



## MATHS

### BOOKS - CENGAGE PUBLICATION

### PRINCIPLE OF MATHEMATICAL INDUCTION

#### Solved Examples

1. Prove the following by using the principle of mathematical induction

for all  $n \in N: 1^3 + 2^3 + 3^3 + \dots + n^3 = \left( \frac{n(n+1)}{2} \right)^2$

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2. Using the principle of mathematical induction prove that

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots$$
$$+ \frac{1}{1+2+3+\dots+n} = \frac{2n}{n+1} \text{ for all } n \in N$$



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3. Using the principle of mathematical induction, prove that

$$1.3 + 2.3^2 + 3.3^2 + \dots + n.3^n = \frac{(2n - 1)(3)^{n+1} + 3}{4} \text{ for all } n \in \mathbb{N}.$$



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4. Using principle of mathematical induction, prove that for all

$n \in \mathbb{N}$ ,  $n(n + 1)(n + 5)$  is a multiple of 3.



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5. Prove the following by the principle of mathematical induction:

$3^{2n+2} - 8n - 9$  is divisible 8 for all  $n \in \mathbb{N}$ .



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6. Using the principle of mathematical induction prove that  $41^n - 14^n$  is a multiple of 27.

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7. Prove the following by using the principle of mathematical induction for all  $n \in \mathbb{N}$ :  $(2n + 7) < (n + 3)^2$ .

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8. Using the principle of mathematical induction, prove that for  $n \in \mathbb{N}$ ,  $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{3n+1} > 1$ .

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9. A sequence  $a_1, a_2, a_3, \dots$  is defined by letting  $a_1 = 3$  and  $a_k = 7a_{k-1}$ , for all natural numbers  $k \geq 2$ . Show that  $a_n = 3 \cdot 7^{n-1}$  for natural

numbers.



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10.

Let

$$A_n = a_1 + a_2 + \dots + a_n, B_n = b_1 + b_2 + b_3 + \dots + b_n, D_n = c_1 + c_2$$

and  $c_n = a_1 b_n + a_2 b_{n-1} + \dots + a_n b_1$   $\forall n \in \mathbb{N}$ . Using mathematical

induction, prove that

(a)

$$D_n = a_1 B_n + a_2 B_{n-1} + \dots + a_n B_1 = b_1 A_n + b_2 A_{n-1} + \dots + b_n A_1 \quad \forall n$$

(b)  $D_1 + D_2 + \dots + D_n = A_1 B_n + A_2 B_{n-1} + \dots + A_n B_1 \quad \forall n \in \mathbb{N}$



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11. Let  $U_1 = 1, U_2 = 1$  and  $U_{n+2} = U_{n+1} + U_n$  for  $n \geq 1$ . Use

mathematical induction to show that:

$$U_n = \frac{1}{\sqrt{5}} \left\{ \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right\} \quad \text{for all } n \geq 1.$$



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12. Using mathematical induction to show that  $p^{n+1} + (p + 1)^{2n-1}$  is divisible by  $p^2 + p + 1$  for all  $n \in \mathbb{N}$

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13. Let  $0 < A_i < \pi$  for  $i = 1, 2, \dots, n$ . Use mathematical induction to prove that

$$\sin A_1 + \sin A_2 + \dots + \sin A_n \leq n \sin \left( \frac{A_1 + A_2 + \dots + A_n}{n} \right)$$

where  $n \geq 1$  is a natural number.

[You may use the fact that  $p \sin x + (1 - p) \sin y \leq \sin[px + (1 - p)y]$ ,

where  $0 \leq p \leq 1$  and  $0 \leq x, y \leq \pi$ ]

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14. Prove the following by the principle of mathematical induction:

$$1 \cdot 3 + 3 \cdot 5 + \dots + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$$

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15. Using the principle of mathematical induction prove that

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

for all  $n \in \mathbb{N}$

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16. Using the principle of mathematical induction, prove that  $(2^{3n} - 1)$  is divisible by 7 for all  $n \in \mathbb{N}$ .

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17. Using the principle of mathematical induction. Prove that  $(x^n - y^n)$  is divisible by  $(x-y)$  for all  $n \in \mathbb{N}$ .

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18. Using principle of mathematical induction prove that  $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}$  for all natural numbers  $n \geq 2$ .

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19. Show that  $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$  is a natural number, for all  $n \in \mathbb{N}$

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20. Using principle of mathematical induction, prove that  $7^{4^n} - 1$  is divisible by  $2^{2n+3}$  for any natural number  $n$ .

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21. Prove by mathematical induction that  $n^5$  and  $n$  have the same unit digit for any natural number  $n$ .

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22. A sequence  $b_0, b_1, b_2, \dots$  is defined by letting  $b_0 = 5$  and  $b_k = 4 + b_{k-1}$ , for all natural number  $k$ . Show that  $b_n = 5 + 4n$ , for all natural number  $n$  using mathematical induction.



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