

**MATHS****BOOKS - CENGAGE****COMPLEX NUMBERS****Single correct Answer**

1. The value of  $\sum_{n=0}^{100} i^{n!}$  equals (where  $i = \sqrt{-1}$ )

A.  $-1$ B.  $i$ C.  $2i + 95$ D.  $97 + i$ **Answer: C**

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2. Suppose  $n$  is a natural number such that

$$|i + 2i^2 + 3i^3 + \dots + ni^n| = 18\sqrt{2} \text{ where } i \text{ is the square root of } -1.$$

Then  $n$  is

A. 9

B. 18

C. 36

D. 72

**Answer: C**

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3. Let  $i = \sqrt{-1}$ . Define a sequence of complex number by

$$z_1 = 0, z_{n+1} = (z_n)^2 + i \text{ for } n \geq 1. \text{ In the complex plane, how far from}$$

the origin is  $z_{111}$ ?

A. 1

B. 2

C. 3

D. 4

**Answer: B**

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4. The complex number,  $z = \frac{(-\sqrt{3} + 3i)(1 - i)}{(3 + \sqrt{3}i)(i)(\sqrt{3} + \sqrt{3}i)}$

A. lies on real axis

B. lies on imaginary axis

C. lies in first quadrant

D. lies in second quadrant

**Answer: B**

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5.  $a, b, c$  are positive real numbers forming a G.P. If  $ax^2 + 2bx + c = 0$  and  $dx^2 + 2ex + f = 0$  have a common root, then prove that  $d/a, e/b, f/c$  are in A.P.

A. A. P.

B. G. P.

C. H. P.

D. None of these

**Answer: C**

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6. Prove that the equation  $Z^3 + iZ - 1 = 0$  has no real roots.

A. three real roots

B. one real roots

C. no real roots

D. no real or complex roots

**Answer: C**



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7. If  $a, b$  are complex numbers and one of the roots of the equation  $x^2 + ax + b = 0$  is purely real whereas the other is purely imaginary, and  $a^2 - \bar{a}^2 = kb$ , then  $k$  is

A. 2

B. 4

C. 6

D. 8

**Answer: B**

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8. If  $Z^5$  is a non-real complex number, then find the minimum value of  $\left| \frac{\text{Im}z^5}{\text{Im}^5z} \right|$

$$\left| \frac{\text{Im}z^5}{\text{Im}^5z} \right|$$

A.  $-1$

B.  $-2$

C.  $-4$

D.  $-5$

**Answer: C**

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9. For any complex numbers

$z_1, z_2$  and  $z_3, z_3\text{Im}(\overline{z_2}z_3) + z_2\text{Im}(\overline{z_3}z_1) + z_1\text{Im}(\overline{z_1}z_2)$  is

A.  $0$

B.  $z_1 + z_2 + z_3$

C.  $z_1 z_2 z_3$

D.  $\left( \frac{z_1 + z_2 + z_3}{z_1 z_2 z_3} \right)$

**Answer: A**

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10. The modulus and amplitude of  $\frac{1 + 2i}{1 - (1 - i)^2}$  are

A.  $\sqrt{2}$  and  $\frac{\pi}{6}$

B. 1 and  $\frac{\pi}{4}$

C. 1 and 0

D. 1 and  $\frac{\pi}{3}$

**Answer: C**

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11. If the argument of  $(z - a)(\bar{z} - b)$  is equal to that of  $\left( (\sqrt{3} + i) \frac{1 + \sqrt{3}i}{1 + i} \right)$  where  $a, b, c$  are two real number and  $z$  is the complex conjugate o the complex number  $z$  find the locus of  $z$  in the rgand diagram. Find the value of  $a$  and  $b$  so that locus becomes a circle having its centre at  $\frac{1}{2}(3 + i)$

A. (3, 2)

B. (2, 1)

C. (2, 3)

D. (2, 4)

**Answer: B**

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12. If a complex number  $z$  satisfies  $|z|^2 + \frac{4}{(|z|)^2} - 2\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right) - 16 = 0$ , then the maximum value of  $|z|$  is



A.  $\sqrt{6} + 1$

B. 4

C.  $2 + \sqrt{6}$

D. 6

**Answer: C**



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13. If  $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$ , then  $\frac{\sin 3\alpha + \sin 3\beta + \sin 3\gamma}{\sin(\alpha + \beta + \gamma)}$  is equal to

A. 1

B. -1

C. 3

D. -3

**Answer: C**



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14. The least value of  $|z - 3 - 4i|^2 + |z + 2 - 7i|^2 + |z - 5 + 2i|^2$  occurs when  $z =$

A.  $1 + 3i$

B.  $3 + 3i$

C.  $3 + 4i$

D. None of these

**Answer: D**



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15. The roots of the equation  $x^4 - 2x^2 + 4 = 0$  are the vertices of a :

A. square inscribed in a circle of radius 2

B. rectangle inscribed in a circle of radius 2

C. square inscribed in a circle of radius  $\sqrt{2}$

D. rectangle inscribed in a circle of radius  $\sqrt{2}$

**Answer: D**



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16. If  $z_1, z_2$  are complex numbers such that  $Re(z_1) = |z_1 - 2|$ ,  $Re(z_2) = |z_2 - 2|$  and  $arg(z_1 - z_2) = \pi/3$ , then  $Im(z_1 + z_2) =$

A.  $2/\sqrt{3}$

B.  $4/\sqrt{3}$

C.  $2/\sqrt{3}$

D.  $\sqrt{3}$

**Answer: B**



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17. If  $z = e^{\frac{2\pi i}{5}}$ , then

$$1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 =$$

A. 0

B.  $4z^3$

C.  $5z^4$

D.  $-4z^2$

**Answer: C**



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18. If  $z = (3 + 7i)(a + ib)$ , where  $a, b \in \mathbb{Z} - \{0\}$ , is purely imaginary, then minimum value of  $|z|^2$  is

A. 74

B. 45

C. 65

D. 58

**Answer: D**



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19. Let  $z$  be a complex number satisfying  $|z + 16| = 4|z + 1|$ . Then

A.  $|z| = 4$

B.  $|z| = 5$

C.  $|z| = 6$

D.  $3 < |z| < 68$

**Answer: A**



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20. If  $|z| = 1$  and  $z' = \frac{1 + z^2}{z}$ , then

A.  $z'$  lie on a line not passing through origin

B.  $|z'| = \sqrt{2}$

C.  $\operatorname{Re}(z') = 0$

D.  $\operatorname{Im}(z') = 0$

**Answer: D**



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21.  $a, b, c$  are three complex numbers on the unit circle  $|z| = 1$ , such that  $abc = a + b + c$ . Then  $|ab + bc + ca|$  is equal to

A. 3

B. 6

C. 1

D. 2

**Answer: C**

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22. If  $|z_1| = |z_2| = |z_3| = 1$  then value of  $|z_1 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$  cannot exceed

- A. 6
- B. 9
- C. 12
- D. none of these

**Answer: B**

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23. Number of ordered pairs  $(s), (a, b)$  of real numbers such that  $(a + ib)^{2008} = a - ib$  holds good is

- A. 2008

B. 2009

C. 2010

D. 1

**Answer: C**



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**24.** The region represented by the inequality  $|2z-3i| < |3z-2i|$  is

A. the unit disc with its centre at  $z = 0$

B. the exterior of the unit circle with its centre at  $z = 0$

C. the interior of a square of side 2 units with its centre at  $z = 0$

D. none of these

**Answer: B**



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25. If  $\omega$  is any complex number such that  $z\omega = |z|^2$  and  $|z - \bar{z}| + |\omega + \bar{\omega}| = 4$ , then as  $\omega$  varies, then the area bounded by the locus of  $z$  is

- A. 4 sq. units
- B. 8 sq. units
- C. 16 sq. units
- D. 12 sq. units

**Answer: B**

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26. If  $az^2 + bz + 1 = 0$ , where  $a, b \in C$ ,  $|a| = \frac{1}{2}$  and have a root  $\alpha$  such that  $|\alpha| = 1$  then  $|a\bar{b} - b| =$

- A.  $1/4$
- B.  $1/2$

C.  $5/4$

D.  $3/4$

**Answer: D**



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27. Let  $p$  and  $q$  are complex numbers such that  $|p| + |q| < 1$ . If  $z_1$  and  $z_2$  are the roots of the  $z^2 + pz + q = 0$ , then which one of the following is correct ?

A.  $|z_1| < 1$  and  $|z_2| < 1$

B.  $|z_1| > 1$  and  $|z_2| > 1$

C. If  $|z_1| < 1$ , then  $|z_2| > 1$  and vice versa

D. Nothing definite can be said

**Answer: A**



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28. If  $z$  and  $w$  are two complex numbers simultaneously satisfying the equations,  $z^3 + w^5 = 0$  and  $z^2 + \bar{w}^4 = 1$ , then

- A.  $z$  and  $w$  both are purely real
- B.  $z$  is purely real and  $w$  is purely imaginary
- C.  $w$  is purely real and  $z$  is purely imaginary
- D.  $z$  and  $w$  both are imaginary

**Answer: A**



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29. All complex numbers ' $z$ ' which satisfy the relation

$|z - |z + 1|| = |z + |z - 1||$  on the complex plane lie on the

- A.  $y = x$
- B.  $y = -x$

C. circle  $x^2 + y^2 = 1$

D. line  $x = 0$  or on a line segment joining  $(-1, 0) \rightarrow (1, 0)$

**Answer: D**



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30. If  $z_1, z_2$  are two complex numbers such that  $\left| \frac{z_1 - z_2}{z_1 + z_2} \right| = 1$  and  $iz_1 = Kz_2$ , where  $K \in R$ , then the angle between  $z_1 - z_2$  and  $z_1 + z_2$  is

A.  $\tan^{-1} \left( \frac{2K}{K^2 + 1} \right)$

B.  $\tan^{-1} \left( \frac{2K}{1 - K^2} \right)$

C.  $-2 \tan^{-1} K$

D.  $2 \tan^{-1} K$

**Answer: D**



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31. If  $z + \frac{1}{z} = 2\cos 6^\circ$ , then  $z^{1000} + \frac{1}{z^{1000}} + 1$  is equal to

A. 0

B. 1

C. -1

D. 2

**Answer: A**



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32. Let  $z_1$  and  $z_2$  be two complex numbers with  $\alpha$  and  $\beta$  as their principal arguments such that  $\alpha + \beta$  then principal  $\arg(z_1 z_2)$  is given by:

A.  $\alpha + \beta + \pi$

B.  $\alpha + \beta - \pi$

C.  $\alpha + \beta - 2\pi$

D.  $\alpha + \beta$

**Answer: C**



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33. Let  $\arg(z_k) = \frac{(2k + 1)\pi}{n}$  where  $k = 1, 2, \dots, n$ . If  $\arg(z_1, z_2, z_3, \dots, z_n) = \pi$ , then  $n$  must be of form ( $m \in \mathbb{Z}$ )

A.  $4m$

B.  $2m - 1$

C.  $2m$

D. None of these

**Answer: B**



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34. Suppose two complex numbers  $z = a + ib$ ,  $w = c + id$  satisfy the equation  $\frac{z + w}{z} = \frac{w}{z + w}$ . Then

- A. both  $a$  and  $c$  are zeros
- B. both  $b$  and  $d$  are zeros
- C. both  $b$  and  $d$  must be non zeros
- D. at least one of  $b$  and  $d$  is non zero

**Answer: D**



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35. If  $|z| = 1$  and  $z \neq \pm 1$ , then one of the possible value of  $arg(z) - arg(z + 1) - arg(z - 1)$ , is

- A.  $-\pi/6$
- B.  $\pi/3$
- C.  $-\pi/2$
- D.  $\pi/4$

**Answer: C**

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36. If  $\arg(z^{3/8}) = \frac{1}{2}\arg(z^2 + \bar{z}^{1/2})$ , then which of the following is not possible ?

A.  $|z| = 1$

B.  $z = \bar{z}$

C.  $\arg(z) = 0$

D. None of these

**Answer: D**

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37.  $z_1, z_2$  are two distinct points in complex plane such that  $2|z_1| = 3|z_2|$

and  $z \in C$  be any point  $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$  such that

A.  $-1 \leq \operatorname{Re} z \leq 1$



B.  $-2 \leq \operatorname{Re} z \leq 2$

C.  $-3 \leq \operatorname{Re} z \leq 3$

D. None of these

**Answer: B**



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38. If  $\alpha, \beta, \gamma \in \{1, \omega, \omega^2\}$  (where  $\omega$  and  $\omega^2$  are imaginary cube roots of unity), then number of triplets  $(\alpha, \beta, \gamma)$  such that  $\left| \frac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha} \right| = 1$  is

A. 3

B. 6

C. 9

D. 12

**Answer: C**

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39. The value of  $\left(3\sqrt{3} + \left(3^{5/6}\right)i\right)^3$  is (where  $i = \sqrt{-1}$ )

- A. 24
- B. - 24
- C. - 22
- D. - 21

**Answer: B**

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40. If  $\omega \neq 1$  is a cube root of unity and  $a + b = 21$ ,  $a^3 + b^3 = 105$ , then the value of  $(a\omega^2 + b\omega)(a\omega + b\omega^2)$  is be equal to

- A. 3
- B. 5

C. 7

D. 35

**Answer: B**



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41. If  $z = \frac{1}{2}(\sqrt{3} - i)$ , then the least possible integral value of  $m$  such that  $(z^{101} + i^{109})^{106} = z^{m+1}$  is

A. 11

B. 7

C. 8

D. 9

**Answer: D**



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42. If  $y_1 = \max ||z - \omega| - |z - \omega^2| |$ , where  $|z| = 2$  and  $y_2 = \max ||z - \omega| - |z - \omega^2| |$ , where  $|z| = \frac{1}{2}$  and  $\omega$  and  $\omega^2$  are complex cube roots of unity, then

A.  $y_1 = \sqrt{3}, y_2 = \sqrt{3}$

B.  $y_1 < \sqrt{3}, y_2 = \sqrt{3}$

C.  $y_1 = \sqrt{3}, y_2 < \sqrt{3}$

D.  $y_1 > 3, y_2 < \sqrt{3}$

**Answer: C**



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43. Let  $1, \omega$  and  $\omega^2$  be the cube roots of unity. The least possible degree of a polynomial, with real coefficients having  $2\omega^2, 3 + 4\omega, 3 + 4\omega^2$  and  $5 - \omega - \omega^2$  as roots is -

A. 4

B. 5

C. 6

D. 7

**Answer: B**



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**44.** Number of imaginary complex numbers satisfying the equation,

$$z^2 = \bar{z}2^{1-|z|} \text{ is}$$

A. 0

B. 1

C. 2

D. 3

**Answer: C**



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45. Least positive argument of the 4th root of the complex number

$$2 - i\sqrt{12}$$
 is

A.  $\pi/6$

B.  $5\pi/12$

C.  $7\pi/12$

D.  $11\pi/12$

**Answer: B**



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46. A root of unity is a complex number that is a solution to the equation,

$z^n = 1$  for some positive integer  $n$  Number of roots of unity that are also

the roots of the equation  $z^2 + az + b = 0$ , for some integer  $a$  and  $b$  is

A. 6

B. 8

C. 9

D. 10

**Answer: B**



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47. If  $z$  is a complex number satisfying the equation  $z^6 + z^3 + 1 = 0$ . If this equation has a root  $re^{i\theta}$  with  $90^\circ < \theta < 180^\circ$  then the value of  $\theta$  is

A.  $100^\circ$

B.  $110^\circ$

C.  $160^\circ$

D.  $170^\circ$

**Answer: C**



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48. Suppose  $A$  is a complex number and  $n \in \mathbb{N}$ , such that  $A^n = (A + 1)^n = 1$ , then the least value of  $n$  is 3 b. 6 c. 9 d. 12

A. 3

B. 6

C. 9

D. 12

**Answer: B**



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49. If  $z_1, z_2, z_3, \dots, z_n$  are in  $G.P$  with first term as unity such that  $z_1 + z_2 + z_3 + \dots + z_n = 0$ . Now if  $z_1, z_2, z_3, \dots, z_n$  represents the vertices of  $n$ -polygon, then the distance between incentre and circumcentre of the polygon is



A. 0

B.  $|z_1|$

C.  $2|z_1|$

D. none of these

**Answer: A**



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50. If  $|z - 1 - i| = 1$ , then the locus of a point represented by the complex number  $5(z - i) - 6$  is

A. circle with centre  $(1, 0)$  and radius 3

B. circle with centre  $(-1, 0)$  and radius 5

C. line passing through origin

D. line passing through  $(-1, 0)$

**Answer: B**



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51. Let  $z \in \mathbb{C}$  and if  $A = \left\{ z : \arg(z) = \frac{\pi}{4} \right\}$  and  $B = \left\{ z : \arg(z - 3 - 3i) = \frac{2\pi}{3} \right\}$ . Then  $n(A \cap B) =$

A. 1

B. 2

C. 3

D. 0

**Answer: D**



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52.  $\theta \in [0, 2\pi]$  and  $z_1, z_2, z_3$  are three complex numbers such that they are collinear and  $(1 + |\sin \theta|)z_1 + (|\cos \theta| - 1)z_2 - \sqrt{2}z_3 = 0$ . If at least one of the complex numbers  $z_1, z_2, z_3$  is nonzero, then number of possible values of  $\theta$  is

A. Infinite

B. 4

C. 2

D. 8

**Answer: B**



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**53.** Let ' $z$ ' be a complex number and ' $a$ ' be a real parameter such that

$z^2 + az + a^2 = 0$ , then which of the following is not true ?

A. locus of  $z$  is a pair of straight lines

B.  $|z| = |a|$

C.  $\arg(z) = \pm \frac{2\pi}{3}$

D. None of these

**Answer: D**



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54. Let  $z = x + iy$  then locus of moving point  $P(z) \frac{1 + \bar{z}}{z} \in R$ , is

A. union of lines with equations  $x = 0$  and  $y = -1/2$  but excluding origin.

B. union of lines with equations  $x = 0$  and  $y = 1/2$  but excluding origin.

C. union of lines with equations  $x = -1/2$  and  $y = 0$  but excluding origin.

D. union of lines with equations  $x = 1/2$  and  $y = 0$  but excluding origin.

**Answer: C**



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55. Let  $A(z_1)$  and  $B(z_2)$  are two distinct non-real complex numbers in the argand plane such that  $\frac{z_1}{z_2} + \frac{\bar{z}_1}{z_2} = 2$ . The value of  $|\angle ABO|$  is

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{4}$

C.  $\frac{\pi}{2}$

D. None of these

**Answer: C**



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56. Complex numbers  $z_1$  and  $z_2$  satisfy  $|z_1| = 2$  and  $|z_2| = 3$ . If the included angle of their corresponding vectors is  $60^\circ$ , then the value of

19  $\left| \frac{z_1 - z_2}{z_1 + z_2} \right|^2$  is

A. 5

B. 6

C. 7

D. 8

**Answer: C**



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57. Let  $A(2, 0)$  and  $B(z)$  are two points on the circle  $|z| = 2$ .  $M(z')$  is the point on  $AB$ . If the point  $\bar{z}'$  lies on the median of the triangle  $OAB$  where  $O$  is origin, then  $\arg(z')$  is

A.  $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$

B.  $\tan^{-1}(\sqrt{15})$

C.  $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$

D.  $\frac{\pi}{2}$

**Answer: A**



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58. If  $A(z_1)$ ,  $B(z_2)$ ,  $C(z_3)$  are vertices of a triangle such that  $z_3 = \frac{z_2 - iz_1}{1 - i}$  and  $|z_1| = 3$ ,  $|z_2| = 4$  and  $|z_2 + iz_1| = |z_1| + |z_2|$ , then area of triangle  $ABC$  is

A.  $\frac{5}{2}$

B. 0

C.  $\frac{25}{2}$

D.  $\frac{25}{4}$

**Answer: D**



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59. Let  $O$ ,  $A$ ,  $B$  be three collinear points such that  $OA \cdot OB = 1$ . If  $O$  and  $B$  represent the complex numbers  $O$  and  $z$ , then  $A$  represents

A.  $\frac{1}{\bar{z}}$

B.  $\frac{1}{z}$

C.  $\bar{z}$

D.  $z^2$

**Answer: A**



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60. If the tangents at  $z_1, z_2$  on the circle  $|z - z_0| = r$  intersect at  $z_3$ , then

$\frac{(z_3 - z_1)(z_0 - z_2)}{(z_0 - z_1)(z_3 - z_2)}$  equals

A. 1

B.  $-1$

C.  $i$

D.  $-i$

**Answer: B**



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61. If  $z_1, z_2$  and  $z_3$  are the vertices of  $\Delta ABC$ , which is not right angled triangle taken in anti-clock wise direction and  $z_0$  is the circumcentre, then

$$\left(\frac{z_0 - z_1}{z_0 - z_2}\right) \frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right) \frac{\sin 2C}{\sin 2B} \text{ is equal to}$$

A. 0

B. 1

C. -1

D. 2

**Answer: C**



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62. Let  $P$  denotes a complex number  $z = r(\cos \theta + i \sin \theta)$  on the Argand's plane, and  $Q$  denotes a complex number  $\sqrt{2|z|^2} \left( \cos \left( \theta + \frac{\pi}{4} \right) + i \sin \left( \theta + \frac{\pi}{4} \right) \right)$ . If ' $O$ ' is the origin, then  $\Delta OPQ$  is

A. isosceles but not right angled

B. right angled but not isosceles

C. right isosceles

D. equilateral

**Answer: C**



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## Multiple Correct Answer

1. Complex numbers whose real and imaginary parts  $x$  and  $y$  are integers

and satisfy the equation  $3x^2 - |xy| - 2y^2 + 7 = 0$

A. do not exist

B. exist and have equal modulus

C. form two conjugate pairs

D. do not form conjugate pairs

**Answer: B::C**



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2. If  $a, b, c, d \in \mathbb{R}$  and all the three roots of  $az^3 + bz^2 + cZ + d = 0$  have negative real parts, then

A.  $ab > 0$

B.  $bc > 0$

C.  $ad > 0$

D.  $bc - ad > 0$

**Answer: A::B::C**



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3. Suppose three real numbers  $a, b, c$  are in *G. P.* Let  $z = \frac{a + ib}{c - ib}$ . Then

A.  $z = \frac{ib}{c}$

B.  $z = \frac{ia}{b}$

C.  $z = \frac{ia}{c}$

D.  $z = 0$

**Answer: A::B**



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4.  $w_1, w_2$  be roots of  $(a + \bar{c})z^2 + (b + \bar{b})z + (\bar{a} + c) = 0$ . If  $|z_1| < 1$ ,

$|z_2| < 1$ , then

A.  $|w_1| < 1$

B.  $|w_1| = 1$

C.  $|w_2| < 1$

D.  $|w_2| = 1$

**Answer: B::D**

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5. A complex number  $z$  satisfies the equation  $|z^2 - 9| + |z^2| = 41$ , then the true statements among the following are

A.  $|Z + 3| + |Z - 3| = 10$

B.  $|Z + 3| + |Z - 3| = 8$

C. Maximum value of  $|Z|$  is 5

D. Maximum value of  $|Z|$  is 6

**Answer: A::C**

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6. Let  $a, b, c$  be distinct complex numbers with  $|a| = |b| = |c| = 1$  and  $z_1, z_2$  be the roots of the equation  $az^2 + bz + c = 0$  with  $|z_1| = 1$ . Let  $P$  and  $Q$  represent the complex numbers  $z_1$  and  $z_2$  in the Argand plane with  $\angle POQ = \theta, 0^\circ < 180^\circ$  (where  $O$  being the origin). Then

$$A. b^2 = ac, \theta = \frac{2\pi}{3}$$

$$B. \theta = \frac{2\pi}{3}, PQ = \sqrt{3}$$

$$C. PQ = 2\sqrt{3}, b^2 = ac$$

$$D. \theta = \frac{\pi}{3}, b^2 = ac$$

**Answer: A:B**



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7. Let  $Z_1 = x_1 + iy_1$ ,  $Z_2 = x_2 + iy_2$  be complex numbers in fourth quadrant of argand plane and  $|Z_1| = |Z_2| = 1$ ,  $Re f(Z_1 Z_2) = 0$ . The complex numbers  $Z_3 = x_1 + ix_2$ ,  $Z_4 = y_1 + iy_2$ ,  $Z_5 = x_1 + iy_2$ ,  $Z_6 = x_6 + iy$ , will always satisfy

$$A. |Z_4| = 1$$

$$B. arg(Z_1 Z_4) = -\pi/2$$

$$C. \frac{Z_5}{\cos(arg Z_1)} + \frac{Z_6}{\sin(arg Z_1)} \text{ is purely real}$$

$$D. Z_5^2 + (\bar{Z}_6)^2 \text{ is purely imaginary}$$

**Answer: A::B::C::D**



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8. If the imaginary part of  $\frac{z-3}{e^{i\theta}} + \frac{e^{i\theta}}{z-3}$  is zero, then  $z$  can lie on

- A. a circle with unit radius
- B. a circle with radius 3 units
- C. a straight line through the point (3, 0)
- D. a parabola with the vertex (3, 0)

**Answer: A::C**



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9. If  $\alpha$  is the fifth root of unity, then :

A.  $|1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4| = 0$

B.  $|1 + \alpha + \alpha^2 + \alpha^3| = 1$

C.  $|1 + \alpha + \alpha^2| = 2 \cos \frac{\pi}{5}$

D.  $|1 + \alpha| = 2 \cos \frac{\pi}{10}$

**Answer: A::B::C**



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10. If  $z_1, z_2, z_3$  are any three roots of the equation  $z^6 = (z + 1)^6$ , then  $\arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right)$  can be equal to

A. 0

B.  $\pi$

C.  $\frac{\pi}{4}$

D.  $-\frac{\pi}{4}$

**Answer: A::B**



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11. Let  $z_1, z_2, z_3$  are the vertices of  $\Delta ABC$ , respectively, such that  $\frac{z_3 - z_2}{z_1 - z_2}$  is purely imaginary number. A square on side  $AC$  is drawn outwardly.  $P(z_4)$  is the centre of square, then

A.  $|z_1 - z_2| = |z_2 - z_4|$

B.  $\arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + \arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = +\frac{\pi}{2}$

C.  $\arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + \arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = 0$

D.  $z_1, z_2, z_3$  and  $z_4$  lie on a circle

Answer: C::D

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## Matching Column

1.  $z_1, z_2, z_3$  are vertices of a triangle. Match the condition in List I with type of triangle in List II.

List I		List II	
(p)	$z_1^2 + z_2^2 + z_3^2 = z_2z_3 + z_3z_1 + z_1z_2$	(1)	right angled but not necessarily isosceles
(q)	$\operatorname{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$	(2)	obtuse angled
(r)	$\operatorname{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) < 0$	(3)	isosceles and right angled
(s)	$\frac{z_3 - z_1}{z_3 - z_2} = i$	(4)	equilateral

Codes

A.  $\begin{matrix} p & q & r & s \\ 3 & 2 & 1 & 4 \end{matrix}$

B.  $\begin{matrix} p & q & r & s \\ 1 & 2 & 4 & 3 \end{matrix}$

C.  $\begin{matrix} p & q & r & s \\ 4 & 1 & 2 & 3 \end{matrix}$

D.  $\begin{matrix} p & q & r & s \\ 2 & 1 & 4 & 3 \end{matrix}$

Answer: C



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1. Consider the region  $R$  in the Argand plane described by the complex number  $Z$  satisfying the inequalities  $|Z - 2| \leq |Z - 4|$ ,  $|Z - 3| \leq |Z + 3|$ ,  $|Z - i| \leq |Z - 3i|$ ,  $|Z + i| \leq |Z + 3i|$

Answer the following questions :

The maximum value of  $|Z|$  for any  $Z$  in  $R$  is

A. 5

B. 3

C. 1

D.  $\sqrt{13}$

**Answer: D**



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2. Consider the region  $R$  in the Argand plane described by the complex number  $Z$  satisfying the inequalities  $|Z - 2| \leq |Z - 4|$ ,  $|Z - 3| \leq |Z + 3|$ ,  $|Z - i| \leq |Z - 3i|$ ,  $|Z + i| \leq |Z + 3i|$

Answer the following questions :

The maximum value of  $|Z|$  for any  $Z$  in  $R$  is

- A. 5
- B. 14
- C.  $\sqrt{13}$
- D. 12

**Answer: A**



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3. Consider the region  $R$  in the Argand plane described by the complex number  $Z$  satisfying the inequalities  $|Z - 2| \leq |Z - 4|$ ,  $|Z - 3| \leq |Z + 3|$ ,  $|Z - i| \leq |Z - 3i|$ ,  $|Z + i| \leq |Z + 3i|$

Answer the following questions :

Minimum of  $|Z_1 - Z_2|$  given that  $Z_1, Z_2$  are any two complex numbers lying in the region  $R$  is

A. 0

B. 5

C.  $\sqrt{13}$

D. 3

**Answer: A**



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4. Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1^2 - 4z_2 = 16 + 20i$  and the roots  $\alpha$  and  $\beta$  of  $x^2 + z_1x + z_2 + m = 0$  for some complex number  $m$  satisfies  $|\alpha - \beta| = 2\sqrt{7}$ .

The locus of the complex number  $m$  is a curve

A. straight line

B. circle

C. ellipse

D. hyperbola

**Answer: B**



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5. Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1^2 - 4z_2 = 16 + 20i$  and the roots  $\alpha$  and  $\beta$  of  $x^2 + z_1x + z_2 + m = 0$  for some complex number  $m$  satisfies  $|\alpha - \beta| = 2\sqrt{7}$ .

The maximum value of  $|m|$  is

A. 14

B.  $2\sqrt{7}$

C.  $7 + \sqrt{41}$

D.  $2\sqrt{6} - 4$

**Answer: C**



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6. Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1^2 - 4z_2 = 16 + 20i$  and the roots  $\alpha$  and  $\beta$  of  $x^2 + z_1x + z_2 + m = 0$  for some complex number  $m$  satisfies  $|\alpha - \beta| = 2\sqrt{7}$ . The value of  $|m|$ , when  $\arg(m)$  is maximum

A. 7

B.  $28 - \sqrt{41}$

C.  $\sqrt{41}$

D.  $2\sqrt{6} - 4$

**Answer: D**



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7. The locus of any point  $P(z)$  on argand plane is  $\arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$ .

Then the length of the arc described by the locus of  $P(z)$  is

A.  $10\sqrt{2}\pi$

B.  $\frac{15\pi}{\sqrt{2}}$

C.  $\frac{5\pi}{\sqrt{2}}$

D.  $5\sqrt{2}\pi$

**Answer: B**



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8. The locus of any point  $P(z)$  on argand plane is  $\arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$ .

Total number of integral points inside the region bounded by the locus of

$P(z)$  and imaginary axis on the argand plane is

A. 62

B. 74



C. 136

D. 138

**Answer: C**



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9. The locus of any point  $P(z)$  on argand plane is  $arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$ .

Area of the region bounded by the locus of a complex number  $Z$

satisfying  $arg\left(\frac{z + 5i}{z - 5i}\right) = \pm \frac{\pi}{4}$

A.  $75\pi + 50$

B.  $75\pi$

C.  $\frac{75\pi}{2} + 25$

D.  $\frac{75\pi}{2}$

**Answer: A**



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10. A person walks  $2\sqrt{2}$  units away from origin in south west direction ( $S45^\circ W$ ) to reach  $A$ , then walks  $\sqrt{2}$  units in south east direction ( $S45^\circ E$ ) to reach  $B$ . From  $B$  he travel is 4 units horizontally towards east to reach  $C$ . Then he travels along a circular path with centre at origin through an angle of  $2\pi/3$  in anti-clockwise direction to reach his destination  $D$ .

Let the complex number  $Z$  represents  $C$  in argand plane. then  $\arg(Z) =$

A.  $-\frac{\pi}{6}$

B.  $\frac{\pi}{4}$

C.  $-\frac{\pi}{4}$

D.  $\frac{\pi}{3}$

**Answer: C**



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11. A person walks  $2\sqrt{2}$  units away from origin in south west direction ( $S45^\circ W$ ) to reach  $A$ , then walks  $\sqrt{2}$  units in south east direction ( $S45^\circ E$ ) to reach  $B$ . From  $B$  he travel is 4 units horizontally towards east to reach  $C$ . Then he travels along a circular path with centre at origin through an angle of  $2\pi/3$  in anti-clockwise direction to reach his destination  $D$ .

Position of  $D$  in argand plane is ( $w$  is an imaginary cube root of unity)

A.  $(3 + i)\omega$

B.  $-(1 + i)\omega^2$

C.  $3(1 - i)\omega$

D.  $(1 - 3i)\omega$

**Answer: C**



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1. Evaluate :

(i)  $i^{135}$

(ii)  $i^{\frac{1}{47}}$

(iii)  $(-\sqrt{-1})^{4n+3}, n \in N$

(iv)  $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$



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2. Find the value of  $i^n + i^{n+1} + i^{n+2} + i^{n+3}$  for all  $n \in N$ .

A. 0

B.  $i$

C.  $-i$

D.  $2i^n$

**Answer: A**



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3. Find the value of  $1 + i^2 + i^4 + i^6 + \dots + i^{2n}$



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4. Show that the polynomial  $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$  is divisible by  $x^3 + x^2 + x + 1$ , where  $p, q, r, s \in n$ .



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5. Solve:

$$ix^2 - 3x - 2i = 0,$$



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6. If  $z = 4 + i\sqrt{7}$ , then find the value of  $z^3 - 4z^2 - 9z + 91$ .

A. 23

B.  $i$

C.  $-1$

D.  $0$

**Answer: C**



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7. Express each of the following in the standard form  $a + ib$

(i)  $\frac{5 + 4i}{4 + 5i}$  (ii)  $\frac{(1 + i)^2}{3 - i}$  (iii)  $\frac{1}{1 - \cos \theta + 2i \sin \theta}$



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8. The root of the equation  $2(1 + i)x^2 - 4(2 - i)x - 5 - 3i = 0$ , where

$i = \sqrt{-1}$ , which has greater modulus is



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9. Find the value of  $(1 + i)^6 + (1 - i)^6$

A.  $16i$

B. 0

C.  $-16i$

D. 1

**Answer: B**



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10. If  $\left(\frac{1+i}{1-i}\right)^m = 1$ , then find the least positive integral value of  $m$ .



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11. Prove that the triangle formed by the points  $1$ ,  $\frac{1+i}{\sqrt{2}}$ , and  $i$  as vertices in the Argand diagram is isosceles.



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12. Find the value of  $\theta$  if  $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$  is purely real or purely imaginary.

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13. If the imaginary part of  $(2z + 1) / (iz + 1)$  is -2, then find the locus of the point representing in the complex plane.

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14. If  $z$  is a complex number such that  $|z - \bar{z}| + |z + \bar{z}| = 4$  then find the area bounded by the locus of  $z$ .

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15. If  $(x + iy)^5 = p + iq$ , then prove that  $(y + ix)^5 = q + ip$ .

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16. If  $z = x + iy$  lies in the third quadrant, then prove that  $\frac{\bar{z}}{z}$  also lies in the third quadrant when  $y < x < 0$

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17. Prove that  $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$  is purely real.

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18. Find the relation if  $z_1, z_2, z_3, z_4$  are the affixes of the vertices of a parallelogram taken in order.

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19. Let  $z_1, z_2, z_3$  be three complex numbers and  $a, b, c$  be real numbers not all zero, such that  $a + b + c = 0$  and  $az_1 + bz_2 + cz_3 = 0$ . Show that

$z_1, z_2, z_3$  are collinear.

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20. Find real values of  $x$  and  $y$  for which the complex numbers  $-3 + ix^2y$  and  $x^2 + y + 4i$  are conjugate of each other.

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21. Given that  $x, y \in R$ . Solve: 
$$\frac{x}{1 + 2i} + \frac{y}{3 + 2i} = \frac{5 + 6i}{8i - 1}$$

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22. If  $(x + iy)^3 = u + iv$ , then show that  $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$ .

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**23.** Let  $z$  be a complex number satisfying the equation  $z^3 - (3 + i)z + m + 2i = 0$ , where  $m \in \mathbb{R}$ . Suppose the equation has a real root. Then root non-real root.

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**24.** Show that the equation  $Z^4 + 2Z^3 + 3Z^2 + 4Z + 5 = 0$  has no root which is either purely real or purely imaginary.

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**25.** Find the square roots of the following:

(i)  $7 - 24i$  (ii)  $5 + 12i$

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**26.** Find all possible values of  $\sqrt{i} + \sqrt{-i}$ .

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27. Solve the following for  $z$ :  $z^2 - (3 - 2i)z = (5i - 5)$

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28. Solve the equation  $(x - 1)^3 + 8 = 0$  in the set  $\mathbb{C}$  of all complex numbers.

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29. If  $n$  is an odd integer that is greater than or equal to 3 but not a multiple of 3, then prove that  $(x + 1)^n = x^n - 1$  is divisible by  $x^3 + x^2 + x$ .

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30.  $\omega$  is an imaginary root of unity.

Prove that

(i)

$$(a + b\omega + c\omega^2)^3 + (a + b\omega^2 + c\omega)^3 = (2a - b - c)(2b - a - c)(2c - a - b)$$

(ii) If  $a + b + c = 0$  then prove that

$$(a + b\omega + c\omega^2)^3 + (a + b\omega^2 + c\omega)^3 = 27abc.$$



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31. Find the complex number  $\omega$  satisfying the equation  $z^3 - 8i$  and lying in the second quadrant on the complex plane.



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32.  $\frac{1}{a + \omega} + \frac{1}{b + \omega} + \frac{1}{c + \omega} + \frac{1}{d + \omega} = \frac{1}{\omega}$  where,  $a, b, c, d, \in \mathbb{R}$  and  $\omega$  is a complex cube root of unity then find the value of  $\sum \frac{1}{a^2 - a + 1}$



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**33.** Write the following complex number in polar form :

(i)  $-3\sqrt{2} + 3\sqrt{2}i$

(ii)  $1 + i$

(iii)  $\frac{1 + 7i}{(2 - i)^2}$

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**34.** Let  $z_1 = \cos 12^\circ + i \sin 12^\circ$  and  $z_2 = \cos 48^\circ + i \sin 48^\circ$ . Write complex number  $(z_1 + z_2)$  in polar form. Find its modulus and argument.

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**35.** Convert the complex number  $z = 1 + \frac{\cos(8\pi)}{5} + i \frac{\sin(8\pi)}{5}$  in polar form. Find its modulus and argument.

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36.  $z$  and  $\omega$  are two nonzero complex number such that  $|z| = |\omega|$  and  $Argz + Arg\omega = \pi$  then  $z$  equals

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37. Find the numbers of non-zero integral solutions of the equation

$$|1 - i|^x = 2^x$$

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38. Let  $z$  be a complex number satisfying  $|z| = 3|z - 1|$ . Then prove that

$$\left| z - \frac{9}{8} \right| = \frac{3}{8}$$

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39. If complex number  $z=x +iy$  satisfies the equation  $Re(z + 1) = |z - 1|$ , then prove that  $z$  lies on  $y^2 = 4x$ .



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40. Solve the equation  $|z| = z + 1 + 2i$



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41. Find the range of real number  $\alpha$  for which the equation  $z + \alpha|z - 1| + 2i = 0$  has a solution.



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42. Find the Area bounded by complex numbers  $\arg|z| \leq \frac{\pi}{4}$  and  $|z - 1| < |z - 3|$



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43. Prove that triangle by complex numbers  $z_1, z_2$  and  $z_3$  is equilateral if

$$|z_1| = |z_2| = |z_3| \text{ and } z_1 + z_2 + z_3 = 0$$

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44. Show that  $e^{2mi\theta} \left( \frac{i \cot \theta + 1}{i \cot \theta - 1} \right)^m = 1$ .

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45.  $Z_1 \neq Z_2$  are two points in an Argand plane. If  $a|Z_1| = b|Z_2|$ , then prove that  $\frac{aZ_1 - bZ_2}{aZ_1 + bZ_2}$  is purely imaginary.

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46. Find the real part of  $(1 - i)^{-i}$ .

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47. If  $(\sqrt{8} + i)^{50} = 3^{49}(a + ib)$ , then find the value of  $a^2 + b^2$ .

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48. Show that  $(x^2 + y^2)^4 = (x^4 - 6x^2y^2 + y^4)^2 + (4x^3y - 4xy^3)^2$ .

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49. If  $\arg(z_1) = 170^\circ$  and  $\arg(z_2) = 70^\circ$ , then find the principal argument of  $z_1 z_2$ .

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50. The value of  $\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \left(\cos \left(\frac{\pi}{2^2}\right) + i \sin \left(\frac{\pi}{2^2}\right)\right) \left(\cos \left(\frac{\pi}{2^3}\right) + i \sin \left(\frac{\pi}{2^3}\right)\right) \dots \infty$  is

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51. Find the principal argument of the complex number

$$\frac{(1+i)^5(1+\sqrt{3}i)^2}{-1i(-\sqrt{3}+i)}$$

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52. If  $z = \frac{(\sqrt{3} + i)^{17}}{(1 - i)^{50}}$ , then find  $\text{amp}(z)$ .

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53. If  $z = x + iy$  and  $w = (1 - iz)/(z - i)$ , then show that  $|w| = 1$  is purely real.

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54. It is given the complex numbers  $z_1$  and  $z_2$ ,  $|z_1| = 2$  and  $|z_2| = 3$ . If the included angle of their corresponding vectors is  $60^\circ$ , then find value of  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right|$



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55. Solve the equation  $z^3 = \bar{z}$  ( $z \neq 0$ )



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56. If  $2z_1/3z_2$  is a purely imaginary number, then find the value of  $|(z_1 - z_2)/(z_1 + z_2)|$ .



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57. Find the complex number satisfying the system of equations  $z^3 + \omega^7 = 0$  and  $z^5 \omega^{11} = 1$ .

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58. Express the following in  $a + ib$  form:

(i)  $\left( \frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta} \right)^4$

(ii)  $\frac{(\cos 2\theta - i \sin 2\theta)^4 (\cos 4\theta + i \sin 4\theta)^{-5}}{(\cos 3\theta + i \sin 3\theta)^{-2} (\cos 3\theta - i \sin 3\theta)^{-9}}$

(iii)  $\frac{(\sin \pi/8 + i \cos \pi/8)^8}{(\sin \pi/8 - i \cos \pi/8)^8}$

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59. If  $z = \left( \frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left( \frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5$ , then prove that  $Im(z) = 0$

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60. Prove that the roots of the equation  $x^4 - 2x^2 + 4 = 0$  forms a rectangle.

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61. If  $z + 1/z = 2 \cos \theta$ , prove that  $\left| \frac{z^{2n} - 1}{z^{2n} + 1} \right| = |\tan n\theta|$

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62. If  $z = x + iy$  is a complex number with  $x, y \in \mathbb{Q}$  and  $|z| = 1$ , then show that  $|z^{2n} - 1|$  is a rational number for every  $n \in \mathbb{N}$ .

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63. If  $z = \cos \theta + i \sin \theta$  is a root of the equation  $a_0 z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n = 0$ , then prove that

$$a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \dots + a_n \cos n\theta = 0$$

$$a_1 \sin \theta + a_2 \sin^2 \theta + \dots + a_n \sin n\theta = 0$$

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64. If  $|z_1| = 1$ ,  $|z_2| = 2$ ,  $|z_3| = 3$ , and  $|9z_1z_2 + 4z_1z_3 + z_2z_3 + 3| = 12$ , then find the value of  $|z_1 + z_2 + z + 3|$ .

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65. If  $\alpha$  and  $\beta$  are complex numbers such that  $|\beta| = 1$ , then  $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| =$

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66. Prove that  $|z_1 + z_2|^2 = |z_1|^2$ , if  $z_1/z_2$  is purely imaginary.

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67. Let  $\left| \frac{(z_1)_1 - 2(z_2)_2}{(2 - z_1(z_2)_2)} \right| = 1$  and  $|z_2| \neq 1$ , where  $z_1$  and  $z_2$  are complex numbers. Show that  $|z_1| = 2$ .

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68. If  $z_1$  and  $z_2$  are two complex numbers and  $c > 0$ , then prove that

$$|z_1 + z_2|^2 \leq (1 + c)|z_1|^2 + (1 + c^{-1})|z_2|^2.$$

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69. If  $z_1, z_2, z_3, z_4$  are the affixes of four points in the Argand plane,  $z$  is the affix of a point such that  $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$ , then  $z_1, z_2, z_3, z_4$  are

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70. if  $|z_1 + z_2| = |z_1| + |z_2|$ , then prove that  $\arg(z_1) = \arg(z_2)$  if  $|z_1 - z_2| = |z_1| + |z_2|$ , then prove that  $\arg(z_1) = \arg(z_2) = \pi$

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71. Show that the area of the triangle on the Argand diagram formed by the complex number  $z$ ,  $iz$  and  $z + iz$  is  $\frac{1}{2}|z|^2$

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72. Find the minimum value of  $|z - 1|$  if  $||z - 3| - |z + 1|| = 2$ .

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73. Find the greatest and the least value of  $|z_1 + z_2|$  if  $z_1 = 24 + 7i$  and  $|z_2| = 6$ .

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74. If  $z$  is a complex number, then find the minimum value of  $|z| + |z - 1| + |2z - 3|$ .

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75. If  $|z_1 - 1| \leq 1$ ,  $|z_2 - 2| \leq 2$ ,  $|z_3| \leq 3$ , then find the greatest value of  $|z_1 + z_2 + z_3|$ .

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76. Prove that following inequalities:

(i)  $\left| \frac{z}{|z|} - 1 \right| \leq |\arg z|$  (ii)  $|z - 1| \leq |z| |\arg z| + |z| - 1$

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77. Identify the locus of  $z$  if  $z = a + \frac{r^2}{z - a}$ ,  $r > 0$ .

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78. If  $z$  is any complex number such that  $|3z - 2| + |3z + 2| = 4$ , then identify the locus of  $z$ .

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79. If  $|z| = 1$  and let  $\omega = \frac{(1 - z)^2}{1 - z^2}$ , then prove that the locus of  $\omega$  is equivalent to  $|z - 2| = |z + 2|$

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80. Let  $z$  be a complex number having the argument  $\theta, 0$

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81. How many solutions the system of equations

$||z + 4| - |z - 3i|| = 5$  and  $|z| = 4$  has ?

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82. Prove that  $|Z - Z_1|^2 + |Z - Z_2|^2 = a$  will represent a real circle [with center  $(|Z_1 + Z_2|/2 + )$ ] on the Argand plane if  $2a \geq |Z_1 - Z_2|^2$



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83. If  $|z - 2 - 3i|^2 + |z - 5 - 7i|^2 = \lambda$  represents the equation of circle with least radius, then find the value of  $\lambda$ .



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84. If  $\frac{|2z - 3|}{|z - i|} = k$  is the equation of circle with complex number 'i' lying inside the circle, find the values of K.



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85. Find the point of intersection of the curves

$$\arg(z - 3i) = \frac{3\pi}{4} \text{ and } \arg(2z + 1 - 2i) = \pi/4.$$

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86. If complex numbers  $z_1, z_2$  and  $z_3$  are such that  $|z_1| = |z_2| = |z_3|$ , then

prove that  $\arg\left(\frac{z_2}{z_1}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)^2$

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87. If the triangle formed by complex numbers  $z_1, z_2$  and  $z_3$  is equilateral

then prove that  $\frac{z_2 + z_3 - 2z_1}{z_3 - z_2}$  is purely imaginary number

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88. Show that the equation of a circle passing through the origin and

having intercepts  $a$  and  $b$  on real and imaginary axis, respectively, on the

argand plane is  $\operatorname{Re}\left(\frac{z - a}{z - ib}\right) = 0$

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89. The triangle formed by  $A(z_1)$ ,  $B(z_2)$  and  $C(z_3)$  has its circumcentre at origin. If the perpendicular from  $A$  to  $BC$  intersects the circumference at  $z_4$  then the value of  $z_1z_4 + z_2z_3$  is

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90. Let vertices of an acute-angled triangle are  $A(z_1)$ ,  $B(z_2)$ , and  $C(z_3)$ . If the origin  $O$  is the orthocentre of the triangle, then prove that  $z_1(z_2)_2 + (z_1)_1z_2 = z_2(z_3)_3 + (z_2)_2z_3 = z_3(z_1)_1 + (z_3)_3z_1$ .

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91. If  $z_1, z_2, z_3$  are three complex numbers such that  $5z_1 - 13z_2 + 8z_3 = 0$ , then prove that  $|z_1(z_1)_1 1z_2(z_2)_2 1z_3(z_3)_3 1| = 0$

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92. If  $z = z_0 + A(z - (z)_0)$ , where  $A$  is a constant, then prove that locus of  $z$  is a straight line.

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93.  $z_1$  and  $z_2$  are the roots of  $3z^2 + 3z + b = 0$ . If  $O(0)$ ,  $(z_1)$ ,  $(z_2)$  form an equilateral triangle, then find the value of  $b$ .

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94. Let  $z_1, z_2$  and  $z_3$  be three complex number such that

$$|z_1 - 1| = |z_2 - 1| = |z_3 - 1| \text{ and } \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \frac{\pi}{6}$$

then prove that  $z_2^3 + z_3^3 + 1 = z_2 + z_3 + z_2z_3$ .

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95. Let the complex numbers  $z_1, z_2$  and  $z_3$  be the vertices of an equilateral triangle. If  $z_0$  is the circumcentre of the triangle, then prove that  $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$ .

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96. In the Argand's plane what is the locus of  $z$  ( $z \neq 1$ ) such that  $\arg\left\{\frac{3}{2}\left(\frac{2z^2 - 5z + 3}{2z^2 - z - 2}\right)\right\} = \frac{2\pi}{3}$ .

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97. If  $\left(\frac{3 - z_1}{2 - z_1}\right)\left(\frac{2 - z_2}{3 - z_2}\right) = k$  ( $k > 0$ ), then prove that points  $A(z_1), B(z_2), C(3)$ , and  $D(2)$  (taken in clockwise sense) are concyclic.

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98. If  $z_1, z_2, z_3$  are complex numbers such that  $(2/z_1) = (1/z_2) + (1/z_3)$ , then show that the points represented by  $z_1, z_2, z_3$  lie on a circle passing through the origin.



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99.  $A(z_1), B(z_2), C(z_3)$  are the vertices of the triangle  $ABC$  (in anticlockwise). If  $\angle ABC = \pi/4$  and  $AB = \sqrt{2}(BC)$ , then prove that  $z_2 = z_3 + i(z_1 - z_3)$ .



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100. If one of the vertices of the square circumscribing the circle  $|z - 1| = \sqrt{2}$  is  $2 + \sqrt{3}i$ . Find the other vertices of square



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**101.** Let  $z_1 = 10 + 6i$  and  $z_2 = 4 + 6i$ . If  $z$  is any complex number such that the argument of  $\frac{(z - z_1)}{(z - z_2)}$  is  $\frac{\pi}{4}$ , then prove that  $|z - 7 - 9i| = 3\sqrt{2}$ .

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**102.** Complex numbers of  $z_1, z_2, z_3$  are the vertices A, B, C respectively, of an isosceles right-angled triangle with right angle at C. Show that  $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$

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**103.** Let  $z_1, z_2$  and  $z_3$  represent the vertices A, B, and C of the triangle ABC, respectively, in the Argand plane, such that  $|z_1| = |z_2| = 5$ . Prove that  $z_1 \sin 2A + z_2 \sin 2B + z_3 \sin 2C = 0$ .

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104. If  $a = \cos(2\pi/7) + is \in (2\pi/7)$ , then find the quadratic equation whose roots are  $\alpha = a + a^2 + a^4$  and  $\beta = a^3 + a^5 + a^7$ .

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105. If  $\omega$  is an imaginary fifth root of unity, then find the value of  $\log_2 |1 + \omega + \omega^2 + \omega^3 - 1/\omega|$ .

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106. If  $1, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_8$  are ninth roots of unity (taken in counter-clockwise sequence in the Argand plane). Then find the value of  $|(2 - \alpha_1)(2 - \alpha_3)(2 - \alpha_5)(2 - \alpha_7)|$ .

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107. find the sum of squares of all roots of the equation.

$$x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$$



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108. Find roots of the equation  $(z + 1)^5 = (z - 1)^5$ .



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109. If the roots of  $(z - 1)^n = i(z + 1)^n$  are plotted in the Argand plane, then prove that they are collinear.



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110. Let  $1, z_1, z_2, z_3, \dots, z_{n-1}$  be the  $n$ th roots of unity. Then prove that

$(1 - z_1)(1 - z_2) \dots (1 - z_{n-1}) = n$ . Also, deduce that

$$\sin \frac{\pi}{n} \sin \frac{2\pi}{n} \sin \frac{3\pi}{n} \dots \sin \frac{(n-1)\pi}{n} = \frac{\pi}{2^{n-1}}$$



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111. If  $\omega$  and  $\omega^2$  are the nonreal cube roots of unity and  $[1/(a + \omega)] + [1/(b + \omega)] + [1/(c + \omega)] = 2\omega^2$  and  $[1/(a + \omega)^2] + [1/(b + \omega)^2] + [1/(c + \omega)^2] = 2\omega$ , then find the value of  $[1/(a + 1)] + [1/(b + 1)] + [1/(c + 1)]$ .

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112. If  $z_1$  and  $z_2$  are complex numbers and  $u = \sqrt{z_1 z_2}$ , then prove that

$$|z_1| + |z_2| = \left| \frac{z_1 + z_2}{2} + u \right| + \left| \frac{z_1 + z_2}{2} - u \right|$$

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113. If  $a$  is a complex number such that  $|a| = 1$ , then find the value of  $a$ , so that equation  $az^2 + z + 1 = 0$  has one purely imaginary root.

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**114.** Let  $z$  and  $z_0$  be two complex numbers. It is given that  $|z| = 1$  and that numbers  $z, z_0, z\bar{z}_0 - 1$ , and  $0$  are represented in a Argand diagram by the points  $P, P_0, Q, A$  and the origin respectively. Show that the triangles  $POP_0$  and  $AOQ$  are congruent. Hence, or otherwise, prove that

$$|z - z_0| = |z\bar{z}_0 - 1|$$



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**115.** Let  $a, b, \text{ and } c$  be any three nonzero complex number. If  $|z| = 1$  and  $z'$  satisfies the equation  $az^2 + bz + c = 0$ , prove that

$$aa = cc \text{ and } |a||b| = \sqrt{ac(b)^2}$$



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**116.** Let  $x_1, x_2$  are the roots of the quadratic equation  $x^2 + ax + b = 0$ , where  $a, b$ , are complex numbers and  $y_1, y_2$  are the roots of the quadratic equation  $y^2 + |a|yy + |b| = 0$ . If  $|x_1| = |x_2| = 1$ , then prove that

$$|y_1| = |y_2| = 1$$



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117. If  $\alpha = (z - i)/(z + i)$  show that, when  $z$  lies above the real axis,  $\alpha$  will lie within the unit circle which has centre at the origin. Find the locus of  $\alpha$  as  $z$  travels on the real axis from  $-\infty$  to  $+\infty$



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118. If  $|z| \leq 1$  and  $|w| < 1$ , then shown that  $|z - w|^2 < (|z| - |w|)^2 + (\arg z - \arg w)^2$



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119. Prove that the distance of the roots of the equation  $|\sin \theta_1|z^3 + |\sin \theta_2|z^2 + |\sin \theta_3|z + |\sin \theta_4| = 3\cos z = 0$  is greater than  $2/3$ .



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**120.** If  $|z - (4 + 3i)| = 1$ , then find the complex number  $z$  for each of the following cases:

(i)  $|z|$  is least

(ii)  $|z|$  is greatest

(iii)  $\arg(z)$  is least

(iv)  $\arg(z)$  is greatest



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**121.** If  $a, b, c$ , and  $u, v, w$  are complex numbers representing the vertices of two triangles such that they are similar, then prove that  $\frac{a - c}{a - b} = \frac{u - w}{u - v}$



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**122.** Let  $z_1$  and  $z_2$  be the roots of the equation  $z^2 + pz + q = 0$  where the coefficient  $p$  and  $q$  may be complex numbers. Let  $A$  and  $B$  represent  $z_1$  and  $z_2$  in the complex plane. If



$\angle AOB = \alpha \neq 0$  and  $OA = OB$ , where  $O$  is the origin prove that  $p^2 = 4q \cos^2\left(\frac{\alpha}{2}\right)$

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**123.** The altitude from the vertices A, B and C of the triangle ABC meet its circumcircle at D, E and F, respectively. The complex number representing the points D, E, and F are  $z_1, z_2$  and  $z_3$ , respectively. If  $(z_3 - z_1)/(z_2 - z_1)$  is purely real, then show that triangle ABC is right-angled at A.

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**124.** Let A, B, C, D be four concyclic points in order in which  $AD:AB=CD:CB$ . If A, B, C are representing by complex numbers  $a, b, c$  respectively find the complex number associated with point D.

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125. If  $n \geq 3$  and  $\alpha_1, \alpha_2, \dots, \alpha_{n-1}$  are  $n$ th roots of unity, then find the

$$\text{sum } \sum_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$$



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### Exercise 3.1

1. Is the following computation correct? If not give the correct

computation:  $\left[ \sqrt{(-2)} \sqrt{(-3)} \right] = \sqrt{(-2)(-3)} = \sqrt{6}$



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2. Find the value of  $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$

A.  $-2$

B.  $0$

C.  $2$

D.  $-1$

**Answer: A**



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3. The value of  $i^{1+3+5+\dots+(2n+1)}$  is, If  $n$  is odd.

A.  $i$

B.  $1$

C.  $-1$

D.  $-i$

**Answer: B**



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4. Find the value of  $x^4 + 9x^3 + 35x^2 - x + 4$  for  $x = -5 + 2\sqrt{-4}$ .

## Exercise 3.2

1. प्रश्न 11 से 13 तक कि सम्मिश्र संख्याओं में प्रत्येक का गुणात्मक प्रतिलोम ज्ञात कीजिए ।

$$4 - 3i$$

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2. Express the following complex numbers in  $a + ib$  form:

$$\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)} \quad \text{(ii)} \quad \frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$$

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3. Find the least positive integer  $n$  such that  $\left(\frac{2i}{1+i}\right)^n$  is a positive integer.

A.  $n = 6$

B.  $n = 5$

C.  $n = 8$

D.  $n = 4$

**Answer: C**



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4. If one root of the equation  $z^2 - az + a - 1 = 0$  is  $(1 + i)$ , where  $a$  is a complex number then find the root.



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5. Prove that quadrilateral formed by the complex numbers which are roots of the equation  $z^4 - z^3 + 2z^2 - z + 1 = 0$  is an equilateral trapezium.



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6. If  $Z^5$  is a non-real complex number, then find the minimum value of

$$\frac{\operatorname{Im} z^5}{\operatorname{Im}^5 z}$$



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7. Find the real numbers  $x$  and  $y$ , if  $(x - iy)(3 + 5i)$  is the conjugate of

$$-6 - 24i$$

A.  $x = -2, y = 2$

B.  $x = -3, y = 3$

C.  $x = 3, y = -3$

D.  $x = -4, y = 1$

**Answer: C**



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8. If  $z_1, z_2, z_3$  are three nonzero complex numbers such that  $z_3 = (1 - \lambda)z_1 + \lambda z_2$  where  $\lambda \in \mathbb{R} - \{0\}$ , then prove that points corresponding to  $z_1, z_2$  and  $z_3$  are collinear.



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9. If  $n_1, n_2$  are positive integers, then  $(1 + i)^{n_1} + (1 + i^3)^{n_1} + (1 + i^5)^{n_2} + (1 + i^7)^{n_2}$  is real if and only if:



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### Exercise 3.3

1. If  $(a + b) - i(3a + 2b) = 5 + 2i$ , then find  $a$  and  $b$

A.  $a = 12, b = -17$

B.  $a = -12, b = -17$

C.  $a = 12, b = 17$

D.  $a = -12, b = 17$

**Answer: D**



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2. Find all non zero complex numbers  $z$  satisfying  $\bar{z} = iz^2$



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3. If  $a, b, c$  are nonzero real numbers and  $az^2 = bz + c + i = 0$  has purely imaginary roots, then prove that  $a = b^2$ .



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4. If the sum of square of roots of equation  $x^2 + (p + iq)x + 3i = 0$  is 8, then find  $|p| + |q|$ , where  $p$  and  $q$  are real.



A. 3

B. 1

C. 4

D. 2

**Answer: C**

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5. Find the square root  $9 + 40i$ .

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6. Simplify:  $\frac{\sqrt{5 + 12i} + \sqrt{5 - 12i}}{\sqrt{5 + 12i} - \sqrt{5 - 12i}}$

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7. If  $\sqrt{x + iy} = \pm (a + ib)$ , then find  $\sqrt{x - iy}$ .

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### Exercise 3.4

1. If  $\alpha$  and  $\beta$  are imaginary cube roots of unity then prove

$$(\alpha)^4 + (\beta)^4 + (\alpha)^{-1} \cdot (\beta)^{-1} = 0$$

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2. If  $\omega$  is a complex cube root of unity, then find the value of the

$$(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \dots \text{to } 2n \text{ factors.}$$

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3. Write the complex number in  $a + ib$  form using cube roots of unity: (a)

$$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{1000} \quad \text{(b) If} \quad z = \frac{(\sqrt{3} + i)^{17}}{(1 - i)^{50}} \quad \text{(c)}$$
$$(i + \sqrt{3})^{100} + (i + \sqrt{3})^{100} + 2^{100}$$

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4. If  $z + z^{-1} = 1$ , then find the value of  $z^{100} + z^{-100}$ .

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5. Find the common roots of  $x^{12} - 1 = 0$  and  $x^4 + x^2 + 1 = 0$

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6. if  $\alpha, \beta, \gamma$  are the roots of  $x^3 - 3x^2 + 3x + 7 = 0$  then

$$\frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$$

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7. Prove that  $t^2 + 3t + 3$  is a factor of  $(t + 1)^{n+1} + (t + 2)^{2n-1}$  for all integral values of  $n \in \mathbb{N}$ .

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### Exercise 3.5

1. Find the principal argument of each of the following:

(a)  $-1 - i\sqrt{3}$

(b)  $\frac{1 + \sqrt{3}i}{3 + i}$

(c)  $\sin \alpha + i(1 - \cos \alpha), 0 > \alpha > \pi$

(d)  $(1 + i\sqrt{3})^2$

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2. Find the modulus, argument, and the principal argument of the complex numbers. (i)  $(\tan 1 - i)^2$

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3. If  $\frac{3\pi}{2} < \alpha < 2\pi$ , find the modulus and argument of  $(1 - \cos 2\alpha) + i \sin 2\alpha$ .

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4. Find the principal argument of the complex number  $\frac{\sin(6\pi)}{5} + i \left( 1 + \frac{\cos(6\pi)}{5} \right)$ .

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5. If  $z = re^{i\theta}$ , then prove that  $|e^{iz}| = e^{-rs \int h\eta}$ .

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6. Find the complex number  $z$  satisfying  $Re(z^2) = 0, |z| = \sqrt{3}$ .

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7. If  $|z - iRe(z)| = |z - Im(z)|$ , then prove that  $z$ , lies on the bisectors of the quadrants.

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8. Find the locus of the points representing the complex number  $z$  for which  $|z + 5|^2 = |z - 5|^2 = 10$ .

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9. Solve :  $z^2 + |z| = 0$ .

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10. Let  $z = x + iy$  be a complex number, where  $x$  and  $y$  are real numbers.

Let  $A$  and  $B$  be the sets defined by

$A = \{z : |z| \leq 2\}$  and  $B = \{z : (1 - i)z + (1 + i)\bar{z} \geq 4\}$ . Find the area

of region  $A \cup B$ .

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11. Real part of  $(e^e)^{i\theta}$  is

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12. Prove that  $z = i^i$ , where  $i = \sqrt{-1}$ , is purely real.

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1. For  $z_1 = \sqrt[6]{(1-i)/(1+i\sqrt{3})}$ ,  $z_2 = \sqrt[6]{(1-i)/(\sqrt{3}+i)}$ ,  
 $z_3 = \sqrt[6]{(1+i)/(\sqrt{3}-i)}$ , prove that  $|z_1| = |z_2| = |z_3|$

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2. If  $\sqrt{3} + i = (a + ib)/(c + id)$ , then find the value of  
 $\tan^{-1}(b/a)\tan^{-1}(d/c)$ .

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3. If  $z_1, z_2$  and  $z_3, z_4$  are two pairs of conjugate complex numbers then  
 $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) =$

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4. Find the modulus, argument, and the principal argument of the  
complex numbers.  $(\tan 1 - i)^2 \frac{i-1}{i\left(1 - \frac{\cos(2\pi)}{5}\right) + s \in n\frac{2\pi}{5}}$

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5. If  $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = x + iy$ , then show that  $2 \times 5 \times 10 \times \dots \times (1 + n^2) = x^2 + y^2$ .

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6. If  $a + ib = \frac{(x + i)^2}{2x + 1}$ , prove that  $a^2 + b^2 = \frac{(x + i)^2}{(2x + 1)^2}$

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7. Let  $z$  be a complex number satisfying the equation  $(z^3 + 3)^2 = -16$ , then find the value of  $|z|$ .

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8. If  $\theta$  is real and  $z_1, z_2$  are connected by  $z_1^2 + z_2^2 + 2z_1z_2 \cos \theta = 0$ , then prove that the triangle formed by vertices  $O, z_1$  and  $z_2$  is isosceles.

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9. If  $|z_1 - z_0| = |z_2 - z_1| = \pi/2$ , then find  $z_0$ .

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10. Show that  $\left| \frac{z-2}{z-3} \right| = 2$  represents a circle. Find its centre and radius.

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### Exercise 3.7

1. Express the following in  $a + ib$  form: (a)  $\frac{(\cos \alpha + i \sin \alpha)^4}{(\sin \beta + i \cos \beta)^5}$  (b)  $\left( \frac{1 + \cos \phi + i \sin \phi}{1 + \cos \phi - i \sin \phi} \right)^n$  (c)  $\frac{(\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta)}{(\cos \gamma + i \sin \gamma)(\cos \delta + i \sin \delta)}$



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2. Find the value of following expression:  $\left[ \frac{1 - \frac{\cos \pi}{10} + i \frac{\sin \pi}{10}}{1 - \frac{\cos \pi}{10} - i \frac{\sin \pi}{10}} \right]^{10}$



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3. If  $iz^4 + 1 = 0$ , then prove that  $z$  can take the value  $\cos \pi/8 + is \in \pi/8$ .



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4. Prove that (a)  $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}} \cdot \cos\left(\frac{n\pi}{4}\right)$ , where  $n$  is a positive integer. (b)  $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$ , where  $n$  is a positive integer



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5. If  $z = (a + ib)^5 + (b + ia)^5$ , then prove that  $Re(z) = Im(z)$ , where  $a, b \in R$ .

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6. If  $\cos \alpha + \cos \beta + \cos \gamma = 0$  and also  $\sin \alpha + \sin \beta + \sin \gamma = 0$ , then prove that. (a)  $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$   
 (b)  $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$  (c)  
 $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$

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### Exercise 3.8

1.  $a, b, c$  are three complex numbers on the unit circle  $|z| = 1$ , such that  $abc = a + b + c$ . Then find the value of  $|ab + bc + ca|$ .

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2. Let  $z$  be not a real number such that  $(1 + z + z^2) / (1 - z + z^2) \in \mathbb{R}$ , then prove that  $|z| = 1$ .

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3. If  $z_1, z_2, z_3$  are distinct nonzero complex numbers and  $a, b, c \in \mathbb{R}^+$  such that  $\frac{a}{|z_1 - z_2|} = \frac{b}{|z_2 - z_3|} = \frac{c}{|z_3 - z_1|}$ . Then find the value of  $\frac{a^2}{|z_1 - z_2|} + \frac{b^2}{|z_2 - z_3|} + \frac{c^2}{|z_3 - z_1|}$

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4. If  $z_1$  and  $z_2$  are two complex numbers such that  $|z_1| < 1 < |z_2|$ , then prove that  $|(1 - z_1 \bar{z}_2) / (z_1 - z_2)| < 1$

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5. if  $|z_1 + z_2| = |z_1| + |z_2|$ , then prove that  $\arg(z_1) = \arg(z_2)$  if  $|z_1 - z_2| = |z_1| + |z_2|$ , then prove that  $\arg(z_1) = \arg(z_2) = \pi$

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6. For any complex number  $z$ , find the minimum value of  $|z| + |z - 2i|$ .

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7. If is any complex number such that  $|z + 4| \leq 3$ , then find the greatest value of  $|z + 1|$ .

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8.  $Z \in \mathbb{C}$  satisfies the condition  $|Z| > 3$ . Then find the least value of  $\left| Z + \frac{1}{Z} \right|$

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9. If  $a, b, c$  are nonzero complex numbers of equal moduli and satisfy  $az^2 + bz + c = 0$ , then prove that  $(\sqrt{5} - 1)/2 \leq |z| \leq (\sqrt{5} + 1)/2$ .

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10. If  $|z| \leq 4$  then find the maximum value of  $|iz + 3 - 4i|$

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11. Let  $z_1, z_2, z_3, \dots, z_n$  be the complex numbers such that  $|z_1| = |z_2| = \dots = |z_n| = 1$ . It is given that if  $z = \left( \sum_{k=1}^n z_k \right) \left( \sum_{k=1}^n \frac{1}{z_k} \right)$  then prove that (a)  $z$  is a real number (b)  $0 < z \leq n^2$

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1. If  $\omega = z/[z - (1/3)i]$  and  $|\omega| = 1$ , then find the locus of  $z$ .

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2. If  $\text{Im}\left(\frac{z-1}{e^{\theta i}} + \frac{e^{\theta i}}{z-1}\right) = 0$ , then find the locus of  $z$ .

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3. For three non-collinear complex numbers  $Z, Z_1$  and  $Z_2$  prove that

$$\left|Z - \frac{Z_1 + Z_2}{2}\right|^2 + \left|\frac{Z_1 - Z_2}{2}\right|^2 = \frac{1}{2}|Z - Z_1|^2 + \frac{1}{2}|Z - Z_2|^2$$

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4. If  $|z - 1| + |z + 3| \leq 8$ , then prove that  $z$  lies on the circle.

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5. If  $z = \frac{3}{2 + \cos \theta + I \sin \theta}$ , then prove that  $z$  lies on the circle.

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6. How many solutions system of equations,  $\arg(z + 3 - 2i) = -\pi/4$  and  $|z + 4| - |z - 3i| = 5$  has ?

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7. Prove that equation of perpendicular bisector of line segment joining complex numbers  $z_1$  and  $z_2$  is  $z(\bar{z}_2 - \bar{z}_1) + \bar{z}(z_2 + z_1) + |z_1|^2 - |z_2|^2 = 0$

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8. If complex number  $z$  lies on the curve  $|z - (-1 + i)| = 1$ , then find the locus of the complex number  $w = \frac{z + i}{1 - i}$ ,  $i = \sqrt{-1}$ .

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### Exercise 3.10

1. If  $z_1, z_2, z_3$  and  $z_4$  taken in order vertices of a rhombus, then proves that

$$\operatorname{Re}\left(\frac{z_3 - z_1}{z_4 - z_2}\right) = 0$$

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2. Find the locus of point  $z$  if  $z, i, \text{ and } iz$ , are collinear.

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3. If  $|z| = 2$  and  $\frac{z_1 - z_3}{z_2 - z_3} = \frac{z - 2}{z + 3}$ , then prove that  $z_1, z_2, z_3$  are vertices of a right angled triangle.

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4. Three vertices of triangle are complex number  $\alpha$ ,  $\beta$  and  $\gamma$ . Then prove that the perpendicular from the point  $\alpha$  to opposite side is given by the equation  $\operatorname{Re}\left(\frac{z - \alpha}{\beta - \gamma}\right) = 0$  where  $z$  is complex number of any point on the perpendicular.

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5. Prove that the complex numbers  $z_1$ ,  $z_2$  and the origin form an equilateral triangle only if  $z_1^2 + z_2^2 - z_1z_2 = 0$ .

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6. The center of a regular polygon of  $n$  sides is located at the point  $z=0$ , and one of its vertex  $z_1$  is known. If  $z_2$  be the vertex adjacent to  $z_1$ , then  $z_2$  is equal to

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7. If one vertex of the triangle having maximum area that can be inscribed in the circle  $|z - i| = 5$  is  $3-3i$ , then find the other vertices of the triangle.

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8. Consider the circle  $|z|=r$  in the Argand plane, which is in fact the incircle of triangle  $ABC$ . If contact points opposite to the vertices  $A, B, C$  are  $A_1(z_1)$ ,  $B_1(z_2)$  and  $C_1(z_3)$ , obtain the complex numbers associated with the vertices  $A, B, C$  in terms of  $z_1, z_2$  and  $z_3$ .

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9.  $P$  is a point on the Argand diagram on the circle with  $OP$  as diameter. Two points are taken such that  $\angle POQ = \angle QOR = \theta$ . If  $O$  is the origin and  $P, Q, R$  are represented by complex numbers  $z_1, z_2, z_3$  respectively then show that

$$z_2^2 \cos 2\theta = z_1 z_3 \cos^2 \theta$$

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10. The center of the arc represented by  $\arg \left[ \frac{z - 3i}{z - 2i + 4} \right] = \frac{\pi}{4}$

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### Exercise 3.11

1. If  $\alpha$  is complex fifth root of unity and  $(1 + \alpha + \alpha^2 + \alpha^3)^{2005} = p + q\alpha + r\alpha^2 + s\alpha^3$  (where  $p, q, r, s$  are real), then find the value of  $p + q + r + s$ .

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2. Find the number of roots of the equation  $z^{15} = 1$  satisfying  $|\arg z| < \pi/2$ .

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3. If  $z$  is nonreal root of  $[-1]^{\frac{1}{7}}$  then, find the value of  $z^{86} + z^{175} + z^{289}$

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4. Given  $\alpha, \beta$ , respectively, the fifth and the fourth non-real roots of units, then find the value of

$$(1 + \alpha)(1 + \beta)(1 + \alpha^2)(1 + \beta^2)(1 + \alpha^4)(1 + \beta^4)$$

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5. If the six roots of  $x^6 = -64$  are written in the form  $a + ib$ , where  $a$  and  $b$  are real, then the product of those roots for which  $a < 0$  is

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6. If  $z_r : r = 1, 2, 3, \dots, 50$  are the roots of the equation  $\sum_{r=0}^{50} z^r = 0$ , then

find the value of  $\sum_{r=1}^{50} 1/(z_r - 1)$



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## Exercise (Single)

1. If  $a < 0, b > 0$ , then  $\sqrt{a}\sqrt{b}$  equal to

A.  $-\sqrt{|a|b}$

B.  $\sqrt{|a|b} i$

C.  $\sqrt{|a|b}$

D. none of these

**Answer: B**



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2. Consider the equation  $10z^2 - 3iz - k = 0$ , where  $z$  is a following complex variable and  $i^2 = -1$ . Which of the following statements is true? For real complex numbers  $k$ , both roots are purely imaginary. For

all complex numbers  $k$ , neither both roots is real. For all purely imaginary numbers  $k$ , both roots are real and irrational. For real negative numbers  $k$ , both roots are purely imaginary.

A. For real positive numbers  $k$ , both roots are purely imaginary

B. For all complex numbers  $k$ , neither root is real .

C. For real negative numbers  $k$ , both roots are real and irrational .

D. For real negative numbers  $k$ , both roots are purely imaginary.

**Answer: D**



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3. The number of solutions of the equation  $z^2 + z = 0$  where  $z$  is a complex number, is

A. 1

B. 2

C. 3



D. 4

**Answer: D**



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4. If center of a regular hexagon is at the origin and one of the vertices on the Argand diagram is  $1 + 2i$ , then its perimeter is  $2\sqrt{5}$  b.  $6\sqrt{2}$  c.  $4\sqrt{5}$  d.  $6\sqrt{5}$

A.  $2\sqrt{5}$

B.  $6\sqrt{5}$

C.  $4\sqrt{5}$

D.  $6\sqrt{5}$

**Answer: D**



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5. If  $x$  and  $y$  are complex numbers, then the system of equations

$$(1 + i)x + (1 - i)y = 1, 2ix + 2y = 1 + i \text{ has}$$

- A. unique solution
- B. no solution
- C. infinite number of solutions
- D. none of these

**Answer: C**

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6. The points  $z_1 = 3 + \sqrt{3}i$  and  $z_2 = 2\sqrt{3} + 6i$  are given on the complex plane. The complex number lying on the bisector of the angle formed by

the vectors  $z_1$  and  $z_2$  is  $z = \frac{(3 + 2\sqrt{3})}{2} + \frac{\sqrt{3} + 2}{2}i$   $z = 5 + 5i$

$z = -1 - i$  none of these

A.  $z = \frac{(3 + 2\sqrt{3})}{2} + \frac{\sqrt{3} + 2}{2}i$

B.  $z = 5 + 5i$

C.  $z = -1 - i$

D. none of these

**Answer: B**



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7. The polynomial  $x^6 + 4x^5 + 3x^4 + 2x^3 + x + 1$  is divisible by \_\_\_\_\_ where  $\omega$  is the cube root of unity  $x + \omega$  b.  $x + \omega^2$  c.  $(x + \omega)(x + \omega^2)$  d.  $(x - \omega)(x - \omega^2)$  where  $\omega$  is one of the imaginary cube roots of unity.

A.  $x + \omega$

B.  $x + \omega^2$

C.  $(x + \omega)(x + \omega^2)$

D.  $(x + \omega)(x - \omega^2)$

**Answer: D**

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8. Dividing  $f(z)$  by  $z - i$ , we obtain the remainder  $i$  and dividing it by  $z + i$ , we get the remainder  $1 + i$ , then remainder upon the division of  $f(z)$  by  $z^2 + 1$  is

A.  $\frac{1}{2}(z + 1) + i$

B.  $\frac{1}{2}(iz + 1) + i$

C.  $\frac{1}{2}(iz - 1) + i$

D.  $\frac{1}{2}(z + i) + 1$

**Answer: B**

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9. The complex number  $\sin(x) + i \cos(2x)$  and  $\cos(x) - i \sin(2x)$  are conjugate to each other for

A.  $x = n\pi, n \in \mathbb{Z}$

B.  $x = 0$

C.  $x = (n + 1/2)\pi, n \in \mathbb{Z}$

D. no value of  $x$

**Answer: D**

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10. If the equation  $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$  where  $a_1, a_2, a_3, a_4$  are real coefficients different from zero has a pure imaginary root then the expression  $\frac{a_1}{a_1a_2} + \frac{a_1a_4}{a_2a_3}$  has the value equal to

A. 0

B. 1

C. -2

D. 2

**Answer: B**



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11. If  $z_1, z_2 \in C$ ,  $z_1^2 \in R$ ,  $z_1(z_1^2 - 3z_2^2) = 2$  and  $z_2(3z_1^2 - z_2^2) = 11$ , then the value of  $z_1^2 + z_2^2$  is

A. 10

B. 12

C. 5

D. 8

**Answer: C**



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12. If  $a^2 + b^2 = 1$  then  $\frac{1 + b + ia}{1 + b - ia} =$

A.  $a + ib$

B.  $a + ia$

C.  $b + ia$

D.  $b + ib$

**Answer: C**



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13. If  $z(1 + a) = b + ic$  and  $a^2 + b^2 + c^2 = 1$ , then

$$[(1 + iz)/(1 - iz)] = \frac{a + ib}{1 + c} \quad \text{b. } \frac{b - ic}{1 + a} \quad \text{c. } \frac{a + ic}{1 + b} \quad \text{d. none of these}$$

A.  $\frac{a + ib}{1 + c}$

B.  $\frac{b - ic}{1 + a}$

C.  $\frac{a + ic}{1 + b}$

D. none of these

**Answer: A**

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14. If  $a$  and  $b$  are complex and one of the roots of the equation  $x^2 + ax + b = 0$  is purely real whereas the other is purely imaginary, then

A.  $a^2 - (\bar{a})^2 = 4b$

B.  $a^2 - (\bar{a})^2 = 2b$

C.  $b^2 - (\bar{a})^2 = 2a$

D.  $b^2 - (\bar{b})^2 = 2a$

**Answer: A**

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15. If  $z = (\lambda + 3) + i\sqrt{(5 - \lambda^2)}$ ; then the locus of  $z$  is

A. ellipse



B. semicircle

C. parabola

D. none of these

**Answer: B**



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16. Let  $z = 1 - t + i\sqrt{t^2 + t + 2}$ , where  $t$  is a real parameter. the locus of the  $z$  in argand plane is

A. a hyperbola

B. an ellipse

C. a straight line

D. none of these

**Answer: A**



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17. If  $z_1$  and  $z_2$  are the complex roots of the equation  $(x - 3)^3 + 1 = 0$ , then  $z_1 + z_2$  equal to

A. 1

B. 3

C. 5

D. 7

**Answer: D**



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18. Which of the following is equal to  $\sqrt[3]{-1}$ ?

A.  $\frac{\sqrt{3} + \sqrt{-1}}{2}$

B.  $\frac{-\sqrt{3} + \sqrt{-1}}{\sqrt{-4}}$

C.  $\frac{\sqrt{3} - \sqrt{-1}}{\sqrt{-4}}$

D.  $-\sqrt{-1}$

**Answer: B**



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19. If  $x^2 + x + 1 = 0$  then the value of  $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \dots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$  is

A. 27

B. 72

C. 45

D. 54

**Answer: D**



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20. Sum of common roots of the equations  $z^3 + 2z^2 + 2z + 1 = 0$  and  $z^{1985} + z^{100} + 1 = 0$  is

- A.  $-1$
- B.  $1$
- C.  $0$
- D.  $1$

**Answer: A**



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21. If  $5x^3 + Mx + N, M, N \in R$  is divisible by  $x^2 + x + 1$ , then the value of  $M + N$  is

- A.  $5$
- B.  $4$
- C.  $-4$

D.  $-5$

**Answer: D**



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22. If  $z = x + iy$  and  $x^2 + y^2 = 16$ , then the range of  $||x| - |y||$  is  
[0, 4] b. [0, 2] c. [2, 4] d. none of these

A. [0, 4]

B. [0, 2]

C. [2, 4]

D. none of these

**Answer: A**



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23. If  $z$  is a complex number satisfying the equation  $z^6 - 6z^3 + 25 = 0$ , then the value of  $|z|$  is

A.  $5^{1/3}$

B.  $25^{1/3}$

C.  $125^{1/3}$

D.  $625^{1/3}$

**Answer: A**



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24. If  $8iz + 12z^2 - 18z + 27i = 0$ , then  $|z| = \frac{3}{2}$  b.  $|z| = \frac{2}{3}$  c.  $|z| = 1$  d.

$|z| = \frac{3}{4}$

A.  $|z| = \frac{3}{2}$

B.  $|z| = \frac{3}{4}$

C.  $|z| = 1$

$$D. |z| = \frac{3}{4}$$

**Answer: A**



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**25.** Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1 \neq z_2$  and  $|z_1| = |z_2|$ . If  $z_1$  has positive real part and  $z_2$  has negative imaginary part, then  $\frac{z_1 + z_2}{z_1 - z_2}$  may be zero (b) real and positive real and negative (d) purely imaginary

A. purely imaginary

B. real and positive

C. real and negative

D. none of these

**Answer: A**



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26.  $|z_1| = |z_2|$  and  $\arg\left(\frac{z_1}{z_2}\right) = \pi$ , then  $z_1 + z_2$  is equal to

- A. 0
- B. purely imaginary
- C. purely real
- D. none of these

**Answer: A**



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27. If for complex numbers  $z_1$  and  $z_2$ ,  $\arg(z_1) - \arg(z_2) = 0$  then

$|z_1 - z_2|$  is equal to

- A.  $|z_1| + |z_2|$
- B.  $|z_1| - |z_2|$
- C.  $||z_1| - |z_2||$



D. 0

**Answer: C**



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28. If  $\left| \frac{z_1}{z_2} \right| = 1$  and  $\arg(z_1 z_2) = 0$ , then

A.  $z_1 = z_2$

B.  $|z_2|^2 = z_1 z_2$

C.  $z_1 z_2 = 1$

D. more than 8

**Answer: B**



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29. Suppose  $A$  is a complex number and  $n \in \mathbb{N}$ , such that  $A^n = (A + 1)^n = 1$ , then the least value of  $n$  is 3 b. 6 c. 9 d. 12

A. 3

B. 6

C. 9

D. 12

**Answer: B**



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30. Let  $z, w$  be complex numbers such that  $\bar{z} + i\bar{w} = 0$  and  $\arg zw = \pi$

Then  $\arg z$  equals

A. 4

B. 6

C. 8

D. more than 8

**Answer: C**



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31. Let  $z, w$  be complex numbers such that  $\bar{z} + i\bar{w} = 0$  and  $\arg zw = \pi$

Then  $\arg z$  equals

A.  $\frac{\pi}{4}$

B.  $\frac{\pi}{2}$

C.  $\frac{3\pi}{4}$

D.  $\frac{5\pi}{4}$

**Answer: C**



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32. If  $z = (3 + 7i)(a + ib)$  where  $a, b \in \mathbb{Z} - \{0\}$ , is purely imaginary, then the minimum value of  $|z|$  is

A. 74

B. 45

C. 58

D. 65

**Answer: C**



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33. If  $(\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots (\cos n\theta + i \sin n\theta) = 1$  then the value of  $\theta$  is :

A.  $4m\pi$

B.  $\frac{2m\pi}{n(n+1)}$

C.  $\frac{4m\pi}{n(n+1)}$

D.  $\frac{m\pi}{n(n+1)}$

**Answer: C**



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34. Given  $z = (1 + i\sqrt{3})^{100}$ , then  $[RE(z) / IM(z)]$  equals  $2^{100}$  b.  $2^{50}$  c.

$\frac{1}{\sqrt{3}}$  d.  $\sqrt{3}$

A.  $2^{100}$

B.  $2^{50}$

C.  $\frac{1}{\sqrt{3}}$

D.  $\sqrt{3}$

**Answer: C**



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35. The expression  $\left[ \frac{1 + \sin\left(\frac{\pi}{8}\right) + i \cos\left(\frac{\pi}{8}\right)}{1 + \sin\left(\frac{\pi}{8}\right) - i \cos\left(\frac{\pi}{8}\right)} \right]^8$  is equal to

- A. 1
- B.  $-1$
- C.  $i$
- D.  $-i$

**Answer: B**



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36. The number of complex numbers  $z$  satisfying

$$|z - 3 - i| = |z - 9 - i| \text{ and } |z - 3 + 3i| = 3$$

are a. one b. two c. four d. none of these

- A. one
- B. two

C. four

D. none of these

**Answer: A**



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37.  $P(z)$  be a variable point in the Argand plane such that  $|z| = m \in i\mu m\{|z - 1, |z + 1|\}$ , then  $z + z$  will be equal to a. -1 or 1 b. 1 but not equal to -1 c. -1 but not equal to 1 d. none of these

A. -1 or 1

B. 1 but not equal to -1

C. -1 but not equal to 1

D. none of these

**Answer: A**



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38. if  $|z^2 - 1| = |z|^2 + 1$  then  $z$  lies on

- A. a circle
- B. a parabola
- C. an ellipse
- D. none of these

**Answer: D**



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39. If  $z = x + iy$  ( $x, y \in R, x \neq -\frac{1}{2}$ ), the number of values of  $z$  satisfying  $|z|^n = z^2|z|^{n-2} + z|z|^{n-2} + 1$ . ( $n \in N, n > 1$ ) is

- A. 0
- B. 1
- C. 2



D. 3

**Answer: B**



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40. Number of solutions of the equation  $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$  where  $z$  is a complex number is

A. 2

B. 3

C. 6

D. 5

**Answer: D**



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41. Number of ordered pairs(s)  $(a, b)$  of real numbers such that  $(a + ib)^{2008} = a - ib$  holds good is

- A. 2008
- B. 2009
- C. 2010
- D. 1

**Answer: C**



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42. The equation  $az^3 + bz^2 + \bar{b}z + \bar{a} = 0$  has a root  $\alpha$ , where  $a, b, z$  and  $\alpha$  belong to the set of complex numbers. The number value of  $|\alpha|$

- A. is  $1/2$
- B. is 1
- C. is 2

D. can't be determined

**Answer: B**



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43. If  $k > 0$ ,  $|z| = w = k$ , and  $\alpha = \frac{z - \bar{w}}{k^2 + z\bar{w}}$ , then  $Re(\alpha)$  (A) 0 (B)  $\frac{k}{2}$  (C)  $k$  (D) None of these

A. 0

B.  $k/2$

C.  $k$

D. none of these

**Answer: A**



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44.  $z_1$  and  $z_2$  are two distinct points in an Argand plane. If  $a|z_1| = b|z_2|$  (where  $a, b \in \mathbb{R}$ ), then the point  $(az_1/bz_2) + (bz_2/az_1)$  is a point on the line segment  $[-2, 2]$  of the real axis line segment  $[-2, 2]$  of the imaginary axis unit circle  $|z| = 1$  the line with  $\arg z = \tan^{-1} 2$

- A. line segment  $[-2, 2]$  of the real axis
- B. line segment  $[-2, 2]$  of the imaginary axis
- C. unit circle  $|z| = 1$
- D. the line with  $\arg z = \tan^{-1} 2$

**Answer: A**



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45. If  $z$  is a complex number such that  $-\frac{\pi}{2} < \arg z \leq \frac{\pi}{2}$ , then which of the following inequalities is true?

- A.  $|z - \bar{z}| \leq |z|(\arg z - \arg \bar{z})$

B.  $|z - \bar{z}| \geq |z|(argz - arg\bar{z})$

C.  $|z - \bar{z}| < (argz - arg\bar{z})$

D. None of these

**Answer: A**



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46. If  $\cos \alpha + 2 \cos \beta + 3 \cos \gamma = \sin \alpha + 2 \sin \beta + 3 \sin \gamma = 0$ , then the value of  $\sin 3\alpha + 8 \sin 3\beta + 27 \sin 3\gamma$  is

A.  $\sin(\alpha + \beta + \gamma)$

B.  $3 \sin(\alpha + \beta + \gamma)$

C.  $18 \sin(\alpha + \beta + \gamma)$

D.  $\sin(\alpha + \beta + \gamma)$

**Answer: C**



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47. If  $\alpha, \beta$  be the roots of the equation  $u^2 - 2u + 2 = 0$  and if  $\cot \theta = x + 1$ , then  $\frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta}$  is equal to (a)  $\begin{pmatrix} \sin n\theta \\ \sin^n \theta \end{pmatrix}$  (b)  $\begin{pmatrix} \cos n\theta \\ \cos^n \theta \end{pmatrix}$  (c)  $((\sin n\theta), \cos^n \theta)$  (d)  $\begin{pmatrix} \cos n\theta \\ \sin^n \theta \end{pmatrix}$

A.  $\frac{\sin n\theta}{\sin^n \theta}$

B.  $\frac{\cos n\theta}{\cos^n \theta}$

C.  $\frac{\sin n\theta}{\cos^n \theta}$

D.  $\frac{\cos n\theta}{\sin^n \theta}$

**Answer: A**



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48. If  $z = (i)^{(i)^i}$  where  $i = \sqrt{-1}$ , then  $|z|$  is equal to

A. 1

B.  $e^{-\pi/2}$

C.  $e^{-\pi}$

D. none of these

**Answer: A**



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49. If  $z = i \log(2 - \sqrt{-3})$ , then  $\cos z =$

A.  $-1$

B.  $-1/2$

C.  $1$

D.  $2$

**Answer: D**



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50. If  $|z| = 1$ , then the point representing the complex number  $-1 + 3z$  will lie on a. a circle b. a parabola c. a straight line d. a hyperbola

A. a circle

B. a straight line

C. a parabola

D. a hyperbola

**Answer: A**



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51. The locus of point  $z$  satisfying  $Re\left(\frac{1}{z}\right) = k$ , where  $k$  is a nonzero real number, is a. a straight line b. a circle c. an ellipse d. a hyperbola

A. a stringht line

B. a circle

C. an ellispe



D. a hyperbola

**Answer: B**



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52. If  $z$  is complex number, then the locus of  $z$  satisfying the condition  $|2z - 1| = |z - 1|$  is perpendicular bisector of line segment joining  $1/2$  and  $1$   
and  $1$  circle parabola none of the above curves

A. perpeciular bisector of line segment joining  $1/2$  and  $1$

B. circle

C. parabola

D. none of the above curves

**Answer: B**



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53. The greatest positive argument of complex number satisfying

$$|z - 4| = \operatorname{Re}(z) \text{ is } \frac{\pi}{3} \text{ b. } \frac{2\pi}{3} \text{ c. } \frac{\pi}{2} \text{ d. } \frac{\pi}{4}$$

A.  $\frac{\pi}{3}$

B.  $\frac{2\pi}{3}$

C.  $\frac{\pi}{2}$

D.  $\frac{\pi}{4}$

Answer: D



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54. If  $t$  and  $c$  are two complex numbers such that

$$|t| \neq |c|, |t| = 1 \text{ and } z = (at + b)/(t - c), z = x + iy. \text{ Locus of } z \text{ is}$$

(where  $a, b$  are complex numbers) a. line segment b. straight line c. circle

d. none of these

A. line segment

B. straight line

C. circle

D. none of these

**Answer: C**



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55. If  $z^2 + z|z| + |z^2| = 0$ , then the locus  $z$  is a. a circle b. a straight line  
c. a pair of straight line d. none of these

A. a circle

B. a straight line

C. a pair of straight line

D. none of these

**Answer: C**



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56. Let  $C_1$  and  $C_2$  are concentric circles of radius 1 and  $\frac{8}{3}$  respectively having centre at  $(3, 0)$  on the argand plane. If the complex number  $z$  satisfies the inequality  $\log_{\frac{1}{3}} \left( \frac{|z - 3|^2 + 2}{11|z - 3| - 2} \right) > 1$ , then

- A.  $z$  lies outside  $C_1$  but inside  $C_2$
- B.  $z$  line inside of both  $C_1$  and  $C_2$
- C.  $z$  line outside both  $C_1$  and  $C_2$
- D. none of these

**Answer: A**

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57. If  $|z - 2 - i| = |z| \sin \left( \frac{\pi}{4} - \arg z \right)$ , where  $i = \sqrt{-1}$ , then locus of  $z$ , is

- A. a pair of straight lines
- B. circle

C. parabola

D. ellipse

**Answer: C**



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58. If  $|z - 1| \leq 2$  and  $|\omega z - 1 - \omega^2| = a$  (where  $\omega$  is a cube root of unity),

then complete set of values of  $a$  is  $0 \leq a \leq 2$  b.  $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$  c.

$\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$  d.  $0 \leq a \leq 4$

A.  $0 \leq a \leq 2$

B.  $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$

C.  $\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$

D.  $0 \leq a \leq 4$

**Answer: D**



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59. If  $|z^2 - 3| = 3|z|$ , then the maximum value of  $|z|$  is 1 b.  $\frac{3 + \sqrt{21}}{2}$  c.

$\frac{\sqrt{21} - 3}{2}$  d. none of these

A. 1

B.  $\frac{3 + \sqrt{21}}{2}$

C.  $\frac{\sqrt{21} - 3}{2}$

D. none of these

**Answer: B**



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60. If  $|2z - 1| = |z - 2|$  and  $z_1, z_2, z_3$  are complex numbers such that

$|z_1 - \alpha| < \alpha, |z_2 - \beta| < \beta, |z_3 - \gamma| < \gamma$

A.  $< |z|$

B.  $< 2|z|$

C.  $> |z|$

D.  $> 2|z|$

**Answer: B**



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61. If  $z_1$  is a root of the equation

$a_0z^n + a_1z^{n-1} + \dots + a_{n-1}z + a_n = 3$ , where  $|a_i| < 2$  for  $i = 0, 1, \dots, n$ , then

$|z| = \frac{3}{2}$  b.  $|z| < \frac{1}{4}$  c.  $|z| > \frac{1}{4}$  d.  $|z| < \frac{1}{3}$

A.  $|z_1| > \frac{1}{2}$

B.  $|z_1| < \frac{1}{2}$

C.  $|z_1| > \frac{1}{4}$

D.  $|z| < \frac{1}{2}$

**Answer: A**



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62. If  $|z| <$

A. less than 1

B.  $\sqrt{2} + 1$

C.  $\sqrt{2 - 1}$

D. none of these

**Answer: A**



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63. Let  $|Z_r - r| \leq r$ , for all  $r = 1, 2, 3, \dots, n$ . Then  $\left| \sum_{r=1}^n z_r \right|$  is less than

A.  $n$

B.  $2n$

C.  $n(n+1)$

D.  $\frac{n(n+1)}{2}$



**Answer: C**



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**64.** All the roots of the equation  $11z^{10} + 10iz^9 + 10iz - 11 = 0$  lie

- A. inside  $|z| = 1$
- B. one  $|z| = 1$
- C. outside  $|z| = 1$
- D. cannot say

**Answer: B**



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**65.** Let  $\lambda \in \mathbb{R}$ . If the origin and the non-real roots of  $2z^2 + 2z + \lambda = 0$  form the three vertices of an equilateral triangle in the Argand plane, then  $\lambda$  is

A. 1

B.  $\frac{2}{3}$

C. 2

D. -1

**Answer: B**



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**66.** The roots of the equation  $t^3 + 3at^2 + 3bt + c = 0$  are  $z_1, z_2, z_3$  which represent the vertices of an equilateral triangle. Then  $a^2 = 3b$  b.  $b^2 = a$  c.  $a^2 = b$  d.  $b^2 = 3a$

A.  $a^2 = 3b$

B.  $b^2 = a$

C.  $a^2 = a$

D.  $b^2 = 3a$

**Answer: C**



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67. The roots of the cubic equation  $(z + ab)^3 = a^3, a \neq 0$  represents the vertices of an equilateral triangle of sides of length

A.  $\frac{1}{\sqrt{3}}|ab|$

B.  $\sqrt{3}|a|$

C.  $\sqrt{3}|b|$

D.  $|a|$

**Answer: B**



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68. If  $|z_1| = |z_2| = |z_3| = 1$  and  $z_1 + z_2 + z_3 = 0$  then the area of the triangle whose vertices are  $z_1, z_2, z_3$  is

A.  $3\sqrt{3/4}$

B.  $\sqrt{3/4}$

C. 1

D. 2

**Answer: A**



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**69.** Let  $z$  and  $\omega$  be two complex numbers such that  $|z| \leq 1$ ,  $|\omega| \leq 1$  and

$|z + i\omega| = |z_1 - z_2|$  is equal to

A.  $\frac{2}{3}$

B.  $\frac{\sqrt{5}}{3}$

C.  $\frac{3}{2}$

D.  $\frac{2\sqrt{5}}{3}$

**Answer: C**

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70. Let  $z_1, z_2, z_3, z_4$  are distinct complex numbers satisfying  $|z| = 1$  and  $4z_3 = 3(z_1 + z_2)$ , then  $|z_1 - z_2|$  is equal to

- A. 1 or  $i$
- B.  $i$  or  $-i$
- C. 1 or  $i$
- D.  $i$  or  $-1$

**Answer: D**

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71.  $z_1, z_2, z_3, z_4$  are distinct complex numbers representing the vertices of a quadrilateral  $ABCD$  taken in order. If  $z_1 - z_4 = z_2 - z_3$  and  $\arg\left[\frac{z_4 - z_1}{z_2 - z_1}\right] = \pi/2$ , the quadrilateral is

A. rectangle

B. rhombus

C. square

D. trapezium

**Answer: A**



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72. If  $k + |k + z^2| = |z|^2$  ( $k \in \mathbb{R}^-$ ), then possible argument of  $z$  is

A. 0

B.  $\pi$

C.  $\pi/2$

D. none of these

**Answer: C**



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73. If  $z_1, z_2, z_3$  are the vertices of an equilateral triangle ABC such that

$|z_1 - i| = |z_2 - i| = |z_3 - i|$ , then  $|z_1 + z_2 + z_3|$  equals to

A.  $3\sqrt{3}$

B.  $\sqrt{3}$

C. 3

D.  $\frac{1}{3\sqrt{3}}$

**Answer: C**



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74. If  $z$  is a complex number having least absolute value and

$|z - 2 + 2i| = \sqrt{2}$ , then  $z =$

A.  $(2 - 1/\sqrt{2})(1 - i)$

B.  $(2 - 1/\sqrt{2})(1 + i)$

C.  $(2 + 1/\sqrt{2})(1 - i)$

D.  $(2 + 1/\sqrt{2})(1 + i)$

**Answer: A**



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75. If  $z$  is a complex number lying in the fourth quadrant of Argand plane and  $|[kz/(k+1)] + 2i| > \sqrt{2}$  for all real value of  $k(k \neq -1)$ , then range of  $\arg(z)$  is  $\left(\frac{\pi}{8}, 0\right)$  b.  $\left(\frac{\pi}{6}, 0\right)$  c.  $\left(\frac{\pi}{4}, 0\right)$  d. none of these

A.  $\left(-\frac{\pi}{8}, 0\right)$

B.  $\left(-\frac{\pi}{6}, 0\right)$

C.  $\left(-\frac{\pi}{4}, 0\right)$

D. None of these

**Answer: C**



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76. If  $|z_2 + iz_1| = |z_1| + |z_2|$  and  $|z_1| = 3$  and  $|z_2| = 4$ , then the area of  $ABC$ , if affixes of  $A, B,$  and  $C$  are  $z_1, z_2,$  and  $[(z_2 - iz_1)/(1 - i)]$  respectively, is  $\frac{5}{2}$  b.  $0$  c.  $\frac{25}{2}$  d.  $\frac{25}{4}$

A.  $\frac{5}{2}$

B.  $0$

C.  $\frac{25}{2}$

D.  $\frac{25}{4}$

**Answer: D**

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77. If a complex number  $z$  satisfies  $|2z + 10 + 10i| \leq 5\sqrt{3} - 5$ , then the least principal argument of  $z$  is : (a)  $-\frac{5\pi}{6}$  (b)  $\frac{11\pi}{12}$  (c)  $-\frac{3\pi}{4}$  (d)  $-\frac{2\pi}{3}$

A.  $-\frac{5\pi}{6}$

B.  $-\frac{11\pi}{12}$

C.  $-\frac{3\pi}{4}$

D.  $-\frac{2\pi}{3}$

**Answer: A**



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78. If 'z', lies on the circle  $|z - 2i| = 2\sqrt{2}$ , then the value of  $\arg\left(\frac{z - 2}{z + 2}\right)$

is the equal to

A.  $\frac{\pi}{3}$

B.  $\frac{\pi}{4}$

C.  $\frac{\pi}{6}$

D.  $\frac{\pi}{2}$

**Answer: B**



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79.  $z_1$  and  $z_2$ , lie on a circle with centre at origin. The point of intersection of the tangents at  $z_1$  and  $z_2$  is given by

A.  $\frac{1}{2}(\bar{z}_1 + \bar{z}_2)$

B.  $\frac{2z_1z_2}{z_1 + z_2}$

C.

D.

**Answer: B**

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80. If  $\arg\left(\frac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \frac{\pi}{2}$  and  $\left|\frac{z}{|z|} - z_1\right| = 3$ , then  $|z_1|$  equals to

A.  $\sqrt{26}$

B.  $\sqrt{10}$

C.  $\sqrt{3}$

D.  $2\sqrt{2}$

**Answer: B**

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**81.** The maximum area of the triangle formed by the complex coordinates

$z, z_1, z_2$  which satisfy the relations  $|z - z_1| = |z - z_2|$  and

$$\left| z - \frac{z_1 + z_2}{2} \right| \leq r, \text{ where } r > |z_1 - z_2| \text{ is}$$

A.  $\frac{1}{2}|z_1 - z_2|^2$

B.  $\frac{1}{2}|z_1 - z_2|r$

C.  $\frac{1}{2}|z_1 - z_2|^2 r^2$

D.  $\frac{1}{2}|z_1 - z_2|^2$

**Answer: B**

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82. Consider the region  $S$  of complex numbers  $z$  such that

$|z^2 - az + 1| = 1$ , where  $|z| = 1$ . Then area of  $S$  in the Argand plane is

A.  $\pi + 8$

B.  $\pi + 4$

C.  $2\pi + 4$

D.  $\pi + 6$

**Answer: A**



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83. The complex number associated with the vertices  $A, B, C$  of  $\Delta ABC$  are  $e^{i\theta}, \omega, \bar{\omega}$ , respectively [ where  $\omega, \bar{\omega}$  are the complex cube roots of unity and  $\cos \theta > \operatorname{Re}(\omega)$  ], then the complex number of the point where angle bisector of  $A$  meets circumcircle of the triangle, is

A.  $e^{i\theta}$

B.  $e^{-i\theta}$

C.  $\omega, \bar{\omega}$

D.  $\omega + \bar{\omega}$

**Answer: D**



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**84.** If  $p$  and  $q$  are distinct prime numbers, then the number of distinct imaginary numbers which are  $p$ th as well as  $q$ th roots of unity are.

a.  $\min(p, q)$  b.  $\min(p, q) - 1$  c. 1 d. zero

A.  $\min(p, q)$

B.  $\max(p, q)$

C. 1

D. zero

**Answer: D**



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85. Given  $z$  is a complex number with modulus 1. Then the equation

$$\left[ \frac{1 + ia}{1 - ia} \right]^4 = z \text{ has (a) all roots real and distinct (b) two real and two}$$

imaginary (c) three roots two imaginary (d) one root real and three imaginary

- A. all roots real and distinct
- B. two real and two imaginary
- C. three roots real and one imaginary
- D. one root real and three imaginary

**Answer: A**



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86. The value of  $z$  satisfying the equation  $\log z + \log z^2 + \dots + \log z^n = 0is$

A.  $\cos. \frac{4m\pi}{n(n+1)} + i \sin. \frac{4m\pi}{n(n+1)}, m = 0, 1, 2, \dots$

B.  $\cos. \frac{4m\pi}{n(n+1)} - i \sin. \frac{4m\pi}{n(n+1)}, m = 0, 1, 2, \dots$

C.  $\sin. \frac{4m\pi}{n} + i \cos. \frac{4m\pi}{n}, m = 0, 1, 2, \dots$

D. 0

**Answer: A**



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87. If  $n \in \mathbb{N} > 1$ , then the sum of real part of roots of  $z^n = (z + 1)^n$  is equal to  $\frac{n}{2}$  b.  $\frac{(n-1)}{2}$  c.  $\frac{n}{2}$  d.  $\frac{(1-n)}{2}$

A.  $\frac{n}{2}$

B.  $\frac{(n-1)}{2}$

C.  $-\frac{n}{2}$

D.  $\frac{(1-n)}{2}$

**Answer: D**



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88. Which of the following represents a points in an Argand pane, equidistant from the roots of the equation  $(z + 1)^4 = 16z^4$ ?  $(0, 0)$  b.

$\left(-\frac{1}{3}, 0\right)$  c.  $\left(\frac{1}{3}, 0\right)$  d.  $\left(0, \frac{2}{\sqrt{5}}\right)$

A.  $(0, 0)$

B.  $\left(-\frac{1}{3}, 0\right)$

C.  $\left(\frac{1}{3}, 0\right)$

D.  $\left(0, \frac{2}{\sqrt{5}}\right)$

**Answer: C**

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89. Let  $a$  be a complex number such that  $|a| < 1$  and  $z_1, z_2, \dots$  be vertices of a polygon such that  $z_k = 1 + a + a^2 + a^3 + \dots + a^{k-1}$ .

Then, the vertices of the polygon lie within a circle.

A.  $\left|z - \frac{1}{1-a}\right| = \frac{1}{|a-1|}$

B.  $\left|z + \frac{1}{a+1}\right| = \frac{1}{|a+1|}$

C.  $\left|z - \frac{1}{1-a}\right| = |a-1|$

D.  $\left|z + \frac{1}{1-a}\right| = |a-1|$

**Answer: A**



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## Exercise (Multiple)

1. If  $z = \omega, \omega^2$  where  $\omega$  is a non-real complex cube root of unity, are two vertices of an equilateral triangle in the Argand plane, then the third vertex may be represented by  $z = 1$  b.  $z = 0$  c.  $z = -2$  d.  $z = -1$

A.  $z = 1$

B.  $z = 0$

C.  $z = -2$

D.  $z = -1$

**Answer: A::C**



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2. If  $\text{amp}(z_1 z_2) = 0$  and  $|z_1| = |z_2| = 1$ , then  $z_1 + z_2 = 0$  b.  $z_1 z_2 = 1$  c.  $z_1 = z_2$  d. none of these

A.  $z_1 + z_2 = 0$

B.  $z_1 z_2 = 1$

C.  $z_1 = \bar{z}_2$

D. none of these

**Answer: B::C**



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3. If  $\sqrt{5 - 12i} + \sqrt{5 - 12i} = z$ , then principal value of  $\arg z$  can be  $\frac{\pi}{4}$  b.

$\frac{\pi}{4}$  c.  $\frac{3\pi}{4}$  d.  $-\frac{3\pi}{4}$

A.  $-\frac{\pi}{4}$

B.  $\frac{\pi}{4}$

C.  $\frac{3\pi}{4}$

D.  $-\frac{3\pi}{4}$

Answer: A::B::C::D



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4. Values  $(s)(-i)^{1/3}$  is/are  $\frac{\sqrt{3} - i}{2}$  b.  $\frac{\sqrt{3} + i}{2}$  c.  $\frac{-\sqrt{3} - i}{2}$  d.

$\frac{-\sqrt{3} + i}{2}$

A.  $s \frac{\sqrt{3} - i}{2}$

B.  $\frac{\sqrt{3} + i}{2}$

C.  $\frac{-\sqrt{3} - i}{2}$

D.  $\frac{-\sqrt{3} + i}{2}$

Answer: A::C



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5. If  $a^3 + b^3 + 6abc = 8c^3$  &  $\omega$  is a cube root of unity then:  $a, b, c$  are in A.P. (b)  $a, b, c$ , are in H.P.  $a + b\omega - 2c\omega^2 = 0$   $a + b\omega^2 - 2c\omega = 0$

A.  $a, c, b$  are in A.P

B.  $a, c, b$  are in H.P

C.  $a + b\omega - 2c\omega^2 = 0$

D.  $a + b\omega^2 - 2c\omega = 0$

Answer: A::C::D



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6. If  $z_1$  and  $z_2$  are two non-zero complex numbers such that  $|z_1 + z_2| = |z_1| + |z_2|$ , then  $\arg\left(\frac{z_1}{z_2}\right)$  is equal to

A.  $1 + \omega$

B.  $1 + \omega^2$

C.  $\omega$

D.  $\omega^2$

Answer: C::D



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7. If  $p = a + b\omega + c\omega^2$ ,  $q = b + c\omega + a\omega^2$ , and  $r = c + a\omega + b\omega^2$ , where  $a, b, c \neq 0$  and  $\omega$  is the complex cube root of unity, then .

A. If  $p, q, r$  lie on the circle  $|z|=2$ , the triangle formed by these points is equilateral.

B.  $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$

$$C. p^2 + q^2 + r^2 = 2(pq + qr + rp)$$

D. none of these

**Answer: A::C**



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8. Let  $P(x)$  and  $Q(x)$  be two polynomials. Suppose that  $f(x) = P(x^3) + xQ(x^3)$  is divisible by  $x^2 + x + 1$ , then

A.  $P(x)$  is divisible by  $(x-1)$ , but  $Q(x)$  is not divisible by  $x-1$

B.  $Q(x)$  is divisible by  $(x-1)$ , but  $P(x)$  is not divisible by  $x-1$

C. Both  $P(x)$  and  $Q(x)$  are divisible by  $x-1$

D.  $f(x)$  is divisible by  $x-1$

**Answer: C::D**



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9. If  $\alpha$  is a complex constant such that  $az^2 + z + \alpha = 0$  has a real root, then  $\alpha + \bar{\alpha} = 1$   $\alpha + \bar{\alpha} = 0$   $\alpha + \bar{\alpha} = -1$  the absolute value of the real root is 1

A.  $\alpha + \bar{\alpha} = 1$

B.  $\alpha + \bar{\alpha} = 0$

C.  $\alpha + \bar{\alpha} = -1$

D. the absolute value of the real root is 1

Answer: A::C::D



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10. If  $z^3 + 3 + 2i(z + (-1 + ia)) = 0$  has one real root, then the value of  $a$  lies in the interval ( $a \in \mathbb{R}$ ) a.  $(-2, 1)$  b.  $(-1, 0)$  c.  $(0, 1)$  d.  $(-2, 3)$

A.  $(2, -1)$

B.  $(-1, 0)$



C. (0, 1)

D. (-2, 3)

**Answer: A::B::D**



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11. Given that the complex numbers which satisfy the equation  $|zz^3| + |zz^3| = 350$  form a rectangle in the Argand plane with the length of its diagonal having an integral number of units, then area of rectangle is 48 sq. units if  $z_1, z_2, z_3, z_4$  are vertices of rectangle, then  $z_1 + z_2 + z_3 + z_4 = 0$  rectangle is symmetrical about the real axis  $\arg(z_1 - z_3) = \frac{\pi}{4}$  or  $\frac{3\pi}{4}$

A. area of rectangle is 48 sq units.

B. if  $z_1, z_2, z_3, z_4$  are vertices of rectangle, then

$$z_1 + z_2 + z_3 + z_4 = 0$$

C. rectangle is symmetrical about the real axis .

D. None of these

**Answer: A::B::C**

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12. If the points  $A(z)$ ,  $B(-z)$ , and  $C(1-z)$  are the vertices of an equilateral triangle  $ABC$ , then sum of possible  $z$  is  $1/2$  sum of possible  $z$  is 1 product of possible  $z$  is  $1/4$  product of possible  $z$  is

- A. sum of possible  $z$  is  $1/2$
- B. sum of possible  $z$  is
- C. product of possible  $z$  is  $1/4$
- D. product of possible  $z$  is  $1/2$ .

**Answer: A::C**

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13. If  $|z - 3| = \min\{|z - 1|, |z - 5|\}$ , then  $Re(z)$  equals to

A. 2

B.  $\frac{5}{2}$

C.  $\frac{7}{2}$

D. 4

Answer: A:D



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14. If  $z_1, z_2$  are two complex numbers ( $z_1 \neq z_2$ ) satisfying

$$|z_1^2 - z_2^2| = |\bar{z}_1^2 + \bar{z}_2^2 - 2\bar{z}_1\bar{z}_2|, \text{ then}$$

A.  $\frac{z_1}{z_2}$  is purely imaginary

B.  $\frac{z_1}{z_2}$  is purely real

C.  $|\arg z_1 - \arg z_2| = \pi$

D.  $|\arg z_1 - \arg z_2| = \frac{\pi}{2}$

**Answer: A::D**



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15. If  $z_1 = a + ib$  and  $z_2 = c + id$  are complex numbers such that  $|z_1| = |z_2| = 1$  and  $Re(z_1\bar{z}_2) = 0$ , then the pair of complex numbers  $\omega = a + ic$  and  $\omega_2 = b + id$  satisfies

A.  $|\omega_1| = 1$

B.  $|\omega_2| = 1$

C.  $Re(\omega_1\bar{\omega}_2) = 0$

D.  $Im(\omega_1\bar{\omega}_2) = 0$

**Answer: A::B::C**



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16. Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1 \neq z_2$  and  $|z_1| = |z_2|$ . If  $z_1$  has positive real part and  $z_2$  has negative imaginary part, then  $\frac{z_1 + z_2}{z_1 - z_2}$  may be zero (b) real and positive real and negative (d) purely imaginary

A. zero

B. real and positive

C. real and negative

D. purely imaginary

Answer: A::D



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17. If  $|z_1| = \sqrt{2}$ ,  $|z_2| = \sqrt{3}$  and  $|z_1 + z_2| = \sqrt{(5 - 2\sqrt{3})}$  then  $\arg\left(\frac{z_1}{z_2}\right)$  (not necessarily principal)

A.  $\frac{3\pi}{4}$

B.  $\frac{2\pi}{3}$

C.  $\frac{5\pi}{4}$

D.  $\frac{5}{2}$

Answer: A::C

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18. Let four points  $z_1, z_2, z_3, z_4$  be in complex plane such that  $|z_2| = 1$ ,

$|z_1| \leq 1$  and  $|z_3| \leq 1$ . If  $z_3 = \frac{z_2(z_1 - z_4)}{\bar{z}_1 z_4 - 1}$ , then  $|z_4|$  can be

A. 2

B.  $\frac{2}{5}$

C.  $\frac{1}{3}$

D.  $\frac{5}{2}$

Answer: B::C

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19. A rectangle of maximum area is inscribed in the circle  $|z - 3 - 4i| = 1$ . If one vertex of the rectangle is  $4 + 4i$ , then another adjacent vertex of this rectangle can be  $2 + 4i$  b.  $3 + 5i$  c.  $3 + 3i$  d.  $3 - 3i$

A.  $2 + 4i$

B.  $3 + 5i$

C.  $3 + 3i$

D.  $3 - 3i$

Answer: B::C



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20. If  $|z_1| = 15$  and  $|z_2 - 3 - 4i| = 5$ , then  $(|z_1 - z_2|)_{\min} = 5$  b.  $(|z_1 - z_2|)_{\min} = 10$  c.  $(|z_1 - z_2|)_{\max} = 20$  d.  $(|z_1 - z_2|)_{\max} = 25$

A.  $|z_1 - z_2|_{\min} = 5$

B.  $|z_1 - z_2|_{\min} = 10$

C.  $|z_1 - z_2|_{\min} = 20$

D.  $|z_1 - z_2|_{\min} = 25$

**Answer: A:D**

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21.  $P(z_1), Q(z_2), R(z_3)$  and  $S(z_4)$  are four complex numbers representing the vertices of a rhombus taken in order on the complex plane, then which one of the following is/are correct?

A.  $\frac{z_1 - z_4}{z_2 - z_3}$  is purely real

B.  $amp \frac{z_1 - z_4}{z_2 - z_4} = amp \frac{z_2 - z_4}{z_3 - z_4}$

C.  $\frac{z_1 - z_3}{z_2 - z_4}$  is purely imaginary

D. is not necessary that  $|z_1 - z_3| \neq |z_2 - z_4|$



Answer: A::B::C::D



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22. If  $\arg(z + a) = \pi/6$  and  $\arg(z - a) = 2\pi/3$  ( $a \in \mathbb{R}^+$ ), then

A.  $|z| = a$

B.  $|z| = 2a$

C.  $\arg(z) = \frac{\pi}{2}$

D.  $\arg(z) = \frac{\pi}{3}$

Answer: A::D



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23. If a complex number  $z$  satisfies  $|z| = 1$  and  $\arg(z - 1) = \frac{2\pi}{3}$ , then ( $\omega$  is complex imaginary number)

A.  $z^2 + z$  is purely imaginary number

B.  $z = -\omega^2$

C.  $z = -\omega$

D.  $|z - 1| = 1$  then,

**Answer: A::B::D**

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24. If  $|z - 1| = 1$ , then

A.  $\arg((z - 1 - i)/z)$  can be equal to  $-\pi/4$

B.  $(z - 2)/z$  is purely imaginary number

C.  $(z - 2)/z$  is purely real number

D. if  $\arg(z) = \theta$ , where  $z \neq 0$  and  $\theta$  is acute, then  $1 - 2/z = i \tan \theta$

**Answer: A::B::D**

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25. If  $z_1 = 5 + 12i$  and  $|z_2| = 4$ , then

A. maximum  $(|z_1 + iz_2|) = 17$

B. minimum  $(|z_1 + (1 + i)z_2|) = 13 - 4\sqrt{2}$

C. minimum  $\left| \frac{z_1}{z_2 + \frac{4}{z_2}} \right| = \frac{13}{4}$

D. maximum  $\left| \frac{z_1}{z_2 + \frac{4}{z_2}} \right| = \frac{13}{3}$

Answer: A::B::D



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26. Let  $z_1, z_2, z_3$  be the three nonzero complex numbers such that

$z_2 \neq 1, a = |z_1|, b = |z_2|$  and  $c = |z_3|$ . Let  $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$  Then

A.  $\arg\left(\frac{z_3}{z_2}\right) = \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$

B. ortho centre of triangle formed by  $z_1, z_2, z_3$  is  $z_1 + z_2 + z_3$

C. if triangle formed by  $z_1, z_2, z_3$  is equilateral, then  $z_1 + z_2 + z_3$  is  $\frac{3\sqrt{3}}{2}|z_1|^2$

D. if triangle formed by  $z_1, z_2, z_3$  is equilateral, then  $z_1 + z_2 + z_3 = 0$

**Answer: A::B::D**



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27.  $z_1$  and  $z_2$  are the roots of the equation  $z^2 - az + b = 0$  where

$|z_1| = |z_2| = 1$  and  $a, b$  are nonzero complex numbers, then

A.  $|a| \leq 1$

B.  $|a| \leq 2$

C.  $2\arg(a) = \arg(b)$

D.  $\arg a = 2\arg(b)$

**Answer: B::C**



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28. If  $|(z - z_1)/(z - z_2)| = 3$ , where  $z_1$  and  $z_2$  are fixed complex numbers and  $z$  is a variable complex complex number, then  $z$  lies on a

- A. circle with  $z_1$  as its interior point
- B. circle with  $z_2$  as its interior point
- C. circle with  $z_1$  as its exterior point
- D. circle with  $z_2$  as its exterior point

**Answer: B::C**



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29. If  $z = x + iy$ , then the equation  $|(2z - i)/(z + 1)| = m$  represents a circle, then  $m$  can be 1/2 b. 1 c. 2 d. 3

- A. 1/2
- B. 1
- C. 2

D. 3

Answer: A::B::C



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30. System of equations  $|z + 3| - |z - 3| = 6$  and  $|z - 4| = r$  where  $r \in R^+$  has

A. one solution if  $r > 1$

B. one solution if  $r > 1$

C. two solutions if  $r = 1$

D. at least one solution

Answer: A::C::D



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31. Let the equation of a ray be  $|z - 2| - |z - 1 - i| = \sqrt{2}$ . If the ray strikes the y-axis, then the equation of the reflected ray (including or excluding the point of incidence) is .

A.  $\arg(z - 2i) = \frac{\pi}{4}$

B.  $|z - 2i| - |z - 1 - i| = \sqrt{2}$

C.  $\arg(z - 2i) = \frac{3\pi}{4}$

D.  $|z - 1i| - |z - 1 - 3i| = 2\sqrt{2}$

Answer: A:B



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32. Given that the two curves  $\arg(z) = \frac{\pi}{6}$  and  $|z - 2\sqrt{3}i| = r$  intersect in two distinct points, then  $[r] \neq 2$  b. 0

A.  $[r] \neq 2$  where  $[.]$  represents greatest integer

B.  $0 < r < 3$

C.  $r = 6$

D.  $3 < r < 2\sqrt{3}$

**Answer: A::D**

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**33.** On the Argand plane ,let  $z_1 = -2 + 3z$ ,  $z_2 = -2 - 3z$  and  $|z| = 1$ .

Then

A.  $z_1$  moves on circle with centre at  $(-2, 0)$  and radius 3

B.  $z_1$  and  $z_2$  describe the same locus

C.  $z_1$  and  $z_2$  move on differenet circles

D.  $z_1 - z_2$  moves on a circle concetric with  $|z| = 1$

**Answer: A::B::D**

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34. Let  $S = \{z: x = x + iy, y \geq 0, |z - z_0| \leq 1\}$ , where  $|z_0| = |z_0 - \omega| = |z_0 - \omega^2|$ ,  $\omega$  and  $\omega^2$  are non-real cube roots of unity.

Then

- A.  $z_0 = -1$
- B.  $z_0 = -1/2$
- C. if  $z \in S$ , then least value of  $|z|$  is 1
- D.  $|\arg(\omega - z_0)| = \pi/3$

Answer: A::D



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35. If P and Q are represented by the complex numbers  $z_1$  and  $z_2$  such that  $|1/z_2 + 1/z_1| = |1/z_2 - 1/z_1|$ , then

- A.  $\triangle OPQ$  (where O is the origin) is equilateral.
- B.  $\triangle OPQ$  is right angled

C. the circumcentre of  $\Delta OPQ$  is  $\frac{1}{2}(z_1 + z_2)$

D. the circumcentre of  $\Delta OPQ$  is  $\frac{1}{2}(z_1 - z_2)$

**Answer: B::C**



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36. Locus of complex number satisfying are

$\arg[(z - 5 + 4i) / (z + 3 - 2i)] = -\pi/4$  is the arc of a circle

A. whose radius is  $5\sqrt{2}$

B. whose radius is 5

C. whose length (of arc) is  $\frac{15\pi}{\sqrt{2}}$

D. whose centre is  $-2-5i$

**Answer: A::B::C**



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37. Equation of tangent drawn to circle  $|z| = r$  at the point  $A(z_0)$ , is

A.  $Re\left(\frac{z}{z_0}\right) = 1$

B.  $z\bar{z}_0 + z_0\bar{z} = 2r^2$

C.  $Im\left(\frac{z}{z_0}\right) = 1$

D.  $Im\left(\frac{z_0}{z}\right) = 1$

Answer: A::B



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38. If  $n$  is a natural number  $> 2$ , such that  $z^n = (z + 1)^n$ , then

A. roots of equation lie on a straight line parallel to the y-axis

B. roots of equation lie on a straight line parallel to the x-axis

C. sum of the real parts of the roots is  $-\frac{(n-1)}{2}$

D. none of these

**Answer: A::C**



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39. If  $|z - (1/z)| = 1$ , then  $(|z|)_{\max} = \frac{1 + \sqrt{5}}{2}$  b.  $(|z|)_{\min} = \frac{\sqrt{5} - 1}{2}$  c.  
 $(|z|)_{\max} = \frac{\sqrt{5} - 2}{2}$  d.  $(|z|)_{\min} = \frac{\sqrt{5} - 1}{\sqrt{2}}$

A.  $|z|_{\max} = \frac{1 + \sqrt{5}}{2}$

B.  $|z|_{\min} = \frac{\sqrt{5} - 1}{2}$

C.  $|z|_{\max} = \frac{\sqrt{4} - 2}{2}$

D.  $|z|_{\min} = \frac{\sqrt{5} - 1}{2}$

**Answer: A::B**



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40. If  $1, z_1, z_2, z_3, \dots, z_{n-1}$  be the  $n$ th roots of unity and  $\omega$  be a non-real complex cube root of unity then the product  $\prod_{r=1}^{n-1} (\omega - z_r)$  can be equal to

- A. 0
- B. 1
- C. -1
- D.  $1 + \omega$

Answer: A::B::C

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41. Let  $z$  be a complex number satisfying equation  $z^p - z^{-q}$ , where  $p, q \in N$ , then if  $p = q$ , then number of solutions of equation will be infinite. if  $p = q$ , then number of solutions of equation will be finite. if  $p \neq q$ , then number of solutions of equation will be  $p + q + 1$ . if  $p \neq q$ , then number of solutions of equation will be  $p + q$ .

- A. if  $p=q$ , then number of solution of equation will infinite.
- B. if  $p=q$ , then number of solutions of equation will finite
- C. if  $p \neq q$ , then number of solutions of equation will  $p + q + 1$ .
- D. if  $p \neq q$ , then number of solutions of equation will be  $p + q$

**Answer: A::B**

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**42.** Which of the following is true?

- A. The number of common roots of  $z^{144} = 1$  and  $z^{24} = 1$  is 24
- B. The number of common roots of  $z^{360} = 1$  and  $z^{315} = 1$  is 45
- C. The number of roots common to  $z^{24} = 1$ ,  $z^{20} = 1$  and  $z^{56} = 1$  is 4
- D. The number of roots common to  $z^{27} = 1$ ,  $z^{125} = 1$  and  $z^{49} = 1$  is 1

**Answer: A::B::C::D**

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43. If from a point P representing the complex number  $z_1$  on the curve  $|z| = 2$ , two tangents are drawn from P to the curve  $|z| = 1$ , meeting at points  $Q(z_2)$  and  $R(z_3)$ , then :

A. complex number  $(z_1 + z_2 + z_3)/3$  will be on the curve  $|z| = 1$

B.  $\left(\frac{4}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right) = 9$

C.  $\arg\left(\frac{z_2}{z_3}\right) = \frac{2\pi}{3}$

D. orthocentre and circumcenter of  $\Delta PQR$  will coincide

Answer: A::B::C::D



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44. A complex number  $z$  is rotated in anticlockwise direction by an angle  $\alpha$  and we get  $z'$  and if the same complex number  $z$  is rotated by an angle  $\alpha$  in clockwise direction and we get  $z''$  then

A.  $z', z', z''$  are in G.P

B.  $z', z', z''$  are H.P

C.  $z' + z'' = 2z \cos \alpha$

D.  $z'^2 + z''^2 = 2z^2 \cos 2\alpha$

**Answer: A::C::D**

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**45.**  $z_1, z_2, z_3$  and  $z_1, z_2, z_3$  are nonzero complex numbers such that  $z_3 = (1 - \lambda)z_1 + \lambda z_2$  and  $z_3 = (1 - \mu)z_1 + \mu z_2$ , then which of the following statements is/are true?

A. If  $\lambda, \mu \in \mathbb{R} - \{0\}$ , then  $z_1, z_2$  and  $z_3$  are collinear and  $z_1, z_2, z_3$  are collinear separately.

B. If  $\lambda, \mu$  are complex numbers, where  $\lambda = \mu$ , then triangles formed by points  $z_1, z_2, z_3$  and  $z'_1, z'_2, z'_3$  are similar.



C. If  $\lambda, \mu$  are distinct complex numbers, then points  $z_1, z_2, z_3$  and

$z'_1, z'_2, z_3$  are not connected by any well defined geometry.

D. If  $0 < \lambda < 1$ , then  $z_3$  divides the line joining  $z_1$  and  $z_2$  internally

and if  $\mu > 1$ , then  $z_3$  divides the following of  $z'_1, z'_2$  externally

Answer: A::B::C::D

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46. Given  $z = f(x) + ig(x)$  where  $f, g: (0, 1) \rightarrow \mathbb{R}$  are real valued functions. Then which of the following does not hold good?

$z = \frac{1}{1-ix} + i\frac{1}{1+ix}$       b.       $z = \frac{1}{1+ix} + i\frac{1}{1-ix}$       c.

$z = \frac{1}{1+ix} + i\frac{1}{1+ix}$       d.  $z = \frac{1}{1-ix} + i\frac{1}{1-ix}$

A.  $z = \frac{1}{1-ix} + i\left(\frac{1}{1+ix}\right)$

B.  $z = \frac{1}{1+ix} + i\left(\frac{1}{1-ix}\right)$

C.  $z = \frac{1}{1+ix} + i\left(\frac{1}{1+ix}\right)$

D.  $z = \frac{1}{1-ix} + i\left(\frac{1}{1-ix}\right)$

Answer: A::C::D



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47. Let  $a, b, c$  be distinct complex numbers with  $|a| = |b| = |c| = 1$  and  $z_1, z_2$  be the roots of the equation  $az^2 + bz + c = 0$  with  $|z_1| = 1$ . Let  $P$  and  $Q$  represent the complex numbers  $z_1$  and  $z_2$  in the Argand plane with  $\angle POQ = \theta, 0^\circ < 180^\circ$  (where  $O$  being the origin). Then

A.  $b^2 = ac$

B.  $PQ = \sqrt{3}$

C.  $\theta = \frac{\pi}{3}$

D.  $\theta = \frac{2\pi}{3}$

Answer: A::B::D



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48. If  $a, b, c, d \in R$  and all the three roots of  $az^3 + bz^2 + cZ + d = 0$  have negative real parts, then

A.  $ab > 0$

B.  $bv > 0$

C.  $ad > 0$

D.  $bc - ad > 0$

Answer: A::B::C::D



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49. If  $\frac{3}{2 + e^{i\theta}} = ax + iby$ , then the locus of  $P(x, y)$  will represent

A. ellipse of  $a=1, b=2$

B. circle if  $a=b=1$

C. pair of straight line if  $a=1, b=0$

D. None of these

**Answer: A::B::C**



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## Exercise (Comprehension)

1. Consider the complex number  $z = (1 - i \sin \theta) / (1 + i \cos \theta)$ .

The value of  $\theta$  for which  $z$  is purely real are

A.  $n\pi - \frac{\pi}{4}, n \in I$

B.  $\pi n + \frac{\pi}{4}, n \in I$

C.  $n\pi, n \in I$

D. None of these

**Answer: A**



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2. Consider the complex number  $z = (1 - i \sin \theta) / (1 + i \cos \theta)$ .

The value of  $\theta$  for which  $z$  is purely imaginary are

A.  $n\pi - \frac{\pi}{4}, n \in I$

B.  $n\pi + \frac{\pi}{4}, n \in I$

C.  $n\pi, n \in I$

D. no real values of  $\theta$

**Answer: D**



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3. Consider the complex number  $z = (1 - i \sin \theta) / (1 + i \cos \theta)$ .

The value of  $\theta$  for which  $z$  is unimodular give by

A.  $n\pi \pm \frac{\pi}{6}, n \in I$

B.  $n\pi \pm \frac{\pi}{3}, n \in I$

C.  $n\pi \pm \frac{\pi}{4}, n \in I$

D. no real values of  $\theta$

**Answer: C**



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4. Consider the complex number  $z = (1 - i \sin \theta) / (1 + i \cos \theta)$ .

If argument of  $z$  is  $\pi/4$ , then

A.  $\theta = n\pi, n \in I$  only

B.  $\theta = (2n + 1)\pi, n \in I$  only

C. both  $\theta = n\pi$  and  $\theta = (2n + 1)\frac{\pi}{2}, n \in I$

D. none of these

**Answer: D**



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5. Consider the complex numbers  $z_1$  and  $z_2$  Satisfying the relation

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$

Complex number  $z_1 \bar{z}_2$  is

- A. purely real
- B. purely imaginary
- C. zero
- D. none of these

**Answer: B**



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6. Consider the complex numbers  $z_1$  and  $z_2$  Satisfying the relation

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$

Complex number  $z_1 / z_2$  is

- A. purely real
- B. purely imaginary

C. zero

D. none of these

**Answer: B**



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7. Consider the complex numbers  $z_1$  and  $z_2$  Satisfying the relation

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$

One of the possible argument of complex number  $i(z_1/z_2)$

A.  $\frac{\pi}{2}$

B.  $-\frac{\pi}{2}$

C. 0

D. none of these

**Answer: C**



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8. Consider the complex numbers  $z_1$  and  $z_2$  Satisfying the relation  $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$  Possible difference between the argument of  $z_1$  and  $z_2$  is

A. 0

B.  $\pi$

C.  $-\frac{\pi}{2}$

D. none of these

**Answer: C**



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9. Let  $z$  be a complex number satisfying  $z^2 + 2z\lambda + 1 = 0$ , where  $\lambda$  is a parameter which can take any real value.

The roots of this equation lie on a certain circle if

A.  $-1 < \lambda < 1$

B.  $\lambda > 1$

C.  $\lambda < 1$

D. none of these

**Answer: A**

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10. Let  $z$  be a complex number satisfying  $z^2 + 2z\lambda + 1 = 0$ , where  $\lambda$  is a parameter which can take any real value.

One root lies inside the unit circle and one outside if

A.  $-1 < \lambda < 1$

B.  $\lambda > 1$

C.  $\lambda < 1$

D. none of these

**Answer: B**

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11. Let  $z$  be a complex number satisfying  $z^2 + 2z\lambda + 1 = 0$ , where  $\lambda$  is a parameter which can take any real value.

For every large value of  $\lambda$  the roots are approximately.

A.  $-2\lambda, 1/\lambda$

B.  $-\lambda, -1/\lambda$

C.  $-2\lambda, -\frac{1}{2\lambda}$

D. none of these

**Answer: C**

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12. The roots of the equation  $z^4 + az^3 + (12 + 9i)z^2 + bz = 0$  (where  $a$  and  $b$  are complex numbers) are the vertices of a square. Then

The value of  $|a - b|$  is

A.  $5\sqrt{5}$

B.  $\sqrt{130}$

C. 12

D.  $\sqrt{175}$

**Answer: B**



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**13.** The roots of the equation  $z^4 + az^3 + (12 + 9i)z^2 + bz = 0$  (where  $a$  and  $b$  are complex numbers) are the vertices of a square. Then The area of the square is

A. 25 sq.units

B. 20 sq.units

C. 5 sq.unit

D. 4 sq .units

**Answer: C**



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14. Consider a quadratic equation  $az^2 + bz + c = 0$ , where  $a, b, c$  are complex numbers.

The condition that the equation has one purely imaginary root is

A.  $(c\bar{a} - a\bar{c})^2 = (b\bar{c} + c\bar{b})(a\bar{a} - \bar{a}b)$

B.  $(c\bar{c} - a\bar{c})^2 = (b\bar{c} - c\bar{a})(a\bar{b} + \bar{a}b)$

C.  $(c\bar{a} - a\bar{c})^2 = (b\bar{c} + c\bar{b})(a\bar{b} + \bar{a}b)$

D. None of these

**Answer: A**



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15. Consider a quadratic equation  $az^2 + bz + c = 0$ , where  $a, b, c$  are complex numbers. If the equation has two purely imaginary roots, then which of the following is not true.

A.  $a\bar{b}$  is purely imaginary

B.  $b\bar{c}$  is purely imaginary

C.  $c\bar{a}$  is purely real

D. none of these

**Answer: D**



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16. Consider a quadratic equation  $az^2 + bz + c = 0$ , where  $a, b, c$  are complex numbers.

The condition that the equation has one purely real root is

A.  $(c\bar{a} - a\bar{c})^2 = (b\bar{c} + c\bar{b})(a\bar{a} - \bar{a}b)$

$$B. (c\bar{c} - a\bar{c})^2 = (b\bar{c} - c\bar{a})^2(a\bar{b} + \bar{a}b)$$

$$C. (c\bar{a} - a\bar{c})^2 = (b\bar{c} + c\bar{b})(a\bar{b} + \bar{a}b)$$

$$D. (c\bar{a} - a\bar{c})^2 = (b\bar{c} - c\bar{b})(a\bar{b} - \bar{a}b)$$

**Answer: D**



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17. Suppose  $z$  and  $\omega$  are two complex number such that  $|z + i\omega| = 2$ .

Which of the following is true about  $|z|$  and  $|\omega|$ ?

$$A. |z| = |\omega| = \frac{1}{2}$$

$$B. |z| = \frac{1}{2}, |\omega|, |\omega| = \frac{3}{4}$$

$$C. |z| = |\omega| = \frac{3}{4}$$

$$D. |z| = |\omega| = 1$$

**Answer: D**



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18. Suppose  $z$  and  $\omega$  are two complex number such that Which of the following is true for  $z$  and  $\omega$ ?

A.  $Re(z) = Re(\omega) = \frac{1}{2}$

B.  $Im(z) = Im(\omega)$

C.  $Re(z) = Im(\omega)$

D.  $Im(z) = Re(\omega)$

**Answer: D**



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19. Suppose  $z$  and  $\omega$  are two complex number such that  $|z| \leq 1$ ,  $|\omega| \leq 1$  and  $|z + i\omega| = |z - i\bar{\omega}| = 2$  The complex number of  $\omega$  can be

A. 1 or -i

B. -1



C.  $I$  or  $-i$

D.  $\omega$  or  $\omega^2$  ( where  $\omega$  is the cube root of unity)

**Answer: C**



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20. Consider the equation of line  $a\bar{z} + a\bar{z} + a\bar{z} + b = 0$ , where  $b$  is a real parameter and  $a$  is fixed non-zero complex number.

The intercept of line on real axis is given by

A.  $\frac{-2b}{a + \bar{a}}$

B.  $\frac{-b}{2(a + \bar{a})}$

C.  $\frac{-b}{a + \bar{a}}$

D.  $\frac{b}{a + \bar{a}}$

**Answer: C**



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21. Consider the equation of line  $a\bar{z} + a\bar{z} + a\bar{z} + b = 0$ , where  $b$  is a real parameter and  $a$  is fixed non-zero complex number.

The intercept of line on imaginary axis is given by

A.  $\frac{b}{\bar{a} - a}$

B.  $\frac{2b}{\bar{a} - a}$

C.  $\frac{b}{2(\bar{a} - a)}$

D.  $\frac{b}{a - \bar{a}}$

**Answer: D**



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22. Consider the equation of line  $a\bar{z} + a\bar{z} + a\bar{z} + b = 0$ , where  $b$  is a real parameter and  $a$  is fixed non-zero complex number.

The locus of mid-point of the line intercepted between real and imaginary axis is given by

A.  $az - \overline{az} = 0$

B.  $az + \overline{az} = 0$

C.  $az - \overline{az} + b = 0$

D.  $az - \overline{az} + 2b = 0$

**Answer: B**

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**23.** Consider the equation  $az + \overline{bz} + c = 0$ , where  $a, b, c \in \mathbb{Z}$

If  $|a| \neq |b|$ , then  $z$  represents

A. circle

B. straight line

C. one point

D. ellipse

**Answer: C**

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24. Consider the equation  $az + \overline{bz} + c = 0$ , where  $a, b, c \in \mathbb{Z}$

If  $|a| = |b|$  and  $\overline{ac} \neq b\overline{c}$ , then  $z$  has

- A. infinite solutions
- B. no solutions
- C. finite solutions
- D. cannot say anything

**Answer: B**

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25. Consider the equation  $az + \overline{bz} + c = 0$ , where  $a, b, c \in \mathbb{Z}$

If  $|a| = |b| \neq 0$  and  $az + b\overline{c} + c = 0$  represents

- A. an ellipse

B. a circle

C. a point

D. a straight line

**Answer: D**



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**26.** Complex numbers  $z$  satisfy the equation  $|z - (4/z)| = 2$

The difference between the least and the greatest moduli of complex number is

A. 2

B. 4

C. 1

D. 3

**Answer: A**

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27. Complex numbers  $z$  satisfy the equation  $|z - (4/z)| = 2$

The value of  $\arg(z_1/z_2)$  where  $z_1$  and  $z_2$  are complex numbers with the greatest and the least moduli, can be

A.  $2\pi$

B.  $\pi$

C.  $\pi/2$

D. none of these

**Answer: B**

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28. Complex numbers  $z$  satisfy the equation  $|z - (4/z)| = 2$

Locus of  $z$  if  $|z - z_1| = |z - z_2|$ , where  $z_1$  and  $z_2$  are complex numbers with the greatest and the least moduli, is

- A. line parallel to the real axis
- B. line parallel to the imaginary axis
- C. line having a positive slope
- D. line having a negative slope

**Answer: B**

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29. In an Argand plane  $z_1, z_2$  and  $z_3$  are, respectively, the vertices of an isosceles triangle ABC with  $AC = BC$  and  $\angle CAB = \theta$ . If  $z_4$  is incentre of triangle, then

The value of  $AB \times AC / (IA)^2$  is

- A.  $\frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2}$
- B.  $\frac{(z_2 - z_1)(z_1 - z_3)}{(z_4 - z_1)^2}$
- C.  $\frac{(z_4 - z_1)^2}{(z_2 - z_1)(z_3 - z_1)}$

D. none of these

**Answer: A**



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30. In an Argand plane  $z_1, z_2$  and  $z_3$  are, respectively, the vertices of an isosceles triangle ABC with  $AC = BC$  and  $\angle CAB = \theta$ . If  $z_4$  is incentre of triangle, then

The value of  $(z_4 - z_1)^2(\cos \theta + 1)\sec \theta$  is

A.  $\frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)}$

B.  $(z_2 - z_1)(z_3 - z_1)$

C.  $(z_2 - z_1)(z_3 - z_1)^2$

D.  $\frac{(z_2 - z_1)(z_1 - z_3)}{(z_4 - z_1)^2}$

**Answer: B**



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31. In an Argand plane  $z_1, z_2$  and  $z_3$  are, respectively, the vertices of an isosceles triangle ABC with  $AC = BC$  and  $\angle CAB = \theta$ . If  $z_4$  is incentre of triangle, then

The value of  $(z_2 - z_1)^2 \tan \theta \tan \theta / 2$  is

- A.  $(z_1 + z_2 - 2z_3)$
- B.  $(z_1 + z_2 - z_3)(z_1 + z_2 - z_4)$
- C.  $-(z_1 + z_2 - 2z_3)(z_1 + z_2 - 2z_4)$
- D.  $z_4 = \sqrt{z_2 z_3}$

**Answer: C**



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32.  $A(z_1), B(z_2)$  and  $C(z_3)$  are the vertices of triangle ABC inscribed in the circle  $|z|=2$ , internal angle bisector of angle A meets the circumcircle again at  $D(z_4)$ . Point D is:

A.  $z_4 = \frac{1}{z_2} + \frac{1}{z_3}$

B.  $\sqrt{\frac{z_2 + z_3}{z_1}}$

C.  $\sqrt{\frac{z_2 z_3}{z_1}}$

D.  $z_4 = \sqrt{z_2 z_3}$

**Answer: D**

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**33.**  $A(z_1), B(z_2)$  and  $C(z_3)$  are the vertices of triangle ABC inscribed in the circle  $|z|=2$ , internal angle bisector of angle A meets the circumcircle again at  $D(z_4)$ . Point D is:

A.  $\frac{\pi}{2}$

B.  $\frac{\pi}{3}$

C.  $\frac{\pi}{2}$

D.  $\frac{2\pi}{3}$

Answer: C



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## MATRIX MATCH TYPE

1. The graph of the quadratic function  $y = ax^2 + bx + c$  is as shown in the following figure.



Now, match the complex numbers given in List I with the corresponding arguments in List II.



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2. Let  $z_1, z_2$  and  $z_3$  be the vertices of triangle. Then match following lists.





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3. Match following lists.



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4. Complex number  $z$  satisfies the equation

$$|z - 5i| + m|z - 12i| = n. \text{ Then match the value of } m \text{ and } n \text{ in List I}$$

with the corresponding locus in List II.



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5. Complex number  $z$  lies on the curve  $S \equiv ar \frac{g(z + 3)}{z + 3i} = -\frac{\pi}{4}$

Now, match the locus in List I with its number of points of intersection

with the curve  $S$  in List II.



- A.           a b c d  
 (1) p q p r
- B.           a b c d  
 (2) s r q p
- C.           a b c d  
 (3) q p q r
- D.           a b c d  
 (4) s p q r

**Answer: A**

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6. Consider sets

$$A = \{z \in C : z^{27} - 1 = 0\} \text{ and } B = \{z \in C : z^{36} - 1 = 0\}$$

Now, match the following lists.



- A.           a b c d  
 (1) p q p r
- B.           a b c d  
 (2) r q s p
- C.           a b c d  
 (3) q p q r
- D.           a b c d  
 (4) s p q r

**Answer: B**



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7. Match the statements in List I with those in List II

[Note: Here  $z$  take the values in the complex place and  $\text{Im}(z)$  and  $\text{Re}(z)$  denote, respectively, the imaginary part and the real part of  $z$ ].



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8. Let  $z_k = \cos\left(\frac{2k\pi}{10}\right) - i \sin\left(\frac{2k\pi}{10}\right)$ ,  $k = 1, 2, \dots, 9$



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9. Match the statements/experssions given in List I with the values given in List II.



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## Exercise (Numerical)

1. If  $x = a + bi$  is a complex number such that  $x^2 = 3 + 4i$  and  $x^3 = 2 + i$ , where  $i = \sqrt{-1}$ , then  $(a + b)$  equal to \_\_\_\_\_.



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2. If the complex numbers  $x$  and  $y$  satisfy  $x^3 - y^3 = 98i$  and  $x - y = 7i$ , then  $xy = a + ib$ , where  $a, b \in \mathbb{R}$ . The value of  $(a + b)/3$  equals \_\_\_\_\_.



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3. If  $x = \omega - \omega^2 - 2$  then , the value of  $x^4 + 3x^3 + 2x^2 - 11x - 6$  is  
(where  $\omega$  is a imaginary cube root of unity)

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4. Let  $z = 9 + bi$ , where  $b$  is nonzero real and  $i^2 = -1$ . If the imaginary part of  $z^2$  and  $z^3$  are equal, then  $b/3$  is \_\_\_\_\_.

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5. Modulus of nonzero complex number  $z$  satisfying  $\bar{z} + z = 0$  and  $|z|^2 - 4iz = z^2$  is \_\_\_\_\_.

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6. If the expression  $(1 + ir)^3$  is of the form of  $s(1 + i)$  for some real 's' where 'r' is also real and  $i = \sqrt{-1}$



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7. If complex number  $z (z \neq 2)$  satisfies the equation  $z^2 = 4z + |z|^2 + \frac{16}{|z|^3}$ , then the value of  $|z|^4$  is\_\_\_\_\_.

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8. The complex number  $z$  satisfies  $z + |z| = 2 + 8i$ . find the value of  $|z| - 8$

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9. Let  $|z| = 2$  and  $w = \frac{z+1}{z-1}$ , where  $z, w, \in C$  (where  $C$  is the set of complex numbers). Then product of least and greatest value of modulus of  $w$  is\_\_\_\_\_.

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10. If  $z$  is a complex number satisfying  $z^4 + z^3 + 2z^2 + z + 1 = 0$  then the set of possible values of  $z$  is

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11. Let  $1, \omega, \omega^2$  be the cube roots of unity. The least possible degree of a polynomial with real coefficients having roots  $2\omega, (2 + 3\omega), (2 + 3\omega^2), (2 - \omega - \omega^2)$  is \_\_\_\_\_.

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12. If  $\omega$  is the imaginary cube roots of unity, then the number of pair of integers  $(a,b)$  such that  $|a\omega + b| = 1$  is \_\_\_\_\_.

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13. Suppose that  $z$  is a complex number the satisfies  $|z - 2 - 2i| \leq 1$ . The maximum value of  $|2iz + 4|$  is equal to \_\_\_\_\_.



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14. If  $|z + 2 - i| = 5$  and maximum value of  $|3z + 9 - 7i|$  is  $M$ , then the value of  $M$  is \_\_\_\_\_.



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15. Let  $Z_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$  and  $Z_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$  are two complex numbers. If  $Z_1 \cdot Z_2 = a + ib$  where  $a, b \in \mathbb{R}$  then the largest value of  $(a + b) \forall \theta \in \mathbb{R}$ , is



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16. Let  $A = \{a \in \mathbb{R}\}$  the equation  $(1 + 2i)x^3 - 2(3 + i)x^2 + (5 - 4i)x + a^2 = 0$  has at least one real root. Then the value of  $\frac{\sum a^2}{2}$  is \_\_\_\_\_.



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17. Find the minimum value of the expression

$$E = |z|^2 + |z - 3|^2 + |z - 6i|^2 \text{ (where } z = x + iy, x, y \in R)$$

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18. If  $z_1$  lies on  $|z - 3| + |z + 3| = 8$  such that  $\arg z_1 = \pi/6$ , then

$$37|z_1|^2 = \text{_____}.$$

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19. If  $z$  satisfies the condition  $\arg(z + i) = \frac{\pi}{4}$ . Then the minimum value of  $|z + 1 - i| + |z - 2 + 3i|$  is \_\_\_\_\_.

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20. Let  $\omega \neq 1$  be a complex cube root of unity. If

$$(4 + 5\omega + 6\omega^2)^{n^2+2} + (6 + 5\omega^2 + 4\omega)^{n^2+2} + (5 + 6\omega + 4\omega^2)^{n^2+2} = 0,$$

and  $n \in N$ , where  $n \in [1, 100]$ , then number of values of  $n$  is \_\_\_\_\_.



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21. Let  $z$  be a non - real complex number which satisfies the equation

$$z^{23} = 1. \text{ Then the value of } \sum_{k=22}^{k=1} \frac{1}{1 + z^{8k} + z^{16k}}$$



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22. If  $z, z_1$  and  $z_2$  are complex numbers such that  $z = z_1 z_2$  and

$$|\bar{z}_2 - z_1| \leq 1, \text{ then maximum value of } |z| - \text{Re}(z) \text{ is } \underline{\hspace{2cm}}.$$



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23. Let  $z_1, z_2$  and  $z_3$  be three complex numbers such that  $z_1 + z_2 + z_3 = z_1z_2 + z_2z_3 + z_1z_3 = z_1z_2z_3 = 1$ . Then the area of triangle formed by points  $A(z_1), B(z_2)$  and  $C(z_3)$  in complex plane is \_\_\_\_\_.

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24. Let  $\alpha$  be the non-real 5 th root of unity. If  $z_1$  and  $z_2$  are two complex numbers lying on  $|z| = 2$ , then the value of  $\sum_{t=0}^4 |z_1 + \alpha^t z_2|^2$  is \_\_\_\_\_.

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25. Let  $z_1, z_2, z_3 \in C$  such that  $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 4$ . If  $|z_1 - z_2| = |z_1 + z_3|$  and  $z_2 \neq z_3$ , then values of  $|z_1 + z_2| \cdot |z_1 + z_3|$  is \_\_\_\_\_.

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26. Let  $A(z_1)$  and  $B(z_2)$  be lying on the curve  $|z - 3 - 4i| = 5$ , where  $|z_1|$  is maximum. Now,  $A(z_1)$  is rotated about the origin in anticlockwise direction through  $90^\circ$  reaching at  $P(z_0)$ . If  $A, B$  and  $P$  are collinear then the value of  $(|z_0 - z_1| \cdot |z_0 - z_2|)$  is \_\_\_\_\_.

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27. If  $z_1, z_2, z_3$  are three points lying on the circle  $|z| = 2$  then the minimum value of the expression  $|z_1 z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2 =$

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28. Minimum value of  $|z_1 + 1| + |z_2 + 1| + |z_1 z_2 + 1|$  if  $|z_1| = 1$  and  $|z_2| = 1$  is \_\_\_\_\_.

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29. If  $|z_1| = 2$  and  $(1 - i)z_2 + (1 + i)\bar{z}_2 = 8\sqrt{2}$ , then the minimum value of  $|z_1 - z_2|$  is \_\_\_\_\_.

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30. Given that  $1 + 2|z|^2 = |z^2 + 1|^2 + 2|z + 1|^2$ , then the value of  $|z(z + 1)|$  is \_\_\_\_\_.

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## JEE Main Previous Year

1. If  $\left|z - \frac{4}{z}\right| = 2$ , then the maximum value of  $|z|$  is equal to (1)  $\sqrt{3} + 1$   
(2)  $\sqrt{5} + 1$  (3) 2 (4)  $2 + \sqrt{2}$

A.  $\sqrt{3} + 1$

B.  $\sqrt{5} + 1$



C. 2

D.  $2 + \sqrt{2}$

**Answer: B**

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2. The number of complex numbers  $z$  such that  $|z| = |z + 1| = |zi|$  equals (1) 1 (2) 2 (3)  $\infty$  (4) 0

A.  $\infty$

B. 0

C. 1

D. 2

**Answer: C**

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3. Let  $\alpha, \beta$  be real and  $z$  be a complex number. If  $z^2 + \alpha z + \beta = 0$  has two distinct roots on the line  $\operatorname{Re} z = 1$ , then it is necessary that : (1)  $b \in (0, 1)$  (2)  $b \in (-1, 0)$  (3)  $|\beta| = 1$  (4)  $b \in (1, \infty)$

A.  $\beta \in (1, \infty)$

B.  $\beta \in (0, 1)$

C.  $\beta \in (-1, 0)$

D.  $|\beta| = 1$

**Answer: A**



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4. If  $\omega \neq 1$  is a cubic root of unit and  $(1 + \omega)^7 = A + B\omega$ , then  $(A, B)$  equals

A.  $(-1, 1)$

B.  $(0, 1)$

C.  $(1, 1)$

D. (1, 0)

**Answer: C**



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5. If  $z \neq 1$  and  $\frac{z^2}{z-1}$  is real, then the point represented by the complex number  $z$  lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis

- A. either on the real axis or on a circle passing through the origin.
- B. on a circle with centre at the origin.
- C. either on the real axis or on a circle not passing through the origin .
- D. on the imaginary axis .

**Answer: A**



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6. If  $z$  is a complex number of unit modulus and argument  $q$ , then

$\arg\left(\frac{1+z}{1+\bar{z}}\right)$  equal (1)  $\frac{\pi}{2} - \theta$  (2)  $\theta$  (3)  $\pi - \theta$  (4)  $-\theta$

A.  $-\theta$

B.  $\frac{\pi}{2} - \theta$

C.  $\theta$

D.  $\pi - \theta$

**Answer: C**



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7. If  $z$  is a complex number such that  $|z| \geq 2$  then the minimum value of

$\left|z + \frac{1}{2}\right|$  is

A. is equal to  $\frac{5}{2}$

B. lies in the interval (1,2)

C. is strictly greater than  $\frac{5}{2}$

D. is strictly greater than  $\frac{3}{2}$  but less than  $\frac{5}{2}$

**Answer: B**



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8. If  $z_1$  and  $z_2$  are two complex numbers such that  $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$  is unimodular whereas  $z_1$  is not unimodular then  $|z_1| =$

A. Straight line parallel to x-axis

B. straight line parallel to y-axis

C. circle of radius 2

D. circle of radius  $\sqrt{2}$

**Answer: C**



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9. A value of  $\theta$  for which  $\frac{2 + 3i \sin \theta}{1 - 2i \sin \theta}$  purely imaginary, is : (1)  $\frac{\pi}{3}$  (2)  $\frac{\pi}{6}$  (3)

$\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$  (4)  $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

A.  $\frac{\pi}{6}$

B.  $\sin^{-1}\left(\frac{\text{Sqrt}(3)}{4}\right)$

C.  $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D.  $\frac{\pi}{3}$

**Answer: C**



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10. If  $\omega \neq 1$  is a cubit root unity and  $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$ , then k is

equal to

A. 1

B.  $\omega$

C.  $-z$

D.  $-1$

**Answer: B**



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11. If  $\alpha, \beta \in C$  are distinct roots of the equation  $x^2 + 1 = 0$  then  $\alpha^{101} + \beta^{107}$  is equal to

A. 2

B.  $-1$

C. 0

D. 1

**Answer: D**



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1. Let  $z = x + iy$  be a complex number where  $x$  and  $y$  are integers. Then, the area of the rectangle whose vertices are the roots of the equation  $zz^3 + zz^3 = 350$  is 48 (b) 32 (c) 40 (d) 80

A. 48

B. 32

C. 40

D. 80

**Answer: A**



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2. Let  $z$  be a complex number such that the imaginary part of  $z$  is nonzero and  $a = z^2 + z + 1$  is real. Then  $a$  cannot take the value (A)  $-1$  (B)  $1/3$  (C)  $1/2$  (D)  $3/4$



A. -1

B.  $\frac{1}{3}$

C.  $\frac{1}{2}$

D.  $\frac{3}{4}$

**Answer: D**



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3. Let complex numbers  $\alpha$  and  $\frac{1}{\alpha}$  lies on circle  $(x - x_0)^2 + (y - y_0)^2 = r^2$  and  $(x - x_0)^2 + (y - y_0)^2 = 4r^2$  respectively.

If  $z_0 = x_0 + iy_0$  satisfies the equation  $2|z_0|^2 = r^2 + 2$  then  $|\alpha|$  is equal

to (a)  $\frac{1}{\sqrt{2}}$  (b)  $\frac{1}{2}$  (c)  $\frac{1}{\sqrt{7}}$  (d)  $\frac{1}{3}$

A.  $1/\sqrt{2}$

B.  $1/2$

C.  $1/\sqrt{7}$

D.  $1/3$

Answer: C



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4. Let  $Z_1$  and  $Z_2$ , be two distinct complex numbers and let  $w = (1 - t)z_1 + tz_2$  for some number "t" with  $0 < t < 1$

A.  $|z - z_1| + |z - z_2| = |z_1 - z_2|$

B.  $(z - z_1) = (z - z_2)$

C.  $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D.  $\arg(z - z_1) = \arg(z_2 - z_1)$

Answer: A::C::D



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5. Let  $w = (\sqrt{3} + \frac{i}{2})$  and  $P = \{w^n : n = 1, 2, 3, \dots\}$ , Further  $H_1 = \left\{ z \in C : \operatorname{Re}(z) > \frac{1}{2} \right\}$  and  $H_2 = \left\{ z \in C : \operatorname{Re}(z) < -\frac{1}{2} \right\}$

Where  $C$  is set of all complex numbers. If  $z_1 \in P \cap H_1$ ,  $z_2 \in P \cap H_2$  and

$O$  represent the origin, then  $\angle Z_1 O Z_2 =$

A.  $\pi/2$

B.  $\pi/6$

C.  $2\pi/3$

D.  $5\pi/6$

**Answer: C::D**



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6. Let  $a, b \in \mathbb{R}$  and  $a^2 + b^2 \neq 0$ . Suppose

$$S = \left\{ z \in C : z = \frac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0 \right\}, \quad \text{where } i = \sqrt{-1}. \quad \text{If}$$

$z = x + iy$  and  $z$  in  $S$ , then  $(x, y)$  lies on

A. the circle with radius  $\frac{1}{2a}$  and centre  $\left( \frac{1}{2a}, 0 \right)$  for  $a > 0, b \neq 0$

B. the circle with radius  $-\frac{1}{2a}$  and centre  $\left( -\frac{1}{2}, 0 \right)$   $a < 0, b \neq 0$

C. the axis for  $a \neq 0, b = 0$

D. the y-axis for  $a = 0, b \neq 0$

**Answer: A::C::D**



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7. Let  $a, b, x$  and  $y$  be real numbers such that  $a - b = 1$  and  $y \neq 0$ . If the complex number  $z = x + iy$  satisfies  $\text{Im}\left(\frac{az + b}{z + 1}\right) = y$ , then which of the following is (are) possible value(s) of  $x$ ? (a)  $-1 - \sqrt{1 - y^2}$  (b)  $1 + \sqrt{1 + y^2}$  (c)  $-1 + \sqrt{1 - y^2}$  (d)  $-1 - \sqrt{1 + y^2}$

A.  $-1 - \sqrt{1 - y^2}$

B.  $1 + \sqrt{1 + y^2}$

C.  $1 - \sqrt{1 + y^2}$

D.  $-1 + \sqrt{1 - y^2}$

**Answer: A::D**



8. For a non-zero complex number  $z$ , let  $\arg(z)$  denote the principal argument with  $-\pi < \arg(z) \leq \pi$ . Then, which of the following statement(s) is (are) FALSE?  $\arg(-1 - i) = \frac{\pi}{4}$ , where  $i = \sqrt{-1}$  (b) The function  $f: \mathbb{R} \rightarrow (-\pi, \pi]$ , defined by  $f(t) = \arg(-1 + it)$  for all  $t \in \mathbb{R}$ , is continuous at all points of  $\mathbb{R}$ , where  $i = \sqrt{-1}$  (c) For any two non-zero complex numbers  $z_1$  and  $z_2$ ,  $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$  is an integer multiple of  $2\pi$  (d) For any three given distinct complex numbers  $z_1, z_2$  and  $z_3$ , the locus of the point  $z$  satisfying the condition  $\arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$ , lies on a straight line

A.  $\arg(-1 - i) = \frac{\pi}{4}$ , where  $i = \sqrt{-1}$

B. The function  $f: \mathbb{R} \rightarrow (-\pi, \pi]$ , defined by  $f(t) = \arg(-1 + it)$

for all  $t \in \mathbb{R}$ , is continuous at all points of  $\mathbb{R}$ , where  $i = \sqrt{-1}$

C. For any two non-zero complex number  $z_1$  and

$z_2$ ,  $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$  is an integer multiple of  $2\pi$

D. For any three given distinct complex numbers  $z_1, z_2$  and  $z_3$  the

locus of the point  $z$  satisfying the condition

$$\left( \frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)} \right) = \pi, \text{ lies on a straight line.}$$

**Answer: A::B::D**



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9. Let  $s, t, r$  be non-zero complex numbers and  $L$  be the set of solutions

$z = x + iy$  ( $x, y \in \mathbb{R}, i = \sqrt{-1}$ ) of the equation  $sz + tz + r = 0$ ,

where  $z = x - iy$ . Then, which of the following statement(s) is (are)

TRUE? (a) If  $L$  has exactly one element, then  $|s| \neq |t|$  (b) If  $|s| = |t|$ , then  $L$

has infinitely many elements (c) The number of elements in

$\ln n\{z: |z - 1 + i| = 5\}$  is at most 2 (d) If  $L$  has more than one element,

then  $L$  has infinitely many elements

A. If  $L$  has exactly one element, then  $|s| \neq |t|$

B. If  $|s| = |t|$  then  $L$  has infinitely many elements

C. The number of elements in  $L \cap \{z: |z - 1 + i| = 5\}$  is at most 2

D. If  $L$  has most than one elements, then  $L$  has infinitely many elements.

**Answer: A::C::D**



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10. Let  $S = S_1 \cap S_2 \cap S_3$ , where  
 $s_1 = \{z \in C: |z| < 4\}$ ,  $S_2 = \left\{ z \in C: \ln \left[ \frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i} \right] > 0 \right\}$  and  $S_3 =$

A.  $\frac{10\pi}{3}$

B.  $\frac{20\pi}{3}$

C.  $\frac{16\pi}{3}$

D.  $\frac{32\pi}{3}$

**Answer: B**



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11. Let  $S = S_1 \cap S_2 \cap S_3$ , where  $S_1 = \{z \in \mathbb{C} : |z| < 4\}$ ,

$$S_2 = \left\{ z \in \mathbb{C} : \operatorname{Im} \left[ \frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i} \right] > 0 \right\} \text{ and } S_3 = \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$$

$$\min_{z \in S} |1 - 3i - z| =$$

A.  $\frac{2 - \sqrt{3}}{2}$

B.  $\frac{2 + \sqrt{3}}{2}$

C.  $\frac{3 - \sqrt{3}}{2}$

D.  $\frac{3 + \sqrt{3}}{2}$

**Answer: C**

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12. Let  $\omega$  be the complex number  $\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right)$ . Then the

number of distinct complex numbers  $z$  satisfying

$$\Delta = \begin{vmatrix} z + 1 & \omega & \omega^2 \\ \omega & z + \omega^2 & 1 \\ \omega^2 & 1 & z + \omega \end{vmatrix} = 0 \text{ is}$$





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13. If  $z$  is any complex number satisfying  $|z - 3 - 2i| \leq 2$  then the maximum value of  $|2z - 6 + 5i|$  is



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14. For any integer  $k$ , let  $\alpha_k = \frac{\cos(k\pi)}{7} + i \sin. \frac{k\pi}{7}$ , where  $I = \sqrt{-1}$ .

Value of the expression  $\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^3 |\alpha_{4k-1} - \alpha_{4k-2}|}$  is \_\_\_\_\_.



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