \to \infty} \cos^{-1} f(x)$

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34. Evaluate the following limits :

$$\operatorname{Lim}_{x \to \infty} \frac{\sqrt{3x^2 - 1} - \sqrt{2x^2 - 1}}{4x + 3}$$

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35. Evaluate $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x + c} - \sqrt{x}\right)$.

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36. Find the value of $\lim_{x \to 0^+} \frac{3(\log_e x)^2 + 5\log_e x + 6}{1 + (\log_e x)^2}$.

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37. Evaluate $\lim_{x \to \infty} \frac{3^{\sin x} + 2x + 1}{\sin x - \sqrt{x^2 + 1}}$.

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38. Evaluate: $\lim_{x \to \infty} \frac{x + 7\sin x}{-2x + 13}$

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39. Evaluate $\lim_{x \to \infty} \frac{\sqrt{x^2+1} - \sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4+1}}$

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40. Evaluate $\lim_{x \to \infty} \left(\sqrt{25x^2 - 3x} + 5x\right).$

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41. Evaluate $\lim_{x \to \infty} \left(\frac{x^2 + x - 1}{3x^2 + 2x + 4}\right)^{\frac{3x^2+x}{x-2}}$

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42. Evaluate $\lim_{n \to \infty} \sin^n\left(\frac{2\pi n}{3n+1}\right), n \in N.$

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43. Evaluate $\lim_{x \to \infty} \left(\sqrt[3]{(x+1)(x+2)(x+3)} - x \right)$.

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44. Evaluate $\lim_{n \to \infty} \frac{1}{1+n^2} + \frac{2}{2+n^2} + \ldots + \frac{n}{n+n^2}$.

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45. If [x] denotes the greatest integer less than or equal to x, then evaluate $\lim_{n \to \infty} \frac{1}{n^3} \left(\left[1^2 x\right] + \left[2^2 x\right] + \left[3^2 x\right] + \ldots + \left[n^2 x\right] \right)$.

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46. If $\lim_{x \to \infty} \left\{ \frac{x^2+1}{x+1} - (ax+b) \right\} = 0$, then find the values of a and b.

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47. Evaluate $\lim_{n \to \infty} \frac{n^p \sin^2(n!)}{n+1}$, where $0 < p < 1$.

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48. Evaluate $\lim_{n \to \infty} (-1)^{n-1} \sin\left(\pi \sqrt{n^2 + 0.5n + 1}\right)$, where $n \in N$

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49. Evaluate $\lim_{x \to 2} \frac{x^{10} - 1024}{x^5 - 32}$

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50. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ and $n \in N$,then find the value of n.

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51. Evaluate $\lim_{x \to 1} \frac{\sqrt{x} + \sqrt{\sqrt{x}} + \sqrt{\sqrt{\sqrt{x}}} + \sqrt{\sqrt{\sqrt{\sqrt{x}}}} - 4}{x - 1}$.

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52. Evaluate: $(\lim)_{x \to a} \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}$

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53. Evaluate $\lim_{x \to 2} \frac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)} - 2\sqrt[3]{(3x-5)}}$.

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54. If $\lim_{x \to 0} \frac{(4x-1)^{\frac{1}{3}} + a + bx}{x} = \frac{1}{3}$ then find the values of a and b.

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55. Evaluate $\lim_{x \to 0} \frac{\sin x - x}{x^3}$.

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56. Evaluate $\lim_{x \to 0} \frac{5 \sin x - 7 \sin 2x + 3 \sin 3x}{x^2 \sin x}$.

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57. Evaluate $\lim_{x \to 0} \frac{\sin x + \log(1 - x)}{x^2}$.

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58. Evaluate $\lim_{x \to 0} \frac{e^{\sin x} - (1 + \sin x)}{(\tan(\sin x))^2}$

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59. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

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60. If $(\lim)_{x \to 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$ is finite, find $aandb$ using expansion formula.

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61. Find the integral value of n for which $(\lim)_{x \to 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$ is a finite nonzero number

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62. Find the limits of the following:

$(i) \lim_{x \to 0} \frac{\sin 3x}{x}$ $(ii) \lim_{x \to 0} \frac{\sin 7x}{\sin 4x}$ $(iii) \lim_{x \to 0} \frac{1 - \cos^2 x}{x^2}$

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63. Find the following limits:

$(i)\ \lim_{x \to 0} \frac{1}{x} \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $(ii)\ \lim_{x \to 0} \frac{1}{x} \sin^{-1}\left(3x - 4x^3\right)$

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64. Solve: $\lim_{x \to \infty} 2^{x-1} \tan\left(\frac{a}{2^x}\right)$

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65. Evaluate: $(\lim)_{x \to \frac{\pi}{2}} \frac{1+\cos 2x}{(\pi - 2x)^2}$

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66. Evaluate: $(\lim)_{x \to 2} \frac{x^2 - x - 2}{x^2 - 2x - \sin(x-2)}$

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67. Evaluate the following limits :

$$\lim_{x \to 0} \sqrt{\frac{\frac{1}{2}(1 - \cos 2x)}{x}}$$

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68. Evaluate $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$.

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69. Evaluate $\lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$.

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70. Evaluate $\lim_{x \to \pi} \frac{\sin^{-1}(1 + \cos x).\sec\left(\frac{x}{2}\right)}{(x - \pi)}$.

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71. Evaluate $\lim_{x \to -\infty} \left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{\left(1 + |x|^3\right)} \right]$.

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72. Evaluate :

$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$

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73. Evaluate: $(\lim)_{x \overrightarrow{\infty}} x \left(\frac{\tan^{-1}(x+1)}{x+4} - \frac{\pi}{4} \right)$

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74. Evaluate $\lim_{n \to \infty} n \sin\left(2\pi\sqrt{1+n^2}\right), (n \in N)$.

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75. Evaluate $\lim_{x \to -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{1+x}}$.

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76. Evaluate: $(\lim)_{n \to \infty} x \left[\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right) \right]$

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77. Evaluate: $(\lim)_{x \to 0} \frac{1 - \cos(1 - \cos x).}{x^4}$

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78. Using $\lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right) = 1$ prove that the area of circle of radius R is πR^2

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79. Evaluate : $\left[\lim_{x \to 0} \frac{\sin x}{x}\right]$, where $[\cdot]$ represents the greatest integer function.

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80. Evaluate : $\left[\lim_{x \to 0} \frac{\tan x}{x}\right]$, where $[\cdot]$ represents the greatest integer function.

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81. If $L = \lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, then find the value of a

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82. If $m, n \in I_0 and (\lim)_{x \to 0} \frac{\tan 2x - n \sin x}{x^3} =$ some integer, then find the value of n and also the value of limit.

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83. Evaluate: $\lim_{x \to 0} \frac{3^{2x} - 2^{3x}}{x}$

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84. Evaluate: $\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$

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85. Evaluate $\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$.

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86. Evaluate: $(\lim)_{x \to 1} \frac{a^{x-1} - 1}{\sin \pi x}$

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87. Evaluate: $(\lim)_{n \to 0} \frac{e^x - e^{x \cos x}}{(x + \sin x)}$

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88. Evaluate: $\lim_{x \to 2} \frac{x - 2}{(\log)_a (x - 1)}$

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89. Evaluate: $\lim_{x \to a} \frac{\log x - \log a}{x - a}$

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90. Evaluate: $\lim_{x \to 0} \frac{\log(5 + x) - \log(5 - x)}{x}$

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91. Evaluate $\lim_{h \to 0} \frac{\log_e(1 + 2h) - 2\log_e(1 + h)}{h^2}$.

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92. Let $P_n = a^{P_{n-1}} - 1, \forall n = 2, 3, \ldots,$ and let $P_1 = a^x - 1,$ where $a \in R^+$. Then evaluate $\lim_{x \to 0} \frac{P_n}{x}$.

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93. If $(\lim)_{x \to 0} \frac{ae^x - b}{x} = 2,$ then find the value of a and b.

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94. Find the following limits:

$(i)\ \lim_{x \to 0} (1 - x)^{\frac{1}{x}}$ $(ii)\ \lim_{x \to 1} (1 + \log_e x)^{\frac{1}{\log_e x}}$

$(iii)\ \lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}}$

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95. Evaluate: $\lim_{x \to 0} (\cos x)^{\cot x}$

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96. The population of a country increases by 2% every year. If it increases k times in a century, then prove that $[k] = 7, where[.]$ represents the greatest integer function.

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97. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{2/x} = e^3$, then the value of a and b, is :

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98. Evaluate $(\lim)_{x \to 0} \left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x - \sin x}\right)}$

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99. Evaluate: $(\lim)_{x \vec{0}}\left(\frac{a^x+b^x+c^x}{3}\right);(a, b, c>0)$

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100. If $f(n)=\lim_{x \to 0}\left\{\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\ldots\left(1+\sin\frac{x}{2^n}\right)\right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

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101. Find the following using L'Hospital's rule

$(i)\ \lim_{x \to 0}\frac{(16+5x)^{1/4}-2}{(32+3x)^{1/5}-2}$ $\quad$ $(ii)\ \lim_{x \to \pi/2}[x\tan x-(\pi/2)\sec x]$

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102. Let $f(x)$ be a twice-differentiable function and $f''(0)=2$. Then evaluate $\lim_{x \to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$.

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103. Let $f(a) = g(a) = k$ and their nth derivatives exist and be not equal for some n.

If $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$ then find the value of k.

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104. Evaluate $\lim_{x \to 0} \left(\log_{\tan^2 x}\left(\tan^2 2x\right)\right.$.

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105. Evaluate $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$.

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106. If the graph of the function $y = f(x)$ has a unique tangent at the point $(a, 0)$ through which the graph passes, then evaluate $(\lim)_{x \to a} \dfrac{(\log)_e\{1 + 6f(x)\}}{3f(x)}$

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107. Evaluate $\left(\lim_{x \to \infty}\right)\left(x(\log)_e\left\{\dfrac{\sin\left(a + \frac{1}{x}\right)}{\sin a}\right\}\right), 0 < a < \dfrac{\pi}{2}$

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108. Find the value of α so that $(\lim)_{x \to 0} \dfrac{1}{x^2}(e^{\alpha x} - e^x - x) = \dfrac{3}{2}$

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109. Find the value of $\lim_{x \to 0} \dfrac{\sin x + \log_e\left(\sqrt{1 + \sin^2 x} - \sin x\right)}{\sin^3 x}$.

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110. Evaluate: $(\lim)_{x \to \infty} x^{\frac{1}{x}}$

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111. Evaluate $\lim_{x \to \frac{\pi}{2}^-} (\cos x)^{\cos x}$.

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112. Evaluate $\lim_{x \to 0} (x)^{\frac{1}{\log_e \sin x}}$.

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Solved Examples

1. Evaluate:

$$\lim_{n\to\infty} n^2\left\{\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)}}..\infty}\right\}$$

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2. Evaluate $\lim_{n\to\infty}\left\{\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)...\cos\left(\frac{x}{2^n}\right)\right\}$.

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3. Evaluate: $\lim_{x\to 1}\sec\left(\frac{\pi}{2x}\right)\log x\cdot$

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4. Evaluate $\lim_{x\to 0^+}\frac{1}{x}\cos^{-1}\left(\frac{\sin x}{x}\right)$.

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5. If $f(x) = \frac{\tan x}{x}$, then find $\lim_{x \to 0} \left([f(x)] + x^2\right)^{\frac{1}{\{f(x)\}}}$, where $[.]$ and $\{.\}$ denotes greatest integer and fractional part function respectively.

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6. Evaluate $\lim_{n \to \infty} \frac{1}{n^{2(\log_e n - \log_e (n+1))} + n}$.

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7. Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{1}{2}ex}{x^2}$.

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8. Evaluate $\lim_{x \to 0^+} x^m (\log x)^m, m, n, \in N.$

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9. If $\alpha_1, , \alpha_2, , \alpha_n$ are the roots of equation $x^n + nax - b = 0$, show that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)...(\alpha_1 - \alpha_n) = n\alpha_1^{n-1} + n\alpha$

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10. Evaluate: $(\lim)_{x \to 0}\left(1^{1/\sin^2 x} + 2^{1/(\sin^2 x)} + + n^{1/\sin^2 x}\right)^{\sin^2 x}$

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11. Evaluate: $(\lim)_{n \to \frac{\pi}{2}} \frac{\sin x - (\sin x)^{\sin x}}{-\sin x + (\log)_e \sin x}$

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12. Slove $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$

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13. Evaluate

$$\lim_{n \to \infty} n^{-n^2}\left[\left(n+2^0\right)\left(n+2^{-1}\right)\left(n+2^{-2}\right)\ldots\left(n+2^{-n+1}\right)\right]^n.$$

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14. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ and h is the altitude from A to BC, then triangle ABC has perimeter $P = 2\left(\sqrt{2hr - h^2} + \sqrt{2hr}\right)$ and area A= _____ and = _____ and also $(\lim)_{h \to 0} \frac{A}{P^3} =$ _ _ _

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15. At the endpoint and midpoint of a circular are AB, tangent lines are drawn, and the points, A and B are jointed with a chord. Prove that the ratio of the areas of the triangles thus formed tends to 4 as the arc AB decreases infinitely.

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EXERCISE 2.1

1. Evaluate $\lim_{x \to -2^+} \frac{x^2 - 1}{2x + 4}$.

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2. Evaluate $\lim_{x \to 2^+} \frac{[x - 2]}{\log(x - 2)}$, where $[.]$ represents the greatest integer function.

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3. Evaluate $\lim_{x \to 0} \frac{\sin[\cos x]}{1 + [\cos x]}$ ($[.]$ denotes the greatest integer function).

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4. If $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$, show that $\lim_{x \to 0} f(x)$ does not exist.

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5. Show that $(\lim)_{x \to 0} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ does not exist

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6. Evaluate $\lim\limits_{x \to 0} \dfrac{3x + |x|}{7x - 5|x|}$

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7. If $f(x) = \begin{cases} x, & x < 0 \\ 1, & x = 0 \\ x^2, & x > 0 \end{cases}$, then find $\lim\limits_{x \to 0} f(x)$ if exists.

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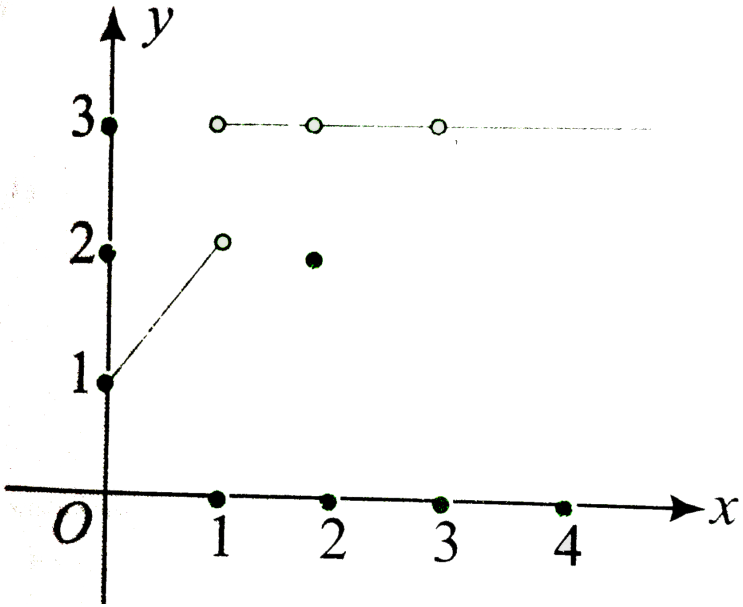
8. Consider the following graph of the function y=f(x). Which of the following is//are correct?



(a) $\lim_{x \to 1} f(x)$ does not exist.

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 3} f(x) = 3$.

(d) $\lim_{x \to 1.99} f(x)$exists.

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9. Evaluate $\lim_{x \to 0} \frac{\tan(sgn(x))}{sgn(x)}$ if exists.

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10. If $f(x) = \{\sin x, x \neq n\pi, n \in I2, otherwise$

$g(x) = \{x^2 + 1, x \neq 0, 4, x = 05, x = 2$ then $(\lim)_{x\,\vec{0}} g\{f(x)\} is =$

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EXERCISE 2.2

1. If $|f(x)| \leq x^2$, then prove that $\lim_{x \to 0} \frac{f(x)}{x} = 0.$

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2. If $f(x) = sgn(x)$ and $g(x) = x^3$,then prove that $\lim_{x \to 0} f(x).\, g(x)$ exists though $\lim_{x \to 0} f(x)$ does not exist.

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3. If $f(x) = \begin{matrix} \sin[x], & \text{for } [x] \neq 0 \\ 0, & \text{for } [x] = 0 \end{matrix}$ where $[x]$ denotes the greatest integer less than or equal to x. Then find $\lim_{x \to 0} f(x)$.

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4. Find the value of $\lim_{x \to 0^+} (\sin x)^{\frac{1}{x}}$.

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5. Let the sequence $< b_n >$ of real numbers satisfy the recurrence relation $b_{n+1} = \frac{1}{3}\left(2b_n + \frac{125}{b_n^2}\right), b_n \neq 0$. Then find $\lim_{n \to \infty} b_n$.

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6. Let $f: (1, 2) \overrightarrow{R}$ satisfies the inequality $\frac{\cos(2x - 4) - 33}{2} < f(x) < \frac{x^2|4x - 8|}{x - 2} \forall x \in (1, 2)$. Then find $\lim_{x \to 2^-} f(x)$.

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7. If $\frac{x^2+x-2}{x+3} \leq \frac{f(x)}{x^2} \leq \frac{x^2+2x-1}{x+3}$ holds for a certain interval containing the value of $\lim_{x \to -1} f(x)$.

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EXERCISE 2.3

1. Evaluate $\lim_{x \to 1} \frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}$.

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2. Evaluate $\lim_{x \to 1} \frac{x^4-3x^4+2}{x^3-5x^2+3x+1}$.

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3. Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{1 + \cos 4x}$

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4. Evaluate : $\lim_{x \to \frac{\pi}{4}} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$

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5. Evaluate the limit: $\lim_{x \to a} \frac{\sqrt{3x - a} - \sqrt{x + a}}{x - a}$

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6. Evaluate the limit: $\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

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7. Evaluate $\lim_{x \to \sqrt{10}} \frac{\sqrt{7+2x} - \left(\sqrt{5}+\sqrt{2}\right)}{x^2 - 10}$.

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8. Evaluate the limit: $(\lim)_{n \to \infty} \left(\frac{1^2 - 2^2 + 3^3 - 4^2 + 5^2 + nterms}{n^2} \right.$

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9. Evaluate the limit: $(\lim)_{x \to \infty} \left[\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1} \right]$

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10. If $[x]$ denotes the greatest integer less than or equal to x, then evaluate $\lim_{n \to \infty} \frac{1}{n^2}([1.\, x] + [2.\, x] + [3.\, x] + \ldots + [n.\, x])$.

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11. Evaluate $\lim_{x \to \infty} x^3 \left\{ \sqrt{x^2 + \sqrt{1 + x^4}} - x\sqrt{2} \right\}$.

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12. Evaluate $\lim_{x \to \infty} \left(\frac{7x^2 + 1}{5x^2 - 1} \right)^{\frac{x^5}{1 - x^3}}$.

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13. Evaluate the limit: $(\lim)_{n \to \infty} \cos\left(\pi\sqrt{n^2 + n}\right) whe \cap isan \in te \geq r$

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14. Evaluate the limit: $\lim_{x \to 1} \frac{\sum_{k=1}^{100} x^k - 100}{x - 1}$

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15. Evaluate $\lim_{h \to 0} \left[\frac{1}{(h)(8+h)^{\frac{1}{3}}} - \frac{1}{2h} \right]$.

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EXERCISE 2.4

1. Evaluate $\lim_{x \to 0} \left\{ \frac{\sin x - x + \frac{x^3}{6}}{x^5} \right\}$.

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2. Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$.

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3. Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

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4. If $\lim_{x \to 0} \frac{1 - \cos x}{e^{ax} - bx - 1} = 2$ then find the values of a and b.

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5. Find the values of $aandb$ in order that $(\lim)_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1\left[us \in gL'Ho\pi tal'sr\underline{e}\right].$

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EXERCISE 2.5

1. Evaluate $\lim_{x \to \infty} \frac{\sin x}{x}$.

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2. Evaluate: $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}$

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3. Evaluate: $\lim_{x \to 0} \frac{\cot 2x - \cos ec2x}{x}$

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4. Evaluate: $(\lim)_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$

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5. Evaluate: $\lim_{n \to \infty} n \cos\left(\frac{\pi}{4n}\right) \sin\left(\frac{\pi}{4n}\right)$

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6. Evaluate: $\lim_{x \to 0} \frac{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)}{\sin^{-1} x}$

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7. Evaluate: $(\lim)_{h \to 0} \dfrac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}$

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8. Evaluate:

$$\lim_{x \to 0} \frac{8}{x^8}\left\{1-\cos\left(\frac{x^2}{2}\right)-\cos\left(\frac{x^2}{4}\right)+\cos\left(\frac{x^2}{2}\right)\cos\left(\frac{x^2}{4}\right)\right\}$$

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9. Evaluate: $(\lim)_{x \to 0, y \to 0} \dfrac{y^2+\sin x}{x^2+\sin y^2}$ where $(x, y) \to (0, 0)$ along the curve $x = y^2$

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10. $\lim_{x \to 1} (1-x)\tan\left(\dfrac{\pi x}{2}\right)$

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11. Evaluate $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$.

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12. If $\lim_{x \to 2} \frac{\tan(x-2).\left(x^2 + (k-2)x - 2k\right)}{(x^2 - 4x + 4)} = 5$, then find the value of k.

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EXERCISE 2.6

1. Evaluate: $\lim_{x \to \infty} \left[x\left(a^{\frac{1}{x}} - 1\right)\right], a > 1$

Watch Video Solution

2. Evalaute $\lim_{x \to 0} \frac{x2^x - x}{1 - \cos x}$

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3. Evaluate $\lim_{x \to 2} \frac{\sin\left(e^{x-2} - 1\right)}{\log(x-1)}$

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$

Watch Video Solution

5. Evaluate $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

Watch Video Solution

6. $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$

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7. Evaluate $\lim_{x \to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}, a > 0$

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8. Evaluate: $\lim_{x \to 0} \frac{(1 - 3^x - 4^x + 12^x)}{\sqrt{(2\cos x + 7)} - 3}$

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9. Evaluate: $(\lim)_{x \to 0} \frac{(729)^x - (243)^x - (811)^x + 9^x + 3^x - 1}{x^3}$

Watch Video Solution

EXERCISE 2.7

1. Evaluate $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$

A. 0

B. 1

C. -1

D. none of these

Answer: e^2

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2. Find the value of $\lim_{x \to 1} (\log_3 3x)^{\log_x 3}$

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3. Evaluate the limit: $\lim_{x \to \infty} \left(\frac{x+2}{x+1} \right)^{x+3}$

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4. Evaluate:

$$(\lim)_{x \to \infty} \left(1 + \frac{1}{a + bx}\right)^{c + dx}, where a, b, c, and d are positive$$

A. 4

B. 2

C. -1

D. 0

Answer: $e^{d/b}$

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5. Evaluate: $(\lim)_{x \to \frac{7}{2}} \left(2x^2 - 9x + 8\right)^{\cot(2x - 7)}$

A. both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist

B. $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ exists

C. neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ may exist

D. $\lim_{x \to a} f(x)$ exists but $\lim_{x \to a} g(x)$ need not exist

Answer: $e^{5/2}$

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6. If x_1 and x_2 are the real and distinct roots of $ax^2 + bx + c = 0$ then prove that $\lim_{x \to x1} \left(1 + \sin\left(ax^2 + bx + c\right)\right)^{\frac{1}{x - x_1}}$. equals to

A. $e^{(x_1 - x_2)}$

B. 1

C. ∞

D. $\frac{1}{2}$

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7. $\lim_{x \to 0} \sin^2\left(\frac{\pi}{2 - px}\right)^{\sec^2\left(\left(\frac{\pi}{2 - px}\right)\right)}$

A. -1

B. 2

C. $\sqrt{5}$

D. e^{-p^2/q^2}

Answer: D

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EXERCISE 2.8

1. Evaluate $\lim_{x \to 1} \frac{\cos \frac{\pi}{2} x}{1 - \sqrt{x}}$

A. -1

B. 1

C. 0

D. does not exist

Answer: π

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2. Evalaute $\lim_{x \to 1} \frac{1 + \log x - x}{1 - 2x + x^2}$

A. 0

B. 1

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: $-1/2$

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3. Evaluate $\lim_{x \to \pi/2} \tan x \log \sin x$

A. -2

B. -1

C. $-\frac{2}{7}$

D. 0

Answer: 0

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4. Evaluate $\lim_{x \to 0} \frac{\log \cos x}{x}$

A. 0

B. $2/3$

C. $-1/4$

D. $3/2$

Answer: 0

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5. Evaluate $\lim_{x \to 0} \dfrac{2^x - 1}{(1+x)^{1/2} - 1}$

A. $\dfrac{1}{8\sqrt{3}}$

B. $\dfrac{1}{4\sqrt{3}}$

C. 0

D. none of these

Answer: $\log 4$

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6. Evaluate $\lim_{x \to \pi/4} (2 - \tan x)^{1/\ln(\tan x)}$

A. 16

B. 8

C. 4

D. 2

Answer: e^{-1}

Watch Video Solution

7. Evaluate: $(\lim)_{x \to 0} x^x$

A. $1/2$

B. 2

C. 1

D. None of these

Answer: $1 \quad \text{and} \quad 0$

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8. If $(\lim)_{x \to a} \frac{a^x - x^a}{x^x - a^a} = -1 and a > 0,$ then find the value of a.

A. 43529

B. 2

C. $\sin\frac{2}{5}$

D. $\sin\frac{1}{5}$

Answer: $a = 1$

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Exercises (Single Correct Answer Type)

1. $\lim_{x \to 0} \left[\frac{\sin(sgn(x))}{(sgn(x))}\right]$, where $[.]$ denotes the greatest integer function, is equal to

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2. Let $\lim_{x \to 0} \frac{[x]^2}{x^2} = m$, where $[.]$ denotes greatest integer. Then, m equals to

A. $-\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: B

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3. $\lim_{x \to 1} \left[\cos ec \frac{\pi x}{2}\right]^{1/(1-x)}$ (where $[.]$ represents the greatest integer function) is equal to

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4. The value of the limit $\lim_{x \to 0} \frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}, a > 1$, is

A. -1

B. $1/3$

C. 0

D. $2/9$

Answer: C

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5. If $\lim_{x \to a} \left\{ \frac{f(x)}{g(x)} \right\}$ exists, then which one of the following correct ?

A. both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist

B. $\lim_{x \to a} f(x)$need not exist $\lim_{x \to a} g(x)$ exist

C. neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ may exist

D. $\lim_{x \to a} f(x)$ exist but $\lim_{x \to a} g(x)$ need not exist

Answer: C

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6. $\lim_{x \to -1} \frac{1}{\sqrt{|x| - \{-x\}}}$ (where $\{x\}$ denotes the fractional part of x) is equal to

A. 16

B. 24

C. 32

D. does not exist

Answer: A

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7. If $x_1 = 3 and x_{n+1} = \sqrt{2 + x_n}, n \geq 1, then (\lim)_{x \overrightarrow{\infty}} x_n$ is -1 (b) 2 (c) $\sqrt{5}$ (d) 3

A. 0

B. $\frac{1}{2}$

C. log2

D. e^4

Answer: B

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8. $\lim_{x \to 0^-} \frac{\sum_{r=1}^{2n+1} [x^r] + (n+1)}{1 + [x] + |x| + 2x}$, where $n \in N$ and $[.]$ denotes the greatest integer function, equals

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9. $\lim_{x \to \infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$ is equal to

(a) 0

(b) 1

(c) $\frac{1}{3}$

(d) $\frac{1}{2}$

A. 0

B. 1

C. 10

D. 100

Answer: B

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10. If $f(x) = \frac{2}{x-3}, g(x) = \frac{x-3}{x+4}$, and $h(x) = -\frac{2(2x+1)}{x^2+x-12}$ then $\lim_{x \to 3} [f(x) + g(x) + h(x)]$ is

(a) -2

(b) -1

(c) $-\frac{2}{7}$

(d)0

A. 1

B. ∞

C. $\sqrt{2}$

D. none of these

Answer: C

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11. The value of $\lim_{x \to \pi} \frac{1+\cos^3 x}{\sin^2 x} is$

(a)$\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $-\frac{1}{4}$

(d) $\frac{3}{2}$

A. [2, 5)

B. (1, 5)

C. (-1, 5)

D. $(-\infty, \infty)$

Answer: D

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12. The value of $\lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$ is

A. 0

B. e^x

C. $\log_e x$

D. none of these

Answer: A

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13. The value of $(\lim)_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} is$ 16 (b) 8 (c) 4 (d) 2

A. $|2x| > \sqrt{3}$

B. $|2x| < \sqrt{3}$

C. $|2x| \geq \sqrt{3}$

D. $|2x| \leq \sqrt{3}$

Answer: B

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14.

$$\lim_{x \to 2} \left(\left(\frac{x^3 - 4x}{x^3 - 8} \right)^{-1} - \left(\frac{x + \sqrt{2x}}{x - 2} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1} \right) isequal \to$$

$\frac{1}{2}$

2

1

none of these

A. 1

B. $1/2$

C. 2

D. none of these

Answer: A

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15. If $\lim_{x \to -2^-} \frac{ae^{1/|x+2|} - 1}{2 - e^{1/|x+2|}} = \lim_{x \to -2^+} \sin\left(\frac{x^4 - 16}{x^5 + 32}\right)$, then a is

A. sin(3/5)

B. sin(2/5)

C. 0

D. none of these

Answer: C

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16. $\lim_{x \to 1} \frac{(1-x)(1-x^2)\ldots(1-x^{2n})}{\{(1-x)(1-x^2)\ldots(1-x^n)\}^2}$, n in N, equals`

A. 0

B. $2n^C n$

C. $2n!$

D. none of these

Answer: B

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17. The value of $\lim_{x \to \frac{1}{\sqrt{2}}} \left(\frac{x - \cos\left(\sin^{-1} x\right)}{1 - \tan\left(\sin^{-1} x\right)} \right)$ is

(a) $-\frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

A. $-\frac{1}{\sqrt{2}}$

B. -1

C. non-existent

D. none of these

Answer: A

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18. Among (i) $\lim_{x \to \infty} \sec^{-1}\left(\frac{x}{\sin x}\right)$ and (ii) $\lim_{x \to \infty} \sec^{-1}\left(\frac{\sin x}{x}\right)$.

A. 0

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. does not exist

Answer: A

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19. $\lim_{x \to \infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} \right)$ is equal to

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20. $\lim_{n \to \infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$ is equal to

(a) 0

(b) 2

(c) 4

(d) ∞

A. 1

B. 0

C. 2

D. none of these

Answer: C

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21. $\lim_{x \to \infty} \frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}}$ is equal to

(a)16

(b) 24

(c) 32

(d) 8

A. $\frac{1}{2\pi}$

B. $\frac{-1}{\pi}$

C. $\frac{-2}{\pi}$

D. none of these

Answer: C

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22. $\lim_{x \to \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}\right]$ is equal to

(a) 0

(b) $\frac{1}{2}$

(c) log 2

(d) e^4

A. 1

B. 0

C. 2

D. none of these

Answer: B

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23. $\lim_{x \to \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$ is equal to

A. $\sec x(x \tan x + 1)$

B. $x \tan x + \sec x$

C. $x \sec x + \tan x$

D. none of these

Answer: D

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24. $\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 10^{10}}$ is equal to

(a) 0

(b) 1

(c) 10

(d) 100

A. 0

B. 1

C. 2

D. 4

Answer: D

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25. $\lim_{x \to \infty} \frac{2\sqrt{x} + 3x3 + 4x4 + \ + nxn}{\sqrt{(2x-3)} + (2x-3)3 + \ + (2x-3)n}$ is equal to

(a)1 (b) ∞ (c) $\sqrt{2}$ (d) none of these

A. 0

B. 1

C. $\sqrt{2}$.

D. $2\sqrt{2}$.

Answer: C

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26. If $\lim_{n \to \infty} \frac{n.3^n}{n(x-2)^n + n.3^{n+1} - 3^n} = \frac{1}{3}$, then the range of x is (where $n \in N$)

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27. $\lim_{n \to \infty} n^2 \left(x^{\frac{1}{n}} - x^{\frac{1}{(n+1)}} \right), x > 0$, is equal to (a)0 (b) e^x (c) $(\log)_e x$ (d) none of these

A. π

B. 2π

C. $\pi / 2$

D. none of these

Answer: C

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28. $Let f(x) = \lim_{n \to \infty} \frac{1}{\left(\frac{3}{\pi} \tan^{-1} 2x \right)^{2n} + 5}$. Then the set of values of x for which $f(x) = 0$ is

A. 199

B. 198

C. 0

D. none of these

Answer: A

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29. $f(x) = \frac{1n\left(x^2 + e^x\right)}{1n\left(x^4 + e^{2x}\right)} \dot{T}hen \lim_{x \to \infty} f(x)$ is equal to

(a) 1

(b) $\frac{1}{2}$

(c) 2

(d) none of these

A. $\frac{2a}{\pi}$

B. $-\frac{2a}{\pi}$

C. $\frac{4a}{\pi}$

D. $-\frac{4a}{\pi}$

Answer: B

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30. The value of $\lim_{n\to\infty}\left[\frac{2n}{2n^2-1}\cos\left(\frac{n+1}{2n-1}\right)-\frac{n}{1-2n}\frac{n(\dot{-}1)^n}{n^2+1}\right] is$ (a) 1 (b) -1 (c) 0 (d) none of these

A. 5

B. 6

C. 7

D. none of these

Answer: C

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31. If $f(x) = 0$ is a quadratic equation such that $f(-\pi) = f(\pi) = 0$ and $f\left(\frac{\pi}{2}\right) = -\frac{3\pi^2}{4}$, then $\lim_{x \to -\pi^+} \frac{f(x)}{\sin(\sin x)}$ is equal to

A. $-\frac{3}{2}\pi$

B. $-\frac{1}{2}$

C. (2π)

D. $\frac{3}{2}\pi$

Answer: C

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32. $\lim_{x \to 1} \frac{x \sin(x - [x])}{x - 1}$, where $[.]$ denotes the greatest integer function, is equal to

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33. $(\lim)_{x \to \infty} \frac{x^2 \frac{\tan 1}{x}}{\sqrt{8x^2+7x+1}} isequa < o: \ -\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}} \ \frac{1}{\sqrt{2}}$ (d) does not exist

A. $-\frac{1}{2\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. 2

D. $1/4$

Answer: A

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34. If $\lim_{x \to 0} \frac{x^a \sin^b x}{\sin(x^c)}$, where a , b , c in R ~{0},exists and has non-zero value. Then,`

A. 0

B. $\pi/2$

C. π

D. 2π

Answer: C

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35. $\lim_{x \to 0} \left(x^4 \frac{\cot^4 x - \cot^2 x + 1}{\tan^4 x - \tan^2 x + 1} \right)$ is equal to (a) 1 (b) 0 (c) 2 (d) none of these

A. -1

B. 1

C. 0

D. none of these

Answer: A

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36. $\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$ is equal to

(a) $\frac{1}{2\pi}$

(b) $-\frac{1}{\pi}$

(c) $\frac{-2}{\pi}$

(d) none of these

A. 0

B. ∞

C. -2

D. 2

Answer: B

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37. $(\lim)_{x \to 0} \frac{1}{x} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is equals to (a)1 (b) 0 (c) 2 (d) none of these

A. $f(x)f(y)$

B. $f(x) + f(y)$

C. $f(x) - f(y)$

D. none of these

Answer: D

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38. $\lim_{y \to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$ is equal to (a) $\sec x(x\tan x + 1)$ (b) $x\tan x + \sec x$ (c) $x\sec x + \tan x$ (d) none of these

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39. $\lim_{x \to 1} \frac{1 + \sin \pi \left(\frac{3x}{1+x^2} \right)}{1 + \cos \pi x}$ is equal to

A. $\log n \left(\frac{2}{3} \right)$

B. 0

C. $n \log n \left(\frac{2}{3} \right)$

D. not defined

Answer: A

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40. $(\lim)(n \to \infty) \sum_{x=1}^{20} \cos^{2n}(x - 10)$ is equal to

A. (a) 2

B. (b) 0

C. (c) 1

D. (d) -1

Answer: B

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41. $\lim_{x \to -1} \left(\frac{x^4 + x^2 + x + 1}{x^2 - x + 1} \right)^{\frac{1 - \cos(x+1)}{(x+1)^2}}$ is equal to (a) 1 (b) $\left(\frac{2}{3} \right)^{\frac{1}{2}}$ (c) $\left(\frac{3}{2} \right)^{\frac{1}{2}}$ (d) $e^{\frac{1}{2}}$

A. 1

B. -1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: B

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42. $\lim_{x \to \infty} \{(x+5)\tan^{-1}(x+5) - (x+1)\tan^{-1}(x+1)\}$ is equal to

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43. The value of $\lim_{x \to 0} \left(\left[\frac{100x}{\sin x} \right] + \left[\frac{99 \sin x}{x} \right] \right)$ (where [.] represents the greatest integral function) is

(a)199

(b) 198

(c)0

(d) none of these

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44. The value of $(\lim)_{x \to a} \sqrt{a^2 - x^2} \cot \left(\frac{\pi}{2} \sqrt{\frac{a - x}{a + x}} \right)$ is (a) $\frac{2a}{\pi}$ (b) $-\frac{2a}{\pi}$ (c) $\frac{4a}{\pi}$ (d) $-\frac{4a}{\pi}$

A. $\frac{5050}{\pi e}$

B. $100\frac{)}{\pi e}$

C. $-\frac{5050}{\pi e}$

D. $-\frac{4950}{\pi e}$

Answer: C

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45. $\lim_{x \to 0}\left[\min\left(y^2-4y+11\right)\frac{\sin x}{x}\right]$ (where $[.]$ denotes the greatest integer function) is

A. -1

B. 1

C. 0

D. 6

Answer: B

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46. The value of $\lim_{x \to 0} \frac{1 - (\cos x)\sqrt{\cos 2x}}{x^2}$ is

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{6}$

D. $\frac{5}{12}$

Answer: D

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47. $\lim_{x \to \infty} \frac{1}{x+1} \tan\left(\frac{\pi x + 1}{2x + 2}\right)$ is equal to

A. a. $\frac{1}{2}$

B. b. $-\frac{1}{2}$

C. c. 0

D. d. none of these

Answer: A

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48. The value of $\lim_{x \to 1} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$ is

(a) 4

(b) $\frac{1}{2}$

(c) 2

(d) $\frac{1}{4}$

A. $1/6$

B. $-1/3$

C. $1/2$

D. 1

Answer: D

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49. $\lim_{x \to \frac{\pi}{2}} \frac{\sin(x \cos x)}{\cos(x \sin x)}$ is equal to (a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

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50. $\lim_{x \to 0} \left[(1 - e^x) \frac{\sin x}{|x|} \right]$ is (where $[.]$ represents the greatest integer function)

A. 1

B. 2

C. 3

D. none of these

Answer: A

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51. Evaluate $\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$ is equal to

A. e

B. $\frac{1}{e}$

C. 1

D. none of these

Answer: D

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52. If $f(x) = \lim_{n \to \infty} n\left(x^{1/n} - 1\right)$, then for $x > 0, y > 0, f(xy)$ is equal to

A. f(x)f(y)

B. f(x) + f(y)

C. f(x) - f(y)

D. none of these

Answer: B

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53. $\lim_{x \to 0} \left\{(1+x)^{\frac{2}{x}}\right\}$ (where {.} denotes the fractional part of x

(a) $e^2 - 7$

(b) $e^2 - 8$

(c) $e^2 - 6$

(d) none of these

A. 1

B. e

C. e^{-1}

D. none of these

Answer: A

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54. The value of $\lim_{x \to \infty} \frac{\left(2^{x^n}\right)e^{\frac{1}{x}} - \left(3^{x^n}\right)e^{\frac{1}{x}}}{x^n}$ (where $n \in N$) is

A. e

B. 0

C. e^{-1}

D. 1

Answer: B

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55. $(\lim)_{x \to 0} \frac{\sin(x^2)}{1n(\cos(2x^2 - x))}$ is equal to (a) 2(b) -2 (c) 1 (d) -1

A. e^a

B. $-a$

C. e^{1-a}

D. e^{1+a}

Answer: B

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56. $\lim_{x \to \infty} \frac{e^{1/x^2} - 1}{2\tan^{-1}(x^2) - \pi}$ is equal to

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57. $\lim_{x \to 0} \frac{(2^m + x)^{1/m} - (2^n + x)^{1/n}}{x}$ is equal to

A. $(n!)^n$

B. $(n!)^{1/n}$

C. $n!$

D. $\ln(n!)$

Answer: C

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58. The value of $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{e^{1/n}}{n}+\frac{e^{2/n}}{n}+\ldots+\frac{e^{(n-1)/n}}{n}\right]$ is

A. $e-1$

B. $e+1$

C. 0

D. 1

Answer: C

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59. $(\lim)_{x \to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}$, $wheren = 100,$ is equal to : $\frac{5050}{\pi e}$ (b) $\frac{100}{\pi e}$ (c) $-\frac{5050}{\pi e}$ (d) $-\frac{4950}{\pi e}$

A. 0

B. -1

C. 1

D. does not exist

Answer: C

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60. $\lim_{x \to 0} \frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x} =$

A. 2

B. 1

C. $\log_a 2$

D. 0

Answer: B

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61. The value of $\lim_{x \to \infty} \left(\sqrt[3]{x^3 + 2x^2} - \sqrt{x^2 + x} \right)$ is

A. e

B. e^2

C. $\sqrt{€}$

D. e^{-1}

Answer: C

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62. The value of $\lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3} is$

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 0

(d) none of these

A. 1

B. -1

C. 2

D. -2

Answer: B

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63. If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$ then $\lim_{x \to a} \{f(x)\}^{g(x)} = e^{\lim_{x \to a} (f(x) - 1) x g(x)}$

$\lim_{x \to 0} \left(\frac{x - 1 + \cos x}{x} \right)^{\frac{1}{x}}$ is equal to

A. $f(1 + 0) = -1, f(1 - 0) = 0$

B. $f(1 + 0) = 0 = f(1 - 0)$

C. $\lim_{x \to 1} f(x)$ exists

D. $f(x)$ does not exist $x \to 1$

Answer: B

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64. If $\lim x \to 0 \left(x^{-3} \sin 3x + a x^{-2} + b \right)$ exists and is equal to 0, then

A. $a = -3$

B. $a = 0$

C. $b = 1$

D. $b = -1$

Answer: A

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65. If $\lim_{x \to 0} \frac{x^n - \sin x^n}{x - \sin^n x}$ is non-zero finite, then n must be equal to 4 (b) 1 (c) 2 (d) 3

A. $1, \quad \text{if} \quad n = m$

B. $0, \quad \text{if} \quad n > m$

C. $\infty, \quad \text{if} \quad n < m$

D. $n/m, \quad \text{if} \quad n < m$

Answer: B

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66. $(\lim)_{x \to 0} \left(\frac{1 + \tan x}{1 + \sin x} \right)^{\cos ecx}$ is equal to

(a)e

(b) $\frac{1}{e}$

(c) 1

(d) none of these

A. $\lim_{x \to 0} f(x)$ exists for $n > 0$

B. $\lim_{x \to 0} f(x)$ does not exists for $n < 0$

C. $\lim_{x \to 0} f(x)$ does not exists for any value of n

D. $\lim_{x \to 0} f(x)$ exists for any value of n

Answer: C

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67. The value of $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ is

(a) $e^{-\frac{2}{\pi}}$

(b) $e^{\frac{1}{\pi}}$

(c) $e^{\frac{2}{\pi}}$

(d) $e^{-\frac{1}{\pi}}$

A. $a = 1/4$

B. $b = 3/4$

C. $L = -1/32$

D. $L = 1/32$

Answer: C

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68. The value of $(\lim)_{x \to \infty}\left(\frac{\cos x}{m}\right)^{m}$ is 1 (b) e (c) e^{-1} (d) none of these

A. $\lim_{x \to \infty} \frac{\log_e x}{\{x\}} = \infty$

B. $\lim_{x \to 2^+} \frac{x}{x^2 - x - 2} = \infty$

C. $\lim_{x \to -1^-} \frac{x}{x^2 - x - 2} = \infty$

D. $\lim_{x \to \infty} \frac{\log_{0.5} x}{\{x\}} = \infty$

Answer: A

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69. $(\lim)_{x \to \infty}\left(\frac{n^2}{n^2}\right)^{n(n-1)isequa<o}$ e (b) e^2 (c) e^{-1} (d) 1

A. $\lim_{x \to 0} \frac{[x + |x|]}{x} = 0$, where $[x]$ denotes the greatest integer functions.

B. $\lim_{x \to 0} \frac{x e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} = 0$

C. $\lim_{x \to 3} (x-3)^{\frac{1}{5}} sgn(x-3) = 0$, where sgn stands for signum function.

D. $\lim_{x \to 0} \frac{\tan^{-1}|x|}{x} = 0$

Answer: B

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70. $\lim_{n \to \infty} \left\{ \left(\frac{n}{n+1} \right)^{\alpha} + \sin \frac{1}{n} \right\}^{n}$ (where $\alpha \in Q$) is equal to

A. $a = 1/3, b = 1$

B. $a = 1, b = -1$

C. $a = 9, b = -9$

D. $a = 2, b = 2/3$

Answer: C

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71. $\lim_{x \to \infty} \left[\left(\frac{e}{1-e} \right) \left(\frac{1}{e} - \frac{x}{1+x} \right) \right]^x$ is :

A. limit does not exist when $a = \pi/6$

B. $L = -1$ when $a = \pi$

C. $L = 1$ when $a = \pi/2$

D. $L = 1$ when $a = 0$

Answer: C

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72. The value of $\lim_{x \to 0} \frac{(1^x + 2^x + 3^x + \ldots + n^x)^{a/x}}{n}$, is:

A. $f(1^+) + f(1^-) = 0$

B. $f(1^+) + f(1^-) + f(1) = 3/2$

C. $f(-1^+) + f(-1^-) = -1$

D. $f(1^+) + f(-1^-) = 0$

Answer: B

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73. The value of $(\lim)_{x \to 1}\left(\frac{p}{1-x^p}-\frac{q}{1-x^q}\right), p, q, \in N,$ equal $\frac{p+q}{2}$ (b) $\frac{pq}{2}$ (c) $\frac{p-q}{2}$ (d) $\sqrt{\frac{p}{q}}$

A. $-\frac{3}{4}$

B. 0 if n is even

C. $-\frac{3}{4}$ if n is odd

D. none of these

Answer: C

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74. $\lim_{x \to \infty} \frac{x(\log x)^3}{1+x+x^2}$ equals

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75. $\lim_{x \to \infty} \frac{\cot^{-1}(x^{-a} \log_a x)}{\sec^{-1}(a^x \log_x a)}, (a > 1)$ is equal to

(a)2

(b)1

(c) $(\log)_a 2$

(d) 0

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76. The value of $\lim_{n \to \infty} \frac{e^n}{\left(1 + \frac{1}{n}\right)^{n^2}}$ is

A. -1

B. 0

C. 1

D. ∞

Answer: C

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Multiple Correct Answers Type

1. Let $f(x) = \begin{cases} 1 + \frac{2x}{a}, & 0 \leq x < 1 \\ ax, & 1 \leq x < 2 \end{cases}$. If $\lim_{x \to 1} f(x)$ exists, then a is

A. $\lim_{x \to 5^-} f(x) = 0$

B. $\lim_{x \to 5^+} f(x) = 1$

C. $\lim_{x \to 5} f(x)$ does not exist

D. none of these

Answer: B::C

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2. If $f(x) = |x - 1| - [x], where\ [x]$ is the greatest integer less then or equal to x , then

(A) $f(1+0) = -1, f(1-0) = 0$

(B) $f(1+0) = 0 = f(1-0)$

(C) $(\lim)_{x \to 1} f(x)$ exists

(D) $(\lim)_{x \to 1} f(x)$ does not exist

A. $\lim_{x \to 0} [f(x)] = 0$

B. $\lim_{x \to 0} [f(x)] = 1$

C. $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ does not exist

D. $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ exists

Answer: A::D

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3. If $\lim_{n \to \infty} \left(an - \frac{1+n^2}{1+n}\right) = b$ a finite number then

A. $f(0) = 1$

B. $f\left(\frac{\pi}{2}\right) = 1$

C. $f(a) = (\cos a)^{\cos^2 a} . (\sin a)^{\sin^2 a}$ if $a \in \left(0, \frac{\pi}{2}\right)$

D. $f(a) = \frac{(\sin a)^{\sin^2 a}}{(\cos a)^{\cos^2 a}}$ if $a \in \left(0, \frac{\pi}{2}\right)$

Answer: A::C

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4. If $m, n \in N$, $\lim_{x \to 0} \frac{\sin x^n}{(\sin x)^m}$ is

A. 1 if $n > m$

B. 0 if $n = m$

C. 0 if $n > m$

D. $\frac{n}{m}$

Answer: A::B::C

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5. If $f(x) = \left\{ x^n \sin\left(\frac{1}{x^2}\right), x \neq 00, x = 0, (n \in I), then \right.$

$(\lim)_{x \to 0} f(x) e\xi st$ or $n > 1$ $(\lim)_{x \to 0} f(x) e\xi st$ or $n < 0$ $(\lim)_{x \to 0} f(x)$

does not exist for any value of n $(\lim)_{x \to 0} f(x)$ cannot be determined

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. $\frac{\pi}{\sqrt{2}}$

D. $\sqrt{2}\pi$

Answer: A::B

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6. If $L = \lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{1+ax}{1+bx}\right)$ exists,then find the value of $\frac{1}{a} - \frac{2}{l} - \frac{3}{b}$

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7. Which of the following true ({.} denotes the fractional part of the function)? $(\lim)_{x \to \infty} \frac{(\log)_e x}{\{x\}} = \infty$ (b) $(\lim)_{x \to 2^+} \frac{x}{x^2 - x - 2} = \infty$ $(\lim)_{x \to 1^-} \frac{x}{x^2 - x - 2} = -\infty$ (d) $(\lim)_{x \to \infty} \frac{(\log)_{0.5} x}{\{x\}} = \infty$

A. always 1

B. always -1

C. $(-1)^{n-m+1}$

D. $(-1)^{n-m}$

Answer: A::B::C

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8. Which of the following is/are correct?

A. always 1

B. always -1

C. $(-1)^{m+1}$

D. $(-1)^{n-m}$

Answer: A::B::C

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9. If $\lim_{x \to 1} (2 - x + a[x-1] + b[1+x])$ exists, then a and b can take the values (where $[.]$ denotes the greatest integer function)

A. a. a is always equal to -1

B. b. b is always equal to +1

C. c.does not exist

D. d. None of these

Answer: B::C

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10. $L = \lim_{x \to a} \frac{|2\sin x - 1|}{2\sin x - 1} \dot{T}hen$

(a) limit does not exist when $a = \frac{\pi}{6}$

(b)$L = -1 whena = \pi$

(c) $L = 1whena = \frac{\pi}{2}$

(d) $L = 1whena = 0$

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11. Let $f(x) = \lim_{n \to \infty} \frac{x}{x^{2n} + 1}$. Then f has

A. real and equal roots

B. complex roots

C. unequal positive real roots

D. unequal roots

Answer: B::C::D

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12. $(\lim)_{x \to \infty} \frac{-3+(-1)^n}{4n-(-1)^n} - \frac{3}{4}$ (b) 0 if n is even $-\frac{3}{4}$ if n is odd (d) none of these

A. $-\frac{3}{4}$

B. 0 if n is even

C. $-\frac{3}{4}$ if n is odd

D. none of these

Answer: A::C

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13. Given a real-valued function f such that $f(x) = \begin{cases} \frac{\tan^2\{x\}}{\left(x^2-[x]^2\right)\sqrt{\{x\}\cot\{x\}}}, f \text{ or } x<0, fx>0 \end{cases}$ Where $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then $(\lim)_{x \to 0^+} f(x) = 1$, $(\lim)_{x \to 0^-} f(x) = \cot 1$, $\cot^{-1}\left((\lim)_{x \to 0^-} f(x)\right)^2 = 1, \tan^{-1}\left((\lim)_{x \to 0^+} f(x)\right) = \frac{\pi}{4}$

A. $p_1 \ \ln \ a_1 + p_2 \ \ln \ a_2 + \ldots + p_n \ \ln \ a_n$

B. $a_1^{p_1} + a_2^{p_2} + \ldots + a_n^{p_n}$

C. $a_1^{p_1} . a_2^{p_2} \ldots a_n^{p_n}$

D. $\sum_{r=1}^{n} a_r p_r$

Answer: A::B::C::D

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14. If $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct

(a) $(\lim)_{x \to 1} f(x) f$ or $a = -2$ (b) $(\lim)(x \to -2) f(x) f$ or $a = 13$

(c) $(\lim)(x \to 1) f(x) = \frac{4}{3}$ (d) $(\lim)(x \to -2) f(x) = -\frac{1}{3}$

A. $\ln \ a_1$

B. e^{a_n}

C. a_(1)`

D. a_n

Answer: A::B::C::D

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15. The value of $\lim_{n \to \infty} \frac{1}{1 + n \sin^2 nx}$ can be $(n \in N)$

A. $\ln \ a_n$

B. 1

C. 0

D. none of these

Answer: B::C

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16. Let $f(x) = \frac{x^2 - 9x + 20}{x - [x]}$ where $[x]$ denotes greatest integer less than or equal to x), then

A. $\lim_{x \to 5} f(x) = 1$

B. $\lim_{x \to 5} f(x) = 0$

C. $\lim_{x \to 5} f(x)$ does not exist

D. none of these

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17. Given $\lim_{x \to 0} \frac{f(x)}{x^2} = 2$, where $[.]$ denotes the greatest integer function, then which options are correct?

A. (a) $\lim_{x \to 0} [f(x)] = 0$

B. (b) $\lim_{x \to 0} [f(x)] = 1$

C. (c) $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ do e s not exist

D. (d) $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ exist

Answer: A::C

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18.

If $f(a) = \lim_{x \to 2} (\sin^x a + \cos^x a)^{\frac{1}{(x-2)}}$ for $a \in \left[0, \frac{\pi}{2}\right]$, then $f(a) = ?$

A. $-np$

B. np

C. n^2p

D. np^2

Answer: A::B::C

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19. Let L= $\lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

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20. Let $f(x) = \left(\frac{1 - x(1 + |1 - x|)}{|1 - x|}\right)\cos\left(\frac{1}{1 - x}\right)$ for $x \neq 1$ then prove that $\lim_{x \to 1^-} f(x) = 0$

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21. If $A = \lim_{x \to 0} \frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B = \lim_{x \to 0} \frac{[|x|]}{x}$, then

A. A = 1

B. A does not exist

C. B = 0

D. B = 1

Answer: B::C

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22. Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

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23. Evaluate $\lim_{x \to 0^+} \left(\frac{x}{a}\right)\left[\frac{b}{x}\right]$ where [.] represents greatest integer function.

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24. Evaluate $\lim_{x \to 0} \left(\frac{x+3}{1+x}\right)^{\frac{x^2+2x-3}{x-1}}$

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Linked Comprehension Type

1. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$L = \lim_{x \to 0-} f(x)$ is equal to

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2. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$L = \lim_{x \to 0-} f(x)$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. (π)

D. $\frac{1}{\sqrt{2}}$

Answer: B

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3. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$L = \lim_{x \to 0-} f(x)$ is equal to

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4. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n, \quad \text{and} \quad a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n, \ \min N, \ \text{then the value of} \ L = \lim_{x \to a_m} (A_1 A_2 ... A_n) \text{is}$

A. 2

B. -1

C. not exist

D. 1

Answer: C

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5. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n, \quad \text{and} \quad a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n, \ \min N, \ \text{then} \ \lim_{x \to a_m} (A_1 A_2 ... A_n)$

A. $e^{-\frac{1}{4}}$

B. $e^{-\frac{1}{2}}$

C. e^{-2}

D. e^{-4}

Answer: D

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6. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n,$ and $a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n,$ min $N,$ then $\lim_{x \to a_m} (A_1 A_2 ... A_n)$

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7. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1 + x)}{x^3}$ exists finitely, then

The value of L is

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8. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

Equation $ax^2 + bx + c = 0$ has

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9. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

The solutions set of $||x + c| - 2a| < 4b$ is

A. $a \quad b \quad c \quad d$
$s \quad r \quad q \quad p$

B. $a \quad b \quad c \quad d$
$q \quad s \quad r \quad p$

C. $a \quad b \quad c \quad d$
$s \quad r \quad p \quad q$

D. $a \quad b \quad c \quad d$
$s \quad p \quad q \quad r$

Answer: C

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10. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + \ldots p_n a_n^x\right)^{1/x}$.

$\lim_{x \to \infty} F(x)$ equals

A. $a \quad b \quad c \quad d$
$s \quad r \quad q \quad p$

B. $a \quad b \quad c \quad d$
$q \quad p \quad s \quad p$

C. $a \quad b \quad c \quad d$
$s \quad r \quad p \quad q$

D. $a \quad b \quad c \quad d$
$p \quad p \quad q \quad r$

Answer: C

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11. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + p_n a_n^x\right)^{1/x}$.

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12. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + \ldots p_n a_n^x\right)^{1/x}$.

$\lim_{x \to \infty} F(x)$ equals

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13. If $L = \lim_{x \to \infty} \left(x + 1 - \sqrt{ax^2 + x + 3}\right)$ exists infinetely then

The value of a is

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14. If $L = \lim_{x \to \infty} \left(x + 1 - \sqrt{ax^2 + x + 3}\right)$ exists finitely then

The value of a is

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15. Let $f: R \to R$ be a real function. The function f is double differentiable. If there exists $n \in N$ and $p \in R$ such that $\lim_{x \to \infty} x^n f(x) = p$ and there exists $\lim_{x \to \infty} x^{n+1} f(x)$, then $\lim_{x \to \infty} x^{n+1} f'(x)$ is equal to

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16. Let $f: R \to R$ be a real function. The function f is double differentiable. If there exists $n \in N$ and $p \in R$ such that $\lim_{x \to \infty} x^n f(x) = p$ and there exists $\lim_{x \to \infty} x^{n+1} f(x)$, then $\lim_{x \to \infty} x^{n+1} f'(x)$ is equal to

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17. $Let f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ and $f(1) = 3, f(3) = 7$ and $f(5) = 11$. Then The value of $f(0)$ is

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18. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ and $f(1) = 3, f(3) = 7$ and $f(5) = 11$. Then The value of $f(0)$ is

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19. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8, \lim_{x \to 0} \frac{g(x)}{2\cos x - ye^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 +$

then $\lambda =$

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20. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8, \lim_{x \to 0} \frac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 + 2f(x))^{\frac{1}{g(x)}} = \frac{1}{e}$, then

$\lim_{x \to 0} (1 + f(x))^{\frac{1}{2g(x)}}$ is equal to

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Matrix Match Type

1. Match the following lists:

List I	List II
a. If $L = \lim_{x \to -1} \frac{\sqrt[3]{(7-x)} - 2}{(x+1)}$, then $12L =$	**p.** -2
b. If $L = \lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$, then $-L/4 =$	**q.** 2
c. If $L = \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2 + x - 3}$, then $20L =$	**r.** 1
d. If $L = \lim_{x \to \infty} \frac{\log x^n - [x]}{[x]}$, where $n \in N$, ($[x]$ denotes greatest integer less than or equal to x), then $-2L =$	**s.** -1

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2. Factorise the expression: $am^2 + bm^2 + bn^2 + an^2$

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3. Compute $\lim x \to -2 (3x^2 + 5x - 9)$

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4. Consider $\lim_{x \to \infty} \left(\frac{x^3 + x^2 + x + \sin x}{x^2 + 2\cos x} - a \sin x - bx + c \right) = 4.$

Now, match the following lists and then choose the correct code.

List I	List II
a. The value of a is	**p.** 1
b. The value of b is	**q.** 3
c. The value of c is	**r.** 2
d. Number of real roots of equation $cx^2 + bx + a = 0$ is	**s.** 0

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5. Match the following lists (where $[x]$ represents the greatest integer function) and then choose the correct code.

List I	List II
a. $\lim_{x\to 0} x(-1)^{[1/x]}$	**p.** Does not exist
b. $\lim_{x\to 2}(-1)^{[x]}$	**q.** is 0
c. $\lim_{x\to \frac{3}{2}}(x-[x])$	**r.** is 1
d. $\lim_{x\to 0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$	**s.** is 2

Codes :

	a	b	c	d
(1)	s	r	q	p
(2)	q	p	s	p
(3)	s	r	p	q
(4)	p	p	q	r

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Numerical Value Type

1. The reciprocal of the value of: $\lim_{n\to\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{n^2}\right) is$

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2. $\lim_{x \to \infty} f(x)$, where $\frac{2x-3}{x} < f(x) < \frac{2x^2+5x}{x^2} \forall x > 0$, is _____.

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3. If $f(x) = \{x-1, x \geq 12x^2 - 2, x < 1, g(x) = \{x+1, x > 0 - x^2 + 1, x \leq$ $= |x|$, *then* $\lim_{x \to 0} f(g(h(x)))$ is___

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4. If $\lim_{x \to \infty} f(x)$ exists and is finite and nonzero and if $\lim_{x \to \infty} \left\{ f(x) + \frac{3f(x)-1}{f^2(x)} \right\} = 3$,then find the value of $\lim_{x \to \infty} f(x)$

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5. If $L - (\lim)_{x \overrightarrow{2}} \frac{(10-x)^{\frac{1}{3}} - 2}{x-2}, then the value of |1(4L)| is\_\_$

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6. If $\lim_{x \to 0} \frac{p \sin 2x + (1 - \cos 2x)}{x + \tan x} = 1$, then the value of p is_____.

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7. The value of $\lim_{x \to \infty} \left(\frac{100}{1 - x^{100}} - \frac{50}{1 - x^{50}} \right)$ is _____.

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8. If $L = \lim_{x \to 2} \frac{\sqrt[3]{60 + x^2} - 4}{\sin(x-2)}$, then the value of $1/L$ is_____.

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9. The value of $\lim_{x \to \infty} \left(\frac{20^x - 1}{19(5^x)} \right)^{1/x}$ is _____.

A. 3

B. 1

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: (4)

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10. The value of $\lim_{n \to \infty} \left[\sqrt[3]{(n+1)^2} - \sqrt[3]{(n-1)^2} \right]$ is _____.

A. Equals $\frac{1}{\sqrt{2}}$

B. Does not exist

C. Equals $\sqrt{2}$

D. Equals -sqrt(2)`

Answer: (0)

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11. If $L = \lim_{n \to \infty} \left(2 \times 3^2 \times 2^3 \times 3^4 \ldots \times 2^{n-1} \times 3^n\right)^{\frac{1}{\left(n^2+1\right)}}$, and n is even then the value of L^4 is _____.

A. a. $-\frac{1}{4}$

B. b. $\frac{1}{2}$

C. c. 6

D. d. 2

Answer: (6)

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12. The value of $\lim_{x \to \infty} \frac{\log_e(\log_e x)}{e\sqrt{x}}$ is _____. (a) π/ 2 (b)0 (c)-π (d)π

A. $\frac{\pi}{2}$

B. 1

C. $-\pi$

D. π

Answer: (0)

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13. Find k if x = 3 is a root of equation $kx^2 - 10x + 3 = 0$

A. 4

B. 3

C. 2

D. $1/2$

Answer: (6)

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14. The value of $\lim_{x \to \infty} \left(x - x^2 \log_e \left(1 + \frac{1}{x} \right) \right)$ is _____.

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 2

Answer: (0.5)

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15. Let $S_n = 1 + 2 + 3 + \ + n$ and $P_n = \frac{S_2}{S_2 - 1} \frac{\dot{S}_3}{S_3 - 1} \frac{\dot{S}_4}{S_4 - 1} \ldots \frac{S_n}{S_n - 1}$ Where $n \in N, (n \geq 2)\cdot$ Then $(\lim)_{n \to \infty} P_n =_{\_} \ _{\_} \ _{\_}$

A. $\frac{1}{4}$

B. $\frac{1}{24}$

C. 3

D. $\frac{1}{8}$

Answer: (3)

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16. If $\lim_{x \to 1} \frac{a \sin(x-1) + b \cos(x-1) + 4}{x^2 - 1} = -2$, then $|a+b|$ is_____.

A. does not exist (in R)

B. is equal to 8

C. is equal to 15

D. is equal to 120

Answer: (8)

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17. Let $\lim_{x \to 1} \frac{x^a - ax + a - 1}{(x-1)^2} = f(a)$. Then the value of $f(4)$ is_____.

A. 4

B. 3

C. 6

D. 2

Answer: (6)

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18. Number of integral values of k for which

$\lim_{x \to 1} \sin^{-1}\left(\frac{k}{\log_e x} - \frac{k}{x-1}\right)$ exists is _____.

A. $a = 1, b = 4$

B. $a = 1b = -4$

C. $a = 2, b = -3$

D. $a = 2, b = 3$

Answer: (5)

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19. If $\lim_{x \to 1} \left(1 + ax + bx^2\right)^{\frac{c}{(x-1)}} = e^3$, then find the value of bc is _____.

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20. Let $f''(x)$ be continuous at $x = 0$

If $\lim_{x \to 0} \frac{2f(x) - 3af(2x) + bf(8x)}{\sin^2 x}$ exists and $f(0) \neq 0, f'(0) \neq 0$, then find the value of $3a/b$ is_____.

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21. If $L = \lim_{x \to 0} \frac{e^{-x^2/2} - \cos x}{x^3 \sin x}$, then find the value of $1/(3L)$ is _____.

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22. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is :

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23. If $\lim_{x \to 0} \left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then the value of $\ln\left(\lim_{x \to 0} \left[1 + \frac{f(x)}{x}\right]^{1/x}\right)$ is _____.

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24. The largest value of the non-negative integer a for which `lim_(xrarr1) {(-ax+sin(x-1)+a)/(x+sin(x-1)-1)}^((1-x)/(1-sqrtx)}= 1/4

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25. Find the 24th term of the sequence: 12, 10, 8, 6,.....

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26. Let $\alpha, \beta \in R$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then find the value of $6(\alpha + \beta)$ _____.

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Archives JEE MAIN

1. Let $f: R \to R$ be a positive, increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ is equal to

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2. $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x - 2} \right)$ =?

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3. $\lim_{x \to 0} \frac{(1 - \cos 2)(3 + \cos x)}{x \tan 4x}$ is equal to

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4. $\lim_{x \to 0} \left(\frac{\sin(\pi \cos^2 x)}{x^2} \right)$ is equal to

Watch Video Solution

5. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

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6. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$. Then $\log_e p$ is equal to

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7. The $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

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8. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then find $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

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Archives JEE ADVANCED

1. If $\lim_{x \to 0} \left[1 + x \ln\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b \sin^2 \theta, b > 0 \text{and} \theta \in (-\pi, \pi]$. then the value of θ is

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2. If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b\right) = 4$,then

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Single Correct Answer Type

1. If $f: R \overrightarrow{R}$ is defined by $f(x) = [x-3] + |x-4|$ for $x \in R$, then $(\lim)_{x \to 3} f(x)$ is equal to (where [.] represents the greatest integer function) a. b. c. d. -1

A. -2

B. -1

C. 0

D. 1

Answer: C

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2. $\lim_{x \to -\frac{\pi}{2}} \left[\frac{[\sin x] - [\cos x] + 1}{3} \right] =$ (where $[.]$ denotes the greatest integer integer function)

A. 0

B. 1

C. -1

D. does not exist

Answer: A

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3. $\lim_{x \to \frac{-1}{3}} \frac{1}{x} \left[\frac{-1}{x} \right] =$ (where [.] denotes the greatest integer function)

A. -9

B. -12

C. -6

D. 0

Answer: C

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4. If $f(x) = \begin{cases} x + \frac{1}{2}, x < 0 \\ 2x + \frac{3}{4}, x > +0 \end{cases}$, then $\left[\left(\lim \right)_{x \to 0} f(x) \right] =$ (where [.] denotes the greatest integer function) a. b. c. does not exist d. none of these

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. does not exist

D. none of these

Answer: C

View Text Solution

5. $\lim_{x \to -7} \frac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)} =$ (where [.] denotes the greatest integer function)

A. is 0

B. is 1

C. is -1

D. does not exist

Answer: A

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6. Evaluate $\lim_{x \to 1^+} (2)^{(-2)^{\frac{1}{1-x}}}$

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7. Evaluate $\lim_{x \to 0} \frac{\sin x - x}{x^3}$

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8. Evaluate $\lim_{x \to 0} \frac{\sin x + \log(1 - x)}{x^2}$

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9. $\lim_{x \to \infty} \left[x - \log_e \left(\frac{e^x + e^{-x}}{2} \right) \right] =$

A. $\log_e 4$

B. 0

C. $\log_e 2$

D. ∞

Answer: C

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10. $\lim_{x \to \infty} \left(\frac{16 - x}{x} \right)^2 =$

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11. If $\frac{\cos x}{\sin ax}$ is periodic function, then $\lim_{m \to \infty} \left(1 + \cos^{2m} n!\pi a\right)$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: C

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12. The value of $\lim_{x \to 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$ is

A. $\frac{1}{2}$

B. 2

C. $\sqrt{2}$

D. none of these

Answer: C

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13. $\lim_{x \to \frac{\pi}{2}} (1 - \sin x)\tan x =$

A. $\frac{\pi}{2}$

B. 1

C. 0

D. ∞

Answer: C

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14. The value of $\lim_{x \to \infty} x^2 \left(1 - \cos. \frac{2}{x}\right)$ is

A. 0

B. $1/4$

C. 2

D. 1

Answer: C

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15. $\lim_{x \to \infty} \sqrt[3]{x}\left(\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2}\right) =$

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$

Answer: D

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16. $\lim_{n \to \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n} =$

A. 0

B. 43529

C. $-4/7$

D. $-20/7$

Answer: D

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17. $\lim_{x \to \infty} \left(\frac{\sin x}{x} \right)$

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18. $\lim_{x \to \infty} \frac{\cot^{-1}\left(\sqrt{x+1}+\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^{x}\right\}} =$

A. 1

B. 0

C. $\pi/2$

D. non existent

Answer: A

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19. $\lim_{x \to 0} \frac{3\tan 3x - 4\tan 2x - \tan x}{4x^2 \tan x}$

A. 0

B. 1

C. 3

D. 4

Answer: D

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20. $\lim_{x \to 1} \left(\frac{2}{1 - x^2} + \frac{1}{x - 1} \right)$

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21. The value of $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$ is

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. 1

D. does not exist

Answer: D

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22. The value of $\lim_{x \to \infty} \left(e^{\sqrt{x^4+}} - e^{(x^2+1)} \right)$ is

A. 0

B. e

C. 1/e

D. $-\infty$

Answer: D

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23. The value of $\lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

A. 8

B. 4

C. -8

D. -2

Answer: C

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24. $\left(\lim\right)_{x \to \frac{\pi}{2}} \frac{(1-\sin x)\left(8x^3-\pi^3\right)\cos x}{(\pi-2x)^4}$

a. $\frac{\pi^2}{6}$ b. $\frac{3\pi^2}{16}$ c. $\frac{\pi^2}{16}$ d. $-\frac{3\pi^2}{16}$

A. $-\frac{\pi^2}{16}$

B. $\frac{3\pi^2}{16}$

C. $\frac{\pi^2}{16}$

D. $-\frac{3\pi^2}{16}$

Answer: D

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25. $\lim_{x \to \infty} \frac{\sum_{r=1}^{10} (x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)} =$

A. 5

B. 2010

C. $\frac{502}{1005}$

D. 0

Answer: A

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26. If $\lim_{x \to 0} \frac{f(x)}{x^2} = a$ and $\lim_{x \to 0} \frac{f(1-\cos x)}{g(x)\sin^2 x} = b$ (where $b \neq 0$), then $\lim_{x \to 0} \frac{g(1-\cos 2x)}{x^4}$ is

A. $\frac{4a}{b}$

B. $\frac{a}{4b}$

C. $\frac{a}{b}$

D. none of these

Answer: C

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27. If

$$f(x) = \begin{cases} \frac{x}{\sin x}, & x > 0 \\ 2 - x, & x \leq 0 \end{cases} \text{ and } g(x) = \begin{cases} x + 3, & x < 1 \\ x^2 - 2x - 2, & 1 \leq x < 2 \\ x - 5, & x \geq 2 \end{cases}$$

Then the value of $\lim_{x \to 0} g(f(x))$

A. is -2

B. is -3

C. is 1

D. does not exist

Answer: B

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28. If $k \in I$ such that $\lim_{n \to \infty} \left(\cos. \frac{k\pi}{4}\right)^{2n} - \left(\cos. \frac{k\pi}{6}\right)^{2n} = 0,$ then

A. k must not be divisible by 24

B. k is divisible by 24 or k is divisible neither by 4 nor by 6

C. k must be divisible by 12 but not necessarity by 24

D. none of these

Answer: B

Watch Video Solution

29. If a_n and b_n are positive integers and $a_n + \sqrt{2}b_n = \left(2 + \sqrt{2}\right)^n$, then $\lim_{n \to \infty} \left(\frac{a_n}{b_n}\right)$=

A. $\sqrt{2}$

B. 2

C. $e^{\sqrt{2}}$

D. e^2

Answer: A

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30. The value of $\lim_{x \to 0} \left(\frac{\tan x^{\frac{1}{5}}}{\left(\tan^{-1}\sqrt{x}\right)^2} \frac{\log(1+5x)}{e^{3\sqrt[5]{x}} - 1} \right)$ is

A. $\frac{3}{5}$

B. $\frac{5}{3}$

C. 1

D. none of these

Answer: B

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31. The value of $\lim_{x \to 3} \frac{\left(x^3 + 27\right)\log_e(x-2)}{x^2 - 9}$ is

A. 9

B. 18

C. 27

D. $5/3$

Answer: A

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32. The value of $\lim_{x \to 0^+} \left(\frac{1 - \cos(\sin^2 x)}{x^2} \right)^{\frac{\log_e (1 - 2x^2)}{\sin^2 x}}$ is

A. 0

B. e

C. -1

D. ∞

Answer: D

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33. $\lim_{x \to 0} \frac{1}{x^2} \begin{vmatrix} 1 - \cos 3x & \log_e(1 + 4x) \\ \sin^{-1}(e^x - 1) & \tan^{-1}(2x) \end{vmatrix}$

A. 2

B. -4

C. 6

D. 4

Answer: B

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34. If graph of the function $y = f(x)$ is continuous and passes through point (3, 1) then $\lim_{x \to 3} \frac{\log_e(3f(x) - 2)}{2(1 - f(x))}$ is equal

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. $-\frac{3}{2}$

D. $-\frac{1}{2}$

Answer: C

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35. Let f(x) be defined for all $x \in R$ such that $\lim_{x \to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right]=0$. Then f(0) is

A. 0

B. 1

C. 2

D. 3

Answer: A

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36. $\lim_{x \to \infty} x^2 \sin\left(\log_e \sqrt{\frac{\cos(\pi)}{x}}\right)$

A. 0

B. $-\frac{\pi^2}{2}$

C. $-\frac{\pi^2}{4}$

D. $-\frac{\pi^2}{8}$

Answer: C

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37. If $\lim_{x \to \infty} \left(\frac{x+c}{x-c}\right)^x = 4$ then the value of e^c is

A. $1/4$

B. $1/2$

C. 1

D. 2

Answer: D

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38. If $\lim_{x \to 0}\left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then $\lim_{x \to 0}\left[1 + \frac{f(x)}{x}\right]^{1/x} =$

A. e

B. e^2

C. e^3

D. none of these

Answer: B

View Text Solution

39. $\lim_{x \to \frac{\pi}{2}}\left[1 + (\cos x)^{\cos x}\right]^2 =$

A. Does not exist

B. 1

C. e

D. 4

Answer: D

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40. If $a > 0, b > 0$ than $\lim_{n \to \infty} \left(\frac{a - 1 + b^{\frac{1}{n}}}{a} \right)^n =$

A. $b^{\frac{1}{a}}$

B. $a^{\frac{1}{b}}$

C. a^b

D. b^a

Answer: A

View Text Solution

41. If $f(x) = \lim_{n \to \infty} \left(\frac{\cos(x)}{\sqrt{n}} \right)^n$, then the value of $\lim_{x \to 0} \frac{f(x) - 1}{x}$ is

A. 0

B. 1

C. 2

D. $3/2$

Answer: A

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42. $\lim_{x \to 0} \frac{\log\left(e^{x^2} + 2\sqrt{x}\right)}{\tan \sqrt{x}}$ is equal to

A. 0

B. 1

C. e^2

D. 2

Answer: D

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43. Let $f: R \to R$ be such that $f(a) = 1, f(a) = 2$. Then $\lim_{x \to 0} \left(\frac{f^2(a+x)}{f(a)} \right)^{1/x}$ is

A. e^2

B. e^4

C. e^{-4}

D. $1/e$

Answer: B

View Text Solution

44. $L = \lim_{n \to \infty} \left(\frac{\sqrt{n^2+n} - 1}{n} \right)^{2\sqrt{n^2+n}-1}$

A. e

B. $1/e$

C. e^2

D. e^{-2}

Answer: B

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45. If $f(n) = \lim_{x \to 0} \left\{ \left(1 + \sin\frac{x}{2}\right)\left(1 + \sin\frac{x}{2^2}\right)...\left(1 + \sin\frac{x}{2^n}\right) \right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

A. 1

B. e

C. 0

D. ∞

Answer: B

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46. $\lim_{n \to \infty} \left(1 - x + x.\sqrt[n]{e}\right)^n$ is equal to

A. e^x

B. e^{-x}

C. e^{2x}

D. none of these

Answer: A

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47. The value of $\lim_{x \to 1} \frac{\sqrt[13]{x} - \sqrt[7]{x}}{\sqrt[5]{x} - \sqrt[3]{x}}$ is

A. $\frac{44}{91}$

B. $\frac{45}{89}$

C. $\frac{45}{89}$

D. $\frac{40}{93}$

Answer: B

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48. The value of $\lim_{x \to 1} \frac{\sqrt[2]{x} - \sqrt[4]{x}}{\sqrt[3]{x} - \sqrt[2]{x}}$ is

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49. The value of $\lim_{x \to 0} \frac{1 - \cos 2x}{e^{x^2} - e^x + x}$ is

A. 0

B. 2

C. 4

D. 8

Answer: C

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50. If $f'(a) = \frac{1}{4}, then(\lim)_{h \vec{0}} \frac{f(a+2h^2) - f(a-2h^2)}{f(a+h^3-h^2) - f(a-h^3+h^2)} =$

A. 0

B. 1

C. -2

D. none of these

Answer: C

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51. $\lim_{x \to 0^+} \frac{1}{x\sqrt{x}} \left(a \tan^{-1}. \frac{\sqrt{x}}{a} - b \tan^{-1}. \frac{\sqrt{x}}{b} \right)$

A. $\frac{a-b}{3}$

B. 0

C. $\frac{(a^2-b^2)}{6a^2b^2}$

D. $\frac{a^2-b^2}{3a^2b^2}$

Answer: D

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52. The value of $\lim_{x\to 0}\left(\frac{1+2x}{1+3x}\right)^{\frac{1}{x^2}} e^{\frac{1}{x}}$ is $e^{\frac{5}{2}}$ b. e^2 c. e^{-2} d. 1

A. $e\left(\frac{5}{2}\right)$

B. e^2

C. e^{-2}

D. 1

Answer: A

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53. Let $f: R \to R$ be a differentiable function at x = 0 satisfying f(0) = 0 and f'(0) = 1, then the value of $\lim_{x \to 0} \frac{1}{x} . \sum_{n=1}^{\infty} (-1)^n . f\left(\frac{x}{n}\right)$, is

A. 0

B. $-\log 2$

C. 1

D. e

Answer: B

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54. The value of $\lim_{x \to \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$ is

A. $-1/2$

B. $-2/3$

C. $-3/2$

D. $-1/3$

Answer: D

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55. $\lim_{x \to 4} \frac{x-2}{x-8}$

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56. $\lim_{x \to 0} \frac{3^x - 1}{x}$

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57. $\lim_{x \to 0} \frac{\log(1+2x)}{x}$

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58. If $\lim_{x \to 0} \frac{x^3}{\sqrt{a+x}(bx - \sin x)} = 1, a > 0$, then $a + b$ is equal to

A. 36

B. 37

C. 38

D. 40

Answer: B

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59. If $\lim_{x \to \infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix} \right) = -5.$ where α, β, γ are finite real numbers, then

A. $\alpha = 2, \beta = 1, \gamma \in R$

B. $\alpha = 2, \beta = 2, \gamma = 5$

C. $\alpha \in R, \beta = 1, \gamma \in R$

D. $\alpha \in R, \beta = 1, \gamma = 5$

Answer: D

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ComprehensionType

1. $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$

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2. Let $f(x)$ be the fourth degree polynomial such that $f'(0) - 6, f(0) = 2 and (\lim)_{x\,1 \to} \frac{f(x)}{(x-1)^2} = 1$ The value of $f(2)$ is 3 b. 1 c. 0 d.2

A. 4

B. 5

C. 6

D. 7

Answer: C

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Multiple Correct Answer Type

1. $\lim_{x \to 8} \frac{x^{\frac{1}{3}} - 2}{x - 8}$

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2. $\lim_{x \to \frac{\pi}{4}} \frac{\tan 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$

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Comprehension Type

1. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

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2. $L = \lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$

The value/values of b is

A. $\in$ R

B. 2

C. 0

D. 1

Answer: B

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3. $\lim_{x \to 0} \frac{\sin 2x \sin 3x}{x^2}$

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4. $\lim_{x \to 0} \frac{\tan x}{x}$

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