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## MATHS

## BOOKS - CENGAGE PUBLICATION

## PROPERTIES AND SOLUTIONS OF TRIANGLE

Example

1. In triangle $A B C, D$ is on $A C$ such that $A D=B C^{\prime}$ and $B D=D C, \angle D B C=2 x, a n d \angle B A D=3 x$, all angles are in degrees, then find the value of ${ }^{\prime} x$
2. In a circle of radius $r$, chords of length aandbcm subtend angles $\theta a n d 3 \theta$, respectively, at the center. Show that $r=a \sqrt{\frac{a}{3 a-b}} c m$

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3. Perpendiculars are drawn from the anglesA, $B, C, o f$ an acute angles $\Delta$ on the opposite sodes and products to meet the circumscribing circle. If these produced parts be $\propto, \beta, \gamma$ respectively, show that $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=2(\tan A+\tan B+\tan C$
4. $D, E, F$ are three points on the sides $B C, C A, A B$, respectively, such that $\angle A D B=\angle B E C=\angle C F A=\theta$. $A^{\prime}, B^{\prime} C^{\prime}$ are the points of intersections of the lines $A D, B E, C F$ inside the triangle. Show that are of $A^{\prime} B^{\prime} C^{\prime}=4 \cos ^{2} \theta$, where is the area of $A B C$.

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5. In $A B C$, as semicircle is inscribed, which lies on the side • If $x$ is the lengthof the angle bisector through angle $C$, then prove that the radius of the semicircle is $x \sin \left(\frac{C}{2}\right)$.

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6. Given the base of a triangle, the opposite angle A, and the product $k^{2}$ of the other two sides, show that it is not possible for $a$ to be less than $2 k \frac{\sin A}{2}$

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7. In a triangle of base a, the ratio of the other sides is $r(<1)$.

Show that the attitude of the triangle is less than or equal to $\frac{a r}{1-r^{2} .}$

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8. Let $A B C$ be a triangle with incentre I. If $P$ and $Q$ are the feet of the perpendiculars from A to BI and Cl , respectively, then prove that $\frac{A P}{B I}+\frac{A Q}{C l}=\cot \frac{A}{2}$
9. Let O be the circumcentre and H be the orthocentre of an acute angled triangle ABC . If $A>B>C$, then show that $\operatorname{Ar}(\Delta B O H)=\operatorname{Ar}(\Delta A O H)+\operatorname{Ar}(\Delta C O H)$

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10. If $I$ is the incenter of $\triangle A B C$ and $R_{1}, R_{2}$, and $R_{3}$ are, respectively, the radii of the circumcircle of the triangle IBC, ICA, and IAB, then prove that $R_{1} R_{2} R_{3}=2 r R^{2}$

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11. Show that the line joining the incenter to the circumcenter of triangle $A B C$ is inclined to the side $B C$ at an angle
$\tan ^{-1}\left(\frac{\cos B+\cos C-1}{\sin C-\sin B}\right)$

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12. In a $\triangle A B C$, the median to the side BC is of length 1
$\sqrt{11-6 \sqrt{3}}$ and it divides t

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13. Three circle touch one another externally. The tangents at their points of contact meet at a point whose distance from the point of contant is 4 . If the ratio of the product of the radii to the sum of the radii of the circle is $\lambda$, then $\frac{\lambda}{2}$ is
14. Let $A B C$ be a triangle with incentre I and inradius $r$. Let $D, E, F$ be the feet of the perpendiculars from I to the sides $B C, C A$ and AB , respectively, If $r_{2}$ and $r_{3}$ are the radii of circles inscribed in the quadrilaterls AFIE, BDIF and CEID respectively, then prove that $\frac{r_{1}}{r-r_{1}}+\frac{r_{2}}{r-r_{2}}+\frac{r_{3}}{r-r_{3}}=\frac{r_{1} r_{2} r_{3}}{\left(r-r_{1}\right)\left(r-r_{2}\right)\left(r-r_{3}\right)}$

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15. 

$A B C D, A B=a, B C=b, C D=c, D A=d$
This
quadrilateral is such that a circle can be inscribed in it and a circle can also be circumscribed about it. Prove that $\frac{\tan ^{2} A}{2}=\frac{b c}{a d}$.

1. If an a triangle $A B C, a=3 b$ and $B-A=90^{\circ}$, then find the value of $\tan A$

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2. In a triangle $A B C$ if $B C=(3)^{\frac{1}{2}}$ and $A C=2$, then what is the maximum possible value of angle $A$ ?

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3. The perimeter of a triangle $A B C$ is saix times the arithmetic mean of the sines of its angles. If the side ais 1 then find angle A.
4. If $A=75^{0}, b=45^{0}$, then prove that $b+c \sqrt{2}=2 a$

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5. If the base angles of triangle are $\frac{22}{12} \operatorname{and} 112 \frac{1}{2^{0}}$, then prove that the altitude of the triangle is equal to $\frac{1}{2}$ of its base.

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6. If $a^{2}, b^{2}, c^{2}$ are in A.P., then prove that $\tan A, \tan B, \tan C$ are in H.P.

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$\frac{a^{2} \sin (B-C)}{\sin B+\sin C}+\frac{b^{2} \sin (C-A)}{\sin C+\sin A}+\frac{c^{2} \sin (A-B)}{\sin A+\sin B}=0$

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8. in $\triangle A B C$ if $\frac{a^{2}+b^{2}}{a^{2}-b^{2}}=\frac{\sin (A+B)}{\sin (A-B)}$ then prove that it is either a right angled or an isosceles triangle.

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9. $A B C D$ is a trapezium such that $A B, D C$.are parallel and $B C$ is perpendicular to them. If $\angle A D B=\theta, B C=p$ and $C D=q$, show that $\mathrm{AB}=\frac{\left(p^{2}+q^{2}\right) \sin \theta}{p \cos \theta+q \sin \theta}$
10. In a DeltaA B C, $\backslash \angle \mathrm{C}=60 \backslash \& \angle A=75$. If $D$ is a point on $A C$ such that area of the"Delta" $B A D$ is $\sqrt{3}$ times the area of the DeltaB C D, then the $\angle \mathrm{ABD}=60^{\circ}$ (b) $30^{\circ}$ (c) $90^{\circ}$ none of these

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11. In a scalene triangle $A B C, D$ is a point on the side $A B$ such that $C D^{2}=A D \cdot D B, \sin \sin A \cdot \sin B=\frac{\sin ^{2} C}{2}$ then prove that CD is internal bisector of $\angle C$.

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12. In a triangle $A B C, \angle A=60^{\circ} a n d b: c=(\sqrt{3}+1): 2$, then
find the value of $(\angle B-\angle C)$.
13. If the median AD of triangle $A B C$ makes an angle $\frac{\pi}{4}$ with the side BC , then find the value of $|\cot B-\cot C|$.

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14. The base of a triangle is divided into three equal parts. If $t_{1}, t_{2}, t_{3}$ are the tangents of the angles subtended by these parts at the opposite vertex, prove that $\left(\frac{1}{t_{1}}+\frac{1}{t_{2}}\right)\left(\frac{1}{t_{2}}+\frac{1}{t_{3}}\right)=4\left(1+\frac{1}{t 22}\right)$.

## (D) Watch Video Solution

$$
\begin{aligned}
& \text { 15. } \begin{array}{c}
\text { In } \\
A B C, \\
(a-b)^{2} \frac{\cos ^{2} C}{2}+(a+b)^{2} \frac{\sin ^{2} C}{2}=c^{2} \text {. }
\end{array} \text { prove that }
\end{aligned}
$$

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16. In $A B C,=$ if $(a+b+c)(a-b+c)=3 a c$, then find $\angle B$.

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17. If $a=\sqrt{3}, b=\frac{1}{2}(\sqrt{6}+\sqrt{2})$. and $c=2$, then find $\angle A$

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18. The sides of a triangle are $x^{2}+x+1,2 x+1, a n d x^{2}-1$. Prove that the greatest angle is $120^{\circ}$.

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19. If the angles $A, B, C$ of a triangle are in A.P. and sides $a, b, c$, are in G.P., then prove that $a^{2}, b^{2}, c^{2}$ are in A.P.

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20. Let $a, b$ and $c$ be the three sides of a triangle, then prove that the equation $b^{2} x^{2}+\left(b^{2}+c^{2}-a^{2}\right) x+c^{2}=0$ has imaginary roots.

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21. Let $a \leq b \leq c$ be the lengths of the sides of a triangle. If ${ }^{\bullet} a^{\wedge} 2+b^{\wedge} 2$
22. In $a$ triangle $A B C$, if the sides $a, b, c$, are roots of $x^{3}-11 x^{2}+38 x-40=0, \quad$ then find the value of $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$

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23. If in a triangle $A B C, \angle C=60^{\circ}$, then prove that $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$.

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24. In a triangle, if the angles $A, B, a n d C$ are in A.P. show that
$2 \frac{\cos 1}{2}(A-C)=\frac{a+c}{\sqrt{a^{2}-a c+c^{2}}}$

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25. If $a=9, b=4 a n d c=8$ then find the distance between the middle point of BC and the foot of the perpendicular form $A$.

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26. Three parallel chords of a circle have lengths $2,3,4$ units and subtend angles $\alpha, \beta, \alpha+\beta$ at the centre, respectively `alpha

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27. In a cyclic quadrilateral $\mathrm{PQRS}, \mathrm{PQ}=2$ units, $\mathrm{QR}=5$ units, $\mathrm{RS}=3$ units and $\angle P Q R=60^{\circ}$, then what is the measure of SP ?

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28. Prove that $a(b \cos C-c \cos B)=b^{2}-c^{2}$

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29. If in a triangle $a \cos ^{2}\left(\frac{C}{2}\right)+c \cos ^{2}\left(\frac{A}{2}\right)=\frac{3 b}{2}$, then find the relation between the sides of the triangle.

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$(b+c) \cos A+(c+a) \cos B+(a+b) \cos C=2 s$.

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31. If $\cos \left(\frac{A}{2}\right)=\sqrt{\frac{b+c}{2 c}}$, then prove that $a^{2}+b^{2}=c^{2}$.
32. If the cotangents of half the angles of a triangle are in A.P., then prove that the sides are in A.P.

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33. If $A, B$ and $C$ are interior angles of a triangle $A B C$, then show that $\sin \left(\frac{B+C}{2}\right)=\cos \left(\frac{A}{2}\right)$

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34. 

In
$A B C,\left(\cot \left(\frac{A}{2}\right)+\cot \left(\frac{B}{2}\right)\right)\left(a \sin ^{2}\left(\frac{B}{2}\right)+b \sin ^{2}\left(\frac{A}{2}\right)\right)=$
(a) $\cot C$ (b) $\cot C$ (c) $\cot \left(\frac{C}{2}\right)$ (d) $\cot \left(\frac{C}{2}\right)$

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35. Find the value of $\tan \mathrm{A}$, if area of $\triangle A B C$ is $a^{2}-(b-c)^{2}$.

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36. Prove that $a^{2} \sin 2 B+b^{2} \sin 2 A=4 \Delta$

## D Watch Video Solution

37. For any triangle $A B C$,
$\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4 b^{2} c^{2}}$ is equal to

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38. If the sides of a triangle are $17,25 a n d 28$, then find the greatest length of the altitude.
39. In equilateral triangle $A B C$ with interior point $D$, if the perpendicular distances from $D$ to the sides of 4,5 , and 6 , respectively, are given, then find the area of $\triangle A B C$.

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40. If area of a triangle is 2 sq. units, then find the value of the product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle.

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41. A triangle has sides 6,7 , and 8 . The line through its incenter parallel to the shortest side is drawn to meet the other two sides at $P$ and $Q$. Then find the length of the segment $P Q$.

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42. Each side of triangle $A B C$ is divided into three equal parts.

Find the ratio of the area of hexagon $P Q R S T U$ to the area of the triangle $A B C$.

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43. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is $60^{\circ}$. If the area of the quadrilateral is $4 \sqrt{3}$, find the remaining two sides.
44. In triangle $A B C, a: b: c=4: 5: 6$. The ratio of the radius of the circumcircle to that of the incircle is $\qquad$ .

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45. Given a triangle $A B C$ with sides $\mathrm{a}=7, \mathrm{~b}=8$ and $\mathrm{c}=5$. Find the value of expression
$(\sin A+\sin B+\sin C)\left(\frac{\cot A}{2}+\frac{\cot B}{2}+\frac{\cot C}{2}\right)$

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46. If $b=3, c=4, \operatorname{and} B=\frac{\pi}{3}$, then find the number of triangles that can be constructed.
47. If $A=30^{\circ}, a=7$, and $b=8$ in $A B C$, then find the number of triangles that can be constructed.

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48. 

If
in
triangle
ABC,
$\left(a=(1+\sqrt{3}) c m, b=2 c m\right.$, and $\angle C=60^{\circ}$, then find the other two angles and the third side.

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49. In $A B C$, sidesb, $c$ and angle $B$ are given such that $a$ has two values $a_{1}$ and $a_{2}$. Then prove that
$\left|a_{1}-a_{2}\right|=2 \sqrt{b^{2}-c^{2} \sin ^{2} B}$
50. In $A B C, a, \operatorname{cand} A$ are given and $b_{1}, b_{2}$ are two values of the third side $b$ such that $b_{2}=2 b_{1}$. Then prove that $\sin A=\sqrt{\frac{9 a^{2}-c^{2}}{8 c^{2}}}$

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51. $O$ is the circumcenter of $A B C a n d R_{1}, R_{2}, R_{3}$ are respectively, the radii of the circumcircles of the triangle $O B C, O C A$ and OAB. Prove that $\frac{a}{R_{1}}+\frac{b}{R_{2}}+\frac{c}{R_{3}}=\frac{a b c}{R_{3}}$

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52. In $A B C, C=60^{\circ}$ and $B=45^{\circ}$. Line joining vertex A of triangle and its circumcenter $(O)$ meets the side $B C$ in $D$ Find the ratio $B D: D C$ and $A O: O D$

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53. The diameters of the circumcirle of triangle ABC drawn from
$A, B$ and $C$ meet $B C, C A$ and $A B$, respectively, in $L, M$ and $N$. Prove
that $\frac{1}{A L}+\frac{1}{B M}+\frac{1}{C N}=\frac{2}{R}$

## (D) Watch Video Solution

54. Find the lengths of chords of the circumcircle of triangle $A B C$, made by its altitudes
55. Let $A B C$ be a triangle with $\angle B=90^{\circ}$. Let AD be the bisector of $\angle A$ with D on BC . Suppose $\mathrm{AC}=6 \mathrm{~cm}$ and the area of the triangle $A D C$ is $10 \mathrm{~cm}^{2}$. Find the length of $B D$.

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56. If the distances of the vertices of a triangle $=A B C$ from the points of contacts of the incercle with sides are $\alpha, \beta a n d \gamma$ then prove that $r^{2}=\frac{\alpha \beta \gamma}{\alpha+\beta+\gamma}$

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57. If $x, y a n d z$ are the distances of incenter from the vertices of the triangle $A B C$, respectively, then prove that $\frac{a b c}{x y z}=\cot \left(\frac{A}{2}\right) \cot \left(\frac{B}{2}\right) \cot \left(\frac{C}{2}\right)$

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58. prove that $\cos A+\cos B+\cos C=1+r / R$.

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59. Prove that $\frac{a \cos A+b \cos B+c \cos C}{a+b+c}=\frac{r}{R}$.

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60. Incircle of $A B C$ touches the sides $\mathrm{BC}, \mathrm{CA}$ and AB at $\mathrm{D}, \mathrm{E}$ and

F, respectively. Let $r_{1}$ be the radius of incircle of $B D F$. Then
prove that $r_{1}=\frac{1}{2} \frac{(s-b) \sin B}{\left(1+\sin \left(\frac{B}{2}\right)\right)}$
61. In an acute angled triangle $A B C$, a semicircle with radius $r_{a}$ is constructed with its base on BC and tangent to the other two sides. $r_{b} a n d r_{c}$ are defined similarly. If $r$ is the radius of the incircle of triangle ABC then prove that $\frac{2}{r}=\frac{1}{r_{a}}+\frac{1}{r_{b}}+\frac{1}{r_{c}}$.

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62. Let the incircle with center I of $A B C$ touch sides $\mathrm{BC}, \mathrm{CA}$ and $A B$ at $D, E, F$, respectively. Let a circle is drawn touching ID, IF and incircle of $A B C$ having radius $r_{2}$. similarly $r_{1} a n d r_{3}$ are defined.

Prove that $\frac{r_{1}}{r-r_{1}} \frac{\dot{r_{2}}}{r-r_{2}} \frac{\dot{r_{3}}}{r-r_{3}}=\frac{a+b+c}{8 R}$

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63. In $A B C$, the bisector of the angle A meets the side BC at D
andthe circumscribed circle at E. Prove that $D E=\frac{a^{2} \frac{\sec A}{2}}{2(b+c)}$

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64. Let I be the incetre of $\triangle \mathrm{ABC}$ having inradius r . $\mathrm{Al}, \mathrm{BI}$ and Ci intersect incircle at D, E and F respectively. Prove that area of
$\Delta D E F$ is $\frac{r^{2}}{2}\left(\cos . \frac{A}{2}+\cos \cdot \frac{B}{2}+\cos \cdot \frac{C}{2}\right)$

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65. In $A B C$, the three bisectors of the angle $\mathrm{A}, \mathrm{B}$ and C are extended to intersect the circumcircle at D,E and F respectively.

## Prove

that
$A D \frac{\cos A}{2}+B E \frac{\cos B}{2}+C F \frac{\cos C}{2}=2 R(\sin A+\sin B+\sin C)$

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66. Given a right triangle with $\angle A=90^{\circ}$. Let M be the midpoint of BC . If the radii of the triangle $A B M$ and $A C M$ are $r_{1}$ and $r_{2}$ then find the range of $\frac{r_{1}}{r_{2}}$.

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67. Prove that the distance between the circumcenter and the incenter of triangle ABC is $\sqrt{R^{2}-2 R r}$

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68. Prove that $a \cos A+b \cos B+\mathrm{os} C \leq s$.
69. about to only mathematics

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70. If in $A B C$, the distances of the vertices from the orthocentre are $\mathrm{x}, \mathrm{y}$, and z , then prove that $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{a b c}{x y z}$

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71. $A B C$ is an acute angled triangle with circumcenter $O$ and orthocentre H . If $\mathrm{AO}=\mathrm{AH}$, then find the angle A .

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72. In a acute angled triangle $A B C$, proint $D, E$ and $F$ are the feet of the perpendiculars from $A, B$ and $C$ onto $B C, A C$ and $A B$, respectively. H is orthocentre. If $\sin A=\frac{3}{5} a n d B C=39$, then find the length of $A H$

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73. Prove that he distance between the circum-centre and the ortho-centre of a triangle ABC is $R \sqrt{1-8 \cos A \cos B \cos C}$.

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74. Let $A B C$ be an acute angled triangle whose orthocentre is at
H. If altitude from $A$ is produced to meet the circumcircle of triangle ABC at $D$, then prove $H D=4 R \cos B \cos C$
75. In $A B C$, let $L, M, N$ be the feet of the altitudes. The prove that

$$
\sin (\angle M L N)+\sin (\angle L M N)+\sin (\angle M N L)=4 \sin A \sin B \sin C
$$

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76. The lengths of the medians through acute angles of a rightangled triangle are 3 and 4 . Find the area of the triangle.

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77. Two medians drawn from the acute angles of a right angled triangle intersect at an angle $\frac{\pi}{6}$. If the length of the hypotenuse
of the triangle is 3 units, then the area of the triangle (in sq. units) is $\sqrt{3}$ (b) 3 (c) $\sqrt{2}$ (d) 9

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78. Prove that $r_{1}+r_{2}+r_{3}-r=4 R$

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79. If $r_{1}=r_{2}+r_{3}+r$, prove that the triangle is right angled.

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80. Prove that $\frac{r_{1+r_{2}}}{1}=2 R$
81. Prove that $(r+r 1) \tan ((B-C) / 2)+(r+r 2) \tan ((C-A) / 2)+(r+r 3) \tan ((A-$
B) $/ 2$ ) $=0$

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82. If the distance between incenter and one of the excenter of an equilateral triangle is 4 units, then find the inradius of the triangle.

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83. If $I_{1}, I_{2}, I_{3}$ are the centers of escribed circles of triangle
$A B C$, show that area of triangle $I_{1} I_{2} I_{3}=\frac{a b c}{2 r}$.
84. Prove that the sum of the radii of the circles, which are, respectively, inscribed and circumscribed about a polygon of $n$ sides, whose side length is $a$, is $\frac{a}{2} \cot \frac{\pi}{2 n}$

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85. If the area of the circle is $A_{1}$ and the area of the regular pentagon inscribed in the circle is $A_{2}$, then find the ratio $\frac{A_{1}}{A_{2}}$.

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86. Prove that the area of a regular polygon hawing $2 n$ sides, inscribed in a circle, is the geometric mean of the areas of the inscribed and circumscribed polygons of $n$ sides.

Concept Application Exercise 51

1. Find the value of $\frac{a^{2}+b^{2}+c^{2}}{R^{2}}$ in any right-angled triangle.

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2. Let the angles $A, B a n d C$ of triangle $A B C$ be in $A P$. and let $b: c$ be $\sqrt{3}: \sqrt{2}$. Find angle $A$.

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3. In a triangle ABC , if $(\sqrt{3}-1) a=2 b, A=3 B$, then $\angle C$ is

## D Watch Video Solution

4. If $\cos \mathrm{A} / \mathrm{a}=\cos \mathrm{B} / \mathrm{b}=\cos \mathrm{C} /$ cand the side $\mathrm{a}=2$, then area of triangle is

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5. In triangle ABC , if $\cos ^{2} A+\cos ^{2} B-\cos ^{2} C=1$, then identify the type of the triangle

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6. Prove that $b^{2} \cos 2 A-a^{2} \cos 2 B=b^{2}-a^{2}$

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7. In any triangle $A B C$, prove that following : $\frac{c}{a+b}=\frac{1-\tan \left(\frac{A}{2}\right) \tan \left(\frac{B}{2}\right)}{1+\tan \left(\frac{A}{2}\right) \tan \left(\frac{B}{2}\right)}$

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8. In any $\quad \triangle A B C, \quad$ prove
$\left(b^{2}-c^{2}\right) \cot A+\left(c^{2}-a^{2}\right) \cot B+\left(c^{2}-b^{2}\right) \cot C=0$

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9. In a triangle $A B C$, prove that $\frac{b+c}{a} \leq \cos e c . \frac{A}{2}$

- Watch Video Solution

10. In any triangle $A B C$, prove that:
$\frac{1+\cos (A-B) \cos C}{1+\cos (A-C) \cos B}=\frac{a^{2}+b^{2}}{a^{2}+c^{2}}$

## - Watch Video Solution

11. In $a$ triangle $A B C$, if $a, b, c$ are in A.P. and $\frac{b}{c} \sin 2 C+\frac{c}{b} \sin 2 B+\frac{b}{a} \sin 2 A+\frac{a}{b} \sin 2 B=2$, then find the value of $\sin B$

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12. 

Prove
that
$a \cos A+b \cos B+c \cos C=4 R \sin A \sin B \sin C$.

Concept Application Exercise 52

1. The lengths of sides of $\triangle A B C$ are a units, b units and $\sqrt{a^{2}+a b+b^{2}}$ units show that greatest angle of the triangle is $120^{\circ}$

## D Watch Video Solution

2. If the segments joining the points $A(a, b)$ and $B(c, d)$ subtends an angle $\theta$ at the origin, prove that : $\cos \theta=\frac{a c+b d}{\sqrt{\left(a^{2}+b^{2}\right)\left(c^{2}+d^{2}\right)}}$

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3. The sides of a triangle are $3 x+4 y, 4 x+3 y$ and $5 x+5 y$, where $x, y>0$. The triangle is

## (-) Watch Video Solution

4. 

In
$\triangle A B C$,
angle
A is
$120^{\circ}, B C+C A=20$, and $A B+B C=21$ Find the length of the side $B C$

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5. In $\triangle A B C, A B=1, B C=1$, and $A C=1 / \sqrt{2}$. In $\Delta M N P, M N=1, N P=1$, and $\angle M N P=2 \angle A B C . \quad$ Find the side MP
6. If in a triangle $A B C, \frac{b c}{2 \cos A}=b^{2}+c^{2}-2 b c \cos A$ then prove that the triangle must be isosceles.

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7. In $\triangle A B C(\mathrm{~b}+\mathrm{c}) / 11=(\mathrm{c}+\mathrm{a}) / 12=(\mathrm{a}+\mathrm{b}) / 13$ then prove that $(\cos \mathrm{A}) / 7=(\cos \mathrm{B}) / 19=(\cos \mathrm{C}) / 25$

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8. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one determine the sides of the triangle

Concept Application Exercise 53

1. In $\triangle A B C$, prove that $c \cos (A-\alpha)+a \cos (C+\alpha)=b \cos \alpha$

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2. Prove that $\frac{\cos C+\cos A}{c+a}+\frac{\cos B}{b}=\frac{1}{b}$

## - Watch Video Solution

3. 

Prove
that
$a\left(b^{2}+c^{2}\right) \cos A+b\left(c^{2}+a^{2}\right) \cos B+c\left(a^{2}+b^{2}\right) \cos C=3 a b c$

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Concept Application Exercise 54

1. In a triangle $A B C$ if $b+c=3 a$ then find the value of $\cot \left(\frac{B}{2}\right) \cot \left(\frac{C}{2}\right)$

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2. Prove that $b c \cos ^{2} \cdot \frac{A}{2}+c a \cos ^{2} \cdot \frac{B}{2}+a b \cos ^{2} \cdot \frac{C}{2}=s^{2}$

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3. If in $\triangle A B C, \tan \cdot \frac{A}{2}=\frac{5}{6}$ and $\tan . \frac{C}{2}=\frac{2}{5}$, then prove that $a, b$, and $c$ are in A.P.
4. Prove that $(b+c-a)\left(\cot . \frac{B}{2}+\cot \cdot \frac{C}{2}\right)=2 a \cot . \frac{A}{2}$

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5. If $\sin ^{2}\left(\frac{A}{2}\right), \sin ^{2}\left(\frac{B}{2}\right)$, and $\sin ^{2}\left(\frac{C}{2}\right)$ are in $H . P$. , then prove that the sides of triangle are in $H . P$.

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Concept Application Exercise 55

1. If $c^{2}=a^{2}+b^{2}$, then prove that

$$
4 s(s-a)(s-b)(s-c)=a^{2} b^{2}
$$

2. If the sides of a triangle are in the ratio $3: 7: 8$, then find $R: r$

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3. In triangle ABC , if $\mathrm{a}=2$ and $\mathrm{bc}=9$, then prove that $R=9 / 2 \Delta$

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4. In $\triangle A B C$, if lengths of medians BE and CF are 12 and 9 respectively, find the maximum value of $\Delta$

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5. Let the lengths of the altitudes drawn from the vertices of
$\Delta A B C$ to the opposite sides are 2,2 and 3 . If the area of
$\Delta A B C$ is $\Delta$, then find the area of triangle

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6. A triangle with equal sides has perimeter 9 cm . Then find the area of the triangle

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7. The sides of a triangle are in A.P. and its area is $\frac{3}{5} t h$ of the an equilateral triangle of the same perimeter, prove that its sides are in the ratio 3:5:7.

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Concept Application Exercise 56

1. In which of the following cases, there exists a triangle $A B C$ ?
(a) $b \sin A=a, A<\pi / 2$
(b) $b \sin A>a, A>\pi / 2$

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2. If in $\triangle A B C, b=3 c m, c=4 c m$ and the length of the perpendicular from $A$ to the side $B C$ is 2 cm , then how many such triangle are possible ?

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3. In a triangle $A B C, \frac{a}{b}=\frac{2}{3}$ and $\sec ^{2} A=\frac{8}{5}$. Find the number of triangle satisfying these conditions
4. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P, then the length of the third side can be (a) $5-\sqrt{6}$ (b) $3 \sqrt{3}$ (c) 5 (d) $5+\sqrt{6}$

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5. If $\mathrm{a}, \mathrm{b}$ and A are given in a triangle and $c_{1}, c_{2}$ are possible values of the third side, then prove that $c_{1}^{2}+c_{2}^{2}-2 c_{1} c_{2} \cos 2 A=4 a^{2} \cos ^{2} A$

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6. In $\triangle A B C, a, b$ and $A$ are given and $c_{1}, c_{2}$ are two values of the third side c. Prove that the sum of the area of two triangles with sides $\mathrm{a}, \mathrm{b}, c_{1}$ and $a, b c_{2}$ is $\frac{1}{2} b^{2} \sin 2 A$

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Concept Application Exercise 57

1. Let $f, g$ and $h$ be the lengths of the perpendiculars from the circumcenter of $\triangle A B C$ on the sides $\mathrm{a}, \mathrm{b}$, and c , respectively.

Prove that $\frac{a}{f}+\frac{b}{g}+\frac{c}{h}=\frac{1}{4} \frac{a b c}{f g h}$

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2. If $\mathrm{AD}, \mathrm{BE}$ and CF are the altitudes of $\triangle A B C$ whose vertex A is $(-4,5)$. The coordinates of points $E$ and $F$ are (4,1) and ( $-1,-4$ ), respectively. Equation of $B C$ is
3. If the sides of triangle are in the ratio $3: 5: 7$, then prove that the minimum distance of the circumcentre from the side of triangle is half the circmradius

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4. If circumradius of triangle $A B C$ is 4 cm , then prove that sum of perpendicular distances from circumcentre to the sides of triangle cannot exceed 6 cm

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Concept Application Exercise 58

1. If the incircle of the triangle $A B C$ passes through its
circumcenter, then find the value of $4 \sin . \frac{A}{2} \sin . \frac{B}{2} \sin . \frac{C}{2}$

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2. In $\triangle A B C, a=10, A=\frac{2 \pi}{3}$, and circle through $B$ and C passes through the incenter. Find the radius of this circle

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3. Let ABC be a triangle with $\angle B A C=2 \pi / 3$ and $A B=x$ such that (AB) $(A C)=1$. If $x$ varies, then find the longest possible length of the angle bisector AD

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4. If the incircle of the $\triangle A B C$ touches its sides at $L, M$ and $N$ and if $x, y, z$ be thecircumradii of the triangles $M I N, N I L$ and
$L I M$ respectively, where $I$ is the incentre, then the product $x y z$ is equal to:
(A) $R r^{2}$
(B) $r R^{2}$
(C) $\frac{1}{2} R r^{2}$
(D) $\frac{1}{2} r R^{2}$

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5. In a triangle $A B C, C D$ is the bisector of the angle C . If $\cos \left(\frac{C}{2}\right)$ has the value $\frac{1}{3}$ and $l(C D)=6$, then $\left(\frac{1}{a}+\frac{1}{b}\right)$ has the value equal to -

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6. In $\triangle A B C, \angle A=\frac{\pi}{3}$ and its incircle of unit radius. Find the radius of the circle touching the sides $\mathrm{AB}, \mathrm{AC}$ internally and the incircle of $\triangle A B C$ externally is x , then find the value of x .
7. In triangle $A B C$, prove that the maximum value of $\frac{\tan A}{2} \frac{\tan B}{2} \frac{\tan C}{2} i s \frac{R}{2 s}$

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## Concept Application Exercise 59

1. Line joining vertex $A$ of triangle $A B C$ and orthocenter (H) meets the side $B C$ in $D$. Then prove that
(a) $B D: D C=\tan C: \tan B$
(b) $A H: H D=(\tan B+\tan C): \tan A$

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2. In a triangle $\mathrm{ABC}, \angle A=30^{\circ}, B C=2+\sqrt{5}$, then find the distance of the vertex A from the orthocenter.

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3. If the perimeter of the triangle formed by feet of altitudes of the triangle $A B C$ is equal to four times the circumradius of $\triangle A B C$, then identify the type of $\triangle A B C$

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4. $A D, B E$ and $C F$ are the medians of triangle $A B C$ whose centroid is $G$. If the points $A, F, G$ and $E$ are concyclic, then prove that $2 a^{2}=b^{2}+c^{2}$
5. Consider an acute angled $\triangle A B C$. Let $\mathrm{AD}, \mathrm{BE}$ and CF be the altitudes drawn from the vertice to the opposite sides. Prove that: $\frac{E F}{a}+\frac{F D}{b}+\frac{D E}{c}=\frac{R+r}{R}$.

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Concept Application Exercise 510

1. In $\triangle A B C$, if $r_{1}<r_{2}<r_{3}$, then find the order of lengths of the sides

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2. The ex-radii $r_{1}, r_{2}, r_{3}$ or $\triangle A B C$ are in H.P. Show that its sides a,b,c are in A.P.
3. If in $\triangle A B C,(a-b)(s-c)=(b-c)(s-a)$, prove that $r_{1}, r_{2}, r_{3}$ are in A.P.

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4. Prove that $2 R \cos A=2 R+r-r_{1}$

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5. If the lengths of the perpendiculars from the vertices of a triangle $A B C$ on the opposite sides are $p_{1}, p_{2}, p_{3}$ then prove that $\frac{1}{p_{1}}+\frac{1}{p_{2}}+\frac{1}{p_{3}}=\frac{1}{r}=\frac{1}{r_{1}}+\frac{1}{r_{2}}+\frac{1}{r_{3}}$.

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6. Prove that $r_{1} r_{2}+r_{2} r_{3}+r_{3} r_{1}=\frac{1}{4}(a+b+c)^{2}$

## D Watch Video Solution

7. In any triangle $A B C$, find the least value of $\frac{r_{1} r_{2} r_{3}}{r^{3}}$

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8. Prove that $\frac{r_{1}-r}{a}+\frac{r_{2}-r}{b}=\frac{c}{r_{3}}$

## D Watch Video Solution

Concept Application Exercise 511

1. Regular pentagons are inscribed in two circles of radius 5and 2 units respectively. The ratio of their areas is

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2. Let $A$ be a point inside a regular polygon of 10 sides. Let $p_{1}, p_{2} \ldots, p_{10}$ be the distances of A from the sides of the polygon. If each side is of length 2 units, then find the value of $p_{1}+p_{2}+\ldots+p_{10}$

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3. If in $\triangle A B C$, Prove that, $a^{2} \sin (B-C)=\left(b^{2}-c^{2}\right) \sin A$
4. If $I_{n}$ is the area of $n-$ sided regular polygon inscribed in a circle of unit radius and $O_{n}$ be the area of the polygon circumscribing the given circle, prove that
$I_{n}=\frac{O_{n}}{2}\left(\sqrt{1+\left(\frac{2 I_{n}}{n}\right)^{2}}\right)$

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## Exercises

1. In $\triangle A B C, \frac{\sin A(a-b \cos C)}{\sin C(c-b \cos A)}=$
A. -2
B. -1
C. 0
D. 1

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2. If in a triangle $A B C, \frac{1+\cos A}{a}+\frac{1+\cos B}{b}+\frac{1+\cos C}{c}$
$=\left(k^{2}(1+\cos A)(1+\cos B) \frac{1+\cos C}{a b c}\right.$, then $k$ is equal to (a)
$\frac{1}{2 \sqrt{2} R}$ (b) $2 R$ (c) $\frac{1}{R}$ (d) none of these
A. $\frac{1}{2 \sqrt{2} R}$
B. $2 R$
C. $\frac{1}{R}$
D. none of these

## Answer: B

3. In triangle $A B C, 2 a c \sin \left(\frac{1}{2}(A-B+C)\right)$ is equal to
A. $a^{2}+b^{2}-c^{2}$
B. $c^{2}+a^{2}-b^{2}$
C. $b^{2}-c^{2}-a^{2}$
D. $c^{2}-a^{2}-b^{2}$

## Answer: B

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4. If the angles of a triangle are in the ratio $4: 1: 1$, then the ratio of the longest side to the perimeter is
A. $\sqrt{3}:(2+\sqrt{3})$
B. 1:6
C. $1: 2+\sqrt{3}$
D. $2: 3$

## Answer: A

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5. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle $A B C(R$ being the radius of the circumcircle)? (a) $a, \sin A, \sin B$ (b) $a, b, c(c) \mathrm{a}, \sin B, \mathrm{R}(d) \mathrm{a}$ , sinA , R'
A. $a, \sin A, \sin B$
B. $a, b, c$
C. $a, \sin B, R$
D. $a, \sin A, R$

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6. The sides of a triangle are in the ratio $1: \sqrt{3}: 2$. Then the angles are in the ratio
A. $1: 3: 5$
B. 2:3:4
C. $3: 2: 1$
D. 1:2:3

## Answer: D

7. In $A B C, a=5, b=12, c=90^{\circ} a n d D$ is a point on $A B$ so that $\angle B C D=45^{\circ}$. Then which of the following is not true?
$C D=\frac{60 \sqrt{2}}{17}$ (b) $B D=\frac{65}{17} A D=\frac{60 \sqrt{2}}{17}$ (d) none of these
A. $C D=\frac{60 \sqrt{2}}{17}$
B. $B D=\frac{65}{17}$
C. $A D=\frac{60 \sqrt{2}}{17}$
D. none of these

## Answer: C

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8. In a triangle $A B C,(a+b+c)(b+c-a)=k b c$ if
A. $k<0$
B. $k>0$
C. $0<k<4$
D. $k<4$

## Answer: C

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9. Let $D$ be the middle point of the side $B C$ of a triangle $A B C$. If the triangle $A D C$ is equilateral, then $a^{2}: b^{2}: c^{2}$ is equal to $1: 4: 3$ (b) $4: 1: 3$ (c) $4: 3: 1$ (d) $3: 4: 1$
A. 1:4:3
B. $4: 1: 3$
C. $4: 3: 1$
D. $3: 4: 1$

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10. In a triangle $A B C$, the altitude from $A$ is not less than $B C$ and the altitude from $B$ is not less than $A C$. (a)The triangle is right angled (b) isosceles obtuse angled (d) equilateral
A. right angled
B. isosceles
C. obtuse angled
D. equilateral

## Answer: A

11. In $A B C$, if $\frac{\sin A}{c \sin B}+\frac{\sin B}{c}+\frac{\sin C}{b}=\frac{c}{a b}+\frac{b}{a c}+\frac{a}{b c}$, then the value of angle $A$ is $120^{\circ}$ (b) $90^{\circ}$ (c) $60^{\circ}$ (d) $30^{\circ}$
A. $120^{\circ}$
B. $90^{\circ}$
C. $60^{\circ}$
D. $30^{\circ}$

## Answer: B

## D Watch Video Solution

12. If in $A B C$, side $a, b, c$ are in A.P. then $B>60^{\circ}$ (b) $B<60^{\circ}$

$$
B \leq 60^{\circ} \text { (d) } B=|A-C|
$$

A. $B>60^{\circ}$
B. $B<60^{\circ}$
C. $B \leq 60^{\circ}$
D. $B=|A-C|$

## Answer: C

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13. In a $\triangle A B C, \mathrm{AD}$ is the altitude from A . Given
$b>c, \angle C=23^{\circ}$ and $A D=\frac{a b c}{\left(b^{2}-c^{2}\right)}$, find $\angle B$.
A. $83^{\circ}$
B. $97^{\circ}$
C. $113^{\circ}$
D. $127^{\circ}$

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14. If the sides $a, b, c$ of a triangle $A B C$ form successive terms of G.P. with common ratio $r(>1)$ then which of the following is correct? (a) $A>\frac{\pi}{3}{ }^{\prime}(b) \mathrm{B} \geq \pi / 3^{\prime}(c) \mathrm{C}<\pi / 3^{\prime}(d) \mathrm{A}<\mathrm{B}<\pi / 3^{\prime}$
A. $A>\pi / 3$
B. $B \geq \pi / 3$
C. $C<\pi / 3$
D. $A<B<\pi / 3$

## Answer: D

$A B C, b^{2} \sin 2 C+c^{2} \sin 2 B=2 b c w h e r e b=20, c=21, \quad$ then inradius $=(\mathrm{a}) 4$ (b) 6 (c) 8 (d) 9
A. 4
B. 6
C. 8
D. 9

## Answer: B

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16. In a $A B C$, if $A B=x, B C=x+1, \angle C=\frac{\pi}{3}$, then the least integer value of $x$ is
A. 6
B. 7
C. 8
D. none of these

## Answer: B

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17. If one side of a triangle is double the other, and the angles on opposite sides differ by $60^{\circ}$, then the triangle is
A. equilateral
B. obtus angled
C. right angled
D. acute angled

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18. If the hypotenuse of a right-angled triangle is four times the length of the perpendicular drawn from the opposite vertex to it, then the difference of the two acute angles will be $60^{\circ}$ (b) $15^{0}$
(c) $75^{0}$ (d) $30^{0}$
A. $60^{\circ}$
B. $15^{\circ}$
C. $75^{\circ}$
D. $30^{\circ}$

## Answer: A

19. If $P$ is a point on the altitude $A D$ of the triangle $A B C$ such the $\angle C B P=\frac{B}{3}$, then AP is equal to $2 a \frac{\sin C}{3}$ (b) $2 b \frac{\sin C}{3}$ (c) $2 c \frac{\sin B}{3}$ (d) $2 c \frac{\sin C}{3}$
A. $2 a \sin$. $\frac{C}{3}$
B. $2 b \sin$. $\frac{C}{3}$
C. $2 c \sin$. $\frac{B}{3}$
D. $2 c \sin$. $\frac{C}{3}$

## Answer: C

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20. With usual notations, in triangle
$A B C, a \cos (B-C)+b \cos (C-A)+c \cos (A-B)$ is equal
to $a b c / R^{2}$ (b) $\frac{a b c}{4 R^{2}} \frac{4 a b c}{R^{2}}$ (d) $\frac{a b c}{2 R^{2}}$
A. $\frac{a b c}{R^{2}}$
B. $\frac{a b c}{4 R^{2}}$
C. $\frac{4 a b c}{R^{2}}$
D. $\frac{a b c}{2 R^{2}}$

## Answer: A

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21. If $8 R^{2}=a^{2}+b^{2}+c^{2}$ then show that the triangle is right angled
A. right angled
B. isosceles
C. equilateral
D. none of these

## Answer: A

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22. Let ABC be a triangle with $\angle A=45^{\circ}$. Let P be a point on side $B C$ with $P B=3$ and $P C=5$. If $O$ is circumcenter of triangle $A B C$, then length $O P$ is $\sqrt{18}$ (b) $\sqrt{17}$ (c) $\sqrt{19}$ (d) $\sqrt{15}$
A. $\sqrt{18}$
B. $\sqrt{17}$
C. $\sqrt{19}$
D. $\sqrt{15}$

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23. In any triangle $A B C, \frac{a^{2}+b^{2}+c^{2}}{R^{2}}$ has the maximum value of 3 (b) 6 (c) 9 (d) none of these
A. 3
B. 6
C. 9
D. none of these

## Answer: C

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24. In triangle $A B C, R(b+c)=a \sqrt{b c}$, where $R$ is the circumradius of the triangle. Then the triangle is isosceles but not right right but not isosceles right isosceles equilateral
A. isosceles but not right
B. right but not isosceles
C. right isosceles
D. equilateral

## Answer: C

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25. In $A B C$, if $b^{2}+c^{2}=2 a^{2}$, then value of $\frac{\cot A}{\cot B+\cot C}$ is
A. $\frac{1}{2}$
B. $\frac{3}{2}$
C. $\frac{5}{2}$
D. $\frac{5}{3}$

## Answer: A

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26. If $\sin \theta$ and $-\cos \theta$ are the roots of the equation $a x^{2}-b x-c=0$, where $a, b a n d c$ are the sides of a triangle
ABC , then $\cos B$ is equal to $1-\frac{c}{2 a}$
(b) $1-\frac{c}{a} 1+\frac{c}{c a}$
$1+\frac{c}{3 a}$
A. $1-\frac{c}{2 a}$
B. $1-\frac{c}{a}$
C. $1+\frac{c}{2 a}$
D. $1+\frac{c}{3 a}$

## Answer: C

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27. If $D$ is the mid-point of the side $B C$ of triangle $A B C$ and $A D$ is perpendicular to $A C$, then $3 b^{2}=a^{2}-c^{\wedge} 2$
$3 a^{2}=b^{2} 3 c^{2} b^{2}=a^{2}-c^{2}$ (d) $a^{2}+b^{2}=5 c^{2}$
A. $3 b^{2}=a^{2}-c^{2}$
B. $3 a^{2}=b^{2}-3 c^{2}$
C. $b^{2}=a^{2}-c^{2}$
D. $a^{2}+b^{2}=5 c^{2}$

Answer: A
28. In a triangle $A B C$, if $\cot A: \cot B: \cot C=30: 19: 6$ then the sides $a, b, c$ are a) in A.P. b) in G.P. c) in H.P. d) None of these
A. in A.P.
B. in G.P.
C. in H.P.
D. none of these

## Answer: A

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29. In $A B C, P$ is an interior point such that $\angle P A B=10^{\circ} \angle P B A=20^{\circ}, \angle P C A=30^{\circ}, \angle P A C=40^{\circ}$ then
$A B C$ is isosoceles (b) right angled equilateral (d) obtuse angled
A. isosceles
B. right angled
C. equilateral
D. obtuse angled

## Answer: A

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30. In $A B C$, if $A B=c$ is fixed, and $\cos A+\cos B+2 \cos C=2$ then the locus of vertex $C$ is ellipse (b) hyperbola (c) circle (d) parabola
A. ellipse
B. hyperbola
C. circle
D. parabola

## Answer: A

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31. If in $A B C, A=\frac{\pi}{7}, B=\frac{2 \pi}{7}, C=\frac{4 \pi}{7}$ then $a^{2}+b^{2}+c^{2}$ must be (a) $R^{2}$ (b) $3 R^{2}$ (c) $4 R^{2}$ (d) $7 R^{2}$
A. $R^{2}$
B. $3 R^{2}$
C. $4 R^{2}$
D. $7 R^{2}$
32. In $\triangle A B C, \cot \frac{A}{2}+\cot \frac{B}{2}+\cot \frac{C}{2}$ is equal to
A. $\frac{\Delta}{r^{2}}$
B. $\frac{(a+b+c)^{2}}{a b c} 2 R$
C. $\frac{\Delta}{r}$
D. $\frac{\Delta}{R r}$

## Answer: A

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A. $\cot C$
B. $c \cot C$
C. $\cot \frac{C}{2}$
D. $c \cot . \frac{C}{2}$

## Answer: D

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34. In a right-angled isosceles triangle, the ratio of the circumradius and inradius is
A. $2(\sqrt{2}+1): 1$
B. $(\sqrt{2}+1): 1$
C. 2:1
D. $\sqrt{2}: 1$

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35. In a $\triangle A B C$, a semicircle is inscribed, whose diameter lies on the side $c$. Then find the radius of the semicircle.(Where $\Delta$ is the area of the triangle $A B C$ )

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36. In $A B C, A \frac{2 \pi}{3}, b-c=3 \sqrt{3} \mathrm{~cm}$ and area of $A B C=\frac{9 \sqrt{3}}{2} \mathrm{~cm}^{2}$, then $B C=6 \sqrt{3}$ (b) 9 cm (c) 18 cm (d) 27 cm
A. $6 \sqrt{3} \mathrm{~cm}$
B. 9 cm
C. 18 cm

## Answer: B

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37. In triangle $A B C$, let $\angle c=\frac{\pi}{2}$. If $r$ is the inradius and $R$ is circumradius of the triangle, then $2(r+R)$ is equal to $a+b$ (b)
$b+c c+a$ (d) $a+b+c$
A. $a+b$
B. $b+c$
C. $c+a$
D. $a+b+c$

Answer: A
38. In the given figure $A B$ is the diameter of the circle, centred at $O$. If $\angle C O A=60^{\circ}, A B=2 r, A C=d$, and $C D=l$
A. $d \sqrt{3}$
B. $d / \sqrt{3}$
C. 3d
D. $\sqrt{3} d / 2$

Answer: A

## D Watch Video Solution

39. In triangle $A B C$, if $P Q, R$ divides sides $B C, A C$, and $A B$, respectively, in the ratio $k: 1(\in$ or $d e r)$. If the ratio
$\left(\frac{\operatorname{ar} E A P Q R}{\operatorname{area} A B C}\right)$ IS $\frac{1}{3}$, thenk is equal to $\frac{1}{3}$ (b) 2 (c) 3 (d) none of these
A. $1 / 3$
B. 2
C. 3
D. none of these

## Answer: B

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40. If the angles of a triangle are $30^{\circ}$ and $45^{\circ}$ and the included side is $(\sqrt{3}+1) \mathrm{cm}$ then the area of the triangle is $\qquad$ .
A. $\frac{\sqrt{3}+1}{2}$ sq. units
B. $(\sqrt{3}+1)$ sq. units
C. $2(\sqrt{3}-1)$ sq. units
D. $\frac{2 \sqrt{3}-1}{2}$ sq. units

## Answer: A

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41. In triangle $A B C$, base $B C$ and area of triangle are fixed. The locus of the centroid of triangle $A B C$ is a straight line that is parallel to side $B C$ right bisector of side $B C$ perpendicular to $B C$ inclined at an angle $\sin ^{-1}\left(\frac{\sqrt{ }}{B C}\right)$ to side $B C$
A. parallel to side $B C$
B. right bisector of side $B C$
C. prependicular to $B C$
D. inclined at an angle $\sin ^{-1}(\sqrt{\Delta} / B C)$ to side $B C$

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42. Let the area of triangle $A B C$ be $(\sqrt{3}-1) / 2, b=2$ and $c=(\sqrt{3}-1)$, and $\angle A$ be acute. The measure of the angle $A$ is
A. $15^{\circ}$
B. $30^{\circ}$
C. $60^{\circ}$
D. $75^{\circ}$

## Answer: A

43. In $\triangle A B C, \Delta=6, a b c=60, r=1$ Then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is nearly
A. 0.5
B. 0.6
C. 0.4
D. 0.8

## Answer: D

## D Watch Video Solution

44. Triangle $A B C$ is isosceles with $A B=A C$ and $B C=65 \mathrm{~cm} . \mathrm{P}$ is a point on $B C$ such that the perpendiculardistances from P to $A B$ and $A C$ are

24 cm and 36 cm , respectively. The area of triangle $A B C$ (in sq cm is)
A. 1254
B. 1950
C. 2535
D. 5070

## Answer: C

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45. In an equilateral triangle, the inradius, circumradius, and one of the exradii are in the ratio
A. $2: 4: 5$
B. 1:2:3
C. 1:2:4
D. 2:4:3

## Answer: B

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46. In triangle $A B C$, if $\cos A+\cos B+\cos C=\frac{7}{4}$, then $\frac{R}{r}$ is equal to $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
A. $\frac{3}{4}$
B. $\frac{4}{3}$
C. $\frac{2}{3}$
D. $\frac{3}{2}$
47. If two sides of a triangle are roots of the equation $x^{2}-7 x+8=0$ and the angle between these sides is $60^{0}$ then the product of inradius and circumradius of the triangle is $\frac{8}{7}$ (b) $\frac{5}{3}$ (c) $\frac{5 \sqrt{2}}{3}$ (d) 8
A. $\frac{8}{7}$
B. $\frac{5}{3}$
C. $\frac{5 \sqrt{2}}{3}$
D. 8

## Answer: B

48. Given $b=2, c=\sqrt{3}$ and $A=30^{\circ}$, then in-radius of triangle $A B C$ is
A. $\frac{\sqrt{3}-1}{2}$
B. $\frac{\sqrt{3}+1}{2}$
C. $\frac{\sqrt{3}-1}{4}$
D. none of these

## Answer: A

## D Watch Video Solution

49. 

In
triangle
$A B C$, if $A-B=120^{\circ}$ and $R=8 r$, whereRandr have their usual meanings, then $\cos C$ equal $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{7}{8}$
A. $3 / 4$
B. $2 / 3$
C. $5 / 6$
D. $7 / 8$

## Answer: D

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50. $A B C$ is an equilateral triangle of side 4 cm . If $R, r, a n d, h$ are the circumradius, inradius, and altitude, respectively, then $\frac{R+r}{h}$ is equal to (a) 4 (b) 2 (c) 1 (d) 3
A. 4
B. 2
C. 1
D. 3

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51. A circle is inscribed in a triangle $A B C$ touching the side $A B$ at $D$ such that $A D=5, B D=3$, if $\angle A=60^{\circ}$ then length
$B C$ equals. (a) 4 (b) $\frac{120}{13}$ (c) 13 (d) 12
A. 9
B. $\frac{120}{13}$
C. 13
D. 12

## Answer: C

52. The rational number which equals the number 2.126 with recurring decimal is

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53. AD is a median of the $\triangle A B C$.If AE and AFare medians of the triangles ABD respectively, and $\mathrm{AD}=m_{1}, A E=m_{2} A F=m_{3}$, then find the value of $\frac{a^{2}}{8}$.
A. $m_{2}^{2}+m_{3}^{2}-2 m_{1}^{2}$
B. $m_{1}^{2}+m_{2}^{2}-2 m_{3}^{2}$
C. $m_{1}^{2}+m_{3}^{2}-2 m_{2}^{2}$
D. none of these

Answer: A
54. For triangle $A B C, R=\frac{5}{2}$ and $r=1$. Let $I$ be the incenter of the triangle and $D, E$ and $F$ be the feet of the perpendiculars from $I \rightarrow B C, C A$ and $A B$, respectively. The value of $\frac{I D \cdot I E \cdot I F}{I A \cdot I B \cdot I C}$ is equal to (a) $\frac{5}{2}$ (b) $\frac{5}{4}$ (c) $\frac{1}{10}$ (d) $\frac{1}{5}$
A. $\frac{5}{2}$
B. $\frac{5}{4}$
C. $\frac{1}{10}$
D. $\frac{1}{5}$

## Answer: C

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55. In triangle $A B C, \angle A=60^{\circ}, \angle B=40^{\circ}$, and $\angle C=80^{\circ}$. If $P$ is the center of the circumcircle of triangle $A B C$ with radius unity, then the radius of the circumcircle of triangle $B P C$ is (a) 1 (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{3} 2$
A. 1
B. $\sqrt{3}$
C. 2
D. $\sqrt{3} / 2$

## Answer: A

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56. If $H$ is the othrocenter of an acute angled triangle $A B C$ whose circumcircle is $x^{2}+y^{2}=16$, then circumdiameter of the
triangle HBC is 1 (b) 2 (c) 4 (d) 8
A. 1
B. 2
C. 4
D. 8

## Answer: D

## D Watch Video Solution

57. In triangle $A B C$, the line joining the circumcenter and incenter is parallel to side BC , then $\cos A+\cos C$ is equal to -1
(b) 1 (c) -2 (d) 2
A. $\frac{1}{2}$
B. 1
C. $\sqrt{3}$
D. 2

## Answer: B

## - Watch Video Solution

58. In triangle $A B C$, line joining the circumcenter and orthocentre is parallel to side AC, then the value of $\tan A \tan C i s e q u a<o \sqrt{3}$ (b) 3 (c) $3 \sqrt{3}$ (d) none of these
A. $\sqrt{3}$
B. 3
C. $3 \sqrt{3}$
D. none of these

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59. In triangle $A B C, \angle C=\frac{2 \pi}{3}$ and $C D$ is the internal angle bisector of $\angle C$ meeting the side $A B$ at $D$. If Length $C D$ is equal to
A. 1
B. 2
C. 3
D. 4

## Answer: B

60. In the given figure $\triangle A B C$ is equilateral on side AB produced. We choose a point such that $A$ lies between $P$ and $B$. We now denote 'a' as the length of sides of $\Delta A B C, r_{1}$ as the radius of incircle $\triangle P A C$ and $r_{2}$ as the ex-radius of $\triangle P B C$ with respect to side BC . Then $r_{1}+r_{2}$ is equal to
A. (a) $\frac{1}{2}$
B. (b) $\frac{3}{2} a$
C. (c) $\frac{\sqrt{3}}{2} a$
D. (d) $a \sqrt{2}$

## Answer: C

## D Watch Video Solution

61. A variable triangle $A B C$ is circumscribed about a fixed circle of unit radius. Side $B C$ always touches the circle at D and has fixed direction. If $B$ and $C$ vary in such a way that (BD) (CD) $=2$, then locus of vertex $A$ will be a straight line. (a)parallel to side $B C$ (b)perpendicular to side $B C$ (c)making an angle ( $\frac{\pi}{6}$ ) with BC (d) making an angle $\sin ^{-1}\left(\frac{2}{3}\right)$ with $B C$
A. parallel to side BC
B. perpendicular to side $B C$
C. making an angle ( $\pi / 6$ ) with BC
D. making an angle $\sin ^{-1}(2 / 3)$ with $B C$

## Answer: A

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62. In $A B C$, if $a=10$ and $b \cot B+c \cot C=2(r+R)$ then the maximum area of $A B C$ will be (a) 50 (b) $\sqrt{50}$ (c) 25 (d) 5
A. 50
B. $\sqrt{50}$
C. 25
D. 5

## Answer: C

## - Watch Video Solution

63. Let $C$ be incircle of $A B C$. If the tangents of lengths $t_{1}, t_{2}$ and $t_{3}$ are drawn inside the given triangle parallel to
sidese $a, b$ and $c$, respectively, the $\frac{t_{1}}{a}+\frac{t_{2}}{b}+\frac{t_{3}}{c}$ is equal to 0 (b) 1 (c) 2 (d) 3
A. 0
B. 1
C. 2
D. 3

## Answer: B

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64. A park is in the form of a rectangle $120 \mathrm{~m} \times 100 \mathrm{~m}$ At the centre of the park there is a circular lawn. The area of park excluding lawn is $8700 \mathrm{~m}^{2}$. Find the radius of the circular lawn.
(Use $\pi=\frac{22}{7}$ )
65. In triangle ABC , if $r_{1}=2 r_{2}=3 r_{3}$, then $b: c$ is equal to
A. $\frac{5}{4}$
B. $\frac{4}{3}$
C. $\frac{7}{4}$
D. $\frac{3}{4}$

Answer: A

## D Watch Video Solution

66. If in a triangle $\left(1-\frac{r_{1}}{r_{2}}\right)\left(1-\frac{r_{1}}{r_{3}}\right)=2$, then the triangle is
A. right angled
B. isosceles
C. equilateral
D. none of these

## Answer: A

## - Watch Video Solution

67. If in a triangle $\frac{r}{r_{1}}=\frac{r_{2}}{r_{3}}$, then
A. $A=90^{\circ}$
B. $B=90^{\circ}$
C. $C=90^{\circ}$
D. none of these

## - Watch Video Solution

68. In $\triangle A B C$, 1 is the incentre, Area of
$\triangle I B C, \triangle I A C$ and $\triangle I A B$ are, respectively, $\Delta_{1}, \Delta_{2}$ and $\Delta_{3}$. If the values of $\Delta_{1}, \Delta_{2}$ and $\Delta_{3}$ are in A.P., then the altitudes of the $\triangle A B C$ are in
A. A.P.
B. G.P.
C. H.P.
D. none of these

## Answer: C

69. In an acute angled triangle $A B C, r+r_{1}=r_{2}+r_{3}$ and $\angle B>\frac{\pi}{3}, \quad$ then
$b+2 c<2 a<2 b+2 c$
(b) $b+4 c<4 a<2 b+4 c$
$b+4 c<4 a<4 b+4 c$ (d) $b+3 c<3 a<3 b+3 c$
A. $b+2 c<2 a<2 b+2 c$
B. $b+4<4 a<2 b+4 c$
C. $b+4 c<4 a<4 b+4 c$
D. $b+3 c<3 a<3 b+3 c$

## Answer: D

70. If in triangle $A B C, \sum \sin \left(\frac{A}{2}\right)=\frac{6}{5}$ and $\sum I I_{1}=9$ (where $I_{1}, I_{2} a n d I_{3}$ are excenters and $I$ is incenter, then circumradius $R$ is equal to $\frac{15}{8}$ (b) $\frac{15}{4}$ (c) $\frac{15}{2}$ (d) $\frac{4}{12}$
A. $\frac{15}{8}$
B. $\frac{15}{4}$
C. $\frac{15}{2}$
D. $\frac{4}{12}$

## Answer: A

## - Watch Video Solution

71. The radii $r_{1}, r_{2}, r_{3}$ of the escribed circles of the triangle ABC are in H.P. If the area of the triangle is $24 \mathrm{~cm}^{2}$ and its perimeter is

24 cm , then the length of its largest side is (a) 10 (b) 9 (c) 8 (d) none of these
A. 10
B. 9
C. 8
D. none of these

## Answer: A

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72. In $A B C$ with usual notations, if $r=1, r_{1}=7$ and $R=3$, the $(a) A B C$ is equilateral (b) acute angled which is not equilateral (c) obtuse angled (d) right angled
B. acute angled which is not equilateral
C. obtuse angled
D. right angled

## Answer: D

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73. Which of the following expresses the circumference of a circle inscribed in a sector $O A B$ with radius $\operatorname{Rand} A B=2 a$ ?
(a) $2 \pi \frac{R a}{R+a}$
(b) $\frac{2 \pi R^{2}}{a}$
$(c) 2 \pi(r-a)^{2}$
(d) $2 \pi \frac{R}{R-a}$
A. $2 \pi \frac{R a}{R+a}$
B. $\frac{2 \pi R^{2}}{a}$
C. $2 \pi(R-a)^{2}$
D. $2 \pi \frac{R}{R-a}$

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74. In $A B C$, the median $A D$ divides $\angle B A C$ such that $\angle B A D: \angle C A D=2: 1$. Then $\cos \left(\frac{A}{3}\right)$ is equal to $\frac{\sin B}{2 \sin C}$ $\frac{\sin C}{2 \sin B} \frac{2 \sin B}{\sin C}$ (d) none of these
A. $\frac{\sin B}{2 \sin C}$
B. $\frac{\sin C}{2 \sin B}$
C. $\frac{2 \sin B}{\sin C}$
D. none of these

## Answer: A

75. The area of the circle and the area of a regular polygon inscribed the circle of $n$ sides and of perimeter equal to that of the circle are in the ratio of
A. $\tan \left(\frac{\pi}{n}\right): \frac{\pi}{n}$
B. $\cos \left(\frac{\pi}{n}\right): \frac{\pi}{n}$
C. $\sin \frac{\pi}{n}: \frac{\pi}{n}$
D. $\cot \left(\frac{\pi}{n}\right): \frac{\pi}{n}$

## Answer: A

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76. The ratio of the area of a regular polygon of $n$ sides inscribed in a circle to that of the polygon of same number of sides
circumscribing the same is 3:4. Then the value of $n$ is 6 (b) 4 (c) 8
(d) 12
A. 6
B. 4
C. 8
D. 12

## Answer: A

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77. In any triangle, the minimum value of $\frac{r^{3}}{r_{1} r_{2} r_{3}}$ is equal to
A. 1
B. $1 / 9$
C. $1 / 27$
D. none of these

## Answer: C

## - Watch Video Solution

78. If $R_{1}$ is the circumradius of the pedal triangle of a given triangle $A B C$, $a n d R_{2}$ is the circumradius of the pedal triangle of the pedal triangle formed, and so on $R_{3}, R_{4}$ then the value of $\sum_{i=1}^{\infty} R_{i}$, where $R$ (circumradius) of $A B C$ is 5 is 8 (b) 10 (c) 12 (d) 15
A. 8
B. 10
C. 12

## Answer: B

## - Watch Video Solution

79. A sector $O A B O$ of central angle $\theta$ is constructed in a circle with centre $O$ and of radius 6 . The radius of the circle that is circumscribed about the triangle $O A B$, is $6 \frac{\cos \theta}{2}$ (b) $6 \frac{\sec \theta}{2}$ $3 \frac{\sec \theta}{2}$ (d) $3\left(\frac{\cos \theta}{2}+2\right)$
A. $6 \cos \cdot \frac{\theta}{2}$
B. 6 sec . $\frac{\theta}{2}$
C. 3 sec. $\frac{\theta}{2}$
D. $3\left(\cos \cdot \frac{\theta}{2}+2\right)$

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80. Triangle $A B C$ is isosceles with
$A B=A C$ and $B C=65 \mathrm{~cm} . \mathrm{P}$ is a point on $B C$ such that the perpendiculardistances from P to $A B$ and $A C$ are 24 cm and 36 cm , respectively. The area of triangle $A B C$ (in sq cm is)
A. 8
B. 12
C. 16
D. 6

## Answer: D

## Multiple Correct Answer Type

1. The sides of $A B C$ satisfy the equation $2 a^{2}+4 b^{2}+c^{2}=4 a b+2 a c$ Then a) the triangle is isosceles b$)$ the triangle is obtuse c) $B=\cos ^{-1}\left(\frac{7}{8}\right)$ d) $A=\cos ^{-1}\left(\frac{1}{4}\right)$
A. the triangle is isosceles
B. the triangle is obtuse
C. $B=\cos ^{-1}(7 / 8)$
D. $A=\cos ^{-1}(1 / 4)$

## Answer: A::C::D

2. If sides of triangle $A B C$ are $a, b a n d c$ such that $2 b=a+c$ then $\frac{b}{c}>\frac{2}{3}$ (b) $\frac{b}{c}>\frac{1}{3} \frac{b}{c}<2$ (d) $\frac{b}{c}<\frac{3}{2}$
A. $\frac{b}{c}>\frac{2}{3}$
B. $\frac{b}{c}>\frac{1}{3}$
C. $\frac{b}{c}<2$
D. $\frac{b}{c}<\frac{3}{2}$

## Answer: A::C

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3. If the sines of the angles $A$ and $B$ of a triangle $A B C$ satisfy the equation $c^{2} x^{2}-c(a+b) x+a b=0$, then the triangle (a) is acute angled (b) is right angled (c) is obtuse angled (d) satisfies the equation $\sin A+\cos A \frac{(a+b)}{c}$
A. is acute angled
B. is right angled
C. is obtus angled
D. satisfies the equation $\sin A+\cos A=\frac{(a+b)}{c}$

## Answer: B::D

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4. There exist a triangle $A B C$ satisfying
A. $\tan A+\tan B+\tan C=0$
B. $\frac{\sin A}{2}=\frac{\sin B}{3}=\frac{\sin C}{7}$
C. $(a+b)^{2}=c^{2}+a b$ and $\sqrt{2}(\sin A+\cos A)=\sqrt{3}$
D.
$\sin A+\sin B=\frac{\sqrt{3}+1}{2}, \cos A \cos B=\frac{\sqrt{3}}{4}=\sin A \sin B$

## Answer: C::D

## - Watch Video Solution

5. In triangle, $A B C$ if $2 a^{2} b^{2}+2 b^{2} c^{2}=a^{2}+b^{4}+c^{4}$, then angle $B$ is equal to $45^{\circ}$ (b) $135^{0} 120^{\circ}$ (d) $60^{\circ}$
A. $45^{\circ}$
B. $135^{\circ}$
C. $120^{\circ}$
D. $60^{\circ}$
6. If in triangle $A B C, a, c$ and angle $A$ are given and $c \sin A<a<c$, then ( $b_{1}$ and $b_{2}$ are values of b) (a)

$$
\begin{align*}
& b_{1}+b_{2}=2 c \cos A \text { (b) } b_{1}+b_{2}=c \cos A \text { (c) } b_{1} b_{2}=c^{2}-a^{2}  \tag{d}\\
& b_{1} b_{2}=c^{2}+a^{2}
\end{align*}
$$

A. $b_{1}+b_{2}=2 c \cos A$
B. $b_{1}+b_{2}=c \cos A$
C. $b_{1} b_{2}=c^{2}-a^{2}$
D. $b_{1} b_{2}=c^{2}+a^{2}$

## Answer: A::C

7. If area of $\Delta A B C(\Delta)$ and angle $C$ are given and if the side $c$ opposite to given angle is minimum, then $a=\sqrt{\frac{2 \Delta}{\sin C}}$
$b=\sqrt{\frac{2 \Delta}{\sin C}} a=\sqrt{\frac{4 \Delta}{\sin C}}$ (d) $b=\sqrt{\frac{4 \Delta}{\sin C}}$
A. $a=\sqrt{\frac{2 \Delta}{\sin C}}$
B. $b=\sqrt{\frac{2 \Delta}{\sin C}}$
C. $a=\frac{4 \Delta}{\sin C}$
D. $b=\frac{4 \Delta}{\sin ^{2} C}$

## Answer: A: B

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8. If $\Delta$ represents the area of acute angled triangle $A B C$, then
$\sqrt{a^{2} b^{2}-4 \Delta^{2}}+\sqrt{b^{2} c^{2}-4 \Delta^{2}}+\sqrt{c^{2} a^{2}-4 \Delta^{2}}=$
$a^{2}+b^{2}+c^{2}$ (b) $\frac{a^{2}+b^{2}+c^{2}}{2}$ (c) $a b \cos C+b c \cos A+c a \cos B$
(d) $a b \sin C+b c \sin A+c a \sin B$
A. $a^{2}+b^{2}+c^{2}$
B. $\frac{a^{2}+b^{2}+c^{2}}{2}$
C. $a b \cos C+b c \cos A+c a \cos B$
D. $a b \sin C+b c \sin A+c a \sin B$

## Answer: B::C

## - Watch Video Solution

9. Sides of a triangle ABC are in A.P. If $a<\min \{b, c\}$, then $\cos \mathrm{A}$ may be equal to
A. $\frac{4 b-3 c}{2 b}$
B. $\frac{3 c-4 b}{2 c}$
C. $\frac{4 c-3 b}{2 b}$
D. $\frac{4 c-3 b}{2 c}$

## Answer: A::D

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10. If the angles of a triangle are $30^{0}$ and $45^{\circ}$ and the included side is $(\sqrt{3}+1) \mathrm{cm}$, then area of the triangle is $\frac{1}{2}(\sqrt{3}+1)$ squnitsareaofthe $\triangle i s 1 / 2($ sqrt(3)-1)sq units`
A. area of the triangle is $\frac{1}{}(\sqrt{3}+1)$ sq. units
B. area of the triangle is $\frac{1}{2}(\sqrt{3}-1)$ sq. units
C. ratio of greater side to smaller side is $\frac{\sqrt{3}+1}{\sqrt{2}}$
D. ratio of greater side to smaller side is $\frac{1}{4 \sqrt{3}}$

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11. Lengths of the tangents from $A, B$ and $C$ to the incircle are in A.P., then (a) $r_{1}, r_{2}, r_{3}$ are in H.P (b) $r_{1}, r_{2}, r_{3}$ are in A.P (c)a, b, c are in A.P (d) $\cos A=\frac{4 c-3 b}{2 c}$
A. $r_{1}, r_{2} r_{3}$ are in H.P
B. $r_{1}, r_{2}, r_{3}$ are in AP
C. a, b, c are in A.P
D. $\cos A=\frac{4 c-3 b}{2 c}$

## Answer: A::C::D

12. $C F$ is the internal bisector of angle $C$ of $A B C$, then $C F$ is equal to (a) $\frac{2 a b}{a+b} \cos \left(\frac{C}{2}\right)$ (b) $\frac{a+b}{2 a b} \frac{\cos C}{2}$ (c) $\frac{b \sin A}{\sin \left(B+\frac{C}{2}\right)}$
(d) none of these
A. $\frac{2 a b}{a+b} \cos \frac{C}{2}$
B. $\frac{a+b}{2 a b} \cos . \frac{C}{2}$
C. $\frac{b \sin A}{\sin \left(B+\frac{C}{2}\right)}$
D. none of these

## Answer: A::C

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13. The incircle of $\triangle A B C$ touches side BC at D . The difference between BD and CD ( R is circumradius of $\triangle A B C$ ) is (a)
$\left|4 R \sin . \frac{A}{2} \sin . \frac{B-C}{2}\right|$
(b) $\left|4 R \cos \cdot \frac{A}{2} \sin \cdot \frac{B-C}{2}\right|$
(c) $|b-c|$
(d) $\left|\frac{b-c}{2}\right|$
A. $\left|4 R \sin . \frac{A}{2} \sin . \frac{B-C}{2}\right|$
B. $\left|4 R \cos . \frac{A}{2} \sin . \frac{B-C}{2}\right|$
C. $|b-c|$
D. $\left|\frac{b-c}{2}\right|$

## Answer: A::C

## - Watch Video Solution

14. A circle of radius 4 cm is inscribed in $\triangle A B C$, which touches side $B C$ at $D$. If $B D=6 \mathrm{~cm}, D C=8 \mathrm{~cm}$ then
A. the triangle is necessarily acute angled
B. $\tan \cdot \frac{A}{2}=\frac{4}{7}$
C. perimeter of the triangle $A B C$ is 42 cm
D. area of $\triangle A B C$ is $84 \mathrm{~cm}^{2}$

## Answer: A::B::C::D

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15. If $H$ is the orthocentre of triangle $A B C, R=$ circumradius and $P=A H+B H+C H$, then (a) $P=2(R+r)$ (b) max. of P is

3R (c) min. of P is 3 R (d) $P=2(R-r)$
A. $P=2(R+r)$
B. max. of $P$ is $3 R$
C. min. of $P$ is $3 R$
D. $P=2(R-r)$

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16. Let $A B C$ be an isosceles triangle with base $B C$. If $r$ is the radius of the circle inscribsed in $\triangle A B C$ and $r_{1}$ is the radius of the circle ecribed opposite to the angle A, then the product $r_{1} r$ can be equal to (where R is the radius of the circumcircle of $\Delta A B C$ )
A. $R^{2} \sin ^{2} A$
B. $R^{2} \sin ^{2} 2 B$
C. $\frac{1}{2} a^{2}$
D. $\frac{a^{2}}{4}$

Answer: A: B::D
17. If inside a big circle exactly $n(n \leq 3)$ small circles, each of radius $r$, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles, then the radius of big circle is $r\left(1+\operatorname{cosec} \frac{\pi}{n}\right)$
$\left(\frac{1+\frac{\tan \pi}{n}}{\frac{\cos \pi}{\pi}}\right) r\left[1+\operatorname{cosec} \frac{2 \pi}{n}\right]$ (d) $\frac{r\left[s \in \frac{\pi}{2 n}+\frac{\cos (2 \pi)}{n}\right]^{2}}{\frac{\sin \pi}{n}}$
A. $r\left(1+\cos e c \cdot \frac{\pi}{n}\right)$
B. $\left(\frac{1+\tan \pi / n}{\cos \pi / n}\right)$
C. $r\left[1+\cos e c . \frac{2 \pi}{n}\right]$
D. $\frac{r\left[\sin \cdot \frac{\pi}{2 n}+\cos \cdot \frac{2 \pi}{n}\right]^{2}}{\sin \pi / n}$

## Answer: A::D

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18. The area of a regular polygon of $n$ sides is (where $r$ is inradius, $R$ is circumradius, and $a$ is side of the triangle (a) $\frac{n R^{2}}{2} \sin \left(\frac{2 \pi}{n}\right)$ (b) $n r^{2} \tan \left(\frac{\pi}{n}\right)$ (c) $\frac{n a^{2}}{4} \frac{\cot \pi}{n}$ (d) $n R^{2} \tan \left(\frac{\pi}{n}\right)$
A. $\frac{n R^{2}}{2} \sin \left(\frac{2 \pi}{n}\right)$
B. $n r^{2} \tan \left(\frac{\pi}{n}\right)$
C. $\frac{n a^{2}}{4} \cot . \frac{\pi}{n}$
D. $n R^{2} \tan \left(\frac{\pi}{n}\right)$

## Answer: A::B::C

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19. In acute angled triangle $A B C, A D$ is the altitude. Circle drawn with $A D$ as its diameter cuts $A B a n d A C a t P a n d Q$,
respectively. Length of $P Q$ is equal to $/(2 R)$ (b) $\frac{a b c}{4 R^{2}}$ $2 R \sin A \sin B \sin C$ (d) $\Delta / R$
A. $\frac{\Delta}{2 R}$
B. $\frac{a b c}{4 R^{2}}$
C. $2 R \sin A \sin B \sin C$
D. $\frac{\Delta}{R}$

## Answer: C::D

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20. If $A$ is the area and $2 s$ is the sum of the sides of a triangle, then
A. $A \leq \frac{s^{2}}{4}$
B. $A \leq \frac{s^{2}}{3 \sqrt{3}}$
C. $A<\frac{s^{2}}{\sqrt{3}}$
D. none of these

## Answer: A::B

## - Watch Video Solution

21. In $A B C$, internal angle bisector of $\angle A$ meets side $B C$ in $D$.
$D E \perp A D$ meets $A C$ at $E$ and $A B$ at $F$. Then (a) $A E$ is in H. P. of $b$ and $c$ (b) $A D=\frac{2 b c}{b+c} \frac{\cos A}{2}$ (c) $E F=\frac{4 b c}{b+c} \frac{\sin A}{2}$
(d) $A E F$ is isosceles
A. AE in H.M of $b$ and $c$
B. $A D=\frac{2 b c}{b+c} \cos . \frac{A}{2}$
C. $E F=\frac{4 b c}{b+c} \sin \cdot \frac{A}{2}$
D. $\triangle A E F$ is isosceles

## - Watch Video Solution

22. In a triangle $A B C, A B=5, B C=7, A C=6$. $A$ point $P$ is in the plane such that it is at distance ' 2 ' units from $A B$ and 3 units form $A C$ then prove its distance from $B C$ is $\frac{12 \sqrt{6}-28}{7}$ when $P$ is inside the trinagle

## D Watch Video Solution

23. The base $B C$ of $A B C$ is fixed and the vertex $A$ moves, satisfying the condition $\cot \left(\frac{B}{2}\right)+\cot \left(\frac{C}{2}\right)=2 \cot \left(\frac{A}{2}\right)$, then (a) $b+c=a$ (b) $b+c=2 a$ (c) vertex $A$ moves along a straight line (d) Vertex $A$ moves along an ellipse
A. $b+c=a$
B. $b+c=2 a$
C. vertex A moves along a straight line
D. vertex A moves along an ellipse

## Answer: B::D

## - Watch Video Solution

24. If $D, E$ and $F$ are the middle points of the sides $B C, C A$ and $A B$ of the $\Delta A B C$, then $\mathrm{AD}+\mathrm{BE}+\mathrm{CF}$ is
A. a zero vector
B. 0
C. a unit vector
D. none of these

## - Watch Video Solution

Linked Comprehension Type

1. Given that $\Delta=6, r_{1}=2, r_{2}=3, r_{3}=6$

Circumradius $R$ is equal to a) 2.5 b) 3.5 c) 1.5 d) none of these
A. 2.5
B. 3.5
C. 1.5
D. none of these

## Answer: A

2. Given that $\Delta=6, r_{1}=2, r_{2}=3, r_{3}=6$ then Inradius is equal to
A. 2
B. 1
C. 1.5
D. 2.5

## Answer: B

## - Watch Video Solution

3. Given that $\Delta=6, r_{1}=2, r_{2}=3, r_{3}=6$ Difference between the greatest and the least angles is
A. $\cos ^{-1} \cdot \frac{4}{5}$
B. $\tan ^{-1} \cdot \frac{3}{4}$
C. $\cos ^{-1} \cdot \frac{3}{5}$
D. none of these

## Answer: C

## - Watch Video Solution

4. Let $\mathrm{a}=6, \mathrm{~b}=3$ and $\cos (A-B)=\frac{4}{5}$

Area (in sq. units) of the triangle is equal to
A. 9
B. 12
C. 11
D. 10

## - Watch Video Solution

5. Let $\mathrm{a}=6, \mathrm{~b}=3$ and $\cos (A-B)=\frac{4}{5}$

Angle $C$ is equal to
A. $\frac{3 \pi}{4}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{2}$
D. none of these

## Answer: C

6. Let $\mathrm{a}=6, \mathrm{~b}=3$ and $\cos (A-B)=\frac{4}{5}$

Value of $\sin C$ is equal to
A. 1
B. $\frac{1}{\sqrt{3}}$
C. $\frac{1}{\sqrt{5}}$
D. $\frac{2}{\sqrt{5}}$

## Answer: D

## - Watch Video Solution

7. Let $A B C$ be an acute angled triangle with orthocenter H.D, E, and $F$ are the feet of perpendicular from $A, B$, and $C$, respectively, on opposite sides. Also, let R be the circumradius of $\triangle A B C$.

Given $A H$. BH. $C H=3$ and $(A H)^{2}+(B H)^{2}+(C H)^{2}=7$

Then answer the following
Value of $\frac{\cos A \cdot \cos B \cdot \cos C}{\cos ^{2} A+\cos ^{2} B+\cos ^{2} C}$ is
A. $\frac{3}{14 R}$
B. $\frac{3}{7 R}$
C. $\frac{7}{3 R}$
D. $\frac{14}{3 R}$

## Answer: A

## - Watch Video Solution

8. Let $A B C$ be an acute angled triangle with orthocenter H.D, E, and $F$ are the feet of perpendicular from $A, B$, and $C$, respectively, on opposite sides. Also, let R be the circumradius of $\triangle A B C$.

Given $A H . B H . C H=3$ and $(A H)^{2}+(B H)^{2}+(C H)^{2}=7$

Then answer the following
Value of $R$ is
A. 1
B. $\frac{3}{2}$
C. $\frac{5}{2}$
D. none

## Answer: B

## - Watch Video Solution

9. Let $A B C$ be an acute angled triangle with orthocenter H.D, E, and $F$ are the feet of perpendicular from $A, B$, and $C$, respectively, on opposite sides. Also, let R be the circumradius of $\triangle A B C$.

Given $A H$. BH. $C H=3$ and $(A H)^{2}+(B H)^{2}+(C H)^{2}=7$

Then answer the following
Value of $H D . H E . H F$ is
A. $\frac{9}{64 R^{3}}$
B. $\frac{9}{8 R^{3}}$
C. $\frac{8}{9 R^{3}}$
D. $\frac{64}{9 R^{3}}$

## Answer: B

## - Watch Video Solution

10. Let $O$ be a point inside a triangle $A B C$ such that $\angle O A B=\angle O B C=\angle O C A=\omega \quad, \quad$ then show that: $\cos e c^{2} \omega=\operatorname{cosec}{ }^{2} A+\operatorname{cosec} c^{2} B+\operatorname{cosec}^{2} C$
11. find the principle value of $\cos ^{-1}\left(\frac{\sqrt{3}}{2}\right)$

## D Watch Video Solution

12. Let $O$ be a point inside a triangle $A B C$ such that $\angle O A B=\angle O B C=\angle O C A=\omega \quad$, then show that: $\cot \omega=\cot A+\cot B+\cot C$

## D Watch Video Solution

13. Given an isoceles triangle with equal side of length $b$ and angle $\alpha<\pi / 4$, then
the circumradius R is given by
A. $\frac{1}{2} b \cos e c \alpha$
B. $b \cos e c \alpha$
C. $2 b$
D. none of these

## Answer: A

## - Watch Video Solution

14. Given an isoceles triangle with equal side of length $b$ and angle $\alpha<\pi / 4$, then
the inradius $r$ is given by
A. $\frac{b \sin 2 \alpha}{2(1-\cos \alpha)}$
B. $\frac{b \sin 2 \alpha}{2(1+\cos \alpha)}$
C. $\frac{b \sin \alpha}{2}$
D. $\frac{b \sin \alpha}{2(1+\sin \alpha)}$

## - Watch Video Solution

15. An isosceles triangle has two equal sides of length 'a' and angle between them is $\propto$. The area of the triangle is
A. $\left|\frac{b \cos (3 \alpha / 2)}{2 \sin \alpha \cos (\alpha / 2)}\right|$
B. $\left|\frac{b \cos 3 \alpha}{\sin 2 \alpha}\right|$
C. $\left|\frac{b \cos 3 \alpha}{\cos \alpha \sin (\alpha / 2)}\right|$
D. $\left|\frac{b}{\sin \alpha \cos \alpha / 2}\right|$

## Answer: A

## - Watch Video Solution

16. Incircle of $\triangle A B C$ touches the sides $\mathrm{BC}, \mathrm{AC}$ and AB at $\mathrm{D}, \mathrm{E}$ and

F, respectively. Then answer the following question
$\angle D E F$ is equal to
A. $\frac{\pi-B}{2}$
B. $\pi-2 B$
C. $A-C$
D. none of these

## Answer: A

## - Watch Video Solution

17. Incircle of $\triangle A B C$ touches the sides $\mathrm{BC}, \mathrm{AC}$ and AB at $\mathrm{D}, \mathrm{E}$ and

F, respectively. Then answer the following question
$\angle D E F$ is equal to
A. $2 r^{2} \sin (2 A) \sin (2 B) \sin (2 C)$
B. $2 r^{2} \cos . \frac{A}{2} \cos . \frac{B}{2} \cos . \frac{C}{2}$
C. $2 r^{2} \sin (A-B) \sin (B-C) \sin (C-A)$
D. none of these

## Answer: B

## - Watch Video Solution

18. Incircle of $\triangle A B C$ touches the sides $\mathrm{BC}, \mathrm{AC}$ and AB at $\mathrm{D}, \mathrm{E}$ and

F, respectively. Then answer the following question

The length of side EF is
A. $r \sin \cdot \frac{A}{2}$
B. $2 r \sin \frac{A}{2}$
C. $r \cos . \frac{A}{2}$
D. $2 r \cos \frac{A}{2}$

## Answer: D

## - Watch Video Solution

19. Bisectors of angles $A, B$ and $C$ of a triangle $A B C$ intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^{\circ}-\frac{1}{2} A, 90^{\circ}-\frac{1}{2} B$ and $90^{\circ}-\frac{1}{2} C$
A. $2 R \cos \frac{A}{2}$
B. $2 R \sin \left(\frac{A}{2}\right)$
C. $R \cos \left(\frac{A}{2}\right)$
D. $2 R \cos \left(\frac{B}{2}\right) \cos \left(\frac{C}{2}\right)$

Answer: A
20. Internal bisectors of $\triangle A B C$ meet the circumcircle at point D, E, and F

Area of $\triangle D E F$ is
A. $2 R^{2} \cos ^{2}\left(\frac{A}{2}\right) \cos ^{2}\left(\frac{B}{2}\right) \cos ^{2}\left(\frac{C}{2}\right)$
B. $2 R^{2} \sin \left(\frac{A}{2}\right) \sin \left(\frac{B}{2}\right) \sin \left(\frac{C}{2}\right)$
C. $2 R^{2} \sin ^{2}\left(\frac{A}{2}\right) \sin ^{2}\left(\frac{B}{2} \sin ^{2}\left(\frac{C}{2}\right)\right.$
D. $2 R^{2} \cos \left(\frac{A}{2}\right) \cos \left(\frac{B}{2}\right) \cos \left(\frac{C}{2}\right)$

## Answer: D

- Watch Video Solution

21. Internal bisectors of $\triangle A B C$ meet the circumcircle at point D , $E$, and $F$

Area of $\triangle D E F$ is
A. $\geq 1$
B. $\leq 1$
C. $\geq 1 / 2$
D. $\leq 1 / 2$

## Answer: B

## - Watch Video Solution

22. The area of any cyclic quadrilateral $A B C D$ is given by
$A^{2}=(s-a)(s-b)(s-c)(s-d)$, where
$2 s=a+b++c+d, a, b, c$ and $d$ are the sides of the

## quadrilateral

Now consider a cyclic quadrilateral $A B C D$ of area 1 sq. unit and answer the following question

The minium perimeter of the quadrilateral is
A. 4
B. 2
C. 1
D. none of these

## Answer: A

## - Watch Video Solution

23. The area of any cyclic quadrilateral $A B C D$ is given by
$A^{2}=(s-a)(s-b)(s-c)(s-d)$,
$2 s=a+b++c+d, a, b, c$ and $d$ are the sides of the

## quadrilateral

Now consider a cyclic quadrilateral $A B C D$ of area 1 sq. unit and answer the following question

The minimum value of the sum of the lenghts of diagonals is
A. $2 \sqrt{2}$
B. 2
C. $\sqrt{2}$
D. none of these

## Answer: A

## - Watch Video Solution

24. The area of any cyclic quadrilateral $A B C D$ is given by
$A^{2}=(s-a)(s-b)(s-c)(s-d)$,
$2 s=a+b++c+d, a, b, c$ and $d$ are the sides of the

## quadrilateral

Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

When the perimeter is minimum, the quadrilateral is necessarily
A. a square
B. a rectangle but not a square
C. a rhombus but not a square
D. none of these

## Answer: A

## - Watch Video Solution

25. In $\triangle A B C, R, r, r_{1}, r_{2}, r_{3}$ denote the circumradius, inradius, the exradii opposite to the vertices $A, B, C$ respectively. Given that
$r_{1}: r_{2}: r_{3}=1: 2: 3$
The sides of the triangle are in the ratio
A. $1: 2: 3$
B. $3: 5: 7$
C. $1: 5: 9$
D. $5: 8: 9$

## Answer: D

## - Watch Video Solution

26. In $\triangle A B C, R, r, r_{1}, r_{2}, r_{3}$ denote the circumradius, inradius, the exradii opposite to the vertices $A, B, C$ respectively. Given that
$r_{1}: r_{2}: r_{3}=1: 2: 3$

The sides of the triangle are in the ratio
A. $5: 2$
B. 5: 4
C. 5:3
D. $3: 2$

## Answer: A

## - Watch Video Solution

27. In $\triangle A B C, R, r, r_{1}, r_{2}, r_{3}$ denote the circumradius, inradius, the exradii opposite to the vertices $A, B, C$ respectively. Given that

$$
r_{1}: r_{2}: r_{3}=1: 2: 3
$$

The sides of the triangle are in the ratio
A. $\cos ^{-1}\left(\frac{1}{30}\right)$
B. $\cos ^{-1}\left(\frac{1}{3}\right)$
C. $\cos ^{-1}\left(\frac{1}{10}\right)$
D. $\cos ^{-1}\left(\frac{1}{5}\right)$

## Answer: C

## - Watch Video Solution

28. In $\triangle A B C, P, Q, R$ are the feet of angle bisectors from the vertices to their opposite sides as shown in the figure. $\triangle P Q R$ is constructed

If $\angle B A C=120^{\circ}$, then measusred of $\angle R P Q$ will be
A. $60^{\circ}$
B. $90^{\circ}$
C. $120^{\circ}$

## Answer: B

## - Watch Video Solution

29. In $\triangle A B C, P, Q, R$ are the feet of angle bisectors from the vertices to their opposite sides as shown in the figure. $\triangle P Q R$ is constructed


If $A B=7$ units, $\mathrm{BC}=8$ units, $\mathrm{AC}=5$ units, then the side PQ will be
A. $\frac{\sqrt{28}}{3}$ units
B. $\frac{\sqrt{88}}{3}$ units
C. $\frac{\sqrt{78}}{3}$ units
D. $\frac{\sqrt{84}}{3}$ units

## Answer: D

## - Watch Video Solution

30. Let $G$ be the centroid of triangle $A B C$ and the circumcircle of triangle AGC touches the side $A B$ at $A$

If $B C=6, A C=8$, then the length of side AB is equal to
A. $\frac{1}{2}$
B. $\frac{2}{\sqrt{3}}$
C. $5 \sqrt{2}$
D. none of these

## Answer: C

## - Watch Video Solution

31. Let $G$ be the centroid of triangle $A B C$ and the circumcircle of triangle $A G C$ touches the side $A B$ at $A$ If $\angle G A C=\frac{\pi}{3}$ and $a=3 b$, then $\sin \mathrm{C}$ is equal to
A. $\frac{3}{4}$
B. $\frac{1}{2}$
C. $\frac{2}{\sqrt{3}}$
D. none of these

Answer: B
32. Let $G$ be the centroid of triangle $A B C$ and the circumcircle of triangle AGC touches the side AB at A

If $A C=1$, then the length of the median of triangle $A B C$ through the vertex $A$ is equal to
A. $\frac{\sqrt{3}}{2}$
B. $\frac{1}{2}$
C. $\frac{2}{\sqrt{3}}$
D. $\frac{5}{\sqrt{2}}$

## Answer: A

33. The inradius in a right angled triangle with integer sides is $r$ If $r=4$, the greatest perimeter (in units) is
A. 96
B. 90
C. 60
D. 48

## Answer: B

## - Watch Video Solution

34. The inradius in a right angled triangle with integer sides is $r$ If $r=5$, the greatest area (in sq. units) is
B. 210
C. 330
D. 450

## Answer: C

## - Watch Video Solution

## Matrix Match Type

1. show that $\tan ^{-1}\left(\frac{1}{2}\right)+\tan ^{-1}\left(\frac{2}{11}\right)=\tan ^{-1}\left(\frac{3}{4}\right)$

Watch Video Solution
2. find the value of $\sin ^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos ^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
3. show
that
$\tan ^{-1} x+\tan ^{-1}\left(\frac{2 x}{1-x^{2}}\right)=\tan ^{-1}\left(\frac{3 x-x^{3}}{1-3 x^{2}}\right),|x|<\frac{1}{\sqrt{3}}$

## D Watch Video Solution

4. Show that $\sin ^{-1}\left(\frac{3}{5}\right)-\sin ^{-1}\left(\frac{8}{17}\right)=\cos ^{-1}\left(\frac{84}{85}\right)$.

## D Watch Video Solution

5. simplify $\cos ^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), \frac{\pi}{4}<\mathrm{x}<\frac{5 \pi}{4}$
6. In a triangle ABC, $a=7, b=8, c=9, B D$ is the median and $B E$ the altitude from the vertex $B$. Match the following lists
a. $B D=p .2$
b. $B E=$ q. 7
c. $E D=\quad r . \sqrt{45}$
d. $A E=s .6$
A. $\begin{array}{llll}a & b & c & d\end{array}$
A. 1.
$\begin{array}{llll}p & r & q & q\end{array}$
B. 2. $\begin{array}{llll}a & b & c & d \\ r & q & s & p\end{array}$
C. 3. $\begin{array}{llll}a & b & c & d \\ q & r & p & s\end{array}$
D. 4. $\begin{array}{llll}a & b & c & d \\ s & p & q & r\end{array}$

## Answer: C

- Watch Video Solution

1. Suppose $\alpha, \beta, \gamma a n d \delta$ are the interior angles of regular pentagon, hexagon, decagon, and dodecagon, respectively, then the value of $|\cos \alpha \sec \beta \cos \gamma \cos e c \delta|$ is $\qquad$

## - Watch Video Solution

2. Let ABCDEFGHIJKL be a regular dodecagon. Then the value of $\frac{A B}{A F}+\frac{A F}{A B}$ is equal to

## - Watch Video Solution

3. In a $\Delta A B C, b=12$ units, $\mathrm{c}=5$ units and $\Delta=30$ sq. units. If d is the distance between vertex A and incentre of the triangle then the value of $d^{2}$ is $\qquad$
4. In $\triangle A B C$, if $r=1, R=3$, and $s=5$, then the value of $a^{2}+b^{2}+c^{2}$ is $\qquad$

## - Watch Video Solution

5. Consider a $\triangle A B C$ in which the sides are $a=(n+1), b=(n+2), c=n$ with $\tan C=4 / 3$, then the value of $\Delta$ is $\qquad$

## - Watch Video Solution

6. In $\triangle A E X, T$ is the midpoint of XE and P is the midpoint of ET .

If $\triangle A P E$ is equilateral of side length equal to unity, then the vaue of $(A X)^{2}$ is
7. In $\triangle A B C$, the incircle touches the sides $\mathrm{BC}, \mathrm{CA}$ and AB , respectively, at D, E,and F. If the radius of the incircle is 4 units and $B D, C E$, and $A F$ are consecutive integers, then
A. Sides are also consecutive integers
B. Perimeter of the triangle is 42 units
C. Area of triangle is 84 sq. units
D. Diameter of circumcircle is 65 units

## Answer: 21

## - Watch Video Solution

8. The altitudes from the angular points $A, B$, and $C$ on the opposite sides $\mathrm{BC}, \mathrm{CA}$ and AB of $\triangle A B C$ are 210,195 and 182
respectively. Then the value of $a$ is $\qquad$

## (D) Watch Video Solution

9. In $\triangle A B C$, If $\angle C=3 \angle A, B C=27$, and $A B=48$. Then the value of $A C$ is $\qquad$

## D Watch Video Solution

10. The area of a right triangle is 6864 sq. units. If the ratio of its
legs is $143: 24$, then the value of $r$ is $\qquad$

## - Watch Video Solution

11. In $\triangle A B C$, if $\cos A+\sin A-\frac{2}{\cos B+\sin B}=0$, then the value of $\left(\frac{a+b}{c}\right)^{4}$ is

## (D) Watch Video Solution

12. In $\triangle A B C, \angle C=2 \angle A$, and $A C=2 B C$, then the value of $\underline{a^{2}+b^{2}+c^{2}}$ (where $R$ is circumradius of triangle) is $R^{2}$

## - Watch Video Solution

13. In $\triangle A B C$, if $b(b+c)=a^{2}$ and $c(c+a)=b^{2}$, then $|\cos A \cdot \cos B \cdot \cos C|$ is $\qquad$

## - Watch Video Solution

14. The sides of triangle $A B C$ satisfy the relations $a+b-c=2$ and $2 a b-c^{2}=4$, then the square of the area of triangle is
15. prove that $\sec ^{2}\left(\tan ^{-1} 3\right)+\operatorname{cosec}\left(\cot ^{-1} 4\right)=27$

## D Watch Video Solution

16. If $a, b$ and $c$ represent the lengths of sides of a triangle then
the possible integeral value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is

## - Watch Video Solution

17. In triangle ABC,
$\sin A \sin B+\sin B \sin C+\sin C \sin A=9 / 4$ and $a=2$, then
the value of $\sqrt{3} \Delta$, where $\Delta$ is the area of triangle, is
18. In a $\triangle A B C, A B=52, B C=56, C A=60$. Let D be the foot of the altitude from $A$ and $E$ be the intersection of the internal angle bisector of $\angle B A C$ with BC . Find the length DE .

## - Watch Video Solution

19. Point $D, E$ are taken on the side $B C$ of an acute angled triangle

$$
\begin{aligned}
& \text { ABC, such that } B D=D E=E C \text {. If } \\
& \angle B A D=x, \angle D A E=y \text { and } \angle E A C=z \text { then the value of } \\
& \frac{\sin (x+y) \sin (y+z)}{\sin x \sin z} \text { is }
\end{aligned}
$$

## - Watch Video Solution

20. For triangle $A B C, R=\frac{5}{2}$ and $r=1$. Let $I$ be the incenter of the triangle and $D, E$ and $F$ be the feet of the
perpendiculars from $I \rightarrow B C, C A$ and $A B$, respectively. The value of $\frac{I D \cdot I E \cdot I F}{I A \cdot I B \cdot I C}$ is equal to (a) $\frac{5}{2}$ (b) $\frac{5}{4}$ (c) $\frac{1}{10}$ (d) $\frac{1}{5}$

## - Watch Video Solution

21. Circumradius of $\triangle A B C$ is 3 cm and its area is $6 \mathrm{~cm}^{2}$. If DEF is the triangle formed by feet of the perpendicular drawn from $A, B$ and $C$ on the sides $B C, C A$ and $A B$, respectively, then the perimeter of $\triangle D E F$ (in cm ) is $\qquad$

## D Watch Video Solution

22. The distance of incentre of the right-angled triangle $A B C$
(right angled at A) from $B$ and $C$ are $\sqrt{10}$ and $\sqrt{5}$, respectively. Then find the perimeter of the triangle.

## Archives Single Correct Answer Type

1. For a regular polygon, let $r$ and $R$ be the radii of the inscribed and the circumscribed circles. A false statement among the following is
A. There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$
B. There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$
C. There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$
D. There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

## Answer: D

2. $A B C D$ is a trapezium such that $A B, D C$.are parallel and $B C$ is perpendicular to them. If $\angle A D B=\theta, B C=p$ and $C D=q$, show that $\mathrm{AB}=\frac{\left(p^{2}+q^{2}\right) \sin \theta}{p \cos \theta+q \sin \theta}$
A. $\left(\frac{p^{2}+q^{2} \sin \theta}{p \cos \theta+q \sin \theta}\right.$
B. $\frac{\left(p^{2}+q^{2}\right) \cos \theta}{p \cos \theta+q \sin t h e t}$
C. $\frac{p^{2}+q^{2}}{p^{2} \cos \theta+q^{2} \sin \theta}$
D. $\frac{\left(p^{2}+q^{2}\right) \sin \theta}{(p \cos \theta+q \sin \theta)}$

## Answer: A

## - Watch Video Solution

## Archives Jee Advanced

1. Let $A B C$ be a triangle such that $\angle A C B=\frac{\pi}{6}$ and let $a, b$ and $c$ denote the lengths of the side opposite to $A, B$,and $C$ respectively. The value(s) of $x$ for which $a=x^{2}+x+1, b=x^{2}-1$, and $c=2 x+1 \quad$ is(are) $-(2+\sqrt{3})$ (b) $1+\sqrt{3}$ (c) $2+\sqrt{3}$ (d) $4 \sqrt{3}$
A. $-(2+\sqrt{3})$
B. $1+\sqrt{3}$
C. $2+\sqrt{3}$
D. $4 \sqrt{3}$

## Answer: B

2. If the angle $A, B a n d C$ of a triangle are in an arithmetic propression and if $a, b a n d c$ denote the lengths of the sides opposite to $A, B a n d C$ respectively, then the value of the expression $\frac{a}{c} \sin 2 C+\frac{c}{a} \sin 2 A$ is (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$
A. $\frac{1}{2}$
B. $\frac{\sqrt{3}}{2}$
C. 1
D. $\sqrt{3}$

## Answer: D

## D Watch Video Solution

3. Let $P Q R$ be a triangle of area $\Delta$ with $a=2, b=\frac{7}{2}$ and $c=\frac{5}{2}$, where $\mathrm{a}, \mathrm{b}$ and c are the lengths of
the sides of the triangle opposite to the angles at $P, Q$ and $R$ respectively. Then $\left(\frac{2 \sin P-\sin 2 P}{2 \sin P+\sin 2 P}\right)$ equals
A. $\frac{3}{4 \Delta}$
B. $\frac{45}{4 \Delta}$
C. $\left(\frac{3}{4 \Delta}\right)^{2}$
D. $\left(\frac{45}{4 \Delta}\right)^{2}$

## Answer: C

## - Watch Video Solution

## Archives Multiple Correct Answer Type

1. a triangle $A B C$ with fixed base $B C$, the vertex $A$ moves such that $\cos B+\cos C=4 \sin ^{2}\left(\frac{A}{2}\right)$. If $a, b$ and $c$, denote the
length of the sides of the triangle opposite to the angles $A, B$ and $C$, respectively, then (a) $b+c=4 a$ (b) $b+c=2 a$ (c) the locus of point $A$ is an ellipse (d) the locus of point $A$ is a pair of straight lines
A. $b+c=4 a$
B. $b+c=2 a$
C. locus of point $A$ is an ellipse
D. locus of point $A$ is a pair of straight lines

## Answer: B::C

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2. about to only mathematics
B. 18
C. 24
D. 22

## Answer: B::D

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3. In a triangle XYZ , let $\mathrm{x}, \mathrm{y}, \mathrm{z}$ be the lengths of sides opposite to the angles $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$, respectively, and $2 \mathrm{~s}=\mathrm{x}+\mathrm{y}+\mathrm{z}$. If $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}$ and area of incircle of the triangle $X Y Z$ is $\frac{8 \pi}{3}$. then the area of triangle $X Y Z$ is _
4. In a triangle PQR , let $\angle P Q R=30^{\circ}$ and the sides PQ and QR have lengths $10 \sqrt{3}$ and 10 , respectively. Then, which of the following statement(s) is (are) TRUE ?
$\angle Q P R=45^{\circ}$ and
The area of the triangle PQR is $25 \sqrt{3}$ and $\angle Q R P=120^{\circ}$

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## Archives Matrix Match Type

1. Solve the equation $2 \tan ^{-1}(\cos x)=\tan ^{-1}(2 \cos e c x)$

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1. Let $A B C a n d A B C$ ' be two non-congruent triangles with sides $A B=4, A C=A C^{\prime}=2 \sqrt{2}$ and angle $B=30^{\circ}$. The absolute value of the difference between the areas of these triangles is

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2. Two parallel chords of a circle of radius 2 are at a distance. $\sqrt{3}+1$ apart. If the chord subtend angles $\frac{\pi}{k}$ and $\frac{2 \pi}{k}$ at the center, where $k>0$, then the value of $[k]$ is

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3. Consider a triangle $A B C$ and let $a, b a n d c$ denote the lengths
of the sides opposite to vertices $A, B, a n d C$, respectively. Suppose $a=6, b=10$, and the area of triangle is $15 \sqrt{3}$. If

## $\angle A C B$ is obtuse and if $r$ denotes the radius of the incircle of

 the triangle, then the value of $r^{2}$ is- Watch Video Solution

