MATHS

BOOKS - CENGAGE PUBLICATION

SOLUTIONS AND PROPERTIES OF TRIANGLE

Single Correct Answer Type

1. In a triangle $\angle A = 55^\circ, \angle B = 15^\circ, \angle C = 110^\circ$. Then $c^2 - a^2$ is equal to

A. ab

B. 2ab

C. $-ab$

D. none of these

Answer: A

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2. In a triangle ABC if $2a = \sqrt{3}b + c$, then possible relation is

A. $c^2 = a^2 + b^2 - ab$

B. $a^2 = b^2 + c^2$

C. $b^2 = a^2 + c^2 - ac\sqrt{3}$

D. a = b = c

Answer: B

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3. A circle of area 20 sq. units is centered at the point O. Suppose ΔABC is inscribed in that circle and has area 8 sq. units. The central angles α, β and γ are as shown in the figure. The value of $(\sin\alpha + \sin\beta + \sin\gamma)$ is equal to

A. $\frac{4\pi}{5}$

B. $\frac{3\pi}{4}$

C. $\frac{2\pi}{5}$

D. $\frac{\pi}{4}$

Answer: A

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4. In $\Delta ABC, bc = 2b^2 \cos A + 2c^2 \cos A - 4bc \cos^2 A,$ then ΔABC is

A. isosceles but not necessarily equilateral

B. equilateral

C. right angled but not necessarily isosceles

D. right angled isosceles

Answer: A

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5. Given a triangle ΔABC such that $\sin^2 A + \sin^2 C = 1001. \sin^2 B.$ Then the value of $\frac{2(\tan A + \tan C) \cdot \tan^2 B}{\tan A + \tan B + \tan C}$ is

A. $\frac{1}{2000}$

B. $\frac{1}{1000}$

C. $\frac{1}{500}$

D. $\frac{1}{250}$

Answer: D

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6. ΔABC has different side lengths a,b,c. If a^2, b^2, c^2 as sides form another ΔPQR, then ΔABC will always be

A. acute angled triangle only

B. obtuse angled triangle only

C. sometimes acute or sometimes obtuse depending on values of a,b nd c

D. none of these

Answer: A

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7. In $\Delta ABC, \angle B = \frac{\pi}{4}, \angle C = \frac{\pi}{6}$. D is a point on BC which divides it in the ratio $1:3, \angle DAB = \beta$, then

A. $\left(\sec. \frac{\pi}{6}\right) AB + \cot\beta = \cot\left(\frac{\pi}{6}\right) AC + \left(\sqrt{3} - 5\right)$

B. $\left(\sec. \frac{\pi}{4} AB \cot\beta = \cot\left(\frac{\pi}{4}\right) AC\left(4\sqrt{3} - 5\right)\right)$

C. $\left(\sec. \frac{\pi}{4}\right) AB + \cot\beta = \cot\left(\frac{\pi}{6}\right) AC + \left(4\sqrt{3} + 5\right)$

D. $\left(\sec. \frac{\pi}{6}\right) AB \cot\beta = \cot\left(\frac{\pi}{4}\right) AC\left(\sqrt{3} + 5\right)$

Answer: B

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8. The acute angle of a rhombus whose side is a mean proportional between its diagonals is

A. 15°

B. 20°

C. 30°

D. 80°

Answer: C

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9. Let side a,b and c of ΔABC be related by the relation a : b : c = 3 : 5 : 4. Altitudes AD,BE and CF are dropped on BC, CA and AB, respectively. If $P_1D + P_2E + P_3F = 42$, then the value of a + b + c is

A. 1200

B. 120

C. 12

D. none of these

Answer: A

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10. Triangle ABC is right angle at A. The points P and Q are on hypotenuse BC such that $BP = PQ = QC$.if $AP = 3$ and $AQ = 4$, then length BC is equal to

A. $3\sqrt{5}$

B. $5\sqrt{3}$

C. $4\sqrt{5}$

D. 7

Answer: A

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11. ABC is a right angled triangle of which A is the right angle, BD is drawn perpendicular to BC meets CA produced in D. If AB = 12, AC = 16, BC = 20, then BD =

A. 15

B. 25

C. 10

D. 225

Answer: A

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12. In a ΔABC, the median AD is perpendicular to AC. If b = 5 and c = 11, then a =

A. 10

B. 12

C. 14

D. $\sqrt{221}$

Answer: C

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13. ABC is an equilateral triangle where AB = a and P is any point in its plane such that PA = PB + PC. Then $\frac{PA^2 + PB^2 + PC^2}{a^2}$ is

A. 3

B. $\frac{\sqrt{3}}{4}$

C. $\frac{3}{4}$

D. 2

Answer: D

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14. In ΔABC if $\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)+\tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right)=\frac{2}{3}$ then $a+c$

A. 3b

B. 2b

C. 3b/2

D. 4b

Answer: B

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15. In a triangle ABC if $\tan.\frac{A}{2}\tan.\frac{B}{2}=\frac{1}{3}$ and ab = 4, then the value of c can be

A. 1

B. 1.5

C. 2.5

D. none of these

Answer: C

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16. In a triangle ABC if $\cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)=c, \cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)=a$ and

$\cot\left(\frac{C}{2}\right)\cot\left(\frac{A}{2}\right) = b$ then $\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} =$

A. -1

B. 0

C. 1

D. 2

Answer: D

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17. The perimeter of a triangle ABC right angled at C is 70 and the inradius is 6, then $|a-b| =$

A. 1

B. 2

C. 8

D. 9

Answer: A

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18. The diagonals of a parallelogram are inclined to each other at an angle of 45°, while its sides a and $b(a > 0)$ are inclined to each other at an angle of 30°, then the value of $\frac{a}{b}$ is

A. $\frac{3}{2}$

B. $\frac{3+\sqrt{5}}{2}$

C. $\frac{3+\sqrt{5}}{4}$

D. $\frac{\sqrt{5}+1}{2}$

Answer: D

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19. In a triangle ABC if $2\Delta^2 = \frac{a^2b^2c^2}{a^2+b^2+c^2}$, then it is

A. equilateral

B. isosceles but not right angled

C. isosceles right angled

D. right angled

Answer: D

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20. If in any triangle, the area $\Delta ABC \leq \frac{b^2+c^2}{\lambda}$, then the largest possible numerical value of λ is

A. 1

B. 2

C. 3

D. 4

Answer: D

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21. In any triangle ABC, if $2\Delta a - b^2c = c^3$, (where Δ is the area of triangle), then which of the following is possible ?

A. B is obtuse

B. A is obtuse

C. C is obtuse

D. B is right angle

Answer: B

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22. In ΔABC, $a^2(s-a)+b^2(s-b)+c^2(s-c)=$

A. $4R\Delta(\cos A+\sin B+\cos C)$

B. $4R\Delta(\sin A+\sin B+\sin C)$

C. $4R\Delta\left(1+4\sin.\frac{A}{2}\sin.\frac{B}{2}\sin.\frac{C}{2}\right)$

D. none of these

Answer: C

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23. Let ABC be an equilateral triangle, let KLMN be a rectangle with K, L on BC , M on AC and N on AB. Suppose $AN/NB=2$ and the area of triangle BKN is 6. The area of the triangle ABC is -

A. 54

B. 108

C. 48

D. none of these

Answer: B

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24. ABC is an acute angled triangle with circumcenter O and orthocentre H. If AO=AH, then find the angle A.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: C

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25. In a triangle ABC if $\angle ABC = 60^\circ$, then $\left(\frac{AB - BC + CA}{r}\right)^2 =$

A. 10

B. 11

C. 12

D. 14

Answer: C

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26. The area of an acute triangle ABC is Δ, the area of its pedal triangle is 'p' , where $\cos B = \frac{2p}{\Delta}$ and $\sin B = \frac{2\sqrt{3}p}{\Delta}$. The value of $8\left(\cos^2 A \cos B + \cos^2 C\right)$ is

A. 1

B. 2

C. 3

D. none of these

Answer: C

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27. In a triangle ABC, D is a point on BC such that AD is the internal bisector of $\angle A$. Let $\angle B = 2\angle C$ and CD = AB. Then $\angle A$ is

A. 18°

B. 36°

C. 54°

D. 72°

Answer: D

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28. In ΔABC, circumrdius is 3 inradius is 1.5 units. The value of $a\cot^2 A + b^2\cot^3 B + c^3\cot^4 C$ is

A. $13\sqrt{3}$

B. $11\sqrt{6}$

C. 21

D. none of these

Answer: A

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29. AD, BE, CF are internal angular bisectors of ΔABC and I is the incentre. If $a(b+c)\sec.\frac{A}{2}ID + b(a+c)\sec.\frac{B}{2}IE + c(a+b)\sec.\frac{C}{2}IF = kabc$, then the value of k is (a) 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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30. In ΔABC it is given distance between the circumcentre (O) and orthocentre (H) is $R\sqrt{1-8\cos A\cos B\cos C}$. If Q is the midopoint of OH, then AQ is

A. $\frac{R}{2}\sqrt{1+8\cos A\sin B\sin C}$

B. $R\sqrt{1+8\cos A\sin B\sin C}$

C. $2R\sqrt{1+8\cos A\sin B\sin C}$

D. $\frac{R}{2}\sqrt{1+8\sin A\cos B\cos C}$

Answer: A

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31. In any ΔABC line joiningcircumcentre (O) and incentre (I) is parallel to AC, then OI is equal to

A. $R\left|\tan\left(\frac{A-C}{2}\right)\right|$

B. $R|\tan(A-C)|$

C. $R\left|\sec\left(\frac{A-C}{2}\right)\right|$

D. $R|\sec(A-C)|$

Answer: A

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32. Let the incircle of a ΔABC touches sides BC, CA and AB at D,E and F, respectively. Let area of ΔABC be Δ and thatof DEF be Δ'. If a, b and c are side of $Detla\ ABC$, then the value of $abc(a+b+c)\frac{\Delta'}{\Delta^3}$ is (a) 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: D

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33. Let H be the orthocentre of triangle ABC. Then angle subtended by side BC at the centre of incircle of ΔCHB is

A. $\frac{A}{2} + 90^\circ$

B. $\frac{B+C}{2} + 90^\circ$

C. $\frac{B-C}{2} + 90^\circ$

D. none of these

Answer: B

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34. If in a triangle $ABC, r_1 + r_2 + r_3 = 9r$, then the triangle is necessarily

A. right angled

B. equilateral

C. obtuse angled

D. none of these

Answer: B

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35. In triangle $ABC, r = \frac{R}{6}$ and $r_1 = 7r$. Then the measure of angle A =

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: D

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36. $(r_2 + r_3)\sqrt{\frac{rr_1}{r_2 r_3}} =$

A. a

B. b

C. c

D. bc

Answer: A

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37. In ΔABC, right angled at A, $\cos^{-1}\left(\frac{R}{r_2+r_3}\right)$ is

A. 30°

B. 60°

C. 90°

D. 45°

Answer: B

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38. In ΔABC if $r_1 = 2r_2 = 3r_3$ and D is the mid point of BC then $\cos\angle ADC$ is (a) $\frac{7}{25}$ (b) $-\frac{7}{25}$ (c) $\frac{24}{25}$ (d) $-\frac{24}{25}$

A. $\frac{7}{25}$

B. $-\frac{7}{25}$

C. $\frac{24}{25}$

D. $-\frac{24}{25}$

Answer: D

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39. In the ambiguous case if the remaining angles of a triangle with given a, b, A and B_1, B_2, C_1, C_2 then $\frac{\sin C_1}{\sin B_1} + \frac{\sin C_2}{\sin B_2} =$

A. 2 cos A

B. 2 sin B

C. 2 tan A

D. 2 cot A

Answer: A

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40. Two sides of a triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is 30. How many such triangles are possible? Find the length of their third side and area.

A. 0

B. 1

C. 2

D. infinite

Answer: C

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41. If circumradius of triangle is 2, then the maximum value of $\frac{abc}{a+b+c}$ is

A. 1

B. 2

C. 3

D. 4

Answer: D

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42. Let ABC and AB'C be two non-congruent triangles with sides BC=B'C=5, AC=6, and $\angle A$ is fixed. If A_1 and A_2 are the area of the two triangles ABC and AB'C, then the value of $\frac{A_1^2 + A_2^2 - 2A_1A_2\cos 2A}{(A_1 + A_2)^2}$ is

A. $9/36$

B. $25/36$

C. $25/16$

D. $16/25$

Answer: B

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Multiple Correct Answers Type

1. Let a,b,c be the sides of a triangle ABC, a=2c,cos(A-C)+cos B=1. then the value of C is

A. $\pi/6$

B. $\pi/3$

C. $2\pi/3$

D. $5\pi/6$

Answer: A::D

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2. If A, B, C are the angles of a triangle such that $\sin^2 A + \sin^2 B = \sin^2 C$, then

A. sin A + sin B >1

B. tan A tan B = 1

C. sin A + sin B = 1

D. tan A. tan B < 1

Answer: A::B

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3. In ΔABC, $\angle C = 2\angle A$ and AC=2BC. Then which of the following is/are True ?

A. Angles A,B,C are in arithmetic progression

B. Angles A,C,B are in arithmetic progression

C. ΔABC is a right angled isosceles triangle

D. $BC^2 + CA^2 + AB^2 = 8R^2$, where R is the circum-radius of ΔABC

Answer: B::D

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4. ΔABC is isosceles with AB=AC=7cm and BC=9cm. The height AD from A to BC, is 6cm. Find the area of ΔABC. What will be the height from C to AB i.e., CE?

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5. If in a triangle ABC, θ is the angle determined by $\cos\theta = (a-b)/c$, then

A. $\frac{(a+b)\sin\theta}{2\sqrt{ab}} = \frac{\cos(A-B)}{2}$

B. $\frac{(a+b)\sin\theta}{2\sqrt{ab}} = \frac{\cos(A+B)}{2}$

C. $\frac{c\sin\theta}{2\sqrt{ab}} = \frac{\cos(A-B)}{2}$

D. $\frac{c\sin\theta}{2\sqrt{ab}} = \frac{\cos(A+B)}{2}$

Answer: A::D

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6. Let a,b,c be the sides BC, CA, AB of ΔABC on xy plane. If abscissa and ordinate of vertices of the triangle are integers and R is the

circumradius, then 2R can be equal to

A. $\frac{8}{9}abc$

B. abc

C. $\frac{9}{8}abc$

D. $\frac{abc}{2}$

Answer: A::B::D

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7. In a ΔABC, if $\tan.\frac{A}{2}=\frac{5}{6}, \tan.\frac{B}{2}=\frac{20}{37}$, then which of the following is/are correct ?

A. $\angle B > \angle C$

B. $\angle B < \angle C$

C. $a > b > c$

D. $a < b < c$

Answer: A::C

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8. If area of $\Delta ABC(\Delta)$ and angle C are given and if c opposite to given angle is minimum, then

A. $a = \sqrt{\frac{2\Delta}{\sin C}}$

B. $b = \sqrt{\frac{2\Delta}{\sin C}}$

C. $a = \frac{4\Delta}{\sin C}$

D. $b = \frac{4\Delta}{\sin^2 C}$

Answer: A::B

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9. Let 'P' be an interior point of ΔABC. If $\angle A = 45^\circ, \angle B = 60^\circ$ and $\angle C = 75^\circ$. If X=area of ΔPBC, $Y =$ area of ΔPAC and Z = area of ΔPAB, then which of the following ratios is/are true ?

A. If P is the centroid, then X : Y : Z is 1 : 1 : 1

B. If P is the incentre, then X : Y : Z is $2 : \sqrt{6} : \left(\sqrt{3}+1\right)$

C. If P the orthocentre, then X : Y : Z is $1 : \sqrt{3} : \left(2+\sqrt{3}\right)$

D. If P is the circumcentre, then X : Y : Z is $2 : \sqrt{3} : 1$

Answer: A::B::C::D

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10. Let 'l' is the length of median from the vertex A to the side BC of a ΔABC. Then

A. $4l^2 = 2b^2 + 2c^2 - a^2$

B. $4l^2 = b^2 + c^2 + 2bc\cos A$

C. $4l^2 = a^2 + 4bc\cos A$

D. $4l^2 = (2s - a)^2 \sin^2 \frac{A}{2}$

Answer: A::B::C::D

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11. A circle having centre as O' and radius r' touches the incircle of ΔABC externally at. F, where F is on BC and also touches its circumcircle internally at G. It O is the circumcentre of ΔABC and I is its incentre, then

A. OO'=R-r'

B. Perpendicular distance from O to line joining IO' is $\left|\frac{b-c}{2}\right|$

C. Projection of OO' on line joining IO'=r'+R cos A

D. $r' = \frac{\Delta}{a}\tan^2 A$

Answer: A::B::C::D

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12. In triangle ABC, if $r_1 + r_2 = 3R$ and $r_2 + r_3 = 2R$, then

A. $\angle A = 90^\circ$

B. $\angle B = 45^\circ$

C. $\angle C = 60^\circ$

D. triangle ABC is right angled isosceles

Answer: A::C

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13. The radii r_1, r_2, r_3 of the escribed circles of the triangle ABC are in H.P. If the area of the triangle is $24cm^2 and$ its perimeter is 24cm, then the length of its largest side is 10 (b) 9 (c) 8 (d) none of these

A. The length of longest side of triangle ABC is equal to 10

B. The radius of circle inscribed in triangle ABC is equal to 4.

C. The circumradius of triangle ABC is equal to 5.

D. The sides of triangle ABC are in A.P.

Answer: A::C::D

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Comprehension Type

1. Let ABC be a triangle in which the line joining the circumecentre and incentre is parallel to base BC of the triangle. Then answer the following questions :

Then range of $\angle A$ is

A. $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

B. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

C. $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right] = \left\{\frac{\pi}{3}\right\}$

D. $\left[0, \frac{\pi}{2}\right]$

Answer: B

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2. Let ABC be a triangle in which the line joining the circumecentre and incentre is parallel to base BC of the triangle. Then answer the

following questions :

If ODEI is a square where O and I stands for circumcentre and incentre, respectively and D and E are the point of perpendicular from O and I on the base BC, then

A. $\frac{r}{R} = \frac{3}{8}$

B. $\frac{r}{R} = 2 - \sqrt{3}$

C. $\frac{r}{R} = \sqrt{2} - 1$

D. $\frac{r}{R} = \frac{1}{4}$

Answer: C

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3. Let ABC be a triangle in which the line joining the circumecentre and incentre is parallel to base BC of the triangle. Then answer the

following questions :

If $\angle A = 60^\circ$, then ΔABC is

A. isoceles

B. right angled

C. right angled isosceles

D. equilateral

Answer: D

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4. Incircle of ΔABC touches AB, BC, CA at R, P, Q, respectively. If $\frac{2}{AR} + \frac{5}{BP} + \frac{5}{CQ} = \frac{6}{r}$ and the perimeter of the triangle is the smallest integer, then answer the following questions :

ΔABC is

A. A. scalene

B. B.isosceles

C. C. equilateral

D. D.right angled

Answer: B

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5. Incircle of ΔABC touches AB, BC, CA at R, P, Q, respectively. If $\frac{2}{AR}+\frac{5}{BP}+\frac{5}{CQ}=\frac{6}{r}$ and the perimeter of the triangle is the smallest integer, then answer the following questions :

The inradius of incircle of ΔABC is

A. 4

B. 3

C. 2

D. 1

Answer: C

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6. Incircle of ΔABC touches AB, BC, CA at R, P, Q, respectively. If $\frac{2}{AR}+\frac{5}{BP}+\frac{5}{CQ}=\frac{6}{r}$ and the perimeter of the triangle is the smallest integer, then answer the following questions :

The area of ΔABC is

A. 15 sq. units

B. 21 sq. units

C. 24 sq. units

D. 27 sq. units

Answer: D

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