MATHS

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VECTOR ALGEBRA

Others

1. In a trapezium ABCD, BC AD and AD = 4 cm. the two diagonals AC and BD intersect at the point O in such a way that AO/OC = DO/OB = 1/2. Calculate the length of BC.

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2. If the vectors $\vec{a} and \vec{b}$ are linearly idependent satisfying $\left(\sqrt{3}\tan\theta + 1\right)\vec{a} + \left(\sqrt{3}sec\theta - 2\right)\vec{b} = 0$, then the most general values of θ

are a. $2n\pi - \frac{\pi}{6}, n \in Z$ b. $2n\pi \pm \frac{11\pi}{6}, n \in Z$ c. $n\pi \pm \frac{\pi}{6}, n \in Z$ d. $2n\pi + \frac{11\pi}{6}, n \in Z$

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3. Given three non-zero, non-coplanar vectors $\vec{a}, \vec{b}$, and $\vec{c}. \vec{r}_1 = p\vec{a} + q\vec{b} + \vec{c} and \vec{r}_2 = \vec{a} + p\vec{b} + q\vec{c}$ If the vectors $\vec{r}_1 + 2\vec{r}_2$ and $2\vec{r}_1 + \vec{r}_2$ are collinear, then (p, q) is `

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4. Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots . \vec{r}_n$ be the position vectors of points $P_1, P_2, P_3, \ldots\ldots P_n$ relative to the origin O. If the vector equation $a_1\vec{r}_1 + a_2\vec{r}_2 + \ldots . a_n\vec{r}_n = 0$ hold, then a similar equation will also hold w.r.t. to any other origin provided a) $a_1 + a_2 + \ldots\ldots a_n = n$ b) $a_1 + a_2 + \ldots . a_n = 1$ c) $a_1 + a_2 + \ldots . a_n = 0$ d) $a_1 = a_2 = a_3 + a_n = 0$

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5. In triangle ABC, $\angle A = 30^0$, H is the orthocenter and D is the midpoint of BC. Segment HD is produced to T such that $HD = DT$ The length AT is equal to

(a). $2BC$

(b). $3BC$

(c). $\frac{4}{2}BC$

(d). none of these

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6. If $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = a\vec{\delta} and \vec{\beta} + \vec{\gamma} + \vec{\delta} = b\vec{\alpha}, \vec{\alpha} and \vec{\delta}$ are non-colliner, then $\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta}$ equals a. $a\vec{\alpha}$ b. $b\vec{\delta}$ c. 0 d. $(a + b)\vec{\gamma}$

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7. Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}, \vec{b} = 2\hat{i} - 6\hat{j} and \vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$ Then the resolution of the vector $\vec{\alpha}$ into components with respect to $\vec{a} and \vec{b}$ is given by a. $3\vec{a} - 2\vec{b}$ b. $3\vec{b} - 2\vec{a}$ c. $2\vec{a} - 3\vec{b}$ d. $\vec{a} - 2\vec{b}$

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8. Let us define the length of a vector $a\hat{i} + b\hat{j} + c\hat{k} as |a| + |b| + |c|$. This definition coincides with the usual definition of length of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is and only if (a) $a = b = c = 0$ (b) any two of $a, b, and c$ are zero (c) any one of $a, b, and c$ is zero (d) $a + b + c = 0$

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9. Vectors $\vec{a} = -4\hat{i} + 3\hat{k}; \vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are laid off from one point. Vector $\hat{d}$, which is being laid of from the same point dividing the angle between vectors $\vec{a} and \vec{b}$ in equal halves and having the magnitude $\sqrt{6},$ is a. $\hat{i} + \hat{j} + 2\hat{k}$ b. $\hat{i} - \hat{j} + 2\hat{k}$ c. $\hat{i} + \hat{j} - 2\hat{k}$ d. $2\hat{i} - \hat{j} - 2\hat{k}$

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10. Vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k},$ are so placed that the end point of one vector is the starting point of the next vector. Then the vector are (A) not coplanar (B) coplanar but cannot form a triangle (C) coplanar and form a triangle (D) coplanar and can form a right angled triangle

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11. The position vectors of the vertices $A, B, andC$ of a triangle are $\hat{i} + \hat{j}, \hat{j} + \hat{k} and \hat{i} + \hat{k}$, respectively. Find the unit vector $\hat{r}$ lying in the plane of ABC and perpendicular to $IA, whereI$ is the incentre of the triangle.

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12. A ship is sailing towards the north at a speed of 1.25 m/s. The current is taking it towards the east at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 0.5 m/s. Find the velocity of the sailor in space.

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13. Given four points $P_1, P_2, P_3 and P_4$ on the coordinate plane with origin O which satisfy the condition $\left(\overrightarrow{OP}\right)_{n-1} + \left(\overrightarrow{OP}\right)_{n+1} = \frac{3}{2}\overrightarrow{OP}_n$. If P1 and P2 lie on the curve xy=1 , then prove that P3 does not lie on the curve

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14. The vectors $\vec{a}$ and $\vec{b}$ are non collinear. If $\vec{p} = (x + 4y)\vec{a} + (2x + y + 1)\vec{b}$ and $\vec{q} = (- 2x + y + 2)\vec{a} + (2x - 3y - 1)\vec{b}$ satisfy the relation $3\vec{p} = 2\vec{q}$ find the values of x and y.

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15. If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three non-coplanar vectors, then prove that points are collinear: $\vec{a} + \vec{b} + \vec{c}, 4\vec{a} + 3\vec{b}, 10\vec{a} + 7\vec{b} - 2\vec{c}$.

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16. If $\vec{a}, \vec{b} and \vec{c}$ are three non-zero non-coplanar vectors, then the value of

$(\vec{a}.\vec{a})\vec{b}\times\vec{c}+(\vec{a}.\vec{b})\vec{c}\times\vec{a}+(\vec{a}.\vec{c})\vec{a}\times\vec{b}.$

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17. Let a, b, c be distinct non-negative numbers an the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}, c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then prove that the quadratic equation $ax^2+2cx+b=0$ has equal roots

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18. A pyramid with vertex at point P has a regular hexagonal base $ABCDEF$, Position vector of points A and B are $\hat{i}$ and $\hat{i}+2\hat{j}$ The centre of base has the position vector $\hat{i}+\hat{j}+\sqrt{3}\hat{k}^{\cdot}$ Altitude drawn from P on the base meets the diagonal AD at point $\overset{\cdot}{G}$ find the all possible position vectors of $G^{\cdot}$ It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP is 5 units.

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19. $ABCD$ is a parallelogram. If L and M be the middle points of BC and CD, respectively express $\overrightarrow{AL}$ and $\overrightarrow{AM}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also show that $\overrightarrow{AL} + \overrightarrow{AM} = (3/2)\overrightarrow{AC}$.

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20. A, B, C and D have position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d},$ respectively, such that $\vec{a} - \vec{b} = 2\left(\vec{d} - \vec{c}\right)^{\cdot}$ Then a. $ABandCD$ bisect each other b. $BDandAC$ bisect each other c. $ABandCD$ trisect each other d. $BDandAC$ trisect each other

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21. If $\vec{a} and \vec{b}$ are two unit vectors and θ is the angle between them, then the unit vector along the angular bisector of $\vec{a}$ and $\vec{b}$ will be given by a. $\dfrac{\vec{a} - \vec{b}}{\cos(\theta/2)}$ b. $\dfrac{\vec{a} + \vec{b}}{2\cos(\theta/2)}$ c. $\dfrac{\vec{a} - \vec{b}}{2\cos(\theta/2)}$ d. none of these

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22. $ABCD$ is a quadrilateral. E is the point of intersection of the line joining the midpoints of the opposite sides. If O is any point and $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = x\vec{O}E, thenx$ is equal to a. 3 b. 9 c. 7 d. 4

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23. If vectors $\vec{A}B = -3\hat{i} + 4\hat{k} and \vec{A}C = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a $\triangle ABC$, then the length of the median through $A is$ a. $\sqrt{14}$ b. $\sqrt{18}$ c. $\sqrt{29}$ d. $\sqrt{5}$

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24. $ABCD$ parallelogram, and $A_1 and B_1$ are the midpoints of sides $BC and CD$, respectivley . If $\vec{A}A_1 + \vec{A}B_1 = \lambda\vec{A}C, then \lambda$ is equal to a. $\frac{1}{2}$ b. 1 c. $\frac{3}{2}$ d. 2 e. $\frac{2}{3}$

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25. The position vectors of the points $P and Q$ with respect to the origin O are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of $\angle POQ$, then $\vec{O}M$ is a. $2\left(\hat{i} - \hat{j} + \hat{k}\right)$ b. $2\hat{i} + \hat{j} - 2\hat{k}$ c. $2\left(-\hat{i} + \hat{j} - \hat{k}\right)$ d. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

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26. A point O is the centre of a circle circumscribed about a triangleABC

Then $\vec{O}A\sin 2A + \vec{O}B\sin 2B + \vec{O}C\sin 2C$ is equal to

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27. If G is the centroid of triangle $ABC, then\vec{G}A + \vec{G}B + \vec{G}C$ is equal to a. $\vec{0}$ b. $3\vec{G}A$ c. $3\vec{G}B$ d. $3\vec{G}C$

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28. Let ABC be triangle, the position vectors of whose vertices are respectively $\hat{i} + 2\hat{j} + 4\hat{k}$, -$2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j}$ - $3\hat{k}$. Then $\mathrm{Delta}ABC$ is a. isosceles b. equilateral c. right angled d. none of these

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29. If $\left|\vec{a} + \vec{b}\right| < \left|\vec{a} - \vec{b}\right|$, then the angle between $\vec{a} and \vec{b}$ can lie in the interval a. $(\pi/2, \pi/2)$ b. $(0, \pi)$ c. $(\pi/2, 3\pi/2)$ d. $(0, 2\pi)$

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30. $'I'$ is the incentre of triangle ABC whose corresponding sides are $a, b, c,$ rspectively. $a\vec{I}A + b\vec{I}B + c\vec{I}C$ is always equal to a. $\vec{0}$ b. $(a+b+c)\vec{B}C$ c. $\left(\vec{a}+\vec{b}+\vec{c}\right)\vec{A}C$ d. $(a+b+c)\vec{A}B$

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31. Let $x^2 + 3y^2 = 3$ be the equation of an ellipse in the $x - y$ plane. $AandB$ are two points whose position vectors are $-\sqrt{3}\hat{i}and - \sqrt{3}\hat{i} + 2\hat{k}$. Then the position vector of a point P on the ellipse such that $\angle APB = \pi/4$ is a. $\pm\hat{j}$ b. $\pm\left(\hat{i}+\hat{j}\right)$ c. $\pm\hat{i}$ d. none of these

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32. Locus of the point P, for which $\overrightarrow{OP}$ represents a vector with direction cosine $\cos\alpha = \frac{1}{2}$ (where O is the origin) is

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33. If $\vec{x} and \vec{y}$ are two non-collinear vectors and ABC isa triangle with side lengths $a, b, and c$ satisfying $(20a - 15b)\vec{x} + (15b - 12c)\vec{y} + (12c - 20a)\left(\vec{x} \times \vec{y}\right) = 0,$ then triangle ABC is

a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. an isosceles triangle

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34. If $\hat{i} - 3\hat{j} + 5\hat{k}$ bisects the angle between $\hat{a}$ and $- \hat{i} + 2\hat{j} + 2\hat{k}, where \hat{a}$ is a unit vector, then a. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} - 40\hat{k}\right)$ b. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} + 40\hat{k}\right)$ c. $\hat{a} = \frac{1}{105}\left(- 41\hat{i} + 88\hat{j} - 40\hat{k}\right)$ d. $\hat{a} = \frac{1}{105}\left(41\hat{i} - 88\hat{j} - 40\hat{k}\right)$

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35. If $4\hat{i} + 7\hat{j} + 8\hat{k}, 2\hat{i} + 3\hat{j} + 4\hat{j} and 2\hat{i} + 5\hat{j} + 7\hat{k}$ are the position vectors of the vertices $A, B and C,$ respectively, of triangle ABC , then the position vecrtor of the point where the bisector of angle A meets BC is a. $\frac{2}{3}\left(-6\hat{i}-8\hat{j}-\hat{k}\right)$ b. $\frac{2}{3}\left(6\hat{i}+8\hat{j}+6\hat{k}\right)$ c. $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$ d. $\frac{1}{3}\left(5\hat{j}+12\hat{k}\right)$

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36. If $\vec{b}$ is a vector whose initial point divides thejoin of $5\hat{i} and 5\hat{j}$ in the ratio $k:1$ and whose terminal point is the origin and $\left|\vec{b}\right| \le \sqrt{37}, thenk$ lies in the interval a. $[-6, -1/6]$ b. $(-\infty, -6] \cup [-1/6, \infty)$ c. $[0, 6]$ d. none of these

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37. Find the value of λ so that the points P, Q, R and S on the sides OA, OB, OC and $AB,$ respectively, of a regular tetrahedron $OABC$ are coplanar. It is given that $\frac{OP}{OA}=\frac{1}{3}, \frac{OQ}{OB}=\frac{1}{2}, \frac{OR}{OC}=\frac{1}{3}$ and $\frac{OS}{AB}=\lambda$. (A) $\lambda=\frac{1}{2}$ (B) $\lambda=-1$ (C) $\lambda=0$ (D) for no value of λ

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38. A uni-modular tangent vector on the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6$ at $t = 2$ is

a. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ b.$\frac{1}{3}\left(\hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{6}\left(2\hat{i} + \hat{j} + \hat{k}\right)$ d. $\frac{2}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

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39. If $\vec{x} and \vec{y}$ are two non-collinear vectors and a, b, and c represent the sides of a ABC satisfying $(a - b)\vec{x} + (b - c)\vec{y} + (c - a)\left(\vec{x} \times \vec{y}\right) = 0,$ then ABC is (where $\overset{\to}{\times} x\vec{y}$ is perpendicular to the plane of $xandy$) a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. a scalene triangle

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40. The position vectors of points $AandB$ w.r.t. the origin are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k},\ \vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ respectively. Determine vector $\vec{OP}$ which bisects angle $AOB,$ where P is a point on AB

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41. What is the unit vector parallel to $\vec{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$? What vector should be added to $\vec{a}$ so that the resultant is the unit vector $\hat{i}$?

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42. ABCD is a quadrilateral and E is the point of intersection of the lines joining the middle points of opposite side. Show that the resultant of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ = 4 $\overrightarrow{OE}$, where O is any point.

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43. A straight line L cuts the lines $AB, ACandAD$ of a parallelogram $ABCD$ at points $B_1, C_1andD_1,$ respectively. If

$\left(\vec{A}B\right)_1, \lambda_1\vec{A}B, \left(\vec{A}D\right)_1 = \lambda_2\vec{A}Dand\left(\vec{A}C\right)_1 = \lambda_3\vec{A}C,$ then prove that $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$.

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44. Find the vector of magnitude 3, bisecting the angle between the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

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45. If $\vec{a}and\vec{b}$ are two vectors of magnitude 1 inclined at 120^0 , then find the angle between $\vec{b}and\vec{b} - \vec{a}$.

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46. If $\vec{r}_1, \vec{r}_2, \vec{r}_3$ are the position vectors of the collinear points and scalar $pandq$ exist such that $\vec{r}_1 = p\vec{r}_2 + q\vec{r}_3,$ then show that $p + q = 1.$

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47. Examine the following vector for linear independence:

(1) $\vec{i} + \vec{j} + \vec{k}, 2\vec{i} + 3\vec{j} - \vec{k}, -\vec{i} - 2\vec{j} + 2\vec{k}$

(2) $3\vec{i} + \vec{j} - \vec{k}, 2\vec{i} - \vec{j} + 7\vec{k}, 7\vec{i} - \vec{j} + 13\vec{k}$

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48. Show that the vectors $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c} and \vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors (where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors)

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49. Let $\vec{a}, \vec{b} and \vec{c}$ be three units vectors such that $2\vec{a} + 4\vec{b} + 5\vec{c} = 0.$ Then which of the following statement is true? a. $\vec{a}$ is parallel to $\vec{b}$ b. $\vec{a}$ is perpendicular to $\vec{b}$ c. $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$ d. none of these

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50. Four non –zero vectors will always be a. linearly dependent b. linearly independent c. either a or b d. none of these

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51. A boat moves in still water with a velocity which is k times less than the river flow velocity. Find the angle to the stream direction at which the boat should be rowed to minimize drifting.

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52. In a triangle $PQR, SandT$ are points on $QRandPR,$ respectively, such that $QS = 3SRandPT = 4TR$. Let M be the point of intersection of $PSandQT$. Determine the ratio $QM : MT$ using the vector method .

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53. In a quadrilateral $PQRS, \vec{PQ} = \vec{a}, \vec{QR} = \vec{b}, \vec{SP} = \vec{a} - \vec{b}, M$ is the midpoint of $\vec{Q}RandX$ is a point on SM such that $SX = \frac{4}{5}SM$. Prove that $P, XandR$ are collinear.

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54. solve the differential equation $\left(1 + x^2\right)\frac{dy}{dx} = x$

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55. If points $\hat{i} + \hat{j}, \hat{i} - \hat{j}$ and $p\hat{i} + q\hat{j} + r\hat{k}$ are collinear, then

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56. The position vector of the points P and Q are $5\hat{i} + 7\hat{j} - 2\hat{k}$ and $-3\hat{i} + 3\hat{j} + 6\hat{k}$, respectively. Vector $\vec{A} = 3\hat{i} - \hat{j} + \hat{k}$ passes through point P

and vector $\vec{B} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through point Q . A third vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ intersects vectors A and B. Find the position vectors of points of intersection.

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57. Consider the vectors $\hat{i} + \cos(\beta - \alpha)\hat{j} + \cos(\gamma - \alpha)\hat{k}, \cos(\alpha - \beta)\hat{i} + \hat{j} + \cos(\gamma - \beta)\hat{k}$ and $\cos(\alpha - \gamma)\hat{i} + \cos(\beta - \gamma)\hat{j} + a\hat{k}$where $\alpha, \beta,$ and γ are different angles. If these vectors are coplanar, show that a is independent of α, β and γ

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58. If $\vec{A}$ and $\vec{B}$ are two vectors and k any scalar quantity greater than zero, then prove that $\left|\vec{A} + \vec{B}\right|^2 \leq (1 + k)\left|\vec{A}\right|^2 + \left(1 + \frac{1}{k}\right)\left|\vec{B}\right|^2$.

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59. The vectors $x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}, (x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k}$ and $(x+6)\hat{i} + (x+7)\hat{j} + (x+8)\hat{k}$ are coplanar if x is equal to a. 1 b. -3 c. 4 d. 0

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60. $\vec{A}$ isa vector with direction cosines $\cos\alpha, \cos\beta and \cos\gamma$ Assuming the y - z plane as a mirror, the directin cosines of the reflected image of $\vec{A}$ in the plane are a. $\cos\alpha, \cos\beta, \cos\gamma$ b. $\cos\alpha, -\cos\beta, \cos\gamma$ c. $-\cos\alpha, \cos\beta, \cos\gamma$ d. $-\cos\alpha, -\cos\beta, -\cos\gamma$

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61. A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system, this system is rotated through a certain clockwise sense, if we write the new system $\vec{a}$ has components (p+1) and 1 then

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62. The sides of a parallelogram are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. The unit vector parallel to one of the diagonals is a. $\frac{1}{7}\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)$ b. $\frac{1}{7}\left(3\hat{i} - 6\hat{j} - 2\hat{k}\right)$ c. $\frac{1}{\sqrt{69}}\left(\hat{i} + 6\hat{j} + 8\hat{k}\right)$ d. $\frac{1}{\sqrt{69}}\left(-\hat{i} - 2\hat{j} + 8\hat{k}\right)$

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63. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + \mu\vec{c} and (2\lambda - 1)\vec{c}$ are coplanar when a. $\mu \in R$ b. $\lambda = \frac{1}{2}$ c. $\lambda = 0$ d. no value of λ

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64. If points $\hat{i} + \hat{j}, \hat{i} - \hat{j}$ and $p\hat{i} + q\hat{j} + r\hat{k}$ are collinear, then

A. a. $p = 1$

B. b. $r = 0$

C. c. $q \in R$

D. d. $q \neq 1$

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65. If the vectors $\hat{i} - \hat{j}, \hat{j} + \hat{k}$ and $\vec{a}$ form a triangle, then $\vec{a}$ may be a. $-\hat{i} - \hat{k}$ b. $\hat{i} - 2\hat{j} - \hat{k}$ c. $2\hat{i} + \hat{j} + \hat{k}$ d. $\hat{i} + \hat{k}$

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66. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \vec{F}_2 = 6\hat{i} - \hat{k} and \vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a parricle has magnitude equal to 5 units, then the value of p is a. -6 b. -4 c. 2 d. 4

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67. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors satisfying the condition $\vec{a} + \vec{b} + \vec{c} = 0$ then show that $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -3/2.$

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68. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$. Then value of x are (a)-$\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

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69. Prove that point $\hat{i}$ +2$\hat{j}$ - 3$\hat{k}$,2$\hat{i}$ - $\hat{j}$ + $\hat{k}$ and 2$\hat{i}$ + 5$\hat{j}$ - $\hat{k}$ from a triangle in space.

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70. Show that the point A, B and C with position vectors $\vec{a}$ =3$\hat{i}$ - 4$\hat{j}$ -4$\hat{k}$ $\vec{b}$= 2 $\hat{i}$ - $\hat{j}$ + $\hat{k}$ and $\vec{c}$ = $\hat{i}$ - 3$\hat{j}$ - 5$\hat{k}$, respectively form the vertices of a right angled triangle.

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71. If $2\overrightarrow{AC} = 3\overrightarrow{CB}$, then prove that $2\overrightarrow{OA} + 3\overrightarrow{OB} = 5\overrightarrow{OC}$ where O is the origin.

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72. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points $(1, 2, 3)$ and $(4, 5, 6)$, respectively.

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73. For given vector, $\vec{a} = 2\hat{i}\ j + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

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74. If the projections of vector $\vec{a}$ on x -, y - and z -axes are 2, 1 and 2 units ,respectively, find the angle at which vector $\vec{a}$ is inclined to the z -axis.

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75. Find a vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

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76. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are the position vector of point A, B, C and D, respectively referred to the same origin O such that no three of these point are collinear and $\vec{a} + \vec{c} = \vec{b} + \vec{d}$, than prove that quadrilateral $ABCD$ is a parallelogram.

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77. Show that the points $A(6, -7, 0), B(16, -19, -4), C(0, 3, -6)$ and $D(2, -5, 10)$ are such that AB and CD intersect at the point $P(1, -1, 2)$.

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78. Statement 1: The direction cosines of one of the angular bisectors of two intersecting line having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2^{.}$ Statement 2: The angle between the two intersection lines having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2^{.}$

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79. Statement 1: In $\Delta ABC, \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

Statement 2: If $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, then\ \overrightarrow{AB} = \vec{a} + \vec{b}$

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80. If $\vec{a} and \vec{b}$ are two vectors of magnitude 1 inclined at 120^0 , then find the angle between $\vec{b} and \vec{b} - \vec{a}^{.}$

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81. $\vec{A}$ isa vector with direction cosines $\cos\alpha, \cos\beta and \cos\gamma$. Assuming the $y - z$ plane as a mirror, the directin cosines of the reflected image of $\vec{A}$ in the plane are a. $\cos\alpha, \cos\beta, \cos\gamma$ b. $\cos\alpha, \; - \cos\beta, \cos\gamma$ c. $-\cos\alpha, \cos\beta, \cos\gamma$ d. $-\cos\alpha, \; - \cos\beta, \; - \cos\gamma$

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82. A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system, this system is rotated through a certain clockwise sense, if we write the new system $\vec{a}$ has components (p+1) and 1 then

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83. Statement 1 : If three point P, Q and R have position vectors $\vec{a}, \vec{b}$ and $\vec{c}$, respectively, and $2\vec{a} + 3\vec{b} - 5\vec{c} = 0$, then the point P, Q and R must be collinear.

Statement 2 : If for three points A, B and C, $\overrightarrow{AB} = \lambda\overrightarrow{AC}$, then points A, B and C must be collinear.

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84. In a four-dimensional space where unit vectors along the axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{l}$, and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as a linear combination of others and $(\lambda - 1)\left(\vec{a}_1 - \vec{a}_2\right) + \mu\left(\vec{a}_2 + \vec{a}_3\right) + \gamma\left(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2\right) + \vec{a}_3 + \delta\vec{a}_4 = 0$, then

A. a. $\lambda = 1$

B. b. $\mu = -2/3$

C. c. $\gamma = 2/3$

D. d. $\delta = 1/3$

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85. Let ABC be a triangle, the position vectors of whose vertices are $-10\hat{i} + 10\hat{k}$, $-\hat{i} + 6\hat{j} + 6\hat{k}$ and $-4\hat{i} + 9\hat{j} + 6\hat{k}$. Then ΔABC is a. isosceles b. equilateral c. right angled d. none of these

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86. If non-zero vectors $\vec{a}$ and $\vec{b}$ are equally inclined to coplanar vector $\vec{c}$, then $\vec{c}$ can be a. $\frac{|\vec{a}|}{|\vec{a}| + 2|\vec{b}|}a + \frac{|\vec{b}|}{|\vec{a}| + |\vec{b}|}\vec{b}$ b. $\frac{|\vec{b}|}{|\vec{a}| + |\vec{b}|}a + \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}\vec{b}$ c. $\frac{|\vec{a}|}{|\vec{a}| + 2|\vec{b}|}a + \frac{|\vec{b}|}{|\vec{a}| + 2|\vec{b}|}\vec{b}$ d. $\frac{|\vec{b}|}{2|\vec{a}| + |\vec{b}|}a + \frac{|\vec{a}|}{2|\vec{a}| + |\vec{b}|}\vec{b}$

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87. If $A(-4, 0, 3)$ and $B(14, 2, -5)$, then which one of the following points lie on the bisector of the angle between $\vec{OA}$ and $\vec{OB}(O$ is the origin of reference)?

a. $(2, 2, 4)$ b. $(2, 11, 5)$ c. $(-3, -3, -6)$ d. $(1, 1, 2)$

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88. Prove that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

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89. Prove that the resultant of two forces acting at point O and represented by $\vec{OB}$ and $\vec{OC}$ is given by $2\vec{OD}$,where D is the midpoint of BC.

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90. Two forces $\vec{AB}$ and $\vec{AD}$ are acting at vertex A of a quadrilateral ABCD and two forces $\vec{CB}$ and $\vec{CD}$ at C prove that their resultant is given by $4\vec{EF}$

, where E and F are the midpoints of AC and BD, respectively.

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91. ABC is a triangle and P any point on BC. If $\overrightarrow{PQ}$ is the sum of $\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PC}$, show that ABQC is a parallelogram and Q, therefore, is a fixed point.

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92. If vector $\vec{a} + \vec{b}$ bisects the angle between $\vec{a}$ and $\vec{b}$, then prove that $\left|\vec{a}\right|$ = $\left|\vec{b}\right|$.

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93. ABCDE is a pentagon. Prove that the resultant of force $\vec{AB}$, $\vec{AE}$,$\vec{BC}$,$\vec{DC}$,$\vec{ED}$ and $\vec{AC}$,is 3$\vec{AC}$.

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94. if $\overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{BO} + \overrightarrow{OC}$,than prove that B is the midpoint of AC.

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95. A unit vector of modulus 2 is equally inclined to x- and y-axes at an angle $\pi/3$. Find the length of projection of the vector on the z-axis.

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96. Find the equations of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

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97. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} - \vec{c} = 0$. If the area of triangle formed by vectors $\vec{a}$ and $\vec{b}$ is A, then what is the value of $4A^2$?

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98. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \vec{F}_2 = 6\hat{i} - \hat{k}$ and $\vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a parricle has magnitude equal to 5 units, then the value of p is a. -6 b. -4 c. 2 d. 4

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99. Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of the four distinct points $A, B, C, D^{.}$ If $\vec{b} - \vec{a} = \vec{c} - \vec{d}$, then show that $ABCD$ is parallelogram.

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100. Statement 1:Let $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 3\hat{k} and \vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$ Then $OABC$ is a tetrahedron. Statement 2: Let $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ be three points such that vectors $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar. Then $OABC$ is a tetrahedron where O is the origin.

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101. Statement 1: If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then $\vec{a}$ and $\vec{b}$ are perpendicular to each other. Statement 2: If the diagonal of a parallelogram are equal magnitude, then the parallelogram is a rectangle. Which of the following Statements is/are correct ?

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102. Statement 1: $\vec{a} = 3\vec{i} + p\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + q\vec{k}$ are parallel vectors if $p = 9/2 and q = 2.$ Statement 2: if

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} and \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ are parallel, then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}.$ Which of the following Statements is/are correct ?

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103. The position vectors of the vertices $A, B and C$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and \vec{c},$ respectively. A vector $\vec{d}$ is such that $\vec{d} \cdot \vec{a} = \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} and \vec{d} = \lambda\left(\vec{b} + \vec{c}\right).$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

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104. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1,$ then find the value of $\left|\vec{a} - \vec{b}\right|.$

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105. Column I, Column II Collinear vectors, p.$\vec{a}$ Coinitial vectors, q. $\vec{b}$ Equal vectors, r. $\vec{c}$ Unlike vectors (same intitial point), s. $\vec{d}$

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106. Statement 1: $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4and\left|\vec{a} + \vec{b}\right| = 5, then\left|\vec{a} - \vec{b}\right| = 5.$

Statement 2: The length of the diagonals of a rectangle is the same.

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107. A man travelling towards east at 8km/h finds that the wind seems to blow directly from the north On doubling the speed, he finds that it appears to come from the north-east. Find the velocity of the wind.

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108. OABCDE is a regular hexagon of side 2 units in the XY-plane in the first quadrant. O being the origin and OA taken along the x-axis. A point P is taken on a line parallel to the z-axis through the centre of the hexagon at a distance of 3 unit from O in the positive Z direction. Then find vector AP.

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109. If $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, determine vector $\vec{c}$ along the internal bisector of the angle between of the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\left|\vec{c}\right| = 5\sqrt{6}$

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110. Find a unit vector $\vec{c}$ if $\vec{-i} + \vec{j} - \vec{k}$ bisects the angle between $\vec{c}$ and $3\vec{i} + 4\vec{j}$.

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111. The vectors $2\hat{i}+3\hat{j}, 5\hat{i}+6\hat{j}$ and $8\hat{i}+\lambda\hat{j}$ have initial points at (1, 1). Find the value of λ so that the vectors terminate on one straight line.

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112. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors, no two of which are collinear, $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then find the value of $\left|\vec{a}+2\vec{b}+6\vec{c}\right|$.

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113. Check whether the given three vectors are coplanar or non-coplanar.

$-2\hat{i}-2\hat{j}+4\hat{k}, \ -2\hat{i}+4\hat{j}, 4\hat{i}-2\hat{j}-2\hat{k}$

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114. Prove that the four points $6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{j} - 6\hat{k}$ and $2\hat{i} + 5\hat{j} + 10\hat{k}$ form a tetrahedron in space.

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115. Show, by vector methods, that the angularbisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.

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116. Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\vec{B}(t) = g(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1], f_1, f_2, g_1, g_2$ are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are non-zero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}, \vec{A}(1) = 6\hat{i} + 2\hat{j}, \vec{B}(0) = 3\hat{i} + 2\hat{i}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$ Then,show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t.

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117. Find the least positive integral value of x for which the angle between vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute.

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118. If vectors $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + 2\hat{k}$ are coplanar, then find the value of $(\lambda - 4)$.

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119. Find the values of λ such that $x, y, z \neq (0, 0, 0)$ and $\left(\hat{i} + \hat{j} + 3\hat{k}\right)x + \left(3\hat{i} - 3\hat{j} + \hat{k}\right)y + \left(-4\hat{i} + 5\hat{j}\right)z = \lambda\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$, where $\hat{i}, \hat{j}, \hat{k}$ are unit vector along coordinate axes.

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120. A vector has component $A_1, A_2 and A_3$ in a right -handed rectangular Cartesian coordinate system $OXYZ$. The coordinate system is rotated about the x-axis through an angel $\pi/2$. Find the component of A in the new coordinate system in terms of $A_1, A_2, and A_3$.

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121. The position vectors of the point A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}, 2\hat{i} + 3\hat{j} - 4\hat{k}, -\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ.

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122. Let $OACB$ be a parallelogram with O at the origin andOC a diagonal. Let D be the midpoint of OA. using vector methods prove that $BD and CO$ intersect in the same ratio. Determine this ratio.

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123. In a triangle $ABC, DandE$ are points on $BCandAC,$ respectivley, such that $BD = 2DCandAE = 3EC$. Let P be the point of intersection of $ADandBE$. Find BP/PE using the vector method.

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124. Prove by vector method that the line segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides and equal to half of their difference.

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125. If the resultant of two forces is equal in magnitude to one of the components and perpendicular to it direction, find the other components using the vector method.

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126. The axes of coordinates are rotated about the z-axis through an angle of $\pi/4$ in the anticlockwise direction and the components of a vector are $2\sqrt{2}, 3\sqrt{2}, 4$. Prove that the components of the same vector in the original system are $-1, 5, 4$.

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127. Prove that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

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128. If two side of a triangle are $\hat{i} + 2\hat{j}$ and $\hat{i} + \hat{k}$, then find the length of the third side.

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129. If in parallelogram ABCD, diagonal vectors are $\vec{AC} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{BD} = -6\hat{i} + 7\hat{j} - 2\hat{k}$, then find the adjacent side vectors $\rightarrow AB$ and $\vec{AD}$

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130. Find the resultant of vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ Find the unit vector in the direction of the resultant vector.

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131. Check whether the three vectors $2\hat{i} + 2\hat{j} + 3\hat{k}, -3\hat{i} + 3\hat{j} + 2\hat{k}and3\hat{i} + 4\hat{k}$ from a triangle or not

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132. The midpoint of two opposite sides of a quadrilateral and the midpoint of the diagonals are the vertices of a parallelogram. Prove that using vectors.

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133. Prove that the lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

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134. Find the angle of vector $\vec{a} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ with x -axis.

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135. If the vectors $\vec{\alpha} = a\hat{i} + a\hat{j} + c\hat{k}, \vec{\beta} = \hat{i} + \hat{k}$ and $\vec{\gamma} = c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then prove that c is the geometric mean of a and b.

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136. The points with position vectors $60i + 3j, 40i - 8j, ai - 52j$ are collinear if a. $a = -40$ b. $a = 40$ c. $a = 20$ d. none of these

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137. Let α, β and γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}; \beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ and $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$ a. are collinear. b. forms an equilateral triangle. c. forms a scalene triangle. d. forms a right angled triangle.

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138. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1 - x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1 + x - y)\vec{k}$. Then $\left[\vec{a}\vec{b}\vec{c}\right]$ depends on (A) only x (B) only y (C) Neither x nor y (D) both x and y

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139. In the $\Delta OAB, M$ is the mid-point of AB,C is a point on OM, such that 2OC=CM. X is a point on the side OB such that OX=2XB. The line XC is produced to meet OA in Y. then, $\frac{OY}{YA}$ is equal to

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140. If $\vec{a}, \vec{b}$ are two non-collinear vectors, prove that the points with position vectors $\vec{a} + \vec{b}, \vec{a} - \vec{b}$ and $\vec{a} + \lambda\vec{b}$ are collinear for all real values of $\dot{\lambda}$

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141. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors & $\left|\vec{c}\right| = \sqrt{3}$, then ordered pair (α, β) is (a)$(1, 1)$ (b) $(1, -1)$ (c) $(-1, 1)$ (d) $(-1, -1)$

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142. The number of distinct real values of λ , for which the vectors $-\lambda^2\hat{i} + \hat{j} + k, \hat{i} - \lambda^2\hat{j} + \hat{k}and\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar is a. zero b. one c. two d. three

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143. If $\vec{A}O + \vec{O}B = \vec{B}O + \vec{O}C$, then A, B and C are (where O is the origin) a. coplanar b. collinear c. non-collinear d. none of these

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144. Find a vector magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}^{\cdot}$

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145. Show that the points $A(1, -2, -8), B(5, 0, -2) and C(11, 3, 7)$ are collinear, and find the ratio in which B divides AC.

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146. The position vectors of $P and Q$ are $5\hat{i}+4\hat{j}+a\hat{k}$ and $-\hat{i}+2\hat{j}-2\hat{k}$, respectively. If the distance between them is 7, then find the value of a.

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147. Given three points are $A(-3, -2, 0), B(3, -3, 1) and C(5, 0, 2)$. Then find a vector having the same direction as that of $\vec{A}B$ and magnitude equal to $\left|\vec{A}C\right|$.

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148. Let $ABCD$ be a p[arallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P$ $\dot{}$

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149. If $ABCD$ is quadrilateral and $EandF$ are the mid-points of $ACandBD$ respectively, prove that $\vec{A}B + \vec{A}D$ +$\vec{C}B$ +$\vec{C}D$ =4 $\vec{E}F$ $\dot{}$

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150. If $ABCD$ is a rhombus whose diagonals cut at the origin $O,$ then proved that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D$ =0

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151. Let $D, EandF$ be the middle points of the sides $BC, CAandAB,$ respectively of a triangle ABC $\dot{}$ Then prove that $\vec{A}D + \vec{B}E + \vec{C}F = \vec{0}$.

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152. Consider the set of eight vectors $V\left[a\hat{i}+b\hat{j}+c\hat{k}: a, b, c \in \{1 - 1\}\right]$. Three non-coplanar vectors cann be chosen from V in 2^p ways, then p is

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153. Find the direction cosines of the vector joining the points $A(1, 2, -3)a \cap B(-1-2, 1)$ directed from $A \rightarrow B^{\cdot}$

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154. Find the direction cosines of the vector $\hat{i}+2\hat{j}+3\hat{k}^{\cdot}$

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155. The median AD of the triangle ABC is bisected at E and BE meets AC at F. Find AF : FC.

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156. Vectors $\vec{a}$ and $\vec{b}$ are non-collinear. Find for what value of n vectors $\vec{c} = (n - 2)\vec{a} + \vec{b}$ and $\vec{d} = (2n + 1)\vec{a} - \vec{b}$ are collinear?

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157. i. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors, prove that vectors $3\vec{a} - 7\vec{b} - 4\vec{c}, 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{a} + \vec{b} + 2\vec{c}$ are coplanar.

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158. Prove that a necessary and sufficient condition for three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ to be coplanar is that there exist scalars l, m, n not all zero simultaneously such that $l\vec{a} + m\vec{b} + n\vec{c} = \vec{0}$.

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159. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} and \vec{a} \times \vec{b} = \vec{c} \times \vec{d}$, prove that $\left(\vec{a} - \vec{d}\right).\left(\vec{b} - \vec{c}\right) \neq 0$,

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160. If $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar vectors, prove that the four points $2\vec{a} + 3\vec{b} - \vec{c}, \vec{a} - 2\vec{b} + 3\vec{c}, 3\vec{a} + 4\vec{b} - 2\vec{c} and \vec{a} - 6\vec{b} + 6\vec{c}$ are coplanar.

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161. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

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162. let P an interioer point of a triangle ABC and AP, BP, CP meets the sides BC, CA, AB in D, E, F, respectively, Show that $\frac{AP}{PD} = \frac{AF}{FB} + \frac{AE}{EC}.$

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163. Let $\vec{a}, \vec{b} and \vec{c}$ be unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}, \vec{a} \cdot \vec{x} = 1, \vec{b} \cdot \vec{x} = \frac{3}{2}, |\vec{x}| = 2.$ Then find the angle between $\vec{c}$ and $\vec{x}$

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164. Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisects the internal angle between $\vec{A}$ and $\vec{B}$, then find the value of α.

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165. If the vectors $3\vec{p} + \vec{q}; 5p - 3\vec{q} and 2\vec{p} + \vec{q}; 4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, then find the angle between vectors $\vec{p} and \vec{q}$.

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166. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(3, -2, -1)$, then find the projection length of $\vec{P}Q on \vec{R}S$.

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167. A, B, C, D are any four points, prove that $\vec{A}B \cdot \vec{C}D + \vec{B}C \cdot \vec{A}D + \vec{C}A \cdot \vec{B}D = 4(Area of \triangle ABC)$.

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168. Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $\left|\vec{w} \cdot \hat{n}\right|$ is

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169. If the angel between unit vectors $\vec{a} and \vec{b} 60^0$, then find the value of $\left|\vec{a} - \vec{b}\right|^{\cdot}$

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170. $\vec{a} + \vec{b} + \vec{c} = \vec{0}, \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 9$,find the angle between $\vec{a}$ and $\vec{c}$.

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171. Constant forces $P_1 = \hat{i} + \hat{j} + \hat{k}, P_2 = -\hat{i} + 2\hat{j} - \hat{k} and P_3 = -\hat{j} - \hat{k}$ act on a particle at a point $A^{\cdot}$ Determine the work done when particle is displaced from position $A\left(4\hat{i} - 3\hat{j} - 2\hat{k}\right)$ to $B\left(6\hat{i} + \hat{j} - 3\hat{k}\right)^{\cdot}$

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172. If $\vec{a}, and \vec{b}$ are unit vectors , then find the greatest value of $\left|\vec{a}+\vec{b}\right|+\left|\vec{a}-\vec{b}\right|^{.}$

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173. Let $G_1, G_2 and G_3$ be the centroids of the triangular faces $OBC, OCA and OAB,$ respectively, of a tetrahedron $OABC^{.}$ If V_1 denotes the volumes of the tetrahedron $OABC and V_2$ that of the parallelepiped with $OG_1, OG_2 and OG_3$ as three concurrent edges, then prove that $4V_1=9V_2^{.}$

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174. Prove that $\hat{i}\times\left(\vec{a}\times\hat{i}\right)+\hat{j}\times\left(\vec{a}\times\hat{j}\right)+\hat{k}\times\left(\vec{a}\times\hat{k}\right)=2\vec{a}$

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175. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] + \hat{j} \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \hat{k} \times \left[\left(\vec{a} - \hat{i}\right) \times \hat{k}\right] = 0$, then find vector $\vec{a}$.

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176. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be any three vectors, then prove that [$\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}$]=$\left[\vec{a} \vec{b} \vec{c}\right]^2$

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177. If $\left[\vec{a} \vec{b} \vec{c}\right] = 2$, then find the value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right]$.

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178. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, find $\left|2\vec{a} + \vec{b} + \vec{c}\right|$.

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179. If $a, bandc$ are three non-cop0lanar vector, non-zero vectors then the value of $\left(\vec{a}.\vec{a}\right)\vec{b}\times\vec{c}+\left(\vec{a}.\vec{b}\right)\vec{c}\times\vec{a}+\left(\vec{a}.\vec{c}\right)\vec{a}\times\vec{b}.$

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180. Prove that vectors $\vec{u}=\left(al+a_1l_1\right)\hat{i}+\left(am+a_1m_1\right)\hat{j}+\left(an+a_1n_1\right)\hat{k}$

$\vec{v}=\left(bl+b_1l_1\right)\hat{i}+\left(bm+b_1m_1\right)\hat{j}+\left(bn+b_1n_1\right)\hat{k}$

$\vec{w}=\left(cl+c_1l_1\right)\hat{i}+\left(cm+c_1m_1\right)\hat{j}+\left(cn+c_1n_1\right)\hat{k}$ are coplanar.

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181. For any four vectors, prove that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\left[\vec{a}\vec{c}\vec{d}\right]\vec{b}-\left[\vec{b}\vec{c}\vec{d}\right]\vec{a}$

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182. If $\vec{b} and \vec{c}$ are two-noncollinear vectors such that $\vec{a} \mid \quad \mid \left(\vec{b} \times \vec{c}\right)$, then prove that $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to $|\vec{a}|^2\left(\overset{\cdot}{\vec{b}\vec{c}}\right)^{\cdot}$

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183. If the vertices A,B,C of a triangle ABC are (1,2,3),(-1,0,0) ,(0,1,2) , respectively, then find $\angle ABC$.

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184. Let $\vec{a}, \vec{b} and \vec{c}$ be pairwise mutually perpendicular vectors, such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 2, \left|\vec{c}\right| = 2.$ Then find the length of $\left|\vec{a} + \vec{b} + \vec{c}\right|$

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185. Show that $\left|\vec{a}\right|\vec{b} + \left|\vec{b}\right|\vec{a}$ is a perpendicular to $\left|\vec{a}\right|\vec{b} - \left|\vec{b}\right|\vec{a}$, for any two non-zero vectors $\vec{a} and \vec{b}^{.}$

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186. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4$ and the angle between $\vec{a}$ and $\vec{b} is 120^{\circ}$. Then find the value of $\left|4\vec{a} + 3\vec{b}\right|$

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187. If $\vec{a}, \vec{b}, and \vec{c}$ be three non-coplanar vector and p, q, r constitute the reciprocal system of vectors, then $(la + mb + nc).(lp + mq + nr)$. is equals to

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188. Find $|\to a|$ and $|\to b|$, if $(\to a + \to b) \to a \overset{\cdot}{-} \to b = 8$ and $|\to a| = 8|\to b|$

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189. Let $\vec{a}, \vec{b}, and \vec{c} and \vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, then prove that $\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \dfrac{\vec{a} + \vec{b} + \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$.

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190. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $\left(\vec{a} + \vec{b} + \vec{c}\right).\left[\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)\right]$ equals a.0 b. $\left[\vec{a}\vec{b}\vec{c}\right]$ c. $2\left[\vec{a}\vec{b}\vec{c}\right]$ d. $-\left[\vec{a}\vec{b}\vec{c}\right]$

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191. Find the vector equation of the plane passing through the points having position vectors $\hat{i} + \hat{j} - 2\hat{k}, 2i - \hat{j} + \hat{k} and \hat{i} + 2\hat{j} + \hat{k}^{.}$

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192. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0, where \vec{a}, \vec{b}, and \vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a} + \vec{c} = k\vec{b}^{.}$

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193. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k} and \vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find thevalue of $\left(\vec{a} \times \vec{b}\right)^{.}\left(\vec{a} \times \vec{c}\right)$.

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194. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right-handed system, then find $\vec{c}$.

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195. Given that $\vec{a}\vec{b} = \vec{a}\vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a}$ is not a zero vector. Show that $\vec{b} = \vec{c}$.

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196. If $|\vec{a}| = 5, |\vec{a} - \vec{b}| = 8$ and $|\vec{a} + \vec{b}| = 10$, then find $|\vec{b}|$.

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197. If A, B, C, D are four distinct point in space such that $\overrightarrow{AB}$ is not perpendicular to $\overrightarrow{CD}$ and satisfies

$\overrightarrow{AB}.\overrightarrow{CD} = k\left(\left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 - \left|\overrightarrow{AC}\right|^2 - \left|\overrightarrow{BD}\right|^2\right)$, then find the value of k.

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198. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$, then find (m, n).

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199. If $\left|\vec{a}\right| = 2\left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$, then find the value of $\vec{a}.\vec{b}$.

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200. Prove that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$ and interpret it geometrically.

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201. $\vec{a}, \vec{b} and \vec{c}$ are unit vectors such that $\left|\vec{a}+\vec{b}+3\vec{c}\right| = 4.$ Angle between $\vec{a} and \vec{b} is \theta_1,$ between $\vec{b} and \vec{c}$ is θ_2 and between $\vec{a} and \vec{c}$ varies $[\pi/6, 2\pi/3]$. Then the maximum of $\cos\theta_1 + 3\cos\theta_2 is$ 3 b. 4 c. $2\sqrt{2}$ d. 6

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202. Prove that $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

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203. Let A, B, C be three unit vectors and $A.B = A.C = 0.$ If the angle between B and C is $\frac{\pi}{6}$, then A is equals to

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204. The position vectors of the four angular points of a tetrahedron are $A\left(\hat{j}+2\hat{k}\right), B\left(3\hat{i}+\hat{k}\right), C\left(4\hat{i}+3\hat{j}+6\hat{k}\right) and D\left(2\hat{i}+3\hat{j}+2\hat{k}\right)$. Find the volume of the tetrahedron $ABCD$

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205. If the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k} and 3\hat{i} - \hat{k}$ form three concurrent edges of a parallelepiped, then find the volume of the parallelepiped.

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206. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then prove that $(\vec{u} + \vec{v} - \vec{w}). \left[\left[(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})\right]\right] = \vec{u}. (\vec{v} \times \vec{w})$

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207. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum.

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208. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$, then find (m, n).

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209. Prove that $\left[\vec{l}\vec{m}\vec{n}\right]\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$.

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210. Find the altitude of a parallelopiped whose three coterminous edges are vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} - \hat{k} and \vec{C} = \hat{i} + \hat{j} + 3\hat{k} with \vec{A}$ and $\vec{B}$ as the sides of the base of the parallopiped.

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211. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a}.\vec{b} = 4,$ then find the value of $\left|\vec{a} - \vec{b}\right|$

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212. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta} \times \left(\overset{\cdot}{\vec{\beta}} \times \vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha} \times \left(\overset{\cdot}{\vec{\alpha}} \times \vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha} \times \vec{\beta}\right)}{\left|\vec{\alpha} \times \vec{\beta}\right|^2}$$

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213. If $\vec{a}, \vec{b},$ and $\vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}, \vec{b}$ and $\vec{c}$ are non-parallel, then prove that the angle between $\vec{a}$ and $\vec{b}, is 3\pi/4.$

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214. If $\left|\vec{a}\right| = 5, \left|\vec{a} - \vec{b}\right| = 8 \text{ and } \left|\vec{a} + \vec{b}\right| = 10$, then find $\left|\vec{b}\right|^{\cdot}$

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215. If $\vec{a}$ and $\vec{b}$ are two given vectors and k is any scalar, then find the vector $\vec{r}$ satisfying $\vec{r} \times \vec{a} + k\vec{r} = \vec{b}$.

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216. $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar ,non-zero vectors and $\vec{r}$ is any vector in space, then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times \vec{b}\right)$ is equal to

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217. If vector $\vec{x}$ satisfying $\vec{x} \times \vec{a} + \left(\vec{x}.\vec{b}\right)\vec{c} = \vec{d}$ is given $\vec{x} = \lambda\vec{a} + \vec{a} \times \dfrac{\vec{a} \times \left(\vec{d} \times \vec{c}\right)}{\left(\vec{a}.\vec{c}\right)|\vec{a}|^2}$, then find the value of λ

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218. Let $\hat{a}, \hat{b}$,and $\hat{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c}$ is α , between $\hat{c}$ and $\hat{a}$ is β and between $\hat{a}$ and $\hat{b}$ is γ . If $A\left(\hat{a}\cos\alpha, 0\right), B\left(\hat{b}\cos\beta, 0\right)$ and $C\left(\hat{c}\cos\gamma, 0\right)$, then show that in triangle ABC, $\dfrac{\left|\hat{a} \times \left(\hat{b} \times \hat{c}\right)\right|}{\sin A} = \dfrac{\left|\hat{b} \times \left(\hat{c} \times \hat{a}\right)\right|}{\sin B} = \dfrac{\left|\hat{c} \times \left(\hat{a} \times \hat{b}\right)\right|}{\sin C}$

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219. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$.

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220. If $\vec{b}$ is not perpendicular to $\vec{c}$, then find the vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ and $\vec{r}.\vec{c} = 0$.

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221. If $\vec{a}, \vec{b} and \vec{c}$ are three non coplanar vectors, then $\left(\vec{a} + \vec{b} + \vec{c}\right)\left[\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)\right]$ is :

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222. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $\lambda\vec{b} \times \vec{a} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$, then find the value of λ

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223. Prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a} \times \hat{i}\right) + \left(\vec{a}.j\right)\left(\vec{a} \times \hat{j}\right) + \left(\vec{a}.\hat{k}\right)\left(\vec{a} \times \hat{k}\right) = 0.$

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224. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2=144$ and $\left|\vec{a}\right|=4,$ then find the value of $\left|\vec{b}\right|^{.}$

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225. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1,1,2)and(1,2,-2)^{.}$ Find the velocity of the particle at point $P(3,6,4)^{.}$

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226. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is acting on point (1, -1, 2).

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227. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1 and \left|\vec{a} + \vec{b}\right| = \sqrt{3}$. If $\vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$, then find the value of $\vec{c}.\vec{b}$

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228. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between a and b is $\frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

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229. Statement 1: $\vec{a}, \vec{b}, and\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are non-coplanar. If $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1, then \vec{d} = \vec{a} + \vec{b} + \vec{c}$. Statement 2:

$\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$; then $\vec{d}$ equally inclined to $\vec{a}$,$\vec{b}$ and $\vec{c}$.

(a) statement 1 is true but statement 2 is false. (b) statement 2 is true but statement 1 is false. (c)both the statements are true. (d) both the statements are false.

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230. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)^{\cdot}$

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231. Prove that $\left[\vec{l}\vec{m}\vec{n}\right]\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$.

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232. Using dot product of vectors, prove that a parallelogram, whose diagonals are equal, is a rectangle

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233. If $a + 2b + 3c = 4,$ then find the least value (to the nearest integer) of $a^2 + b^2 + c^2$.

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234. In any triangle $ABC,$ prove the projection formula$a = b\cos C + c\cos B$ using vector method.

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235. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

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236. If $\vec{a}.\hat{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$, then find the unit vector $\vec{a}$

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237. Prove by vector method that $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

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238. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}$ on vector $2\hat{i} - \hat{j} + 5\hat{k}, is\frac{1}{\sqrt{30}}$,then find the value of x.

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239. If $\vec{a} = x\hat{i} + (x - 1)\hat{j} + \hat{k}$ and $\vec{b} = (x + 1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a.

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240. A unit vector a makes an angle $\frac{\pi}{4}$ with z-axis. If $a+i+j$ is a unit vector, then a can be equal to

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241. if $\vec{a}, \vec{b}$ and $\vec{c}$ are there mutually perpendicular unit vectors and $\vec{a}$ ia a unit vector then find the value of $\left|2\vec{a}+\vec{b}+\vec{c}\right|^2$

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242. If $\vec{a}, \vec{b},$ and $\vec{c}$ be non-zero vectors such that no two are collinear or $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}^{\cdot}$ If θ is the acute angle between vectors $\vec{b}$ and $\vec{c}$, then find the value of $\sin\theta^{\cdot}$

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243. If $\vec{p}, \vec{q}, \vec{r}$ denote vector $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$, respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}, \vec{b}$ is parallel $\vec{r} \times \vec{p}, \vec{c}$ is parallel to $\vec{p} \times \vec{q}$.

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244. If $\vec{a}$ and $\vec{b}$ be two non-collinear unit vector such that $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \frac{1}{2}\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

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245. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)^{\cdot}$

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246. Prove that $\left(\vec{a}.\left(\vec{b} \times \hat{i}\right)\right)\hat{i} + \left(\vec{a}.\left(\vec{b} \times \hat{j}\right)\right)\hat{j} + \left(\vec{a}.\left(\vec{b} \times \hat{k}\right)\right)\hat{k} = \vec{a} \times \vec{b}.$

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247. For any four vectors, $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that $\vec{d}.\left(\vec{a} \times \left(\vec{b} \times \left(\vec{c} \times \vec{d}\right)\right)\right) = \left(\vec{b}.\vec{d}\right)\left[\vec{a}\ \vec{c}\ \vec{d}\right]$.

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248. If $\vec{a}, \vec{b},$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b},$ then prove that $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$.

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249. If $\vec{a} = \vec{p} + \vec{q}, \vec{p} \times \vec{b} = 0and\vec{q}\overset{\cdot}{}\vec{b} = 0,$ then prove that $\dfrac{\vec{b} \times \left(\vec{a} \times \vec{b}\right)}{\overset{\cdot}{\vec{b}\vec{b}}} = \vec{q}^{\cdot}$

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250. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, then find vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}^{\cdot}$

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251. If non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then the solution of the equation $\vec{r} \times \vec{a} = \vec{b}$ is given by

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252. If $\vec{a}, \vec{b},$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, then find the angle between vectors $\vec{a}$ and $\vec{a} + \vec{b} + \vec{c}^{\cdot}$

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253. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors satisfying the condition $\vec{a} + \vec{b} + \vec{c} = 0$ then show that $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -3/2.$

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254. If three unit vectors $\vec{a}, \vec{b},$ and $\vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = 0,$ then find the angle between $\vec{a}$ and $\vec{b}$.

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255. If $\left|\vec{a}\right| + \left|\vec{b}\right| = \left|\vec{c}\right|$ and $\vec{a} + \vec{b} = \vec{c},$ then find the angle between $\vec{a}$ and $\vec{b}$.

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256. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

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257. If $\vec{r}.\hat{i} = \vec{r}.\hat{j} = \vec{r}.\hat{k} and \left|\vec{r}\right| = 3,$ then find the vector $\vec{r}$

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258. If $\vec{a}, \vec{b}, and \vec{c}$ are non-zero vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c},$ then find the geometrical relation between the vectors.

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259. Find the projection of vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

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260. If θ is th angle between the unit vectors a and b , then prove that $\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} + \vec{b}\right|$,and $\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

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261. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is pi/3 . If veca × vecb + vecb × vecc =p veca +q vecb +r vecc , where p,q and r are scalars, then the value of p 2 +2q 2 +r 2 /q2 is

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262. Given unit vectors $\hat{m}, \hat{n}$ and $\hat{p}$ such that angel between $\hat{m}$ and $\hat{n}$ is α and angle between $\hat{p}$ and $\left(\hat{m} \times \hat{n}\right)$ is also $\alpha, then\left[\hat{n}\hat{p}\hat{m}\right] =$

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263. Let $\vec{a}, \vec{b}, and\vec{c}$ be non-coplanar vectors and let the equation $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c},$ then prove that $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$ is a null vector.

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264. Vector $\vec{OA} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is $\dfrac{4\hat{i} - \hat{j} - \hat{k}\cdot}{\sqrt{2}}$

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265. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}^{\cdot}$

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266. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \dfrac{\sqrt{2}}{3}$, *then*, $\vec{a} \times \vec{b}$ is a unit vector, if the angel between $\vec{a}$ and $\vec{b}$ is?

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267. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)^{\cdot}$

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268. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k} and \vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}^{\cdot}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

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269. If $A, BandC$ are the vetices of a triangle $ABC,$ then prove sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}^{\cdot}$

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270. Using cross product of vectors , prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

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271. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1)$ and $(0, 2, 1)^{.}$

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272. If $\vec{a}$ and $\vec{b}$ are two vectors, then prove that $\left(\vec{a}\times\vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$.

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273. In isosceles triangles ABC, $\left|\vec{A}B\right| = \left|\vec{B}C\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}Eand\vec{C}A\left(where\left|\vec{C}A\right| = 12\right)^{.}$

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274. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular , then the third pair is also perpendicular.

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275. Let $\vec{a}, \vec{b},$ and $\vec{c}$ are vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5,$ and $(\vec{a} + \vec{b})$ is perpendicular to $\vec{c}, (\vec{b} + \vec{c})$ is perpendicular to $\vec{a}$ and $(\vec{c} + \vec{a})$ is perpendicular to $\vec{b}^{\cdot}$ Then find the value of $|\vec{a} + \vec{b} + \vec{c}|$.

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276. If $|\vec{a}| = |\vec{b}| = |\vec{a} + \vec{b}| = 1,$ then find the value of $|\vec{a} - \vec{b}|^{\cdot}$

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277. If $\vec{A} = 4\hat{i} + 6\hat{j}$ and $\vec{B} = 3\hat{j} + 4\hat{k},$ then find the component of $\vec{A}$ along $\vec{B}$

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278. A particle acted by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{9}j - \hat{k}$ is displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \hat{k}.$ find the total work done by the forces in SI units.

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279. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that $\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{3}$

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280. Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k}and\vec{c} = \hat{i} + \hat{k}.$ If the ordered set $\left[\vec{b}\vec{c}\vec{a}\right]$ is left handed, then find the values of $x.$

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281. If $\vec{a}, \vec{b}, and \vec{c}$ are three non-coplanar vectors, then find the value of

$$\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\vec{b}.\left(\vec{c}\times\vec{a}\right)}+\frac{\vec{b}.\left(\vec{c}\times\vec{a}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}+\frac{\vec{c}.\left(\vec{b}\times\vec{a}\right)}{\vec{a}.\left(\vec{b}\times\vec{c}\right)}$$

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282. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are the position vectors of the vertices of a cyclic quadrilateral $ABCD,$ prove that

$$\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}\right|}{\left(\vec{b}-\vec{a}\right).\left(\vec{d}-\vec{a}\right)}+\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\vec{d}\times\vec{b}\right|}{\left(\vec{b}-\vec{c}\right).\left(\vec{d}-\vec{c}\right)}=0^{.}$$

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283. The position vectors of the vertices of a quadrilateral with A as origin are $B\left(\vec{b}\right), D\left(\vec{d}\right) and C\left(l\vec{b}+m\vec{d}\right)^{.}$ Prove that the area of the quadrialateral is $\frac{1}{2}(l+m)\left|\vec{b}\times\vec{d}\right|^{.}$

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284. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d} and \vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$, is parallel to $\vec{b} - \vec{c}$

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285. Show by a numerical example and geometrically also that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does not imply $\vec{b} = \vec{c}$

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286. In triangle ABC ,points $D, EandF$ are taken on the sides $BC, CAandAB$, respectively, such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$. Prove that $\triangle DEF = \frac{n^2 - n + 1}{(n+1)^2} \triangle (ABC)$.

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287. Let A, B, C be points with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}and3\hat{i} + \hat{j} + 2\hat{k}$ respectively. Find the shortest distance between point B and plane OAC.

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288. Let $\vec{a}$ and $\vec{b}$ be unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. Then find the value of $\left(2\vec{a} + 5\vec{b}\right). \left(\left(3\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)\right)$.

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289. If u and v are two non-collinear unit vectors such that $\left|\vec{u} \times \vec{v}\right| = | \frac{\vec{u} - \vec{v}}{2} \Bigg|$, then the value of $| \vec{u} \times \left(\vec{u} \times \vec{v}\right)|^2$ is equal to

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290. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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291. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}; \vec{r} \times \vec{b} = \vec{a} \times \vec{b}; \vec{a} \neq \vec{0}; \vec{b} \neq \vec{0}; \vec{a} \neq \lambda \vec{b}, and \vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a} and \vec{b}^{\cdot}$

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292. If $\left|\vec{a}\right| = 2,$ then find the value of $\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^{2^{\cdot}}$

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293. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}\left[\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right]$. Also deduce the condition for collinearity of the points A, B and C.

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294. $A, B, CandD$ are any four points in the space, then prove that

$\left|\vec{A}B \times \vec{C}D + \vec{B}C \times \vec{A}D + \vec{C}A \times \vec{B}D\right| = 4$ (area of ABC).

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295. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}and\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}^{\cdot}$

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296. Using vectors, find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

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297. Let $\vec{a}, \vec{b} and \vec{c}$ be three verctors such that $\vec{a} \neq 0, \left|\vec{a}\right| = \left|\vec{c}\right| = 1, \left|\vec{b}\right| = 4$ and $\left|\vec{b} \times \vec{c}\right| = \sqrt{15}^{\cdot}$ If $\vec{b} - 2\vec{c} = \lambda\vec{a},$ then find the value of λ

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298. Find the area of a parallelogram whose diagonals are $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}^{\cdot}$

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299. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a} + \vec{b}\right).\left[\left(2\vec{a} + 3\vec{b}\right) \times \left(3\vec{a} - 2\vec{b}\right)\right] = 0$, then angle between $\vec{a}$ and $\vec{b}$ is

A. a.0

B. b. $\pi/2$

C. c. π

D. d. indeterminate

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300. If $\vec{a} and \vec{b}$ are any two unit vectors, then find the greatest positive integer in the range of $\frac{3\left|\vec{a}+\vec{b}\right|}{2}+2\left|\vec{a}-\vec{b}\right|$.

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301. If the vectors $\vec{a}, \vec{b}, and \vec{c}$ form the sides$BC, CA and AB,$ respectively, of triangle $ABC, then$

A. (a)$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$

B. (b)$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

C. (c). $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}$

D. (d). $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

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302. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60^{0}. Suppose that $\left|\vec{u}-\hat{i}\right|$ is geometric mean of $\left|\vec{u}\right| and \left|\vec{u}-2\hat{i}\right|, where \hat{i}$ is the unit vector along the x-axis. Then find the value of $\left(\sqrt{2}+1\right)\left|\vec{u}\right|$.

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303. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B=2\hat{i}+10\hat{j}+11\hat{k} and \vec{A}D=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

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304. Let $\vec{a}, \vec{b}, and\vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angleθ then $\left[\vec{a}\vec{b}\vec{c}\right]$ in terms of θ is equal to :

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305. Given three vectors $\vec{a}, \vec{b}, and\vec{c}$ two of which are non-collinear. Further if $\left(\vec{a} + \vec{b}\right)$ is collinear with $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is collinear with $\vec{a}, \left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = \sqrt{2}$. Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ a. 3 b. -3 c. 0 d. cannot be evaluated

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306. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum.

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307. $A_1, A_2, \dots, A_n$ are the vertices of a regular plane polygon with n sides and O as its centre. Show that $\sum_{i=1}^{n} \overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}} = (1 - n)\left(\overrightarrow{OA_2} \times \overrightarrow{OA_1}\right)$

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308. If c is a given non-zero scalar, and $\vec{A}$ and $\vec{B}$ are given non-zero vector such that $\vec{A} \perp \vec{B}$, then find vector $\vec{X}$ which satisfies the equation $\vec{A}.\vec{X} = c$ and $\vec{A} \times \vec{X} = \vec{B}$.

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309. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}\right| = 4$ (area of ABC).

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310. If $a, bandc$ are three non-cop0lanar vector, non-zero vectors then the value of $\left(\vec{a}.\vec{a}\right)\vec{b}\times\vec{c} + \left(\vec{a}.\vec{b}\right)\vec{c}\times\vec{a} + \left(\vec{a}.\vec{c}\right)\vec{a}\times\vec{b}.$

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311. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}\ \ \vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ Determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\vec{R}.\vec{A} = 0.$

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312. Determine the value of c so that for all real x , vectors $cx\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

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313. If $\vec{r} = x_1\left(\vec{a}\times\vec{b}\right) + x_2\left(\vec{b}\times\vec{c}\right) + x_3\left(\vec{c}\times\vec{a}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1,$ then $x_1 + x_2 + x_3$ is equal to (A) $\frac{1}{2}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (B) $\frac{1}{4}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (C)

$2\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$ (D) $4\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

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314. $\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right)\right]$ is equal to (where $\vec{a}, \vec{b} and \vec{c}$ are nonzero non-coplanar vector) a. $\left[\vec{a}\vec{b}\vec{c}\right]^2$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^3$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^4$ d. $\left[\vec{a}\vec{b}\vec{c}\right]$

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315. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV^{'}, thenK$ is equal to a. 9 b. 12 c. 27 d. 81

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316. $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar ,non-zero vectors and $\vec{r}$ is any vector in space, then

$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{r}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{r}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{r}\times\vec{b}\right)$ is equal to

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317. $A(\vec{a}), B\left(\vec{b}\right), C(\vec{c})$ are the vertices of the triangle ABC and $R(\vec{r})$ is any point in the plane of triangle ABC , then $\vec{r}.\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)$ is always equal to

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318. Let $\vec{a}, \vec{b}and\vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}and\vec{r}$ the vectors defined by the relation $\vec{p}=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}and\vec{r}=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}\cdot$ Then the value of the expression $\left(\vec{a}+\vec{b}\right)\vec{p}+\left(\vec{b}+\vec{c}\right)\vec{q}+\left(\vec{c}+\vec{a}\right)\vec{r}$ is a.0 b. 1 c. 2 d. 3

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319. $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar ,non-zero vectors and $\vec{r}$ is any vector in space, then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times \vec{b}\right)$ is equal to

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320. The position vectors of point $A, B, and C$ are $\hat{i} + \hat{j} + \hat{k}, \hat{i} + 5\hat{j} - \hat{k} and 2\hat{i} + 3\hat{j} + 5\hat{k},$ respectively. Then greatest angel of triangle ABC is 120^0 b. 90^0 c. $\cos^{-1}(3/4)$ d. none of these

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321. Let $\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j} and \vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)^{.}$ Then $\vec{a}(x) and \vec{b}(x)$ are a. collinear for unique value of x b. perpendicular for infinite values of x c. zero vectors for unique value of x d. none of these

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322. If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $(1+\alpha)\hat{i} + \beta(1+\alpha)\hat{j} + \gamma(1+\alpha)(1+\beta)\hat{k} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$, $then \alpha, \beta and \gamma$ are

a. $-2, -4, -\frac{2}{3}$ b. $2, -4, \frac{2}{3}$ c. $-2, 4, \frac{2}{3}$ d. $2, 4, -\frac{2}{3}$

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323. If $\vec{a}, \vec{b} and \vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9,$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is.

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324. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then

a. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$

c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar

d. none of these

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325. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to vector $\hat{i} + \hat{j} + \hat{k}$, is/are a. $\hat{j} - \hat{k}$ b. $-\hat{i} + \hat{j}$ c. $\hat{i} - \hat{j}$ d. $-\hat{j} + \hat{k}$

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326. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j} and \vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then find the value of $\vec{r}.\vec{b}$

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327. Let $\vec{a}, \vec{b}, and \vec{c}$ be vectors forming right-hand traid. Let $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}, and \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. If $x \in R^+$, then a. $x\left[\vec{a}\vec{b}\vec{c}\right] + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x}$ has least value $= 2$. b. $x^4\left[\vec{a}\vec{b}\vec{c}\right]^2 + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x^2}$ has least value $= \left(\frac{3}{2}\right)^{2/3}$ c. $\left[\vec{p}\vec{q}\vec{r}\right] > 0$ d. none of these

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328. If the vectors $\vec{a}, \vec{b}, and \vec{c}$ form the sides $BC, CA and AB$, respectively, of triangle $ABC, then$

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329. Find $\vec{a} \times \vec{b}$, if $\vec{a} = 2\hat{i} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

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330. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acrting on a particle such that the particle is displaced from point $A(-3, -4, 1) \rightarrow B(-1, -1, -2)$.

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331. Prove that $\left[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

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332. find the angle between the vectors$\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

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333. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge AC. Prove that DE is equal to half the edge of tetrahedron.

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334. In the quadrilateral ABCD, the diagonals AC and BD are equal and perpendicular to each other. What type of a quadrilateral is ABCD?

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335. If $\vec{e}_1, \vec{e}_2, \vec{e}_3$ and $\vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i . \vec{E}_j = 1$, if $i = j$ and $\vec{e}_i . \vec{E}_j = 0$ and if $i \neq j$, the prove that $[\vec{e}_1 \vec{e}_2 \vec{e}_3][\vec{E}$

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336. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

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337. Given the vectors $\vec{A}, \vec{B}, and\vec{C}$ form a triangle such that $\vec{A} = \vec{B} + \vec{C}$. find $a, b, c, andd$ such that the area of the triangle is $5\sqrt{6}$ where $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}\ \vec{B} = d\hat{i} + 3\hat{j} + 4\hat{k}\ \vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$

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338. If $\vec{a}, \vec{b}and\vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is $\vec{a} + \vec{b} + \vec{c}$ b. $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$ c. $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$ d. $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

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339. Let a three dimensional vector $\vec{V}$ satisfy the condition, $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}$. If $3|\vec{V}| = \sqrt{m}$. Then find the value of m.

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340. If $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, find $\left|3\vec{a} - 2\hat{b} + 4\hat{c}\right|^{\cdot}$

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341. Let $\vec{O}A = \vec{a}, \hat{O}B = 10\vec{a} + 2\vec{b}and\vec{O}C = \vec{b}, whereO, AandC$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OAandOC$ as adjacent sides. If $p = kq$, then find$k^{\cdot}$

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342. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$.

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343. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a-2)\alpha^2+(b-3)\alpha+c\right]\vec{x}+\left[(a-2)\beta^2+(b-3)\beta+c\right]\vec{y}+\left[(a-2)\gamma^2+(b-3)\gamma+c\right.$ are three distinct real numbers, then find the value of $\left(a^2+b^2+c^2-4\right)^{.}$

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344. Let $\vec{a}=\alpha\hat{i}+2\hat{j}-3\hat{k}, \vec{b}=\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}, and\vec{c}=2\hat{i}-\alpha\hat{j}+\hat{k}^{.}$ Find thevalue of 6α, such that $\left\{\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\right\}\times\left(\vec{c}\times\vec{a}\right)=0.$

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345. Let $\vec{a}, \vec{b}and\vec{c}$ be three vectors having magnitudes 1, 5and 3, respectively, such that the angel between $\vec{a}and\vec{b}is\theta$ and $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=c$. Then $tan\theta$ is equal to a. 0 b. 2/3 c. 3/5 d. 3/4

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346. Two vectors in space are equal only if they have equal component in

a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

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347. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k} and \vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d} = 0 = \left[\vec{b}\vec{c}\vec{d}\right]$, then d equals a.$\pm\frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ b. $\pm\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ c. $\pm\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ d. $\pm\hat{k}$

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348. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is a.$\vec{b} + \frac{\vec{b}\times\vec{a}}{|\vec{a}|^2}$ b. $\frac{\overset{.}{\vec{a}\vec{b}}}{|\vec{b}|^2}$ c. $\vec{b} - \frac{\overset{.}{\vec{b}\vec{a}}}{|\vec{a}|^2}$ d. $\frac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{|\vec{b}|^2}$

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349. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is perpendicular to $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$, we may have a. $\left(\vec{a}.\vec{c}\right)\left|\vec{b}\right|^2 = \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)\left(\vec{c}.\vec{a}\right)$ b. $\vec{a}\cdot\vec{b} = 0$ c. $\vec{a}\cdot\vec{c} = 0$ d. $\vec{b}\cdot\vec{c} = 0$

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350. $\left[\left(\vec{a} \times \vec{b}\right)\left(\vec{c} \times \vec{d}\right)\left(\vec{e} \times \vec{f}\right)\right]$is equal to

(a)$\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

(b)$\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right] - \left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

(c)$\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$

(d)$\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

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351. $\vec{a}, \vec{b} and \vec{c}$ are non-collinear if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a}.\vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}$ and $\left(\vec{c}.\vec{c}\right)\vec{a} = \vec{c}$ Then

a. $x = 1$ b. $x = -1$ c. $y = (4n + 1)\pi/2, n \in I$ d. $y = (2n + 1)\pi/2, n \in I$

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352. If $\vec{a}$ and $\vec{b}$ are unit vectors, then angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be unit vector is

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353. If $\vec{a} \perp \vec{b}$, then vector $\vec{v}$ in terms of $\vec{a} and \vec{b}$ satisfying the equation s $\dot{\vec{v}}\vec{a} = 0 and \dot{\vec{v}}\vec{b} = 1 and \left[\vec{v}\vec{a}\vec{b}\right] = 1$ is a. $\frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ b. $\frac{\vec{b}}{\left|\vec{b}\right|^{\square}} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ c. $\frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^{\square}}$ d. none of these

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354. If $\vec{a}' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}' = 2\hat{i} + \hat{j} - \hat{k},$ then the altitude of the parallelepiped formed by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is reciprocal vector $\vec{a}$)

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355. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i},$ then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is a. $\frac{\hat{i} + \hat{j} + \hat{k}}{2}$ b. $\frac{\hat{i} - \hat{j} + \hat{k}}{2}$ c. $\frac{-\hat{i} - \hat{j} + \hat{k}}{2}$ d. $\frac{\hat{i} + \hat{j} - \hat{k}}{2}$

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356. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1 and 0 \le \theta \le \pi, then$ θ lies in interval a.$[0, \pi/6)$ b. $(5\pi/6, \pi]$ c. $[\pi/6, \pi/2]$ d. $[\pi/2, 5\pi/6]$

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357. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q} and \vec{r}$ the vectors defined by the relation $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]} and \vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}\cdot$ Then the value of the expression $\left(\vec{a} + \vec{b}\right)\cdot\vec{p} + \left(\vec{b} + \vec{c}\right)\cdot\vec{q} + \left(\vec{c} + \vec{a}\right)\cdot\vec{r}$ is 0 b. 1 c. 2 d. 3

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358. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between a and b is $\frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

A. (a) 0

B. (b) 1

C. (c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

D. (d) $\frac{3}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)\left(c_1^2+c_2^2+c_3^2\right)$

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359. $A, B, CandD$ are four points such that $\vec{AB}=m\left(2\hat{i}-6\hat{j}+2\hat{k}\right), \vec{BC}=\left(\hat{i}-2\hat{j}\right)and\vec{CD}=n\left(-6\hat{i}+15\hat{j}-3\hat{k}\right)^{\cdot}$ If CD intersects AB at some point E, then a. $m \geq 1/2$ b.$n \geq 1/3$ c. $m=n$ d. $m<n$

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360. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}and\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}and\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by a. $\hat{i}-3\hat{j}+3\hat{k}$ b. $-3\hat{i}-3\hat{j}+3\hat{k}$ c. $3\hat{i}-\hat{j}+3\hat{k}$ d. $\hat{i}+3\hat{j}-3\hat{k}$

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361. If $\hat{a}, \hat{b}, and\hat{c}$ are unit vectors, then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\hat{c} - \hat{a}\right|^2$ does not exceed

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362. Which of the following expressions are meaningful? a. $\vec{u}.\left(\vec{v} \times \vec{w}\right)$ b. $\vec{u}.\vec{v}.\vec{w}$ c. $\left(\vec{u}\vec{v}\right).\vec{w}$ d. $\vec{u} \times \left(\vec{v}.\vec{w}\right)$

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363. Find the value of λ if the volume of a tetrahedron whose vertices are with position vectors $\hat{i} - 6\hat{j} + 10\hat{k}, \ -\hat{i} - 3\hat{j} + 7\hat{k}, 5\hat{i} - \hat{j} + \lambda\hat{k}$ and $7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic unit.

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364. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} = \hat{k} and \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b} and \vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is a.$2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $2\hat{i} - 3\hat{j} + 3\hat{k}$ c. $-2\hat{i} - \hat{j} + 5\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$

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365. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right).\left(\vec{a} \times \vec{d}\right) = 0$, then which of the following may be true? (a) $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are necessarily coplanar (b) $\vec{a}$ lies in the plane of $\vec{c} and \vec{d}$ (c) $\vec{b}$ lies in the plane of $\vec{a} and \vec{d}$ (d) $\vec{c}$ lies in the plane of $\vec{a} and \vec{d}$

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366. Vector $\frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ is

(A) a unit vector (B) makes an angle $\pi/3$ with vector $\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)$ (C) parallel to vector $\left(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$ (D) perpendicular to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

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367. Let $\vec{u}$ and $\vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$. Find the value of $\left[\vec{u}\ \vec{v}\ \vec{w}\right]$.

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368. The scalars l and m such that $l\vec{a} + m\vec{b} = \vec{c}$,where $\vec{a}, \vec{b}$ and $\vec{c}$ are given vectors, are equal to

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369. If $OABC$ is a tetrahedron where O is the origin and $A, B, andC$ are the other three vertices with position vectors, $\vec{a}, \vec{b}, and\vec{c}$ respectively, then prove that the centre of the sphere circumscribing the tetrahedron is given by position vector $\dfrac{a^2\left(\vec{b} \times \vec{c}\right) + b^2\left(\vec{c} \times \vec{a}\right) + c^2\left(\vec{a} \times \vec{b}\right)}{2\left[\vec{a}\vec{b}\vec{c}\right]}$.

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370. If K is the length of any edge of a regular tetrahedron, then the distance of any vertex from the opposite face is

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371. In $\triangle ABC,$ a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP,$ using vector method, find the area of ABC if the area of BRC is 1 unit

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372. Let $ABCD$ be a parallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P$

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373. Find $\vec{a}\vec{b}$ when: $\vec{a} = \hat{j} - \hat{k}\ and\ \vec{b} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

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374. if $\vec{a}=2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b}=\hat{i} + 2\hat{j} - 3\hat{k}$ then $\vec{a}X\vec{b}$ is

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375. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

A. (a) parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. (b) orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. (c) orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. (d) orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

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376. Find $| \rightarrow a \times \rightarrow b|$, if $\rightarrow a = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\rightarrow b = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

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377. find the value of x,y and z so that the vectors $\vec{a} = x\hat{i} + 2\hat{j} + z\hat{k}$ and $\vec{b}$= $2\hat{i} + y\hat{j} + \hat{k}$ are equal

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378. The lengths of two opposite edges of a tetrahedron are a and b; the shortest distane between these edges is d, and the angel between them is θ Prove using vectors that the volume of the tetrahedron is $\frac{abdsin\theta}{6}$.

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379. Volume of the parallelopiped whose adjacent edges are vectors $\vec{a}, \vec{b}, \vec{c}$ is $\vec{a} = 2\hat{i} - 3\hat{j} - 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k} and \vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$

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380. Column I, Column II If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} + 2\vec{b}\right|$, then angel between $\vec{a} and \vec{b}$ is, p. 90^0 If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - 2\vec{b}\right|$, then angel between $\vec{a} and \vec{b}$ is, q. obtuse If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then angel between $\vec{a} and \vec{b}$ is, r.0^0 Angle between $\vec{a} \times \vec{b}$ and a vector perpendicular to the vector $\vec{c} \times \left(\vec{a} \times \vec{b}\right)$ is, s. acute

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381. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + \hat{j} + 5\hat{k}$ and $\vec{C}$ form a left-handed system, then $\vec{C}$ is a.$11\hat{i} - 6\hat{j} - \hat{k}$ b.$-11\hat{i} + 6\hat{j} + \hat{k}$ c. $11\hat{i} - 6\hat{j} + \hat{k}$ d. $-11\hat{i} + 6\hat{j} - \hat{k}$

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382. Let $a = 2i - j + k$, $b = i + 2j - k$ and $c = i + j - 2k$ be three vectors. A vector r in the plane of b and c whose projection on a is of magnitude $\sqrt{\frac{2}{3}}$ is

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383. Vectors $\vec{A} and \vec{B}$ satisfying the vector equation $\vec{A} + \vec{B} = \vec{a}, \vec{A} \times \vec{B} = \vec{b} and \vec{A} \cdot \vec{a} = 1, where \vec{a} and \vec{b}$ are given vectors, are a. $\vec{A} = \frac{\left(\vec{a} \times \vec{b}\right) - \vec{a}}{a^2}$ b. $\vec{B} = \frac{\left(\vec{b} \times \vec{a}\right) + \vec{a}\left(a^2 - 1\right)}{a^2}$ c. $\vec{A} = \frac{\left(\vec{a} \times \vec{b}\right) + \vec{a}}{a^2}$ d. $\vec{B} = \frac{\left(\vec{b} \times \vec{a}\right) - \vec{a}\left(a^2 - 1\right)}{a^2}$

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384. if $\vec{\alpha}$ | | $\left(\vec{\beta} \times \vec{\gamma}\right)$, then $\left(\vec{\alpha} \times \beta\right).\left(\vec{\alpha} \times \vec{\gamma}\right)$ equals to a.$\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$ b. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$ c. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$ d. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

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385. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k} and \vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ are three coplanar vectors with $a \neq b, and \vec{v} = \hat{i} + \hat{j} + \dot{\hat{k}}$ Then v is perpendicular to $\vec{\alpha}$ b.$\vec{\beta}$ c. $\vec{\gamma}$ d. none of these

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386. $a_1, a_2, a_3, \in R - \{0\}$ and $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all $x \in R$, then

A. (a) vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. (b) vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$ are parallel to each other

C. (c) If vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then one of the ordered triplet is $\left(a_1, a_2, a_3\right) = (1, -1, -2)$

D. (d) If $2a_1 + 3a_2 + 6a_3 = 26$, then $\left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right|$ is $2\sqrt{6}$

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387. If P is any arbitrary point on the circumcircle of the equilateral triangle of side length l units, then $\left|\vec{P}A\right|^2 + \left|\vec{P}B\right|^2 + \left|\vec{P}C\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

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388. Let $\vec{a} and \vec{b}$ be two non-zero perpendicular vectors. A vecrtor $\vec{x}$ satisfying the equation $\vec{x} \times \vec{b} = \vec{a}$ is $\vec{x} = \beta\vec{b} - \frac{1}{|b|^2}\vec{a} \times \vec{b}$ then β can be

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389. If $\vec{a} and \vec{b}$ are two vectors and angle between them is $\theta,$ then

$\left|\vec{a}\times\vec{b}\right|^2+\left(\overset{\cdot}{\vec{a}\vec{b}}\right)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$ $\left|\vec{a}\times\vec{b}\right|=\left(\overset{\cdot}{\vec{a}\vec{b}}\right),\quad \text{if}\quad \theta=\pi/4$

$\vec{a}\times\vec{b}=\left(\overset{\cdot}{\vec{a}\vec{b}}\right)\hat{n},$ $\left(where \hat{n}\right.$ is unit vector,) if $\theta=\pi/4$ $\left(\vec{a}\times\vec{b}\right)\overset{\cdot}{}\vec{a}+\vec{b}=0$

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390. Let $\vec{r}$ be a unit vector satisfying $\vec{r}\times\vec{a}=\vec{b}, where\left|\vec{a}\right|=\sqrt{3} and \left|\vec{b}\right|=\sqrt{2}.$ Then $\vec{r}$ =?

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391. If vector $\vec{b}=\left(tan\alpha,\ -1,2\sqrt{\sin\alpha/2}\right) and \vec{c}=\left(tan\alpha, tan\alpha,\ -\dfrac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a}=(1,3,\sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is

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392. Let $\vec{a}, \vec{b}, and\vec{c}$ be non-zero vectors and $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right) and\vec{V}_2 = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$. Vectors $\vec{V}_1 and\vec{V}_2$ are equal. Then (a).$\vec{a} an\vec{b}$ are orthogonal (b). $\vec{a} and\vec{c}$ are collinear (c). $\vec{b} and\vec{c}$ are orthogonal (d). $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right) when\lambda$ is a scalar

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393. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} = \hat{k} and\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b} and\vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is $2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $2\hat{i} - 3\hat{j} + 3\hat{k}$ c. $-2\hat{i} - \hat{j} + 5\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$

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394. Let $\vec{PR} = 3\hat{i} + \hat{j} - 2\hat{k} and\vec{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS, and\vec{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determine by the vectors $\vec{PT}$, $\vec{PQ}$ and $\vec{PS}$ is 5 b. 20 c. 10 d. 30

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395. If in a right-angled triangle ABC, the hypotenuse $AB = p$,then $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

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396. If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \vec{a}.\vec{b} = 1 and \vec{a} \times \vec{b} = \hat{j} - \hat{k}, then \hat{b}$ is $\hat{i} - \hat{j} + \hat{k}$ b. $2\hat{j} - \hat{k}$ c. $\hat{i}$ d. $2\hat{i}$

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397. If $\vec{a}$ satisfies $\vec{a} \times \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - \hat{k}$, then $\vec{a}$ is equal to a. $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda \in R$ b. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$ c. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda \in R$ d. $\lambda\hat{i} - (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

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398. If $\vec{r}.\vec{a}=\vec{r}.\vec{b}=\vec{r}.\vec{c}=0, where \vec{a}, \vec{b},$ and $\vec{c}$ are non-coplanar, then a. $\vec{r}\perp\left(\vec{c}\times\vec{a}\right)$ b. $\vec{r}\perp\left(\vec{a}\times\vec{b}\right)$ c. $\vec{r}\perp\left(\vec{b}\times\vec{c}\right)$ d. $\vec{r}=\vec{0}$

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399. The unit vector orthogonal to vector $-\hat{i}+\hat{j}+2\hat{k}$ and making equal angles with the x and y-axis a.$\pm\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$ b. $\pm\frac{1}{3}\left(\hat{i}+\hat{j}-\hat{k}\right)$ c. $\pm\frac{1}{3}\left(2\hat{i}-2\hat{j}-\hat{k}\right)$ d. none of these

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400. Vectors $3\vec{a}-5\vec{b}$ and $2\vec{a}+\vec{b}$ are mutually perpendicular. If $\vec{a}+4\vec{b}$ and $\vec{b}-\vec{a}$ are also mutually perpendicular, then the cosine of the angle between a and b is a.$\frac{19}{5\sqrt{43}}$ b. $\frac{19}{3\sqrt{43}}$ c. $\frac{19}{2\sqrt{45}}$ d. $\frac{19}{6\sqrt{43}}$

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401. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is a.$\vec{b}$ + $\frac{\vec{b} \times \vec{a}}{\left|\vec{a}\right|^2}$ b. $\frac{\vec{a}\overset{\cdot}{}\vec{b}}{\left|\vec{b}\right|^2}$ c. $\vec{b}$ - $\frac{\vec{b}\overset{\cdot}{}\vec{a}}{\left|\vec{a}\right|^2}$ d. $\frac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

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402. The value of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k} and \vec{b} = 7\hat{i} - 2\hat{j} + \hat{k}$ is obtuse and the angle between b and the z-axis acute and less than $\pi/6$ is given by

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403. Let $\vec{a} \cdot \vec{b} = 0, where \vec{a} and \vec{b}$ are unit vectors and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a} and \overset{\cdot}{\vec{b}}$ If

$\vec{c} = m\vec{a} + n\vec{b} + p\left(\vec{a} \times \vec{b}\right), (m, n, p \in R)$, then a.- $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ b. $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$ c. $0 \leq \theta \leq \frac{\pi}{4}$ d. $0 \leq \theta \leq \frac{3\pi}{4}$

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404. A parallelogram is constructed on $3\vec{a} + \vec{b} and \vec{a} - 4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is 40 b. 64 c. 32 d. 48

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405. Let the position vectors of the points $P and Q$ be $4\hat{i} + \hat{j} + \lambda\hat{k} and 2\hat{i} - \hat{j} + \lambda\hat{k}$, respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $P and Q$. Then λ equals a $-1/2$ b. $1/2$ c. 1 d. none of these

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406. If a and c are unit vectors and $|b| = 4$. The angle between a and c is $\cos^{-1}(1/4)$ and $a \times b = 2a \times c$ then, $b - 2c = \lambda a$ The value of λ is

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407. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then

a. $|\vec{a}| = \left|\vec{b}\right| = |\vec{c}|$

b. $|\vec{a}| + \left|\vec{b}\right| + |\vec{c}| = |d|$

c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar

d. none of these

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408. If $\vec{a} + 2\vec{b} + 3\vec{c} = 0, then \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$ a. $2\left(\vec{a} \times \vec{b}\right)$ b. $6\left(\vec{b} \times \vec{c}\right)$ c. $3\left(\vec{c} \times \vec{a}\right)$ d. $\vec{0}$

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409. If $\vec{a}\ and\ \vec{b}$ are two non-collinear unit vector, and $\left|\ \vec{a} + \vec{b}\ \right| = 3\ then\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right)$=

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410. The angles of triangle, two of whose sides are represented by vectors $\sqrt{3}\left(\vec{a} \times \vec{b}\right)and\ \vec{b}$ - $\left(\overset{\cdot}{\hat{a}\vec{b}}\right)\hat{a},\ where\vec{b}$ is a non zero vector and $\hat{a}$ is unit vector in the direction of $\vec{a},$ are

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411. $\vec{a}, \vec{b}, and\vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicular to then. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k},$ and the angel between $\vec{a}and\vec{b}$ is $30^{0}, then\vec{c}$ is a. $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$ b. $\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right)/3$ c. $\left(2\hat{i} + 2\hat{j} - \hat{k}\right)/3$ d. $\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

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412. Vectors perpendicular to $\hat{i}-\hat{j}-\hat{k}$ and in the plane of $\hat{i}+\hat{j}+\hat{k} and -\hat{i}+\hat{j}+\vec{k}$ are $\hat{i}+\hat{k}$ b. $2\hat{i}+\hat{j}+\hat{k}$ c. $3\hat{i}+2\hat{j}+\hat{k}$ d. $-4\hat{i}-2\hat{j}-2\hat{k}$

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413. If side $\vec{AB}$ of an equilateral trangle ABC lying in the x-y plane $3\hat{i}$, then side $\vec{CB}$ can be a. $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$ b. $\frac{3}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$ c. $-\frac{3}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ d. $\frac{3}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

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414. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right).\vec{c}\times\vec{d}=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$ then a) $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar b) $\vec{b}, \vec{c}, \vec{d}$ are non -coplanar c)$\vec{b}, \vec{d}$ are non parallel d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

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415. Let two non-collinear unit vector $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time $t,$ the position vector $OP(whereO$ is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t\overset{.}{W}henP$ is farthest from origin $O,$ let M be the length of $OPand\hat{u}$ be the unit vector along $O\overset{.}{P}$ Then (a) $\hat{u} = \dfrac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}andM = \left(1 + \hat{a}\overset{.}{\hat{b}}\right)^{1/2}$ (b) $\hat{u} = \dfrac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}andM = \left(1 + \hat{a}\overset{.}{\wedge}\right)^{1/2}$ (c) $\hat{u} = \dfrac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}andM = \left(1 + 2\hat{a}\overset{.}{\hat{b}}\right)^{1/2}$ (d) $\hat{u} = \dfrac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}andM = \left(1 + 2\hat{a}\overset{.}{\hat{b}}\right)^{1/2}$

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416. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}and\vec{c} = \hat{i} + \hat{j} - \overset{.}{\hat{k}}$ Then find $\left[\vec{a}\vec{b}\vec{c}\right]$

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417. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-zero, non coplanar vector $\vec{b}_1 = \vec{b} - \dfrac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1 = \vec{c} - \dfrac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \dfrac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $, c_2 = \vec{c} - \dfrac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \dfrac{\vec{b}.\vec{c}}{|\vec{b}_1|^2}$,

$b_1, \vec{c}_3 = \vec{c} - \dfrac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} + \dfrac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c} - \dfrac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} = \dfrac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$ then the set of orthogonal vectors is

a. $\left(\vec{a}, \vec{b}_1, \vec{c}_3\right)$

b. $\left(\vec{a}, \vec{b}_1, \vec{c}_2\right)$

c. $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$

d. $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

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418. The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}and\hat{i} - \hat{j} + \hat{k}$ is $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ b. $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ c. $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$

d. $\dfrac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

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419. If $\vec{a}$ and $\vec{b}$ are unequal unit vectors such that $\left(\vec{a} - \vec{b}\right) \times \left[\left(\vec{b} + \vec{a}\right) \times \left(2\vec{a} + \vec{b}\right)\right] = \vec{a} + \vec{b}$, then angle θ between $\vec{a} and \vec{b}$ is

0 b. $\pi/2$ c. $\pi/4$ d. π

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420. If $\vec{a}, \vec{b}, \vec{c}$ are 3 unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b}}{2}$then $(\vec{b}$ and $\vec{c}$ being non parallel). (a)angle between $\vec{a} \& \vec{b}$ is $\frac{\pi}{3}$ (b)angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{3}$ (c)angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$ (d)angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{2}$

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421. Prove that $\left[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

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422. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} and \vec{c} = 3\hat{j} - 2\hat{k}$ Let $\vec{x}, \vec{y}, and \vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a},$ respectively. Then a.$\vec{x}.\vec{d} = -1$ b. $\vec{y}.\vec{d} = 1$ c. $\vec{z}.\vec{d} = 0$ d. $\vec{r}.\vec{d} = 0, where \vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

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423. If $a \times (b \times c) = (a \times b) \times c,$ then

a. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$

b.$\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$

c. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = 0$

d. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

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424. If $\hat{a}, \hat{b}, and \hat{c}$ are three unit vectors inclined to each other at angle θ, then the minimum value of θ is $\frac{\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{2\pi}{3}$ d. $\frac{5\pi}{6}$

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425. Let the pairs $a, b,$ and c, d each determine a plane. Then the planes are parallel if a.$(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{d})=\vec{0}$ b. $(\vec{a}\times\vec{c}).(\vec{b}\times\vec{d})=\vec{0}$ c. $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$ d. $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=\vec{0}$

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426. $P(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the position vectorvariable point. If R moves such that $(\vec{r}-\vec{p})\times(\vec{r}-\vec{q})=0$ then the locus of R is

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427. Two adjacent sides of a parallelogram $ABCD$ are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. Then the value of $|AC\times BD|$ is a. $20\sqrt{5}$ b. $22\sqrt{5}$ c. $24\sqrt{5}$ d. $26\sqrt{5}$

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428. If $\hat{a}, \hat{b}, and \hat{c}$ are three unit vectors, such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are angles between the vectors $\hat{a}, \hat{b}; \hat{b}, \hat{c} and \hat{c}, \hat{a}$ respectively, then among θ_1, θ_2 and θ_3. a. all are acute angles b. all are right angles c. at least one is obtuse angle d. none of these

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429. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$.

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430. Let $\vec{a} = \hat{i} + \hat{j}; \vec{b} = 2\hat{i} - \hat{k}$ Then vector $\vec{r}$ satisfying $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ then $\vec{r}$ is a.$\hat{i} - \hat{j} + \hat{k}$ b. $3\hat{i} - \hat{j} + \hat{k}$ c. $3\hat{i} + \hat{j} - \hat{k}$ d. $\hat{i} - \hat{j} - \hat{k}$

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431. If $\vec{a}, \vec{b}$ are two vectors such that $\vec{a}.\vec{b} < 0$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between $\vec{a}$ and $\vec{b}$ is

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432. $\vec{a}, \vec{b}$, and $\vec{c}$ are three vectors of equal magnitude. The angle between each pair of vectors is $\pi/3$ such that $\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{6}$. Then $\left|\vec{a}\right|$ is equal to a.2 b. -1 c. 1 d. $\sqrt{6}/3$

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433. Let $\vec{p} and \vec{q}$ be any two orthogonal vectors of equal magnitude 4 each. Let $\vec{a}, \vec{b}, and \vec{c}$ be any three vectors of lengths $7\sqrt{15} and 2\sqrt{33}$, mutually perpendicular to each other. Then find the distance of the vector

$$\left(\overset{\cdot}{\vec{a}\vec{p}}\right)\vec{p} + \left(\overset{\cdot}{\vec{a}\vec{q}}\right)\vec{q} + \left(\overset{\cdot}{\vec{a}\vec{p} \times \vec{q}}\right)(\vec{p} \times \vec{q}) + \left(\overset{\cdot}{\vec{b}\vec{p}}\right)\vec{p}\left(\overset{\cdot}{\vec{b}\vec{q}}\right)\vec{q} + \left(\overset{\cdot}{\vec{b}\vec{p} \times \vec{q}}\right)(\vec{p} \times \vec{q}) + \Big($$

from the origin.

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434. Let $\vec{a} and \vec{b}$ be two non-collinear unit vector. If $\vec{u} = \vec{a} - \left(\vec{a}\vec{b}\right)\vec{b} and \vec{v} = \vec{a} \times \vec{b}, then\left|\vec{v}\right|$ is a. $\left|\vec{u}\right|$ b. $\left|\vec{u}\right| + \left|\vec{u}\vec{a}\right|$ c. $\left|\vec{u}\right| + \left|\vec{u}\vec{b}\right|$ d. $\left|\vec{u}\right| + \hat{u}\left|\vec{a} + \vec{b}\right|$

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435. The vertex A triangle ABC is on the line $\vec{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i}and\hat{j}$ Let Δ be the area of the triangle and $\Delta\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to A is a.$[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

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436. If a is real constant A, B and C are variable angles and $\sqrt{a^2 - 4\tan A} + a\tan B + \sqrt{a^2 + 4\tan C} = 6a,$ then the least value of $\tan^2 A + \tan^2 B + \tan^2 C$ is a. 6 b. 10 c. 12 d. 3

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437. The position vectors of the vertices $A, BandC$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and\vec{c},$ respectively. A vector $\vec{d}$ is such that $\vec{d}\cdot\vec{a} = \vec{d}\cdot\vec{b} = \vec{d}\cdot\vec{c}and\vec{d} = \lambda\left(\vec{b} + \vec{c}\right)^{\cdot}$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

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438. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b}\cdot\vec{q} = 0and\left|\vec{b}\right|^2 = 1, where\mu$ is a scalar. Then $\left|\left(\vec{a}\overset{\cdot}{}\vec{q}\right)\vec{p} - \left(\vec{p}\overset{\cdot}{}\vec{q}\right)\vec{a}\right|$ is equal to (a) $2\left|\vec{p}.\vec{q}\right|$ (b) $(1/2)\left|\vec{p}.\vec{q}\right|$ (c) $\left|\vec{p}\times\vec{q}\right|$ (d) $\left|\vec{p}.\vec{q}\right|$

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439. In AB, DE and GF are parallel to each other and AD, BG and EF ar parallel to each other . If CD: CE = CG:CB = 2:1 then the value of area $(\triangle AEG): area(\triangle ABD)$ is equal to (a) 7/2 (b)3 (c)4 (d)9/2

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440. In a quadrilateral $ABCD, \vec{A}C$ is the bisector of $\vec{A}Band\vec{A}D$, angle between $\vec{A}Band\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|^{.}$ Then the angle between $\vec{B}Aand\vec{C}D$ is $(a)\cos^{-1}\left(\frac{\sqrt{14}}{7\sqrt{2}}\right)$ b. $\cos^{-1}\left(\frac{\sqrt{21}}{7\sqrt{3}}\right)$ c. $\cos^{-1}\left(\frac{2}{\sqrt{7}}\right)$ d. $\cos^{-1}\left(\frac{2\sqrt{7}}{14}\right)$

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441. Position vector $\hat{k}$ is rotated about the origin by angle 135^0 in such a way that the plane made by it bisects the angle between $\hat{i}$ and $\hat{j}$. Then its new position is

A. a. $\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}$

B. b. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$

C. c. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$

D. d. none of these

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442. A non-zero vector $\vec{a}$ is such that its projections along vectors $\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal, then unit vector along $\vec{a}$ is a. $\frac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$ b. $\frac{\hat{j}-\sqrt{2}\hat{k}}{\sqrt{3}}$ c. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

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443. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is a.$\frac{1}{\sqrt{2}}\left(- \hat{j} + \hat{k}\right)$ b. $\frac{1}{\sqrt{3}}\left(- \hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{\sqrt{5}}\left(- \hat{k} - 2\hat{j}\right)$ d. $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} - \hat{k}\right)$

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444. Let $\vec{a} = 2i + j - 2kand\vec{b} = i + j^{\cdot}$ If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = \left|\vec{c}\right|, \left|\vec{c} - \vec{a}\right| = 2\sqrt{2}$ between $\vec{a} \times \vec{b}$ and $\vec{c}is30^0, then\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ I equal to a. $2/3$ b. $3/2$ c. 2 d. 3

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445. Let $ABCD$ be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be $3, 4$ and $5sq. units$, respectively. Then the area of triangle BCD is

a.$5\sqrt{2}$

b. 5

c. $\frac{\sqrt{5}}{2}$

d. $\frac{5}{2}$

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446. Vector $\vec{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j} and \vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b} and \vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}^{\cdot}$ The value of$\vec{a}$ is a. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$ b. $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ c. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$ d. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

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447. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is a. $3\pi/4$ b. $\pi/4$ c. $\pi/2$ d. π

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448. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=0.$ If $\left|\vec{u}\right|=3, \left|\vec{v}\right|=4$ and $\left|\vec{w}\right|=5,$ then $\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}$ is a.47 b. -25 c. 0 d. 25

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449. If $\vec{a}, \vec{b}, and\vec{c}$ are three non-coplanar non-zero vecrtors, then prove that $\left(\vec{a}\overset{\cdot}{\ }\vec{a}\right)\vec{b}\times\vec{c}+\left(\vec{a}\overset{\cdot}{\ }\vec{b}\right)\vec{c}\times\vec{a}+\left(\vec{a}\overset{\cdot}{\ }\vec{c}\right)\vec{a}\times\vec{b}=\left[\vec{b}\vec{c}\vec{a}\right]\vec{a}^{\,\cdot}$

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450. Let $\vec{p}and\vec{q}$ be any two orthogonal vectors of equal magnitude 4 each. Let $\vec{a}, \vec{b}, and\vec{c}$ be any three vectors of lengths $7\sqrt{15}and2\sqrt{33},$ mutually perpendicular to each other. Then find the distance of the vector $\left(\vec{a}\overset{\cdot}{\ }\vec{p}\right)\vec{p}+\left(\vec{a}\overset{\cdot}{\ }\vec{q}\right)\vec{q}+\left(\vec{a}\vec{p}\overset{\cdot}{\ }\times\vec{q}\right)\left(\vec{p}\times\vec{q}\right)+\left(\vec{b}\overset{\cdot}{\ }\vec{p}\right)\vec{p}\left(\vec{b}\overset{\cdot}{\ }\vec{q}\right)\vec{q}+\left(\vec{b}\vec{p}\overset{\cdot}{\ }\times\vec{q}\right)\left(\vec{p}\times\vec{q}\right)+($ from the origin.

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451. $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar ,non-zero vectors and $\vec{r}$ is any vector in space, then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{r}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{r}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{r}\times\vec{b}\right)$ is equal to

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ b. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ c. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ d. none of these

B. null

C. null

D. null

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452. Find a unit vector perpendicular to each of the vectors $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

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453. Prove that $\left(\vec{a}.\left(\vec{b}\times\hat{i}\right)\right)\hat{i}+\left(\vec{a}.\left(\vec{b}\times\hat{j}\right)\right)\hat{j}+\left(\vec{a}.\left(\vec{b}\times\hat{k}\right)\right)\hat{k}=\vec{a}\times\vec{b}.$

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454. Column I, Column II The possible value of $\vec{a}$ if $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+2\hat{i}-\hat{k}\right)$ and $\vec{r}=\left(\hat{i}+2\hat{j}\right)+\mu\left(-\hat{i}+\hat{j}+a\hat{k}\right)$ are not consistent, where $\lambda and \mu$ are scalars, is, p. -4 The angel between vectors $\vec{a}=\lambda\hat{i}-3\hat{j}-\hat{k} and \vec{b}=2\lambda\hat{i}+\lambda\hat{j}-\hat{k}$ is acute, whereas vecrtor $\vec{b}$ makes an obtuse angel with the axes of coordinates. Then λ may be, q. -2 The possible value of a such that $2\hat{i}-\hat{j}+\hat{k},\hat{i}+2\hat{j}+(1+a)\hat{k} and 3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar is, r. 2 If $\vec{A}=2\hat{i}+\lambda\hat{j}+3\hat{k},\vec{B}=2\hat{i}+\lambda\hat{j}+\hat{k},\vec{C}=3\hat{i}+\hat{j} and \vec{A}+\lambda\vec{B}$ is perpendicular to $\vec{C}$ then $|2\lambda|$ is, s. 3

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455. If $\vec{A},\vec{B}$ and $\vec{C}$ are vectors such that $\left|\vec{B}\right|=\left|\vec{C}\right|$. Prove that

$$\left[\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)\right]\times\left(\vec{B}+\vec{C}\right).\left(\vec{B}+\vec{C}\right)=0$$

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456. A parallelogram is constructed on $3\vec{a} + \vec{b}and\vec{a} - 4\vec{b}, where\left|\vec{a}\right| = 6and\left|\vec{b}\right| = 8, and\vec{a}and\vec{b}$ are anti-parallel. Then the length of the longer diagonal is 40 b. 64 c. 32 d. 48

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457. Statement 1: Vector $\vec{c} = -5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angel between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}and\vec{b} = 8\hat{i} + \hat{j} - 4\hat{k}^{\cdot}$ Statement 2: $\vec{c}$ is equally inclined to $\vec{a}and\vec{b}^{\cdot}$

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458. Statement 1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}is\hat{i} - \hat{j}^{\cdot}$ Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is $2\hat{i} + 2\hat{j} + 2\hat{k}^{\cdot}$

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459. Statement 1 : Points $A(1, 0), B(2, 3), C(5, 3), and D(6, 0)$ are concyclic. Statement 2 : Points $A, B, C, and D$ form an isosceles trapezium or $AB and CD$ meet at E. Then $EA \cdot EB = EC \cdot ED$.

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460. Let $\vec{r}$ be a non-zero vector satisfying $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for given non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Statement 1: $\left[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}\right] = 0$ Statement 2: $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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461. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both

$\vec{a}$ and $\vec{b}$. If the angle between a and b is $\frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

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462. Statement-I $A = 2\hat{i} + 3\hat{j} + 6\hat{k}, B = \hat{i} + \hat{j} - 2\hat{k}$ and $C = \hat{i} + 2\hat{j} + \hat{k}$, then $|A \times (A \times (A \times B)) \cdot C| = 243$

Statement-II $|A \times (A \times (A \times B)) \cdot C| = |A|^2|[ABC]|$

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463. If $\vec{a}, \vec{b}, and \vec{c}$ are mutually perpendicular vectors and $\vec{a} = \alpha\left(\vec{a} \times \vec{b}\right) + \beta\left(\vec{b} \times \vec{c}\right) + \gamma\left(\vec{c} \times \vec{a}\right) and \left[\vec{a}\vec{b}\vec{c}\right] = 1$, then find the value of $\alpha + \beta + \gamma$.

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464. Let vectors $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ be such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = 0.$ Let $P_1 and P_2$ be planes determined by the pair of vectors $\vec{a}, \vec{b}, and \vec{c}, \vec{d}$, respectively. Then the angle between $P_1 and P_2$ is a.0 b. $\pi/4$ c. $\pi/3$ d. $\pi/2$

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465. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

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466. Prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a} \times \hat{i}\right) + \left(\vec{a}.j\right)\left(\vec{a} \times \hat{j}\right) + \left(\vec{a}.\hat{k}\right)\left(\vec{a} \times \hat{k}\right) = 0.$

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467. Let $\vec{f}(t) = [t]\hat{i} + (t - [t])\hat{j} + [t + 1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors $\vec{f}\left(\frac{5}{4}\right) and f(t), 0 < t < 1$ are(a) parallel to each other(b) perpendicular(c) inclined at $\cos^{-1}2\left(\sqrt{7\left(1 - t^2\right)}\right)$ (d)inclined at $\cos^{-1}\left(\frac{8 + t}{9\sqrt{1 + t^2}}\right)$;

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468. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, then $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to a. $\left|\vec{a}\right|^2\left(\vec{b}.\vec{c}\right)$ b. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$ c. $\left|\vec{c}\right|^2\left(\vec{a}.\vec{b}\right)$ d. none of these

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469. The three vectors $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume: ____________

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470. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then

a. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$

c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar

d. none of these

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471. If $|a| = 2and|b| = 3$ and $a\dot{b} = 0, then(a \times (a \times (a \times (a \times b))))$ is equal to $48\hat{b}$ b. $16\hat{b}$ c. $48\hat{a}$ d. - $48\hat{a}$

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472. If the two diagonals of one its faces are $6\hat{i} + 6\hat{k} and 4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $c = 4\hat{j} - 8\hat{k}$, then the volume of a parallelepiped is a. 60 b. 80 c. 100 d. 120

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473. The volume of a tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, and \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a} + \vec{b}, \vec{b} + \vec{c} and \vec{c} + \vec{a}$ is 6 b. 18 c. 36 d. 9

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474. If $\vec{a}, \vec{b}, \text{ and } \vec{c}$ are three mutually orthogonal unit vectors, then the triple product $\left[\vec{a} + \vec{b} + \vec{c} \vec{a} + \vec{b} \vec{b} + \vec{c}\right]$ equals: (a.) 0 (b.) $1 \text{ or } -1$ (c.) 6 (d.) 3

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475. Vector $\vec{c}$ is perpendicular to vectors $\vec{a} = (2, -3, 1) and \vec{b} = (1, -2, 3)$ and satisfies the condition $\vec{x} \cdot \left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 10$. Then vector $\vec{c}$ is equal to a.$(7, 5, 1)$ b. $-7, -5, -1$ c. $1, 1, -1$ d. none of these

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476. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j}; \vec{a} \perp \vec{b}, \vec{a}\vec{c} = 4$. Then $\left[\vec{a}\vec{b}\vec{c}\right]^2 = |\vec{a}|$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^{=}|\vec{a}|$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^{=}0$ d. $\left[\vec{a}\vec{b}\vec{c}\right]^{=}|\vec{a}|^2$

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477. $\vec{a} and \vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that is equally inclined to $\vec{a}, \vec{b} and \vec{a} \times \vec{b}$ is a.$\frac{1}{\sqrt{2}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$ b. $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{a} + \vec{b}\right)$ c. $\frac{1}{\sqrt{3}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$ d. $\frac{1}{3}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

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478. If $\vec{r} and \vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r} + b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2 + \left|\vec{r} + b\vec{s}\right|^2$ is equal to a.$2\left|\vec{r}\right|^2$ b. $\left|\vec{r}\right|^2/2$ c. $3\left|\vec{r}\right|^2$ d. $|r|^2$

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479. The scalar $\vec{A}\left(\left(\vec{B} + \vec{C}\right) \times \left(\vec{A} + \vec{B} + \vec{C}\right)\right)$ equals

a.0 b. $\left[\vec{A}\vec{B}\vec{C}\right] + \left[\vec{B}\vec{C}\vec{A}\right]$ c. $\left[\vec{A}\vec{B}\vec{C}\right]$ d. none of these

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480. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is a.$\frac{4}{13}$ b. 4 c. $\frac{2}{7}$ d. 2

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481. For non-zero vectors $\vec{a}, \vec{b}, and\vec{c}$, $\left|\left(\vec{a}\times\vec{b}\right)\dot{\vec{c}}\right| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if a.$\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0$ b. $\vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$ c. $\vec{c}\cdot\vec{a}=0, \vec{a}\cdot\vec{b}=0$ d. $\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$

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482. For three vectors $\vec{u}, \vec{v}and\vec{w}$ which of the following expressions is not equal to any of the remaining three ? a.$\vec{u}\dot{\vec{v}}\times\vec{w}$ b. $\left(\vec{v}\times\vec{w}\right)\dot{\vec{u}}$ c. $\vec{v}\dot{\vec{u}}\times\vec{w}$ d. $\left(\vec{u}\times\vec{v}\right)\dot{\vec{w}}$

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483. Let $\vec{A}$ be a vector parallel to the line of intersection of planes P_1andP_2. Plane P_1 is parallel to vectors $2\hat{j}+3\hat{k}and4\hat{j}-3kandP_2$ is parallel to $\hat{j}-\hat{k}and3\hat{i}+3\hat{j}$. Then the angle betweenvector $\vec{A}$ and a given vector $2\hat{i}+\hat{j}-2\hat{k}$ is a.$\pi/2$ b. $\pi/4$ c. $\pi/6$ d. $3\pi/4$

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484. If $\vec{a}.\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}, then\vec{b}$ is $\dfrac{\left(\beta\vec{a}-\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^{2}}$ b. $\dfrac{\left(\beta\vec{a}+\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^{2}}$ c. $\dfrac{\left(\beta\vec{c}-\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^{2}}$ d. $\dfrac{\left(\beta\vec{a}+\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^{2}}$

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485. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vecrors and $\vec{r}$ be any arbitrary vector. Then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{r}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{r}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{r}\times\vec{b}\right)$ is always equal to $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ b. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ c. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ d. none of these

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486. Let $\vec{a} and \vec{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$, a. $\vec{r}=\left(\vec{r}\overset{.}{\hat{a}}\right)\hat{a}+\left(\vec{r}\overset{.}{\hat{b}}\right)\hat{b}+\left(\vec{r}\overset{.}{\hat{a}}\times\hat{b}\right)\left(\hat{a}\times\hat{b}\right)$ b.

$\vec{r} = \left(\vec{r}\overset{\cdot}{\,}\hat{a}\right) - \left(\vec{r}\overset{\cdot}{\,}\hat{b}\right)\hat{b} - \left(\vec{r}\overset{\cdot}{\,}\hat{a} \times \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$ c.

$\vec{r} = \left(\vec{r}\overset{\cdot}{\,}\hat{a}\right)\hat{a} - \left(\vec{r}\overset{\cdot}{\,}\hat{b}\right)\hat{b} + \left(\vec{r}\overset{\cdot}{\,}\hat{a} \times \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$ none of these

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487. Value of $\left[\vec{a} \times \vec{b}, \vec{a} \times \vec{c}, \vec{d}\right]$ is always equal to a. $\left(\vec{a}\overset{\cdot}{\,}\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$ b. $\left(\vec{a}\overset{\cdot}{\,}\vec{c}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ c. $\left(\vec{a}\overset{\cdot}{\,}\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ d. none of these

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488. Let $\vec{a} and \vec{b}$ be unit vectors that are perpendicular to each other. Then $\left[\vec{a} + \left(\vec{a} \times \vec{b}\right)\vec{b} + \left(\vec{a} \times \vec{b}\right)\vec{a} \times \vec{b}\right]$ will always be equal to 1 b. 0 c. -1 d. none of these

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489. Let $\vec{r}, \vec{a}, \vec{b} and \vec{c}$ be four nonzero vectors such that

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$\vec{r}\vec{a} = 0, \left|\vec{r} \times \vec{b}\right| = |\vec{r}|\left|\vec{b}\right| and \left|\vec{r} \times \vec{c}\right| = |\vec{r}||\vec{c}|^{\cdot}$ Then $[abc]$ is equal to $|a||b||c|$

b. -$|a||b||c|$ c. 0 d. none of these

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490. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between a and b is $\frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

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491. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0,$ then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to

a. vector perpendicular to the plane ofa, b, c b. a scalar quantity c. $\vec{0}$ d. none of these

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492. If $\vec{a}, \vec{b}, and\vec{c}$ are such that $\left[\vec{a}\vec{b}\vec{c}\right] = 1, \vec{c} = \lambda\vec{a} \times \vec{b},$, $\left|\vec{a}\right| = \sqrt{2}, \left|\vec{b}\right| = \sqrt{3}and\left|\vec{c}\right| = \frac{1}{\sqrt{3}}$, then the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$

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493. A vector of magnitude $\sqrt{2}$ coplanar with the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k},$ and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k},$ is a.$-\hat{j} + \hat{k}$ b. $\hat{i} - \hat{k}$ c. $\hat{i} - \hat{j}$ d. $\hat{i} - \hat{j}$

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494. Let P be a point interior to the acute triangle ABC If $PA + PB + PC$ is a null vector, then w.r.t triangle ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

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495. G is the centroid of triangle ABC and A_1 and B_1 are the midpoints of sides AB and AC , respectively. If Δ_1 is the area of quadrilateral GA_1AB_1 and Δ is the area of triangle ABC , then $\frac{\Delta}{\Delta_1}$ is equal to

a. $\frac{3}{2}$

b. 3

c. $\frac{1}{3}$

d. none of these

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496. Points $\vec{a}, \vec{b}, \vec{c}, and\vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = 0.$ Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma is$ a. $\frac{1}{14}$ b. 14 c. 6 d. $1/\sqrt{6}$

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497. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 1 and 2, respectively, and $\left(1 - 3\vec{a}.\vec{b}\right)^2 + \left|2\vec{a} + \vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2 = 47,$ then the angel between $\vec{a} and \vec{b}$ is $\pi/3$ b. $\pi - \cos^{-1}(1/4)$ c. $\frac{2\pi}{3}$ d. $\cos^{-1}(1/4)$

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498. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 2 and 3, respectively, such that $\left|2\left(\vec{a} \times \vec{b}\right)\right| + \left|3\left(\vec{a}.\vec{b}\right)\right| = k,$ then the maximum value of k is a. $\sqrt{13}$ b. $2\sqrt{13}$ c. $6\sqrt{13}$ d. $10\sqrt{13}$

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499. If $\vec{a}\ and\ \vec{b}$ are two vectors such that $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}\ and\ \vec{a}\vec{b} = 1,$ find the angle between $\vec{a}\ and\ \vec{b}$.

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500. If the vector product of a constant vector $\vec{O}A$ with a variable vector $\vec{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is a. a straight line perpendicular to $\vec{O}A$ b. a circle with centre O and radius equal to $\left|\vec{O}A\right|$ c. a straight line parallel to $\vec{O}A$ d. none of these

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501. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $\left|\vec{u}\right| = 1, \left|\vec{v}\right| = 2 and \left|\vec{w}\right| = 3.$ If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $\left|\vec{u} - \vec{v} + \vec{w}\right|$ equals 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. 2

B. sqrt(7)`

C. sqrt(14)`

D. 14`

Answer: 3

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502. If the two adjacent sides of two rectangles are represented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} + \vec{b}$, respectively, then the angel between the vector $\vec{x} = \frac{1}{3}\left(\vec{p} + \vec{r} + \vec{s}\right)$ and $\vec{y} = \frac{1}{5}\left(\vec{r} + \vec{s}\right)$ is a.$-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ b. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ c. $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ d. cannot be evaluate

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503. Let P, Q, R and S be the points on the plane with position vectors $-2i - j, 4i, 3i + 3j and - 3i + 2j,$ respectively. The quadrilateral $PQRS$ must be (a) Parallelogram, which is neither a rhombus nor a rectangle (b) Square (c) Rectangle but not a square (d) Rhombus, but not a square

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504. $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar unit vecrtors and α, β and γ are the angles between$\vec{u}$ and $\vec{v}, \vec{v}$and $\vec{w}, and \vec{w}$ and $\vec{u},$ respectively, and $\vec{x}, \vec{y}$ and $\vec{z}$ are unit vectors along the bisectors of the angles $\alpha, \beta and \gamma$, respectively. Prove that

$$\left[\vec{x} \times \vec{y}\,\vec{y} \times \vec{z}\,\vec{z} \times \vec{x}\right] = \frac{1}{16}\left[\vec{u}\,\vec{v}\,\vec{w}\right]^2 \sec^2\left(\frac{\alpha}{2}\right)\sec^2\left(\frac{\beta}{2}\right)\sec^2\left(\frac{\gamma}{2}\right).$$

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505. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}; \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, . \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ and $\left[3\vec{a} + \vec{b}\;\; 3\vec{b} + \vec{c}\;\; 3\vec{c} + \vec{a}\right] = \lambda\left[\vec{a}\vec{b}\vec{c}\right]$, then find the value of $\frac{\lambda}{4}$.

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506. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(t, 1, 1)$ is minimum.

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507. The condition for equations $\vec{r}\times\vec{a}=\vec{b} and \vec{r}\times\vec{c}=\vec{d}$ to be consistent is a. $\vec{b}\cdot\vec{c}=\vec{a}\cdot\vec{d}$ b. $\vec{a}\cdot\vec{b}=\vec{c}\cdot\vec{d}$ c. $\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{d}=0$ d. $\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{d}=0$

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508. If $a and b$ are nonzero non-collinear vectors, then $\left[\vec{a}\vec{b}\hat{i}\right]\hat{i}+\left[\vec{a}\vec{b}\hat{j}\right]\hat{j}+\left[\vec{a}\vec{b}\hat{k}\right]\hat{k}$ is equal to a. $\vec{a}\times\vec{b}$ b. $\vec{a}+\vec{b}$ c. $\vec{a}-\vec{b}$ d. $\vec{b}\times\vec{a}$

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509. $\left(\vec{a}+\vec{b}\right)\vec{b}+\vec{c}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=$

a. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

b. 0

c. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

d. $-\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

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510. A vector of magnitude 10 along the normal to the curve $3x^2+8xy+2y^2-3=0$ at its point $P(1,0)$ can be (A) $6\hat{i}+8\hat{j}$ (B) $-8\hat{i}+3\hat{j}$ (C) $6\hat{i}-8\hat{j}$ (D) $8\hat{i}+6\hat{j}$

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511. If $a\left(\vec{\alpha}\times\vec{\beta}\right)+b\left(\vec{\beta}\times\vec{\gamma}\right)+c\left(\vec{\gamma}\times\vec{\alpha}\right)=0$ and at least one of $a, bandc$ is nonzero, then vectors $\vec{\alpha}, \vec{\beta}and\vec{\gamma}$ are a. parallel b. coplanar c. mutually perpendicular d. none of these

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512. If $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)=\vec{b}, where\vec{a}, \vec{b}, and\vec{c}$ are nonzero vectors, then 1. $\vec{a}, \vec{b}, and\vec{c}$ can be coplanar 2.$\vec{a}, \vec{b}, and\vec{c}$ must be coplanar 3.$\vec{a}, \vec{b}, and\vec{c}$ cannot be coplanar 4.none of these

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513. If $\vec{a}, \vec{b}and\vec{c}$ are three non coplanar vectors, then $\left(\vec{a}+\vec{b}+\vec{c}\right)\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)\right]$ is :

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514. If $\vec{x}+\vec{c}\times\vec{y}=\vec{a}and\vec{y}+\vec{c}\times\vec{x}=\vec{b}, where\vec{c}$ is a nonzero vector, then which of the following is not correct? a.$\vec{x}=\dfrac{\vec{b}\times\vec{c}+\vec{a}+\left(\overset{\cdot}{\vec{c}\vec{a}}\right)\vec{c}}{1+\overset{\cdot}{\vec{c}\vec{c}}}$ b.

$\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + \left(\vec{c} \cdot \vec{a}\right)\vec{c}}{1 + \vec{c} \cdot \vec{c}}$ c. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + \left(\vec{c} \cdot \vec{b}\right)\vec{c}}{1 + \vec{c} \cdot \vec{c}}$ d. none of these

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515. If $\vec{a} and \vec{b}$ are two unit vectors incline at angle $\pi/3$, then $\left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\} \cdot \vec{b}$ is equal to $\dfrac{-3}{4}$ b. $\dfrac{1}{4}$ c. $\dfrac{3}{4}$ d. $\dfrac{1}{2}$

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516. Value of $\left[\vec{a} \times \vec{b}, \vec{a} \times \vec{c}, \vec{d}\right]$ is always equal to a. $\left(\vec{a} \cdot \vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$ b. $\left(\vec{a} \cdot \vec{c}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ c. $\left(\vec{a} \cdot \vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ d. none of these

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517. Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r and c_r, where r = 1, 2, 3, are non-negative real numbers and $\sum_{r=1}^{3}\left(a_r + b_r + c_r\right) = 3L$ show that $V \le L^3$

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518. Find 3-dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying

$\vec{v}_1 \cdot \vec{v}_1 = 4, \vec{v}_1 \cdot \vec{v}_2 = -2, \vec{v}_1 \cdot \vec{v}_3 = 6,$

$\vec{v}_2 \cdot \vec{v}_2 = 2, \vec{v}_2 \cdot \vec{v}_3 = -5, \vec{v}_3 \cdot \vec{v}_3 = 29$

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519. Let $\vec{u}$ and $\vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$. Find the value of $\left[\vec{u}\ \vec{v}\ \vec{w}\right]$.

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520. For any two vectors $\vec{u} and \vec{v}$ prove that $\left(\vec{u}.\ \vec{v}\right)^2 + \left|\vec{u} \times \vec{v}\right|^2 = \left|\vec{u}\right|^2\left|\vec{v}\right|^2$

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521. If the incident ray on a surface is along the unit vector $\vec{v}$, the reflected ray is along the unit vector $\vec{w}$ and the normal is along the unit vector $\vec{a}$ outwards, express $\vec{w}$ in terms of $\vec{a}$ and $\vec{v}$

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522. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} and \vec{a} \times \vec{b} = \vec{c} \times \vec{d}$, prove that $\left(\vec{a} - \vec{d}\right).\left(\vec{b} - \vec{c}\right) \neq 0,$

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523. Given two vectors $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k} and \vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$ Column I, Column II A vector coplanar with $\vec{a} and \vec{b}$, p. $-3\hat{i} + 3\hat{j} + 4\hat{k}$ A vector which is perpendicular to both $\vec{a} and \vec{b}$, q. $2\hat{i} - 2\hat{j} + 3\hat{k}$ A vector which is equally inclined to $\vec{a} and \vec{b}$, r. $\hat{i} + \hat{j}$ A vector which forms a triangle with $\vec{a} and \vec{b}$, s. $\hat{i} - \hat{j} + 5\hat{k}$

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524. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is a.-1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

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525. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and l,m,n are distinct real numbers, then $[(l\vec{a} + m\vec{b} + n\vec{c}\Big)\Big(l\vec{b} + m\vec{c} + n\vec{a}\Big)\Big(l\vec{c} + m\vec{a} + n\vec{b})]$ = 0, implies (A) $lm + mn + nl = 0$ (B) $l + m + n = 0$ (C) $l^2 + m^2 + n^2 = 0$

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526. If $\vec{a}, \vec{b} and \vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b} 2\vec{b} - \vec{c} 2\vec{c} - \vec{a}\right]$ is 0 b. 1 c. $-\sqrt{3}$ d. $\sqrt{3}$

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