MATHS

BOOKS - CENGAGE

Complex Numbers

Single correct Answer

1. The value of $\sum_{n=0}^{100} i^{n!}$ equals (where $i = \sqrt{-1}$)

A. -1

B. i

C. $2i + 95$

D. $97 + i$

Answer: C

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2. Suppose n is a natural number such that $\left|i + 2i^2 + 3i^3 + \ldots\ldots + ni^n\right| = 18\sqrt{2}$ where i is the square root of -1. Then n is

A. 9

B. 18

C. 36

D. 72

Answer: C

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3. Let $i = \sqrt{-1}$ Define a sequence of complex number by $z_1 = 0, z_{n+1} = \left(z_n\right)^2 + i$ for $n \geq 1$. In the complex plane, how far from the origin is z_{111}?

A. 1

B. 2

C. 3

D. 4

Answer: B

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4. The complex number, $z = \frac{\left(-\sqrt{3}+3i\right)(1-i)}{\left(3+\sqrt{3}i\right)(i)\left(\sqrt{3}+\sqrt{3}i\right)}$

A. lies on real axis

B. lies on imaginary axis

C. lies in first quadrant

D. lies in second quadrant

Answer: B

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5. a, b, c are positive real numbers forming a G.P. ILf $ax62 + 2bx + c = 0 and dx^2 + 2ex + f = 0$ have a common root, then prove that $d/a, e/b, f/c$ are in A.P.

A. $A. P.$

B. $G. P.$

C. $H. P.$

D. None of these

Answer: C

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6. Prove that the equation $Z^3 + iZ - 1 = 0$ has no real roots.

A. three real roots

B. one real roots

C. no real roots

D. no real or complex roots

Answer: C

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7. If a, b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real whereas the other is purely imaginery, and $a^2 - \bar{a}^2 = kb$, then k is

A. 2

B. 4

C. 6

D. 8

Answer: B

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8. If Z^5 is a non-real complex number, then find the minimum value of $\left|\frac{Imz^5}{Im^5z}\right|$

A. -1

B. -2

C. -4

D. -5

Answer: C

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9. For any complex numbers z_1, z_2 and $z_3, z_3Im\left(\overline{z_2}z_3\right) + z_2Im\left(\overline{z_3}z_1\right) + z_1Im\left(\overline{z_1}z_2\right)$ is

A. 0

B. $z_1 + z_2 + z_3$

C. $z_1 z_2 z_3$

D. $\left(\dfrac{z_1 + z_2 + z_3}{z_1 z_2 z_3}\right)$

Answer: A

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10. The modulus and amplitude of $\dfrac{1 + 2i}{1 - (1 - i)^2}$ are

A. $\sqrt{2}$ and $\dfrac{\pi}{6}$

B. 1 and $\dfrac{\pi}{4}$

C. 1 and 0

D. 1 and $\dfrac{\pi}{3}$

Answer: C

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11. If the argument of $(z - a)(\bar{z} - b)$ is equal to that $\left(\left(\sqrt{3} + i\right)\frac{1 + \sqrt{3}i}{1 + i}\right)$ where a,b,c are two real number and z is the complex conjugate o the complex number z find the locus of z in the rgand diagram. Find the value of a and b so that locus becomes a circle having its centre at $\frac{1}{2}(3 + i)$

A. $(3, 2)$

B. $(2, 1)$

C. $(2, 3)$

D. $(2, 4)$

Answer: B

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12. If a complex number z satisfies $|z|^2 + \frac{4}{(|z|)^2} - 2\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right) - 16 = 0$, then the maximum value of $|z|$ is

A. $\sqrt{6}+1$

B. 4

C. $2+\sqrt{6}$

D. 6

Answer: C

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13. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then $\dfrac{\sin 3\alpha+\sin 3\beta+\sin 3\gamma}{\sin(\alpha+\beta+\gamma)}$ is equal to

A. 1

B. -1

C. 3

D. -3

Answer: C

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14. The least value of $|z - 3 - 4i|^2 + |z + 2 - 7i|^2 + |z - 5 + 2i|^2$ occurs when z=

A. $1 + 3i$

B. $3 + 3i$

C. $3 + 4i$

D. None of these

Answer: D

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15. The roots of the equation $x^4 - 2x^2 + 4 = 0$ are the vertices of a :

A. square inscribed in a circle of radius 2

B. rectangle inscribed in a circle of radius 2

C. square inscribed in a circle of radius $\sqrt{2}$

D. rectangle inscribed in a circle of radius $\sqrt{2}$

Answer: D

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16. If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 2|$, $Re(z_2) = |z_2 - 2|$ and $arg(z_1 - z_2) = \pi/3$, then $Im(z_1 + z_2) =$

A. $2/\sqrt{3}$

B. $4/\sqrt{3}$

C. $2/\sqrt{3}$

D. $\sqrt{3}$

Answer: B

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17. If $z = e^{\frac{2\pi i}{5}}$, then $1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 =$

A. 0

B. $4z^3$

C. $5z^4$

D. $-4z^2$

Answer: C

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18. If $z = (3 + 7i)(a + ib)$, where $a, b \in Z - \{0\}$, is purely imaginery, then minimum value of $|z|^2$ is

A. 74

B. 45

C. 65

D. 58

Answer: D

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19. Let z be a complex number satisfying $|z+16| = 4|z+1|$. Then

A. $|z| = 4$

B. $|z| = 5$

C. $|z| = 6$

D. $3 < |z| < 68$

Answer: A

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20. If $|z| = 1$ and $z' = \dfrac{1+z^2}{z}$, then

A. z' lie on a line not passing through origin

B. $|z'| = \sqrt{2}$

C. $Re(z') = 0$

D. $Im(z') = 0$

Answer: D

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21. a, b,c are three complex numbers on the unit circle $|z| = 1$, such that $abc = a + b + c$. Then $|ab + bc + ca|$ is equal to

A. 3

B. 6

C. 1

D. 2

Answer: C

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22. If $|z_1| = |z_2| = |z_3| = 1$ then value of $|z_1 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$ cannot exceed

A. 6

B. 9

C. 12

D. none of these

Answer: B

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23. Number of ordered pairs (s), (a, b) of real numbers such that $(a + ib)^{2008} = a - ib$ holds good is

A. 2008

B. 2009

C. 2010

D. 1

Answer: C

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24. The region represented by the inequality |2z-3i|<|3z-2i| is

A. the unit disc with its centre at $z = 0$

B. the exterior of the unit circle with its centre at $z = 0$

C. the inerior of a square of side 2 units with its centre at $z = 0$

D. none of these

Answer: B

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25. If ω is any complex number such that $z\omega = |z|^2$ and $|z - \bar{z}| + |\omega + \bar{\omega}| = 4$, then as ω varies, then the area bounded by the locus of z is

A. 4 sq. units

B. 8 sq. units

C. 16 sq. units

D. 12 sq. units

Answer: B

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26. If $az^2 + bz + 1 = 0$, where $a, b \in C$, $|a| = \frac{1}{2}$ and have a root α such that $|\alpha| = 1$ then $|a\bar{b} - b| =$

A. $1/4$

B. $1/2$

C. $5/4$

D. $3/4$

Answer: D

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27. Let p and q are complex numbers such that $|p|+|q|<1$. If z_1 and z_2 are the roots of the $z^2+pz+q=0$, then which one of the following is correct ?

A. $|z_1|<1$ and $|z_2|<1$

B. $|z_1|>1$ and $|z_2|>1$

C. If $|z_1|<1$, then $|z_2|>1$ and vice versa

D. Nothing definite can be said

Answer: A

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28. If z and w are two complex numbers simultaneously satisfying te equations, $z^3 + w^5 = 0$ and $z^2 + \bar{w}^4 = 1$, then

A. z and w both are purely real

B. z is purely real and w is purely imaginery

C. w is purely real and z is purely imaginery

D. z and w both are imaginery

Answer: A

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29. All complex numbers 'z' which satisfy the relation $|z - |z + 1|| = |z + |z - 1||$ on the complex plane lie on the

A. $y = x$

B. $y = -x$

C. circle $x^2 + y^2 = 1$

D. line $x = 0$ or on a line segment joining $(-1, 0) \rightarrow$ (1,0)`

Answer: D

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30. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $iz_1 = Kz_2$, where $K \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is

A. $\tan^{-1}\left(\frac{2K}{K^2 + 1}\right)$

B. $\tan^{-1}\left(\frac{2K}{1 - K^2}\right)$

C. $-2\tan^{-1}K$

D. $2\tan^{-1}K$

Answer: D

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31. If $z + \frac{1}{z} = 2\cos 6^{\circ}$, then $z^{1000} + \frac{1}{z^{1000}} + 1$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: A

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32. Let z_1 and z_2q, be two complex numbers with α and β as their principal arguments such that $\alpha + \beta$ then principal $arg(z_1 z_2)$ is given by:

A. $\alpha + \beta + \pi$

B. $\alpha + \beta - \pi$

C. $\alpha + \beta - 2\pi$

D. $\alpha + \beta$

Answer: C

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33. Let $arg(z_k) = \frac{(2k+1)\pi}{n}$ where $k = 1, 2, \ldots\ldots\ldots n.$ If $arg(z_1, z_2, z_3, \ldots\ldots\ldots\ldots z_n) = \pi$, then n must be of form $(m \in z)$

A. $4m$

B. $2m - 1$

C. $2m$

D. None of these

Answer: B

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34. Suppose two complex numbers $z = a + ib,\ w = c + id$ satisfy the equation $\frac{z + w}{z} = \frac{w}{z + w}$. Then

A. both a and c are zeros

B. both b and d are zeros

C. both b and d must be non zeros

D. at least one of b and d is non zero

Answer: D

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35. If $|z| = 1$ and $z \neq \pm 1$, then one of the possible value of $arg(z) - arg(z + 1) - arg(z - 1)$, is

A. $-\pi/6$

B. $\pi/3$

C. $-\pi/2$

D. $\pi/4$

Answer: C

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36. If $arg\left(z^{3/8}\right) = \frac{1}{2} arg\left(z^2 + \bar{z}^{1/2}\right)$, then which of the following is not possible ?

A. $|z| = 1$

B. $z = \bar{z}$

C. $arg(z) = 0$

D. None of these

Answer: D

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37. z_1, z_2 are two distinct points in complex plane such that $2|z_1| = 3|z_2|$ and $z \in C$ be any point $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$ such that

A. $-1 \le Rez \le 1$

B. $-2 \le Rez \le 2$

C. $-3 \le Rez \le 3$

D. None of these

Answer: B

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38. If $\alpha, \beta, \gamma \in \{1, \omega, \omega^2\}$ (where ω and ω^2 are imaginery cube roots of unity), then number of triplets (α, β, γ) such that $\left|\frac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha}\right| = 1$ is

A. 3

B. 6

C. 9

D. 12

Answer: C

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39. The value of $\left(3\sqrt{3} + \left(3^{5/6}\right)i\right)^3$ is (where $i = \sqrt{-1}$)

A. 24

B. -24

C. -22

D. -21

Answer: B

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40. If $\omega \neq 1$ is a cube root of unity and $a + b = 21, a^3 + b^3 = 105$, then the value of $\left(a\omega^2 + b\omega\right)\left(a\omega + b\omega^2\right)$ is be equal to

A. 3

B. 5

C. 7

D. 35

Answer: B

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41. If $z = \frac{1}{2}\left(\sqrt{3} - i\right)$, then the least possible integral value of m such that $\left(z^{101} + i^{109}\right)^{106} = z^{m+1}$ is

A. 11

B. 7

C. 8

D. 9

Answer: D

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42. If $y_1 = \max ||z - \omega| - |z - \omega^2||$, where $|z| = 2$ and $y_2 = \max ||z - \omega| - |z - \omega^2||$, where $|z| = \frac{1}{2}$ and ω and ω^2 are complex cube roots of unity, then

A. $y_1 = \sqrt{3}, y_2 = \sqrt{3}$

B. $y_1 < \sqrt{3}, y_2 = \sqrt{3}$

C. $y_1 = \sqrt{3}, y_2 < \sqrt{3}$

D. $y_1 > 3, y_2 < \sqrt{3}$

Answer: C

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43. Let I, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial, with real coefficients having $2\omega^2, 3 + 4\omega, 3 + 4\omega^2$ and $5 - \omega - \omega^2$ as roots is -

A. 4

B. 5

C. 6

D. 7

Answer: B

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44. Number of imaginary complex numbers satisfying the equation, $z^2 = \bar{z}2^{1 - |z|}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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45. Least positive argument ofthe 4th root ofthe complex number $2 - i\sqrt{12}$ is

A. $\pi/6$

B. $5\pi/12$

C. $7\pi/12$

D. $11\pi/12$

Answer: B

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46. A root of unity is a complex number that is a solution to the equation, $z^n = 1$ for some positive integer nNumber of roots of unity that are also the roots of the equation $z^2 + az + b = 0$, for some integer a and b is

A. 6

B. 8

C. 9

D. 10

Answer: B

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47. If z is a complex number satisfying the equation $z^6 + z^3 + 1 = 0$. If this equation has a root $re^{i\theta}$ with $90° < 0 < 180°$ then the value of θ is

A. $100°$

B. 110°

C. 160°

D. 170°

Answer: C

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48. Suppose A is a complex number and $n \in N$, such that $A^n = (A+1)^n = 1$, then the least value of n is 3 b. 6 c. 9 d. 12

A. 3

B. 6

C. 9

D. 12

Answer: B

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49. If $z_1, z_2, z_3 \ldots\ldots\ldots\ldots z_n$ are in $G.P$ with first term as unity such that $z_1 + z_2 + z_3 + \ldots + z_n = 0$. Now if $z_1, z_2, z_3 \ldots\ldots. z_n$ represents the vertices of n-polygon, then the distance between incentre and circumcentre of the polygon is

A. 0

B. $|z_1|$

C. $2|z_1|$

D. none of these

Answer: A

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50. If $|z - 1 - i| = 1$, then the locus of a point represented by the complex number $5(z - i) - 6$ is

A. circle with centre $(1, 0)$ and radius 3

B. circle with centre $(- 1, 0)$ and radius 5

C. line passing through origin

D. line passing through $(- 1, 0)$

Answer: B

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51. Let $z \in C$ and if $A = \left\{z : \arg(z) = \frac{\pi}{4}\right\}$ and $B = \left\{z : \arg(z - 3 - 3i) = \frac{2\pi}{3}\right\}$.

Then $n(A \quad B) =$

A. 1

B. 2

C. 3

D. 0

Answer: D

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52. $\theta \in [0, 2\pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1 + |\sin\theta|)z_1 + (|\cos\theta| - 1)z_2 - \sqrt{2}z_3 = 0$. If at least one of the complex numbers z_1, z_2, z_3 is nonzero, then number of possible values of θ is

A. Infinite

B. 4

C. 2

D. 8

Answer: B

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53. Let $'z'$ be a comlex number and $'a'$ be a real parameter such that $z^2 + az + a^2 = 0$, then which is of the following is not true ?

A. locus of z is a pair of straight lines

B. $|z| = |a|$

C. $arg(z) = \pm \frac{2\pi}{3}$

D. None of these

Answer: D

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54. Let $z = x + iy$ then locus of moving point P(z) $\frac{1 + \bar{z}}{z} \in R$, is

A. union of lines with equations $x = 0$ and $y = -1/2$but excluding origin.

B. union of lines with equations $x = 0$ and $y = 1/2$but excluding origin.

C. union of lines with equations $x = -1/2$ and $y = 0$but excluding origin.

D. union of lines with equations $x = 1/2$ and $y = 0$but excluding origin.

Answer: C

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55. Let $A(z_1)$ and $B(z_2)$ are two distinct non-real complex numbers in the argand plane such that $\frac{z_1}{z_2} + \frac{\bar{z}_1}{z_2} = 2$. The value of $|\angle ABO|$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. None of these

Answer: C

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56. Complex numbers z_1 and z_2 satisfy $|z_1| = 2$ and $|z_2| = 3$. If the included angle of their corresponding vectors is 60°, then the value of

$19\left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2$ is

A. 5

B. 6

C. 7

D. 8

Answer: C

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57. Let $A(2, 0)$ and $B(z)$ are two points on the circle $|z| = 2$. $M(z')$ is the point on AB. If the point $\bar{z}'$ lies on the median of the triangle OAB where O is origin, then $arg(z')$ is

A. $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$

B. $\tan^{-1}\left(\sqrt{15}\right)$

C. $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$

D. $\frac{\pi}{2}$

Answer: A

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58. If $A(z_1), B(z_2), C(z_3)$ are vertices of a triangle such that $z_3 = \frac{z_2 - iz_1}{1 - i}$ and $|z_1| = 3$, $|z_2| = 4$ and $|z_2 + iz_1| = |z_1| + |z_2|$, then area of triangle ABC is

A. $\frac{5}{2}$

B. 0

C. $\frac{25}{2}$

D. $\frac{25}{4}$

Answer: D

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59. Let O, A, B be three collinear points such that $OA. OB = 1$. If O and B represent the complex numbers O and z, then A represents

A. $\frac{1}{\bar{z}}$

B. $\frac{1}{z}$

C. $\bar{z}$

D. z^2

Answer: A

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60. If the tangents at z_1, z_2 on the circle $|z - z_0| = r$ intersect at z_3, then $\frac{(z_3 - z_1)(z_0 - z_2)}{(z_0 - z_1)(z_3 - z_2)}$ equals

A. 1

B. -1

C. i

D. $-i$

Answer: B

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61. If z_1, z_2 and z_3 are the vertices of ΔABC, which is not right angled triangle taken in anti-clock wise direction and z_0 is the circumcentre, then

$\left(\dfrac{z_0 - z_1}{z_0 - z_2}\right)\dfrac{\sin 2A}{\sin 2B} + \left(\dfrac{z_0 - z_3}{z_0 - z_2}\right)\dfrac{\sin 2C}{\sin 2B}$ is equal to

A. 0

B. 1

C. -1

D. 2

Answer: C

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62. Let P denotes a complex number $z = r(\cos\theta + i\sin\theta)$ on the Argand's plane, and Q denotes a complex number $\sqrt{2|z|^2}\left(\cos\left(\theta + \frac{\pi}{4}\right) + i\sin\left(\theta + \frac{\pi}{4}\right)\right)$. If $'O'$ is the origin, then ΔOPQ is

A. isosceles but not right angled

B. right angled but not isosceles

C. right isosceles

D. equilateral

Answer: C

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Multiple Correct Answer

1. Complex numbers whose real and imaginary parts x and y are integers and satisfy the equation $3x^2 - |xy| - 2y^2 + 7 = 0$

A. do not exist

B. exist and have equal modulus

C. form two conjugate pairs

D. do not form conjugate pairs

Answer: B::C

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2. If $a, b, c, d \in R$ and all the three roots of $az^3 + bz^2 + cZ + d = 0$ have negative real parts, then

A. $ab > 0$

B. $bc > 0$

C. $ad > 0$

D. $bc - ad > 0$

Answer: A::B::C

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3. Suppose three real numbers a, b, c are in $G.P.$ Let $z = \frac{a + ib}{c - ib}$. Then

A. $z = \frac{ib}{c}$

B. $z = \frac{ia}{b}$

C. $z = \frac{ia}{c}$

D. $z = 0$

Answer: A::B

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4. w_1, w_2 be roots of $(a+\bar{c})z^2+(b+\bar{b})z+(\bar{a}+c)=0$. If $|z_1|<1$, $|z_2|<1$, then

A. $|w_1|<1$

B. $|w_1|=1$

C. $|w_2|<1$

D. $|w_2|=1$

Answer: B::D

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5. A complex number z satisfies the equation $|Z^2-9|+|Z^2|=41$, then the true statements among the following are

A. $|Z+3|+|Z-3|=10$

B. $|Z+3|+|Z-3|=8$

C. Maximum value of $|Z|$ is 5

D. Maximum value of $|Z|$ is 6

Answer: A::C

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6. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $\left|z_1\right| = 1$. Let P and Q represent the complex numbers z_1 and z_2 in the Argand plane with $\angle POQ = \theta, o^\circ < 180^\circ$ (where O being the origin).Then

A. $b^2 = ac, \theta = \frac{2\pi}{3}$

B. $\theta = \frac{2\pi}{3}, PQ = \sqrt{3}$

C. $PQ = 2\sqrt{3}, b^2 = ac$

D. $\theta = \frac{\pi}{3}, b^2 = ac$

Answer: A::B

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7. Let $Z_1 = x_1 + iy_1, Z_2 = x_2 + iy_2$ be complex numbers in fourth quadrant of argand plane and $|Z_1| = |Z_2| = 1,\ Ref(Z_1Z_2) = 0$. The complex numbers $Z_3 = x_1 + ix_2, Z_4 = y_1 + iy_2, Z_5 = x_1 + iy_2, Z_6 = x_6 + iy$, will always satisfy

A. $|Z_4| = 1$

B. $arg(Z_1Z_4) = -\pi/2$

C. $\dfrac{Z_5}{\cos(argZ_1)} + \dfrac{Z_6}{\sin(argZ_1)}$ is purely real

D. $Z_5^2 + (\bar{Z}_6)^2$ is purely imaginergy

Answer: A::B::C::D

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8. If the imaginery part of $\dfrac{z-3}{e^{i\theta}} + \dfrac{e^{i\theta}}{z-3}$ is zero, then z can lie on

A. a circle with unit radius

B. a circle with radius 3 units

C. a straight line through the point $(3, 0)$

D. a parabola with the vertex $(3, 0)$

Answer: A::C

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9. If α Is the fifth root of unity, then :

A. $\left|1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4\right| = 0$

B. $\left|1 + \alpha + \alpha^2 + \alpha^3\right| = 1$

C. $\left|1 + \alpha + \alpha^2\right| = 2\cos\frac{\pi}{5}$

D. $|1 + \alpha| = 2\cos\frac{\pi}{10}$

Answer: A::B::C

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10. If z_1, z_2, z_3 are any three roots of the equation $z^6 = (z+1)^6$, then $arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right)$ can be equal to

A. 0

B. π

C. $\frac{\pi}{4}$

D. $-\frac{\pi}{4}$

Answer: A::B

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11. Let z_1, z_2, z_3 are the vertices of ΔABC, respectively, such that $\frac{z_3 - z_2}{z_1 - z_2}$ is purely imaginery number. A square on side AC is drawn outwardly. $P\left(z_4\right)$ is the centre of square, then

A. $\left|z_1 - z_2\right| = \left|z_2 - z_4\right|$

B. $arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = +\frac{\pi}{2}$

C. $arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = 0$

D. z_1, z_2, z_3 and z_4 lie on a circle

Answer: C::D

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Matching Column

1. z_1, z_2, z_3 are vertices of a triangle. Match the condition in List I with type of triangle in List II.

List I		List II	
(p)	$z_1^2 + z_2^2 + z_3^2 = z_2 z_3 + z_3 z_1 + z_1 z_2$	(1)	right angled but not necessarily iscosceles
(q)	$\operatorname{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$	(2)	obtuse angled
(r)	$\operatorname{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) < 0$	(3)	isosceles and right angled
(s)	$\frac{z_3 - z_1}{z_3 - z_2} = i$	(4)	equilateral

Codes

A. $\begin{matrix} p & q & r & s \\ 3 & 2 & 1 & 4 \end{matrix}$

B. $\begin{matrix} p & q & r & s \\ 1 & 2 & 4 & 3 \end{matrix}$

C. $\begin{matrix} p & q & r & s \\ 4 & 1 & 2 & 3 \end{matrix}$

D. $\begin{matrix} p & q & r & s \\ 2 & 1 & 4 & 3 \end{matrix}$

Answer: C

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Comprehension

1. Consider the region R in the Argand plane described by the complex number. Z satisfying the inequalities $|Z - 2| \le |Z - 4|$, $|Z - 3| \le |Z + 3|$, $|Z - i| \le |Z - 3i|$, $|Z + i| \le |Z + 3i|$

Answer the followin questions :

The maximum value of $|Z|$ for any Z in R is

A. 5

B. 3

C. 1

D. $\sqrt{13}$

Answer: D

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2. Consider the region R in the Argand plane described by the complex number. Z satisfying the inequalities $|Z - 2| \leq |Z - 4|$, $|Z - 3| \leq |Z + 3|$, $|Z - i| \leq |Z - 3i|$, $|Z + i| \leq |Z + 3i|$

Answer the followin questions :

The maximum value of $|Z|$ for any Z in R is

A. 5

B. 14

C. $\sqrt{13}$

D. 12

Answer: A

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3. Consider the region R in the Argand plane described by the complex number. Z satisfying the inequalities $|Z - 2| \leq |Z - 4|$, $|Z - 3| \leq |Z + 3|$, $|Z - i| \leq |Z - 3i|$, $|Z + i| \leq |Z + 3i|$

Answer the followin questions :

Minimum of $\left|Z_1 - Z_2\right|$ given that Z_1, Z_2 are any two complex numbers lying in the region R is

A. 0

B. 5

C. $\sqrt{13}$

D. 3

Answer: A

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4. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The locus of the complex number m is a curve

A. straight line

B. circle

C. ellipse

D. hyperbola

Answer: B

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5. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1 x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

The maximum value of $|m|$ is

A. 14

B. $2\sqrt{7}$

C. $7 + \sqrt{41}$

D. $2\sqrt{6} - 4$

Answer: C

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6. Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$. The value of $|m|$, when $are(m)$ is maximum

A. 7

B. $28 - \sqrt{41}$

C. $\sqrt{41}$

D. $2\sqrt{6} - 4$

Answer: D

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7. The locus of any point $P(z)$ on argand plane is $arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$.

Then the length of the arc described by the locus of $P(z)$ is

A. $10\sqrt{2}\pi$

B. $\frac{15\pi}{\sqrt{2}}$

C. $\frac{5\pi}{\sqrt{2}}$

D. $5\sqrt{2}\pi$

Answer: B

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8. The locus of any point $P(z)$ on argand plane is $arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$.

Total number of integral points inside the region bounded by the locus of $P(z)$ and imaginery axis on the argand plane is

A. 62

B. 74

C. 136

D. 138

Answer: C

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9. The locus of any point $P(z)$ on argand plane is $arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$.

Area of the region bounded by the locus of a complex number Z satisfying $arg\left(\frac{z + 5i}{z - 5i}\right) = \pm\frac{\pi}{4}$

A. $75\pi + 50$

B. 75π

C. $\frac{75\pi}{2} + 25$

D. $\frac{75\pi}{2}$

Answer: A

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10. A person walks $2\sqrt{2}$ units away from origin in south west direction $\left(S45\,^\circ W\right)$ to reach A, then walks $\sqrt{2}$ units in south east direction $\left(S45\,^\circ E\right)$ to reach B. From B he travel is 4 units horizontally towards east to reach C. Then he travels along a circular path with centre at origin through an angle of $2\pi/3$ in anti-clockwise direction to reach his destination D.

Let the complex number Z represents C in argand plane. then $arg(Z) =$

A. $-\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $-\dfrac{\pi}{4}$

D. $\dfrac{\pi}{3}$

Answer: C

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11. A person walks $2\sqrt{2}$ units away from origin in south west direction $\left(S45^{\circ}W\right)$ to reach A, then walks $\sqrt{2}$ units in south east direction $\left(S45^{\circ}E\right)$ to reach B. From B he travel is 4 units horizontally towards east to reach C. Then he travels along a circular path with centre at origin through an angle of $2\pi/3$ in anti-clockwise direction to reach his destination D.

Position of D in argand plane is (w is an imaginary cube root of unity)

A. $(3 + i)\omega$

B. $-(1 + i)\omega^2$

C. $3(1 - i)\omega$

D. $(1 - 3i)\omega$

Answer: C

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Examples

1. Evaluate :

(i) i^{135}

(ii) $i^{\frac{1}{47}}$

(iii) $\left(-\sqrt{-1}\right)^{4n+3}, n \in N$

(iv) $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$

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2. Find the value of $i^{n}+i^{n+1}+i^{n+2}+i^{n+3}$ for all $n \in \dot{N}$

A. 0

B. i

C. $-i$

D. $2i^{n}$

Answer: A

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3. Find the value of $1 + i^2 + i^4 + i^6 + \quad + i^{2n}$

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4. Show that the polynomial $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$ is divisible by $x^3 + x^2 + x + 1, wherep, q, r, s \in \dot{n}$

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5. Solve:

$ix^2 - 3x - 2i = 0,$

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6. If $z = 4 + i\sqrt{7}$, then find the value of $z^3 - 4z^2 - 9z + 91.$

A. 23

B. i

C. -1

D. 0

Answer: C

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7. Express each of the following in the standerd from $a + ib$

(i) $\frac{5 + 4i}{4 + 5i}$ (ii) $\frac{(1 + i)^2}{3 - i}$ (iii) $\frac{1}{1 - \cos\theta + 2i\sin\theta}$

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8. The root of the equation $2(1 + i)x^2 - 4(2 - i)x - 5 - 3i = 0$, where $i = \sqrt{-1}$, which has greater modulus is

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9. Find the value of $(1 + i)^6 + (1 - i)^6$

A. $16i$

B. 0

C. $-16i$

D. 1

Answer: B

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10. If $\left(\frac{1 + i}{1 - i}\right)^m = 1$, then find the least positive integral value of m.

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11. Prove that the triangle formed by the points $1, \frac{1+i}{\sqrt{2}}, and i$ as vertices in the Argand diagram is isosceles.

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12. Find the value of θ if $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real or purely imaginary.

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13. If the imaginary part of $(2z+1)/(iz+1)$ is -2, then find the locus of the point representing in the complex plane.

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14. If z is a complex number such that $|z-\bar{z}|+|z+\bar{z}|=4$ then find the area bounded by the locus of z.

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15. If $(x+iy)^5 = p+iq$, then prove that $(y+ix)^5 = q+ip$

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16. If z = x + iy lies in the third quadrant, then prove that $\frac{\bar{z}}{z}$ also lies in the third quadrant when $y < x < 0$

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17. Prove that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ is purely real.

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18. Find the relation if z_1, z_2, z_3, z_4 are the affixes of the vertices of a parallelogram taken in order.

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19. Let z_1, z_2, z_3 be three complex numbers and a, b, c be real numbers not all zero, such that $a + b + c = 0 and az_1 + bz_2 + cz_3 = 0$. Show that z_1, z_2, z_3 are collinear.

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20. Find real values of x and y for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

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21. Given that x, $y \in R$. Solve: $\frac{x}{1 + 2i} + \frac{y}{3 + 2i} = \frac{5 + 6i}{8i - 1}$

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22. If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$.

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23. Let z be a complex number satisfying the equation $z^3-(3+i)z+m+2i=0, wherem \in R$. Suppose the equation has a real root. Then root non-real root.

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24. Show that the equation $Z^4+2Z^3+3Z^2+4Z+5=0$ has no root which is either purely real or purely imaginary.

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25. Find the square roots of the following:

(i) $7-24i$ (ii) $5+12i$

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26. Find all possible values of $\sqrt{i} + \sqrt{-i}$.

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27. Solve the following for z: $z^2 - (3 - 2i)z = (5i - 5)$

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28. Solve the equation $(x - 1)^3 + 8 = 0$ in the set C of all complex numbers.

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29. If n is n odd integer that is greater than or equal to 3 but not la ultiple of 3, then prove that $(x + 1)^n = x^n - 1$ is divisible by $x^3 + x^2 + x$.

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30. ω is an imaginary root of unity.

Prove that

(i) $\left(a + b\omega + c\omega^2\right)^3 + \left(a + b\omega^2 + c\omega\right)^3 = (2a - b - c)(2b - a - c)(2c - a - b)$

(ii) If $a + b + c = 0$ then prove that $\left(a + b\omega + c\omega^2\right)^3 + \left(a + b\omega^2 + c\omega\right)^3 = 27abc.$

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31. Find the complex number ω satisfying the equation $z^3 - 8i$ and lying in the second quadrant on the complex plane.

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32. $\frac{1}{a + \omega} + \frac{1}{b + \omega} + \frac{1}{c + \omega} + \frac{1}{d + \omega} = \frac{1}{\omega}$ where, a,b,c,d, $\in$ R and ω is a complex cube root of unity then find the value of $\sum \frac{1}{a^2 - a + 1}$

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33. Write the following complex number in polar form :

(i) $-3\sqrt{2} + 3\sqrt{2}i$

(ii) $1 + i$

(iii) $\dfrac{1 + 7i}{(2 - i)^2}$

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34. Let $z_1 = \cos 12^\circ + I\sin 12^\circ$ and $z_2 = \cos 48^\circ + i.\sin 48^\circ$. Write complex number $\left(z_1 + z_2\right)$ in polar form. Find its modulus and argument.

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35. Covert the complex number $z = 1 + \dfrac{\cos(8\pi)}{5} + i.\dfrac{\sin(8\pi)}{5}$ in polar form. Find its modulus and argument.

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36. z and ω are two nonzero complex number such that $|z| = |\omega|$ and $Argz + Arg\omega = \pi$ then z equals

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37. Find the numbers of non-zero integral solutions of the equation $|1 - i|^x = 2^x$

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38. Let z be a complex number satisfying $|z| = 3|z - 1|$. Then prove that $\left|z - \frac{9}{8}\right| = \frac{3}{8}$

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39. If complex number z=x +iy satisfies the equation $Re(z + 1) = |z - 1|$, then prove that z lies on $y^2 = 4x$.

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40. Solve the equation $|z| = z + 1 + 2i$

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41. Find the range of real number α for which the equation $z + \alpha|z - 1| + 2i = 0$ has a solution.

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42. Find the Area bounded by complex numbers $arg|z| \leq \frac{\pi}{4}$ and $|z - 1| < |z - 3|$

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43. Prove that traingle by complex numbers z_1, z_2 and z_3 is equilateral if $|z_1| = |z_2| = |z_3|$ and $z_1 + z_2 + z_3 = 0$

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44. Show that $e^{2mi\theta}\left(\dfrac{i\cot\theta + 1}{i\cot\theta - 1}\right)^m = 1.$

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45. $Z_1 \neq Z_2$ are two points in an Argand plane. If $a|Z_1| = b|Z_2|$, then prove that $\dfrac{aZ_1 - bZ_2}{aZ_1 + bZ_2}$ is purely imaginary.

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46. Find the real part of $(1 - i)^{-i}$

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47. If $\left(\sqrt{8}+i\right)^{50} = 3^{49}(a+ib)$, then find the value of a^2+b^2.

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48. Show that $\left(x^2+y^2\right)^4 = \left(x^4-6x^2y^2+y^4\right)^2+\left(4x^3y-4xy^3\right)^2$.

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49. If $arg\left(z_1\right) = 170^0 and arg\left(z_2\right) 70^0$, then find the principal argument of z_1z_2.

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50. The value of

$$\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\left(\frac{\pi}{2^2}\right)+i\sin\left(\frac{\pi}{2^2}\right)\right)\left(\cos\left(\frac{\pi}{2^3}\right)+i\sin\left(\frac{\pi}{2^3}\right)\right)$$

.... ∞ is

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51. Find the principal argument of the complex number $\dfrac{(1+i)^5\left(1+\sqrt{3i}\right)^2}{-1i\left(-\sqrt{3}+i\right)}$

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52. If $z = \dfrac{\left(\sqrt{3}+i\right)^{17}}{(1-i)^{50}}$, then find $amp(z)$.

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53. If $z = x + iyandw = (1 - iz)/(z - i)$, then show that $|w| = 1z$ is purely real.

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54. It is given the complex numbers z_1 and z_2, $\left|z_1\right| = 2$ and $\left|z_2\right| = 3$. If the included angle of their corresponding vectors is 60° , then find value of

$$\left|\frac{z_1 + z_2}{z_1 - z_2}\right|$$

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55. Solve the equation $z^3 = \bar{z}(z \neq 0)$

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56. If $2z_1/3z_2$ is a purely imaginary number, then find the value of

$$|\left(z_1 - z_2)/\left(z_1 + z_2\right) \mid$$

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57. Find the complex number satisfying the system of equations $z^3 + \omega^7 = 0 and z^5 \omega^{11} = 1.$

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58. Express the following in $a + ib$ form:

(i) $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4$

(ii) $\frac{(\cos 2\theta - i\sin 2\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2} (\cos 3\theta - i\sin 3\theta)^{-9}}$

(iii) $\frac{(\sin\pi/8 + i\cos\pi/8)^8}{(\sin\pi/8 - i\cos\pi/8)^8}$

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59. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then prove that $Im(z) = 0$

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60. Prove that the roots of the equation $x^4 - 2x^2 + 4 = 0$ forms a rectangle.

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61. If $z + 1/z = 2\cos\theta,$ prove that $\left|\left(z^{2n} - 1\right)/\left(z^{2n} + 1\right)\right| = |\tan n\theta|$

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62. If $z = x + iy$ is a complex number with $x, y \in Q$ and $|z| = 1$, then show that $\left|z^{2n} - 1\right|$ is a rational numberfor every $n \in N$.

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63. If $z = \cos\theta + i\sin\theta$ is a root of the equation $a_0 z^n + a_2 z^{n-2} + \ + a_{n-1} z^+ a_n = 0,$ then prove that $a_0 + a_1\cos\theta + a_2^{\cos 2}\theta + \ + a_n\cos n\theta = 0$ $a_1\sin\theta + a_2^{\sin 2}\theta + \ + a_n\sin n\theta = 0$

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64. If $\left|z_1\right| = 1, \left|z_2\right| = 2, \left|z_3\right| = 3, and \left|9z_1z_2 + 4z_1z_3 + z_2z_3 + 3\right| = 12,$ then find the value of $\left|z_1 + z_2 + z + 3\right|^{\cdot}$

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65. If α and β are complex numbers such that $|\beta| = 1,$ then $\left|\dfrac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| =$

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66. Prove that $\left|z_1 + z_2\right|^2 = \left|z_1\right|^2, \text{ if } z_1/z_2$ is purely imaginary.

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67. Let $\left|\left((z)_1 - 2(z)_2\right)/\left(2 - z_1(z)_2\right)\right| = 1 and \left|z_2\right| \neq 1, where z_1 and z_2$ are complex numbers. Show that $\left|z_1\right| = 2.$

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68. If $z_1 and z_2$ are two complex numbers and $c > 0$, then prove that $\left|z_1 + z_2\right|^2 \leq (1 + c)\left|z_1\right|^2 + \left(1 + c^{-1}\right)\left|z_2\right|^2$.

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69. If z_1, z_2, z_3, z_4 are the affixes of four point in the Argand plane, z is the affix of a point such that $\left|z - z_1\right| = \left|z - z_2\right| = \left|z - z_3\right| = \left|z - z_4\right|$, then z_1, z_2, z_3, z_4 are

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70. if $\left|z_1 + z_2\right| = \left|z_1\right| + \left|z_2\right|$, then prove that $arg\left(z_1\right) = arg\left(z_2\right)$ if $\left|z_1 - z_2\right| = \left|z_1\right| + \left|z_2\right|$, then prove that $arg\left(z_1\right) = arg\left(z_2\right) = \pi$

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71. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz + iz$ is $\frac{1}{2}|z|^2$

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72. Find the minimum value of $|z - 1$ if $||z - 3| - |z + 1 \,|\, | = 2.$

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73. Find the greatest and the least value of $\left|z_1 + z_2\right|$ if $z_1 = 24 + 7iand\left|z_2\right| = 6.$

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74. If z is a complex number, then find the minimum value of $|z| + |z - 1| + |2z - 3|$.

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75. If $\left|z_1 - 1\right| \leq , \left|z_2 - 2\right| \leq 2, \left|z_{33}\right| \leq 3,$ then find the greatest value of $\left|z_1 + z_2 + z_3\right|$.

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76. Prove that following inequalities:

(i) $\left|\frac{z}{|z|} - 1\right| \leq |argz|$ (ii) $|z - 1| \leq |z||argz| + |z| - 1|$

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77. Identify the locus of z if $z = a + \frac{r^2}{z - a}, > 0.$

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78. If z is any complex number such that $|3z - 2| + |3z + 2| = 4$, then identify the locus of $\dot{z}$

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79. If $|z| = 1$ and let $\omega = \frac{(1 - z)^2}{1 - z^2}$, then prove that the locus of ω is equivalent to $|z - 2| = \mid z + 2$

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80. Let z be a complex number having the argument `theta,0

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81. How many solutions the system of equations $||z+4|-|z-3i||=5$ and |z| = 4 ` has ?

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82. Prove that $|Z-Z_1|^2+|Z-Z_2|^2=a$ will represent a real circle [with center $\left(|Z_1+Z_2|^{/}2+\right)$] on the Argand plane if $2a \geq |Z_1-Z_1|^2$

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83. If $|z-2-3i|^2+|z-5-7i|^2=\lambda$ respresents the equation of circle with least radius, then find the value of λ.

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84. If $\frac{|2z-3|}{|z-i|}=k$ is the equation of circle with complex number 'I' lying inside the circle, find the values of K.

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85. Find the point of intersection of the curves $arg(z - 3i) = \frac{3\pi}{4} and arg(2z + 1 - 2i) = \pi/4$.

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86. If complex numbers $z_1 z_2$ and z_3 are such that $|z_1| = |z_2| = |z_3|$, then prove that $arg\left(\frac{z_2}{z_1}\right) = arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)^2$

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87. If the triangle fromed by complex numbers z_1, z_2 and z_3 is equilateral then prove that $\frac{z_2 + z_3 - 2z_1}{z_3 - z_2}$ is purely imaginary number

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88. Show that the equation of a circle passings through the origin and having intercepts a and b on real and imaginary axis, respectively, on the argand plane is $Re\left(\frac{z - a}{z - ib}\right) = 0$

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89. The triangle formed by $A(z_1), B(z_2)$ and $C(z_3)$ has its circumcentre at origin .If the perpendicular form A to BC intersect the circumference at z_4 then the value of $z_1 z_4 + z_2 z_3$ is

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90. Let vertices of an acute-angled triangle are $A(z_1), B(z_2), andC(z_3)^{\cdot}$ If the origin O is he orthocentre of the triangle, then prove that

$z_1(z)_2 + (z)_1 z_2 =_2(z)_3 + (z)_2 z_3 = z_3(z)_1 + (z)_3 z_1^{\cdot}$

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91. If z_1, z_2, z_3 are three complex numbers such that $5z_1 - 13z_2 + 8z_3 = 0$, then prove that $\left|z_1(z)_1 1 z_2(z)_2 1 z_3(z)_3 1\right| = 0$

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92. If $z = z_0 + A\left(z - (z)_0\right)$, $where A$ is a constant, then prove that locus of z is a straight line.

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93. $z_1 and z_2$ are the roots of $3z^2 + 3z + b = 0.$ if $O(0), \left(z_1\right), \left(z_2\right)$ form an equilateral triangle, then find the value of b

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94. Let z_1, z_2 and z_3 be three complex number such that

$$\left|z_1 - 1\right| = \left|z_2 - 1\right| = \left|z_3 - 1\right| \text{ and } arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \frac{\pi}{6}$$

then prove that $z_2^3 + z_3^3 + 1 = z_2 + z_3 + z_2 z_3$.

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95. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equailateral triangle. If z_0 is the circumcentre of the triangle , then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$.

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96. In the Argands plane what is the locus of $z(\neq 1)$ such that

$$arg\left\{\frac{3}{2}\left(\frac{2z^2 - 5z + 3}{2z^2 - z - 2}\right)\right\} = \frac{2\pi}{3}.$$

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97. If $\left(\frac{3 - z_1}{2 - z_1}\right)\left(\frac{2 - z_2}{3 - z_2}\right) = k(k > 0)$, then prove that points $A\left(z_1\right), B\left(z_2\right), C(3), andD(2)$ (taken in clockwise sense) are concyclic.

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98. If z_1, z_2, z_3 are complex numbers such that $\left(2/z_1\right) = \left(1/z_2\right) + \left(1/z_3\right)$, then show that the points represented by $z_1, z_2()_, z_3$ lie one a circle passing through e origin.

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99. $A\left(z_1\right), B\left(z_2\right), C\left(z_3\right)$ are the vertices of he triangle ABC (in anticlockwise). If $\angle ABC = \pi/4$ and $AB = \sqrt{2}(BC)$, then prove that $z_2 = z_3 + i\left(z_1 - z_3\right)^{.}$

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100. If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\iota$. Find the other vertices of square

View Text Solution

101. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$ If z is any complex number such that the argument of $\frac{(z - z_1)}{(z - z_2)}$ is $\frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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102. Complex numbers of z_1, z_2, z_3 are the vertices A, B, C respectively, of on isosceles right-angled triangle with right angle at C. show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$

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103. Let $z_1, z_2 and z_3$ represent the vertices $A, B, and C$ of the triangle ABC, respectively, in the Argand plane, such that $\left|z_1\right| = \left|z_2\right| = 5$. Prove that $z_1 \sin 2A + z_2 \sin 2B + z_3 \sin 2C = 0$.

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104. F $a = \cos(2\pi/7) + is \in (2\pi/7)$, then find the quadratic equation whose roots are $\alpha = a = a^2 + a^4 and \beta = a^3 = a^5 + a^7$.

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105. If ω is an imaginary fifth root of unity, then find the value of $loe_2\left|1 + \omega + \omega^2 + \omega^3 - 1/\omega\right|^{.}$

View Text Solution

106. If 1, $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_s$ are ninth roots of unity (taken in counter - clockwise sequence in the Argard plane). Then find the value of $|(2-\alpha_1)(2-\alpha_3), (2-\alpha_5)(2-\alpha_7)|$.

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107. find the sum of squares of all roots of the equation. $x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$

View Text Solution

108. Find roots of the equation $(z+1)^5 = (z-1)^5$.

View Text Solution

109. If the roots of $(z-1)^n = i(z+1)^n$ are plotted in ten Argand plane, then prove that they are collinear.

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110. Let 1, $z_1, z_2, z_3, \ldots, z_{n-1}$ be the nth roots of unity. Then prove that $(1 - z_1)(1 - z_2)\ldots(1 - z_{n-1}) = n.$ Also,deduce that $\sin.\frac{\pi}{n}\sin.\frac{2\pi}{\pi}\sin.\frac{3\pi}{n}\ldots\sin.\frac{(n-1)\pi}{n} = \frac{\pi}{2^{n-1}}$

View Text Solution

111. if$\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)] + [1/(b+\omega)] + [1/(c+\omega)] = 2\omega^2$ and $\left[1/(a+\omega)^2\right] + \left[1/(b+\omega)^2\right] + \left[1/(c+\omega)^2\right] = 2\omega$, then find the value of $[1/(a+1)] + [1/(b+1)] + [1/(c+1)]$

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112. If $z_1 and z_2$ are complex numbers and $u = \sqrt{z_1 z_2}$, then prove that

$$|z_1| + |z_2| = \left|\frac{z_1 + z_2}{2} + u\right| + \left|\frac{z_1 + z_2}{2} - u\right|$$

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113. If a is a complex number such that $|a| = 1$, then find thevalue of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root.

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114. Let z and z_0 be two complex numbers. It is given that $|z| = 1$ and that numbers $z, z_0, z\bar{z}_0$ 1, and 0 are represented in a Argand diagram by the points P, P_0, Q, A and the origin respectively. Show that the triangles POP_0 and AOQ are congruent . Hence, or otherwise, prove that $\left|z - z_0\right| = \left|z\bar{z}_0 - 1\right|$

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115. Let $a, b, andc$ be any three nonzero complex number. If $|z| = 1and' z'$ satisfies the equation $az^2 + bz + c = 0,$ prove that

$aa = ccand|a||b| = \sqrt{ac(b)^2}$

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116. Let x_1, x_2 are the roots of the quadratic equation $x^2 + ax + b = 0$, where a,b, are complex numbers and y_1, y_2 are the roots of the quadratic equation $y^2 + |a|yy + |b| = 0$. If $\left|x_1\right| = \left|x_2\right| = 1$, then prove that $\left|y_1\right| = \left|y_2\right| = 1$

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117. If $\alpha = (z - i)/(z + i)$ show that, when z lies above the real axis, αwill lie within the unit circle which has centre at the origin. Find the locus of α as z travels on the real axis form $-\infty \text{to} + \infty$

View Text Solution

118. If $|z| \leq 1$ and $|w| < 1,$ then shown that $|z - w|^2 < (|z| - |w|)^2 + (argz - argw)^2$

View Text Solution

119. Prove that the distance of the roots of the equation $|\sin\theta_1|z^3 + |\sin\theta_2|z^2 + |\sin\theta_3|z + |\sin\theta_4| = 3omz = 0$ is greater than $2/3$.

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120. If $|z - (4 + 3i)| = 1,$ then find the complex number z for each of the following cases:

(i) $|z|$ is least

(ii) $|z|$ is greatest

(iii) $arg(z)$ is least

(iv) arg(z) is greatest

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121. If a ,b,c, and u,v,w are complex numbers repersenting the vertices of two triangle such that they are similar, then prove that $\frac{a - c}{a - b} = \frac{u - w}{u - v}$

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122. Let z_1 and z_2 be the root of the equation $z^2 + pz + q = 0$ where the coefficient p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \neq 0$ and 0 and $OA = OB, where O$ is the origin prove that $p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$

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123. The altitude form the vertices A, B and C of the triangle ABC meet its circumcircle at D,E and F, respectively . The complex number representing the points D,E, and F are z_1, z_2 and z_3, respectively. If $\left(z_3 - z_1\right)/\left(z_2 - z_1\right)$ is purely real, then show that triangle ABC is right-angled at A.

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124. Let A,B, C,D be four concyclic points in order in which AD:AB=CD: CB. If A,B,C are representing by complex numbers a,b,c respectively find the complex number associated with point D.

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125. If $n \geq 3$ and , $\alpha_1, \alpha_2......, \alpha_{n-1}$ are nth roots of unity , then find the sum $\sum_{1 \leq i \leq j \leq n-1} \alpha_i lpha_j$

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Exercise 3.1

1. Is the following computation correct? If not give the correct computation: $\left[\sqrt{(-2)}\sqrt{(-3)}\right] = \sqrt{(-2)\cdot 3} = \sqrt{6}$

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2. Find the value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}-1$

A. -2

B. 0

C. 2

D. -1

Answer: A

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3. The value of $i^{1+3+5++(2n+1)}$ is, If n is odd.

A. i

B. 1

C. -1

D. $-i$

Answer: B

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4. Find the value of $x^4 + 9x^3 + 35x^2 - x + 4$ for $x = -5 + 2\sqrt{-4}$.

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Exercise 3.2

1. प्रश्न 11 से 13 तक कि सम्मिश्र संख्याओं में प्रत्येक का गुणात्मक प्रतिलोम ज्ञात कीजिए।

$4 - 3i$

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2. Express the following complex numbers in $a + ib$ form: $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$

(ii) $\frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$

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3. Find the least positive integer n such that $\left(\frac{2i}{1 + i}\right)^n$ is a positive integer.

A. n =6

B. n =5

C. n =8

D. n =4

Answer: C

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4. If one root of the equation $z^2 - az + a - 1 = 0 is (1 + i), where a$ is a complex number then find the root.

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5. Prove that quadrilateral formed by the complex numbers which are roots of the equation $z^4 - z^3 + 2z^2 - z + 1 = 0$ is an equailateral trapezium.

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6. If Z^5 is a non-real complex number, then find the minimum value of

$$\frac{Imz^5}{Im^5 z}$$

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7. Find the real numbers x and y, if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$

A. $x = -2, y = 2$

B. $x = -3, y = 3$

C. $x = 3, y = -3$

D. $x = -4, y = 1$

Answer: C

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8. If z_1, z_2, z_3 are three nonzero complex numbers such that $z_3 = (1 - \lambda)z_1 + \lambda z_2 where \lambda \in R - \{0\}$, then prove that points corresponding to $z_1, z_2 and z_3$ are collinear .

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9. If n_1, n_2 are positive integers, then $(1+i)^{n_1} + \left(1+i^3\right)^{n_1} + \left(1+i_5\right)^{n_2} + \left(1+i^7\right)^{n_2}$ is real if and only if :

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Exercise 3.3

1. If $(a+b)-i(3a+2b)=5+2i,$ then find a and b

A. $a=12, b=-17$

B. $a=-12, b=-17$

C. $a=12, b=17$

D. $a=-12, b=17$

Answer: D

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2. Find all non zero complex numbers z satisfying $\bar{z}=iz^2$

View Text Solution

3. If a, b, c are nonzero real numbers and $az^2 = bz + c + i = 0$ has purely imaginary roots, then prove that $a = b^2 \cdot$

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4. If the sum of square of roots of equation $x^2 + (p + iq)x + 3i = 0$ is 8, then find |p|+|q| , where p and q are real.

A. 3

B. 1

C. 4

D. 2

Answer: C

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5. Find the square root $9 + 40i$

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6. Simplify: $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$

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7. If $\sqrt{x+iy} = \pm(a+ib)$, then find $\sqrt{x-iy}$.

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Exercise 3.4

1. if α and β are imaginary cube root of unity then prove $(\alpha)^4 + (\beta)^4 + (\alpha)^{-1}.(\beta)^{-1} = 0$

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2. If ω is a complex cube roots of unity, then find the value of the $(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)$... to 2n factors.

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3. Write the comple number in a + ib form unsing cube roots of unity: (a) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{1000}$ (b)If $z=\frac{\left(\sqrt{3}+i\right)^{17}}{(1-i)^{50}}$ (c) $\left(i+\sqrt{3}\right)^{100}+\left(i+\sqrt{3}\right)^{100}+2^{100}$

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4. If $z+z^{-1}=1$, then find the value of $z^{100}+z^{-100}$.

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5. Find the common roots of $x^{12}-1=0$ and $x^4+x^2+1=0$

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6. if $\alpha, \beta, \gamma$are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ then $\frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$

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7. Prove that $t^2 + 3t + 3$ is a factor of $(t + 1)^{n+1} + (t + 2)^{2n-1}$ for all intergral values of n $\in$ N.

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Exercise 3.5

1. Find the pricipal argument of each of the following:

(a) $-1 - i\sqrt{3}$

(b) $\frac{1 + \sqrt{3}i}{3 + i}$

(c) $\sin\alpha + i(1 - \cos\alpha), 0 > \alpha > \pi$

(d) $\left(1 + i\sqrt{3}\right)^2$

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2. Find the modulus, argument, and the principal argument of the complex numbers. (i) $(\tan 1 - i)^2$

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3. If $\frac{3\pi}{2} < \alpha < 2\pi$, find the modulus and argument of $(1 - \cos 2\alpha) + i\sin 2\alpha$.

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4. Find the principal argument of the complex number $\frac{\sin(6\pi)}{5} + i\left(1 + \frac{\cos(6\pi)}{5}\right)$.

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5. If $z = re^{i\theta}$, then prove that $\left|e^{iz}\right| = e^{-rs \int h\eta}$.

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6. Find the complex number z satisfying $Re\left(z^2 0 = 0, |z| = \sqrt{3.}\right.$

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7. If $|z - iRe(z)| = |z - Im(z)|$, then prove that z , lies on the bisectors of the quadrants.

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8. Find the locus of the points representing the complex number z for which$|z + 5|^2 = |z - 5|^2 = 10.$

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9. Solve : $+ z^2 + |z| = 0$.

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10. Let $z = x + iy$ be a complex number, where $xandy$ are real numbers. Let $AandB$ be the sets defined by $A = \{z : |z| \le 2\} andB = \{z : (1 - i)z + (1 + i)z \ge 4\}$. Find the area of region $A \cup B$.

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11. Real part of $\left(e^{e}\right)^{\iota\theta}$ is

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12. Prove that $z = i^{i}, wherei = \sqrt{-1}$, is purely real.

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Exercise 3.6

1. For $z_1 = \sqrt[6]{(1 - i)/\left(1 + i\sqrt{3}\right)}, z_2 = \sqrt[6]{(1 - i)/\left(\sqrt{3} + i\right)}$, $z_3 = \sqrt[6]{(1 + i)/\left(\sqrt{3} - i\right)}$, prove that $\left|z_1\right| = \left|z_2\right| = \left|z_3\right|$

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2. If $\sqrt{3} + i = (a + ib)/(c + id)$, then find the value of $\tan^{-1}(b/a)\tan^{-1}(d/c)$

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3. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers then $arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right) =$

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4. Find the modulus, argument, and the principal argument of the complex numbers. $(tan1 - i)^2 \dfrac{i - 1}{i\left(1 - \frac{\cos(2\pi)}{5}\right) + s \in n\frac{2\pi}{5}}$

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5. If $(1 + i)(1 + 2i)(1 + 3i)1 + m) = (x + iy)$, then show that $2 \times 5 \times 10 \times \times \left(1 + n^2\right) = x^2 + y^2$.

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6. If $a + ib = \dfrac{(x + i)^2}{2x + 1}$, prove that $a^2 + b^2 = \dfrac{(x + i)^2}{(2x + 1)^2}$

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7. Let z be a complex number satisfying the equation $\left(z^3 + 3\right)^2 = -16$, then find the value of $|z|$.

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8. If θ is real and z_1, z_2 are connected by $z12 + z22 + 2z_1z_2\cos\theta = 0$, then prove that the triangle formed by vertices $O, z_1 and z_2$ is isosceles.

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9. If $\left|z_1 - z_0\right| = z_2 - z_1 = \pi/2$, then find z_0

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10. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represents a circle. Find its centre and radius.

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Exercise 3.7

1. Express the following in $a + ib$ form: (a) $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^5}$ (b) $\left(\frac{1 + \cos\phi + i\sin\phi}{1 + \cos\phi - i\sin\phi}\right)^n$ (c) $\frac{(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)}{(\cos\gamma + i\sin\gamma)(\cos\delta + i\sin\delta)}$

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2. Find the value of following expression: $\left[\frac{1 - \frac{\cos\pi}{10} + i\frac{\sin\pi}{10}}{1 - \frac{\cos\pi}{10} - i\frac{\sin\pi}{10}}\right]^{10}$

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3. If $iz^4 + 1 = 0$, then prove that z can take the value $\cos\pi/8 + is \in \pi/8$.

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4. Prove that $(a)(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}}.\cos\left(\frac{n\pi}{4}\right)$, where n is a positive integer. $(b)\left(1+i\sqrt{3}\right)^n + \left(1-i\sqrt{3}^n = 2^{n+1}\cos\left(\frac{n\pi}{3}\right)\right.$, where n is a positive integer

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5. If $z = (a+ib)^5 + (b+ia)^5$, then prove that $Re(z) = Im(z), wherea, b \in R$

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6. If cos $\alpha + \cos\beta + \cos\gamma = 0$ and alos $\sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that. (a) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$ (b) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$ (c) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

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Exercise 3.8

1. a, b, c are three complex numbers on the unit circle $|z| = 1$, such that $abc = a + b + \cdot$ Then find the value of $|ab + bc + ca|$

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2. Let z be not a real number such that $\left(1 + z + z^2\right)/\left(1 - z + z^2\right) \in R$, then prove tha $|z| = 1$.

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3. If z_1, z_2, z_3 are distinct nonzero complex numbers and $a, b, c \in R^+$ such that $\frac{a}{|z_1 - z_2|} = \frac{b}{|z_2 - z_3|} = \frac{c}{|z_3 - z_1|}$ Then find the value of $\frac{a^2}{|z_1 - z_2|} + \frac{b^2}{|z_2 - z_3|} + \frac{c^2}{|z_3 - z_1|}$

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4. If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$, then prove that $\left|\left(1 - z_1\bar{z}_2\right)/\left(z_1 - z_2\right)\right| < 1$

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5. if $|z_1 + z_2| = |z_1| + |z_2|$, then prove that $arg(z_1) = arg(z_2)$ if $|z_1 - z_2| = |z_1| + |z_2|$, then prove that $arg(z_1) = arg(z_2) = \pi$

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6. For any complex number z, find the minimum value of $|z| + |z - 2i|$

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7. If is any complex number such that $|z + 4| \leq 3$, then find the greatest value of $|z + 1|$

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8. $Z \in C$ satisfies the condition $|Z| > 3$. Then find the least value of

$$\left|Z + \frac{1}{Z}\right|$$

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9. If a, b, c are nonzero complex numbers of equal moduli and satisfy $az^2 + bz + c = 0$, hen prove that $\left(\sqrt{5} - 1\right)/2 \leq |z| \leq \left(\sqrt{5} + 1\right)/2$.

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10. If $|z| \leq 4$ then find the maximum value of $|iz + 3 - 4i|$

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11. Let $z_1, z_2, z_3, \ldots\ldots z_n$ be the complex numbers such that $|z_1| = |z_2| = \ldots\ldots = |z_n| = 1$. ltbgt If $z = \left(\sum_{k=1}^{n} Z_k\right)\left(\sum_{k=1}^{n} \frac{1}{z_k}\right)$ then prove that (a) z is a real number (b) $0 < z \leq n^2$

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Exercise 3.9

1. If $\omega = z/[z - (1/3)i]$ and $|\omega| = 1$, then find the locus of z.

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2. If $Im\left(\frac{z - 1}{e^{\theta i}} + \frac{e^{\theta i}}{z - 1}\right) = 0$, then find the locus of z

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3. For three non-colliner complex numbers Z, Z_1 and Z_2 prove that

$$\left|Z - \frac{Z_1 + z_2}{2}\right|^2 + \left|\frac{Z_1 - Z_2}{2}\right| = \frac{1}{2}\left|Z - Z_1\right|^2 + \frac{1}{2}\left|Z - Z_2\right|^2$$

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4. If $|z - 1| + |z + 3| \leq 8$, then prove that z lies on the circle.

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5. If $z = \dfrac{3}{2 + \cos\theta + I\sin\theta}$, then prove that z lies on the circle.

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6. How many solutions system of equations, $arg(z + 3 - 2i) = -\pi/4$ and $|z + 4| - |z - 3i| = 5$ has ?

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7. Prove that equation of perpendicular bisector of line segment joining complex numbers z_1 and z_2 is $z\left(\bar{z}_2 - \bar{z}_1\right) + \bar{z}\left(z_2 + z_1\right) + \left|z_1\right|^2 - \left|z_2\right|^2 = 0$

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8. If complex number z lies on the curve $|z - (-1 + i)| = 1$, then find the locus of the complex number $w = \dfrac{z + i}{1 - i}, i = \sqrt{-1}$.

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Exercise 3.10

1. If $z_1 z_2, z_3$ and z_4 taken in order vertices of a rhombus, then proves that

$$Re\left(\frac{z_3 - z_1}{z_4 - z_2}\right) = 0$$

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2. Find the locus of point z if $z, i, andiz,$ are collinear.

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3. If $|z| = 2and\frac{z_1 - z_3}{z_2 - z_3} = \frac{z - 2}{z + 3}$, then prove that z_1, z_2, z_3 are vertices of a right angled triangle.

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4. Three vertices of triangle are complex number α, β and γ. Then prove that the perpendicular form the point α to opposite side is given by the equation $Re\left(\frac{z - \alpha}{\beta - \gamma}\right) = 0$ where z is complex number of any point on the perpendicular.

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5. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1 z_2 = 0$.

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6. The center of a regular polygon of n sides is located at the point z=0, and one of its vertex z_1 is known. If z_2 be the vertex adjacent to z_1, then z_2 is equal to

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7. If one vertices of the triangle having maximum area that can be inscribed in the circle $|z - i| = 5$ is 3-3i, then find the other verticles of the traingle.

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8. Consider the circle |z|=r in the Argand plane, which is in fact the incircle of trinagle ABC. If contact points opposite to the vertices A,B,C are $A_1(z_1), B_1(z_2)$ and $C_1(z_3)$, obtain the complex numbers associate with the vertices A,B,C in terms of z_1, z_2 and z_3.

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9. P is a point on the argand diagram on the circle with OP as diameter two points taken such that $\angle POQ = \angle QOR = 0$ If O is the origin and P, Q, R are are represented by complex z_1, z_2, z_3 respectively then show that $z_2^2 \cos 2\theta = z_1 z_3 \cos^2\theta$

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10. The center of the arc represented by $arg\left[\frac{z - 3i}{z - 2i + 4}\right] = \frac{\pi}{4}$

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Exercise 3.11

1. If α is complex fifth root of unity and $\left(1+\alpha+\alpha^2+\alpha^3\right)^{2005}=p+q\alpha+r\alpha^2+s\alpha^3$ (where p,q,r,s are real), then find the value of $p+q+r+s$.

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2. Find the number of roots of the equation $z^{15}=1$ satisfying $|argz|<\pi/2$.

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3. If z is nonreal root of $[-1]^{\frac{1}{7}}$ then,find the value of $z^{86}+z^{175}+z^{289}$

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4. Given $\alpha, \beta,$ respectively, the fifth and the fourth non-real roots of units, then find the value of $(1+\alpha)(1+\beta)\left(1+\alpha^2\right)\left(1+\beta^2\right)\left(1+\alpha^4\right)\left(1+\beta^4\right)$

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5. If the six roots of $x^6 = -64$ are written in the form $a + ib$, where a and b are real, then the product ofthose roots for which $a < 0$ is

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6. If $z_r : r = 1, 2, 3, \ldots .50$ are the roots of the equaiton $\sum_{r=0}^{50} z^r = 0$, then find the value of $\sum_{r=1}^{50} 1/\left(z_r - 1\right)$

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Exercise (Single)

1. If $a < 0, b > 0$, then $\sqrt{a}\sqrt{b}$ equal to

A. $-\sqrt{|a|b}$

B. $\sqrt{|a|b}$ i

C. $\sqrt{|a|b}$

D. none of these

Answer: B

View Text Solution

2. Consider the equation $10z^2 - 3iz - k = 0, wherez$ is a following complex variable and $i^2 = -1.$ Which of the following statements ils true? For real complex numbers k , both roots are purely imaginary. For all complex numbers k , neither both roots is real. For all purely imaginary numbers k , both roots are real and irrational. For real negative numbers k , both roots are purely imaginary.

A. For real positive numbers k, both roots are purely imaginary

B. For all complex numbers k, neither root is real .

C. For real negative numbers k, both roots are real and irrational .

D. For real negative numbers k, both roots are purely imaginary.

Answer: D

View Text Solution

3. The number of solutions of the equation $z^2 + z = 0$ where z is a a complex number, is

A. 1

B. 2

C. 3

D. 4

Answer: D

View Text Solution

4. If center of a regular hexagon is at the origin and one of the vertices on the Argand diagram is $1 + 2i$, then its perimeter is $2\sqrt{5}$ b. $6\sqrt{2}$ c. $4\sqrt{5}$ d. $6\sqrt{5}$

A. $2\sqrt{5}$

B. $6\sqrt{5}$

C. $4\sqrt{5}$

D. $6\sqrt{5}$

Answer: D

View Text Solution

5. If x and y are complex numbers, then the system of equations $(1 + i)x + (1 - i)y = 1, 2ix + 2y = 1 + i$ has

A. unique solution

B. no solution

C. infinte number of solutions

D. none of theses

Answer: C

View Text Solution

6. The point $z_1 = 3 + \sqrt{3} iadnz_2 = 2\sqrt{3} + 6i$ are given on la complex plane. The complex number lying on the bisector of the angel formed by the vectors $z_1 andz_2$ is $z = \frac{\left(3 + 2\sqrt{3}\right)}{2} + \frac{\sqrt{3} + 2}{2} i$ $z = 5 + 5i$ $z = -1 - i$ none of these

A. $z = \frac{\left(3 + 2\sqrt{3}\right)}{2} + \frac{\sqrt{3} + 2}{2} i$

B. $z = 5 + 5i$

C. $z = -1 - i$

D. none of these

Answer: B

View Text Solution

7. The polynomial $x^6 + 4x^5 + 3x64 + 2x^3 + x + 1$ is divisible by______ where w is the cube root of units $x + \omega$ b. $x + \omega^2$ c. $(x + \omega)\left(x + \omega^2\right)$ d. $(x - \omega)\left(x - \omega^2\right)$ where ω is one of the imaginary cube roots of unity.

A. $x + \omega$

B. $x + \omega^2$

C. $(x + \omega)\left(x + \omega^2\right)$

D. $(x + \omega)\left(x - \omega^2\right)$

Answer: D

View Text Solution

8. Dividing $f(z)$ by $z - i$, we obtain the remainder i and dividing it by $z + i$, we get the remainder 1 + i, then remainder upon the division of $f(z)$ by $z^2 + 1$ is

A. $\frac{1}{2}(z + 1) + i$

B. $\frac{1}{2}(iz + 1) + i$

C. $\frac{1}{2}(iz - 1) + i$

D. $\frac{1}{2}(z + i) + 1$

Answer: B

View Text Solution

9. The complex number $\sin(x) + i\cos(2x)$ and $\cos(x) - i\sin(2x)$ are conjugate to each other for

A. $x = n\pi, n \in Z$

B. $x = 0$

C. $x = (n + 1/2)\pi, n \in Z$

D. no value of x

Answer: D

View Text Solution

10. If the equation $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$ where a_1, a_2, a_3, a_4 are real coefficients different from zero has a pure imaginary root then the expression $\frac{a_1}{a_1a_2} + \frac{a_1a_4}{a_2a_3}$ has the value equal to

A. 0

B. 1

C. -2

D. 2

Answer: B

View Text Solution

11. If $z_1, z_2 \in C, z_1^2 \in R, z_1\left(z_1^2 - 3z_2^2\right) = 2$ and $z_2\left(3z_1^2 - z_2^2\right) = 11,$ then the value of $z_1^2 + z_2^2$ is

A. 10

B. 12

C. 5

D. 8

Answer: C

View Text Solution

12. If $a^2 + b^2 = 1$ then $\dfrac{1 + b + ia}{1 + b - ia} =$

A. a + ib

B. a + ia

C. b+ ia

D. b + ib

Answer: C

View Text Solution

13. If $z(1+a)=b+icanda^2+b^2+c^2=1,$ then $[(1+iz)/(1-iz)=\frac{a+ib}{1+c}$ b. $\frac{b-ic}{1+a}$ c. $\frac{a+ic}{1+b}$ d. none of these

A. $\frac{a+ib}{1+c}$

B. $\frac{b-ic}{1+a}$

C. $\frac{a+ic}{1+b}$

D. none of these

Answer: A

View Text Solution

14. If a and b are complex and one of the roots of the equation $x^2 + ax + b = 0$ is purely real whereas the other is purely imaginary, then

A. $a^2 - \left(\bar{a}\right)^2 = 4b$

B. $a^2 - \left(\bar{a}\right)^2 = 2b$

C. $b^2 - \left(\bar{a}\right)^2 = 2a$

D. $b^2 - \left(\bar{b}\right)^2 = 2a$

Answer: A

View Text Solution

15. If $z = (\lambda + 3) + i\sqrt{\left(5 - \lambda^2\right)}$; then the locus of z is

A. ellispe

B. semicircle

C. parabola

D. none of these

Answer: B

View Text Solution

16. Let $z = 1 - t + i\sqrt{t^2 + t + 2}$, where t is a real parameter.the locus of the z in argand plane is

A. a hyperbola

B. an ellipse

C. a striaght line

D. none of these

Answer: A

View Text Solution

17. If z_1 and z_2 are the complex roots of the equation $(x - 3)^3 + 1 = 0$, then $z_1 + z_2$ equal to

A. 1

B. 3

C. 5

D. 7

Answer: D

View Text Solution

18. Which of the following is equal to $\sqrt[3]{-1}$?

A. $\dfrac{\sqrt{3} + \sqrt{-1}}{2}$

B. $\dfrac{-\sqrt{3} + \sqrt{-1}}{\sqrt{-4}}$

C. $\dfrac{\sqrt{3} - \sqrt{-1}}{\sqrt{-4}}$

D. $-\sqrt{-1}$

Answer: B

View Text Solution

19. If $x^2 + x + 1 = 0$ then the value of

$$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \ldots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$$ is

A. 27

B. 72

C. 45

D. 54

Answer: D

View Text Solution

20. Sum of common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ is

A. -1

B. 1

C. 0

D. 1

Answer: A

View Text Solution

21. If $5x^3 + Mx + N, M, N \in R$ is divisible by $x^2 + x + 1$, then the value of $M + N$ is

A. 5

B. 4

C. -4

D. -5

Answer: D

View Text Solution

22. If $z = x + iyandx^2 + y^2 = 16,$ then the range of $||x| - |y||$ is $[0, 4]$ b. $[0, 2]$ c. $[2, 4]$ d. none of these

A. $[0, 4]$

B. $[0, 2]$

C. $[2, 4]$

D. none of these

Answer: A

View Text Solution

23. If z is a complex number satisfying the equaiton $z^6 - 6z^3 + 25 = 0$, then the value of $|z|$ is

A. $5^{1/3}$

B. $25^{1/3}$

C. $125^{1/3}$

D. $625^{1/3}$

Answer: A

View Text Solution

24. If $8iz + 12z^2 - 18z + 27i = 0, then\ |z| = \frac{3}{2}$ b. $|z| = \frac{2}{3}$ c.$|z| = 1$ d. $|z| = \frac{3}{4}$

A. $|z| = \frac{3}{2}$

B. $|z| = \frac{3}{4}$

C. $|z| = 1$

D. $|z| = \frac{3}{4}$

Answer: A

View Text Solution

25. Let $z_1 and z_2$ be complex numbers such that $z_1 \neq z_2$ and $\left|z_1\right| = \left|z_2\right|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be zero (b) real and positive real and negative (d) purely imaginary

A. purely imaginary

B. real and positive

C. real and negative

D. none of these

Answer: A

View Text Solution

26. $|z_1| = |z_2|$ and $arg\left(\frac{z_1}{z_2}\right) = \pi$, then $z_1 + z_2$ is equal to

A. 0

B. purely imaginary

C. purely real

D. none of these

Answer: A

View Text Solution

27. If for complex numbers z_1 and z_2, $arg(z_1) - arg(z_2) = 0$ then $|z_1 - z_2|$ is equal to

A. $|z_1| + |z_2|$

B. $|z_1| - |z_2|$

C. $||z_1| - |z_2||$

D. 0

Answer: C

View Text Solution

28. If $\left|\frac{z_1}{z_2}\right| = 1$ and $arg\left(z_1 z_2\right) = 0$, then

A. $z_1 = z_2$

B. $\left|z_2\right|^2 = z_1 z_2$

C. $z_1 z_2 = 1$

D. more than 8

Answer: B

View Text Solution

29. Suppose A is a complex number and $n \in N$, such that $A^n = (A+1)^n = 1$, then the least value of n is 3 b. 6 c. 9 d. 12

A. 3

B. 6

C. 9

D. 12

Answer: B

View Text Solution

30. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $argzw = \pi$ Then $argz$ equals

A. 4

B. 6

C. 8

D. more than 8

Answer: C

View Text Solution

31. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $argzw = \pi$ Then $argz$ equals

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

Answer: C

View Text Solution

32. If $z = (3 + 7i)(a + ib)$ where a,$b \in \ \in Z$ - $\{0\}$, is purely imaginary , then the minimum value of $|z|$ is

A. 74

B. 45

C. 58

D. 65

Answer: C

View Text Solution

33. If $(\cos\theta + i\sin\theta)(\cos2\theta + i\sin2\theta)\ldots.\,.(\cos n\theta + i\sin n\theta) = 1$ then the value of θ is :

A. $4m\pi$

B. $\dfrac{2m\pi}{n(n+1)}$

C. $\dfrac{4m\pi}{n(n+1)}$

D. $\frac{m\pi}{n(n+1)}$

Answer: C

View Text Solution

34. Given $z = \left(1 + i\sqrt{3}\right)^{100}$, then $[RE(z)/IM(z)]$ equals 2^{100} b. 2^{50} c. $\frac{1}{\sqrt{3}}$ d. $\sqrt{3}$

A. 2^{100}

B. 2^{50}

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Answer: C

View Text Solution

35. The expression $\left[\dfrac{1+\sin\left(\frac{\pi}{8}\right)+i\cos\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)-i\cos\left(\frac{\pi}{8}\right)}\right]^8$ is equal is

A. 1

B. -1

C. i

D. $-i$

Answer: B

View Text Solution

36. The number of complex numbers z satisfying $|z-3-i|=|z-9-i| and |z-3+3i|=3$ are a. one b. two c. four d. none of these

A. one

B. two

C. four

D. none of these

Answer: A

View Text Solution

37. $P(z)$ be a variable point in the Argand plane such that $|z| = m \in i\mu m\{|z - 1, |z + 1|\}, thenz + z$ will be equal to a. -1 or 1 b. 1 but not equal to-1 c. -1 but not equal to 1 d. none of these

A. -1 or 1

B. 1 but not equal to -1

C. -1 but not equal to 1

D. none of these

Answer: A

View Text Solution

38. if $\left|z^2 - 1\right| = |z|^2 + 1$ then z lies on

A. a circle

B. a parabola

C. an ellipse

D. none of these

Answer: D

View Text Solution

39. If $z = x + iy\left(x, y \in R, x \neq -\frac{1}{2}\right)$, the number of values of z satisfying $|z|^n = z^2|z|^{n-2} + z|z|^{n-2} + 1.\ (n \in N, n > 1)$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

View Text Solution

40. Number of solutions of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$ where z is a complex number is

A. 2

B. 3

C. 6

D. 5

Answer: D

View Text Solution

41. Number of ordered pairs(s) (a, b) of real numbers such that $(a + ib)^{2008} = a - ib$ holds good is

A. 2008

B. 2009

C. 2010

D. 1

Answer: C

View Text Solution

42. The equation $az^3 + bz^2 + \bar{b}z + \bar{a} = 0$ has a root α, where a, b,z and α belong to the set of complex numbers. The number value of $|\alpha|$

A. is 1/2

B. is 1

C. is 2

D. can't be determined

Answer: B

View Text Solution

43. If $k > 0$, $|z| = w = k$, and $\alpha = \frac{z - \bar{w}}{k^2 + z\bar{w}}$, then $Re(\alpha)$ (A) 0 (B) $\frac{k}{2}$ (C) k (D) None of these

A. 0

B. $k/2$

C. k

D. none of these

Answer: A

View Text Solution

44. $z_1 and z_2$ are two distinct points in an Argand plane. If $a|z_1| = b|z_2|(where a, b \in R)$, then the point $(az_1/bz_2) + (bz_2/az_1)$ is a point on the line segment [-2, 2] of the real axis line segment [-2, 2] of the imaginary axis unit circle $|z| = 1$ the line with $arg z = \tan^{-1}2$

A. line segment $[-2, 2]$ of the real axis

B. line segment $[-2, 2]$ of the imaginary axis

C. unit circle $|z| = 1$

D. the line with arg $z = \tan^{-1}2$

Answer: A

View Text Solution

45. If z is a comple number such that $-\frac{\pi}{2} < \arg z \leq \frac{\pi}{2}$, then which of the following inequalities is ture ?

A. $|z - \bar{z}| \leq |z|(arg z - arg \bar{z})$

B. $\left|z - \bar{z}\right| \geq |z|\left(argz - arg\bar{z}\right)$

C. $\left|z - \bar{z}\right| < \left(argz - arg\bar{z}\right)$

D. None of these

Answer: A

View Text Solution

46. If $\cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma\gamma = 0$, then the value of $\sin3\alpha + 8\sin3\beta + 27\sin3\gamma$ is

A. $\sin(a + b + \gamma)$

B. $3\sin(\alpha + \beta + \gamma)$

C. $18\sin(\alpha + \beta + \gamma)$

D. $\sin(\alpha + \beta + \gamma)$

Answer: C

View Text Solution

47. If α, β be the roots of the equation $u^2 - 2u + 2 = 0$ and if $\cot\theta = x + 1$,

then $\frac{(x + \alpha)^n - (x + \beta)^n}{\alpha - \beta}$ is equal to (a) $\left(\frac{\sin n\theta}{\sin^n\theta}\right)$ (b) $\left(\frac{\cos n\theta}{\cos^n\theta}\right)$ (c) $\left((\sin n\theta), \cos^n\theta\right)\Big)$ (d) $\left(\frac{\cos n\theta}{\sin\theta^n\theta}\right)$

A. $\frac{\sin nn\theta}{\sin^n\theta}$

B. $\frac{\cos n\theta}{\cos^n\theta}$

C. $\frac{\sin n\theta}{\cos^n\theta}$

D. $\frac{\cos n\theta}{\sin^n\theta}$

Answer: A

View Text Solution

48. If $z = (i)^{(i)^i} where i = \sqrt{-1}, then |z|$ is equal to

A. 1

B. $e^{-\pi/2}$

C. $e^{-\pi}$

D. none of these

Answer: A

View Text Solution

49. If $z = i\log\left(2 - \sqrt{-3}\right)$, then $\cos z =$

A. -1

B. $-1/2$

C. 1

D. 2

Answer: D

View Text Solution

50. If $|z| = 1$, then the point representing the complex number -$1 + 3z$ will lie on a. a circle b. a parabola c. a straight line d. a hyperbola

A. a circle

B. a straight line

C. a parabola

D. a hyperbola

Answer: A

View Text Solution

51. The locus of point z satisfying $Re\left(\frac{1}{z}\right) = k, wherek$ is a nonzero real number, is a. a straight line b. a circle c. an ellipse d. a hyperbola

A. a stringht line

B. a circle

C. an ellipse

D. a hyperbola

Answer: B

View Text Solution

52. If z is complex number, then the locus of z satisfying the condition $|2z - 1| = |z - 1|$ is perpendicular bisector of line segment joining 1/2 and 1 circle parabola none of the above curves

A. perpeciular bisector of line segment joining 1/2 and 1

B. circle

C. parabola

D. none of the above curves

Answer: B

View Text Solution

53. The greatest positive argument of complex number satisfying $|z - 4| = Re(z)$ is $\frac{\pi}{3}$ b. $\frac{2\pi}{3}$ c.$\frac{\pi}{2}$ d. $\frac{\pi}{4}$

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: D

View Text Solution

54. If $t and c$ are two complex numbers such that $|t| \neq |c|, |t| = 1 and z = (at + b)/(t - c), z = x + iy$ Locus of z is (where a, b are complex numbers) a. line segment b. straight line c. circle d. none of these

A. line segment

B. straight line

C. circle

D. none of these

Answer: C

View Text Solution

55. If $z^2 + z|z| + \left|z^2\right| = 0,$ then the locus z is a. a circle b. a straight line c. a pair of straight line d. none of these

A. a circle

B. a straight line

C. a pair of straing line

D. none of these

Answer: C

View Text Solution

56. Let C_1 and C_2 are concentric circles of radius 1 and $\frac{8}{3}$ respectively having centre at $(3, 0)$ on the argand plane. If the complex number z satisfies the inequality $\log_{\frac{1}{3}}\left(\frac{|z - 3|^2 + 2}{11|z - 3| - 2}\right) > 1,$ then

A. z lies outside C_1 but inside C_2

B. z line inside of both C_1 and C_2

C. z line outside both C_1 and C_2

D. none of these

Answer: A

View Text Solution

57. If $|z - 2 - i| = |z|\sin\left(\frac{\pi}{4} - argz\right)|$, where $i = \sqrt{-1}$,then locus of z, is

A. a pair of straing lines

B. circle

C. parabola

D. ellispe

Answer: C

View Text Solution

58. If $|z - 1| \leq 2 and \left|\omega z - 1 - \omega^2\right| = a$ $\left(where \omega is a cube \sqrt[o]{f} unity\right)$, then complete set of values of a is $0 \leq a \leq 2$ b. $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$ d. $0 \leq a \leq 4$

A. $0 \leq a \leq 2$

B. $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$

D. $0 \leq a \leq 4$

Answer: D

View Text Solution

59. If $\left|z^2 - 3\right| = 3|z|$, then the maximum value of $|z|$ is 1 b. $\frac{3 + \sqrt{21}}{2}$ c. $\frac{\sqrt{21} - 3}{2}$ d. none of these

A. 1

B. $\frac{3 + \sqrt{21}}{2}$

C. $\frac{\sqrt{21} - 3}{2}$

D. none of these

Answer: B

View Text Solution

60. If $|2z - 1| = |z - 2| and z_1, z_2, z_3$ are complex numbers such that `|z_1-(alpha)|< alpha,|z_2-beta||z|$d.$ >2|z|`

A. $< |z|$

B. $< 2|z|$

C. $> |z|$

D. $> 2|z|$

Answer: B

View Text Solution

61. If z_1 is a root of the equation $a_0 z^n + a_1 z^{n-1} + \quad + a_{n-1} z + a_n = 3$, $where \left|a_i\right| < 2f$ or $i = 0, 1, , n$, $then$ $|z| = \frac{3}{2}$
b. $|z| < \frac{1}{4}$ c. $|z| > \frac{1}{4}$ d. $|z| < \frac{1}{3}$

A. $\left|z_1\right| > \frac{1}{2}$

B. $\left|z_1\right| < \frac{1}{2}$

C. $\left|z_1\right| > \frac{1}{4}$

D. $|z| < \frac{1}{2}$

Answer: A

View Text Solution

62. If `|z|<

A. less than 1

B. $\sqrt{2} + 1$

C. $\sqrt{2 - 1}$

D. none of these

Answer: A

View Text Solution

63. Let $\left|Z_r - r\right| \leq r$, for all $r = 1, 2, 3\ldots, n$. Then $\left|\sum_{r=1}^{n} z_r\right|$ is less than

A. n

B. 2n

C. n(n+1)

D. $\frac{n(n+1)}{2}$

Answer: C

View Text Solution

64. All the roots of the equation $1lz^{10} + 10iz^9 + 10iz - 11 = 0$ lie

A. inside $|z| = 1$

B. one $|z| = 1$

C. outside $|z| = 1$

D. cannot say

Answer: B

View Text Solution

65. Let $\lambda \in R$. If the origin and the non-real roots of $2z^2 + 2z + \lambda = 0$ form the three vertices of an equilateral triangle in the Argand lane, then λ is

A. 1

B. $\frac{2}{3}$

C. 2

D. -1

Answer: B

View Text Solution

66. The roots of the equation $t^3 + 3at^2 + 3bt + c = 0 are z_1, z_2, z_3$ which represent the vertices of an equilateral triangle. Then $a^2 = 3b$ b. $b^2 = a$ c. $a^2 = b$ d. $b^2 = 3a$

A. $a^2 = 3b$

B. $b^2 = a$

C. $a^2 = a$

D. $b^2 = 3a$

Answer: C

View Text Solution

67. The roots of the cubic equation $(z + ab)^3 = a^3, a \neq 0$ represents the vertices of an equilateral triangle of sides of length

A. $\frac{1}{\sqrt{3}}|ab|$

B. $\sqrt{3}|a|$

C. $\sqrt{3}|b|$

D. $|a|$

Answer: B

View Text Solution

68. If $\left|z_1\right| = \left|z_2\right| = \left|z_3\right| = 1$ and $z_1 + z_2 + z_3 = 0$ then the area of the triangle whose vertices are z_1, z_2, z_3 is

A. $3\sqrt{3/4}$

B. $\sqrt{3/4}$

C. 1

D. 2

Answer: A

View Text Solution

69. Let z and ω be two complex numbers such that $|z| \le 1, |\omega| \le 1$ and $|z + i\omega| = \left|z_1 - z_2\right|$ is equal to

A. $\frac{2}{3}$

B. $\frac{\sqrt{5}}{3}$

C. $\frac{3}{2}$

D. $\frac{2\sqrt{5}}{3}$

Answer: C

View Text Solution

70. Let z_1, z_2, z_3, z_4 are distinct complex numbers satisfying $|z| = 1$ and $4z_3 = 3(z_1 + z_2)$, then $|z_1 - z_2|$ is equal to

A. 1 or i

B. i or $-i$

C. 1 or i

D. i or -1

Answer: D

View Text Solution

71. z_1, z_2, z_3, z_4 are distinct complex numbers representing the vertices of a quadrilateral $ABCD$ taken in order. If $z_1 - z_4 = z_2 - z_3$ and $\arg\left[\left(z_4 - z_1\right)/\left(z_2 - z_1\right)\right] = \pi/2$, the quadrilateral is

A. rectangle

B. rhombus

C. square

D. trapezium

Answer: A

View Text Solution

72. If $k + \left|k + z^2\right| = |z|^2\left(k \in R^-\right)$, then possible argument of z is

A. 0

B. π

C. $\pi/2$

D. none of these

Answer: C

View Text Solution

73. If z_1, z_2, z_3 are the vertices of an equilational triangle ABC such that $|z_1 - i| = |z_2 - i| = |z_3 - i|$, then $|z_1 + z_2 + z_3|$ equals to

A. $3\sqrt{3}$

B. $\sqrt{3}$

C. 3

D. $\frac{1}{3\sqrt{3}}$

Answer: C

View Text Solution

74. If z is a complex number having least absolute value and $|z - 2 + 2i| = |$, then $z =$

A. $\left(2 - 1/\sqrt{2}\right)(1 - i)$

B. $\left(2 - 1/\sqrt{2}\right)(1 + i)$

C. $\left(2 + 1/\sqrt{2}(1 - i)\right.$

D. $\left(2 + 1/\sqrt{2}\right)(1 + i)$

Answer: A

View Text Solution

75. If z is a complex number lying in the fourth quadrant of Argand plane and $|[kz/(k + 1)] + 2i| > \sqrt{2}$ for all real value of $k(k \neq -1)$, then range of $a\,r\,g(z)$ is $\left(\frac{\pi}{8}, 0\right)$ b. $\left(\frac{\pi}{6}, 0\right)$ c. $\left(\frac{\pi}{4}, 0\right)$ d. none of these

A. $\left(-\frac{\pi}{8}, 0\right)$

B. $\left(-\frac{\pi}{6}, 0\right)$

C. $\left(-\frac{\pi}{4}, 0\right)$

D. None of these

Answer: C

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76. If $\left|z_2 + iz_1\right| = \left|z_1\right| + \left|z_2\right| and \left|z_1\right| = 3 and \left|z_2\right| = 4,$ then the area of $ABC,$ if affixes of $A, B, and C are z_1, z_2, and \left[\left(z_2 - iz_1\right)/(1 - i)\right]$ respectively, is $\frac{5}{2}$ b. 0 c. $\frac{25}{2}$ d. $\frac{25}{4}$

A. $\frac{5}{2}$

B. 0

C. $\frac{25}{2}$

D. $\frac{25}{4}$

Answer: D

View Text Solution

77. If a complex number z satisfies $|2z + 10 + 10i| \leq 5\sqrt{3} - 5,$ then the least principal argument of z is : (a) $-\frac{5\pi}{6}$ (b) $\frac{11\pi}{12}$ (c) $-\frac{3\pi}{4}$ (d) $-\frac{2\pi}{3}$

A. $-\frac{5\pi}{6}$

B. $-\frac{11\pi}{12}$

C. $-\frac{3\pi}{4}$

D. $-\frac{2\pi}{3}$

Answer: A

View Text Solution

78. If 'z, lies on the circle $|z - 2i| = 2\sqrt{2}$, then the value of $arg\left(\frac{z - 2}{z + 2}\right)$ is the equal to

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: B

View Text Solution

79. z_1 and z_2, lie on a circle with centre at origin. The point of intersection of the tangents at z_1 and z_2 is given by

A. $\frac{1}{2}\left(\bar{z}_1 + \bar{z}_2\right)$

B. $\frac{2z_1z_2}{z_1 + z_2}$

C.

D.

Answer: B

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80. If arg $\left(\dfrac{z_1 - \dfrac{z}{|z|}}{\dfrac{z}{|z|}}\right) = \dfrac{\pi}{2}$ and $\left|\dfrac{z}{|z|} - z_1\right| = 3$, then $\left|z_1\right|$ equals to

A. $\sqrt{26}$

B. $\sqrt{10}$

C. $\sqrt{3}$

D. $2\sqrt{2}$

Answer: B

View Text Solution

81. The maximum area of the triangle formed by the complex coordinates z, z_1, z_2 which satisfy the relations $\left|z - z_1\right| = \left|z - z_2\right|$ and $\left|z - \dfrac{z_1 + z_2}{2}\right| \leq r$,where $r > \left|z_1 - z_2\right|$ is

A. $\frac{1}{2}\left|z_1 - z_2\right|^2$

B. $\frac{1}{2}\left|z_1 - z_2\right|r$

C. $\frac{1}{2}\left|z_1 - z_2\right|^2 r^2$

D. $\frac{1}{2}\left|z_1 - z_2\right|^2$

Answer: B

View Text Solution

82. Consider the region S of complex numbers a such that $\left|z^2 - az + 1\right| = 1$, where $|z| = 1$. Then area of S in the Argand plane is

A. $\pi + 8$

B. $\pi + 4$

C. $2\pi + 4$

D. $\pi + 6$

Answer: A

View Text Solution

83. The complex number associated with the vertices A, B, C of ΔABC are $e^{i\theta}, \omega, \bar{\omega}$, respectively [where $\omega, \bar{\omega}$ are the com plex cube roots of unity and $\cos\theta > Re(\omega)$], then the complex number of the point where angle bisector of A meets cumcircle of the triangle, is

A. $e^{i\theta}$

B. $e^{-i\theta}$

C. $\omega, \bar{\omega}$

D. $\omega + \bar{\omega}$

Answer: D

View Text Solution

84. If $pandq$ are distinct prime numbers, then the number of distinct imaginary numbers which are pth as well as qth roots of unity are.

min (p, q) b. min (p, q) c. 1 d. *zero*

A. min(p,q)

B. max(p,q)

C. 1

D. zero

Answer: D

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85. Given z is a complex number with modulus 1. Then the equation $\left[\frac{1+ia}{1-ia}\right]^4 = z$ has (a) all roots real and distinct (b)two real and two imaginary (c) three roots two imaginary (d)one root real and three imaginary

A. all roots real and distinct

B. two real and tw imaginary

C. three roots real and one imaginary

D. one root real and three imaginary

Answer: A

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86. The value of z satisfying the equation $\log z + \log z^2 + \dot{} + \log z^n = 0 is$

A. $\cos.\frac{4m\pi}{n(n+1)} + i\sin.\frac{4m\pi}{n(n+1)}, m = 0, 1, 2, \ldots$

B. $\cos.\frac{4m\pi}{n(n+1)} - i\sin.\frac{4m\pi}{n(n+1)}, m = 0, 1, 2, \ldots$

C. $\sin.\frac{4m\pi}{n} + i\cos.\frac{4m\pi}{n}, m = 0, 1, 2, \ldots$

D. 0

Answer: A

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87. If $n \in N > 1$, then the sum of real part of roots of $z^n = (z + 1)^n$ is equal to $\frac{n}{2}$ b. $\frac{(n - 1)}{2}$ c. $\frac{n}{2}$ d. $\frac{(1 - n)}{2}$

A. $\frac{n}{2}$

B. $\frac{(n - 1)}{2}$

C. $-\frac{n}{2}$

D. $\frac{(1 - n)}{2}$

Answer: D

View Text Solution

88. Which of the following represents a points in an Argand pane, equidistant from the roots of the equation $(z + 1)^4 = 16z^4$? $(0, 0)$ b. $\left(-\frac{1}{3}, 0\right)$ c. $\left(\frac{1}{3}, 0\right)$ d. $\left(0, \frac{2}{\sqrt{5}}\right)$

A. $(0, 0)$

B. $\left(-\frac{1}{3}, 0\right)$

C. $\left(\frac{1}{3}, 0\right)$

D. $\left(0, \frac{2}{\sqrt{5}}\right)$

Answer: C

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89. Let a be a complex number such that $|a| < 1$ and $z_1, z_2 \ldots .$ be vertices of a polygon such that $z_k = 1 + a + a^2 + a^3 + a^{k-1}$.

Then, the vertices of the polygon lie within a circle.

A. $\left|z - \frac{1}{1-a}\right| = \frac{1}{|a-1|}$

B. $\left|z + \frac{1}{a+1}\right| = \frac{1}{|a+1|}$

C. $\left|z - \frac{1}{1-a}\right| = |a-1|$

D. $\left|z + \frac{1}{1-a}\right| = |a-1|$

Answer: A

View Text Solution

Exercise (Multiple)

1. If $z = \omega, \omega^2 where \omega$ is a non-real complex cube root of unity, are two vertices of an equilateral triangle in the Argand plane, then the third vertex may be represented by $z = 1$ b. $z = 0$ c. $z = -2$ d. $z = -1$

A. $z = 1$

B. $z = 0$

C. $z = -2$

D. $z = -1$

Answer: A::C

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2. If $amp\left(z_1z_2\right) = 0 and \left|z_1\right| = \left|z_2\right| = 1, then\ z_1 + z_2 = 0$ b. $z_1z_2 = 1$ c. $z_1 = z_2$ d. none of these

A. $z_1 + z_2 = 0$

B. $z_1z_2 = 1$

C. $z_1 = \bar{z}_2$

D. none of these

Answer: B::C

View Text Solution

3. If $\sqrt{5 - 12i} + \sqrt{5 - 12i} = z$, then principal value of $argz$ can be $\frac{\pi}{4}$ b. $\frac{\pi}{4}$ c. $\frac{3\pi}{4}$ d. $-\frac{3\pi}{4}$

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $-\frac{3\pi}{4}$

Answer: A::B::C::D

View Text Solution

4. Values $(s)(-i)^{1/3}$ is/are $\frac{\sqrt{3}-i}{2}$ b. $\frac{\sqrt{3}+i}{2}$ c. $\frac{-\sqrt{3}-i}{2}$ d. $\frac{-\sqrt{3}+i}{2}$

A. $s\frac{\sqrt{3}-i}{2}$

B. $\frac{\sqrt{3}+i}{2}$

C. $\frac{-\sqrt{3}-i}{2}$

D. $\frac{-\sqrt{3}+i}{2}$

Answer: A::C

View Text Solution

5. If $a^3 + b^3 + 6abc = 8c^3 \& \omega$ is a cube root of unity then: a, b, c are in $\ddot{AP}$

(b) $a, b, c,$ are in $\ddot{HP}$ $a + b\omega - 2c\omega^2 = 0$ $a + b\omega^2 - 2c\omega = 0$

A. a, c, b are in A.P

B. a,c,b are in H.P

C. $a + b\omega - 2c\omega^2 = 0$

D. $a + b\omega^2 - 2c\omega = 0$

Answer: A::C::D

View Text Solution

6. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $arg\left(\frac{z_1}{z_2}\right)$ is equal to

A. $1 + \omega$

B. $1 + \omega^2$

C. ω

D. ω^2

Answer: C::D

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7. If $p = a + b\omega + c\omega^2, q = b + c\omega + a\omega^2,$ and $r = c + a\omega + b\omega^2,$ where $a, b, c \neq 0$ and ω is the complex cube root of unity , then .

A. If p,q,r lie on the circle |z|=2, the trinagle formed by these point is equilateral.

B. $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$

C. $p^2 + q^2 + r^2 = 2(pq + qr + rp)$

D. none of these

Answer: A::C

View Text Solution

8. Let P(x) and Q(x) be two polynomials.Suppose that $f(x) = P\left(x^3\right) + xQ\left(x^3\right)$ is divisible by $x^2 + x + 1$, then

A. P(x) is divisible by (x-1),but Q(x) is not divisible by x -1

B. Q(x) is divisible by (x-1), but P(x) is not divisible by x-1

C. Both P(x) and Q(x) are divisible by x-1

D. f(x) is divisible by x-1

Answer: C::D

View Text Solution

9. If α is a complex constant such that $az^2 + z + \alpha = 0$ has a ral root, then $\alpha + \alpha = 1$ $\alpha + \alpha = 0$ $\alpha + \alpha =$ - 1 the absolute value of the real root is 1

A. $alph + \bar{\alpha} = 1$

B. $\alpha + \bar{\alpha} = 0$

C. $\alpha + \bar{\alpha} = -1$

D. the absolute value of the real root is 1

Answer: A::C::D

View Text Solution

10. If $z^3 + 3 + 2i(z + (-1 + ia) = 0$ has on ereal roots, then the value of a lies in the interval $(a \in R)$ $(-2, 1)$ b. $(-1, 0)$ c. $(0, 1)$ d. $(-2, 3)$

A. $(2, -1)$

B. $(-1, 0)$

C. $(0, 1)$

D. $(-2, 3)$

Answer: A::B::D

View Text Solution

11. Given that the complex numbers which satisfy the equation $\left|zz^3\right| + \left|zz^3\right| = 350$ form a rectangle in the Argand plane with the length of its diagonal having an integral number of units, then area of rectangle is 48 sq. units if z_1, z_2, z_3, z_4 are vertices of rectangle, then $z_1 + z_2 + z_3 + z_4 = 0$ rectangle is symmetrical about the real axis $arg\left(z_1 - z_3\right) = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

A. area of rectangle is 48 sq units.

B. if z_1, z_2, z_3, z_4 are vertices of rectangle, then $z_1 + z_2 + z_3 + z_4 = 0$

C. rectangle is symmetrical about the real axis .

D. None of these

Answer: A::B::C

View Text Solution

12. If the points $A(z), B(-z), andC(1-z)$ are the vertices of an equilateral triangle $ABC,$ then sum of possible z is 1/2 sum of possible z is 1 product of possible z is 1/4 product of possible z is

A. sum of possible z is $1/2$

B. sum of possible z is

C. product of possible z is $1/4$

D. product of possibble z is $1/2$.

Answer: A::C

View Text Solution

13. If $|z - 3| = \min\{|z - 1|, |z - 5|\}$, then $Re(z)$ equals to

A. 2

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. 4

Answer: A::D

View Text Solution

14. If z_1, z_2 are tow complex numberes $(z_1 \neq z_2)$ satisfying $\left|z_1^2 - z_2^2\right| = \left|\bar{z}_1^2 + \bar{z}_2^2 - 2\bar{z}_1\bar{z}_2\right|$, then

A. $\frac{z_1}{z_2}$ is purely imaginary

B. $\frac{z_1}{z_2}$ is purely real

C. $\left|\arg z_1 - \arg z_2\right| = \pi$

D. $\left|\arg z_1 - \arg z_2\right| = \frac{\pi}{2}$

Answer: A::D

View Text Solution

15. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $\left|z_1\right| = \left|z_2\right| = 1$ and $Re\left(z_1\bar{z}_2\right) = 0$, then the pair ofcomplex nunmbers $\omega = a + ic$ and $\omega_2 = b + id$ satisfies

A. $\left|\omega_1\right| = 1$

B. $\left|\omega_2\right| = 1$

C. $Re\left(\omega_1\bar{\omega}_2\right) = 0$

D. $Im\left(\omega_1\bar{\omega}_2\right) = 0$

Answer: A::B::C

View Text Solution

16. Let $z_1 and z_2$ be complex numbers such that $z_1 \neq z_2$ and $\left|z_1\right| = \left|z_2\right|^{\cdot}$ If z_1 has positive real part and z_2 has negative imaginary part, then $\dfrac{z_1 + z_2}{z_1 - z_2}$ may be zero (b) real and positive real and negative (d) purely imaginary

A. zero

B. real and positive

C. real and negative

D. purely imaginary

Answer: A::D

View Text Solution

17. If $|z_1| = \sqrt{2}$, $|z_2| = \sqrt{3}$ and $|z_1 + z_2| = \sqrt{(5 - 2\sqrt{3})}$ then arg $\left(\frac{z_1}{z_2}\right)$ (not neccessarily principal)

A. $\frac{3\pi}{4}$

B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{4}$

D. $\frac{5}{2}$

Answer: A::C

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18. Let four points z_1, z_2, z_3, z_4 be in complex plane such that $|z_2| = 1$, $|z_1| \leq 1$ and $|z_3| \leq 1$. If $z_3 = \frac{z_2(z_1 - z_4)}{\bar{z}_1 z_4 - 1}$, then $|z_4|$ can be

A. 2

B. $\frac{2}{5}$

C. $\frac{1}{3}$

D. $\frac{5}{2}$

Answer: B::C

View Text Solution

19. A rectangle of maximum area is inscribed in the circle $|z - 3 - 4i| = 1$. If one vertex of the rectangle is $4 + 4i$, then another adjacent vertex of this rectangle can be $2 + 4i$ b. $3 + 5i$ c. $3 + 3i$ d. $3 - 3i$

A. $2 + 4i$

B. $3 + 5i$

C. $3 + 3i$

D. $3 - 3i$

Answer: B::C

View Text Solution

20. If $|z_1| = 15adn|z_2 - 3 - 4i| = 5, then \left(|z_1 - z_2|\right)_{m \in} = 5$ b. $\left(|z_1 - z_2|\right)_{m \in} = 10$ c. $\left(|z_1 - z_2|\right)_{max} = 20$ d. $\left(|z_1 - z_2|\right)_{max} = 25$

A. $|z_1 - z_2|_{\min} = 5$

B. $|z_1 - z_2|_{\min} = 10$

C. $|z_1 - z_2|_{\min} = 20$

D. $|z_1 - z_2|_{\min} = 25$

Answer: A::D

View Text Solution

21. $P(z_1), Q(z_2), R(z_3)$ and $S(z_4)$ are four complex numbers representing the vertices of a rhombus taken in order on the comple plane, then which one of the following is/are correct?

A. $\frac{z_1 - z_4}{z_2 - z_3}$ is purely real

B. $amp\frac{z_1 - z_4}{z_2 - z_4} = amp\frac{z_2 - z_4}{z_3 - z_4}$

C. $\frac{z_1 - z_3}{z_2 - z_4}$ is pureluy imaginary

D. is not necessary that $|z_1 - z_3| \neq |z_2 - z_4|$

Answer: A::B::C::D

View Text Solution

22. If $arg(z + a) = \pi/6$ and $arg(z - a) = 2\pi/3\left(a \in R^+\right)$, then

A. $|z| = a$

B. $|z| = 2a$

C. $arg(z) = \frac{\pi}{2}$

D. $arg(z) = \frac{\pi}{3}$

Answer: A::D

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23. If a complex number z satisfies $|z| = 1$ and $arg(z - 1) = \frac{2\pi}{3}$, then ($\omega$ is complex imaginary number)

A. $z^2 + z$ is purely imaginary number

B. $z = -\omega^2$

C. $z = -\omega$

D. $|z - 1| = 1$ then,

Answer: A::B::D

View Text Solution

24. If $|z - 1| = 1$, then

A. $arg((z - 1 - i)/z)$ can be equal to $-\pi/4$

B. $(z - 2)/z$ is purely imaaginary number

C. $(z - 2)/z$ is purely real number

D. if $arg(z) = \theta,$ where $z \neq 0$ and θ is acute, then $1 - 2/z = i\tan\theta$

Answer: A::B::D

View Text Solution

25. If $z_1 = 5 + 12i$ and $\left|z_2\right| = 4$, then

A. maximum $\left(\left|z_1 + iz_2\right|\right) = 17$

B. minimum $\left(\left|z_1 + (1 + i)z_2\right|\right) = 13 - 4\sqrt{2}$

C. minimum $\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{4}$

D. maximum $\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{3}$

Answer: A::B::D

View Text Solution

26. Let z_1, z_2, z_3 be the three nonzero comple numbers such that $z_2 \neq 1, a = |z_1|, b = |z_2|$ and $c = |z_3|$. Let $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ Then

A. $arg\left(\dfrac{z_3}{z_2}\right) = arg\left(\dfrac{z_3 - z_1}{z_2 - z_1}\right)$

B. ortho centre of triangle formed by z_1, z_2, z_3 is $z_1 + z_2 + z_3$

C. if trinagle formed by z_1, z_2, z_3 is $z_1 + z_2 + z_3$ is $\dfrac{3\sqrt{3}}{2}|z_1|^2$

D. if triangle formed by z_1, z_2, z_3 is equlateral, then $z_1 + z_2 + z_3 = 0$

Answer: A::B::D

View Text Solution

27. z_1 and z_2 are the roots of the equaiton $z^2 - az + b = 0$ where $|z_1| = |z_2| = 1$ and a,b are nonzero complex numbers, then

A. $|a| \leq 1$

B. $|a| \leq 2$

C. $2arg(a) = arg(b)$

D. $agra = 2arg(b)$

Answer: B::C

View Text Solution

28. If $|(z - z_1)/(z - z_2)| = 3$, where z_1 and z_2 are fixed complex numbers and z is a variable complex complex number, then z lies on a

A. circle with z_1 as its interior point

B. circle with z_2 as its interior point

C. circle with z_1 as its exterior point

D. circle with z_2 as its exterior point

Answer: B::C

View Text Solution

29. If $z = x + iy$, then he equation $|(2z - i)/(z + 1)| = m$ represents a circle, then m can be $1/2$ b. 1 c. 2 d. `3

A. $1/2$

B. 1

C. 2

D. 3

Answer: A::B::C

View Text Solution

30. System of equaitons $|z + 3| - |z - 3| = 6$ and $|z - 4| = r$ where $r \in R^+$ has

A. one solution if $r > 1$

B. one solution if $r > 1$

C. two solutions if $r = 1$

D. at leat one solution

Answer: A::C::D

View Text Solution

31. Let the equaiton of a ray be $|z - 2| - |z - 1 - i| = \sqrt{2}$. If the is strik the y-axis, then the equation of relfected ray (including or excluding the point of incidence) is .

A. $arg(z - 2i) = \frac{\pi}{4}$

B. $|z - 2i| - |z - 1 - i| = \sqrt{2}$

C. $arg(z - 2i) = \frac{3\pi}{4}$

D. $|z - 1i| - |z - 1 - 3i| = 2\sqrt{2}$

Answer: A::B

View Text Solution

32. Given that the two curves $arg(z) = \frac{\pi}{6} and \left|z - 2\sqrt{3}i\right| = r$ intersect in two distinct points, then $[r] \neq 2$ b. `0

A. $[r] \neq 2$ where [.] represents greatest integer

B. $0 < r < 3$

C. $r = 6$

D. $3 < r < 2\sqrt{3}$

Answer: A::D

View Text Solution

33. On the Argand plane ,let $z_1 = -2 + 3z, z_2 = -2 - 3z$ and $|z| = 1$. Then

A. z_1 moves on circle with centre at $(-2, 0)$ and radius 3

B. z_1 and z_2 describe the same locus

C. z_1 and z_2 move on differenet circles

D. $z_1 - z_2$ moves on a circle concetric with $|z| = 1$

Answer: A::B::D

View Text Solution

34. Let $S = \{z : x = x + iy, y \geq 0, |z - z_0| \leq 1\}$, where $|z_0| = |z_0 - \omega| = |z_0 - \omega^2|$, ω and ω^2 are non-real cube roots of unity. Then

A. $z_0 = -1$

B. $z_0 = -1/2$

C. if $z \in S$, then least value of $|z|$ is 1

D. $\left|arg\left(\omega - z_0\right)\right| = \pi/3$

Answer: A::D

View Text Solution

35. If P andn Q are represented by the complex numbers z_1 and z_2 such that $\left|1/z_2 + 1/z_1\right| = \left|1/z_2 - 1/z_1\right|$, then

A. ΔOPQ (where O is the origin) is equilateral.

B. ΔOPQ is right angled

C. the circumcentre of ΔOPQ is $\frac{1}{2}\left(z_1 + z_2\right)$

D. the circumcentre of ΔOPQ is $\frac{1}{2}\left(z_1 - z_2\right)$

Answer: B::C

View Text Solution

36. Loucus of complex number satifying are $arg[(z - 5 + 4i)/(z + 3 - 2i)] = -\pi/4$ is the are of a circle

A. whose radius is $5\sqrt{2}$

B. whose radius is 5

C. whose length (of arc) is $\dfrac{15\pi}{\sqrt{2}}$

D. whose centre is -2-5i

Answer: A::B::C

View Text Solution

37. Equation of tangent drawn to circle $|z| = r$ at the point $A(z_0)$, is

A. $Re\left(\dfrac{z}{z_0}\right) = 1$

B. $z\bar{z}_0 + z_0\bar{z} = 2r^3$

C. $Im\left(\dfrac{z}{z_0}\right) = 1$

D. $Im\left(\frac{z_0}{z}\right) = 1$

Answer: A::B

View Text Solution

38. If n is a natural number > 2, such that $z^n = (z + 1)^n$, then

A. roots of equation lie on a straight line parallel to the y-axis

B. roots of equaiton lie on a straight line parallel to the x-axis

C. sum of the real parts of the roots is $-[(n - 1)/2]$

D. none of these

Answer: A::C

View Text Solution

39. If $|z - (1/z)| = 1$, *then* $(|z|)_{max} = \dfrac{1+\sqrt{5}}{2}$ b. $(|z|)_{m\in} = \dfrac{\sqrt{5}-1}{2}$ c. $(|z|)_{max} = \dfrac{\sqrt{5}-2}{2}$ d. $(|z|)_{m\in} = \dfrac{\sqrt{5}-1}{\sqrt{2}}$

A. $|z|_{\max} = \dfrac{1+\sqrt{5}}{2}$

B. $|z|_{\min} = \dfrac{\sqrt{5}-1}{2}$

C. $|z|_{\max} = \dfrac{\sqrt{4}-2}{2}$

D. $|z|_{\min} = \dfrac{\sqrt{5}-1}{2}$

Answer: A::B

View Text Solution

40. If $1, z_1, z_2, z_3, \ldots\ldots, z_{n-1}$ be the nth roots of unity and ω be a non-real complex cube root of unity then the product $\prod_{r=1}^{n-1}\left(\omega - z_r\right)$ can be equal to

A. 0

B. 1

C. -1

D. $1 + \omega$

Answer: A::B::C

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41. Let z be a complex number satisfying equation $z^p - z^{-q}, where p, q \in N, then$ if $p = q$, then number of solutions of equation will be infinite. if $p = q$, then number of solutions of equation will be finite. if $p \neq q$, then number of solutions of equation will be $p + q + 1$. if $p \neq q$, then number of solutions of equation will be $p + q$

A. if p=q, then number of solution of equation will infinte.

B. if p=q, then number of solutions of equaiton will finite

C. if $p \neq q$, then number of solutions of equaiton will $p + q + 1$.

D. if $p \neq q$, then number of solutions of equaiton will be $p + q$

Answer: A::B

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42. Which of the following is true?

A. The number of common roots of $z^{144} = 1$ and $z^{24} = 1$ is 24

B. The number of common roots of $z^{360} = 1$ and $z^{315} = 1$ is 45

C. The number of roots common to $z^{24} = 1, z^{20} = 1$ and $z^{56} = 1$ is 4

D. The number of roots common to $z^{27} = 1, z^{125} = 1$ and $z^{49} = 1$ is 1

Answer: A::B::C::D

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43. If from a point P representing the complex number z_1 on the curve $|z| = 2$, two tangents are drawn from P to the curve $|z| = 1$, meeting at points $Q(z_2)$ and $R(z_3)$, then :

A. complex number $\left(z_1+z_2+z_3\right)/3$ will be on the curve $|z| = 1$

B. $\left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)=9$

C. $arg\left(\frac{z_2}{z_3}\right)=\frac{2\pi}{3}$

D. orth ocenre and circumcenter of ΔPQR wil coincide

Answer: A::B::C::D

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44. A complex number z is rotated in anticlockwise direction by an angle α and we get z' and if the same complex number z is rotated by an angle α in clockwise direction and we get z'' then

A. $z', z'.z''$ are in G.P

B. z',z',z'' are H.P

C. $z'+z''=2z\cos\alpha$

D. $z'^2+z''^2=2z^2\cos 2\alpha$

Answer: A::C::D

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45. z_1, z_2, z_3 and z_1, z_2, z_3 are nonzero complex numbers such that $z_3 = (1 - \lambda)z_1 + \lambda z_2$ and $z_3 = (1 - \mu)z_1 + \mu z_2$, then which of the following statements is/are ture?

A. If $\lambda, \mu \in R - \{0\}$, then z_1, z_2 and z_3 are colliner and z_1, z_2, z_3 are colliner separately.

B. If λ, μ are complex numbers, where $\lambda = \mu$, then triangles formed by points z_1, z_2, z_3 and z'_1, z'_2, z'_3 are similare.

C. If λ, μ are distinct complex numbers, then points z_1, z_2, z_3 and z'_1, z'_2, z_3 are not connectd by any well defined gemetry.

D. If $0 < \lambda < 1$, then z_3 divides the line joining z_1 and z_2 internally and if $\mu > 1$, then z_3 divides the following of z'_1, z'_2 extranlly

Answer: A::B::C::D

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46. Given $z = f(x) + ig(x) where f, g : (0, 1) \overset{\rightarrow}{0}, 1$ are real valued functions. Then which of the following does not hold good? $z = \frac{1}{1 - ix} + i\frac{1}{1 + ix}$ b. $z = \frac{1}{1 + ix} + i\frac{1}{1 - ix}$ c. $z = \frac{1}{1 + ix} + i\frac{1}{1 + ix}$ d. $z = \frac{1}{1 - ix} + i\frac{1}{1 - ix}$

A. $z = \frac{1}{1 - ix} + i\left(\frac{1}{1 + ix}\right)$

B. $z = \frac{1}{1 + ix} + i\left(\frac{1}{1 - ix}\right)$

C. $z = \frac{1}{1 + ix} + i\left(\frac{1}{1 + ix}\right)$

D. $z = \frac{1}{1 - ix} + i\left(\frac{1}{1 - ix}\right)$

Answer: A::C::D

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47. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $\left|z_1\right| = 1$. Let P and Q represent the complex numbers z_1 and z_2 in the Argand plane with $\angle POQ = \theta, o^\circ < 180^\circ$ (where O being the origin).Then

A. $b^2 = ac$

B. $PQ = \sqrt{3}$

C. $\theta = \frac{\pi}{3}$

D. $\theta = \frac{2\pi}{3}$

Answer: A::B::D

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48. If $a, b, c, d \in R$ and all the three roots of $az^3 + bz^2 + cZ + d = 0$ have negative real parts, then

A. $ab > 0$

B. $bv > 0$

C. $ad > 0$

D. $bc - ad > 0$

Answer: A::B::C::D

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49. If $\frac{3}{2 + e^{i\theta}} = ax + iby$, then the locous of $P(x, y)$ will represent

A. ellipse of a =1,b=2

B. circle if a=b=1

C. pair of straight line if a=1,b=0

D. None of these

Answer: A::B::C

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Exercise (Comprehension)

1. Consider the complex number $z = (1 - i\sin\theta)/(1 + i\cos\theta)$.

The value of θ for which z is purely real are

A. $n\pi - \frac{\pi}{4}, n \in I$

B. $\pi n + \frac{\pi}{4}, n \in I$

C. $n\pi, n \in I$

D. None of these

Answer: A

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2. Consider the complex number $z = (1 - i\sin\theta)/(1 + i\cos\theta)$.

The value of θ for which z is purely imaginary are

A. $n\pi - \frac{\pi}{4}, n \in I$

B. $\pi n + \frac{\pi}{4}, n \in I$

C. $n\pi, n \in I$

D. no real values of θ

Answer: D

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3. Consider the complex number $z = (1 - i\sin\theta)/(1 + i\cos\theta)$.

The value of θ for which z is unimodular give by

A. $n\pi \pm \frac{\pi}{6}, n \in I$

B. $n\pi \pm \frac{\pi}{3}, n \in I$

C. $n\pi \pm \frac{\pi}{4}, n \in I$

D. no real values of θ

Answer: C

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4. Consider the complex number $z = (1 - i\sin\theta)/(1 + i\cos\theta)$.

If agrument of z is $\pi/4$, then

A. $\theta = n\pi, n \in I$ only

B. $\theta = (2n + 1), n \in I$only

C. both $\theta = n\pi$ and $\theta = (2n + 1)\frac{\pi}{2}, n \in I$

D. none of these

Answer: D

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5. Consider the complex numbers z_1 and z_2 Satisfying the relation $|z_1 + z_2|^2 = |z_1| + |z_2|^2$ Complex number $z_1\bar{z}_2$ is

A. purely real

B. purely imaginary

C. zero

D. none of theses

Answer: B

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6. Consider the complex numbers z_1 and z_2 Satisfying the relation

$$\left|z_1 + z_2\right|^2 = \left|z_1\right| + \left|z_2\right|^2$$

Complex number z_1/z_2 is

A. purely real

B. purely imaginary

C. zero

D. none of these

Answer: B

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7. Consider the complex numbers z_1 and z_2 Satisfying the relation $|z_1 + z_2|^2 = |z_1| + |z_2|^2$

One of the possible argument of complex number $i(z_1/z_2)$

A. $\frac{\pi}{2}$

B. $-\frac{\pi}{2}$

C. 0

D. none of these

Answer: C

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8. Consider the complex numbers z_1 and z_2 Satisfying the relation $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ Possible difference between the argument of z_1 and z_2 is

A. 0

B. π

C. $-\frac{\pi}{2}$

D. none of these

Answer: C

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9. Let z be a complex number satisfying $z^2 + 2z\lambda + 1 = 0$, where λ is a parameter which can take any real value.

The roots of this equation lie on a certain circle if

A. $-1 < \lambda < 1$

B. $\lambda > 1$

C. $\lambda < 1$

D. none of these

Answer: A

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10. Let z be a complex number satisfying $z^2 + 2z\lambda + 1 = 0$, where λ is a parameter which can take any real value.

One root lies inside the unit circle and one outside if

A. -$1 < \lambda < 1$

B. $\lambda > 1$

C. $\lambda < 1$

D. none of these

Answer: B

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11. Let z be a complex number satisfying $z^2 + 2z\lambda + 1 = 0$, where λ is a parameter which can take any real value.

For every large value of λ the roots are approximately.

A. $-2\lambda, 1/\lambda$

B. $-\lambda, -1/\lambda$

C. $-2\lambda, -\frac{1}{2\lambda}$

D. none of these

Answer: C

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12. The roots of the equation $z^4 + az^3 + (12 + 9i)z^2 + bz = 0$ (where a and b are complex numbers) are the vertices of a square. Then

The value of $|a - b|$ is

A. $5\sqrt{5}$

B. $\sqrt{130}$

C. 12

D. $\sqrt{175}$

Answer: B

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13. The roots of the equation $z^4 + az^3 + (12 + 9i)z^2 + bz = 0$ (where a and b are complex numbers) are the vertices of a square. Then The area of the square is

A. 25 sq.units

B. 20 sq.units

C. 5 sq.unit

D. 4 sq .units

Answer: C

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14. Consider a quadratic equaiton $az^2 + bz + c = 0$, where a,b,c are complex number.

The condition that the equation has one purely imaginary root is

A. $\left(c\bar{a} - a\bar{c}\right)^2 = \left(b\bar{c} + c\bar{b}\right)\left(a\bar{a} - \bar{a}b\right)$

B. $\left(c\bar{c} - a\bar{c}\right)^2 = \left(b\bar{c} - c\bar{a}\right)^2\left(a\bar{b} + \bar{a}b\right)$

C. $\left(c\bar{a} - a\bar{c}\right)^2 = \left(b\bar{c} + c\bar{b}\right)\left(a\bar{b} + a\bar{b}\right)$

D. None of these

Answer: A

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15. Consider a quadratic equaiton $az^2 + bz + c = 0$, where a,b,c are complex number. If equaiton has two purely imaginary roots, then which of the following is not ture.

A. $a\bar{b}$ is purely imaginary

B. $b\bar{c}$ is purely imaginary

C. $c\bar{a}$ is purely real

D. none of these

Answer: D

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16. Consider a quadratic equaiton $az^2 + bz + c = 0$, where a,b,c are complex number.

The condition that the equaiton has one purely real roots is

A. $\left(c\bar{a} - a\bar{c}\right)^2 = \left(b\bar{c} + c\bar{b}\right)\left(a\bar{a} - \bar{a}b\right)$

B. $\left(c\bar{c} - a\bar{c}\right)^2 = \left(b\bar{c} - c\bar{a}\right)^2\left(a\bar{b} + \bar{a}b\right)$

C. $\left(c\bar{a} - a\bar{c}\right)^2 = \left(b\bar{c} + c\bar{b}\right)\left(a\bar{b} + a\bar{b}\right)$

D. $\left(c\bar{a} - a\bar{c}\right)^2 = \left(b\bar{c} - c\bar{b}\right)\left(a\bar{b} - \bar{a}b\right)$

Answer: D

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17. Suppose z and ω are two complex number such that $|z + i\omega| = 2$. Which of the following is ture about $|z|$ and $|\omega|$?

A. $|z| = |\omega| = \frac{1}{2}$

B. $|z| = \frac{1}{2}, |\omega|, |\omega| = \frac{3}{4}$

C. $|z| = |\omega| = \frac{3}{4}$

D. $|z| = |\omega| = 1$

Answer: D

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18. Suppose z and ω are two complex number such that Which of the following is true for z and ω?

A. $Re(z) = Re(\omega) = \frac{1}{2}$

B. $Im(z) = Im(\omega)$

C. $Re(z) = Im(\omega)$

D. $Im(z) = Re(\omega)$

Answer: D

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19. Suppose z and ω are two complex number such that $|z| \leq 1$, $|\omega| \leq 1$ and $|z + i\omega| = \left|z - i\bar{\omega}\right| = 2$ The complex number of ω can be

A. 1 or -i

B. -1

C. I or $-i$

D. ω or ω^2 (where ω is the cube root of unity)

Answer: C

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20. Consider the equaiton of line $a\bar{z} + a\bar{z} + a\bar{z} + b = 0$, where b is a real parameter and a is fixed non-zero complex number.

The intercept of line on real axis is given by

A. $\frac{-2b}{a + \bar{a}}$

B. $\frac{-b}{2\left(a + \bar{a}\right)}$

C. $\frac{-b}{a + \bar{a}}$

D. $\frac{b}{a + \bar{a}}$

Answer: C

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21. Consider the equaiton of line $a\bar{z} + a\bar{z} + a\bar{z} + b = 0$, where b is a real parameter and a is fixed non-zero complex number.

The intercept of line on imaginary axis is given by

A. $\dfrac{b}{\bar{a} - a}$

B. $\dfrac{2b}{\bar{a} - a}$

C. $\dfrac{b}{2\left(\bar{a} - a\right)}$

D. $\dfrac{b}{a - \bar{a}}$

Answer: D

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22. Consider the equaiton of line $a\bar{z} + a\bar{z} + a\bar{z} + b = 0$, where b is a real parameter and a is fixed non-zero complex number.

The locus of mid-point of the line intercepted between real and imaginary axis is given by

—

A. $az - az = 0$

—

B. $az + az = 0$

—

C. $az - az + b = 0$

D. $az - a\bar{z} + 2b = 0$

Answer: B

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23. Consider the equation $az + b\bar{z} + c = 0$, where a,b,c $\in$ Z

If $|a| \neq |b|$, then z represents

A. circle

B. straight line

C. one point

D. ellispe

Answer: C

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—

24. Consider the equation $az + bz + c = 0$, where a,b,c $\in$ Z

If $|a| = |b|$ and $\bar{a}c \neq b\bar{c}$, then z has

A. infnite solutions

B. no solutions

C. finite solutions

D. cannot say anything

Answer: B

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25. Consider the equation $az + bz + c = 0$, where a,b,c $\in$ Z

If $|a| = |b| \neq 0$ and $az + b\bar{c} + c = 0$ represents

A. an ellipse

B. a circle

C. a point

D. a straight line

Answer: D

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26. Complex numbers z satisfy the equaiton $|z - (4/z)| = 2$

The difference between the least and the greatest moduli of complex number is

A. 2

B. 4

C. 1

D. 3

Answer: A

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27. Complex numbers z satisfy the equaiton $|z - (4/z)| = 2$

The value of $arg\left(z_1/z_2\right)$ where z_1 and z_2 are complex numbers with the greatest and the least moduli, can be

A. 2π

B. π

C. $\pi/2$

D. none of these

Answer: B

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28. Complex numbers z satisfy the equaiton $|z - (4/z)| = 2$

Locus of z if $\left|z - z_1\right| = \left|z - z_2\right|$, where z_1 and z_2 are complex numbers with the greatest and the least moduli, is

A. line parallel to the real axis

B. line parallel to the imaginary axis

C. line having a positive slope

D. line having a negative slope

Answer: B

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29. In an Agrad plane z_1, z_2 and z_3 are, respectively, the vertices of an isosceles trinagle ABC with AC= BC and $\angle CAB = \theta$. If z_4 is incentre of triangle, then

The value of $AB \times AC/(IA)^2$ is

A. $\dfrac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2}$

B. $\dfrac{(z_2 - z_1)(z_1 - z_3)}{(z_4 - z_1)^2}$

C. $\dfrac{(z_4 - z_1)^2}{(z_2 - z_1)(z_3 - z_1)}$

D. none of these

Answer: A

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30. In an Agrad plane z_1, z_2 and z_3 are, respectively, the vertices of an isosceles trinagle ABC with AC= BC and $\angle CAB = \theta$. If z_4 is incentre of triangle, then

The value of $\left(z_4 - z_1\right)^2(\cos\theta + 1)\sec\theta$ is

A. $\dfrac{\left(z_2 - z_1\right)\left(z_3 - z_1\right)}{\left(z_4 - z_1\right)}$

B. $\left(z_2 - z_1\right)\left(z_3 - z_1\right)$

C. $\left(z_2 - z_1\right)\left(z_3 - z_1\right)^2$

D. $\dfrac{\left(z_2 - z_1\right)\left(z_1 - z_3\right)}{\left(z_4 - z_1\right)^2}$

Answer: B

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31. In an Agrad plane z_1, z_2 and z_3 are, respectively, the vertices of an isosceles trinagle ABC with AC= BC and $\angle CAB = \theta$. If z_4 is incentre of triangle, then

The value of $\left(z_2 - z_1\right)^2 \tan\theta \tan\theta/2$ is

A. $\left(z_1 + z_2 - 2z_3\right)$

B. $\left(z_1 + z_2 - z_3\right)\left(z_1 + z_2 - z_4\right)$

C. $-\left(z_1 + z_2 - 2z_3\right)\left(z_1 + z_2 - 2z_4\right)$

D. $z_4 = \sqrt{z_2 z_3}$

Answer: C

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32. $A\left(z_1\right), B\left(z_2\right)$ and $C\left(z_3\right)$ are the vertices of triangle ABC inscribed in the circle |z|=2,internal angle bisector of angle A meets the circumcircle again at $D\left(z_4\right)$.Point D is:

A. $z_4 = \frac{1}{z_2} + \frac{1}{z_3}$

B. $\sqrt{\frac{z_2 + z_3}{z_1}}$

C. $\sqrt{\frac{z_2 z_3}{z_1}}$

D. $z_4 = \sqrt{z_2 z_3}$

Answer: D

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33. $A\left(z_1\right)$,$B\left(z_2\right)$ and $C\left(z_3\right)$ are the vertices of triangle ABC inscribed in the circle |z|=2,internal angle bisector of angle A meets the circumcircle again at $D\left(z_4\right)$.Point D is:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer: C

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MATRIX MATCH TYPE

1. The graph of the quadrationc funtion $y = ax^2 + bx + c$ is as shown in the following figure.

Now,match the complex numbers given in List I with the corresponding arguments in List II.

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2. Let z_1, z_2 and z_3 be the vertices of trinagle . Then match following lists.

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3. Match following lists.

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4. Complex number z satisfies the equation $||z - 5i| + m|z - 12i| \;|\;\;| = n.$

Then match the value of m and n in List I with the corresponding locus in List II.

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5. Complex number z lies on the curve $S \equiv ar\frac{g(z + 3)}{z + 3i} = -\frac{\pi}{4}$

Now, match the locus in List I with its number of points of intersection with the curve S in List II.

A. $\begin{matrix} & a & b & c & d \\ (1) & p & q & p & r \end{matrix}$

B. $\begin{matrix} & a & b & c & d \\ (2) & s & r & q & p \end{matrix}$

C. $\begin{matrix} & a & b & c & d \\ (3) & q & p & q & r \end{matrix}$

D. $\begin{matrix} & a & b & c & d \\ (4) & s & p & q & r \end{matrix}$

Answer: A

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6. Consider sets $A = \{z \in C : z^{27} - 1 = 0\}$ and $B = \{z \in C : z^{36} - 1 = 0\}$

Now ,match the following lists.

A.

	a	b	c	d
(1)	p	q	p	r

B.

	a	b	c	d
(2)	r	q	s	p

C.

	a	b	c	d
(3)	q	p	q	r

D.

	a	b	c	d
(4)	s	p	q	r

Answer: B

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7. Match the statements in List I with those in List II

[Note: Here z take the values in the complex place and Im(z) and Re(z) denote, repectively, the imaginary part and the real part of z].

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8. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) - i\sin\left(\frac{2k\pi}{10}\right), k = 1, 2, \ldots . . , 9$

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9. Match the statements/experssions given in List I with the values given in List II.

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Exercise (Numerical)

1. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4iadnx^3 = 2 + 1i, wherei = \sqrt{-1}, then(a + b)$ equal to _____.

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2. If the complex numbers $x and y$ satisfy $x^3 - y^3 = 98i and x - y = 7i, then xy = a + ib, where a, b, \in R$. The value of $(a + b)/3$ equals _____.

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3. If $x = \omega - \omega^2 - 2$ then , the value of $x^4 + 3x^3 + 2x^2 - 11x - 6$ is (where ω is a imaginary cube root of unity)

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4. Let $z = 9 + bi, where b$ is nonzero real and $i^2 = -1$. If the imaginary part of $z^2 and z^3$ are equal, then $b/3$ is _____.

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5. Modulus of nonzero complex number z satifying $\bar{z} + z = 0$ and $|z|^2 - 4iz = z^2$ is ____.

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6. If the expression $(1 + ir)^3$ is of the form of $s(1 + i)$ for some real 's' where 'r' is also real and $i = \sqrt{-1}$

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7. If complex number $z(z \neq 2)$ satisfies the equation $z^2 = 4z + |z|^2 + \frac{16}{|z|^3}$,then the value of $|z|^4$ is_____.

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8. The complex number z satisfies $z + |z| = 2 + 8i$. find the value of $|z| - 8$

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9. Let $|z| = 2andw - \frac{z + 1}{z - 1}, wherez, w, \in C$ (where C is the set of complex numbers). Then product of least and greatest value of modulus of w is________.

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10. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$ then the set of possible values of z is

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11. Let $1, \omega, \omega^2$ be the cube roots of unity. The least possible degree of a polynomial with real coefficients having roots $2\omega, (2 + 3\omega), \left(2 + 3\omega^2\right), (2 - \omega - \omega)$ is____.

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12. If ω is the imaginary cube roots of unity, then the number of pair of integers (a,b) such that $|a\omega + b| = 1$ is _____.

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13. Suppose that z is a complex number the satisfies $|z - 2 - 2i| \leq 1$. The maximum value of $|2iz + 4|$ is equal to ______.

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14. If $|z + 2 - i| = 5$ and maxium value of $|3z + 9 - 7i|$ is M, then the value of M is _____.

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15. Let $Z_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $Z_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ are two complex numbers. If $Z_1 \cdot Z_2 = a + ib$ where $a, b \in R$ then the largest value of $(a + b) \forall \theta \in R$, is

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16. Let $A = \{a \in R\}$ the equation $(1 + 2i)x^3 - 2(3 + i)x^2 + (5 - 4i)x + a^2 = 0$ has at least one real root. Then the value of $\frac{\sum a^2}{2}$ is______.

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17. Find the minimum value of the expression $E = |z|^2 + |z - 3|^2 + |z - 6i|^2$ (where $z = x + iy, x, y \in R$)

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18. If z_1 lies on $|z - 3| + |z + 3| = 8$ such that arg $z_1 = \pi/6$, then $37\left|z_1\right|^2 =$ ________.

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19. If z satisfies the condition $\arg(z + i) = \frac{\pi}{4}$. Then the minimum value of $|z + 1 - i| + |z - 2 + 3i|$ is ______.

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20. Let $\omega \neq 1$ be a complex cube root of unity. If

$$\left(4 + 5\omega + 6\omega^2\right)^{n^2+2} + \left(6 + 5\omega^2 + 4\omega\right)^{n^2+2} + \left(5 + 6\omega + 4\omega^2\right)^{n^2+2} = 0,$$ and

$n \in N$, where $n \in [1, 100]$, then number of values of n is ______.

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21. Let z be a non - real complex number which satisfies the equation $z^{23} = 1$. Then the value of $\sum_{22}^{k=1} \frac{1}{1 + z^{8k} + z^{16k}}$

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22. If z, z_1 and z_2 are complex numbers such that $z = z_1 z_2$ and $\left|\bar{z}_2 - z_1\right| \leq 1$, then maximum value of $|z|$ - Re(z) is ____.

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23. Let z_1, z_2 and z_3 be three complex numbers such that $z_1 + z_2 + z_3 = z_1 z_2 + z_2 z_3 + z_1 z_3 = z_1 z_2 z_3 = 1$. Then the area of triangle formed by points $A(z_1), B(z_2)$ and $C(z_3)$ in complex plane is ______.

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24. Let α be the non-real 5 th root of unity. If z_1 and z_2 are two complex numbers lying on $|z| = 2$, then the value of $\sum_{t=0}^{4} \left|z_1 + \alpha^t z_2\right|^2$ is ______ .

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25. Let $z_1, z_2, z_3 \in C$ such that $\left|z_1\right| = \left|z_2\right| = \left|z_3\right| = \left|z_1 + z_2 + z_3\right| = 4.$

If $\left|z_1 - z_2\right| = \left|z_1 + z_3\right|$ and $z_2 \neq z_3$, then values of $\left|z_1 + z_2\right| \cdot \left|z_1 + z_3\right|$ is

_____.

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26. Let $A\left(z_1\right)$ and $B\left(z_2\right)$ be lying on the curve $|z - 3 - 4i| = 5$, where $\left|z_1\right|$ is maximum. Now, $A\left(z_1\right)$ is rotated about the origin in anticlockwise direction through 90° reaching at $P\left(z_0\right)$. If A, B and P are collinear then the value of $\left(\left|z_0 - z_1\right| \cdot \left|z_0 - z_2\right|\right)$ is _______.

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27. If z_1, z_2, z_3 are three points lying on the circle $|z| = 2$ then the minimum value of the expression $\left|z_1\right|z_2| \wedge 2 + |z_2 + z_3| \wedge 2 + |z_3 + z_1 +^2 =$

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28. Minimum value of $\left|z_1+1\right|+\left|z_2+1\right|+\left|z_1z_2+1\right|$ if $\left[z_1\right|=1$ and $|z_2|=1$ is _______.

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29. If $\left|z_1\right|=2$ and $(1-i)z_2+(1+i)\bar{z}_2=8\sqrt{2}$, then the minimum value of $\left|z_1-z_2\right|$ is _____.

View Text Solution

30. Given that $1+2|z|^2=\left|z^2+1\right|^2+2|z+1|^2$, then the value of $|z(z+1)|$ is _____.

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JEE Main Previous Year

1. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|Z|$ is equal to (1) $\sqrt{3} + 1$ (2) $\sqrt{5} + 1$ (3) 2 (4) $2 + \sqrt{2}$

A. $\sqrt{3} + 1$

B. $\sqrt{5} + 1$

C. 2

D. $2 + \sqrt{2}$

Answer: B

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2. The number of complex numbers z such that $|z1| = |z + 1| = |zi|$ equals

(1) 1 (2) 2 (3) ∞ (4) 0

A. ∞

B. 0

C. 1

D. 2

Answer: C

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3. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that : (1) $b \in (0, 1)$ (2) $b \in (- 1, 0)$ (3) $|b| = 1$ (4) $b \in (1, \infty)$

A. $\beta \in (1, \infty)$

B. $\beta \in (0, 1)$

C. $\beta \in (- 1, 0)$

D. $|\beta| = 1$

Answer: A

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4. If $\omega \neq 1$ is a cubic root of unit and $(1 + \omega)^7$ = A + Bω, then (A, B) equals

A. $(-1, 1)$

B. $(0, 1)$

C. $(1, 1)$

D. $(1, 0)$

Answer: C

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5. If $z \neq 1$ and $\dfrac{z^2}{z - 1}$ is real, then the point represented by the complex number z lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis

A. either on the real axis or on a circle passing thorugh the origin.

B. on a circle with centre at the origin.

C. either on the real axis or an a circle not possing through the origin .

D. on the imaginary axis .

Answer: A

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6. If z is a complex number of unit modulus and argument q, then $arg\left(\frac{1+z}{1+\bar{z}}\right)$ equal (1) $\frac{\pi}{2}$ - θ (2) θ (3) π - θ (4) -θ

A. -θ

B. $\frac{\pi}{2}$ - θ

C. θ

D. π - θ

Answer: C

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7. If z is a complex number such that $|z| \geq 2$ then the minimum value of $\left|z + \frac{1}{2}\right|$ is

A. is equal to $\frac{5}{2}$

B. lies in the interval (1,2)

C. is strictly gerater than $\frac{5}{2}$

D. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

Answer: B

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8. If z_1 and z_2 are two complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular whereas z_1 is not unimodular then $\left|z_1\right|$=

A. Straight line parallel to x-axis

B. sraight line parallel to y-axis

C. circle of radius 2

D. circle of radius $\sqrt{2}$

Answer: C

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9. A value of for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ purely imaginary, is : (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

A. $\frac{\pi}{6}$

B. $\sin^{-1}\left(\frac{Sqrt(3)}{4}\right)$

C. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D. $\frac{\pi}{3}$

Answer: C

View Text Solution

10. If $\omega \neq 1$ is a cubit root unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3$ k, then k is equal to

A. 1

B. z

C. $-z$

D. -1

Answer: B

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11. If $\alpha, \beta \in C$ are distinct roots of the equation $x^2 + 1 = 0$ then $\alpha^{101} + \beta^{107}$ is equal to

A. 2

B. -1

C. 0

D. 1

Answer: D

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JEE Advanced Previous Year

1. Let $z = x + iy$ be a complex number where $xandy$ are integers. Then, the area of the rectangle whose vertices are the roots of the equation $zz^3 + zz^3 = 350$ is 48 (b) 32 (c) 40 (d) 80

A. 48

B. 32

C. 40

D. 80

Answer: A

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2. Let z be a complex number such that the imaginary part of z is nonzero and a = z2 + z + 1 is real. Then a cannot take the value (A) –1 (B) 1 3 (C) 1 2 (D) 3 4

A. -1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: D

View Text Solution

3. Let complex numbers α and $\frac{1}{\alpha}$ lies on circle $\left(x-x_0\right)^2\left(y-y_0\right)^2=r^2$ and $\left(x-x_0\right)^2+\left(y-y_0\right)^2=4r^2$ respectively. If $z_0=x_0+iy_0$ satisfies the equation $2\left|z_0\right|^2=r^2+2$ then $|\alpha|$ is equal to (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

A. $1/\sqrt{2}$

B. $1/2$

C. $1/\sqrt{7}$

D. $1/3$

Answer: C

View Text Solution

4. Let Z_1 and Z_2, be two distinct complex numbers and let $w=(1-t)z_1+tz_2$for some number "t" with o

A. $\left|z-z_1\right|+\left|z-z_2\right|=\left|z_1-z_2\right|$

B. $\left(z - z_1\right) = \left(z - z_2\right)$

C. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D. $arg\left(z - z_1\right) = arg\left(z_2 - z_1\right)$

Answer: A::C::D

View Text Solution

5. Let w = $\left(\sqrt{3} + \frac{\iota}{2}\right)$ and $P = \left\{w^n : n = 1, 2, 3, \ldots.\right\}$, Further $H_1 = \left\{z \in C : Re(z) > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in c : Re(z) < -\frac{1}{2}\right\}$ Where C is set of all complex numbers. If $z_1 \in P \cap H_1, z_2 \in P \cap H_2$ and O represent the origin, then $\angle Z_1OZ_2$ =

A. $\pi/2$

B. $\pi/6$

C. $2\pi/3$

D. $5\pi/6$

Answer: C::D

View Text Solution

6. Let a,b $\in$ R and $a^2 + b^2 \neq 0$. Suppose $S = \left\{ z \in C : z = \frac{1}{a + ibt}, t \in R, t \neq 0 \right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and z in S, then (x,y) lies on

A. the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0 be \neq 0$

B. the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2}, 0\right) a < 0, b \neq 0$

C. the axis for $a \neq 0, b = 0$

D. the y-axis for $a = 0, b \neq 0$

Answer: A::C::D

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7. Let $a, b, xandy$ be real numbers such that $a - b = 1andy \neq 0$. If the complex number $z = x + iy$ satisfies $Im\left(\frac{az + b}{z + 1}\right) = y$, then which of the following is (are) possible value9s) of x?| $-1 - \sqrt{1 - y^2}$ (b) $1 + \sqrt{1 + y^2}$ $-1 + \sqrt{1 - y^2}$ (d) $-1 - \sqrt{1 + y^2}$

A. $-1 - \sqrt{1 - y^2}$

B. $1 + \sqrt{1 + y^2}$

C. $1 - \sqrt{1 + y^2}$

D. $-1 + \sqrt{1 - y^2}$

Answer: A::D

View Text Solution

8. For a non-zero complex number z , let $arg(z)$ denote the principal argument with $\pi < arg(z) \leq \pi$ Then, which of the following statement(s) is (are) FALSE? $arg(- 1, - i) = \frac{\pi}{4},$ where $i = \sqrt{-1}$ (b) The function

$f: R \to (-\pi, \pi]$, defined by $f(t) = arg(-1 + it)$ for all $t \in R$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$ (c) For any two non-zero complex numbers z_1 and z_2, $arg\left(\frac{z_1}{z_2}\right) - arg(z_1) + arg(z_2)$ is an integer multiple of 2π (d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$, lies on a straight line

A. $arg(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

B. The function$f: R \to (-\pi, \pi]$, defined by $f(t) = arg(-1 + it)$ for all $t \in R$, is continous at all points of R, where $i = \sqrt{-1}$

C. For any tow non-zero complex number z_1 and z_2, $arg\left(\frac{z_1}{z_2} - arg(z_1) + arg(z_2)\right.$ is an integer multiple of 2π

D. For any three given distinct complex numbers z_1, z_2 and z_3 the locus of the point z satisfying the condition $\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$

, lies on a strainght line.

Answer: A::B::D

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9. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy \left(x, y \in \mathbb{R}, i = \sqrt{-1}\right)$ of the equation $sz + tz + r = 0$, where $z = x - iy$. Then, which of the following statement(s) is (are) TRUE? If L has exactly one element, then $|s| \neq |t|$ (b) If $|s| = |t|$, then L has infinitely many elements (c) The number of elements in $\ln n\{z : |z - 1 + i| = 5\}$ is at most 2 (d) If L has more than one element, then L has infinitely many elements

A. If L has exactly one element, then $|s| \neq |t|$

B. If $|s| = |t|$ then L has infinitely many elements

C. The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

D. If L has most than one elements, then L has infinitely many elements.

Answer: A::C::D

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10. Let $S = S_1 \cap S_2 \cap S_3$, where

$s_1 = \{z \in C : |z| < 4\}, S_2 = \left\{z \in C : \ln\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{31}}\right] > 0\right\}$ and $S_3 = \{z \in C : Re$

A. $\frac{10\pi}{3}$

B. $\frac{20\pi}{3}$

C. $\frac{16\pi}{3}$

D. $\frac{32\pi}{3}$

Answer: B

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11. Let $S = S_1 \cap S_2 \cap S_3$, where $S_1 = \{zinC:|z| < 4\}$,

$S_2 = \left\{z \ \mathrm{in}C:Im\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{zinC : Rez > 0\}$

$\min_{z \in S} |1 - 3i - z| =$

A. $\frac{2 - \sqrt{3}}{2}$

B. $\frac{2 + \sqrt{3}}{2}$

C. $\frac{3 - \sqrt{3}}{2}$

D. $\frac{3 + \sqrt{3}}{2}$

Answer: C

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12. Let ω be the complex number $\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$. Then the number of distinct complex cos numbers z satisfying

$\Delta = \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is

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13. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$ then the maximum value of $|2z - 6 + 5i|$ is

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14. For any integer k, let $\alpha_k = \frac{\cos(k\pi)}{7} + i\sin.\frac{k\pi}{7}$, where $I = \sqrt{-1}$. Value of the expression. $\frac{\sum_{k=1}^{12}\left|\alpha_{k+1} - \alpha_k\right|}{\sum_{k=1}^{3}\left|\alpha_{4k-1} - \alpha_{4k-2}\right|}$ is_______.

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