MATHS

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METHODS OF DIFFERENTIATION

Single Correct Answer Type

1. The right hand derivative of $f(x) = [x]tan\pi xatx = 7$ is (where [.] denotes the greatest integer function) 0 b. 7π c. -7π d. none of these

A. 0

B. 7π

C. -7π

D. None of these

Answer: B

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2. If $f(x-y)=f(x).g(y)-f(y).g(x)$ and $g(x-y)=g(x).g(y)+f(x).f(y)$ for all $x \in R$. If right handed derivative at x=0 exists for f(x) find the derivative of g(x) at x =0

A. -1

B. 0

C. 1

D. none of these

Answer: B

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3. If $xe^{xy} - y = \sin^2 x$ then $\frac{dy}{dx}$ at x = 0 is

A. 0

B. 1

C. -1

D. none of these

Answer: B

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4. about to only mathematics

A. 2

B. 4

C. 8

D. 16

Answer: C

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5. If for a continuous function $f, f(0) = f(1) = 0, f'(1) = 2 and y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to a. 1 b. 2 c. 0 d. none of these

A. 1

B. 2

C. 0

D. none of these

Answer: B

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6. The derivative of $\cos\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) - 2\cos^{-1}\sqrt{\frac{1-x}{2}}$ w.r.t. x is

A. $1 - \frac{1}{\sqrt{1-x^2}}$

B. $1 - \frac{1}{\sqrt{1+x^2}}$

C. $2 - \frac{1}{\sqrt{1-x^2}}$

D. $2 - \frac{1}{\sqrt{1+x^2}}$

Answer: A

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7. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2+1} + \ln\sqrt{x+\sqrt{x^2+1}}$ then the value of $xy' + \log y'$ is

A. y

B. 2y

C. 0

D. $-2y$

Answer: B

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8. Let $g(x) = f(x)\sin x, where f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. The value of $\left|g^{-\pi}\right|$ equals _____

A. 1

B. 2

C. -2

D. 0

Answer: C

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9. If $f(x) = \frac{\log_e(\log_e x)}{\log_e x}$ then $f'(x)$ at x = e is

A. 0

B. 1

C. e

D. $1/2$

Answer: D

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10. Let $g(x) = e^{f(x)} and f(x+1) = x + f(x) \forall x \in R$. If $n \in I^+, then \frac{g'\left(n+\frac{1}{2}\right)}{g\left(n+\frac{1}{2}\right)} - \frac{g'\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)} =$

$2\left(1+\frac{1}{2}+\frac{1}{3}+ +\frac{1}{n}\right) 2\left(1+\frac{1}{3}+\frac{1}{5}+\frac{1}{2n-1}\right) n\,1$

A. $2\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right)$

B. $2\left(1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-n}\right)$

C. n

D. 1

Answer: C

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11. $\frac{d}{dx}\left[\cos^{-1}\left(x\sqrt{x}-\sqrt{(1-x)\left(1-x^2\right)}\right)\right]=$

$\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$ $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

$\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ $\frac{1}{\sqrt{1-x^2}}$ 0 b. $1/4$ c. $-1/4$ d. none of these

A. $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

B. $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

C. $\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$

D. $\frac{1}{\sqrt{1-x^2}}$

Answer: B

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12. If $t\left(1+x^2\right)=x$ and $x^2+t^2=y$, then at $x=2$ the value of $\frac{dy}{dx}$ is equal to

A. $\frac{24}{5}$

B. $\frac{101}{125}$

C. $\frac{488}{125}$

D. $\frac{358}{125}$

Answer: C

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13. if $x = \frac{1+t}{t^3}, y = \frac{3}{2t^2} + \frac{2}{t}$ satisfies $f(x) \cdot \left\{\frac{dy}{dx}\right\}^3 = 1 + \frac{dy}{dx}$ then $f(x)$ is:

A. x

B. $\frac{x^2}{1+X^2}$

C. $x + x + \frac{1}{x}$

D. $x - \frac{1}{x}$

Answer: A

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14. Let $y = x^3 - 8x + 7 and x = f(t)\cdot$ If $\frac{dy}{dt} = 2$ and $x = 3$ at $t = 0$, then $\frac{dx}{dt}$ at $t = 0$ is given by 1 (b) $\frac{19}{2}$ (c)

$\frac{2}{19}$ (d) none of these

A. 1

B. $\frac{19}{2}$

C. $\frac{2}{19}$

D. None of these

Answer: C

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15. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$ then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

A. 8

B. 16

C. 64

D. 49

Answer: C

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16. The derivative of the function represented parametrically as $x = 2t = |t|, y = t^3 + t^2|t|a$ = 0 is a. -1 b. 1 c. 0 d. does not exist

A. -1

B. 0

C. 1

D. does not exist

Answer: B

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17. If $y = \tan^{-1}\left(\frac{u}{\sqrt{1-u^2}}\right)$ and $x = \sec^{-1}\left(\frac{1}{2u^2-1}\right)$, $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$, prove that $2\frac{dy}{dx} + 1 = 0$.

A. y

B. xy

C. 0

D. 1

Answer: C

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18. The differential coefficient of $\sin^{-1}\left(\frac{5\cos x - 4s \in x}{\sqrt{41}}\right)$ is -2 b.-1 c. 1 d. 2

A. -2

B. -1

C. 1

D. 2

Answer: D

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19. $xy = (x + y)6nand\frac{dy}{dx} = \frac{y}{x}the \cap = 1$ b.2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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20. If $x + y = 3e^2the\frac{d}{dx}(x^y) = 0f$ or $x = e^2$ b. e^e c. e d. $2e^2$

A. e

B. e^2

C. e^e

D. $2e^2$

Answer: B

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21. If $f(x) = (x-1)^{100}(x-2)^{2(99)}(x-3)^{3(98)}\ldots(x-100)^{100}$, then the value of $\frac{f'(101)}{f(101)}$ is

A. 5050

B. 2575

C. 3030

D. 1250

Answer: A

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22. The function $f: R\overrightarrow{R}$ satisfies $f\left(x^2\right)\dot{f}^{x} = f'(x)\dot{f}^{'}\left(x^2\right)$ for all real x. Given that $f(1) = 1$ and $f^{1} = 8$, then the value of $f'(1) + f^{1}$ is 2 b. 4 c. 6 d. 8

A. 2

B. 4

C. 6

D. 8

Answer: C

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23. The second derivative of a single valued function parametrically represented by $x = \phi(t)$ and $y = \psi(t)$ (where $\phi(t)$ and $\psi(t)$ are different function and $\phi'(t) \neq 0$) is given by

A. $\frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\left(\frac{d^2y}{dt^2}\right) - \left(\frac{d^2x}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)^3}$

B. $\frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\left(\frac{d^2y}{dt^2}\right) - \left(\frac{d^2x}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)^2}$

C. $\frac{d^2y}{dx^2} = \frac{\left(\frac{d^2x}{dt}\right)\left(\frac{dy}{dt}\right) - \frac{dx}{dt}\left(\frac{d^2y}{dt^2}\right)}{\left(\frac{dx}{dt}\right)^3}$

D. $\frac{d^2y}{dx^2} = \frac{\left(\frac{d^2x}{dt}\right)\left(\frac{dy}{dt}\right) - \left(\frac{d^2y}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dy}{dt}\right)^3}$

Answer: A

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24. For the curve $\sin x + \sin y = 1$ lying in first quadrant.

If $\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2}$ exists and non-zero than $2\alpha =$

A. 3

B. 4

C. 5

D. 1

Answer: A

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25. If $y = \left(\frac{\alpha x + \beta}{\gamma x + \delta}\right)$, then $2\frac{dy}{dx}.\frac{d^3y}{dx^3}$ is

A. $7\left(\frac{d^2y}{dx^2}\right)^2$

B. $5\left(\frac{d^2y}{dx^2}\right)^2$

C. $3\left(\frac{d^2y}{dx^2}\right)^2$

D. $\left(\frac{d^2y}{dx^2}\right)^2$

Answer: C

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26. If $f(1)=3, f'(1)=2, f''(1)=4,$ then $(f^{-})''(3)=$ (where $f^{-1}=$ inverse of $y=f(x)$)

A. 1

B. $-\frac{1}{2}$

C. -2

D. None of these

Answer: B

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27. If the third derivative of $\frac{x^4}{(x-1)(x-2)}$ is $\frac{-12k}{(x-2)^4} + \frac{6}{(x-1)^4}$, then the value of k is

A. 2

B. 4

C. 8

D. 16

Answer: C

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28. If $(a+bx)e^{\frac{y}{x}} = x$, Prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

A. $\left(\frac{dy}{dx}+x\right)^2$

B. $\left(x\frac{dy}{dx}-y\right)^2$

C. $\left(\frac{dy}{dx}-y\right)^2$

D. $\left(x\frac{dy}{dx}+y\right)^2$

Answer: B

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29. If $R=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-3/2}}{\frac{d^2y}{dx2}}$, then $R^{2/3}$ can be put in the form of $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}}+\frac{1}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$ b.

$\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} - \frac{1}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$ c. $\frac{2}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{2}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$ d. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} \frac{\dot{1}}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$

A. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

B. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

C. $\frac{2}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{2}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

D. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} \cdot \frac{1}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

Answer: A

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30. If $x = 2\cos t - \cos 2t, \quad y = 2\sin t - \sin 2t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.

A. $1/2$

B. $5/2$

C. $-3/2$

D. 2

Answer: C

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31. If $y^3 - y = 2x, then\left(x^2 - \frac{1}{27}\right)\frac{d^2y}{dx^2} + x\frac{dy}{d} = y$ b. $\frac{y}{3}$ c. $\frac{y}{9}$ d. $\frac{y}{27}$

A. y

B. $\frac{y}{3}$

C. $\frac{y}{9}$

D. $\frac{y}{27}$

Answer: C

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32. Let $f(x) = \frac{g(x)}{x} when x \neq 0$ and $f(0) = 0.$ If $g(0) = g'(0) = 0 and g^0 = 17$ then $f'(0) = 3/4$ b. $-1/2$ c. $17/3$ d. $17/2$

A. $3/4$

B. $-1/2$

C. $17/3$

D. $17/2$

Answer: D

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33. Let $f:(-\infty,\infty)\overrightarrow{0,\infty}$ be a continuous function such that $f(x+y)=f(x)+f(y)+f(x)f(y),\ \forall x\in R.$ Also $f'(0)=1.$ Then $[f(2)]$ equal ($[.]$ represents the greatest integer function) 5 b. 6 c. 7 d. 8

A. 5

B. 6

C. 7

D. 8

Answer: B

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34. Let $f: R \to R$ be a function satisfying $f(x+y) = f(x) = \lambda xy + 3x^2y^2 \text{for all } x, y \in R$

If f(3)=4 and f(5)=52, then f'(x) is equal to

A. $10x$

B. $-10x$

C. $20x$

D. $128x$

Answer: B

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35. A function $f: R\overrightarrow{1, \infty}$ satisfies the equation $f(xy) = f(x)f(y) - f(x) - f(y) + 2$. If differentiable on $R - \{0\} and f(2) = 5, f'(x) = \frac{f(x) - 1}{x} \dot{\lambda} then \lambda =$ $2' f(1)$ b. $3f'(1)$ c. $\frac{1}{2}f'(1)$ d. $f'(1)$

A. $2f'(1)$

B. $3f'(1)$

C. $\frac{1}{2}f'(1)$

D. $f'(1)$

Answer: D

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36. Let $\frac{f(x+y)-f(x)}{2}=\frac{f(y)-a}{2}+xy$ for all real $xandy$. If $f(x)$ is differentiable and $f'(0)$ exists for all real permissible value of a and is equal to $\sqrt{5a-1-a^2}$. Then $f(x)$ is positive for all real x $f(x)$ is negative for all real x $f(x)=0$ has real roots Nothing can be said about the sign of $f(x)$

A. f(x) is positive for all real x

B. f(x) is negative for all real x

C. f(x)=0 has real roots

D. nothing can be said about the sign of f(x)

Answer: A

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37. Let $f(3) = 4$ and $f'(3) = 5$. Then $\lim_{x \to 3} [f(x)]$ (where [.] denotes the greatest integer function) is

A. 3

B. 4

C. 5

D. non-existent

Answer: D

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38. Let f(x) be a function which is differentiable any number of times and $f\left(2x^2-1\right)=2x^3f(x),\ \forall x\in R.$ Then $f^{(2010)}(0)=$ (Here $f^{(n)}(x)=n^{th}$ order derivative of f at x)

A. -1

B. 1

C. 0

D. data is insufficient

Answer: C

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39. If $f(x)=\begin{vmatrix}(x-a)^4 & (x-a)^3 & 1\\ (x-b)^4 & (x-b)^3 & 1\\ (x-c)^4 & (x-c)^3 & 1\end{vmatrix}$ then $f'(x)=\lambda\begin{vmatrix}(x-a)^4 & (x-a)^2 & 1\\ (x-b)^4 & (x-b)^2 & 1\\ (x-c)^4 & (x-c)^2 & 1\end{vmatrix}$. Find the value of λ

A. 1

B. 2

C. 3

D. None of these

Answer: C

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40. Suppose $\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x) \end{vmatrix} = 0$ where f(x) is continuously differentiable function with $f'(x) \neq 0$ and satisfies f(0) = 1 and f'(0) = 2 then $\lim_{x \to 0} \frac{f(x) - 1}{x}$ is

A. 1

B. 2

C. $1/2$

D. 0

Answer: B

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41. A nonzero polynomial with real coefficient has the property that $f(x) = f'(x)\dot{f}'(x)$. If a is the leading

coefficient of $f(x)$, then the value of $1/2a)$ is___

A. $1/3$

B. 6

C. 12

D. $1/18$

Answer: D

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42. If $'f'$ is an increasing function from $R\overrightarrow{R}$ such that $f^x > 0 and f^{-1}$ exists then $\frac{d^2\left(f^{-1}(x)\right)}{dx^2}$ is < 0 b. > 0 c. $= 0$ d. cannot be determined

A. lt 0

B. gt 0

C. 0

D. cannot be determined

Answer: A

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43. Vertices of a variable acute angled triangle ABC lies on a fixed circle. Also a, b, c and A, B, C are lengths of sides and angles of triangle ABC, respectively. If x_1, x_2 and x_3 are distances of orthocentre from A, B and C, respectively, then the maximum value of $\left(\frac{dx_1}{da}+\frac{dx_2}{db}+\frac{dx_3}{dc}\right)$ is

A. $-\sqrt{3}$

B. $-3\sqrt{3}$

C. $\sqrt{3}$

D. $3\sqrt{3}$

Answer: B

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44. In a question a student was given to find the derivative of the product of two functions $f and g.$ The student y misstate thought $(fg)' = f'g'$ for his question $f(x) = x^3$ and he got the correct answer. Given that

$g(4) = 1$. Then which of the following is false? $g(5) = \frac{1}{8}$

b. $f'(x) < 0$ c.$f(0) < 0$ d. none of these

A. $g(5) = \frac{1}{8}$

B. $f'(x) < 0$

C. $f(0) < 0$

D. None of these

Answer: A

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45. f is a strictly monotonic differentiable function with $f'(x) = \frac{1}{\sqrt{1+x^3}}$. If g is the inverse of f, then $g^x = \frac{2x^2}{2\sqrt{1+x^3}}$ b. $\frac{2g^2(x)}{2\sqrt{1+g^2(x)}}$ c. $\frac{3}{2}g^2(x)$ d. $\frac{x^2}{\sqrt{1+x^3}}$

A. $\frac{3x^2}{2\sqrt{1+x^3}}$

B. $\frac{3g^2(x)}{2\sqrt{1+g^2(x)}}$

C. $\frac{3}{2}g^2(x)$

D. $\frac{x^2}{\sqrt{1+x^3}}$

Answer: C

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46. Suppose $f: R\vec{R}^{+}$ be a differentiable function such that $3f(x+y) = f(x)f(y)\,\forall x, y \in R$ with $f(1) = 6$. Then the value of $f(2)$ is 6 b. 9 c. 12 d. 15

A. 6

B. 9

C. 12

D. 15

Answer: C

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Multiple Correct Answer Type

1. If $y = \cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}, then \frac{dy}{dx} isequa < o$

$\frac{1}{2(1+x^2)}, x \in R$ (b) $\frac{1}{2(1+x^2)}, x > 0$

$\frac{-1}{2(1+x^2)}, x < 0$ (d) $\frac{1}{2(1+x^2)}, x < 0$

A. $\frac{1}{2(1+x^2)}, x \in R$

B. $\frac{1}{2(1+x^2)}, x > 0$

C. $\frac{1}{2(1+x^2)}, x < 0$

D. $\frac{-1}{2(1+x^2)}, x < 0$

Answer: B::D

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2. Suppose that $f(x)$ is differentiable invertible function $f'(x) \neq 0 and h'(x) = f(x)$. Given that $f(1) = f'(1) = 1, h(1) = 0$ and $g(x)$ is inverse of $f(x)$. Let $G(x) = x^2 g(x) - x h(g(x)) \forall x \in R$. Which of the

following is/are correct? $G'(1) = 2$ b. $G'(1) = 3$ c. $G^1 = 2$ d. $G^1 = 3$

A. G''(1) = 2

B. G'(1) = 3

C. G''(1) = 2

D. G''(1) = 3

Answer: A::D

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3. If $f(x) = (x-1)^4(x-2)^3(x-3)^2(x-4)$, then the value of $f'''(1) + f''(2) + f'(3) + f'(4)$ equals

A. $\frac{1}{3}$

B. 2

C. 3

D. 1

Answer: A::D

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4. If $y = x^{(\log x)^{\log(\log, x)}}$, then $\frac{dy}{dx} is$

A. $\frac{y}{x}\left(\ln x^{\log x - 1}\right) + 2\ln x \ln(\ln x)$

B. $\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x) + 1)$

C. $\left[(\ln x)^2 + 2\ln(\ln x)\right]$

D. $\frac{y}{x}\frac{\log y}{\log x}(2\log(\log x)+1)$

Answer: B::D

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5. If $y = e^{-x}\cos x$ and $y_n + k_n y = 0$ where $yn = \frac{d^n y}{dx^n}$ and k_n are constant $n \in N$ then

A. $k_4 = 4$

B. $k_8 = -16$

C. $k_{12} = 20$

D. $k_{16} = -24$

Answer: A::B

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6. If $y = y(x)$ and it follows the relation $e^{xy} + y\cos x = 2$, then find (i) $y'(0)$ and (ii) $y(0)$.

A. $y'(0) = -1$

B. $y''(0) = 2$

C. $y'(0) = 3/2$

D. $y''(0) = -2$

Answer: A::B

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7. A twice differentiable function f(x)is defined for all real numbers and satisfies the following conditions $f(0) = 2; f'(0) - -5$ and $f(0) = 3$. The function $g(x)$ is defined by $g(x) = e^{ax} + f(x) \forall x \in R$, where 'a' is any constant If $g'(0) + g(0) = 0$. Find the value(s) of 'a'

A. 1

B. -1

C. 2

D. -2

Answer: A::D

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