



MATHS

BOOKS - CENGAGE

CONTINUITY AND DIFFERENTIABILITY

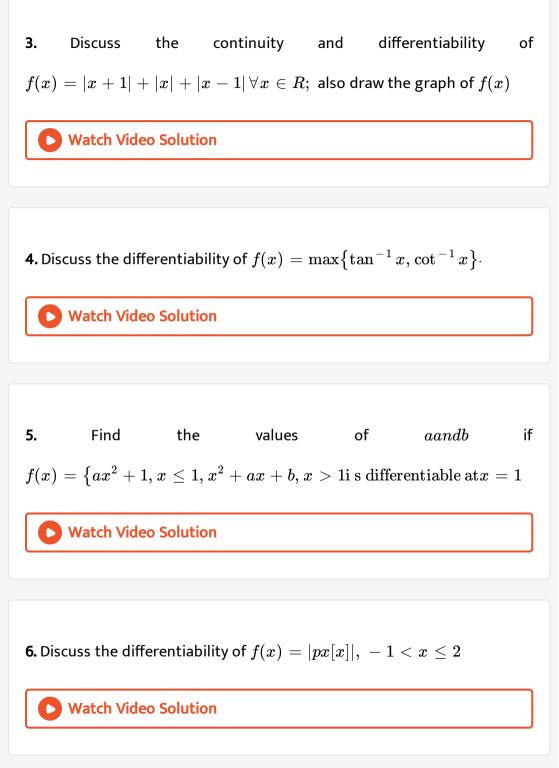
Solved Examples And Exercises

1. Find
$$x$$
 where $f(x)=ma\xi\mu m\Big\{\sqrt{x(2-x)},\,2-x\Big\}$ is non-

differentiable.

Watch Video Solution

2. Discuss the differentiability of $f(x) = \sin \lvert x \rvert$



7. Find the values of
$$aandb$$
 if
 $f(x) = \{ax^2 + 1, x \le 1x^2 + ax + b, x > 1isd \Leftrightarrow erentiableatx = 1$
Watch Video Solution

8. Discuss the differentiability of
$$f\left(x=\cos^{-1}\left(rac{1-x^2}{1+x^2}
ight)$$

Watch Video Solution

9. Which of the following function is non-differentiable in domain? $f(x)=rac{x-2}{x^2+3}$ (b) $f(x)=\log \lvert x
vert f(x)=x^3\log x$ (d) $f(x)=(x-3)^{rac{3}{5}}$

Watch Video Solution

10. Discuss the continuity of
$$f(x) = \left\{ rac{1}{e^{x-1}}, x
eq 10, x = 1
ight.$$

11. Which of the following functions is not continuous $\,\,orall x \in R ?\,$ (a)

$$\sqrt{2\sin x+3}$$
 (b) $rac{e^x+1}{e^x+3}$ (c) $\left(rac{2^{3x}+1}{2^{3x}+5}
ight)^{rac{5}{7}}$ (d) $\sqrt{sgnx+1}$

12. Find the value of
$$f(0)$$
 so that the function. $f(x)=rac{\sqrt{1+x}-1+x3}{x}becomescont\in uousatx=0$

13. If the function
$$f(x)=rac{x^2-(A+2)x+A}{x-2}, f ext{ or } x
eq 2and f(2)=2, ext{ is continuous at }$$

 $x=2, ext{ then find the value of } A \cdot$

14. If the function $f\!:\!R\!\setminus\;\{0\}^{
ightarrow}$ given by $f(x)=rac{1}{x}-rac{2}{e^{2x}-1}$ is

continuous at $x=0,\,$ then find the value of f(0)

15. Let
$$f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$$
, If $f(x)$ is continuous in $\left[0, \frac{\pi}{4}\right]$, then find the value of $f\left(\frac{\pi}{4}\right)$.

Watch Video Solution

16. If
$$f(x) = \left(an \left(rac{\pi}{4} + (\log)_e x
ight)
ight)^{(\log)_x e}$$
 is to be made continuous at

 $x=1, ext{ then what is the value of } f(1)$?

$$f(x)=\Big\{2x an x-rac{\pi}{\cos x},x
eqrac{\pi}{2}kx=rac{\pi}{2}iscont\in uousatx=rac{\pi}{2},$$

then find the value of k_{\cdot}



18. Discuss the continuity of
$$f(x) = \left\{ rac{x^2}{|x|}, x
eq 00, x = 0
ight.$$

Watch Video Solution

19. Let $f(x)=ig\{(1+3x)^{rac{1}{x}},x
eq 0e^3,x=0$ Discuss the continuity of f(x) at (a) x=0, (b) x=1

Watch Video Solution

20. Which of the following function is not differentiable at x = 0?

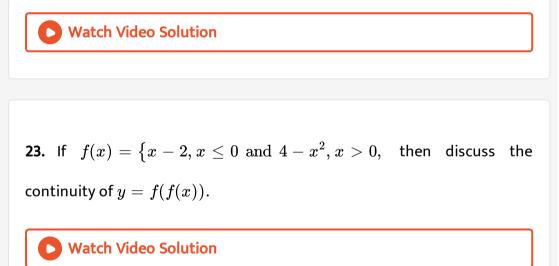
$$f(x) = \min \left\{x, \sin x
ight\} f(x) = \left\{0, x \geq 0x^2, x < 0$$
f(x)= x^2 sgn(x)

21. Discuss the differentiability of $f(x) = \left|\left|x^2 - 4\right| - 12\right|$.

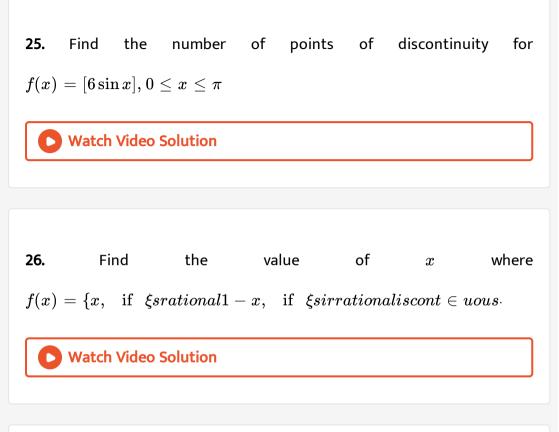


22. Find the value of x in [1,3] where the function $\left[x^2+1
ight]([.]$ represents

the greatest integer function) is discontinuous.



24. Discuss the continuity of $f(x) = [\tan^{-1} x]([.]$ represents the greatest integer function).



27. Discuss the continuity of the following function : f(x)={1 if x rational ; 0

if x is irrational



28. If $f(x) = rac{x+1}{x-1} and g(x) = rac{1}{x-2},$ then discuss the continuity of f(x), g(x), and fog(x).

Watch Video Solution

29. For
$$x > 0$$
, $leth(x) = \left\{\frac{1}{q}, \text{ if } x = \frac{p}{q}0, \text{ if } \xi sirrational where $p, q > 0$ are relatively prime integers. Then prove that $f(x)$ is continuous for all irrational values of x .$

Watch Video Solution

30. Discuss the continuity of $f(x) = (\lim)_{n \stackrel{\longrightarrow}{\infty}} \cos^{2n} x$.

Watch Video Solution

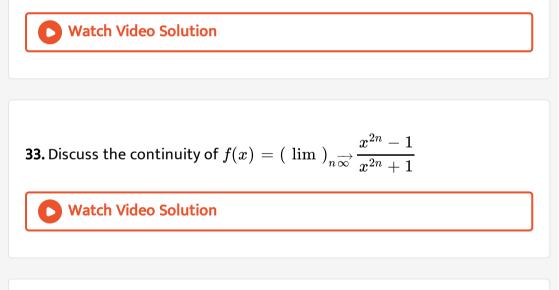
31. Find the values of a if $f(x) = (\lim)_{n o \infty} rac{a x^{2n} + 2}{x^{2n} + a + 1}$ is continuous

at x = 1.

32.

$$f(x) = igg\{(\ln x). \ sgnigg(igg\{x - rac{1}{2}igg\}igg); 1 < x \leq 3 \ ext{and} \ ig\{x^2ig\}; 3 < x \leq 3.5.$$

Find the pointswhere the continuity of f(x), should be checked, where [.] is greatest integer function and {.} fractional part function.

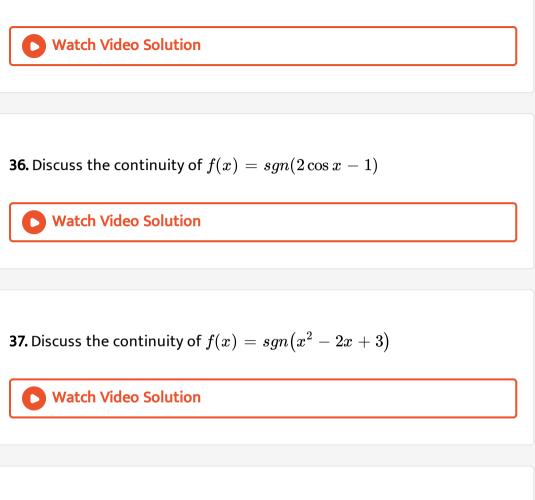


34. If $f(x) = sgn(2\sin x + a)$ is continuous for all x, then find the possible values of a.



If

35. Discuss the continuity of $f(x) = sgnig(x^3-xig)$



38. Discuss the continuity of $f(x)=\{x\{x\}+1, 0\leq x<1 ext{ and } 2-\{x\}, 1\leq x<2 ext{ where } \{x\}$

denotes the fractional part function.



39. Discuss the continuity of $f(x) = sgnig(x^3-xig)$



40. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $rac{a+b}{2}$ for some value of $x \in [a,b]$.

Watch Video Solution

41. Prove that
$$f(x)=rac{x^3}{4}-\sin\pi x+3$$
 takes the value of $rac{7}{3}$ for $x\in [-2,2]$.

Watch Video Solution

42. Suppose f is a continuous map from R to R and f(f(a)) = a for some a. Show that there is some b such that f(b) = b.

43. Let $f\colon [0,1] o [0,1]$ be a continuous function. Then prove that f(x)=x for at least one $0\leq x\leq 1.$

Watch Video Solution

44. Discuss the differentiability of
$$f(x) = \begin{cases} x \sin(\ln x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$$
 at x=0
Watch Video Solution

45. Discuss the differentiability of
$$f(x) = \left\{ \frac{\sin x^2}{x}, x \neq 00, x = 0 atx = 0 \right\}$$
 Watch Video Solution

46. If f is an even function such that $(\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then prove that f(x) is not differentiable at x = 0.

47. Using intermediate value theorem, prove that there exists a number x

such that
$$x^{2005} + rac{1}{1+\sin^2 x} = 2005.$$

Watch Video Solution

48. Which of the function is non-differential at x=0? $f(x)=|\mathbf{x}|$

Watch Video Solution

49. Which of the function is non-differential at $x=0? \; f(x)=\cos \lvert x \rvert$

Watch Video Solution

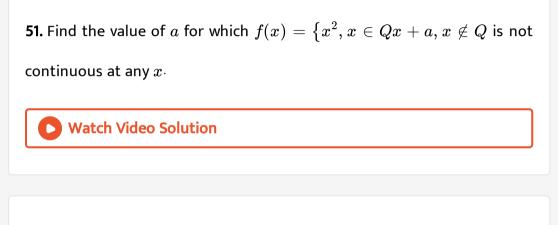
50.
$$f(x) = \left\{ [\sin \pi x], 0 \le x < 1 sgn\left(x - \frac{5}{4}\right)x\left\{x - \frac{2}{3}
ight\}, 1 \le x \le 2$$

where [.] denotes the greatest integer function and {.} represents the

fractional part function. At what points should the continuity be checked?

Hence, find the points of discontinuity.

Watch Video Solution



52. Discuss the continuity of $f(x) = (\log \lvert x
vert) sgnig(x^2-1ig), x
eq 0.$

Watch Video Solution

53. Find the number of integers lying in the interval (0,4) where the

function
$$f(x)=(\lim_{n
ightarrow})_{n
ightarrow}\left(rac{\cos(\pi x)}{2}
ight)^{2n}$$
 is discontinuous

54. If f(x) is continuous function $\forall x \in R$ and the range of $f(x)is(2,\sqrt{26})andg(x) = \left[\frac{f(x)}{c}\right]$ is continuous $\forall x \in R$, then find the least positive integral value of c, where [.] denotes the greatest integer function.

Watch Video Solution

55. Discuss the continuity of $f(x) = \{x^2, x \text{ is rational}\}$

-x^2,x is irrational

Watch Video Solution

56. If
$$y = rac{1}{t^2+t-2},$$
 $where t = rac{1}{x-1},$ then find the number of points

where f(x) is discontinuous.

57. Let f be a continuous function defined onto [0,1] with range [0,1]. Show that there is some c in [0,1] such that f(c) = 1 - c



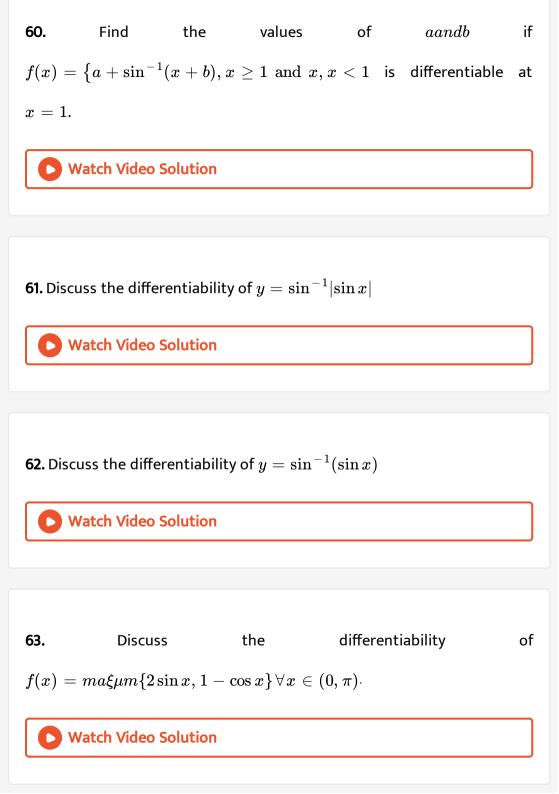
58. Leg f be continuous on the interval [0,1] to R such that f(0) = f(1). Prove that there exists a point c in $\left[\frac{0,1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.

Watch Video Solution

59.

$$f(x) = ig\{ax(x-1)+b, x < 1x-1, 1 \leq x \leq 3. \ px^2+qx+2, x > 3ig\}$$

Find the values of the constants a, b, pandq so that all the following conditions are satisfied f(x) is continuous for all $x \cdot f(1)$ does not exist. f'(x) is continuous at x = 3



64. Discuss the differentiability of
$$f(x) = \max\{x^2 - 3x + 2, 2 - |x - 1|\}$$

• Watch Video Solution
65. If $f(x) = \max\{x^2 + 2ax + 1, b\}$ has two points of non-
differentiability, then prove that $a^2 > 1 - b$
• Watch Video Solution

66. Discuss the differentiability of $f(x) = e^{-|x|}$

Watch Video Solution

67. If $f(x)=ig\{x,x\leq 1,x^2+bx+c,x>1\,
angle$ find b and c if function is

continuous and differentiable at x=1

68. Test the continuity and differentiability of the function $f(x) = \left| \left(x + \frac{1}{2} \right) [x] \right|$ by drawing the graph of the function when $-2 \le x < 2$, where [.] represents the greatest integer function.

Watch Video Solution

69. Let
$$f(x) = \left\{ rac{\log(1+x)^{1+x}-x}{x^2}
ight\}$$
. Then find the value of $f(0)$ so

that the function f is continuous at x = 0.

Watch Video Solution

70. Find the points of discontinuity of the function: f(x) = [[x]] - [x - 1], where[.] represents the greatest integer function

71. let
$$f(x) = rac{\ln\cos x}{\left(1+x^2
ight)^{rac{1}{4}}-1}$$
 if $x>0$ and $f(x) = rac{e^{\sin 4x}-1}{\ln(1+\tan 2x)}$ if

x < 0 Is it possible to difine f(0) to make the function continuous at x = 0. If yes what is the value of f(0), if not then indicate the nature of discontinuity.



72. What value must be assigned to k so that the function $f(x) = \left\{ \frac{x^4 - 256}{x - 4}, x \neq 4 \text{ and } k, x = 4 \text{ is continuous at x=4.} \right.$ Watch Video Solution

73. Let
$$f(x)$$
 be a function defined as $f(x) = \left\{ rac{x^2-1}{x^2-2|x-1|-1}, x
eq 1rac{1}{2}, x = 1 ext{ Discuss the continuity of }
ight.$

the function at x = 1.

74. If the function `f(x)={A x-B ,xlt=1 3x ,1



75. Let
$$: f(x) = iggl\{ rac{a+3\cos x}{x^2}, x < 0b aniggl(rac{\pi}{[x+3]}iggr), x \ge 0 ext{ If } f(x) ext{ is }$$

continuous at x = 0, then find aandb, where [.] denotes the greatest integer function.

Watch Video Solution

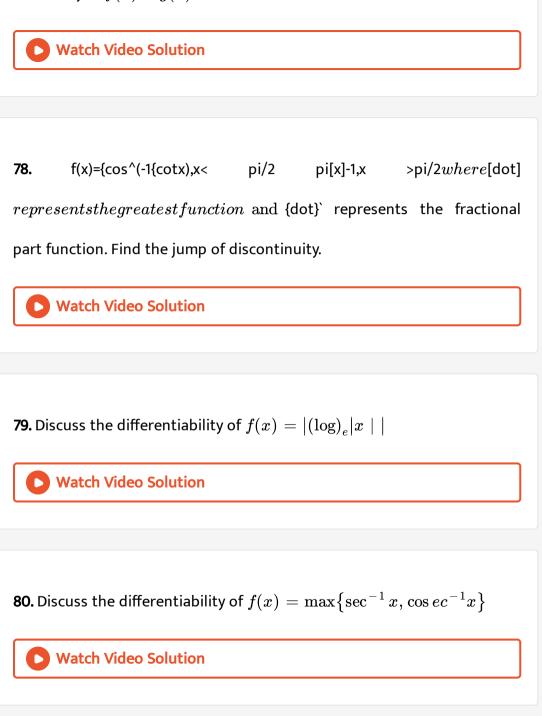
76. Let
$$: f(x) = \left\{ rac{\sin ax^2}{x^2}, x
eq 0 rac{3}{4} + rac{1}{4a}, x = 0 ext{ for what values of } a ext{ is } f(x) ext{ continuous at } x = 0 ?
ight.$$

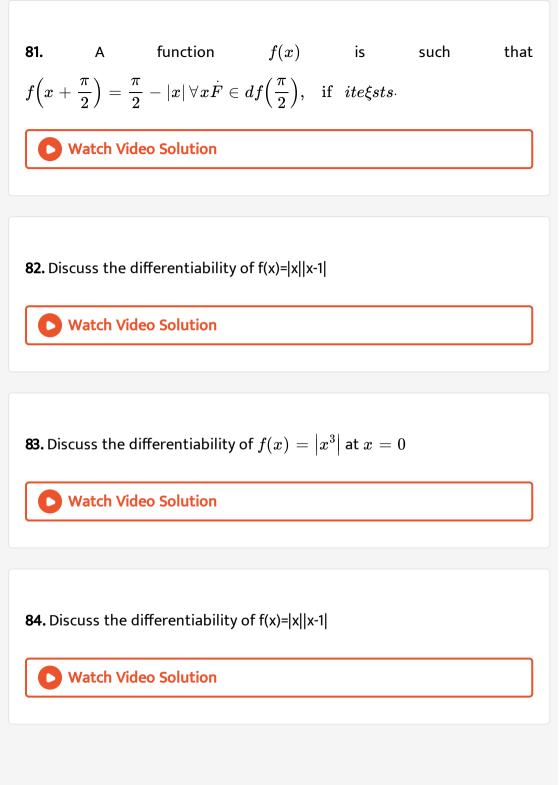
Watch Video Solution

77.
$$f(x) = \{|x+1|; x \le 0x; x > 0$$
 and

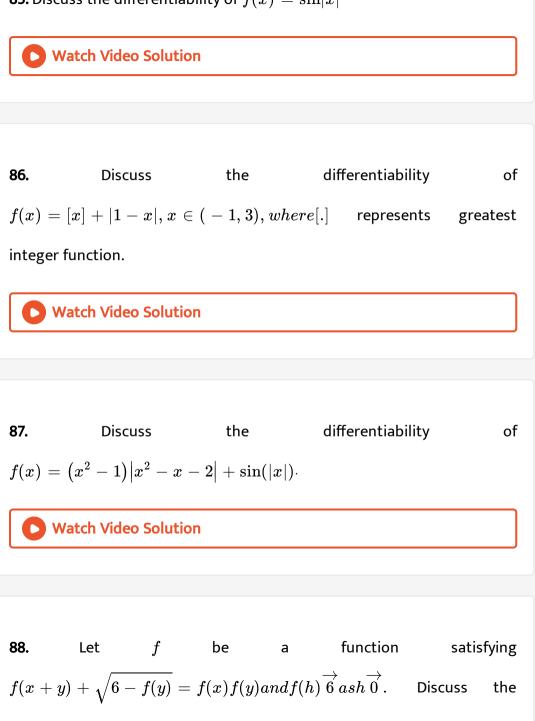
 $g(x)=\{|x|+1;x\leq 1-|x-2|;x>1$ Draw its graph and discuss the

continuity of f(x) + g(x).

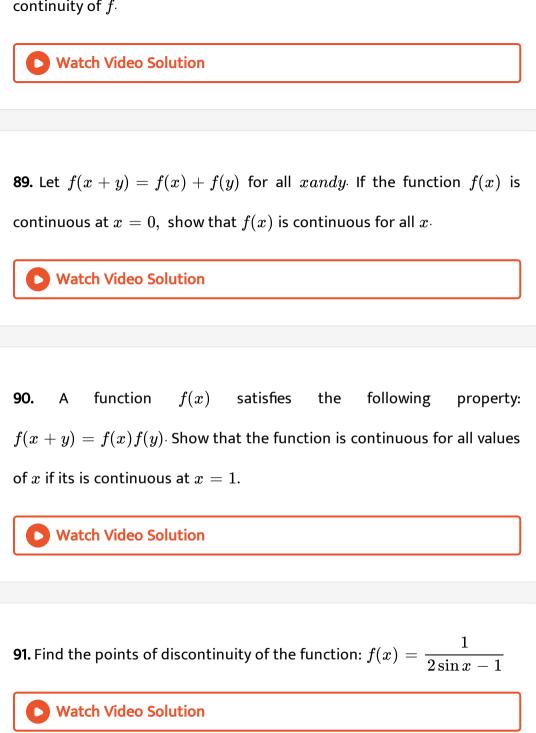


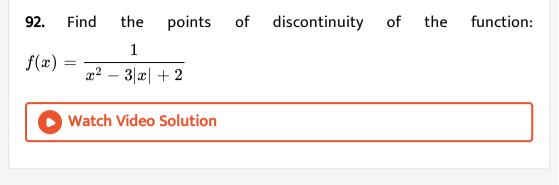


85. Discuss the differentiability of $f(x) = \sin \lvert x \rvert$









93. A function f(x) satisfies the following property: $f(x\dot{y}) = f(x)f(y)$. Show that the function f(x) is continuous for all values of x if it is continuous at x = 1.

Watch Video Solution

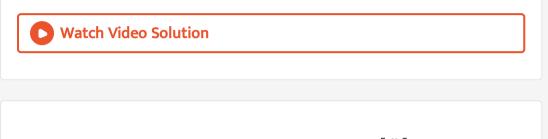
94. Find the points of discontinuity of the function: $f(x) = rac{1}{x^4 + x^2 + 1}$



95. Find the points of discontinuity of the function: $f(x) = rac{1}{1-e^{rac{x-1}{x-2}}}$

96. Draw the graph and find the points of discontinuity $f(x) = [2\cos x]$,

 $x\in [0,2\pi]$. ([.] represents the greatest integer function.)



97. Find the number of points where $f(x) = \left[\frac{x}{3}\right], x \in [0, 30]$, is discontinuous (where [.] represents greatest integer function).

Watch Video Solution

98. Discuss the continuity of the function ([.] represents the greatest

integer function):
$$f(x) = iggl[rac{2}{1+x^2} iggr], x \ge 0$$

99. Discuss the continuity of the function ([.] represents the greatest integer function): $f(x) = [\sin^{-1} x]$

Watch Video Solution

100. Discuss the continuity of the function ([.] represents the greatest integer function): $f(x) = ig[(\log)_e xig]$

Watch Video Solution

101. Match the following for the type of discontinuity at x = 1 in column II for the function in column I. $f(x) = \frac{1}{x-1}$, p. Removable discontinuity $f(x) = \frac{x^3 - x}{x^2 - 1}$, q. Non-removable discontinuity $f(x) = \frac{|x-1|}{x-1}$, r. Jump of discontinuity $f(x) = \sin\left(\frac{1}{x-1}\right)$, s. Discontinuity due to vertical asymptote, t. Missing point discontinuity, u. Oscillating discontinuity

102. Discuss the continuity of
$$f(x) = iggl\{ rac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|}, x
eq 1, 2, 6, x = 1, 12, x = 2 iggr\}$$



103. If
$$x=\sqrt{5}+2$$
 , then find the value of $x^2+rac{1}{x^2}$

Watch Video Solution

104. If the function
$$f(x) = \left[\frac{(x-2)^3}{a} \right] \sin(x-2) + a \cos(x-2), [.]$$

denotes the greatest integer function, is continuous in [4, 6], then find the values of a.

105. Draw the graph and discuss the continuity of $f(x)=[\sin x+\cos x], x\in [0,2\pi),$ where [.] represents the greatest integer function.

Watch Video Solution

Single Correct Answer Type

1. If
$$f(x)=\left\{\left(rac{\sin(2x^2)}{a}+\cos\left(rac{3x}{b}
ight)
ight)^ab/x^2,x
eq 0e^3,x=0$$
 is

continuous at $x=0\,orall b\in R$ then minimum value of a is -1/8 b. -1/4

 $\mathsf{c.}-1/2\,\mathsf{d.}\,\mathsf{0}$

A. -1/8

B. - 1/4

C. - 1/2

D. 0

Answer: B



2. Let $f: R\overline{R}$ be any function. Also $g: R\overline{R}$ is defined by g(x) = |f(x)| for all x. Then is Onto if f is onto One-one if f is one-one Continuous if f is continuous None of these

A. onto if f is onto

B. one-one if f is one-one

C. continuous if f is continuous

D. None of these

Answer: C



3. about to only mathematics

A. 1

B.1/4

C. 4

D. none of these

Answer: D

Watch Video Solution

4. Let
$$f(x) = x^3 - x^2 - 3x - 1, g(x) = (x+1)a$$
 and $h(x) = \frac{f(x)}{g(x)}$

where h is a rational function such that

(i) It is continuous everywhere except when x = -1,

(ii) $\lim_{x
ightarrow\infty} h(x)=rac{1}{2}.$

The value of h(1) is

A. 1/2

B.1/4

C. - 1/2

Answer: C

View Text Solution

5. If the function
$$f(x)=rac{3x^2ax+a+3}{x^2+x-2}$$
 is continuous at $x=-2,$ then the value of $f(-2)$ is

A. 0

 $\mathsf{B.}-1$

C. 1

D. 2

Answer: B

View Text Solution

6. Let $f(x)=egin{cases} 8^{rac{1}{x}}, & x<0\ a[x], & a\in R-\{0\}, & x\geq 0 \end{cases}$ (where [.] denotes the

greatest integer function).

Then f(x) is

A. continuous only at a finite number of points.

B. discontinuous at a finite number of points.

C. discontinuous at an infinite number of points.

D. discontinuous at x = 0.

Answer: C

View Text Solution

7. If
$$f(x) = \left\{rac{2\cos x, s \in 2x}{\left(\pi - 2x
ight)^2}, x \leq rac{\pi}{2} rac{e^{-\cot x} - 1}{8x - 4\pi}, x > rac{\pi}{2}
ight.$$
 , then which

of the following holds? f is continuous at $x = \pi/2 f$ has an irremovable discontinuity at $x = \pi/2 f$ has a removable discontinuity at $x = \pi/2$ None of these A. f is continuous at $x=\pi/2$

B. f has an irremovable discontinuity at $x=\pi/2$

C. f has a removable discontinuity at $x=\pi/2$

D. none of these

Answer: B

View Text Solution

8.

$$f(x)=igg\{\sin\Bigl(rac{\pi}{2}\Bigr)(x-[x]),x<55(b-1),x=5rac{ab^2ig|x^2-11x+24ig|}{x-3},x>$$

If

is continuous at $x=5, a, b \in R$ then ([.] denotes the greatest integer

function)
$$a = \frac{25}{108}, b = \frac{6}{5}$$
 b. $a = \frac{6}{13}, b = \frac{17}{29}$ c. $a = \frac{1}{2}, b = \frac{25}{36}$ d. $a = \frac{23}{100}, b = \frac{6}{5}$

A.
$$a = \frac{25}{108}, b = \frac{6}{5}$$

B. $a = \frac{6}{13}, b = \frac{17}{29}$
C. $a = \frac{1}{2}, b = \frac{25}{36}$

D.
$$a = (23), (100), b = rac{6}{5}$$

Answer: A



9. The function f(x) is discontinuous only at x = 0 such that $f^2(x) = 1 \, orall \, x \in R.$ The total number of such functions is

A. 2

B. 3

C. 6

D. none of these

Answer: C

View Text Solution

10. $f(x) = \left\{ \left(x^2 + e^{rac{1}{2-x}}
ight)^{-1}k, x = 2, x
eq 2$ is continuous from right at

the point $x\,=\,2,\,$ then k equals 0 b. 1/4 c. -1/4 d. none of these

A. 0

B. 1\4

C. -1/4

D. none of these

Answer: B

Watch Video Solution

11. Let g(x)=f(f(x)) where $f(x)=\{1+x; 0\leq x\leq 2\}$ and $f(x)=\{3-x; 2< x\leq 3\}$ then the number of points of discontinuity of g(x) in [0,3] is :

A. 0

B. 1

C. 2

D. gt 2

Answer: C

View Text Solution

12. If the function
$$f(x) = rac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$$
 is continuous at $x = 0$, then the value of a/b is $rac{3}{5}f(0)$ b. $2^{8/5}f(0)$ c. $rac{64}{5}f(0)$ d. none of these

A.
$$\frac{3}{5}f(0)$$

B. $2^{8/5}f(0)$

C.
$$\frac{64}{5}f(0)$$

D. none of these

Answer: C

13. If
$$f(x)= egin{cases} \displaystyle rac{1-\cos\left(1-rac{\cos x}{2}
ight)}{2^m x^n} 1x=0, x
eq 0$$
 is continous at $x=0$

then the value of m+n is a. 2 b. 3 c. -3 d. 7

A. 2

B. 3

C. - 3

D. 7

Answer: C

Niew Text Solution

14. Let
$$f(x)=egin{cases}rac{lpha\cot x}{x}+rac{eta}{x^2}, & 0< x\leq 1\ rac{1}{3}, & x=0 \end{cases}$$

If f(x) is continuous at x =0, then the value of $lpha^2+eta^2$ is

A. 1

B. 2

C. 5

D. 9

Answer: B



15. Let
$$f(x) = \left\{ rac{2}{1+x^2}, \xi stationalb, \xi stational ext{ has exactly two points }
ight.$$

of continuity then the value of b are (0,3] b. [0,1] c. (0,2] d. φ

A. (0,3]

B. [0,1]

C. (0,2]

D. ϕ

Answer: C

View Text Solution

16. If $f(x)=\{sin((a-x)/2)t \ h \ a \ n[(pix)/(2a)]forx > a([cos((pix)/2)])/(a-x)forx0, then f(a^-)<0b.$ fhas a removab $\leq discont \in uityatx=ac.$ f has an irremovab $\leq discont \in uityatx=ad.$ f(a^+)<0`

A. $fig(a^{\,-}ig) < 0$

B. f has a removable discontinuity at x = a

C. f has an irremovable discontinuity at x = a

D. $fig(a^+ig) < 0$

Answer: B

Watch Video Solution

17. Let $f(x) = [tanx[\cot x]], x\left[\frac{\pi}{12}, \frac{\pi}{12}\right]$, (where [.] denotes the greatest integer less than or equal to x). Then the number of points, where f(x) is discontinuous is a. one b. zero c. three d. infinite

B. zero

C. three

D. infinite

Answer: C

View Text Solution

18. Let $f:[a,b] \to R$ be any function which is such that f(x) is rational for irrational x and that f(x) is iirrational for rational x, then in [a,b]

A. f is discontinuous everywhere

B. f is discontinuous only at x = 0 and discontinuous everywhere

C. f is continuous for all irrational x and discontinuous for rational x

D. f is continuous for rational x and discontinuous for irrational x

Answer: A

19. If $f(x) = [x](\sin kx)^p$ is continuous for real x, then (where [.] represents the greatest integer function)

A.
$$k \in [n\pi, n \in I], p > 0$$

B. $k \in \{2n\pi, n \in I\}, p > 0$
C. $k \in \{n\pi, n \in I\}, p \in R - \{0\}$
D. $k \in \{n\pi, nI, n \neq 0\}, p \in R - \{0\}$

Answer: A

View Text Solution

20. Statement 1: Minimum number of points of discontinuity of the function $f(x) = (g(x)[2x - 1] \forall x \in (-3, -1))$, where [.] denotes the greatest integer function and $g(x) = ax^3 = x^2 + 1$ is zero. Statement 2: f(x) can be continuous at a point of discontinuity, say $x = c_1 of[2x - 1]$ if $g(c_1) = 0$. Statement 1 is True, Statement 2 is

True, Statement 2 isa correct explanation for Statement 1. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for statement 1. Statement 1 is True, Statement 2 is False Statement 1 is False, Statement 2 is True.

A. Statement 1 is True, Statement 2 is True, Statement 2 is a correct

explaination for Statement 1.

B. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a

correct explanation for Statement 1/

C. Statement 1 is True, Statement 2 is False.

D. Statement 1 is False, Statement 2 is True.

Answer: D

View Text Solution

21. Number of points of discontinuity of $f(x) = [\sin^{-1} x] - [x]$ in its domain is equal to (where [.] denotes the greatest integer function) a. O

b. 1 c. 2 d. 3	
A. 0	
B. 1	
C. 2	
D. 3	

Answer: D

Watch Video Solution

22. If
$$g(x) = (\lim_{m \to \infty} \frac{x^m f(x) + h(x) + 3}{2x^m + 4x + 1}$$
 when $x \neq 1$ and $g(1) = e^3$ such that $f(x), g(x)$ and $h(x)$ are continuous functions at $x = 1$ then the value of $5f(1) - 2h(1)$ is 7 b. 6 c. 9 d. 8

A. 7

B. 6

C. 9

Answer: B

Watch Video Solution

23. The number of points of discontinuity of fx = $[2x^2] - {2x2}^2$ (where [] denotes the greatest integer function and {} is fractional part of x) in the interval (-2, 2), is 1 b. 6 c. 2 d. 4

A. 1

B. 6

C. 2

D. 5

Answer: B

24. Let
$$f(x) = egin{cases} -3+|x|, & -\infty < x < 1 \ a+|2-x|, & 1 \leq x < \infty \end{cases}$$
 and $g(x) = egin{cases} 2-|-x|, & -\infty < x < 2 \ -b+\operatorname{sgn}(\mathrm{x}), & 2 \leq x < \infty \end{cases}$

where sgn(x) denotes signum function of x. If h(x) = f(x) + g(x) is discontinuous at exactly one point, then which of the following is not possible?

A. a = -3, b = 0

B. a = 0, b = 1

C. a = 2, b = 1

D. a = -3, b = 1

Answer: D

View Text Solution

25. The function $f(x)=rac{x^3}{8}-s\in\pi x+4\in[-4,4]$ does not take the value -4 b. 10 c. 18 d. 12

 $\mathsf{A}.-4$

B. 10

C. 18

D. 12

Answer: C

View Text Solution

26. Let f(x) be continuous functions f: RR satisfying f(0) = 1 and f(2x) - f(x) = x. Then the value of f(3) is 2 b. 3 c. 4 d. 5

- A. 2
- B. 3

C. 4

D. 5

Answer: C

27. about to only mathematics

A. a = b = 4

B.
$$a = b = -4$$

C. a = 4 and b = -4

D.a = -4 and b = 4

Answer: C

Watch Video Solution

28. If
$$f(x) = \left\{ [x] + \sqrt{\{x\}}, x < 1 \frac{1}{[x] + \{x\}^2}, x \ge 1 \text{ , then [where [.]} \right\}$$

and {.] represent the greatest integer and fractional part functions respectively] $f(x)$ is continuous at $x = 1$ $f(x)$ is not continuous at $x = 1$ $f(x)$ is not continuous at $x = 1$ $f(x)$ is differentiable at $x = 1$ $(\lim_{x \to 1} f(x)$ does not exist

A. f(x) is continuous at x = 1 but not differentiable

- B. f(x) is not continuous at x = 1
- C. f(x) is differentiable at x = 1
- D. $\lim_{x o 1} f(x)$ does not exist

Answer: A

Watch Video Solution

29. If f is an even function such that $(\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then prove that f(x) is not differentiable at x = 0.

A. f is continuous and derivable at x =0

B. f is continuous but not differentiable at x = 0

C. f may be discontinuous at x = 0

D. None of these

Answer: B

30. Let f(x) be differentiable for real x such that $f'(x) > 0on(-\infty, -4),$ f'(x) < 0on(-4, 6), $f'(x) > 0on(6, \infty),$ If g(x) = f(10 - 2x), then the value of g'(2) is a. 1 b. 2 c. 0 d. 4

- A. 1
- B. 2
- C. 0
- D. 4

Answer: C



31. Number of points where $f(x) = x^2 - |x^2 - 1| + 2||x| - 1| + 2|x| - 7$ is non-differentiable is a. 0 b. 1 c. 2 d. 3

A. 0	
B. 1	
C. 2	
D. 3	

Answer: A

View Text Solution

32. If f(x) = |x - 1|. ([x] = [-x]), then (where [.] represents greatest integer function)

A. f(a) is continuous and differentiable at x = 1

B. f(x) is discontinuous at x = 1

C. f(x) is continuous at x = 2

D. f(x) is continuous but non-differentiable at x= 1

Answer: D

33. Number of points where function f(x) defined as $f:[0,2]\pi \overrightarrow{R}, f(x) = \left\{3 - \left|\cos x - \frac{1}{\sqrt{2}}\right|, \left|\sin x < \frac{1}{\sqrt{2}}\right|2 + \left|\cos x + \frac{1}{\sqrt{2}}\right|, \right|$

is non-differentiable is a. 2 b. 4 c. 6 d. 0

A. 2

B. 4

C. 6

D. 0

Answer: B

Watch Video Solution

34. Let $f(x) = \begin{cases} [x] & x \not \in I \\ x - 1 & x \in I \end{cases}$ (where, [.] denotes the greatest integer function) and $g(x) = \begin{cases} \sin x + \cos x, & x < 0 \\ 1, & x \ge 0 \end{cases}$ Then for f(g(x)) at x = 0

- A. $\lim_{x o 0} \, g(g(x))$ exists but not continuous
- B. continuous but not differentiable at x = 0
- C. differentiable at x = 0
- D. $\lim_{x o 0} \, f(g(x))$ does not exist

Answer: C

View Text Solution

35. If
$$f(x) = \begin{cases} \sin(\cos^{-1}x) + \cos(\sin^{-1}x), & x \le 0 \\ \sin(\cos^{-1}x) - \cos(\sin^{-1}x), & x > 0 \end{cases}$$
 then at x = 0

A. f(x) is continuous and differentiable

- B. f(x) is continuous but not differentiable
- C. f(x) not continuous but differentiable
- D. f(x) is neither continuous nor differentiable

Answer: D

36. If $f(x) = \max \{ \tan x, \sin x, \cos x \}$ where $x \in \left[-rac{\pi}{2}, rac{3\pi}{2}
ight)$ then

the number of points, where f(x) is non -differentiable, is

- A. 2
- B. 3
- C. 4
- D. 5

Answer: B

View Text Solution

37. The number of points at which
$$g(x) = \frac{1}{1 + \frac{2}{f(x)}}$$
 is not differentiable, where $f(x) = \frac{1}{1 + \frac{1}{x}}$, is 1 b. 2 c. 3 d. 4

C. 3

D. 4

Answer: C

Watch Video Solution

38. Let
$$f(x)=\lim_{n
ightarrow\infty}\;\sum_{r=0}^{n-1}rac{x}{(rx+1)\{(r+1)x+1\}}.$$
 Then

A. f(x) is continuous but not differentiable at x = 0

B. f(x) is both continuous and differentiable at x = 0

C. f(x) is neither continuous not differentiable at x = 0

D. f(x) is a periodic function

Answer: C

39. Let the given function is differentiable at x = 1.

$$f(x) = \left\{ egin{array}{c} \lim_{n o \infty} \; rac{ax\,(x-1)\,ig(\cot . rac{\pi x}{4}ig)^n + (px^2+2)}{ig(\cot , rac{\pi x}{4}ig)^n + 1}, \;\; x \in (0,1) \cup (1,2) \ 0, \;\; x = 1 \end{array}
ight.$$

Then the value of $\left| a+q
ight|$ is

A. 4 B. 6 C. 8

D. 10

Answer: B

Watch Video Solution

Multiple Correct Answer Type

1. Which of the following functions is/are not discontinuous at x = 1?

$$\begin{array}{l} \mathsf{A}.\,f(x)=\frac{1}{1+2^{\tan x}}\\\\ \mathsf{B}.\,g(x)=\lim_{n\to\infty}\,\frac{1}{1+n\sin^2(\pi x)}\\\\ \mathsf{C}.\,h(x)=2^{-2^{\left(\frac{1}{1-x}\right)}},x\neq 1\,\,\mathrm{and}\,\,h(1)=1\\\\\\ \mathsf{D}.\,\phi(x)=\frac{x-1}{|x-1|+2(x-1)^2},x\neq 1\,\,\mathrm{and}\,\,\phi(1)=1\end{array}$$

Answer: A



2. about to only mathematics

A. h(x) may or may not be continuous in [a, c]

$$\texttt{B}.\,h\big(b^+\big)=g\big(b^-\big) \ \text{and} \ h\big(b^-\big)=f\big(b^+\big)$$

$$\mathsf{C}.\,hig(b^-ig)=gig(b^+ig) \,\, ext{and}\,\,hig(b^+ig)=fig(b^-ig)$$

D. h(x) has a removable discontinuity at x = b

Answer: C::D

3. If the function f(x) defined as f(x) defined as $f(x) = \left\{3, x = 0\left(1 + \frac{ax + bx^3}{x^2}\right), x > 0$ is continuous at x = 0, then a = 0 b. $b = e^3$ c. a = 1 d. $b = (\log)_e 3$

A. a = 0

 $\mathsf{B}.\,b=e^3$

C. a = 1

 $\mathsf{D}. b = \log_e 3$

Answer: A::D

View Text Solution

4.

Given

$$f(x) = \left\{3 - \left[\cot^{-1}\left(\frac{2x^3 - 3}{x^2}\right)\right]f \text{ or } x > 0\left\{x^2\right\}\cos\left(e^{\frac{1}{x}}\right)f \text{ or } x < 0$$

(where {} and [] denotes the fractional part and the integral part

functions respectively). Then which of the following statements do/does

not hold good? $f(0^-) = 0$ b. $f(0^+) = 3$ c. if f(0) = 0 , then f(x) is continuous at x = 0 d. irremovable discontinuity of f at x = 0

A. $fig(0^-ig)=0$

B.
$$fig(0^+ig)=3$$

C. If f(0) = 0, then f(x) is continuous at x = 0

D. Irremovable discontinuity of f at x = 0

Answer: B::D

View Text Solution

5. Let
$$f(x) = \begin{cases} x \left[rac{1}{x}
ight] + x[x] & ext{if} & x
eq 0 \\ 0 & ext{if} & x = 0 \end{cases}$$
 (where [x] denotes the

greatest integer function). Then the correct statement is/are

A. Limit exists for x = -1.

B. f(x) has a removable discontinuity at x = 1.

C. f(x) has a non removable discontinuity at x = 2.

D. f(x) is discontinuous at all positive integers.

Answer: A::B::C::D



6. A function f: R o R is defined as $f(x) = \lim_{n o \infty} \frac{ax^2 + bx + c + e^{nx}}{1 + c e^{nx}}$ where f is continuous on R, then

A. point (a, b, c) lies on line in space

B. point (a, b) represents the 2-dimensional Cartesian plane

C. Locus of point (a, c) and (c, b) intersect at one point

D. point (a, b, c) lies on the plane in space

Answer: A::B::C

View Text Solution

7. Let f be a function with continuous second derivative and f(0) = f'(0) = 0. Determine a function g by $g(x) = \begin{cases} \frac{f(x)}{x}, & x \neq 00, x = 0 \end{cases}$ Then which of the following statements is correct? g has a continuous first derivative g has a first derivative g is continuous but g fails to have a derivative g has a first derivative but the first derivative is not continuous

A. g has a continuous first derivative

B. g has a first derivative

C. g is continuous but g fails to have a derivative

D. g has a first derivative but the first derivative is not continuous

Answer: A::B

f(x)



8. Let f(x) be a function defined on (-a, a) with a > 0. Assume that

x = 0 and $(\lim_{x \to 0} \frac{f(x) - f(kx)}{x} = \alpha$, where $k \in (0, 1)$ then $f'(0^+) = 0$ b. $f'(0^-) = \frac{\alpha}{1-k}$ c. f(x) is differentiable at x = 0 d. f(x) is non-differentiable at x = 0

B.
$$f'(0^-) = \frac{\alpha}{1-k}$$

 $f'(0^+) = 0$

C. f(x) is defferentiable at x = 0

D. f(x) is non-differentiable at x = 0

Answer: B::C::D

Watch Video Solution

9. If
$$f(x)=x^{1/3}(x-2)^{2/3}$$
 for all $x,$ then the domain of f' is $x\in R-\{0\}$ b. $\{x\mid x
angle 0\}$ c. $x\in R-\{0,2\}$ d. $x\in R$

A. $x \in R-\{0\}$

B. $\{x \mid x > 0\}$

 $\mathsf{C}.\,x\in R-\{0,2\}$

D. $x \in R$

Answer: C

Watch Video Solution

Comprehension Type

1. Consider two function y = f(x) and y = g(x) defined as

$$f(x) = egin{cases} ax^2+b & 0 \leq x \leq 1 \ bx+2b & 1 < x \leq 3 \ (a-1)x+2c-3 & 3 < x \leq 4 \ and & g(x) = egin{cases} cx+d & 0 \leq x \leq 2 \ ax+3-c & 2 < x < 3 \ x^2+b+1 & 3 \geq x \leq 4 \ \end{cases}$$

Let f be differentiable at x = 1 and g(x) be continuous at x = 3. If the roots of the quadratic equation $x^2 + (a + b + c)\alpha x + 49(k + k\alpha) = 0$ are real distinct for all values of α then possible values of k will be

A. $k\in(\,-1,0)$

B. $k\in(\infty,0)$

 $\mathsf{C}.\,k\in(1,\,5)$

D. $k\in(\,-1,1)$

Answer: A



2. Consider two function y = f(x) and y = g(x) defined as

$$f(x) = egin{cases} ax^2+b & 0 \leq x \leq 1 \ bx+2b & 1 < x \leq 3 \ (a-1)x+2c-3 & 3 < x \leq 4 \ ax+3-c & 2 < x < 3 \ x^2+b+1 & 3 \geq x \leq 4 \end{cases}$$

 $\lim_{x o 2} \; rac{f(x)}{|g(x)|+1}$ exists and f is differentiable at x = 1. The value of limit

will be

A. - 2

B. -1

C. 0

D. 2

Answer: C



Examples

1. A function f(x) satisfies the following property: $f(x\dot{y}) = f(x)f(y)$. Show that the function f(x) is continuous for all values of x if it is continuous at x = 1.

Watch Video Solution

2. Let
$$f(x) = \left\{ rac{\log(1+x)^{1+x}-x}{x^2}
ight\}$$
. Then find the value of $f(0)$ so

that the function f is continuous at x = 0.

3. Let $: f(x) = \left\{ \frac{a+3\cos x}{x^2}, x < 0b\tan\left(\frac{\pi}{[x+3]}\right), x \ge 0 \text{ If } f(x) \text{ is continuous at } x = 0, \text{ then find } aandb \text{, where } [.] \text{ denotes the greatest becomes the set of the set$

integer function.

Watch Video Solution

4. Let
$$f(x)$$
 be a function defined as $f(x)=iggl\{ rac{x^2-1}{x^2-2|x-1|-1},x
eq1rac{1}{2},x=1 ext{ Discuss the continuity of }
ight.$

the function at x = 1.

Watch Video Solution

5. about to only mathematics

6. Draw the graph and discuss the continuity of $f(x)=[\sin x+\cos x], x\in [0,2\pi],$ where [.] represents the greatest integer function.

Watch Video Solution

7. If the function
$$f(x) = \left[\frac{(x-2)^3}{a} \right] \sin(x-2) + a \cos(x-2), [.]$$

denotes the greatest integer function, is continuous in [4, 6], then find the values of a.

Watch Video Solution

8. If $f(x) = sgn(2\sin x + a)$ is continuous for all x, then find the

possible values of a.

9. Discuss the continuity of $f(x)=(\ \lim \)_{n \overrightarrow{\infty}} rac{x^{2n}-1}{x^{2n}+1}$

Watch Video Solution

10. Find the values of a if $f(x)=(\lim)_{n \overrightarrow{\infty}} rac{ax^{2n}+2}{x^{2n}+a+1}$ is continuous at

x = 1.

11. For
$$x > 0$$
, $leth(x) = \left\{\frac{1}{q}, \text{ if } x = \frac{p}{q}0, \text{ if } \xi sirrational where $p, q > 0$ are relatively prime integers. Then prove that $f(x)$ is continuous for all irrational values of x .$

12. If
$$f(x) = rac{x+1}{x-1} and g(x) = rac{1}{x-2},$$
 then discuss the continuity of $f(x), g(x), and fog(x).$

13. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value

$$\displaystyle rac{a+b}{2}$$
 for some value of $x\in [a,b]$.

Watch Video Solution

14. Using intermediate value theorem, prove that there exists a number \boldsymbol{x}

such that $x^{2005} + rac{1}{1+\sin^2 x} = 2005.$

Watch Video Solution

15. Discuss the differentiability of
$$f(x) = \left\{ \frac{\sin x^2}{x}, x
eq 00, x = 0 atx = 0
ight.$$

16.Discussthedifferentiabilityof
$$f(x) = \{x \sin(1nx^2), x \neq 00, x = 0atx = 0$$
 \checkmark Watch Video Solution17. If $f(x) = \{x, x \leq 1x^2 + bx + c, x > 1'$ \checkmark Watch Video Solution

18. Find the values of
$$aandb$$
 if $f(x) = \left\{ a + \sin^{-1}(x+b), x \ge 1x, x < 1 ext{ is differentiable at } x = 1.
ight.$

19.

$$f(x)=ig\{ax(x-1)+b,x<1x-1,1\leq x\leq 3.\ px^2+qx+2,x>3$$
 Find the values of the constants $a,b,pandq$ so that all the following

conditions are satisfied f(x) is continuous for all $x \cdot f(1)$ does not exist.

 $f^{\,\prime}(x)$ is continuous at x=3



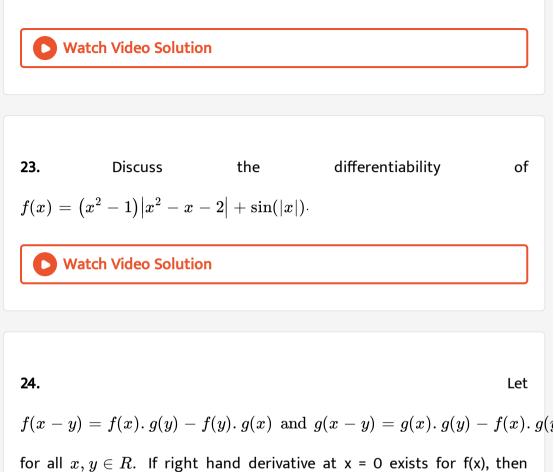
20. If $f(x)=\max\left\{x^2+2ax+1,b
ight\}$ has two points of nondifferentiability, then prove that $a^2>1-b$

Watch Video Solution

21. Test the continuity and differentiability of the function $f(x) = \left| \left(x + \frac{1}{2} \right) [x] \right|$ by drawing the graph of the function when $-2 \le x < 2$, where [.] represents the greatest integer function.

22. Discuss the differentiability of
$$f(x) = [x] + igl(1 - x \mid \dot{x} \in (-1, 3), where [.]$$
 represents greatest

integer function.



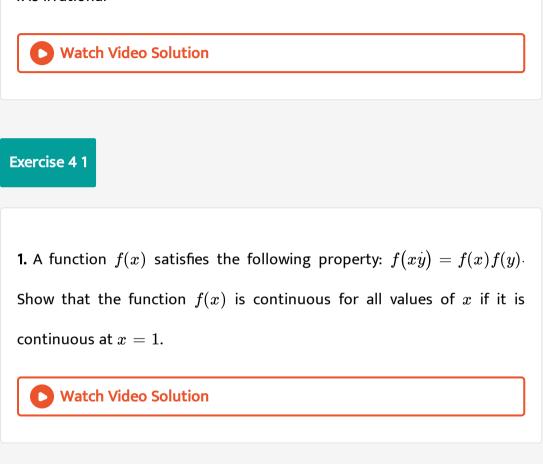
derivative of g(x) at x = 0





1. Discuss the continuity of the following function : f(x)={1 if x rational ; 0 if

x is irrational



Exercise Single

1. If
$$f(x) = rac{a\cos x - \cos bx}{x^2}$$
, $x \neq 0$ and $f(0) = 4$ is continous at $x = 0$,
then the ordered pair (a,b) is $(\pm 1,3)$ b. $(1,\pm 3)$ c. $(-1,-3)$ d.

 $(\,-1,3)$

A. $\pm 1, 3)$

B. $(1, \pm 3)$

- C. (-1, -3)
- D.(1,3)

Answer: B



2. Let
$$f$$
 be a continuous function on R such that $f\left(\frac{1}{4n}\right) = \frac{\sin e^n}{e^{n^2}} + \frac{n^2}{n^2 + 1}$ Then the value of $f(0)$ is
A.1
B.1/2
C.0

D. none of these

Answer: A



3. If
$$f(x) = \frac{x^2 - bx + 25}{x^2 - 7x + 10}$$
 for $x \neq 5$ and f is continuous at $x = 5$ then
 $f(5) =$
A. 0
B. 5
C. 10
D. 25

Answer: A

4. The function $f(x) = rac{ anlambda m [x-\pi]|}{1+{[x]}^2}$, where [x] denotes the greatest

integer less than or equal to x, is

- A. f(x) is discontinuous at some x
- B. f(x) is continuous at all x, but the derivative f'(x) does not exist for

some x

C. f'(x) exists for all x, but f"(x) does not exist for some x

D. f'(x) exists for all x

Answer: D

Watch Video Solution

5. if
$$f(x) = \begin{cases} \frac{1 - |x|}{1 + x} & x \neq -1 \\ 1 & x = -1 \end{cases}$$
 then f([2x]), where [.] represents the

greatest integer function, is

A. discontinuous at x=-1

B. continuous at x=0

C. continuous at x=1/2

D. continuous at x=1

Answer: B



6. Let
$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4a + b, \frac{x-4}{|x-4|} + b, x > 4$$
 Then $f(x)$
is continous at $x = 4$ when $a = 0, b = 0$ b. $a = 1, b = 1$ c.
 $a = -1, b = 1$ d. $a = -1, b = -1$
A. a=0,b=0
B. a=1,b=1
C. a=-1,b=1
D. a=1,b=-1

Answer: D

7. Which of the following is true about

$$Letf(x) = \left\{ egin{array}{c} rac{x-2}{|x-2|} \Big(rac{x^2-1}{x^2+1}\Big) & x
eq 2 \ rac{3}{5} & x = 2 \end{array}
ight.$$

A. f(X) is continuous at x=2

B. f(x) has removable discontinuity ata x=2.

C. f(x) has non-removable discontinuity at x=2.

D. Discontiuity at x=2 can be removed by redefining the function at

x=2.

Answer: C

Watch Video Solution

$${f 8.} \, {
m lf} \, f(x) = egin{cases} x+2 & x < 0 \ -x^2-2 & 0 \le x < 1 \ x & x \ge 1 \end{cases}$$

then the number of points of discontinuity of |f(x)| is

B. 2

C. 3

D. none of these

Answer: A

Watch Video Solution

9. Which of the following statements is always true? ([.] represents the greatest integer function. a) If f(x) is discontinuous then |f(x)| is discontinuous b) If f(x) is discontinuous then f(|x|) is discontinuous c) f(x)=[g(x)] is discontinuous when 'g(x) is an integer d) none of these

A. if f(x) is discontinuous then |f(x)| is discontinuous

B. if f(x) is discontinuous , then f(|x|) is discontinuous .

C. f(x) = [g(x)] is discontinuous, when g(x) is an integer

D. none of these

Answer: D



10. The number of point $f(x)=egin{cases} [\cos\pi x]&0\leq x<1\ |2x-3|[x-2]&1< x\leq 2 \end{bmatrix}$ is

discontinuous at Is ([.] denotes the greatest intgreal function)

A. two

B. three

C. four

D. zero

Answer: B





1. Let $f(x) = rac{\sqrt{x^2 + px + 1}}{x^2 - p}$. If f(x) is discontinuous at exactly 2 values

of x then number of integers in the range of p is

