MATHS

BOOKS - CENGAGE

CONTINUITY AND DIFFERENTIABILITY

Solved Examples And Exercises

1. Find x where $f(x) = max\left\{\sqrt{x(2-x)}, 2-x\right\}$ is non-differentiable.

Watch Video Solution

2. Discuss the differentiability of $f(x) = \sin|x|$

Watch Video Solution

3. Discuss the continuity and differentiability of $f(x) = |x+1| + |x| + |x-1| \, \forall x \in R;$ also draw the graph of $f(x)$

Watch Video Solution

4. Discuss the differentiability of $f(x) = \max\{\tan^{-1} x, \cot^{-1} x\}.$

Watch Video Solution

5. Find the values of $aandb$ if $f(x) = \{ax^2 + 1, x \leq 1, x^2 + ax + b, x > 1\text{i s differentiable at}x = 1$

Watch Video Solution

6. Discuss the differentiability of $f(x) = |px[x]|, \ -1 < x \leq 2$

Watch Video Solution

7. Find the values of $aandb$ if

$f(x) = \{ax^2 + 1, x \leq 1x^2 + ax + b, x > 1isd \Leftrightarrow erentiableatx = 1$

Watch Video Solution

8. Discuss the differentiability of $f\left(x = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)\right.$

Watch Video Solution

9. Which of the following function is non-differentiable in domain?

$f(x) = \frac{x - 2}{x^2 + 3}$ (b) $f(x) = \log|x|$ $f(x) = x^3 \log x$ (d) $f(x) = (x - 3)^{\frac{3}{5}}$

Watch Video Solution

10. Discuss the continuity of $f(x) = \left\{\frac{1}{e^{x-1}}, x \neq 10, x = 1\right.$

Watch Video Solution

11. Which of the following functions is not continuous $\forall x \in R$? (a) $\sqrt{2\sin x + 3}$ (b) $\frac{e^x + 1}{e^x + 3}$ (c) $\left(\frac{2^{3x} + 1}{2^{3x} + 5}\right)^{\frac{5}{7}}$ (d) $\sqrt{sgnx + 1}$

Watch Video Solution

12. Find the value of $f(0)$ so that the function. $f(x) = \frac{\sqrt{1+x} - 1 + x3}{x} becomescont \in uousatx = 0$

Watch Video Solution

13. If the function $f(x) = \frac{x^2 - (A+2)x + A}{x - 2}, f \text{ or } x \neq 2 and f(2) = 2,$ is continuous at $x = 2,$ then find the value of A.

Watch Video Solution

14. If the function $f: R \setminus \{0\}^{\rightarrow}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ is continuous at $x = 0$, then find the value of $f(0)$

Watch Video Solution

15. *Let* $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$, If $f(x) is$ continuous in $\left[0, \frac{\pi}{4}\right]$, then find the value of $f\left(\frac{\pi}{4}\right)$.

Watch Video Solution

16. If $f(x) = \left(\tan\left(\frac{\pi}{4} + (\log)_e x\right)\right)^{(\log)_x e}$ is to be made continuous at $x = 1$, then what is the value of $f(1)$?

Watch Video Solution

17.

$f(x) = \left\{2x \tan x - \frac{\pi}{\cos x}, x \neq \frac{\pi}{2} kx = \frac{\pi}{2} iscont \in uousatx = \frac{\pi}{2},\right.$

then find the value of k.

Watch Video Solution

18. Discuss the continuity of $f(x) = \begin{cases} \frac{x^2}{|x|}, x \neq 00, x = 0 \end{cases}$

Watch Video Solution

19. Let $f(x) = \begin{cases} (1+3x)^{\frac{1}{x}}, x \neq 0 e^3, x = 0 \end{cases}$ Discuss the continuity of $f(x)$ at (a) $x = 0$, (b) $x = 1$

Watch Video Solution

20. Which of the following function is not differentiable at $x = 0$?

$f(x) = \min\{x, \sin x\}$ $f(x) = \begin{cases} 0, x \geq 0 x^2, x < 0 \end{cases}$f(x)= x^2 sgn(x)

Watch Video Solution

21. Discuss the differentiability of $f(x) = \left|\left|x^2 - 4\right| - 12\right|$.

Watch Video Solution

22. Find the value of x in $[1, 3]$ where the function $\left[x^2 + 1\right]$ ([.] represents the greatest integer function) is discontinuous.

Watch Video Solution

23. If $f(x) = \{x - 2, x \leq 0 \text{ and } 4 - x^2, x > 0,$ then discuss the continuity of $y = f(f(x))$.

Watch Video Solution

24. Discuss the continuity of $f(x) = \left[\tan^{-1} x\right]$ ([.] represents the greatest integer function).

Watch Video Solution

25. Find the number of points of discontinuity for $f(x) = [6 \sin x], 0 \leq x \leq \pi$

Watch Video Solution

26. Find the value of x where $f(x) = \{x, \quad \text{if} \quad \xi srational 1 - x, \quad \text{if} \quad \xi sirrationaliscont \in uous \cdot$

Watch Video Solution

27. Discuss the continuity of the following function : f(x)={1 if x rational ; 0 if x is irrational

Watch Video Solution

28. If $f(x) = \frac{x+1}{x-1} and g(x) = \frac{1}{x-2}, then$ discuss the continuity of $f(x), g(x), and fog(x).$

Watch Video Solution

29. For $x > 0, leth(x) = \begin{cases} \frac{1}{q}, & \text{if } x = \frac{p}{q} 0, \text{ if } \xi sirrational \end{cases}$ where $p, q > 0$ are relatively prime integers. Then prove that $f(x)$ is continuous for all irrational values of $x.$

Watch Video Solution

30. Discuss the continuity of $f(x) = (\lim)_{n \overrightarrow{\infty}} \cos^{2n} x.$

Watch Video Solution

31. Find the values of a if $f(x) = (\lim)_{n \to \infty} \frac{ax^{2n} + 2}{x^{2n} + a + 1}$ is continuous at $x = 1.$

Watch Video Solution

32. If

$$f(x)=\left\{(\ln x).\, sgn\left(\left\{x-\frac{1}{2}\right\}\right);1<x\leq 3 \text{ and } \{x^2\};3<x\leq 3.5.\right.$$

Find the pointswhere the continuity of $f(x),$ should be checked, where [.] is greatest integer function and {.} fractional part function.

Watch Video Solution

33. Discuss the continuity of $f(x)=(\lim)_{n\overrightarrow{\infty}}\frac{x^{2n}-1}{x^{2n}+1}$

Watch Video Solution

34. If $f(x)=sgn(2\sin x+a)$ is continuous for all $x,$ then find the possible values of $a\cdot$

Watch Video Solution

35. Discuss the continuity of $f(x) = sgn\left(x^3 - x\right)$

Watch Video Solution

36. Discuss the continuity of $f(x) = sgn(2\cos x - 1)$

Watch Video Solution

37. Discuss the continuity of $f(x) = sgn\left(x^2 - 2x + 3\right)$

Watch Video Solution

38. Discuss the continuity of $f(x) = \{x\{x\} + 1, 0 \leq x < 1 \text{ and } 2 - \{x\}, 1 \leq x < 2$ where $\{x\}$ denotes the fractional part function.

Watch Video Solution

39. Discuss the continuity of $f(x) = sgn\left(x^3 - x\right)$

Watch Video Solution

40. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$.

Watch Video Solution

41. Prove that $f(x) = \frac{x^3}{4} - \sin \pi x + 3$ takes the value of $\frac{7}{3}$ for $x \in [-2, 2]$.

Watch Video Solution

42. Suppose f is a continuous map from R to R and $f(f(a)) = a$ for some a. Show that there is some b such that $f(b) = b$.

Watch Video Solution

43. Let $f:[0,1] \to [0,1]$ be a continuous function. Then prove that $f(x) = x$ for at least one $0 \leq x \leq 1$.

Watch Video Solution

44. Discuss the differentiability of f(x)=$\begin{cases} x\sin\left(\ln x^2\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ at x=0

Watch Video Solution

45. Discuss the differentiability of $f(x) = \left\{ \frac{\sin x^2}{x}, x \neq 00, x = 0 at x = 0 \right.$

Watch Video Solution

46. If f is an even function such that $\left(\lim\right)_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then prove that $f(x)$ is not differentiable at $x = 0$.

Watch Video Solution

Watch Video Solution

47. Using intermediate value theorem, prove that there exists a number x such that $x^{2005} + \frac{1}{1+\sin^2 x} = 2005.$

Watch Video Solution

48. Which of the function is non-differential at $x = 0$? $f(x) = |\mathrm{x}|$

Watch Video Solution

49. Which of the function is non-differential at $x = 0$? $f(x) = \cos|x|$

Watch Video Solution

50. $f(x) = \left\{[\sin \pi x], 0 \leq x < 1 sgn\left(x - \frac{5}{4}\right) x\left\{x - \frac{2}{3}\right\}, 1 \leq x \leq 2\right.$ where [.] denotes the greatest integer function and {.} represents the

fractional part function. At what points should the continuity be checked?

Hence, find the points of discontinuity.

Watch Video Solution

51. Find the value of a for which $f(x) = \{x^2, x \in Qx + a, x \notin Q$ is not continuous at any x.

Watch Video Solution

52. Discuss the continuity of $f(x) = (\log|x|)sgn(x^2 - 1), x \neq 0.$

Watch Video Solution

53. Find the number of integers lying in the interval (0,4) where the function $f(x) = (\lim)_{n \to \infty} \left(\frac{\cos(\pi x)}{2}\right)^{2n}$ is discontinuous

Watch Video Solution

54. If $f(x)$ is continuous function $\forall x \in R$ and the range of $f(x) is (2, \sqrt{26}) and g(x) = \left[\frac{f(x)}{c}\right]$ is continuous $\forall x \in R$, then find the least positive integral value of c, where $[.]$ denotes the greatest integer function.

Watch Video Solution

55. Discuss the continuity of $f(x) =$ {x^2, x is rational`

-x^2,x is irrational

Watch Video Solution

56. If $y = \frac{1}{t^2 + t - 2}, where t = \frac{1}{x - 1}$, then find the number of points where $f(x)$ is discontinuous.

Watch Video Solution

57. Let f be a continuous function defined onto [0,1] with range [0,1]. Show that there is some c in [0,1] such that $f(c) = 1 - c$

Watch Video Solution

58. Leg f be continuous on the interval [0,1] to R such that $f(0) = f(1)$. Prove that there exists a point c in $\left[\frac{0,1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.

Watch Video Solution

59.

$f(x) = \{ax(x-1) + b, x < 1x - 1, 1 \leq x \leq 3. px^2 + qx + 2, x > 3$

Find the values of the constants $a, b, pandq$ so that all the following conditions are satisfied $f(x)$ is continuous for all x. $f(1)$ does not exist. $f'(x)$ is continuous at $x = 3$

Watch Video Solution

60. Find the values of $aandb$ if $f(x) = \{a + \sin^{-1}(x + b), x \geq 1 \text{ and } x, x < 1$ is differentiable at $x = 1$.

Watch Video Solution

61. Discuss the differentiability of $y = \sin^{-1}|\sin x|$

Watch Video Solution

62. Discuss the differentiability of $y = \sin^{-1}(\sin x)$

Watch Video Solution

63. Discuss the differentiability of $f(x) = ma\xi\mu m\{2\sin x, 1 - \cos x\} \forall x \in (0, \pi)$.

Watch Video Solution

64. Discuss the differentiability of $f(x) = \max\{x^2 - 3x + 2, 2 - |x - 1|\}$

Watch Video Solution

65. If $f(x) = \max\{x^2 + 2ax + 1, b\}$ has two points of non-differentiability, then prove that $a^2 > 1 - b$

Watch Video Solution

66. Discuss the differentiability of $f(x) = e^{-|x|}$

Watch Video Solution

67. If $f(x) = \{x, x \leq 1, x^2 + bx + c, x > 1'$ find b and c if function is continuous and differentiable at $x = 1$

Watch Video Solution

68. Test the continuity and differentiability of the function $f(x) = \left|\left(x + \frac{1}{2}\right)[x]\right|$ by drawing the graph of the function when $-2 \leq x < 2$, where $[.]$ represents the greatest integer function.

Watch Video Solution

69. Let $f(x) = \left\{\frac{\log(1+x)^{1+x} - x}{x^2}\right\}$. Then find the value of $f(0)$ so that the function f is continuous at $x = 0$.

Watch Video Solution

70. Find the points of discontinuity of the function: $f(x) = [[x]] - [x-1], where[.]$ represents the greatest integer function

Watch Video Solution

71. let $f(x) = \frac{\ln \cos x}{(1+x^2)^{\frac{1}{4}} - 1}$ if $x > 0$ and $f(x) = \frac{e^{\sin 4x} - 1}{\ln(1 + \tan 2x)}$ if $x < 0$ Is it possible to difine $f(0)$ to make the function continuous at $x = 0$. If yes what is.the value of $f(0)$, if not then indicate the nature of discontinuity.

Watch Video Solution

72. What value must be assigned to k so that the function $f(x) = \begin{cases} \frac{x^4 - 256}{x - 4}, & x \neq 4 \text{ and } k, x = 4 \end{cases}$ is continuous at x=4.

Watch Video Solution

73. Let $f(x)$ be a function defined as $f(x) = \begin{cases} \frac{x^2 - 1}{x^2 - 2|x-1| - 1}, & x \neq 1\frac{1}{2}, x = 1 \end{cases}$ Discuss the continuity of the function at $x = 1$.

Watch Video Solution

74. If the function `f(x)={A x-B ,xlt=1 3x ,1

Watch Video Solution

75. Let : $f(x) = \begin{cases} \frac{a + 3\cos x}{x^2}, x < 0 b \tan\left(\frac{\pi}{[x+3]}\right), x \geq 0 \end{cases}$ If $f(x)$ is continuous at $x = 0,$ then find $aandb$, where $[.]$ denotes the greatest integer function.

Watch Video Solution

76. Let : $f(x) = \begin{cases} \frac{\sin ax^2}{x^2}, x \neq 0 \frac{3}{4} + \frac{1}{4a}, x = 0 \end{cases}$ for what values of a is $f(x)$ continuous at $x = 0$?

Watch Video Solution

77. $f(x) = \{|x+1|; x \leq 0 x; x > 0$ and $g(x) = \{|x| + 1; x \leq 1 - |x-2|; x > 1$ Draw its graph and discuss the

continuity of $f(x) + g(x)$.

Watch Video Solution

78. f(x)={cos^(-1{cotx),x< pi/2 pi[x]-1,x >pi/2$where$[dot] $represents the greatest function$ and {dot}` represents the fractional part function. Find the jump of discontinuity.

Watch Video Solution

79. Discuss the differentiability of $f(x) = |(\log)_e|x||$

Watch Video Solution

80. Discuss the differentiability of $f(x) = \max\{\sec^{-1}x, \cos ec^{-1}x\}$

Watch Video Solution

81. A function $f(x)$ is such that $f\left(x+\frac{\pi}{2}\right)=\frac{\pi}{2}-|x| \forall x \dot{F} \in df\left(\frac{\pi}{2}\right), \quad \text{if} \quad ite\xi sts.$

Watch Video Solution

82. Discuss the differentiability of f(x)=|x||x-1|

Watch Video Solution

83. Discuss the differentiability of $f(x)=\left|x^3\right|$ at $x=0$

Watch Video Solution

84. Discuss the differentiability of f(x)=|x||x-1|

Watch Video Solution

85. Discuss the differentiability of $f(x) = \sin|x|$

Watch Video Solution

86. Discuss the differentiability of $f(x) = [x] + |1 - x|, x \in (-1, 3), where[.]$ represents greatest integer function.

Watch Video Solution

87. Discuss the differentiability of $f(x) = \left(x^2 - 1\right)\left|x^2 - x - 2\right| + \sin(|x|)$.

Watch Video Solution

88. Let f be a function satisfying $f(x + y) + \sqrt{6 - f(y)} = f(x)f(y) and f(h) \overrightarrow{6} ash \overrightarrow{0}$. Discuss the

continuity of f.

Watch Video Solution

89. Let $f(x+y)=f(x)+f(y)$ for all $xandy$. If the function $f(x)$ is continuous at $x=0$, show that $f(x)$ is continuous for all x.

Watch Video Solution

90. A function $f(x)$ satisfies the following property: $f(x+y)=f(x)f(y)$. Show that the function is continuous for all values of x if its is continuous at $x=1$.

Watch Video Solution

91. Find the points of discontinuity of the function: $f(x)=\frac{1}{2\sin x-1}$

Watch Video Solution

92. Find the points of discontinuity of the function: $f(x) = \frac{1}{x^2 - 3|x| + 2}$

Watch Video Solution

93. A function $f(x)$ satisfies the following property: $f(xy) = f(x)f(y)$. Show that the function $f(x)$ is continuous for all values of x if it is continuous at $x = 1$.

Watch Video Solution

94. Find the points of discontinuity of the function: $f(x) = \frac{1}{x^4 + x^2 + 1}$

Watch Video Solution

95. Find the points of discontinuity of the function: $f(x) = \frac{1}{1 - e^{\frac{x-1}{x-2}}}$

Watch Video Solution

96. Draw the graph and find the points of discontinuity $f(x) = [2\cos x]$, $x \in [0, 2\pi]$. ([.] represents the greatest integer function.)

Watch Video Solution

97. Find the number of points where $f(x) = \left[\frac{x}{3}\right], x \in [0, 30]$, is discontinuous (where [.] represents greatest integer function).

Watch Video Solution

98. Discuss the continuity of the function ([.] represents the greatest integer function): $f(x) = \left[\frac{2}{1+x^2}\right], x \geq 0$

Watch Video Solution

99. Discuss the continuity of the function ([.] represents the greatest integer function): $f(x) = \left[\sin^{-1} x\right]$

Watch Video Solution

100. Discuss the continuity of the function ([.] represents the greatest integer function): $f(x) = \left[(\log)_e x\right]$

Watch Video Solution

101. Match the following for the type of discontinuity at $x = 1$ in column II for the function in column I. $f(x) = \dfrac{1}{x-1}$, p. Removable discontinuity $f(x) = \dfrac{x^3 - x}{x^2 - 1}$, q. Non-removable discontinuity $f(x) = \dfrac{|x-1|}{x-1}$, r. Jump of discontinuity $f(x) = \sin\left(\dfrac{1}{x-1}\right)$, s. Discontinuity due to vertical asymptote , t. Missing point discontinuity , u. Oscillating discontinuity

Watch Video Solution

102. Discuss the continuity of $f(x) = \begin{cases} \dfrac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|}, & x \neq 1, 2, 6, \\ x = 1, 12, & x = 2 \end{cases}$

Watch Video Solution

103. If $x = \sqrt{5} + 2$, then find the value of $x^2 + \dfrac{1}{x^2}$

Watch Video Solution

104. If the function $f(x) = \left[\dfrac{(x-2)^3}{a}\right] \sin(x-2) + a\cos(x-2)$, $[.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

Watch Video Solution

105. Draw the graph and discuss the continuity of $f(x) = [\sin x + \cos x], x \in [0, 2\pi)$, where [.] represents the greatest integer function.

Watch Video Solution

Single Correct Answer Type

1. If $f(x) = \left\{\left(\frac{\sin(2x^2)}{a} + \cos\left(\frac{3x}{b}\right)\right)^a b/x^2, x \neq 0 e^3, x = 0\right.$ is continuous at $x = 0 \forall b \in R$ then minimum value of a is $-1/8$ b. $-1/4$ c. $-1/2$ d. 0

A. $-1/8$

B. $-1/4$

C. $-1/2$

D. 0

Answer: B

Watch Video Solution

2. Let $f: R\overrightarrow{R}$ be any function. Also $g: R\overrightarrow{R}$ is defined by $g(x) = |f(x)|$ for all x. Then is Onto if f is onto One-one if f is one-one Continuous if f is continuous None of these

A. onto if f is onto

B. one-one if f is one-one

C. continuous if f is continuous

D. None of these

Answer: C

Watch Video Solution

3. about to only mathematics

A. 1

B. $1/4$

C. 4

D. none of these

Answer: D

Watch Video Solution

4. Let $f(x) = x^3 - x^2 - 3x - 1, g(x) = (x+1)a$ and $h(x) = \frac{f(x)}{g(x)}$

where h is a rational function such that

(i) It is continuous everywhere except when $x = -1$,

(ii) $\lim_{x \to \infty} h(x) = \frac{1}{2}$.

The value of h(1) is

A. $1/2$

B. $1/4$

C. $-1/2$

D. 1

Answer: C

View Text Solution

5. If the function $f(x) = \frac{3x^2ax + a + 3}{x^2 + x - 2}$ is continuous at $x = -2$, then the value of $f(-2)$ is

A. 0

B. -1

C. 1

D. 2

Answer: B

View Text Solution

6. Let $f(x)=\begin{cases}8^{\frac{1}{x}}, & x<0\\ a[x], & a\in R-\{0\}, \quad x\geq 0\end{cases}$ (where [.] denotes the greatest integer function).

Then f(x) is

A. continuous only at a finite number of points.

B. discontinuous at a finite number of points.

C. discontinuous at an infinite number of points.

D. discontinuous at x = 0.

Answer: C

View Text Solution

7. If $f(x)=\begin{cases}\dfrac{2\cos x,\, s\in 2x}{(\pi-2x)^2}, & x\leq \dfrac{\pi}{2}\\ \dfrac{e^{-\cot x}-1}{8x-4\pi}, & x>\dfrac{\pi}{2}\end{cases}$, then which of the following holds? f is continuous at $x=\pi/2$ f has an irremovable discontinuity at $x=\pi/2$ f has a removable discontinuity at $x=\pi/2$

None of these

A. f is continuous at $x = \pi/2$

B. f has an irremovable discontinuity at $x = \pi/2$

C. f has a removable discontinuity at $x = \pi/2$

D. none of these

Answer: B

View Text Solution

8. If

$$f(x) = \begin{cases} \sin\left(\frac{\pi}{2}\right)(x - [x]), & x < 55(b-1) \\ x = 5 \\ \frac{ab^2|x^2 - 11x + 24|}{x-3}, & x > \end{cases}$$

is continuous at $x = 5, a, b \in R$ then ([.] denotes the greatest integer function) $a = \frac{25}{108}, b = \frac{6}{5}$ b. $a = \frac{6}{13}, b = \frac{17}{29}$ c. $a = \frac{1}{2}, b = \frac{25}{36}$ d. $a = \frac{23}{100}, b = \frac{6}{5}$

A. $a = \frac{25}{108}, b = \frac{6}{5}$

B. $a = \frac{6}{13}, b = \frac{17}{29}$

C. $a = \frac{1}{2}, b = \frac{25}{36}$

D. $a = (23), (100), b = \frac{6}{5}$

Answer: A

Watch Video Solution

9. The function f(x) is discontinuous only at x = 0 such that $f^2(x) = 1 \forall x \in R$. The total number of such functions is

A. 2

B. 3

C. 6

D. none of these

Answer: C

View Text Solution

10. $f(x)=\left\{\left(x^2+e^{\frac{1}{2-x}}\right)^{-1} k, x=2, x \neq 2\right.$ is continuous from right at the point $x=2,$ then k equals 0 b. $1/4$ c. $-1/4$ d. none of these

A. 0

B. 1\4

C. $-1/4$

D. none of these

Answer: B

Watch Video Solution

11. Let $g(x)=f(f(x))$ where $f(x)=\{1+x; 0 \leq x \leq 2\}$ and $f(x)=\{3-x; 2<x \leq 3\}$ then the number of points of discontinuity of $g(x)$ in [0,3] is :

A. 0

B. 1

C. 2

D. gt 2

Answer: C

View Text Solution

12. If the function $f(x) = \frac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$ is continuous at $x = 0$, then the value of a/b is $\frac{3}{5}f(0)$ b. $2^{8/5}f(0)$ c. $\frac{64}{5}f(0)$ d. none of these

A. $\frac{3}{5}f(0)$

B. $2^{8/5}f(0)$

C. $\frac{64}{5}f(0)$

D. none of these

Answer: C

Watch Video Solution

13. If $f(x) = \begin{cases} \dfrac{1-\cos\left(1-\frac{\cos x}{2}\right)}{2^m x^n} 1x = 0, x \neq 0 \end{cases}$ is continous at $x = 0$ then the value of $m + n$ is a. 2 b. 3 c. -3 d. 7

A. 2

B. 3

C. -3

D. 7

Answer: C

View Text Solution

14. Let $f(x) = \begin{cases} \frac{\alpha \cot x}{x} + \frac{\beta}{x^2}, & 0 < x \leq 1 \\ \frac{1}{3}, & x = 0 \end{cases}$

If f(x) is continuous at x =0, then the value of $\alpha^2 + \beta^2$ is

A. 1

B. 2

C. 5

D. 9

Answer: B

View Text Solution

15. Let $f(x) = \begin{cases} \dfrac{2}{1+x^2}, \xi srationalb, \xi srational \end{cases}$ has exactly two points of continuity then the value of b are $(0, 3]$ b. $[0, 1]$ c. $(0, 2]$ d.φ

A. (0,3]

B. [0,1]

C. (0,2]

D. ϕ

Answer: C

View Text Solution

16. If `f(x)={sin((a-x)/2)t h a n[(pix)/(2a)]forx > a([cos((pix)/2)])/(a-x)forx0, $then$f(a^-)<0$b.$ f$hasaremovab \leq discont \in uityat$x=a$c.$ f

$hasanirremovab \leq discont \in uityat$x=a$d.$ f(a^+)<0`

A. $f\left(a^{-}\right)<0$

B. f has a removable discontinuity at x = a

C. f has an irremovable discontinuity at x = a

D. $f\left(a^{+}\right)<0$

Answer: B

Watch Video Solution

17. Let $f(x)=[tanx[\cot x]], x\left[\frac{\pi}{12}, \frac{\pi}{12}\right]$, (where [.] denotes the greatest integer less than or equal to$x$). Then the number of points, where $f(x)$ is discontinuous is a. one b. zero c. three d. infinite

A. one

B. zero

C. three

D. infinite

Answer: C

View Text Solution

18. Let $f:[a,b] \to R$ be any function which is such that $f(x)$ is rational for irrational x and that $f(x)$ is iirrational for rational x, then in [a,b]

A. f is discontinuous everywhere

B. f is discontinuous only at x = 0 and discontinuous everywhere

C. f is continuous for all irrational x and discontinuous for rational x

D. f is continuous for rational x and discontinuous for irrational x

Answer: A

View Text Solution

19. If $f(x) = [x](\sin kx)^p$ is continuous for real x, then (where [.] represents the greatest integer function)

A. $k \in [n\pi, n \in I], p > 0$

B. $k \in \{2n\pi, n \in I\}, p > 0$

C. $k \in \{n\pi, n \in I\}, p \in R - \{0\}$

D. $k \in \{n\pi, nI, n \neq 0\}, p \in R - \{0\}$

Answer: A

View Text Solution

20. Statement 1: Minimum number of points of discontinuity of the function $f(x) = (g(x)[2x - 1] \forall x \in (-3, -1)$, where [.] denotes the greatest integer function and $g(x) = ax^3 = x^2 + 1$ is zero. Statement 2: $f(x)$ can be continuous at a point of discontinuity, say $x = c_1 of[2x - 1]$ if $g(c_1) = 0$. Statement 1 is True, Statement 2 is

True, Statement 2 isa correct explanation for Statement 1. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for statement 1. Statement 1 is True, Statement 2 is False Statement 1 is False, Statement 2 is True.

A. Statement 1 is True, Statement 2 is True, Statement 2 is a correct explaination for Statement 1.

B. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for Statement 1/

C. Statement 1 is True, Statement 2 is False.

D. Statement 1 is False, Statement 2 is True.

Answer: D

View Text Solution

21. Number of points of discontinuity of $f(x) = \left[\sin^{-1} x\right] - [x]$ in its domain is equal to (where [.] denotes the greatest integer function) a. 0

b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: D

Watch Video Solution

22. If $g(x) = (\lim)_{m \to \infty} \frac{x^m f(x) + h(x) + 3}{2x^m + 4x + 1}$ when $x \neq 1 and g(1) = e^3$ such that $f(x), g(x) and h(x)$ are continuous functions at $x = 1$ then the value of $5f(1) - 2h(1)$ is 7 b. 6 c. 9 d. 8

A. 7

B. 6

C. 9

D. 8

Answer: B

Watch Video Solution

23. The number of points of discontinuity of $fx\Big) = \left[2x^2\right] - \{2x2\}^2$ (where [] denotes the greatest integer function and {} is fractional part of x) in the interval $(-2, 2),$ is 1 b. 6 c. 2 d. 4

A. 1

B. 6

C. 2

D. 5

Answer: B

Watch Video Solution

24. Let $f(x) = \begin{cases} -3 + |x|, & -\infty < x < 1 \\ a + |2 - x|, & 1 \leq x < \infty \end{cases}$ and $g(x) = \begin{cases} 2 - |-x|, & -\infty < x < 2 \\ -b + \text{sgn(x)}, & 2 \leq x < \infty \end{cases}$

where sgn(x) denotes signum function of x. If $h(x) = f(x) + g(x)$ is discontinuous at exactly one point, then which of the following is not possible?

A. $a = -3, b = 0$

B. $a = 0, b = 1$

C. $a = 2, b = 1$

D. $a = -3, b = 1$

Answer: D

View Text Solution

25. The function $f(x) = \frac{x^3}{8} - s \in \pi x + 4 \in [-4, 4]$ does not take the value -4 b. 10 c. 18 d. 12

A. -4

B. 10

C. 18

D. 12

Answer: C

View Text Solution

26. Let $f(x)$ be continuous functions $f: R\overrightarrow{R}$ satisfying $f(0) = 1 and f(2x) - f(x) = x$. Then the value of $f(3)$ is 2 b. 3 c. 4 d. 5

A. 2

B. 3

C. 4

D. 5

Answer: C

Watch Video Solution

27. about to only mathematics

A. $a = b = 4$

B. $a = b = -4$

C. $a = 4$ and $b = -4$

D. $a = -4$ and $b = 4$

Answer: C

Watch Video Solution

28. If $f(x) = \begin{cases} [x] + \sqrt{\{x\}}, x < 1 \\ \dfrac{1}{[x] + \{x\}^2}, x \geq 1 \end{cases}$, then [where [.] and {.] represent the greatest integer and fractional part functions respectively] $f(x)$ is continuous at $x = 1$ $f(x)$ is not continuous at $x = 1$ $f(x)$ is differentiable at $x = 1$ $(\lim)_{x \to 1} f(x)$ does not exist

A. f(x) is continuous at x = 1 but not differentiable

B. f(x) is not continuous at x = 1

C. f(x) is differentiable at x = 1

D. $\lim_{x \to 1} f(x)$ does not exist

Answer: A

Watch Video Solution

29. If f is an even function such that $\left(\lim \right)_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then prove that $f(x)$ is not differentiable at $x = 0$.

A. f is continuous and derivable at x =0

B. f is continuous but not differentiable at x = 0

C. f may be discontinuous at x = 0

D. None of these

Answer: B

Watch Video Solution

30. Let $f(x)$ be differentiable for real x such that $f'(x) > 0 on (-\infty, -4)$, $f'(x) < 0 on (-4, 6)$, $f'(x) > 0 on (6, \infty)$, If $g(x) = f(10 - 2x)$, then the value of $g'(2)$ is a. 1 b. 2 c. 0 d. 4

A. 1

B. 2

C. 0

D. 4

Answer: C

Watch Video Solution

31. Number of points where $f(x) = x^2 - |x^2 - 1| + 2||x| - 1| + 2|x| - 7$ is non-differentiable is a. 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: A

View Text Solution

32. If $f(x) = |x - 1|. ([x] = [-x]),$ then (where [.] represents greatest integer function)

A. f(a) is continuous and differentiable at x = 1

B. f(x) is discontinuous at x = 1

C. f(x) is continuous at x = 2

D. f(x) is continuous but non-differentiable at x= 1

Answer: D

View Text Solution

33. Number of points where function $f(x)$ defined as $f:[0,2]\pi\overrightarrow{R}, f(x)=\left\{3-\left|\cos x-\frac{1}{\sqrt{2}}\right|,\left|\sin x<\frac{1}{\sqrt{2}}\right|2+\left|\cos x+\frac{1}{\sqrt{2}}\right|,\right.$ is non-differentiable is a. 2 b. 4 c. 6 d. 0

A. 2

B. 4

C. 6

D. 0

Answer: B

Watch Video Solution

34. Let $f(x)=\begin{cases}[x] & x \not\in I \\ x-1 & x \in I\end{cases}$ (where, [.] denotes the greatest integer function) and $g(x)=\begin{cases}\sin x+\cos x, & x<0 \\ 1, & x \geq 0\end{cases}$ Then for f(g(x)) at x = 0

A. $\lim_{x \to 0} g(g(x))$ exists but not continuous

B. continuous but not differentiable at x = 0

C. differentiable at x = 0

D. $\lim_{x \to 0} f(g(x))$ does not exist

Answer: C

View Text Solution

35. If $f(x) = \begin{cases} \sin\left(\cos^{-1} x\right) + \cos\left(\sin^{-1} x\right), & x \leq 0 \\ \sin\left(\cos^{-1} x\right) - \cos\left(\sin^{-1} x\right), & x > 0 \end{cases}$ then at x = 0

A. f(x) is continuous and differentiable

B. f(x) is continuous but not differentiable

C. f(x) not continuous but differentiable

D. f(x) is neither continuous nor differentiable

Answer: D

View Text Solution

View Text Solution

36. If $f(x) = \max\{\tan x, \sin x, \cos x\}$ where $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right)$ then the number of points, where f(x) is non -differentiable, is

A. 2

B. 3

C. 4

D. 5

Answer: B

View Text Solution

37. The number of points at which $g(x) = \frac{1}{1+\frac{2}{f(x)}}$ is not differentiable, where $f(x) = \frac{1}{1+\frac{1}{x}}$, is 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: C

Watch Video Solution

38. Let $f(x) = \lim_{n \to \infty} \sum_{r=0}^{n-1} \frac{x}{(rx+1)\{(r+1)x+1\}}$. Then

A. f(x) is continuous but not differentiable at x = 0

B. f(x) is both continuous and differentiable at x = 0

C. f(x) is neither continuous not differentiable at x = 0

D. f(x) is a periodic function

Answer: C

Watch Video Solution

39. Let the given function is differentiable at x = 1.

$$f(x)=\begin{cases}\lim\limits_{n\to\infty}\dfrac{ax(x-1)\left(\cot.\frac{\pi x}{4}\right)^n+(px^2+2)}{\left(\cot,\frac{\pi x}{4}\right)^n+1}, & x\in(0,1)\cup(1,2)\\ 0, & x=1\end{cases}$$

Then the value of $|a+q|$ is

A. 4

B. 6

C. 8

D. 10

Answer: B

Watch Video Solution

Multiple Correct Answer Type

1. Which of the following functions is/are not discontinuous at $x=1$?

A. $f(x) = \frac{1}{1+2^{\tan x}}$

B. $g(x) = \lim_{n \to \infty} \frac{1}{1+n\sin^2(\pi x)}$

C. $h(x) = 2^{-2^{\left(\frac{1}{1-x}\right)}}, x \neq 1$ and $h(1) = 1$

D. $\phi(x) = \frac{x-1}{|x-1|+2(x-1)^2}, x \neq 1$ and $\phi(1) = 1$

Answer: A

Watch Video Solution

2. about to only mathematics

A. h(x) may or may not be continuous in [a, c]

B. $h(b^+) = g(b^-)$ and $h(b^-) = f(b^+)$

C. $h(b^-) = g(b^+)$ and $h(b^+) = f(b^-)$

D. h(x) has a removable discontinuity at x = b

Answer: C::D

Watch Video Solution

3. If the function $f(x)$ defined as $f(x)$ defined as $f(x) = \left\{3, x = 0\left(1 + \frac{ax + bx^3}{x^2}\right), x > 0\right.$ is continuous at $x = 0$, then $a = 0$ b. $b = e^3$ c. $a = 1$ d. $b = (\log)_e 3$

A. a = 0

B. $b = e^3$

C. $a = 1$

D. $b = \log_e 3$

Answer: A::D

View Text Solution

4. Given $f(x) = \left\{3 - \left[\cot^{-1}\left(\frac{2x^3 - 3}{x^2}\right)\right] f \text{ or } x > 0\{x^2\}\cos\left(e^{\frac{1}{x}}\right) f \text{ or } x < 0\right.$ (where {} and [] denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does

not hold good? $f(0^-) = 0$ b. $f(0^+) = 3$ c. if $f(0) = 0$, then $f(x)$ is continuous at $x = 0$ d. irremovable discontinuity of f at $x = 0$

A. $f(0^-) = 0$

B. $f(0^+) = 3$

C. If $f(0) = 0,$ then f(x) is continuous at x = 0

D. Irremovable discontinuity of f at x = 0

Answer: B::D

View Text Solution

5. Let $f(x) = \begin{cases} x\left[\frac{1}{x}\right] + x[x] & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$ (where [x] denotes the greatest integer function). Then the correct statement is/are

A. Limit exists for $x = -1$.

B. f(x) has a removable discontinuity at x = 1.

C. f(x) has a non removable discontinuity at x = 2.

D. f(x) is discontinuous at all positive integers.

Answer: A::B::C::D

View Text Solution

6. A function $f: R \to R$ is defined as

$f(x) = \lim_{n \to \infty} \frac{ax^2 + bx + c + e^{nx}}{1 + c.\, e^{nx}}$ where f is continuous on R, then

A. point (a, b, c) lies on line in space

B. point (a, b) represents the 2-dimensional Cartesian plane

C. Locus of point (a, c) and (c, b) intersect at one point

D. point (a, b, c) lies on the plane in space

Answer: A::B::C

View Text Solution

7. Let f be a function with continuous second derivative and $f(0) = f'(0) = 0.$ Determine a function g by $g(x) = \begin{cases} \frac{f(x)}{x}, x \neq 00, x = 0 \end{cases}$ Then which of the following statements is correct? g has a continuous first derivative g has a first derivative g is continuous but g fails to have a derivative g has a first derivative but the first derivative is not continuous

A. g has a continuous first derivative

B. g has a first derivative

C. g is continuous but g fails to have a derivative

D. g has a first derivative but the first derivative is not continuous

Answer: A::B

Watch Video Solution

8. Let $f(x)$ be a function defined on $(-a, a)$ with $a > 0.$ Assume that $f(x)$ is continuous at

$x = 0 and (\lim)_{x \to 0} \frac{f(x) - f(kx)}{x} = \alpha, where k \in (0, 1)$ then

$f'(0^+) = 0$ b. $f'(0^-) = \frac{\alpha}{1-k}$ c. $f(x)$ is differentiable at $x = 0$ d. $f(x)$ is non-differentiable at $x = 0$

A. $f'(0^+) = 0$

B. $f'(0^-) = \frac{\alpha}{1-k}$

C. f(x) is defferentiable at x = 0

D. f(x) is non-differentiable at x = 0

Answer: B::C::D

Watch Video Solution

9. If $f(x) = x^{1/3}(x-2)^{2/3}$ for all x, then the domain of f' is $x \in R - \{0\}$ b. $\{x \mid x \rangle 0\}$ c. $x \in R - \{0, 2\}$ d. $x \in R$

A. $x \in R - \{0\}$

B. $\{x \mid x > 0\}$

C. $x \in R - \{0, 2\}$

D. $x \in R$

Answer: C

Watch Video Solution

Comprehension Type

1. Consider two function $y = f(x)$ and $y = g(x)$ defined as

$$f(x) = \begin{cases} ax^2 + b & 0 \le x \le 1 \\ bx + 2b & 1 < x \le 3 \\ (a-1)x + 2c - 3 & 3 < x \le 4 \end{cases}$$

and

$$g(x) = \begin{cases} cx + d & 0 \le x \le 2 \\ ax + 3 - c & 2 < x < 3 \\ x^2 + b + 1 & 3 \ge x \le 4 \end{cases}$$

Let f be differentiable at x = 1 and g(x) be continuous at x = 3. If the roots of the quadratic equation $x^2 + (a+b+c)\alpha x + 49(k + k\alpha) = 0$ are real distinct for all values of α then possible values of k will be

A. $k \in (-1, 0)$

B. $k \in (\infty, 0)$

C. $k \in (1, 5)$

D. $k \in (-1, 1)$

Answer: A

Watch Video Solution

2. Consider two function $y = f(x)$ and $y = g(x)$ defined as

$$f(x) = \begin{cases} ax^2 + b & 0 \le x \le 1 \\ bx + 2b & 1 < x \le 3 \\ (a-1)x + 2c - 3 & 3 < x \le 4 \end{cases}$$

and $$g(x) = \begin{cases} cx + d & 0 \le x \le 2 \\ ax + 3 - c & 2 < x < 3 \\ x^2 + b + 1 & 3 \ge x \le 4 \end{cases}$$

$\lim_{x \to 2} \frac{f(x)}{|g(x)| + 1}$ exists and f is differentiable at x = 1. The value of limit will be

A. -2

B. -1

C. 0

D. 2

Answer: C

Watch Video Solution

Examples

1. A function $f(x)$ satisfies the following property: $f(xy) = f(x)f(y)$. Show that the function $f(x)$ is continuous for all values of x if it is continuous at $x = 1$.

Watch Video Solution

2. Let $f(x) = \left\{ \frac{\log(1+x)^{1+x} - x}{x^2} \right\}$. Then find the value of $f(0)$ so that the function f is continuous at $x = 0$.

Watch Video Solution

3. Let : $f(x)=\begin{cases}\dfrac{a+3\cos x}{x^2}, & x<0 \\ b\tan\left(\dfrac{\pi}{[x+3]}\right), & x\geq 0\end{cases}$ If $f(x)$ is continuous at $x=0,$ then find $aandb$, where $[.]$ denotes the greatest integer function.

Watch Video Solution

4. Let $f(x)$ be a function defined as $f(x)=\begin{cases}\dfrac{x^2-1}{x^2-2|x-1|-1}, & x\neq 1\dfrac{1}{2} \\ , & x=1\end{cases}$ Discuss the continuity of the function at $x=1.$

Watch Video Solution

5. about to only mathematics

Watch Video Solution

6. Draw the graph and discuss the continuity of $f(x) = [\sin x + \cos x], x \in [0, 2\pi]$, where [.] represents the greatest integer function.

Watch Video Solution

7. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right] \sin(x-2) + a\cos(x-2)$, $[.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

Watch Video Solution

8. If $f(x) = sgn(2\sin x + a)$ is continuous for all x, then find the possible values of a.

Watch Video Solution

9. Discuss the continuity of $f(x) = (\lim)_{n\to\infty} \frac{x^{2n}-1}{x^{2n}+1}$

Watch Video Solution

10. Find the values of a if $f(x) = (\lim)_{n\to\infty} \frac{ax^{2n}+2}{x^{2n}+a+1}$ is continuous at $x = 1$.

Watch Video Solution

11. For $x > 0, leth(x) = \begin{cases} \frac{1}{q}, & \text{if } x = \frac{p}{q}0, \\ \text{if } \xi sirrational & \end{cases}$ where $p, q > 0$ are relatively prime integers. Then prove that $f(x)$ is continuous for all irrational values of x.

Watch Video Solution

12. If $f(x) = \frac{x+1}{x-1} and g(x) = \frac{1}{x-2}, then$ discuss the continuity of $f(x), g(x), and fog(x)$.

Watch Video Solution

13. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$.

Watch Video Solution

14. Using intermediate value theorem, prove that there exists a number x such that $x^{2005} + \frac{1}{1+\sin^2 x} = 2005.$

Watch Video Solution

15. Discuss the differentiability of $f(x) = \left\{ \frac{\sin x^2}{x}, x \neq 00, x = 0 at x = 0 \right.$

Watch Video Solution

16. Discuss the differentiability of

$f(x) = \{x \sin(1nx^2), x \neq 00, x = 0atx = 0$

Watch Video Solution

17. If $f(x) = \{x, x \leq 1x^2 + bx + c, x > 1'$

Watch Video Solution

18. Find the values of $aandb$ if

$f(x) = \{a + \sin^{-1}(x + b), x \geq 1x, x < 1$ is differentiable at $x = 1.$

Watch Video Solution

19.

$f(x) = \{ax(x - 1) + b, x < 1x - 1, 1 \leq x \leq 3. px^2 + qx + 2, x > 3$

Find the values of the constants $a, b, pandq$ so that all the following

conditions are satisfied $f(x)$ is continuous for all $x\cdot f(1)$ does not exist.

$f'(x)$ is continuous at $x = 3$

Watch Video Solution

20. If $f(x) = \max\left\{x^2 + 2ax + 1, b\right\}$ has two points of non-differentiability, then prove that $a^2 > 1 - b$

Watch Video Solution

21. Test the continuity and differentiability of the function $f(x) = \left|\left(x + \frac{1}{2}\right)[x]\right|$ by drawing the graph of the function when $-2 \leq x < 2,$ where $[.]$ represents the greatest integer function.

Watch Video Solution

22. Discuss the differentiability of $f(x) = [x] + \backslash 1 - x \mid \dot{x} \in (-1, 3), where[.]$ represents greatest

integer function.

Watch Video Solution

23. Discuss the differentiability of $f(x) = \left(x^2 - 1\right)\left|x^2 - x - 2\right| + \sin(|x|)$.

Watch Video Solution

24. Let $f(x-y) = f(x).\, g(y) - f(y).\, g(x)$ and $g(x-y) = g(x).\, g(y) - f(x).\, g($

for all $x, y \in R$. If right hand derivative at x = 0 exists for f(x), then derivative of g(x) at x = 0

Watch Video Solution

Illustration

1. Discuss the continuity of the following function : f(x)={1 if x rational ; 0 if x is irrational

Watch Video Solution

Exercise 4 1

1. A function $f(x)$ satisfies the following property: $f(xy) = f(x)f(y)$. Show that the function $f(x)$ is continuous for all values of x if it is continuous at $x = 1$.

Watch Video Solution

Exercise Single

1. If $f(x) = \frac{a\cos x - \cos bx}{x^2}, x \neq 0 and f(0) = 4$ is continous at $x = 0$, then the ordered pair (a, b) is $(\pm 1, 3)$ b. $(1, \pm 3)$ c. $(-1, -3)$ d.

$(-1, 3)$

A. $\pm 1, 3)$

B. $(1, \pm 3)$

C. $(-1, -3)$

D. $(1, 3)$

Answer: B

Watch Video Solution

2. Let f be a continuous function on R such that $f\left(\frac{1}{4n}\right) = \frac{\sin e^n}{e^{n^2}} + \frac{n^2}{n^2+1}$ Then the value of $f(0)$ is

A. 1

B. $1/2$

C. 0

D. none of these

Answer: A

Watch Video Solution

3. If $f(x) = \frac{x^2 - bx + 25}{x^2 - 7x + 10}$ for $x \neq 5$ and f is continuous at $x = 5$ then $f(5) =$

A. 0

B. 5

C. 10

D. 25

Answer: A

Watch Video Solution

4. The function $f(x) = \frac{\tan|\pi[x - \pi]|}{1 + [x]^2}$, where [x] denotes the greatest integer less than or equal to x, is

A. f(x) is discontinuous at some x

B. f(x) is continuous at all x, but the derivative f'(x) does not exist for some x

C. $f'(x)$ exists for all x, but f''(x) does not exist for some x

D. f'(x) exists for all x

Answer: D

Watch Video Solution

5. if $f(x) = \begin{cases} \frac{1-|x|}{1+x} & x \neq -1 \\ 1 & x = -1 \end{cases}$ then f([2x]) , where [.] represents the greatest integer function , is

A. discontinuous at x=-1

B. continuous at x=0

C. continuous at x=1/2

D. continuous at x=1

Answer: B

Watch Video Solution

6. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4a + b, \\ \frac{x-4}{|x-4|} + b, x > 4 \end{cases}$ Then $f(x)$ is continous at $x = 4$ when $a = 0, b = 0$ b. $a = 1, b = 1$ c. $a = -1, b = 1$ d. $a = -1, b = -1$

A. a=0,b=0

B. a=1,b=1

C. a=-1,b=1

D. a=1,b=-1

Answer: D

Watch Video Solution

7. Which of the following is true about

$$Let f(x) = \begin{cases} \frac{x-2}{|x-2|}\left(\frac{x^2-1}{x^2+1}\right) & x \neq 2 \\ \frac{3}{5} & x = 2 \end{cases} ?$$

A. f(X) is continuous at x=2

B. f(x) has removable discontinuity ata x=2.

C. f(x) has non-removable discontinuity at x=2 .

D. Discontiuity at x=2 can be removed by redefining the function at x=2.

Answer: C

Watch Video Solution

8. If $f(x) = \begin{cases} x+2 & x < 0 \\ -x^2 - 2 & 0 \leq x < 1 \\ x & x \geq 1 \end{cases}$

then the number of points of discontinuity of |f(x)|is

A. 1

B. 2

C. 3

D. none of these

Answer: A

Watch Video Solution

9. Which of the following statements is always true? ([.] represents the greatest integer function. a) If f(x) is discontinuous then $|f(x)|$ is discontinuous b) If $f(x)$ is discontinuous then f(|x|) is discontinuous c) f(x)=[g(x)] is discontinous when `g(x) is an integer d) none of these

A. if f(x) is discontinuous then |f(x)| is discontinuous

B. if f(x) is discontinuous , then f(|x|) is discontinuous .

C. f(x) =[g(x)] is discontinuous , when g(x) is an integer

D. none of these

Answer: D

Watch Video Solution

10. The number of point $f(x) = \begin{cases} [\cos \pi x] & 0 \leq x < 1 \\ |2x - 3|[x - 2] & 1 < x \leq 2 \end{cases}$ is discontinuous at Is ([.] denotes the greatest intgreal function)

A. two

B. three

C. four

D. zero

Answer: B

Watch Video Solution

Question Bank

1. Let $f(x) = \frac{\sqrt{x^2 + px + 1}}{x^2 - p}$. If f(x) is discontinuous at exactly 2 values of x then number of integers in the range of p is

Watch Video Solution