# MATHS

# BOOKS - CENGAGE

# CONIC SECTIONS

## Solved Examples And Exercises

**1.** Given that $A(1, 1)$ and $B(2, -3)$ are two points and $D$ is a point on $AB$ produced such that $AD = 3AB$. Find the coordinates of $D$.

Watch Video Solution

**2.** Find the coordinates of the point which divides the line segments joining the points $(6, 3)$ and $(-4, 5)$ in the ratio $3 : 2$ (i) internally and (ii) externally.

Watch Video Solution

**3.** Four points $A(6, 3), B(-3, 5), C(4, -2)$ and $D(x, 2x)$ are given in such a way that $\frac{(Area of DBC)}{(Area of ABC)} = \frac{1}{2}$.

Watch Video Solution

**4.** If the points $(1, 1) : (0, \sec^2\theta);$ and $(\cos ec^2\theta, 0)$ are collinear, then find the value of $\theta$

Watch Video Solution

**5.** If $P$ divides $OA$ internally in the ratio $\lambda_1 : \lambda_2$ and $Q$ divides $OA$ externally in the ratio $\lambda_1 ; \lambda_2,$ then prove that $OA$ is the harmonic mean of $OP$ and $OQ$.

Watch Video Solution

**6.** Prove that the point $(-2, -1), (1, 0), (4, 3) and (1, 2)$ are the vertices of parallel-gram. Is it a rectangle?

Watch Video Solution

**7.** Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1,3) and $(2, 7)$.

Watch Video Solution

**8.** Find the orthocentre of the triangle whose vertices are $(0, 0)$, $(3, 0)$, and $(0, 4)$.

Watch Video Solution

**9.** If a vertex of a triangle is $(1, 1)$ , and the middle points of two sides passing through it are $-2, 3)$ and $(5, 2)$, then find the centroid and the incenter of the triangle.

Watch Video Solution

**10.** The vertices of a triangle are $A(-1, -7), B(5, 1) and C(1, 4)$. If the internal angle bisector of $\angle B$ meets the side $AC$ in $D$, then find the length $AD$.

Watch Video Solution

**11.** If $ABC$ having vertices $A(a\cos\theta_1, a\sin\theta_1), B(a\cos\theta_2 a\sin\theta_2), and C(a\cos\theta_3, a\sin\theta_3)$ is equilateral, then prove that $\cos\theta_1+\cos\theta_2+\cos\theta_3=\sin\theta_1+\sin\theta_2+\sin\theta_3=0.$

Watch Video Solution

**12.** If the point $(x, -1), (3, y), (-2, 3), and(-3, -2)$ taken in order are the vertices of a parallelogram, then find the values of $x and y$.

Watch Video Solution

**13.** If the midpoints of the sides of a triangle are $(2, 1), (-1, -3), and(4, 5),$ then find the coordinates of its vertices.

Watch Video Solution

**14.** If the circumcenter of an acute-angled triangle lies at the origin and the centroid is the middle point of the line joining the points $\left(a^2+1, a^2+1\right)$ and $(2a, -2a),$ then find the orthocentre.

Watch Video Solution

**15.** If a vertex, the circumcenter, and the centroid of a triangle are (0, 0), (3,4), and (6, 8), respectively, then the triangle must be (a) a right-angled triangle (b) an equilateral triangle (c) an isosceles triangle (d) a right-angled isosceles triangle

Watch Video Solution

**16.** Orthocenter and circumcenter of a $\mathrm{Delta}ABC$ are $(a, b)and(c, d)$ , respectively. If the coordinates of the vertex $A$ are $(x_1, y_1)$, then find the coordinates of the middle point of $BC$.

Watch Video Solution

**17.** The points $(a, b), (c, d)$, and $\left(\frac{kc+la}{k+l}, \frac{kd+lb}{k+l}\right)$ are (a) vertices of an equilateral triangle (b) vertices of an isosceles triangle (c) vertices of a right-angled triangle (d) collinear

Watch Video Solution

**18.** The line joining $A(b\cos\alpha b\sin\alpha)$ and $B(a\cos\beta, a\sin\beta)$ is produced to the point $M(x, y)$ so that $AM$ and $BM$ are in the ratio $b : a\cdot$ Then prove that $x + y\tan\left(\alpha + \frac{\beta}{2}\right) = 0.$

Watch Video Solution

**19.** If the middle points of the sides of a triangle are $(-2, 3), (4, -3), and(4, 5)$ , then find the centroid of the triangle.

Watch Video Solution

**20.** In what ratio does the x=axis divide the line segment joining the points $(2, -3)$ and (5, 6)?

Watch Video Solution

**21.** If $(1, 4)$ is the centroid of a triangle and the coordinates of its any two vertices are $(4, -8)$ and $(-9, 7)$, find the area of the triangle.

Watch Video Solution

**22.** If $(x_i, y_i), i = 1, 2, 3,$ are the vertices of an equilateral triangle such that

$$(x_1 + 2)^2 + (y_1 - 3)^2 = (x_2 + 2)^2 + (y_2 - 3)^2 = (x_3 + 2)^2 + (y_3 - 3)^2,$$

then find the value of $\dfrac{x_1 + x_2 + x_3}{y_1 + y_2 + y_3}$.

Watch Video Solution

**23.** Find the locus of a point, so that the join of (-5, 1) and (3, 2) subtends a right angle at the moving point.

Watch Video Solution

**24.** The sum of the squares of the distances of a moving point from two fixed points (a,0) and $(-a, 0)$ is equal to a constant quantity $2c^2$. Find the equation to its locus.

Watch Video Solution

**25.** $AB$ is a variable line sliding between the coordinate axes in such a way that $A$ lies on the x-axis and $B$ lies on the y-axis. If $P$ is a variable point on $AB$ such that $PA = b, Pb = a$ , and $AB = a + b,$ find the equation of the locus of $P$.

Watch Video Solution

**26.** A rod of length $l$ slides with its ends on two perpendicular lines. Find the locus of its midpoint.

Watch Video Solution

**27.** Find the locus of the point $\left(t^2 - t + 1, t^2 + t + 1\right), t \in R$.

Watch Video Solution

**28.** Find the locus of a point such that the sum of its distance from the points (2, 2) and $(2, \ -2)$ is 6.

Watch Video Solution

**29.** Two points P(a,0) and Q(-a,0) are given. $R$ is a variable point on one side of the line $PQ$ such that $\angle RPQ - \angle RQP$ is a positive constant $2\alpha$. Find the locus of the point $R$.

Watch Video Solution

**30.** If the coordinates of a variable point $P$ are $(a\cos\theta, b\sin\theta)$, where $\theta$ is a variable quantity, then find the locus of $P$.

Watch Video Solution

**31.** Find the locus of a point whose distance from (a, 0) is equal to its distance from the y-axis.

Watch Video Solution

**32.** The coordinates of the point $AandB$ are (a,0) and $(-a, 0)$, respectively. If a point $P$ moves so that $PA^2 - PB^2 = 2k^2$, when $k$ is constant, then find the equation to the locus of the point $P$.

Watch Video Solution

**33.** Find the locus of the foot of perpendicular from the point (2, 1) on the variable line passing through the point (0, 0).

Watch Video Solution

**34.** A variable line through the point $P(2, 1)$ meets the axes at $A and B$ . Find the locus of the centroid of triangle $OAB$ (where $O$ is the origin).

Watch Video Solution

**35.** If $A(\cos\alpha, \sin\alpha), B(\sin\alpha, -\cos\alpha), C(1, 2)$ are the vertices of $ABC,$ then as $\alpha$ varies, find the locus of its centroid.

Watch Video Solution

**36.** Let A (2, -3) and B (-2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x + 3y=1, then the locus of the vertex C is the line

Watch Video Solution

**37.** A straight line is drawn through $P(3, 4)$ to meet the axis of $x$ and $y$ at $A and B$ , respectively. If the rectangle $OACB$ is completed, then find the locus of $C$.

Watch Video Solution

**38.** A variable line through point $P(2, 1)$ meets the axes at $A and B$ . Find the locus of the circumcenter of triangle $OAB$ (where $O$ is the origin).

Watch Video Solution

**39.** A point moves such that the area of the triangle formed by it with the points (1,5) and (3,-7) is 21 sq. units. Then locus of the point is

Watch Video Solution

**40.** Find the locus of the point of intersection of lines $x \cos \alpha + y \sin \alpha = a$ and $x \sin \alpha - y \cos \alpha = b$($\alpha$ is a variable).

Watch Video Solution

**41.** Find the locus of the middle point of the portion of the line $x \cos \alpha + y \sin \alpha = p$ which is intercepted between the axes, given that $p$ remains constant.

Watch Video Solution

**42.** $Q$ is a variable point whose locus is $2x + 3y + 4 = 0$; corresponding to a particular position of $Q$, $P$ is the point of section of $OQ$, $O$ being the origin, such that $OP : PQ = 3 : 1$. Find the locus of $P$.

Watch Video Solution

**43.** Convert $y = 10$ into a polar equation.

Watch Video Solution

**44.** Find the minimum distance of any point on the line $3x + 4y - 10 = 0$ from the origin using polar coordinates.

Watch Video Solution

**45.** Express the polar equation $r = 2\cos\theta$ in rectangular coordinates.

Watch Video Solution

**46.** Convert $x^2 - y^2 = 4$ into a polar equation.

Watch Video Solution

**47.** Convert $r\sin\theta = r\cos\theta + 4$ into its equivalent Cartesian equation.

Watch Video Solution

**48.** Convert $r = \cos ec\theta e^{r\cos\theta}$ into its equivalent Cartesian equation.

Watch Video Solution

**49.** Find the maximum distance of any point on the curve $x^2 + 2y^2 + 2xy = 1$ from the origin.

Watch Video Solution

**50.** Convert $r = 4\tan\theta\sec\theta$ into its equivalent Cartesian equation.

Watch Video Solution

**51.** Given the equation $4x^2 + 2\sqrt{3}xy + 2y^2 = 1$ . Through what angle should the axes be rotated so that the term $xy$ is removed from the transformed equation.

Watch Video Solution

**52.** The equation of a curve referred to a given system of axes is $3x^2 + 2xy + 3y^2 = 10.$ Find its equation if the axes are rotated through an angle $45^0$ , the origin remaining unchanged.

Watch Video Solution

**53.** Determine $x$ so that the line passing through $(3, 4) and (x, 5)$ makes an angle of $135^0$ with the positive direction of the x-axis.

Watch Video Solution

**54.** What does the equation $2x^2 + 4xy - 5y^2 + 20x - 22y - 14 = 0$ become when referred to the rectangular axes through the point $(-2, -3)$, the new axes being inclined at an angle at $45^0$ with the old axes?

Watch Video Solution

**55.** Shift the origin to a suitable point so that the equation $y^2 + 4y + 8x - 2 = 0$ will not contain a term in $y$ and the constant term.

Watch Video Solution

**56.** At what point should the origin be shifted if the coordinates of a point $(4, 5)$ become $(-3, 9)$?

Watch Video Solution

**57.** Find the equation to which the equation $x^2 + 7xy - 2y^2 + 17x - 26y - 60 = 0$ is transformed if the origin is shifted to the point $(2, -3)$, the axes remaining parallel to the original axies.

Watch Video Solution

**58.** The equation of curve referred to the new axes, axes retaining their directions, and origin $(4, 5)$ is $X^2 + Y^2 = 36$ . Find the equation referred to the original axes.

Watch Video Solution

**59.** If the point $(2, 3), (1, 1), and(x, 3x)$ are collinear, then find the value of $x$, using slope method.

Watch Video Solution

**60.** Find the orthocentre of $\Delta ABC$ with vertices $A(1, 0), B(-2, 1)$, and $C(5, 2)$

Watch Video Solution

**61.** The angle between the line joining the points (1,-2) , (3,2) and the line x+2y -7 =0 is

Watch Video Solution

**62.** The line joining the points $A(2, 1), andB(3, 2)$ is perpendicular to the line $(a^2)x + (a + 2)y + 2 = 0$. Find the values of $a$.

Watch Video Solution

**63.** For what value of $k$ are the points $(k, 2 - 2k), (-k + 1, 2k) and (-4 - k, 6 - 2k)$ collinear?

Watch Video Solution

**64.** Find the area of the quadrilateral $ABCD$ having vertices $A(1, 1), B(7, -3), C(12, 2),$ and $D(7, 21)$.

Watch Video Solution

**65.** Given that $P(3, 1), Q(6. 5),$ and $R(x, y)$ are three points such that the angle $PQR$ is a right angle and the area of $RQP$ is 7, find the number of such points $R$.

Watch Video Solution

**66.** If $O$ is the origin and if the coordinates of any two points $Q_1 and Q_2$ are $(x_1, y_1) and (x_2, y_2)$, respectively, prove that $OQ_1 \dot{O} Q_2 \cos\angle Q_1 O Q_2 = x_1 x_2 + y_1 y_2$.

Watch Video Solution

**67.** Prove that the area of the triangle whose vertices are $(t, t-2), (t+2, t+2),$ and $(t+3, t)$ is independent of $t$.

Watch Video Solution

**68.** Find the area of a triangle having vertices $A(3, 2), B(11, 8),$ and $C(8, 12)$.

Watch Video Solution

**69.** In $ABC$ Prove that $AB^2 + AC^2 = 2(AO^2 + BO^2)$ , where $O$ is the middle point of $BC$

Watch Video Solution

**70.** Find the coordinates of the circumcenter of the triangle whose vertices are $(A(5, -1), B(-1, 5),$ and $C(6, 6)$. Find its radius also.

Watch Video Solution

**71.** Find the orthocentre of $ABC$ with vertices $A(1, 0), B(-2, 1),$ and $C(5, 2)$

Watch Video Solution

**72.** If $(b_2 - b_1)(b_3 - b_1) + (a_2 - a_1)(a_3 - a_1) = 0$ , then prove that the circumcenter of the triangle having vertices $(a_1, b_1), (a_2, b_2)$ and $(a_3, b_3)$ is $\left(\frac{a_{2+a_3}}{2}, \frac{b_{2+}b_3}{2}\right)$

Watch Video Solution

**73.** If line $3x - ay - 1 = 0$ is parallel to the line $(a + 2)x - y + 3 = 0$ then find the value of $a$.

Watch Video Solution

**74.** If $A(2, -1) and B(6, 5)$ are two points, then find the ratio in which the food of the perpendicular from $(4, 1)$ to $AB$ divides it.

Watch Video Solution

**75.** Angle of a line with the positive direction of the x-axis is $\theta$ . The line is rotated about some point on it in anticlockwise direction by angle $45^0$ and its slope becomes $3.$ Find the angle $\theta$.

Watch Video Solution

**76.** Let $A(6, 4) and B(2, 12)$ be two given point. Find the slope of a line perpendicular to $AB$.

Watch Video Solution

**77.** If the points $(a, 0), (b, 0), (0, c)$, and $(0, d)$ are concyclic $(a, b, c, d > 0)$, then prove that $ab = cd$.

Watch Video Solution

**78.** If three points are $A(-2, 1)B(2, 3), andC(-2, -4)$ , then find the angle between $ABandBC$.

Watch Video Solution

**79.** The line joining the points $(x, 2x)and(3, 5)$ makes an obtuse angle with the positive direction of the x-axis. Then find the values of $x$.

Watch Video Solution

**80.** If the line passing through $(4, 3)and(2, k)$ is parallel to the line $y = 2x + 3,$ then find the value of $k$.

Watch Video Solution

**81.** Find the area of the pentagon whose vertices are $A(1, 1), B(7, 21), C(7, -3), D(12, 2),$ and $E(0, -3)$

Watch Video Solution

**82.** Let $A=(3,4)$ and $B$ is a variable point on the lines $|x|$ =6. IF $AB\leq 4$ , then find the number of position of $B$ with integral coordinates.

Watch Video Solution

**83.** The three points $(-2,2)(9,-2), and(-4,-3)$ are the vertices of (a) an isosceles triangle (b) an equilateral triangle (c) a right-angled triangle (d) none of these

Watch Video Solution

**84.** The points $(-a, -b), (a, b), \left(a^2, ab\right)$ are (a) vertices of an equilateral triangle (b) vertices of a right angled triangle (c) vertices of an isosceles triangle (d) collinear

Watch Video Solution

**85.** Find the length of altitude through $A$ of the triangle $ABC,$ where $A \equiv (-3, 0)B \equiv (4, -1), C \equiv (5, 2)$

Watch Video Solution

**86.** If the coordiantes of two points A and B are (3,4) and (5, -2) respectively. Find the coordinates of any point P if PA = PB and area of $\Delta PAB = 10$ sq. units.

Watch Video Solution

**87.** The vertices of a triangle have integer co- ordinates then the triangle cannot be

Watch Video Solution