MATHS

BOOKS - CENGAGE

APPLICATION OF DERIVATIVES

Solved Examples And Exercises

1. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ cuts orthogonally

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2. Find the angle of intersection of $y = a^x andy = b^x$

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3. If $y = x + 2$ is normal to the parabola $y^2 = 4ax$, then find the value of a.

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4. Find the cosine of the angle of intersection of curves $f(x) = 2^x (\log)_e x and g(x) = x^{2x} - 1$.

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5. The acute angle between the curves $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their points of intersection when when x> 0, is

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6. In the curve $x^{m+n} = a^{m-n} y^{2n}$, prove that the mth power of the sub-tangent varies as the nth power of the sub-normal.

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7. Find the length of the tangent for the curve $y = x^3 + 3x^2 + 4x - 1$ at point $x = 0$.

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8. Find the equation of a curve passing through the origin, given that the slope of the tangent of the curve at any point (x, y) is equal to tha sum of the coordinates of the point.

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9. Sketch the curve $y = f(x) = x^2 - 5x + 6$

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10. The condition that the equation $ax^2 + bx + c = 0$may have one root is the double the other is

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11. Find c of Lagranges mean value theorem for the function $f(x) = 3x^2 + 5x + 7$ in the interval $[1, 3]$.

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12. Let $0 < a < b < \frac{\pi}{2}. \ If \ f(x) = \begin{vmatrix} \tan x & \tan a & \tan b \\ \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \end{vmatrix}$, then find the minimum possible number of roots of $f'(x) = 0$ in (a , b)

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13. Let $f(x) and g(x)$ be differentiable for $0 \leq x \leq 2$ such that $f(0) = 2, g(0) = 1, and f(2) = 8.$ Let there exist a real number c in

$[0, 2]$ such that $f'(c) = 3g'(c)$. Then find the value of $g(2)$.

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14. Let f be continuous on $[a, b], a > 0, and$ differentiable on (a, b). Prove that there exists $c \in (a, b)$ such that $\frac{bf(a) - af(b)}{b - a} = f(c) - cf'(c)$

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15. Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x + y + z - xyz}{1 - xy - yz - zx}\right)$

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16. Using Lagranges mean value theorem, prove that $|\cos a - \cos b| < |a - b|$.

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17. If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

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18. Let f be differentiable for all x, If $f(1) = -2 and f'(x) \geq 2$ for all $x \in [1, 6]$, then find the range of values of $f(6)$.

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19. The equation of the tangent to the curve $y = x^2 - 4x + 2$ at (4, 2) is

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20. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$.

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21. If the equetion of tangent to the curve $y^3 = ax^3 +$ b at point (2,3) is y = 4x = 5, then find the values of a and b.

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22. Find the length of the tangent for the curve $y = x^3 + 3x^2 + 4x - 1$ at point $x = 0$.

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23. For the curve $xy = c,$ prove that the portion of the tangent intercepted between the coordinate axes is bisected at the point of contact.

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24. The line y = x + 1 is a tangent to the curve y^2 = 4x at the poin

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25. If $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ then prove that at $\theta = \frac{\pi}{2}, y'' = \frac{1}{a}$.

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26. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point (1, 2). .

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27. If $f(x)$ is continuous in $[a, b]$ and differentiable in (a,b), then prove that there exists at least one $c \in (a, b)$ such that $\frac{f'(c)}{3c^2} = \frac{f(b) - f(a)}{b^3 - a^3}$

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28. Let $f(x) and g(x)$ be two functions which are defined and differentiable for all $x \geq x_0$. If $f(x_0) = g(x_0) and f'(x) > g'(x)$ for all $x > x_0$, then prove that $f(x) > g(x)$ for all $x > x_0$.

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29. A particle moves along the curve 6y = $x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

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30. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle.

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31. Find the minimum value of $(x_1 - x_2)^2 + \left(\frac{x_1^2}{20} - \sqrt{(17 - x_2)(x_2 - 13)}\right)^2$ where $x_1 \in R^+, x_2 \in (13, 17)$.

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32. Displacement s of a particle at time t is expressed as $s = \frac{1}{2}t^3 - 6t$. Find the acceleration at the time when the velocity vanishes (i.e., velocity tends to zero).

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33. Find the distance of the point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line $y = 2x - 1$

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34. In how many points graph of $y = x^3 - 3x2 + 5x - 3$ interest the x-axis?

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35. The tangent at any point P on the circle $x^2 + y^2 = 4$ meets the coordinate axes at $AandB$. Then find the locus of the midpoint of AB.

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36. Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4)

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37. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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38. A lamp is $50ft\cdot$ above the ground. A ball is dropped from the same height from a point $30ft\cdot$ away from the light pole. If ball falls a distance $s = 16t^2 ft\cdot$ in t second, then how fast is the shadow of the ball moving along the ground $\frac{1}{2}s$ later?

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39. Find the values of p so that the equation $2\cos^2 x - (p+3)\cos x + 2(p-1) = 0$ has a real solution.

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40. Find the angle between the curves $2y^2 = x^3 and y^2 = 32x\cdot$

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41. Find the locus of point on the curve $y^2 = 4a\left(x + as \in \frac{x}{a}\right)$ where tangents are parallel to the axis of $x\cdot$

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42. Find the values of a if equation $1 - \cos x = \frac{\sqrt{3}}{2}|x| + a, x \in (0, \pi),$ has exactly one solution.

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43. Find the angle at which the curve $y = Ke^{Kx}$ intersects the y-axis.

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44. Find the angle of intersection of the curves $xy = a^2 and x^2 + y^2 = 2a^2$

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45. Find the angle between the curvesC1: $x^2 - \frac{y^2}{3} = a^2 and C_2 : xy^3 = c$

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46. If the two circles $2x^2 + 2y^2 - 3x + 6y + k = 0$ and $x^2 + y^2 - 4x + 10y + 16 = 0$ cut orthogonally, then find the value of k .

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47. Find the point on the curve $3x^2 - 4y^2 = 72$ which is nearest to the line $3x + 2y + 1 = 0.$

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48. Find the area between the line $y = x + 1$ and the curve $y = x^2 - 1$

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49. Using differential find the approximate value of cos 61, if it is given that $\sin 60^{\circ} 0.86603$ and $1^{\circ} = 0.01745$ radians.

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50. If an a triangle $ABC, b = 3cand\ C - B = 90^{0},$ then find the value of $\tan B$

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51. Find the approximate value of $(26)^{\frac{1}{3}}$.

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52. Using the approximation to find approximate value of

$(123)^{\frac{2}{3}}$

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53. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

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54. Find approximate value of f (5.001) where $f(x) = x^3 - 7x^2 + 15$.

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55. In an acute triangle ABC if sides a, b are constants and the base angles $AandB$ vary, then show that $\frac{dA}{\sqrt{a^2 - b^2 \sin^2 A}} = \frac{dB}{\sqrt{b^2 - a^2 \sin^2 B}}$

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56. Find the approximate value of f(3.02) where $f(x) = 3x^2 + 5x + 3.$

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57. If the radius of the sphere is measured as 9 cm with an error of 0.03 cm, the approximate error in calculating its volume is

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58. Find the approximate value of $(1.999)^6$.

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59. Let f be differentiable for all x, If $f(1) = -2 and f'(x) \geq 2$ for all $x \in [1, 6]$, then find the range of values of $f(6)$.

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60. Let $f:[2,7]\overrightarrow{0,\infty}$ be a continuous and differentiable function. Then show that $(f(7)-f(2))\dfrac{(f(7))^2+(f(2))^2+f(2)f(7)}{3}=5f^2(c)f'(c)$, where $c\in[2,7]\cdot$

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61. Let $f(x)andg(x)$ be differentiable functions such that $f'(x)g(x)\neq f(x)g'(x)$ for any real $x\cdot$ Show that between any two real solution of $f(x)=0,$ there is at least one real solution of $g(x)=0.$

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62. Consider the function $f(x)=8x^2-7x+5$ on the interval $[-6,6]\cdot$ Find the value of c that satisfies the conclusion of Lagranges mean value theorem.

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63. Using mean value theorem, show that $\frac{\beta-\alpha}{1+\beta^2} < \tan^{-1}\beta - \tan^{-1}\alpha < \frac{\beta-\alpha}{1+\alpha^2}, \beta > \alpha > 0.$

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64. Let $f(x) and g(x)$ be two differentiable functions in $R and f(2) = 8, g(2) = 0, f(4) = 10, and g(4) = 8.$ Then prove that $g'(x) = 4f'(x)$ for at least one $x \in (2, 4)\cdot$

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65. Using Lagranges mean value theorem, prove that $|\cos a - \cos b| \leq |a-b|\cdot$

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66. Let $f(x) and g(x)$ be differentiable function in (a, b), continuous at $a and b, and g(x) \neq 0$ in $[a, b]\cdot$ Then prove that

$$\frac{g(a)f(b)-f(a)g(b)}{g(c)f'(c)-f(c)g'(c)}=\frac{(b-a)g(a)g(b)}{(g(c))^2}$$

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67. Suppose $\alpha, \beta and th\eta$ are angles satisfying 0

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68. Let f be continuous on $[a, b], a > 0, and$ differentiable on (a, b). Prove that there exists $c \in (a, b)$ such that $\frac{bf(a)-af(b)}{b-a}=f(c)-cf'(c)$

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69. Two men $PandQ$ start with velocity u at the same time from the junction of two roads inclined at 45^0 to each other. If they travel by different roads, find the rate at which they are being separated.

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70. $x and y$ are the sides of two squares such that $y = x - x^2$. Find the rate of the change of the area of the second square with respect to the first square.

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71. A spherical iron ball 10cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50 cm^3 / m \in$. When the thickness of ice is 5cm, then find the rate at which the thickness of ice decreases.

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72. Two cyclists start from the junction of two perpendicular roads, there velocities being $3 um / m \in$ and $4 um / m \in$, respectively. Find the rate at which the two cyclists separate.

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73. Tangent of an angle increases four times as the angle itself. At what rate the sine of the angle increases w.r.t. the angle?

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74. The distance covered by a particle moving in a straight line from a fixed point on the line is s, where $s^2 = at^2 + 2bt + c$. Then prove that acceleration is proportional to s^{-3}.

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75. A horse runs along a circle with a speed of $20 km/h$. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. Find the speed with which the shadow of the horse moves along the fence at the moment when it covers 1/8 of the circle in km/h.

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76. Let x be the length of one of the equal sides of an isosceles triangle, and let θ be the angle between them. If x is increasing at the rate (1/12) m/h, and θ is increasing at the rate of $\frac{\pi}{180}$ radius/h, then find the rate in m^3 / h at which the area of the triangle is increasing when $x = 12m$ and $\theta = \pi/4$.

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77. If water is poured into an inverted hollow cone whose semi-vertical angel is 30^0, and its depth (measured along the axis) increases at the rate of 1 cm/s. Find the rate at which the volume of water increases when the depth is 24 cm.

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78. If $f:[-5,5] \rightarrow R$ is a differentiable function function and if f'(x) does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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79. Discuss the applicability of Rolles theorem for the following functions on the indicated intervals:

$f(x)=|x| \in [-1,1]$

$f(x)=3+(x-2)^{2/3}$ in [1,3]

$f(x)=\tan x \in [0,\pi]$

$f(x)=\log\left\{\dfrac{x^2+ab}{x(a+b)}\right\}$ in [a,b], where 0 less than a less than b

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80. How many roots of the equation $(x-1)(x-2)(x-3)+(x-1)(x-2)(x-4)+(x-2)(x-3)(x-4)$ are positive?

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81. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on [1,3] satisfies Rolles theorem for $c = \frac{2\sqrt{3}+1}{\sqrt{3}}$ then find the value of $aandb$

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82. If $\varphi(x)$ is differentiable function $\forall x \in R$ and $a \in R^+$ such that $\varphi(0) = \varphi(2a), \varphi(a) = \varphi(3a) and \varphi(0) \neq \varphi(a)$ then show that there is at least one root of equation $\varphi'(x+a) = \varphi'(x) \in (0, 2a)$

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83. Let $f(x)$ be differentiable function and $g(x)$ be twice differentiable function. Zeros of $f(x), g'(x)$ be a, b, respectively, `

(a

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84. Show that between any two roots of $e^{-x} - \cos x = 0$, there exists at least one root of $\sin x - e^{-x} = 0$

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85. If $2a + 3b + 6c = 0$, then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

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86. The equation $ax^2 - bx + c = 0$ has real and positive roots. Prove that the roots of the equation $ad^2x^2 + a(3b - 2c)x + (2b - c)(b - c) + ac = 0$ re real and positive.

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87. Let $P(x)$ be a polynomial with real coefficients, Let `a , b in R ,a

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88. If the curve $y = ax^2 - 6x + b$ pass through $(0, 2)$ and has its tangent parallel to the x-axis at $x = \frac{3}{2}$, then find the values of $a and b$.

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89. Find the equation of the tangent to the curve $(1 + x^2)y = 2 - x$, where it crosses the x-axis.

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90. A curve is given by the equations $x = \sec^2\theta, y = \cot\theta$. If the tangent at $P where \theta = \frac{\pi}{4}$ meets the curve again at $Q, then [PQ]$ is, where [.] represents the greatest integer function, ______.

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91. Find the point on the curve where tangents to the curve $y^2 - 2x^3 - 4y + 8 = 0$ pass through (1,2).

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92. At the point $P(a, a^n)$ on the graph of $y = x^n, (n \in N)$, in the first quadrant, a normal is drawn. The normal intersects the y-axis at the point $(0, b)$. If $(\lim)_{a \to 0} b = \frac{1}{2}$, then n equals ____.

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93. Find the equation of the normal to the curve $x^3 + y^3 = 8xy$ at the point where it meets the curve $y^2 = 4x$ other than the origin.

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94. If the slope of line through the origin which is tangent to the curve $y = x^3 + x + 16$ is m, then the value of $m - 4$ is___.

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95. For the curve $xy = c,$ prove that the portion of the tangent intercepted between the coordinate axes is bisected at the point of contact.

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96. Water is dropped at the rate of 2 m^3/s into a cone of semi-vertical angle is 45° . If the rate at which periphery of water surface changes when the height of the water in the cone is 2m is d. Then the value of 5d is ____ m/sec

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97. Find the equation of the normal to the curve x^2 = 4y which passes through the point (1, 2).

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98. Suppose a, b, c are such that the curve $y = ax^2 + bx + c$ is tangent to $y = 3x - 3at(1, 0)$ and is also tangent to $y = x + 1at(3, 4)$. Then the value of $(2a - b - 4c)$ equals ____

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99. Find the normal to the curve $x = a(1 + \cos\theta), y = a\sin\theta a\mathrm{h}\eta$. Prove that it always passes through a fixed point and find that fixed point.

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100. If the curve C in the xy plane has the equation $x^2 + xy + y^2 = 1$, then the fourth power of the greatest distance of a point on C from the origin is___.

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101. Show that the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $xy = a^2$, if $p^2 = 4a^2\cos\alpha\sin\alpha$.

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102. Let C be a curve defined by $y = e^a + bx^2$. The curve C passes through the point $P(1, 1)$ and the slope of the tangent at P is (-2). Then the value of $2a - 3b$ is____.

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103. If the line $x\cos\theta + y\sin\theta = P$ is the normal to the curve $(x+a)y = 1$, then show $\theta \in \left(2n\pi + \frac{\pi}{2}, (2n+1)\pi\right) \cup \left(2n\pi + \frac{3\pi}{2}, (2n+2)\pi\right), n \in Z$

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104. Let f defined on $[0, 1]$ be twice differentiable such that $|f(x)| \leq 1$ for $x \in [0, 1]$. if $f(0) = f(1)$ then show that $|f'(x) < 1$ for all $x \in [0, 1]$.

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105. If the tangent at any point $(4m^2, 8m^3)$ of $x^3 - y^2 = 0$ is a normal to the curve $x^3 - y^2 = 0$, then find the value of m.

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106. If a, b are two real numbers with $a < b$, then a real number c can be found between a and b such that the value of $\frac{a^2 + ab + b^2}{c^2} is\_\_\_$

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107. For the curve y =$4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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108. Find the total number of parallel tangents of $f_1(x) = x^2 - x + 1 and f_2(x) = x^3 - x^2 - 2x + 1.$

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109. There is a point (p,q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = \frac{-8}{x}, where p > 0 and r > 0.$ If the line through $(p, q) and (r, s)$ is also tangent to both the curves at these points, respectively, then the value of $P + r$ is_______.

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110. A curve is defined parametrically be equations $x = t^2 and y = t^3$. A variable pair of perpendicular lines through the origin O meet the curve of $P and Q$. If the locus of the point of intersection of the tangents at $P and Q$ is $ay^2 = bx - 1,$ then the value of $(a + b)$ is___

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111. Find the equation of the tangent to the curve$y = \left\{x^2 \frac{\sin 1}{x}, x \neq 00, x = 0\right.$at the origin

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112. Statement 1: If $f(x)$ is differentiable in $[0, 1]$ such that $f(0) = f(1) = 0,$ then for any $\lambda \in R,$ there exists c such that f'(c) $= \lambda$f(c),$0 < c < 1.$ statement 2: if $g(x)$ is differentiable in [0,1], where $g(0) = g(1),$ then there exists c such that g'(c)=0,

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113. Find the equation of tangent to the curve $y = \frac{\sin^{-1}(2x)}{1 + x^2} at x = \sqrt{3}$

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114. Statement 1: For the function $f(x) = x^2 + 3x + 2, LMVT$ is applicable in [1, 2] and the value of c is $3/2$. Statement 2: If LMVT is known to be applicable for any quadratic polynomial in $[a, b]$, then c of $LMVT$ is $\frac{a+b}{2}$.

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115. Find the equation of the normals to the curve y = x^3 + 2x + 6 which are parallel to the line x + 14y + 4 = 0.

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116. Let $y = f(x)$ be a polynomial of odd degree (≥ 3) with real coefficients and (a, b) be any point. Statement 1: There always exists a line passing through (a, b) and touching the curve $y = f(x)$ at some point. Statement 2: A polynomial of odd degree with real coefficients has at least one real root.

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117. Find the equation of tangent and normal to the curve $x = \frac{2at^2}{(1+t^2)}, y = \frac{2at^3}{(1+t^2)}$ at the point for which $t = \frac{1}{2}$.

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118. If d is the minimum distance between the curves $f(x) = e^x and g(x) = (\log)_e x,$ then the value of d^6 is

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119. Let $f(x0$ be a non-constant thrice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(6-x) and f'(0) = 0 = f'(x)^2 = f(5)$. If n is the minimum number of roots of $\left(f'(x)^2 + f'(x) f^x = 0\right.$ in the interval [0,6], then the value of $\frac{n}{2}$ is__

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120. Points on the curve $f(x) = \frac{x}{1 - x^2}$ where the tangent is inclined at an angle of $\frac{\pi}{4}$ to the x-axis are (a)(0,0) (b) $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$ $\left(-2, \frac{2}{3}\right)$ (d) $\left(-\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

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121. In the curve $y = ce^{\frac{x}{a}}$, the sub-tangent is constant sub-normal varies as the square of the ordinate tangent at (x_1, y_1) on the curve intersects the x-axis at a distance of $(x_1 - a)$ from the origin equation of the normal at the point where the curve cuts $y - a\xi s$ is $cy + ax = c^2$

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122. Let $f'(x) = e^x \hat{} 2 \text{ and } f(0) = 10.$

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123. If f is a continuous function on $[0, 1]$, differentiable in (0, 1) such that $f(1) = 0$, then there exists some $c \in (0, 1)$ such that

$cf'(c) - f(c) = 0$

$cf'(c) + cf(c) = 0$

$f'(c) - cf(c) = 0$

$cf'(c) + f(c) = 0$

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124. Given $g(x) = \dfrac{x+2}{x-1}$ and the line $3x + y - 10 = 0$. Then the line is (a)tangent to $g(x)$ (b) normal to $g(x)$ (c)chord of $g(x)$ (d) none of these

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125. Let f be a continuous, differentiable, and bijective function. If the tangent to $y = f(x) at x = a$ is also the normal to $y = f(x) at x = b$, then there exists at least one $c \in (a, b)$ such that

(a) $f'(c) = 0$

(b) $f'(c) > 0$

(c) $f'(c) < 0$

(d) none of these

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126. If $f(x) and g(x)$ are differentiable functions for $0 \leq x \leq 1$ such that $f(0) = 10, g(0) = 2, f(1) = 2, g(1) = 4,$ then in the interval $(0, 1)$. (a) $f'(x) = 0 f \text{ or } all x$ (b) $f'(x) + 4g'(x) = 0$ for at least one x (c) $f(x) = 2g'(x)$ for at most one x (d) none of these

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127. A continuous and differentiable function $y = f(x)$ is such that its graph cuts line $y = mx + c$ at n distinct points. Then the minimum number of points at which $f''(x) = 0$ is/are (a) $n - 1$ (b) $n - 3$ $n - 2$ (d) cannot say

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128. If $f(x)$ is continuous in $[a, b]$ and differentiable in (a,b), then prove that there exists at least one $c \in (a, b)$ such that $\frac{f'(c)}{3c^2} = \frac{f(b) - f(a)}{b^3 - a^3}$

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129. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface when the radius is 7 cm and altitude is 24cm is (a) $108\pi cm^2 / \min$ (b) $54\pi cm^2 / \min$ (c) $27\pi cm^2 / \min$ (d) none of these

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130. Let $f(x) and g(x)$ be differentiable for $0 \leq x \leq 2$ such that $f(0) = 2, g(0) = 1, and f(2) = 8.$ Let there exist a real number c in $[0, 2]$ such that $f'(c) = 3g'(c)\cdot$ Then find the value of $g(2)\cdot$

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131. If $3(a+2c)=4(b+3d)$, then the equation $ax^3+bx^2+cx+d=0$ will have (a)no real solution (b)at least one real root in $(-1,0)$ (c)at least one real root in $(0,1)$ (d)none of these

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132. If $f(x)=x^3+7x-1$, then $f(x)$ has a zero between $x=0 and x=1$. The theorem that best describes this is a. mean value theorem b. maximum-minimum value theorem c. intermediate value theorem none of these

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133. Consider the function $f(x)=\left\{x\sin\left(\frac{\pi}{x}\right), f \text{ or } x>0, 0 f \text{ or } x=0\right.$ The, the number of point in (0,1) where the derivative $f'(x)$ vanishes is 0 (b) 1 (c) 2 (d) infinite

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134. Let $f(x)$ be a twice differentiable function for all real values of x and satisfies $f(1) = 1$, $f(2) = 4$, $f(3) = 9$. Then which of the following is definitely true? (a).$f''(x) = 2 \forall x$ in (1,3) (b) $f''(x) = 5$ for some x in (2,3) (c) $f''(x) = 3 \forall x$ in (2,3) (d) $f''(x) = 2$ for some x in (1,3)

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135. The value of c in Lagranges theorem for the function $f(x) = \log \sin x$ in the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c)$\frac{2\pi}{3}$ (d) none of these

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136. If the function $f(x) = ax^3 + bx^2 + 11x - 6$ satisfies conditions of Rolles theorem in $[1, 3]$ and $f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$, then values of a and b, respectively, are

(A) $-3, 2$

(B) $2, \ -4$

(C) $1, \ -6$

(D) none of these

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137. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = (\log)_e x$ on the interval [1, 3] is (1) $2(\log)_3 e$ (2) $\frac{1}{2}(\log)_e 3$ (3) $(\log)_3 e$ (4) $(\log)_e 3$

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138. Each question has four choices, a,b,c and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. If both the statement are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1. If both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. If STATEMENT 1 is TRUE and STATEMENT 2 is FLASE. If STATEMENT 1 is FALSE and STATEMENT 2 is TURE. Statement 1: Lagrange mean value theorem is not applicable to

$f(x) = |x-1|(x-1)$ **Statement 2:** $|x-1|$ **is not differentiable at** $x = 1.$

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139. The abscissa of the point on the curve $\sqrt{xy} = a + x$ the tangent at which cuts off equal intercepts from the coordinate axes is $-\dfrac{a}{\sqrt{2}}$ (b) $a/\sqrt{2}$ (c) $-a\sqrt{2}$ (d) $a\sqrt{2}$

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140. In which of the following functions is Rolles theorem applicable?

$(a) f(x) = \{x, 0 \leq x < 10, x = 1 on[0, 1]$

$(b) f(x) = \left\{\frac{\sin x}{x}, -\pi \leq x < 00, x = 0 on[-\pi, 0)\right.$

$(c) f(x) = \frac{x^2 - x - 6}{x - 1} on[-2, 3]$

$(d) f(x) = \left\{\frac{x^3 - 2x^2 - 5x + 6}{x - 1} \text{ if } x \neq 1, -6 \text{ if } x = 1 on[-2, 3]\right.$

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141. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is (a)(2,6) (b) $(2, -6)$ (c) $\left(\frac{9}{8}, -\frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

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142. Statement 1: If $g(x)$ is a differentiable function, $g(2) \neq 0, g(-2) \neq 0,$ and Rolles theorem is not applicable to $f(x) = \frac{x^2 - 4}{g(x)} \in [-2, 2], theng(x)$ has at least one root in $(-2, 2)\cdot$

Statement 2: If $f(a) = f(b), theng(x)$ has at least one root in $(-2, 2)\cdot$

Statement 2: If $f(a) = f(b),$ then Rolles theorem is applicable for $x \in (a, b)\cdot$

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143. Statement 1: The maximum value of $\left(\sqrt{-3 + 4x - x^2} + 4\right)^2 + (x - 5)^2 (where 1 \leq x \leq 3) is 36.$ Statement

2: The maximum distance between the point $(5, -4)$ and the point on the circle $(x-2)^2 + y^2 = 1$ is 6

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144. Statement 1: If both functions $f(t) and g(t)$ are continuous on the closed interval [1,b], differentiable on the open interval (a,b) and $g'(t)$ is not zero on that open interval, then there exists some c in (a, b) such that $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$ Statement 2: If $f(t) and g(t)$ are continuou and differentiable in [a, b], then there exists some c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b - a} and g'(c) \frac{g(b) - g(a)}{b - a}$ from Lagranes mean value theorem.

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145. Statement 1: If $27a + 9b + 3c + d = 0,$ then the equation $f(x) = 4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one real root lying between $(0, 3)\cdot$ Statement 2: If $f(x)$ is continuous in [a,b], derivable in

(a, b) such that $f(a) = f(b)$, then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$.

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146. Find the angle of intersection of curves $y = [|\sin x| + |\cos x|] and x^2 + y^2 = 5$, where [.] denotes the greatest integral function.

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147. If the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

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148. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then find the value of n.

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149. If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2), then prove that $\frac{x_2}{x_1} + \frac{y_2}{y_1} = -1$

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150. Show that the segment of the tangent to the curve $y = \frac{a}{2} In\left(\frac{a + \sqrt{a^2 - x^2}}{a - \sqrt{a^2 - x^2}}\right) - \sqrt{a^2 - x^2}$ contained between the y=axis and the point of tangency has a constant length.

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151. Prove that the equation of the normal to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $y \cos\theta - x \sin\theta = a \cos 2\theta$, where θ is the angle which the normal makes with the axis of $x\cdot$

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152. Prove that the curves $y=f(x),[f(x)>0], and y=f(x)\sin x, where f(x)$ is differentiable function, have common tangents at common points.

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153. Tangents are drawn from the origin to curve $y=\sin x\cdot$ Prove that points of contact lie on $y^2=\dfrac{x^2}{1+x^2}$

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154. Given $f(x)=4-\left(\dfrac{1}{2}-x\right)^{\frac{2}{3}}, g(x)=\left\{\dfrac{\tan[x]}{x}, x\neq 01, x=0\right.$ $h(x)=\{x\}, k(x)=5^{(\log)_2(x+3)}$ Then in [0,1], lagranges mean value theorem is not applicable to (where [.] and {.} represents the greatest integer functions and fractional part functions, respectively). f (b) g (c) k (d) h

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155. Show that the angle between the tangent at any point P and the line joining P to the origin O is same at all points on the curve $\log(x^2+y^2)=k\tan^{-1}\left(\frac{y}{x}\right)$

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156. The angle between the tangents to the curves $y=x^2 and x=y^2 at(1,1)$ is $\cos^{-1}\left(\frac{4}{5}\right)$ (b) $\sin^{-1}\left(\frac{3}{5}\right)$ $\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{1}{3}\right)$

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157. If the tangent at any point $(4m^2, 8m^3)$ of $x^3-y^2=0$ is a normal to the curve $x^3-y^2=0$, then find the value of $m\cdot$

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158. The angle formed by the positive y - axis and the tangent to $y = x^2 + 4x - 17at\left(\frac{5}{2}, -\frac{3}{4}\right)$ is: (a) $\tan^{-1}(9)$ (b) $\frac{\pi}{2} - \tan^{-1}(9)$ $\frac{\pi}{2} + \tan^{-1}(9)$ (d) none of these

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159. The abscissa of a point on the curve $xy = (a + x)^2$, the normal which cuts off numerically equal intercepts from the coordinate axes, is $-\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}a$ (c) $\frac{a}{\sqrt{2}}$ (d) $-\sqrt{2}a$

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160. The corrdinate of the points(s) on the graph of the function, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 7x - 4$ where the tangent drawn cuts offintercepts from the coordinate axes which are equal in magnitude but opposite is sign, is

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161. Which of the following pair(s) of curves is/are orthogonal?

$y^2 = 4ax; y = e^{-\frac{x}{2a}}$ $y^2 = 4ax; x^2 = 4ayat(0, 0)$ $xy = a^2; x^2 - y^2 = b^2$

$y = ax; x^2 + y^2 = c^2$

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162. Let the parabolas $y = x(c - x)andy = x^2 + ax + b$ touch each other at the point (1,0). Then (a)$a + b + c = 0$ (b) $a + b = 2$ (c)$b - c = 1$ (d) $a + c = -2$

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163. Let $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x,$ where $a_i's$ are real and $f(x) = 0$ has a positive root α_0. Then (a)$f'(x) = 0$ has a positive root α_1 such that $0\alpha_1\alpha_0$

A. (b)$f'(x) = 0$ has at least two real roots (c)$f^x = 0$ has at least one real root (d)$noneofthese$

B. null

C. null

D. null

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164. If there is an error of $k\ \%$ in measuring the edge of a cube, then the percent error in estimating its volume is (a)k (b) $3k$ (c)$\frac{k}{3}$ (d) none of these

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165. The rate of change of the volume of a sphere w.r.t. its surface area, when the radius is 2 cm, is 1 (b) 2 (c) 3 (d) 4

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166. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30m from the foot of the tower. Assume that the eye level of the man is 1.6m from the ground.

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167. A lamp of negligible height is placed on the ground l_1 away from a wall. A man $l_2 m$ tall is walking at a speed of $\frac{l_1}{10} m / s$ from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is $-\frac{5l_2}{2} m / s$ (b) $-\frac{2l_2}{5} m / s$ $-\frac{l_2}{2} m / s$ (d) $-\frac{l_2}{5} m / s$

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168. At the point $P(a, a^n)$ on the graph of $y = x^n, (n \in N)$, in the first quadrant, a normal is drawn. The normal intersects the y-axis at the point $(0, b)$. If $(\lim)_{a \vec{0}} b = \frac{1}{2},$ then n equals ____.

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169. The coordinates of a point on the parabola $y^2 = 8x$ whose distance from the circle $x^2 + (y+6)^2 = 1$ is minimum is (a)$(2, 4)$ (b) $(2, -4)$ (c) $(18, -12)$ (d) $(8, 8)$

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170. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in $cm^2 / m \in$, when the radius is $2cm$ and the height is 3cm is $-2p$ (b) $-\frac{8\pi}{5} - \frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

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171. Suppose that f is differentiable for all x and that $f'(x) \leq 2f \text{ or } allx \cdot$ If $f(1) = 2andf(4) = 8, thenf(2)$ has the value equal to 3 (b) 4 (c) 6 (d) 8

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172. The tangent to the curve $y = e^{kx}$ at a point (0,1) meets the x-axis at (a,0), where $a \in [-2, -1]$. Then $k \in \left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right]$ $[0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

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173. A cube of ice melts without changing its shape at the uniform rate of $4\frac{cm^3}{m \in} \cdot$ The rate of change of the surface area of the cube, in $\frac{cm^2}{m \in}$, when the volume of the cube is $125cm^3$, is -4 (b) $-\frac{16}{5}$ (c) $-\frac{16}{6}$ (d) $-\frac{8}{15}$

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174. Using Rolles theorem, prove that there is at least one root in $\left(45^{\frac{1}{100}}, 46\right)$ of the equation. $P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035 = 0$.

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175. if $|f(x_1) - f(x_2)| \leq (x_1 - x_2)^2$ Find the equation of gent to the curve $y = f(x)$ at the point $(1, 2)$.

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176. If f(x) is a twice differentiable function such that f(a)=0, f(b)=2, f(c)=-1,f(d)=2, f(e)=0 where a < b < c < d e, then the minimum number of zeroes of $g(x) = f'(x)^2 + f''(x)f(x)$ in the interval [a, e] is

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177. A function $y - f(x)$ has a second-order derivative $f^x = 6(x-1)$. It its graph passes through the point (2,1) and at that point tangent to the graph is $y = 3x - 5$, then the value of $f(0)$ is 1 (b) -1 (c) 2 (d) 0

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178. If $x + 4y = 14$ is a normal to the curve $y^2 = \alpha x^3 - \beta$ at (2,3), then the value of $\alpha + \beta$ is 9 (b) -5 (c) 7 (d) -7

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179. In the curve represented parametrically by the equations $x = 2\log\cot t + 1$ and $y = \tan t + \cot t$, A. tangent and normal intersect at the point (2,1) B. normal at $t = \frac{\pi}{4}$ is parallel to the y-axis. C. tangent at $t = \frac{\pi}{4}$ is parallel to the line $y = x$ D. tangent at $t = \frac{\pi}{4}$ is parallel to the x-axis.

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180. The abscissas of point $PandQ$ on the curve $y = e^x + e^{-x}$ such that tangents at $PandQ$ make 60^0 with the x-axis are. $1n\left(\frac{\sqrt{3}+\sqrt{7}}{7}\right) and 1n\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$ $1n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$ (c) $1n\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$ $\pm 1n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

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181. The normal to the curve $2x^2 + y^2 = 12$ at the point $(2, 2)$ cuts the curve again at (A) $\left(-\frac{22}{9}, -\frac{2}{9}\right)$ (B) $\left(\frac{22}{9}, \frac{2}{9}\right)$ (C) $(-2, -2)$ (D) none of these

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182. At what point of curve $y = \frac{2}{3}x^3 + \frac{1}{2}x^2$, the tangent makes equal angle with the axis? (a) $\left(\frac{1}{2}, \frac{2}{24}\right) and \left(-1, -\frac{1}{6}\right)$ (b) $\left(\frac{1}{2}, \frac{4}{9}\right) and (-1, 0)$ (c) $\left(\frac{1}{3}, \frac{1}{7}\right) and \left(-3, \frac{1}{2}\right)$ (d) $\left(\frac{1}{3}, \frac{4}{47}\right) and \left(-1, -\frac{1}{3}\right)$

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183. The equation of the tangent to the curve $y = be^{-x/a}$ at the point where it crosses the y-axis is $a)\frac{x}{a} - \frac{y}{b} = 1$ **(b)** $ax + by = 1$ $c)ax - by = 1$ **(d)** $\frac{x}{a} + \frac{y}{b} = 1$

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184. Then angle of intersection of the normal at the point $\left(-\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ of the curves $x^2 - y^2 = 8$ and $9x^2 + 25y^2 = 225$ is 0 **(b)** $\frac{\pi}{2}$ **(c)** $\frac{\pi}{3}$ **(d)** $\frac{\pi}{4}$

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185. If a variable tangent to the curve $x^2y = c^3$ makes intercepts $a, bonx - andy - axes,$ respectively, then the value of a^2b is $27c^3$ **(b)** $\frac{4}{27}c^3$ **(c)** $\frac{27}{4}c^3$ **(d)** $\frac{4}{9}c^3$

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186. Let C be the curve $y = x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A, then K is equal to 4 (b) 2 (c) -2 (d) $\frac{1}{4}$

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187. If H is the number of horizontal tangents and V is the number of vertical tangents to the curve $y^3 - 3xy + 2 = 0$, then the value of $(H + V)$ equals

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188. Let $f(1) = -2 and f'(x) \geq 4.2 f \text{ or } 1 \leq x \leq 6.$ The smallest possible value of $f(6)$ is 9 (b) 12 (c) 15 (d) 19

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189. The curves $4x^2 + 9y^2 = 72$ and $x^2 - y^2 = 5at(3, 2)$ touch each other (b) cut orthogonally intersect at 45^0 (d) intersect at 60^0

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190. If the length of sub-normal is equal to the length of sub-tangent at any point (3,4) on the curve $y = f(x)$ and the tangent at (3,4) to $y = f(x)$ meets the coordinate axes at $AandB$, then the maximum area of the triangle OAB, where O is origin, is 45/2 (b) 49/2 (c) 25/2 (d) 81/2

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191. At any point on the curve $2x^2y^2 - x^4 = c$, the mean proportional between the abscissa and the difference between the abscissa and the sub-normal drawn to the curve at the same point is equal to $\text{or } d \in ate$ (b) radius vector $x - \in tercep \to ftan \geq nt$ (d) sub-tangent

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192. The x-intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^2} + \frac{b}{y^2} = 1$ is proportional to (a)square of the abscissa of the point of tangency (b)square root of the abscissa of the point of tangency (c)cube of the abscissa of the point of tangency (d)cube root of the abscissa of the point of tangency

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193. A curve is represented by the equations $x = \sec^2 tandy = \cot t$, where t is a parameter. If the tangent at the point P on the curve where $t = \frac{\pi}{4}$ meets the curve again at the point Q, then $|PQ|$ is equal to $\frac{5\sqrt{3}}{2}$ (b) $\frac{5\sqrt{5}}{2}$ (c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{3\sqrt{5}}{2}$

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194. The two curves $x = y^2, xy = a^3$ cut orthogonally at a point. Then a^2 is equal to $\frac{1}{3}$ (b) 3 (c) 2 (d) $\frac{1}{2}$

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195. The line tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^2 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

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196. Tangent of acute angle between the curves $y = |x^2 - 1|$ and $y = \sqrt{7 - x^2}$ at their points of intersection is $\frac{5\sqrt{3}}{2}$ (b) $\frac{3\sqrt{5}}{2}$ $\frac{5\sqrt{3}}{4}$ (d) $\frac{3\sqrt{5}}{4}$

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197. The number of point in the rectangle $\{(x, y)\} - 12 \leq x \leq 12 and - 3 \leq y \leq 3\}$ which lie on the curve $y = x + \sin x$ and at which in the tangent to the curve is parallel to the x-axis is 0 (b) 2 (c) 4 (d) 8

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198. Statement 1: The tangent at $x = 1$ to the curve $y = x^3 - x^2 - x + 2$ again meets the curve at $x = 0$. Statement 2: When the equation of a tangent is solved with the given curve, repeated roots are obtained at point of tangency.

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199. An aeroplane is flying horizontally at a height of $\frac{2}{3} km$ with a velocity of 15 km/h. Find the rate at which it is receding from a fixed point on the ground which it passed over 2 min ago.

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200. Use the mean value theorem to prove $e^x \geq 1 + x \forall x \in R$

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201. Find the condition for the line $y = mx$ to cut at right angles the conic $ax^2 + 2hxy + by^2 = 1$.

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202. Show that for the curve $by^2 = (x + a)^3$, the square of the sub-tangent varies as the sub-normal.

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203. Let a, b, c be three real numbers such that `a

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204. Prove that the portion of the tangent to the curve $\frac{x + \sqrt{a^2 - y^2}}{a} = (\log)_e \frac{a + \sqrt{a^2 - y^2}}{y}$ intercepted between the point of contact and the x-axis is constant.

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205. Let a, b, c be non-zero real numbers such that

$$\int_0^1 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$
$$=\int_0^2 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$

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206. If f is continuous and differentiable function and $f(0)=1, f(1)=2,$ then prove that there exists at least one $c\in[0,1] f \text{ or } which f'(c)(f(c))^{n-1}>\sqrt{2^{n-1}}$, where $n\in N$.

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207. Sand is pouring from a pipe at the rate of 12 cm^3 /s. The falling sand forms a cone on the ground in such a way that the height of the cone is

always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

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208. Let $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + + \frac{a_{n-1}}{2} + a_n = 0.$ Show that there exists at least one real x between 0 and 1 such that $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + + a_n = 0$

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209. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1,$ then $a > 0, b > 0$ $a > 0, b < 0$ $a\langle 0, b\rangle 0$ **(d)** $a < 0, b < 0$ none of these

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210. Which one of the following curves cut the parabola at right angles?

(a)$x^2 + y^2 = a^2$ **(b)** $y = e^{-x/2a}$ **(c)**$y = ax$ **(d)** $x^2 = 4ay$

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211. Let $f, g: [-1, 2] \rightarrow \mathbb{R}$ be continuous functions which are twice differentiable on the interval (-1, 2). Let the values of f and g at the points –1, 0 and 2 be as given in the following table : $x = -1x = 0x = 2f(x)360g(x)01-1$ In each of the intervals (-1,0) and (0, 2) the function (f – 3g)" never vanishes. Then the correct statement(s) is(are)

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212. Which of the following is/are correct?

(A) Between any two roots of $e^x \cos x = 1,$ there exists at least one root of $\tan x = 1.$

(B) Between any two roots of $e^x \sin x = 1,$ there exists at least one root of $\tan x = -1.$

(C) Between any two roots of $e^x \cos x = 1,$ there exists at least one root of $e^x \sin x = 1.$

(D) Between any two roots of $e^x \sin x = 1$, there exists at least one root of $e^x \cos x = 1$.

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213. Which of the following pair(s) of curves is/are orthogonal?

$y^2 = 4ax; y = e^{-\frac{x}{2a}}$ $y^2 = 4ax; x^2 = 4ayat(0, 0)$ $xy = a^2; x^2 - y^2 = b^2$

$y = ax; x^2 + y^2 = c^2$

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214. Find the equation of tangents to the curve

$y = \cos(x + y), -2\pi \leq x \leq 2\pi$

that are parallel to the line x + 2y = 0.

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215. Find the equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x) at$**x=0.`**

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216. Let $f and g$ be differentiable on [0,1] such that $f(0) = 2, g(0), f(1) = 6 and g(1) = 2.$ Show that there exists $c \in (0, 1)$ such that $f'(c) = 2g'(c)\cdot$

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217. Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $0 \leq c \leq 5$.

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218. The distance between the origin and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$ is

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219. Let $f(x) = \{ -x^2, f \text{ or } x < 0, x^2 + 8, f \text{ or } x \geq 0$

the x intercept of the line, that is, the tangent to the graph of f(x) , is zero

(b) -1 (c) -2 (d) -4

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220. The curve $y = ax^3 + bx^2 + cx + 5$ touches the x-axis at $P(-2, 0)$ and cuts the y-axis at the point Q where its gradient is 3. Find the equation of the curve completely.

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221. The slope of the tangent to the curve $y = \sqrt{4 - x^2}$ at the point where the ordinate and the abscissa are equal is -1 (b) 1 (c) 0 (d) none of these

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222. If at each point of the curve $y = x^3 - ax^2 + x + 1,$ the tangent is inclined at an acute angle with the positive direction of the x-axis, then $a > 0$ (b) $a < -\sqrt{3} - \sqrt{3} \leq a \leq \sqrt{3}$ (d) $noneofthese$

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223. If the line joining the points $(0, 3) and (5, -2)$ is a tangent to the curve $y = \frac{C}{x + 1}$, then the value of c is 1 (b) -2 (c) 4 (d) none of these

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224. The curve given by $x + y = e^{xy}$ has a tangent parallel to the y- axis at the point (a)$(0, 1)$ (b) $(1, 0)$ (c)$(1, 1)$ (d) none of these

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225. The number of tangents to the curve $x^{\frac{3}{2}} + y^{\frac{3}{2}} = 2a^{\frac{3}{2}}, a > 0,$ which are equally inclined to the axes, is 2 (b) 1 (c) 0 (d) 4

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226. Show that the square roots of two successive natural numbers greater than N^2 differ by less than $\frac{1}{2N}$.

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227. If m is the slope of a tangent to the curve $e^y = 1 + x^2,$ then $|m| > 1$ (b) $m > 1$ $m \succ 1$ (d) $|m| \leq 1$

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228. The angle made by the tangent of the curve $x = a(t + sint \cos t), y = a(1 + \sin t)^2$ **with the x-axis at any point on it is (A)** $\frac{1}{4}(\pi + 2t)$ **(B)** $\frac{1 - \sin t}{\cos t}$ **(C)** $\frac{1}{4}(2t - \pi)$ **(D)** $\frac{1 + \sin t}{\cos 2t}$

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229. If $f(x) = \begin{cases} x^\alpha \log x & x > 0 \\ 0 & x = 0 \end{cases}$ and Rolle's theorem is applicable to $f(x)$ **for** $x \in [0, 1]$ **then** α **may equal to (A) -2 (B) -1 (C) 0 (D)** $\frac{1}{2}$

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230. In $[0, 1]$ Lagranges Mean Value theorem in NOT applicable to $f(x) = \left\{ \frac{1}{2} - x; x < \frac{1}{2}\left(\frac{1}{2} - x\right)^2; x \geq \frac{1}{2} \right.$ **b.** $f(x) = \left\{ \frac{\sin x}{x}, x \neq 01, x = 0 \right.$ **c.** $f(x) = x|x|$ **d.** $f(x) = |x|$

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231. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are) ?? $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (b) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ $(0, 0)$ (d) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

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232. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point $(1, 1)$ and the coordinate axes, lies in the first quadrant. If its area is 2, then the value of b is (a) -1 (b) 3 (c) -3 (d) 1

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233. If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$ (a) -1 (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 1

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234. The slope of the tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point $(1, 2)$ then the area of the region bounded by the curve, the x-axis and the line $x = 1$ is (A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) $\frac{1}{6}$ (D) 1

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235. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta, y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

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236. If $a, b, c \in R$ and $a + b + c = 0,$ then the quadratic equation $3ax^2 + 2bx + c = 0$ has (a) at least one root in $[0, 1]$ (b) at least one root in $[1, 2]$ (c) at least one root in $\left[\frac{3}{2}, 2\right]$ (d) none of these

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237. The tangent to the curve $y = e^{x}$ drawn at the point (c, e^{c}) intersects the line joining $\left(c - 1, e^{c-1}\right)$ and $\left(c + 1, e^{c+1}\right)$ (a) on the left of $x = c$ (b) on the right of $x = c$ (c) at no points (d) at all points

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238. Let S denote the set of all polynomials $P(x)$ of degree ≤ 2 such that $P(1) = 1, P(0) = 0 and P'(x) > 0 \forall x \in [0, 1]$, then $S = \varphi$ b. `S= {(1-a)x^2+a x ;0

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Examples

1. Find the total number of parallel tangents of $f_1(x) = x^2 - x + 1 and f_2(x) = x^3 - x^2 - 2x + 1.$

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2. Find the equation of tangent to the curve $y = \frac{\sin^{-1}(2x)}{1+x^2} at x = \sqrt{3}$

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3. The equation of the tangent tothe curve $y = \left\{x^2 \sin\left(\frac{1}{x}\right), x \ne 0 \text{ and } 0, x = 0\right.$ at the origin is

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4. Find the equation of the normals to the curve y = x^3 + 2x + 6 which are parallel to the line x + 14y + 4 = 0.

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5. If the equation of the tangent to the curve $y^2 = ax^3 + b$ at point $(2, 3) is y = 4x - 5$, then find the values of $a and b$.

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6. For the curve y $=4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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7. For the curve $xy = c$, prove that the portion of the tangent intercepted between the coordinate axes is bisected at the point of contact.

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8. If the tangent at any point $(4m^2, 8m^3)$ of $x^3 - y^2 = 0$ is a normal to the curve $x^3 - y^2 = 0$, then find the value of m.

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9. Find the equation of tangents to the curve

$y = \cos(x + y), -2\pi \leq x \leq 2\pi$

that are parallel to the line x + 2y = 0.

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10. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0.$

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11. Find the acute angle between the curves $y = \left|x\hat{2} - 1\right| and$ $y = \left|x^2 - 3\right|$ at their points of intersection.

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12. Find the angle between the curves $2y^2 = x^3 andy^2 = 32x\cdot$

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13. Find the cosine of the angle of intersection of curves $f(x) = 2^x (\log)_e xandg(x) = x^{2x} - 1.$

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14. The length of subtangent to the curve, $y = e^{x/a}$ is

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15. Find the length of normal to the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$.

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16. In the curve $x^{m+n} = a^{m-n}y^{2n}$, prove that the mth power of the sub-tangent varies as the nth power of the sub-normal.

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17. Find the possible values of p such that the equation $px^2 = (\log)_e x$ has exactly one solution.

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18. Find the shortest distance between the line $y = x - 2$ and the parabola $y = x^2 + 3x + 2$.

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20. Prove that points of the curve $y^2 = 4a\left\{x + a\sin\left(\frac{x}{a}\right)\right\}$ at which tangents are parallel to x-axis lie on the parabola.

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21. Displacement s of a particle at time t is expressed as $s = \frac{1}{2}t^3 - 6t$. Find the acceleration at the time when the velocity vanishes (i.e., velocity tends to zero).

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22. If $f(x) = 2x + 3, g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $fo(goh) = (fog)oh$.

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23. Find the derivative of $y = \tan^{-1}(x^2 - 1)$

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24. Let x be the length of one of the equal sides of an isosceles triangle, and let θ be the angle between them. If x is increasing at the rate (1/12) m/h, and θ is increasing at the rate of $\frac{\pi}{180}$ radius/h, then find the rate in m^3 / h at which the area of the triangle is increasing when $x = 12mandth\eta = \pi / 4$.

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25. A lamp is $50 ft.$ above the ground. A ball is dropped from the same height from a point $30 ft.$ away from the light pole. If ball falls a distance $s = 16t^2 ft.$ in t second, then how fast is the shadow of the ball moving along the ground $\frac{1}{2}s$ later?

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26. If water is poured into an inverted hollow cone whose semi-vertical angel is 30^0, and its depth (measured along the axis) increases at the rate of 1 cm/s. Find the rate at which the volume of water increases when the depth is 24 cm.

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27. A horse runs along a circle with a speed of $20 km/h$. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. Find the speed with which the shadow of

the horse moves along the fence at the moment when it covers 1/8 of the circle in km/h.

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28. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface when the radius is 7 cm and altitude is 24 cm is

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29. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

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30. Find approximate value of f (5.001) where $f(x) = x^3 - 7x^2 + 15$.

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31. Find the approximate change in the volume V of a cube of side x meters caused by increasing side by $1\,\%$.

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32. Discuss the applicability of Rolles theorem for the following functions on the indicated intervals:

$f(x) = |x| \in [-1, 1]$

$f(x) = 3 + (x-2)^{2/3}$ in [1,3]

$f(x) = \tan x \in [0, \pi]$

$f(x) = \log\left\{\dfrac{x^2 + ab}{x(a+b)}\right\}$ in [a,b], where 0 less than a less than b

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33. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on [1,3] satisfies Rolles theorem for $c = \dfrac{2\sqrt{3}+1}{\sqrt{3}}$ then find the value of $a and b$

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34. How many roots of the equation $(x-1)(x-2)(x-3)+(x-1)(x-2)(x-4)+(x-2)(x-3)(x-4)$ are positive?

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35. If 2a+3b+6c = 0, then show that the equation $ax^2+bx+c=0$ has atleast one real root between 0 to 1.

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36. Let $f(x)$ be differentiable function and $g(x)$ be twice differentiable function. Zeros of $f(x), g'(x)$ be a, b , respectively, `

(a

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37. Let $f(x)$ be differentiable function and $g(x)$ be twice differentiable function. Zeros of $f(x), g'(x)$ be a, b, respectively,`

(a

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38. If $f:[-5,5] \rightarrow R$ is a differentiable function function and if f'(x) does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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39. Let f be differentiable for all x, If $f(1)=-2 and f'(x) \geq 2$ for all $x \in [1,6]$, then find the range of values of $f(6)\cdot$

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40. Let $f:[2,7]\overrightarrow{0,\infty}$ be a continuous and differentiable function. Then show that $(f(7)-f(2))\frac{(f(7))^2+(f(2))^2+f(2)f(7)}{3}=5f^2(c)f'(c)$, where $c\in[2,7]\cdot$

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41. Let $f(x) and g(x)$ be differentiable function in (a,b), continuous at $a and b, and g(x)\neq 0$ in $[a,b]\cdot$ Then prove that $\frac{g(a)f(b)-f(a)g(b)}{g(c)f'(c)-f(c)g'(c)}=\frac{(b-a)g(a)g(b)}{(g(c))^2}$

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42. Using Lagranges mean value theorem, prove that $|\cos a-\cos b|<|a-b|\cdot$

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43. Let $f(x) and g(x)$ be two differentiable functions in $R and f(2) = 8, g(2) = 0, f(4) = 10, and g(4) = 8.$ Then prove that $g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$.

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44. Suppose $\alpha, \beta and th\eta$ are angles satisfying 0

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45. Let f be continuous on $[a, b], a > 0, and$ differentiable on (a, b). Prove that there exists $c \in (a, b)$ such that

$$\frac{bf(a) - af(b)}{b - a} = f(c) - cf'(c)$$

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46. Prove that the equation of the normal to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $y\cos\theta - x\sin\theta = a\cos 2\theta$, where θ is the angle which the normal makes with the axis of x.

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47. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then find the value of n.

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48. Show that the segment of the tangent to the curve $y = \frac{a}{2} In\left(\frac{a + \sqrt{a^2 - x^2}}{a - \sqrt{a^2 - x^2}}\right) - \sqrt{a^2 - x^2}$ contained between the y=axis and the point of tangency has a constant length.

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49. If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2), then prove that $\frac{x_2}{x_1} + \frac{y_2}{y_1} = -1$

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50. Find the condition for the line $y = mx$ to cut at right angles the conic $ax^2 + 2hxy + by^2 = 1$.

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51. If the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

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52. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is

changing, when he is at a distance of 30m from the foot of the tower. Assume that the eye level of the man is 1.6m from the ground.

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53. If f is continuous and differentiable function and $f(0) = 1, f(1) = 2,$ then prove that there exists at least one $c \in [0, 1] f$ or $which f'(c)(f(c))^{n-1} > \sqrt{2^{n-1}}$, where $n \in N.$

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54. Let a, b, c be three real numbers such that `a

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55. Use the mean value theorem to prove $e^x \geq 1 + x \forall x \in R$

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56. Show that the square roots of two successive natural numbers greater than N^2 differ by less than $\frac{1}{2N}$.

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57. Using Rolles theorem, prove that there is at least one root in $\left(45^{\frac{1}{100}}, 46\right)$ of the equation. $P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035 = 0$.

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58. For a twice differentiable function $f(x), g(x)$ is defined as $g(x) = f'(x)^2 + f'(x)f(x)on[a, e]$. If for `a

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ILLUSTRATION

1. Find Distance between the points for which lines that pass through the point $(1, 1)$ and are tangent to the curve represent parametrically as $x = 2t - t^2$ and $y = t + t^2$

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2. Find the approximate value of sin 3.

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SOLVED EXAMPLES

1. If f"(x) exists for all points in $[a, b]$ and

$\frac{f(c)-f(a)}{c-a}=\frac{f(b)-f(c)}{b-c}, \text{ where} a < c < b,$ then show that there exists a number 'k' such that f"(k)=0.

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Exercise 5.1

1. Find the equation of the tangent to the curve $(1+x^2)y=2-x$, where it crosses the x-axis.

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2. Find the equation of tangent and normal to the curve $x=\frac{2at^2}{(1+t^2)}, y=\frac{2at^3}{(1+t^2)}$ at the point for which $t=\frac{1}{2}$.

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3. Find the normal to the curve $x = a(1 + \cos\theta), y = a\sin\theta a\mathrm{h}\eta$. Prove that it always passes through a fixed point and find that fixed point.

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4. If the curve $y = ax^2 - 6x + b$ pass through $(0, 2)$ and has its tangent parallel to the x-axis at $x = \frac{3}{2}$, then find the values of $aandb$.

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5. Does there exists line/lines which is/are tangent to the curve $y = \sin xat(x_1, y_1)$ and normal to the curve at (x_2, y_2)?

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Exercise 5.2

1. Find the angle of intersection of $y = a^x and y = b^x$

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2. Find the angle of intersection of the curves $xy = a^2 and x^2 + y^2 = 2a^2$

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3. Find the angle at which the curve $y = Ke^{Kx}$ intersects the y-axis.

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Exercise 5.3

1. Find the length of the tangent for the curve $y = x^3 + 3x^2 + 4x - 1$ at point $x = 0$.

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Exercise 5.4

1. Minimum integral value of k for which the equation $e^x = kx^2$ has exactly three real distinct solution,

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2. Find the point on the curve $3x^2 - 4y^2 = 72$ which is nearest to the line $3x + 2y + 1 = 0.$

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3. Find the possible values of 'a' such that the inequality $3 - x^2 > |x - a|$ has atleast one negative solution

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4. Tangents are drawn from the origin to curve $y = \sin x$. Prove that points of contact lie on $y^2 = \frac{x^2}{1 + x^2}$

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5. Find the distance of the point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line $y = 2x - 1$

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Exercise 5.5

1. The distance covered by a particle moving in a straight line from a fixed point on the line is s, where $s^2 = at^2 + 2bt + c$. Then prove that acceleration is proportional to s^{-3}.

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2. Two cyclists start from the junction of two perpendicular roads, there velocities being $3um / m \in$ and $4um / m \in$, respectively. Find the rate at which the two cyclists separate.

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3. A spherical iron ball 10cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50cm^3 / m \in$. When the thickness of ice is 5cm, then find the rate at which the thickness of ice decreases.

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4. $x and y$ are the sides of two squares such that $y = x - x^2$. Find the rate of the change of the area of the second square with respect to the first square.

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5. Two men $P and Q$ start with velocity u at the same time from the junction of two roads inclined at 45^0 to each other. If they travel by different roads, find the rate at which they are being separated.

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6. Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always 1/6th of the radius of the base. How fast does the height of the sand cone increase when the height in 4 cm?

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7. An aeroplane is flying horizontally at a height of $\frac{2}{3}km$ with a velocity of 15 km/h. Find the rate at which it is receding from a fixed point on the ground which it passed over 2 min ago.

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Exercise 5.6

1. Find the approximate value of $(1.999)^6$.

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2. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

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Exercise 5.7

1. Let $0 < a < b < \frac{\pi}{2}. If\ f(x) = \begin{vmatrix} \tan x & \tan a & \tan b \\ \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \end{vmatrix}$, then find the minimum possible number of roots of $f'(x) = 0$ in (a , b)

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2. Let $f(x) and g(x)$ be differentiable for $0 \leq x \leq 2$ such that $f(0) = 2, g(0) = 1, and f(2) = 8.$ Let there exist a real number c in $[0, 2]$ such that $f'(c) = 3g'(c)$. Then find the value of $g(2)$.

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3. Prove that if $2a_0^2 < 15a,$ all roots of $x^5 - a_0x^4 + 3ax^3 + bx^2 + cx + d = 0$ cannot be real. It is given that $a_0, a, b, c, d \in R$.

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4. Let f and g be function continuous in $[a, b]$ and differentiable on $[a, b]$ .If $f(a) = f(b) = 0$ then show that there is a point $c \in (a, b)$ such that $g'(c)f(c) + f'(c) = 0$.

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5. If $\varphi(x)$ is differentiable function $\forall x \in R$ and $a \in R$ such that $\varphi(0) = \varphi(2a), \varphi(a) = \varphi(3a) and \varphi(0) \neq \varphi(a)$ then show that there is at least one root of equation $\varphi'(x + a) = \varphi'(x) \in (0, 2a)$

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Exercise 5.8

1. Find c of Lagranges mean value theorem for the function $f(x) = 3x^2 + 5x + 7$ in the interval $[1, 3]$.

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2. If f(x) is differentiate in [a,b], then prove that there exists at least one $c \in (a, b) \text{such that} \left(a^2 - b^2\right) f'(c) = 2c(f(a) - f(b))$.

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Exercise (Single)

1. The number of tangents to the curve $x^{\frac{3}{2}} + y^{\frac{3}{2}} = 2a^{\frac{3}{2}}, a > 0,$ which are equally inclined to the axes, is 2 (b) 1 (c) 0 (d) 4

A. 2

B. 1

C. 0

D. 4

Answer: B

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2. The angle made by the tangent of the curve $x = a(t + s \in t \cos t), y = a(1 + s \in t)^2$ with the x-axis at any point on it is $\frac{1}{4}(\pi + 2t)$ (b) $\frac{1 - s \in t}{\cos t}$ $\frac{1}{4}(2t - \pi)$ (d) $\frac{1 + s \in t}{\cos 2t}$

A. $\frac{1}{4}(\pi + 2t)$

B. $\dfrac{1-\sin t}{\cos t}$

C. $\dfrac{1}{4}(2t-\pi)$

D. $\dfrac{1+\sin t}{\cos 2t}$

Answer: A

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3. If m is the slope of a tangent to the curve $e^y = 1 + x^2$, then $|m| > 1$

(b) $m > 1$ $m \succ 1$ **(d)** $|m| \leq 1$

A. $|m| > 1$

B. $m > 1$

C. $m > \ -1$

D. $|m| \leq 1$

Answer: D

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4. If at each point of the curve $y = x^3 - ax^2 + x + 1,$ the tangent is inclined at an acute angle with the positive direction of the x-axis, then $a > 0$ (b) $a < -\sqrt{3} - \sqrt{3} \le a \le \sqrt{3}$ (d) $none of these$

A. $a > 0$

B. $a \le \sqrt{3}$

C. $-\sqrt{3} \le a \le \sqrt{3}$

D. none of these

Answer: C

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5. The slope of the tangent to the curve $y = \sqrt{4 - x^2}$ at the point where the ordinate and the abscissa are equal is -1 (b) 1 (c) 0 (d) none of these

A. -1

B. 1

C. 0

D. none of these

Answer: A

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6. The curve given by $x + y = e^{xy}$ has a tangent parallel to the y- axis at the point (a)$(0, 1)$ (b) $(1, 0)$ (c)$(1, 1)$ (d) none of these

A. $(0, 1)$

B. $(1, 0)$

C. $(1, 1)$

D. none of these

Answer: B

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7. If the line joining the points $(0, 3) and (5, -2)$ is a tangent to the curve $y = \frac{C}{x+1}$, then the value of c is 1 (b) -2 (c) 4 (d) none of these

A. 1

B. -2

C. 4

D. none of these

Answer: C

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8. The distance between the origin and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$ is $\left(1, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, 1\right)$ $\left(2, -\frac{28}{3}\right)$ (d) none of these

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{-1}{\sqrt{5}}$

D. $\frac{2}{\sqrt{3}}$

Answer: A

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9. The normal to the curve $2x^2 + y^2 = 12$ at the point $(2, 2)$ cuts the curve again at $\left(-\frac{22}{9}, -\frac{2}{9}\right)$ (b) $\left(\frac{22}{9}, \frac{2}{9}\right)$ $(-2, -2)$ (d) none of these

A. $\left(-\frac{22}{9}, -\frac{2}{9}\right)$

B. $\left(\frac{22}{9}, \frac{2}{9}\right)$

C. $(-2, -2)$

D. none of these

Answer: A

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10. At what point of curve $y=\frac{2}{3}x^3+\frac{1}{2}x^2$, the tangent makes equal angle with the axis? (a) $\left(\frac{1}{2}, \frac{2}{24}\right)and\left(-1, -\frac{1}{6}\right)$ (b) $\left(\frac{1}{2}, \frac{4}{9}\right)and(-1, 0)$ (c) $\left(\frac{1}{3}, \frac{1}{7}\right)and\left(-3, \frac{1}{2}\right)$ (d) $\left(\frac{1}{3}, \frac{4}{47}\right)and\left(-1, -\frac{1}{3}\right)$

A. $\left(\frac{1}{2}, \frac{4}{24}\right)$ and $\left(-1, -\frac{1}{6}\right)$

B. $\left(\frac{1}{2}, \frac{4}{9}\right)$ and $(-1, 0)$

C. $\left(\frac{1}{3}, \frac{1}{7}\right)$ and $\left(-3, \frac{1}{2}\right)$

D. $\left(\frac{1}{3}, \frac{4}{47}\right)$ and $\left(-1, -\frac{1}{3}\right)$

Answer: A

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11. The equation of the tangent to the curve $y=be^{-x/a}$ at the point where it crosses the y-axis is $\frac{x}{a}-\frac{y}{b}=1$ (b) $ax+by=1$ $ax-by=1$

(d) $\frac{x}{a}+\frac{y}{b}=1$

A. $\frac{x}{a}-\frac{y}{b}=1$

B. $ax+by+1$

C. $ax-by=1$

D. $\frac{x}{a}+\frac{y}{b}=1$

Answer: D

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12. Then angle of intersection of the normal at the point $\left(-\frac{5}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ of the curves $x^2-y^2=8$ and $9x^2+25y^2=225$ is 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: B

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13. A function $y - f(x)$ has a second-order derivative $f^x = 6(x - 1)$. It its graph passes through the point (2,1) and at that point tangent to the graph is $y = 3x - 5$, then the value of $f(0)$ is 1 (b) -1 (c) 2 (d) 0

A. 1

B. -1

C. 2

D. 0

Answer: B

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14. If $x+4y=14$ is a normal to the curve $y^2=\alpha x^3-\beta$ at (2,3), then the value of $\alpha+\beta$ is 9 (b) -5 (c) 7 (d) -7

A. 9

B. -5

C. 7

D. -7

Answer: A

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15. The abscissas of point $P and Q$ on the curve $y=e^x+e^{-x}$ such that tangents at $P and Q$ make 60^0 with the x-axis are. $1n\left(\frac{\sqrt{3}+\sqrt{7}}{7}\right) and 1n\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$ $1n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$ (c) $1n\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$ $\pm 1n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

A. $\ln\left(\frac{\sqrt{3}+\sqrt{7}}{7}\right)$ and $\ln\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$

B. $\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

C. $\ln\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$

D. $\pm\ln\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

Answer: B

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16. Let C be the curve $y = x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A, then K is equal to 4 (b) 2 (c) -2 (d) $\frac{1}{4}$

A. 4

B. 2

C. -2

D. $\frac{1}{4}$

Answer: A

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17. The x-intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^2}+\frac{b}{y^2}=1$ is proportional to (a)square of the abscissa of the point of tangency (b)square root of the abscissa of the point of tangency (c)cube of the abscissa of the point of tangency (d)cube root of the abscissa of the point of tangency

A. square of the abscissa of the point of tangency

B. square root of the absciss of the point of tangency

C. cube of the abscissa of the point of tangency

D. cube root of the abscissa of the point of tangency

Answer: C

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18. Given $g(x) = \frac{x+2}{x-1}$ and the line $3x + y - 10 = 0$. Then the line is (a)tangent to $g(x)$ (b) normal to $g(x)$ (c)chord of $g(x)$ (d) none of these

A. tangent to g(x)

B. normal to g(x)

C. chord ofg(x)

D. none of these

Answer: A

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19. The number of point in the rectangle $\{(x, y)\} - 12 \leq x \leq 12 and - 3 \leq y \leq 3\}$ which lie on the curve $y = x + \sin x$ and at which in the tangent to the curve is parallel to the x-axis is 0 (b) 2 (c) 4 (d) 8

A. 0

B. 2

C. 4

D. 8

Answer: A

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20. Tangent of acute angle between the curves $y = |x^2 - 1|$ and $y = \sqrt{7 - x^2}$ at their points of intersection is $\frac{5\sqrt{3}}{2}$ (b) $\frac{3\sqrt{5}}{2}$ $\frac{5\sqrt{3}}{4}$ (d) $\frac{3\sqrt{5}}{4}$

A. $\frac{5\sqrt{3}}{2}$

B. $\frac{3\sqrt{5}}{2}$

C. $\frac{5\sqrt{3}}{4}$

D. $\frac{3\sqrt{5}}{4}$

Answer: C

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21. The line tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^2 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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22. The two curves $x = y^2, xy = a^3$ cut orthogonally at a point. Then a^2 is equal to $\frac{1}{3}$ (b) 3 (c) 2 (d) $\frac{1}{2}$

A. $\frac{1}{3}$

B. 3

C. 2

D. $\frac{1}{2}$

Answer: D

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23. The tangent to the curve $y = e^{kx}$ at a point (0,1) meets the x-axis at (a,0), where $a \in [-2, -1]$. Then $k \in$ $\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right]$ $[0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

A. $[-1/2, 0]$

B. $[-1, -1/2]$

C. $[0, 1]$

D. $[1/2, 1]$

Answer: D

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24. The curves $4x^2 + 9y^2 = 72$ and $x^2 - y^2 = 5at(3, 2)$ touch each other (b) cut orthogonally intersect at 45^0 (d) intersect at 60^0

A. touch each other

B. cut orthogonally

C. intersect at 45°

D. intersect at 60°

Answer: B

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25. The coordinates of a point on the parabola $y^2 = 8x$ whose distance from the circle $x^2 + (y+6)^2 = 1$ is minimum is $(2, 4)$ (b) $(2, -4)$

$(18, -12)$ **(d)** $(8, 8)$

A. $(2, 4)$

B. $(2, -4)$

C. $(18, -12)$

D. $(8, 8)$

Answer: B

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26. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant at normal is drawn. The normal intersects the Y-axis at the point (0, b). If $\lim_{a \to 0} b = \frac{1}{2}$, then n equals ....

A. 1

B. 3

C. 2

D. 4

Answer: C

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27. Let f be a continuous, differentiable, and bijective function. If the tangent to $y = f(x) at x = a$ is also the normal to $y = f(x) at x = b$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$ **(b)** $f'(c) > 0$ $f'(c) < 0$ **(d) none of these**

A. $f'(c) = 0$

B. $f'(c) > 0$

C. $f'(c) < 0$

D. none of these

Answer: A

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28. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is (2,6) (b) $(2, -6)$ $\left(\frac{9}{8}, -\frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

A. $(2, 6)$

B. $(2, -6)$

C. $\left(\frac{9}{8}, \frac{9}{2}\right)$

D. $\left(\frac{9}{8}, \frac{9}{2}\right)$

Answer: D

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29. Find the rate of change of volume of a sphere with respect to its surface area when the radius is 2cm.

A. 1

B. 2

C. 3

D. 4

Answer: A

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30. If there is an error of $k\%$ in measuring the edge of a cube, then the percent error in estimating its volume is k (b) $3k$ $\frac{k}{3}$ (d) none of these

A. k

B. 3k

C. $\frac{k}{3}$

D. none of these

Answer: B

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31. A lamp of negligible height is placed on the ground l_1 away from a wall. A man $l_2 m$ tall is walking at a speed of $\frac{l_1}{10} m/s$ from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is $-\frac{5l_2}{2} m/s$ (b) $-\frac{2l_2}{5} m/s$ $-\frac{l_2}{2} m/s$ (d) $-\frac{l_2}{5} m/s$

A. $-\frac{5l_2}{2} m/s$

B. $-\frac{2l_2}{5} m/s$

C. $-\frac{l_2}{2} m/s$

D. $-\frac{l_2}{5} m/s$

Answer: B

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32. The function $f(x) = x(x+3)e^{-\left(\frac{1}{2}\right)x}$ satisfies the conditions of Rolle's theorem in (-3,0). The value of c, is

A. -2

B. -1

C. 0

D. 3

Answer: A

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33. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in $cm^2/m \in$, when the radius is $2cm$ and the height is 3cm is $-2p$ (b) $-\frac{8\pi}{5}$ $-\frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

A. -2π

B. $-\frac{8\pi}{5}$

C. $16/6$

D. $-8/15$

Answer: D

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34. A cube of ice melts without changing its shape at the uniform rate of $4\frac{cm^3}{m \in}$. The rate of change of the surface area of the cube, in $\frac{cm^2}{m \in}$, when the volume of the cube is $125cm^3$, is -4 (b) $-\frac{16}{5}$ (c) $-\frac{16}{6}$ (d) $-\frac{8}{15}$

A. -4

B. $-16/5$

C. $-16/6$

D. $-8/15$

Answer: B

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35. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface when the radius is 7 cm and altitude is 24cm is $108\pi cm^2 / \min$ (b) $7\pi cm^2 / \min$ $27\pi cm^2 / \min$ (d) none of these

A. $108\pi cm^2 / \min$

B. $7\pi cm^2 / \min$

C. $27\pi cm^2 / \min$

D. none of these

Answer: A

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36. If $f(x) = x^3 + 7x - 1$, then $f(x)$ has a zero between $x = 0 and x = 1$. The theorem that best describes this is mean value theorem maximum-minimum value theorem intermediate value theorem none of these

A. mena value theorem

B. maximum-minimum value theorem

C. intermediate value theorem

D. none of these

Answer: C

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37. Consider the function $f(x) = \begin{cases} x^{\frac{\sin(\pi)}{x}} & \text{for} x > 0 \\ 0 & \text{for} x = 0 \end{cases}$

Then, the number of points in $(0, 1)$ where the derivative f'(x) vanishes is

A. 0

B. 1

C. 2

D. infinite

Answer: D

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38. Let $f(x) and g(x)$ be differentiable for $0 \leq x \leq 1$, such that $f(0), g(0), f(1) = 6$. Let there exists real number c in (0,1) such taht $f'(c) = 2g'(c)$. Then the value of $g(1)$ must be 1 (b) 3 (c) -2 (d) -1

A. 1

B. 3

C. -2

D. $1-$

Answer: B

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39. If $3(a+2c) = 4(b+3d)$, then the equation $ax^3 + bx^2 + cx + d = 0$ will have (a)no real solution (b)at least one real root in $(-1, 0)$ (c)at least one real root in $(0, 1)$ (d)none of these

A. no real solution

B. at least one real root in $(-1, 0)$

C. at least one real root in $(0, 1)$

D. none of these

Answer: B

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40. A value of c for which the conclusion of Mean value theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

A. $\frac{1}{2}\log_e 3$

B. $\log_3 e$

C. $\log_e 3$

D. $2\log_3 e$

Answer: D

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41. Let $f(x)$ be a twice differentiable function for all real values of x and satisfies $f(1) = 1$, $f(2) = 4$, $f(3) = 9$. Then which of the following is definitely true? $f^{x} = 2\forall x \in (1, 3)$ $f^{x} = f(x) = 5f$ or $somex \in (2, 3)$ $f^{x} = 3\forall x \in (2, 3)$ $f^{x} = 2f$ or $somex \in (1, 3)$

A. $f''(x) = 2\forall x \in (1, 3)$

B. $f''(x) = f(x)5\text{for some}x \in (2, 3)$

C. $f''(x) = 3\forall x \in (2, 3)$

D. $f''(x) = 2\text{for some}x \in (1, 3)$

Answer: D

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42. The value of c in Lagranges theorem for the function $f(x) = \log\sin x$ in the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c)$\frac{2\pi}{3}$ (d) none of these

A. $\pi / 4$

B. $\pi / 2$

C. $2\pi / 3$

D. none of these

Answer: B

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43. In which of the following function Rolle's theorem is applicable ?

A. $f(x) = \begin{cases} x & 0 \le x < 1 \\ 0 & x = 1 \end{cases} on[0, 1]$

B. $f(x) = \begin{cases} \frac{\sin x}{x} & -\pi \le x < 0 \\ 0 & x = 0 \end{cases} on[-\pi, 0]$

C. $f(x)\frac{x^2 - x - 6}{x - 1} on[-2, 3]$

D. $f(x) = \begin{cases} \frac{x^3 - 2x^3 - 5x + 6}{x - 1} & \text{if } x \ne 1 \\ -6 & \text{if } x = 1 \end{cases} on[-2, 3]$

Answer: D

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44. Let $f'(x) = e^{x2}$ and $f(0) = 10. If A < f(1) < B$ can be concluded from the mean value theorem, then the largest volume of $(A - B)$ equals

A. e

B. $1 - e$

C. $e - 1$

D. $1 + e$

Answer: B

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45. If $f(x) and g(x)$ are differentiable functions for $0 \le x \le 1$ such that $f(0) = 10, g(0) = 2, f(1) = 2, g(1) = 4,$ then in the interval $(0, 1)\cdot$ (a) $f'(x) = 0 f \text{ or } all x$ (b)$f'(x) + 4g'(x) = 0$ for at least one x (c) $f(x) = 2g'(x)$ for at most one x (d)none of these

A. $f(x) = 0$ for all x

B. $f(x) + 4g'(x) = 0$ for at least one x

C. $f(x) = 2g'(x)$ for at most one x

D. none of these

Answer: B

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46. A continuous and differentiable function $y = f(x)$ is such that its graph cuts line $y = mx + c$ at n distinct points. Then the minimum number of points at which $f(x) = 0$ is/are

(a) $n - 1$

(b) $n - 3$

(c) $n - 2$

(d) cannot say

A. $n - 1$

B. $n-3$

C. $n-2$

D. cannot say

Answer: C

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47. Given $f'(1)=1 \text{and} \frac{d}{dx}(f(2x))=f'(x)\forall x>0$.If $f'(x)$ is differentiable then there exies a number $c \in (2,4)$ such that $f''(c)$ equals

A. $\frac{1}{4}$

B. $\frac{-1}{2}$

C. $-1\frac{1}{4}$

D. $-\frac{1}{8}$

Answer: D

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48. If (x) is differentiable in $[a, b]$ such that $f(a) = 2, f(b) = 6,$ then there exists at least one $c, a < c \leq b,$ such that $(b^3 - a^3) f'(c) =$

A. c^2

B. $2c^2$

C. $-3c^2$

D. $12c^2$

Answer: D

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Exercise (Multiple)

1. Points on the curve $f(x) = \frac{x}{1 - x^2}$ where the tangent is inclined at an angle of $\frac{\pi}{4}$ to the x-axis are (a)(0,0) (b) $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$ $\left(-2, \frac{2}{3}\right)$ (d)

$\left(-\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

A. $(0, 0)$

B. $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$

C. $\left(-2, \frac{2}{3}\right)$

D. $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$

Answer: A::B::D

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2. For the curve $y = ce^{x/a}$, which one of the following is incorrect?

A. sub-tangent is constant

B. sub-normal varies as the square of the ordinate

C. tangent at (x_1, y_1) on the curve intersects the x-axis at a distance of $(x_1 - a)$ from the origin

D. equaltion of the normal at the point where the curve cuts

$y-\text{axis is} cy+ax=c^2$

Answer: A::B::C::D

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3. Let the parabolas $y=x(c-x)andy=x^2+ax+b$ touch each other at the point (1,0). Then (a)$a+b+c=0$ (b) $a+b=2$ (c)$b-c=1$ (d) $a+c=\;-2$

A. $a+b+c=0$

B. $a+b=2$

C. $b-c=1$

D. $a+c=\;-2$

Answer: A::C::D

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4. **The angle formed by the positive y - axis and the tangent to** $y = x^2 + 4x - 17at\left(\frac{5}{2}, -\frac{3}{4}\right)$ **is: (a)** $\tan^{-1}(9)$ **(b)** $\frac{\pi}{2} - \tan^{-1}(9)$ $\frac{\pi}{2} + \tan^{-1}(9)$ **(d) none of these**

A. $\tan^{-1}(9)$

B. $\frac{\pi}{2} - \tan^{-1}(9)$

C. $\frac{\pi}{2} + \tan^{-1}(9)$

D. none of these

Answer: B::C

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5. **Which of the following pair(s) of curves is/are orthogonal?** $y^2 = 4ax; y = e^{-\frac{x}{2a}}$ $y^2 = 4ax; x^2 = 4ayat(0, 0)$ $xy = a^2; x^2 - y^2 = b^2$ $y = ax; x^2 + y^2 = c^2$

A. $y^2 = 4ax, y = e^{-x/2a}$

B. $y^2 = 4ax, x^2 = 4ayat(0, 0)$

C. $xy = a^2, x^2 - y^2 = b^2$

D. $y = ax, x^2 + y^2 = c^2$

Answer: A::B::C::D

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6. The coordinates of the point(s) on the graph of the function $f(x) = \frac{x^3}{x} - \frac{5x^2}{2} + 7x - 4$, where the tangent drawn cuts off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, are $\left(2, \frac{8}{3}\right)$ (b) $\left(3, \frac{7}{2}\right)$ $\left(1, \frac{5}{6}\right)$ (d) none of these

A. $(2, 8/3)$

B. $(3, 7/2)$

C. $(1, 5/6)$

D. none of these

Answer: A::B

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7. The abscissa of a point on the curve $xy = (a+x)^2$, the normal which cuts off numerically equal intercepts from the coordinate axes, is $-\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}a$ (c) $\frac{a}{\sqrt{2}}$ (d) $-\sqrt{2}a$

A. $-\frac{a}{\sqrt{2}}$

B. $\sqrt{2}a$

C. $\frac{a}{\sqrt{2}}$

D. $-\sqrt{2}a$

Answer: A::C

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8. Given $f(x)=4-\left(\frac{1}{2}-x\right)^{\frac{2}{3}}, g(x)=\left\{\frac{\tan[x]}{x}, x \neq 01, x=0\right.$ $h(x)=\{x\}, k(x)=5^{(\log)_2(x+3)}$ Then in [0,1], lagranges mean value theorem is not applicable to (where [.] and {.} represents the greatest integer functions and fractional part functions, respectively). f (b) g (c) k (d) h

A. f

B. g

C. k

D. h

Answer: A::B::D

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9. Let $f(x)=a_5x^5+a_4x^4+a_3x^3+a_2x^2+a_1x,$ where $a_i{}'s$ are real and $f(x)=0$ has a positive root $\alpha_0\cdot$ Then $f'(x)=0$ has a positive root α_1 such that `0

A. f'(x) = 0 has a root $\alpha_1$such that $< \alpha_1 < \alpha_0$

B. f' (x) = 0 has at least one real root

C. f''(x) = 0 has at least one real root

D. none of these

Answer: A::B::C

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10. Which of the following is/are correct ?

A. Between any two root of $e^x \cos x = 1,$ there exists at least one root of tan x = 1.

B. Between any two roots of $e^x \sin x = 1,$ there exists at least one root of tan x= -1.

C. Between any two roots of $e^x \cos x = 1,$ there exists at least one root of $e^x \sin x = 1.$

D. Between any two roots of $e^x \sin x = 1$, then exists at least one root of $e^x \cos x = 1$.

Answer: A::B::C

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Exercise (Numerical)

1. There is a point (p,q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = \frac{-8}{x}, wherep > 0andr > 0$. If the line through $(p, q)and(r, s)$ is also tangent to both the curves at these points, respectively, then the value of $P + r$ is______.

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2. A curve is defined parametrically be equations $x = t^2 andy = t^3$. A variable pair of perpendicular lines through the origin O meet the curve

of $PandQ$. If the locus of the point of intersection of the tangents at $PandQ$ is $ay^2 = bx - 1,$ then the value of $(a + b)$ is___

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3. If d is the minimum distance between the curves $f(x) = e^x andg(x) = (\log)_e x,$ then the value of d^0 is

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4. Let $f(x0$ be a non-constant thrice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(6 - x)andf'(0) = 0 = f'(x)^2 = f(5)$. If n is the minimum number of roots of $\left(f'(x)^2 + f'(x)f^x = 0\right.$ in the interval [0,6], then the value of $\frac{n}{2}$ is__

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5. At the point $P(a, a^n)$ on the graph of $y = x^n, (n \in N)$, in the first quadrant, a normal is drawn. The normal intersects the y-axis at the point $(0, b)$. If $(\lim)_{a \vec{0}} b = \frac{1}{2}$, then n equals ____.

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6. A curve is given by the equations $x = \sec^2 \theta, y = \cot \theta$. If the tangent at $P where \theta = \frac{\pi}{4}$ meets the curve again at $Q, then [PQ]$ is, where [.] represents the greatest integer function, _______.

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7. about to only mathematics

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8. If the slope of line through the origin which is tangent to the curve $y = x^3 + x + 16$ is m, then the value of $m - 4$ is___.

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9. Suppose a, b, c are such that the curve $y = ax^2 + bx + c$ is tangent to $y = 3x - 3at(1, 0)$ and is also tangent to $y = x + 1at(3, 4)$. Then the value of $(2a - b - 4c)$ equals ____

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10. Let C be a curve defined by $y = e^a + bx^2$. The curve C passes through the point $P(1, 1)$ and the slope of the tangent at P is (-2). Then the value of $2a - 3b$ is____.

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11. If the curve C in the xy plane has the equation $x^2 + xy + y^2 = 1$, then the fourth power of the greatest distance of a point on C from the origin is___.

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12. If a, b are two real numbers with `a

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JEE Previous Year

1. The shortest distance between line y-x=1 and curve $x = y^2$ is

A. $\frac{3\sqrt{2}}{8}$

B. $\frac{2\sqrt{3}}{8}$

C. $\frac{3\sqrt{2}}{5}$

D. $\frac{\sqrt{3}}{4}$

Answer: A

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2. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the $x - \text{axis}$, is

A. $y = 8$

B. $y = 0$

C. $y = 3$

D. $y = 2$

Answer: C

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3. Consider the function $f(x) = |x-2| + |x-5|, c \in R.$

Statement 1: $f'(4) = 0$

Statement 2: f is continuous in $[2, 5]$, differentiable in

A. Statement 1 is false, statement 2 is true.

B. Statement 1 is true , Statement 2 is true, statement 2 is correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is trur, Statement2 is no a correct explanation for statement 1.

D. Statement 1 is true, Statement 2 is false.

Answer: C

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4. Let $fandg$ be differentiable on [0,1] such that $f(0) = 2, g(0), f(1) = 6andg(1) = 2.$ Show that there exists $c \in (0, 1)$

such that $f'(c) = 2g'(c)$.

A. 2f'(c)=g'(c)`

B. 2f'(c)=3g'(c)`

C. f'(c)=g'(c)`

D. f'(c)=2g'(c)`

Answer: D

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5. The norma to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1, 1)$

A. does not meet the curve again.

B. meets the curve again in the second quadrant.

C. meets the curve again in the third quadrant.

D. meets the curve again in the fourth quadrant.

Answer: D

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6. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point: (1) (0, 0) (2) $\left(0, \frac{2\pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)$ (4) $\left(\frac{\pi}{4}, 0\right)$

A. $\left(0, \frac{2\pi}{3}\right)$

B. $\left(\frac{\pi}{6}0, \right)$

C. $\left(\frac{\pi}{4}, 0\right)$

D. $(0, 0)$

Answer: A

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7. The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the $y - a\xi s,$ passes through the point : $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(2) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(\frac{\frac{1}{2,1}}{2}\right)$

A. $\left(\frac{1}{2}, \frac{1}{3}\right)$

B. $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

C. $\left(\frac{1}{2}, \frac{1}{2}\right)$

D. $\left(\frac{1}{2}, \frac{1}{3}\right)$

Answer: C

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8. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

A. $9/2$

B. 6

C. $7/2$

D. 4

Answer: A

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9. Let $f, g: [-1, 2]\vec{R}$ be continuous functions which are twice differentiable on the interval $(-1, 2)\cdot$ Let the values of $fandg$ at the points $-1, 0and2$ be as given in the following table: , $x = -1$, $x = 0$, $x = 2$ $f(x)$, 3, 6, 0 $g(x)$, 0, 1, -1 In each of the intervals $(-1, 0)and(0, 2)$ the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is (are) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0) \cup (0, 2)\cdot$ $f'(x) - 3g'(x) = 0$ has exactly one solutions in $(-1, 0)\cdot$ $f'(x) - 3g'(x) = 0$ has exactly one solutions in $(-1, 2)\cdot$ $f'(x) - 3g'(x) = 0$ has exactly two solutions in $(-1, 0)$ and exactly two solutions in $(0, 2)\cdot$

A. $f'(x) - 3g'(x) = 0$ has exactly three solution in $(-1, 0) \cup (0, 2)$

B. $f'(x) - 3g'(x) = 0$ has exactly one solution in (-1,0)

C. $f'(x) - 3g'(x) = 0$ has exactly one solution in (0,2)

D. $f'(x) - 3g'(x) = 0$ has excatly two solutions in (-1,0) and exactly two solution in (0,2)

Answer: B::C

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