MATHS

BOOKS - CENGAGE

DEFINITE INTEGRATION

Examples

1. Evaluate the following definite integrals as limit of sum $\int_2^1 x^2 dx.$

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2. Evaluate: $\int_a^b e^x dx$ using limit of sum

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3. Evaluate: $\int_a^b \sin x dx$ using limit of sum

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4. Evaluate $\int_a^b \frac{dx}{\sqrt{x}}, where a, b > 0.$

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5. If $f(x) = \min\left(|x|, 1 - |x|, \frac{1}{4}\right) \forall x \in R,$ then find the value of $\int_{-1}^{1} f(x)dx.$

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6. Evaluate: $\int_{-\frac{\pi}{2}}^{2\pi} \sin^{-1}(\sin x)dx$

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7. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}\sin^{-1}\left(2x\sqrt{1-x^2}\right)dx}$.

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8. Evaluate: $\int_0^{2\pi}[\sin x]dx, where[x]$ denotes the greatest integer function.

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9. Prove that $\frac{1+\sqrt{2}}{2} < \int_0^{\pi/2}\frac{\sin x}{x}dx < \frac{\pi+2\sqrt{2}}{4}$

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10. Evaluate: $\int_{-1}^{0}\frac{dx}{x^2+2x+2}$

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11. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1, -1), (1.1)$and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of 60°. Then find the value of $\int_0^1 p(x)dx$.

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12. Let f be a continuous function on $[a, b]\cdot$ Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dx = \int_x^b f(t)dt\cdot$

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13. $\int_0^1 \frac{dx}{e^x + e^{-x}}$

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14. Evaluate $\left(\int_0^{\frac{\pi}{2}}\right)\frac{\tan x dx}{1 + m^2 \tan^2 x}$

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15. Find the mistake of the following evaluation of the integral $I=\int_0^{\pi}\frac{dx}{1+2\sin^2 x}$ $I=\int_0^{\pi}\frac{dx}{\cos^2 x+3\sin^2 x}$ $=\int_0^{\pi}\frac{\sec^2 x dx}{1+3\tan^2 x}=\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]\pi 0=0$

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16. Let $\frac{d}{dx}(F(x))=\frac{e^{\sin x}}{x}, x>0$. If $\int_1^4 2\frac{e^{\sin\left(x^2\right)}}{x}dx=F(k)-F(1)$, then possible value of k is:

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17. If $\int_a^b (f(x)-3x)dx=a^2-b^2$ then the value of $f\left(\frac{\pi}{6}\right)$ is ___

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18. If $f(0)=1, f(2)=3, f(2)=5, then f \in d the value of$ `int_0^1xf"

(2x)dx

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19. $F \in d the value of \int_0^1 \log x dx.$

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20. Evaluate: $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

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21. If $\lambda = \int_0^1 \frac{e^t}{1+t}$, then $\int_0^1 e^t \log_e(1+t) dt$ is equal to

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22. If $\int_0^1 e^{-x} \ ^\wedge \ 2dx = a,$ then find the value of $\int_0^1 x^2 e^{-x} \ ^\wedge \ 2dx$ in terms of a .

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23. If $f(x) = x + \sin x,$ then find the value of $\int_\pi^{2\pi} f^{-1}(x)dx$.

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24. Find the value of $\int_0^{\pi/2} \cos^5 x \sin^7 x dx$

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25. Evalute $\lim_{n \to \infty} \left[\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + \ldots\ldots + \frac{1}{6n^2}\right]$

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26. Evaluate: $(\lim)_{n \overrightarrow{\infty}} n\left[\frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \ + \frac{1}{nb}\right]$

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27. Evaluate: $(\lim)_{n \overrightarrow{\infty}} \left(\frac{(n+1)(n+2)(n+n)^{\frac{1}{n}}}{n}\right.$

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28. Evaluate: $(\lim)_{n \overrightarrow{\infty}} \frac{\left(1^2+2^2+3^3+\ +n^2\right)\left(1^3+2^3+3^3+\ +n^3\right)}{1^6+2^6+3^6+\ +n^6}$

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29. `P rov et h a t0

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30. $Provethat\frac{1}{2} \leq \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^{2n}}} \leq \frac{\pi}{6} f \text{ or } n \geq 1.$

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31.

$$Let I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx,\ I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\sin \frac{\sin x}{\sin x} dx,\ I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{\sin(\tan x)}{\tan x} dx\right.\right.$$

Then arrange in the decreasing order in which values I_1, I_2, I_3 lie.

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32. Prove that $1 \leq \int\left(\frac{5-x}{9-x^2}\right) dx \leq \frac{6}{5}$ for $x \in [0, 2]$.

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33. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

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34. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)}\,dx$ cannot exceed $\sqrt{\frac{15}{8}}$.

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35. Prove that $\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)x+a)dx$

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36. Evaluate $\int_{-1}^{2} \left|x^3 - x\right| dx$

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37. Evaluate $\int_{-1}^{\frac{3}{2}} |x\sin(\pi x)|dx$

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38. Show that $\int_a^b \frac{|x|}{x} dx = |b| = |a| \cdot$

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39. If $f(n) = \int_0^{2015} \frac{e^x}{1 + x^n} dx$, then find the value of $\lim_{n \to \infty} f(n)$

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40. Let: $f(x) = \int_0^x |2t - 3| dt \cdot$ Then discuss continuity and differentiability of $f(x) at x = \frac{3}{2}$

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41. A continuous real function f satisfies $f(2x) = 3\Big(f(x) \forall x \in R \dot{I} f \int_0^1 f(x) dx = 1,$ then find the value of $\int_1^2 f(x) dx$

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42. Let $g(x)=\int_0^x f(t)dt$, where f is such that $\frac{1}{2}\leq f(t)\leq 1$, for $t\varepsilon[0,1]$ and $0\leq f(t)\leq\frac{1}{2}$, for $t\varepsilon[1,2]$. Then prove that $\frac{1}{2}\leq g(2)\leq\frac{3}{2}$.

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43. If $[x]$ denotes the greatest integer less than or equal to x, then find the value of the integral $\int_0^2 x^2[x]dx$.

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44. Evaluate: $\int_0^{\frac{5\pi}{12}}[\tan x]dx, where[.]$ deontoes the greatest integer function.

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45. Evaluate: $\int_0^{10\pi}[\tan^{-1}x]dx, where[x]$ represents greatest integer function.

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46. Evaluate: $\int_0^2[x^2-x+1]dx, where[.]$ denotos the greatest integer function.

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47. Prove that $\int_0^{\infty}[ne^{-x}]dx=1n\left(\frac{n^n}{n!}\right), wheren$ is a natural number greater than 1 and [.] denotes the greatest integer function..

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48. Evaluate: $\int_0^{\sqrt{3}}\frac{1}{1+x^2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

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49. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

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50. Evaluate: $\int_0^a \frac{dx}{x + \sqrt{(a^2 - x^2)}}$ or $\int_0^{\frac{\pi}{2}} \frac{dth\eta}{1 + \tan\theta}$

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51. Evaluate $\int_0^{\pi} \frac{\sin 6x}{\sin x} dx.$

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52. The value $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is

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53. Evaluate: $\int_{-\pi}^{3\pi} \log(sec\theta - tan\theta)dth\eta$

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54. Prove that $\int_0^{2a} f(x)dx = \int_a^a [f(a-x) + f(a+x)]dx$

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55. Evaluate $\int_0^{\pi/4} In(1 + \tan x)dx$

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56. Evaluate: $\int_{-5}^{5} x^2 \left[x + \frac{1}{2}\right] dx(where[.]$ denotes the greatest integer function).

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57. Evaluate: $\int_{-\pi}^{\pi} \frac{x \sin x dx}{e^x + 1}$

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58. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$

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59. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1 - x + x^2}\right) dx = 2\int_0^1 \tan^{-1} x dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}\left(1 - x + x^2\right) dx$

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60. Show that $\int_0^{\frac{\pi}{2}} \sqrt{(\sin 2\theta)} \sin \theta d th \eta = \frac{\pi}{4}$

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61. For

$$\theta \in \left(= , \frac{\pi}{2} \right), provethat \int_0^{\theta} \log(1 + \tan\theta \tan x) dx = \theta \log(\sec\theta)$$

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62. Evaluate the definite integral: $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1 - x^4} \right) \cos^{01} \left(\frac{2x}{1 + x^2} \right) dx.$

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63. Evaluate $\int_0^1 \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2 - 4x})} dx$

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64. Evaluate $\int_0^{2\pi} \frac{dx}{1 + 3\cos^2 x}$

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65. Evaluate $\int_0^{2\pi} \frac{x \cos^{2n} x}{\cos^{2n} x + \sin^{2n} x} dx$

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66. Evaluate $\int_0^{\pi} e^{|\cos x|} \left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right) \sin x dx.$

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67. $\int_0^{\pi} x \log \sin x dx$

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68. Evaluate: $\int_{\pi/4}^{\pi/4} \log(\sin x + \cos x) dx$

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69. Evaluate: $\int_0^{\frac{\pi}{2}} x \cot x dx$

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70. Evaluate: $\int_0^{\infty} \log\left(x + \frac{1}{x}\right)\frac{dx}{1 + x^2}$

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71. Evaluate $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\,dx.$

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72. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{a - s\int h\eta}{a + \sin\theta}\right)dth\eta,\ a > 0$

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73. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left\{\frac{ax^2 + bx + c}{ax^2 - bx + c}(a + b)|\sin x|\right\}dx$

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74. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^9 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} dx$

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75. If f is an odd function, then evaluate $I = \int_{-a}^{a} \left(\frac{f(\sin x)}{f(\cos x) + f\left(\sin^2 x\right) dx} \right.$

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76. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right]^{\frac{1}{2}} dx$

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77. Find the value of $\int_{-2}^{2} \frac{\sin^{-1}(\sin x) + \cos^{-1}(\cos x)}{(1+x^2)\left(1 + \left[\frac{x^2}{5}\right]\right)} dx$, where [.] represents the greatest integer function.

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78. Determine the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx.$

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79. Evalaute $\int_0^{16\pi/3} |\sin x| dx.$

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80. Evaluate $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$ (where $[x]$ and $\{x\}$ are integral and fractional parts of x respectively and $n \varepsilon N$).

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81. Let $f(x)$ be a continuous and periodic function such that $f(x) = f(x+T)$ for all $x\varepsilon R, T > 0$.If $\int_{-2T}^{a+5T} f(x)dx = 19(ag0)$ and $\int_0^T f(x)dx = 2$, then find the value of $\int_0^a f(x)dx$.

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82. If $g(x) = \int_0^x \cos^4 tdt$, then prove that $g(x+\pi) = g(x) + g(\pi)$.

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83. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi - \frac{\pi}{4}} |\sin x + \cos x| dx$

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84. Evaluate: $\int_0^x [\cos t] dt where n \in \left(2n\pi, \left(4n + 1\frac{\pi}{2}\right), n \in N, and [.]$ denotes the greatest integer function.

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85. Let f be a real-valued function satisfying $f(x)+f(x+4)=f(x+2)+f(x+6)$ Prove that $\int_x^{x+8} f(t)dt$ is constant function.

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86. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers a, b and two unequal non-zero positive integers $mandn$ $\int_a^{a+n} f(x)dx = \int_b^{b+m} f(x)dx \dot{C}alc\underline{a}tethevalueof \int_m^n f(x)dx$

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87. If $y=\int_{x^2}^{x^3} \frac{1}{\log t} dt (x>0)$, then find $\frac{dy}{dx}$

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88. If $\int_0^y \cos t^2\, dt = \int_0^{x^2} \frac{\sin t}{t}\, dt$, then $\frac{dy}{dx}$ is equal to

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89. $I f x = \int_0^y \frac{dt}{\sqrt{1+9t^2}} and \frac{d^2 y}{dx^2} = ay, then f \in da$

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90. If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x, x \varepsilon \left(0, \frac{\pi}{2}\right)$ then find the value of $f\left(\frac{1}{\sqrt{3}}\right)$

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91. Let $f: R\vec{R}$ be a differentiable function having $f(2) = 6, f'(2) = \frac{1}{48}$. Then evaluate $(\lim)_{x \vec{2}} \int_6^{f(x)} \frac{4t^3}{x-2} dt$

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92. Evaluate: $(\lim)_{x \xrightarrow{} \infty} \frac{\left(\int 0 x e^{x} \hat{} 2 d x\right)^{2}}{\int 0 x e^{2 x} \hat{} 2 d x}$

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93. Prove that: $y=\int_{\frac{1}{8}}^{\sin ^{2} x} \sin ^{-1} \sqrt{t} d t+\int_{\frac{1}{8}}^{\cos ^{2} x} \cos ^{-1}, where 0 \leq x \leq \frac{\pi}{2}$, is the equation of a straight line parallel to the x-axis. Find the equation.

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94. If $f(x)=\int_{x^{2}}^{x^{2}+1} e^{-t^{2}} d t$, then find the interval in which $f(x)$ increases.

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95. Evaluate $\lim_{n \rightarrow \infty}\left[\frac{1+2+3+\ldots+n}{4 n^{2}-3 n+2}\right]$

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96. Let $f: R-(0, \infty)$ be a real valued function satisfying $\int_0^x tf(x-t)dt = e^{2x}-1$ then $f(x)$ is

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97. Find the shortest distance between the lines whose vector equations are $\overrightarrow{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\overrightarrow{r} = \left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

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98. Find $fog(2)$ and $gof(1)$, when $f: R \to R; f(x) = x^2+8$ and $g: R \to R; g(x) = 3x^3+1.$

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99. Prove that $\int_0^x e^{xt} e^{-t}\ \hat{}\ 2dt = e^{\frac{x^2}{4}} \int_0^x e^{-\left(\frac{t^2}{4}\right)} dt$

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100. Evaluate: $\int_{-4}^{-5} e^{x+5}\ \hat{}\ 2dx + 3\int_{\frac{1}{3}}^{\frac{2}{3}} e^9 \left(x - \frac{2}{3}\right)^2 dx$

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101. Compute the integrals: $\int_0^\infty f\left(x^n + x^{-n}\right) \log x \frac{dx}{x}$

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102. Compute the integrals: $\int_0^\infty f\left(x^n + x^{-n}\right) \log x \frac{dx}{x}$

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103. Compute the integrals: $\int_{\frac{1}{e}}^{e} \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$

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104. $LetA = \int_0^{\infty} \frac{\log x}{1+x^3} dx\dot{T}henf \in dthevalueof \int_0^{\infty} \frac{x \log x}{1+x^3} dx$ in terms of $A\cdot$

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105. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, then find the value of $\int_{4\pi-2}^{4\pi} \frac{\sin\frac{t}{2}}{4\pi|2-t|} dt$

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106. Prove that $\int_0^1 \left(\frac{\tan^{-1} x}{x}\right) dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx.$

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107. For $x > 0, let f(x) = \int_1^x \frac{\log t}{1+t} dt$. Find the function $f(x) + f\left(\frac{1}{x}\right)$ and find the value of $f(e) + f\left(\frac{1}{e}\right)$.

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108. Determine a positive integer n such that $\int_0^{\frac{\pi}{2}} x^n \sin x dx = \frac{3}{4}(\pi^2 - 8)$

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109. Determine a positive integer $n \leq 5$ such that $\int_0^1 e^x (x-1)^n = 16 - 6e$

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110. Prove that: $I_n = \int_0^\infty x^{2n+1} e^{-x} \hat{} 2dx = \frac{n!}{2}, n \in N$.

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111. $IfI_n = \int_0^1 \frac{dx}{(1+x^2)^n}, where n \in N,$ which of the following statements hold good? $2nI_{n+1} = 2^{-n} + (2n-1)I_n$ $I_2 = \frac{\pi}{8} + \frac{1}{4}$ (c) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

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112. $IfI_n = \int_0^{\frac{\pi}{2}} \sin^n x dx, then show that I_n = \left(\frac{n-1}{n}\right) I_{n-2}$ Hence, Prove that

$I_n = f(x) = \left\{ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \ddot{i} \frac{1}{2}\frac{\pi}{2} \text{ if } n is even \left(\frac{n-1}{n}\right)\left(\right.\right.$

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113. $f, g, h,$ are continuous in $[0, a], f(a-x) = f(x), g(a-x) = -g(x), 3h(x) - 4h(a-x) = 5.$ Then prove that $\int_0^a f(x)g(x)h(x)dx = 0$

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114. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin 3x}{\sin x + \cos x} dx.$

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115. Given a function $f:[0, 4] \rightarrow R$ is differentiable ,then prove that for some $\alpha, \beta \varepsilon (0, 2), \int_0^4 f(t)dt = 2\alpha f(\alpha^2) + 2\beta f(\beta^2).$

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116. Prove that $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \int_0^{\infty} \frac{\sin x}{x} dx$

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117. If $\int_0^{\frac{\pi}{2}} \log \sin \theta d th\eta = k,$ then find the value of $\int_{\pi}^{\frac{\pi}{2}} \left(\frac{\theta}{s \int h\eta}\right)^2 d th\eta$ in terms of k

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118. Evaluate : $\int_0^{\pi} \frac{x^2 \sin 2x \cdot \sin\left(\frac{\pi}{2} \cdot \cos x\right)}{2x - \pi} dx$

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119. Functions $f, g : R \rightarrow R$ are defined, respectively, by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$, find fof

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120. The function $g(x + 2)$ is equal to

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121. Evaluate: $\int_0^{\frac{\pi}{4}} \left(\tan^{-1}\left(\frac{2\cos^2\theta}{2 - \sin 2\theta}\right)\right) \sec^2\theta d\theta.$

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122. If $f(x)=\frac{\sin x}{x}\forall x\in(0,\pi],$ prove that $\frac{\pi}{2}\int_0^{\frac{\pi}{2}}f(x)f\left(\frac{\pi}{2}-x\right)dx=\int_0^{\pi}f(x)dx$

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123. Let $f(x)$ be a continuous function $\forall x\in R,$ except at $x=0,$ such that g(x)=$\int_x^a\frac{f(t)}{t}dt$, prove that $\int_0^a f(x)dx=\int_0^a g(x)dx$

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124. If $\xi nt_0^x\sin(f(t))dt=(x+2)\int_0^x t\sin(f(t))dt, wherex>0,$ then show that $f'(x)\cot f(x)+\frac{3}{1+x}0.$

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125. Show that: $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx.$

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126. Let $a+b=4, where a<2, and let g(x)$ be a differentiable function.

If $\frac{dg}{dx}>0$ for all x, prove that

$\int_0^a g(x)dx+\int_0^b g(x)dx \in crerasesas(b-a) \in crerases.$

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127. about to only mathematics

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128. If $f(x+f(y))=f(x)+y\forall x, y\in R and f(0)=1,$ then prove that

$\int_0^2 f(2-x)dx=2\int_0^1 f(x)dx.$

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129. Suppose f is a real-valued differentiable function defined on $[1, \infty]$ with $f(x1) = 1.$ Moreover, suppose that f satisfies $f'(x) = \frac{1}{x^2 + f^2(x)} Show that f(x) < 1 + \frac{\pi}{4} \forall x \geq 1.$

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130. Let f be a continuous function on $[a, b]\cdot$ If $F(x) = \left(\int_a^x f(t)dt - \int_x^b f(t)dt\right)(2x - (a + b)),$ then prove that there exist some $c \in (a, b)$ such that $\int_a^c f(t)dt - \int_c^b f(t)dt = f(c)(a + b - 2c)\cdot$

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131. $f(x)$ is a continuous and bijective function on $R\cdot$ If $\forall t \in R,$ then the area bounded by $y = f(x), x = a - t, x = a,$ and the x-axis is equal to

the area bounded by $y = f(x), x = a + t, x = a,$ and the x-axis. Then prove that $\int_{-\lambda}^{\lambda} f^{-1}(x)dx = 2a\lambda(given that f(a) = 0)\cdot$

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132. $Iff(x) = x + \int_0^1 t(x+t)f(t)dt,$ $thenf \in dthevalueofthedef \in ite \in tegral \int_0^1 f(x)dx\cdot$

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Exercise 8.1

1. Evaluate the following integrals using limit of sum.

$$\int_a^b \cos x dx$$

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2. Evaluate the following integrals using limit of sum.

$$\int_a^b x^3 dx$$

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3. Find the value of $\int_0^4 [x]dx$, where $[.]$ represents the gretest integer function.

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4. If

$f(x) = \{1 - |x|, |x| \leq 10, |x| > 1' and g(x) = f(x-1) + f(x+1),$

find the value of $\int_{-3}^{3} g(x)dx$.

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Exercise 8.2

1. Consider the integral $I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x}$

Making the substitution $\tan\frac{1}{2}x = t$, we have

$$I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x} = \int_0^{0} \frac{2dt}{(1 + t^2)[5 - 2(1 - t^2)/(1 + t^2)]} = 0$$

The result is obviously wrong, since the integrand is positive and consequently the integral of this function cannot be equal to zero. Find the mistake.

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2. Evaluate the following : $\int_0^{\pi} \frac{dx}{1 + \sin x}$

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3. Evaluate: $\int_1^{\infty} \left(e^{x+1} + e^{3-1}\right)^{-1} dx$

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4. Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)\sqrt{1-x^2}} dx$

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5. Evaluate: $\int_0^1 \frac{2-x^2}{(1+x)\sqrt{1-x^2}} dx$

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6. Evaluate $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

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7. Evaluate: $\int_{\pi/6}^{\pi/4} \frac{1+\cot x}{e^x \sin x} dx$

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8. Evaluate $\int_0^1 \frac{e^{-x} dx}{1+e^x}$

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9. prove that $\int_0^{102} (x-1)(x-2)..(x-100) \times \frac{1}{\frac{1}{x-1}+\frac{1}{x-2}+.\frac{.1}{x-100}} dx = 101! - 10$

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10. Show that : $\int_0^1 \frac{\log x}{(1+x)} dx = -\int_0^1 \frac{\log(1+x)}{x} dx$

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11. If $\int_0^1 \frac{e^t}{1+t} dt = a$, then find the value of $\int_0^1 \frac{e^t}{(1+t)^2} dt$ in terms of a .

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12. Let f be a one-to-one continuous function such that $f(2) = 3 and f(5) = 7. Given \int_2^5 f(x)dx = 17,$ then find the value of $\int_3^7 f^{-1}(x)dx$

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13. Evaluate: $(\lim)_{n \overrightarrow{\infty}} \left(\frac{1}{\sqrt{4n^2 - 1}} + \frac{1}{\sqrt{4n^2 - 2^2}} + + \frac{1}{\sqrt{3n^2}} \right)$

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14. Evaluate: $(\lim)_{n \overrightarrow{\infty}} \left[\frac{1}{n^2} \frac{\sec^2 1}{n^2} + 2/n^2 \frac{\sec^2 4}{n^2} + + \frac{1}{n} \sec^2 1 \right]$

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15. Evaluate $(\lim)_{n \overrightarrow{\infty}} \sum_{k=1}^{n} \frac{k}{n^2 + k^2}$

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16. Evaluate the following limit:

$$\lim_{n \to \infty} \frac{\sum_{r=1}^{n} \sqrt{r} \sum_{r=1}^{h} \frac{1}{\sqrt{r}}}{\sum_{r=1}^{n} r}$$

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17. Evaluate: $(\lim)_{n \overrightarrow{\infty}} \left[\frac{n!}{n^n}\right]^{1/n}$

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Exercise 8.3

1. $Provethat4 \leq \int_1^3 \sqrt{3+x^2} \leq 4\sqrt{3}$

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2.

$If I_1 = \int_0^1 2^x \hat{} 3dx, \ I_2 = \int_0^1 2^x \hat{} 2dx, \ I_3 = \int_1^2 2^x \hat{} 2dx, \ I_4 = \int_1^2 2^x \hat{} 3$

then which of the following is/are ture? $I_1 > I_2$ (b) $I_2 > I_2$ $I_3 > I_4$ (d)

`I_3

A. $I_1 > I_2$

B. $I_2 > I_1$

C. $I_3 > I_4$

D. $I_3 < I_4$

Answer: A::D

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3.

$If I_I = \int_0^{\pi/2} \cos(\sin x) dx, \ I_2 = \int_0^{\frac{\pi}{2}} \sin(\cos x) d, \ and I_3 = \int_0^{\frac{\pi}{2}} \cos x dx,$

then find the order in which the values $I_1, I_2, I_3,$ exist.

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4. If $f(x) = x + 7$ and $g(x) = x - 7, x\epsilon R,$ then find $fog(7)$.

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Exercise 8.4

1. Evaluate : int_0^(pi/2) (sinx+cosx) dx

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2. Evaluate: $\int_{-1}^{4} f(x)dx = 4and\int_{2}^{4} (3 - f(x))dx = 7,$ then find the value of $\int_{2}^{-1} f(x)dx\cdot$

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3. Find the derivative of y

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4. Evaluate: $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

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5. Evaluate $\int_1^a x.\, a^{-[\log_e x]} dx, (a > 1)$.Here $[.]$ represents the greatest integer function.

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6. Evaluate: $\int_1^{e^6} \left[\frac{\log x}{3}\right] dx, where[.]$ denotes the greatest integer function.

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7. Find the value of $\int_{-1}^{1}\left[x^2+\{x\}\right]dx, where[.]and\{.\}$ denote the greatest function and fractional parts of x, respectively.

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8. Prove that $\int_{0}^{\infty}\left[\cot^{-1}x\right]dx, where[.]$ denotes the greatest integer function.

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9. Find the derivative of $y=\ln(1-2x)^3$.

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10. Evaluate: $\int_{0}^{\infty}\left[2e^{-e}\right]dx, where[x]$ represents greatest integer function.

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Exercise 8.5

1. If $f(a+b-x)=f(x)$, then $\int_a^b xf(x)dx$ is equal to

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2. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

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3. Find the value of $\int_0^1 \sqrt[3]{2x(3)-3x^2-x+1}dx.$

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4. Show that $\int_0^\pi fx(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx\cdot$

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5. Find the value of $\int_0^1 x(1-x)^n dx$

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6. If a continuous function f on [0, a] satisfies f(x)f(a-x)=1, a >0, then find the value of $\int_0^a \frac{dx}{1+f(x)}$

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7. If $fandg$ are continuous function on $[0, a]$ satisfying $f(x)=f(a-x)andg(x)(a-x)=2,$ then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx.$

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8. Find the value of $\int_0^{\frac{\pi}{2}} \sin 2x \log \tan x dx$

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9. Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx$

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10. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

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11. Check whether the equation is quadratic equation $x^3 - 4x^2 - x + 1 = (x - 2)^2$.

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Exercise 8.6

1. Find the value of $\int_0^{2\pi} \frac{1}{1+\tan^4 x} dx$

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2. STATEMENT 1: $\int_0^{\pi} \sin^{100} x \cos^{99} x dx is zer \odot$ STATEMENT 2: $\int_a^b f(x) dx = \int_{a+c}^{b+c} f(x-c) dx, and$ for odd function, $\int_{-a}^{a} f(x) dx = 0$

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3. If $U_n = \int_0^1 x^n (2-x)^n dx and V_n = \int_0^1 x^n (1-x)^n dx, n \in N,$ and if $\frac{V_n}{U_n} = 1024,$ then the value of n is______

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4. By using the properties of definite integrals, evaluate the integrals

$$\int_0^{\pi} \log(1+\cos x)dx$$

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5. Find the value of $\int_0^1 \{(\sin^{-1} x)/x\}dx$

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6. Evaluate $\int_{-\infty}^{0} \frac{te^t}{\sqrt{1-e^{2t}}}dt$

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7. If $I_1 = \int_0^{\pi} x f(\sin^3 x + \cos^2 x)dx and$ $I_2 = \int_0^{\frac{\pi}{2}} f(\sin^3 x + \cos^2 x)dx, then relate I_1 and I_2$

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Exercise 8.7

1. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x(\sin x + \cos x)dx$

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2. Evaluate: $\int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1}dx$

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3. Evaluate: $\int_{-\pi}^{\pi} (1 - x^2)\sin x \cos^2 x dx$

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4. Evaluate: $\int_{-1}^{1} \frac{\sin x - x^2}{3 - |x|}dx$

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5. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \frac{\log(1-x)}{1+x} dx$

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6. Evaluate: $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right] dx$

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Exercise 8.8

1. Evaluate: $\int_0^{100} (x - [x] dx (where[.]$ represents the greatest integer function).

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2. Evaluate: $\int_0^{100\pi} \sqrt{(1-\cos 2x)} dx.$

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3. $If\int_0^{n\pi} f(\cos^2 x)\,dx = k\int_0^{\pi} f(c0s^2 x)\,dx, thenf \in dthevaluek\cdot$

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4. Evaluate $\int_0^{\pi} \left(\frac{\cos x}{1+\sin x}\right) dx$

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5. Evaluate $\int_{-5}^{5} x^2\left[x+\frac{1}{2}\right] dx$ (where [.] denotes the greatest integer function).

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6. If $f(x)$ is a function satisfying $f(x+a)+f(x)=0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x)dx$ is independent of b, then find the least positive value of ·

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7. Show that $\int_0^{n\pi+v}|\sin x|dx = 2n+1-\cos v,$ where n is a positive integer and `,lt=v

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Exercise 8.9

1. $If\int_{\frac{\pi}{3}}^{x}\sqrt{(3-\sin^2 t)}dt+\int_0^y \cos tdt = 0, the \neq valuate\frac{dy}{dx}$

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2. $Iff(x) = e^{g(x)} andg(x) = \int_2^x \frac{tdt}{1+t^4},$ then find the value of $f'(2)$

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3. Let $f: R \rightarrow R$ be a differentiable function having $f(2)=6, f'(2)=\frac{1}{48}$. Then evaluate $\lim_{x \rightarrow 2} \int_{6}^{f(x)} \frac{4t^3}{x-2} dt.$

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4. Evaluate: $(\lim)_{x \overrightarrow{2}} \frac{\int 0 \mathrm{xcos}\, t^2 dt}{x}$

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5. Find the points of minima for $f(x)=\int_0^x t(t-1)(t-2)dt$

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6. Find the equation of tangent to $y=\int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}} at x=1.$

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7. $Iff(x) = \int_{\frac{x^2}{16}}^{x^2} \frac{\sin x \sin \sqrt{\theta}}{1 + \cos^2 \sqrt{\theta}} dth\eta$, then find the value of $f'\left(\frac{\pi}{2}\right)$.

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8. Let $f(x)$ be a continuous and differentiable function such that $f(x) = \int_0^x \sin(t^2 - t + x) dt$ Then prove that $f^{''}(x) + f(x) = \cos x^2 + 2x\sin x^2$

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9. If f' is a differentiable function satisfying $f(x) = \int_0^x \sqrt{1 - f^2(t)} dt + \frac{1}{2}$ then the value of $f(\pi)$ is equal to

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Exercise 8.10

1. $If \int_0^1 \frac{e^t dt}{t+1} = a, the \ne valuate \int_{b-1}^{b} \frac{e^{-t} dt}{t-b-1}$

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2. $If f(x) = \int_1^x \frac{\log t}{1+t+t^2} dx \, \forall x \le 1, then prove that f(x) f\left(\frac{1}{x}\right).$

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3. $f(x) = \int_1^x \frac{\tan^{-1}(t)}{t} dt \, \forall x \in R^+, then f \in d the value of$
$f(e^2) - f\left(\frac{1}{e^2}\right)$

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4. Evaluate: $\int_{\sqrt{2}}^{\sqrt{2}+1} \frac{(x^2-1)}{(x^2+1)^2} dx$

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5. Evaluate: $\int_{-1}^{0} \frac{dx}{x^2 + 2x + 2}$

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6. Find the value of $\int_{\frac{1}{2}}^{2} e^{\left|x - \frac{1}{x}\right|} dx.$

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7. If $I_1 = \int_0^1 \frac{dx}{e^x(1+x)}$ and $I_2 = \int_0^{\pi/4} \frac{e^{\tan^7\theta}\sin\theta}{(2-\tan^2\theta)\cos^3\theta d\theta}$,then find the value of $\frac{l_1}{l_2}$.

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Exercise 8.11

1. If $I_K = \int_1^e (1nx)^k dx \left(k \in I^+\right) dx \left(k \in I^+\right)$, then find the value of I_4.

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2. Given $I_m = \int_1^e (\log x)^m dx, thenprovethat \frac{I_m}{1-m} + mI_{m-2} = e$

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3. $IfI_n = \int_0^{\frac{\pi}{2}} \sin^n x dx, thenshowthatI_n = \left(\frac{n-1}{n}\right) I_{n-2}$ Hence, Prove that

$I_n = f(x) = \left\{ \left(\frac{n-1}{n}\right) \left(\frac{n-3}{n-2}\right) \left(\frac{n-5}{n-4}\right) \ddot{\frac{1}{2}} \frac{\pi}{2} \text{ if } niseven \left(\frac{n-1}{n}\right) \left(\right. \right.$

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5. $IfI_n = \int_0^1 x^n (\tan^{-1} x) dx, thenprovethat$ $(n+1)I_n + (n-1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$

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6. $IfI_n = \int_0^{\frac{\pi}{2}} \sin^n x dx, thenshowthatI_n = \left(\frac{n-1}{n}\right) I_{n-2}$ Hence, Prove that $I_n = f(x) = \left\{ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \frac{\ddot{1}}{2} \frac{\pi}{2} \text{ if } niseven \left(\frac{n-1}{n}\right)\left(\right.\right.$

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Exercise (Single)

1. $\lim_{n \to \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots\ldots\ldots + \frac{n}{n^2}\right)$ is

A. $1/4$

B. $1/6$

C. $1/9$

D. $1/12$

Answer: D

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2. $IfS_n = \left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + + \frac{1}{n+\sqrt{n^2}}\right], then(\lim)_{n \overrightarrow{\infty}} S_n$ is equal to log 2 (b) log 4 log 8 (d) none of these

A. $\log 2$

B. $\log 4$

C. $\log 8$

D. none of these

Answer: B

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3. The value of $(\lim)_{n\overrightarrow{\infty}} \sum_{r=1}^{4n} \frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+\sqrt{n}\right)^2}$ is equal to $\frac{1}{35}$ (b) $\frac{1}{4}$ (c) $\frac{1}{10}$ (d) $\frac{1}{5}$

A. $\frac{1}{35}$

B. $\frac{1}{14}$

C. $\frac{1}{10}$

D. $\frac{1}{5}$

Answer: C

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4. $\lim_{x \to \infty} \frac{1^2 + 2^2 + \ldots + n^2}{n^3}$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$

Answer: A

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5. $The value of \left(\lim\right)_{n \overrightarrow{\infty}} \left[tan\frac{\pi}{2n}\frac{\tan(2\pi)}{2n}\frac{\tan(\dot{n}\pi)}{2n}\right]^{1/n} is\ e$ (b) e^2 (c) 1

(d) e^3

A. e

B. e^2

C. 1

D. e^3

Answer: C

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6. $\int_{2-a}^{2+a} f(x) dx is equa < o[where f(2-\alpha) = f(2+\alpha) \forall \alpha \in R]$ 2 $\int_{2}^{2+a} f(x) dx$ (b) $2\int_{0}^{a} f(x) dx$ $2\int_{2}^{2} f(x) dx$ (d) none of these

A. $2\int_{2}^{2+a} f(x) dx$

B. $2\int_{0}^{a} f(x) dx$

C. $2\int_{2}^{2} f(x) dx$

D. none of these

Answer: A

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7. about to only mathematics

A. 50

B. 100

C. 200

D. none of these

Answer: A

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8. Which of the following is incorrect ?

A. $\int_{a+c}^{b+c} f(x)dx = \int_a^b f(x+c)dx$

B. $\int_{ac}^{bc} f(x)dx = c\int_a^b f(cx)dx$

C. $\int_{-a}^{a} f(x)dx = \frac{1}{2}\int_{-a}^{a} f(x) + f(-x)dx$

D. none of these

Answer: D

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9. $\int_{-1}^{\frac{1}{2}} \frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}} isequa < o\ \frac{\sqrt{e}}{2}(\sqrt{3}+1)$ (b) $\frac{\sqrt{3e}}{2}$ $\sqrt{3e}$ (d) $\sqrt{\frac{e}{3}}$

A. $\frac{\sqrt{e}}{2}(\sqrt{3}+1)$

B. $\frac{\sqrt{3e}}{2}$

C. $\sqrt{3e}$

D. $\sqrt{\frac{e}{3}}$

Answer: C

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10. $If\int_{\log 2}^{x}\frac{dx}{\sqrt{e^{x}-1}}=\frac{\pi}{6}, \text{then } \ x \ \text{ is equalto}$ (a)4 (b) log 8 (c) log 4 (d) none of these

A. $\log 4$

B. $In8$

C. $In4$

D. none of these

Answer: C

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11. $\int_{\frac{5}{2}}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4} dx$ is equal to (a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{3}$

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{3}$

Answer: D

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12. If $f(x)$ is integrable over $[1, 2]$ then $\int_{1}^{2} f(x)dx$ is equal to

A. 1

B. 3

C. 0

D. none of these

Answer: C

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13. $\int \frac{e^x}{e^{2x}+4}dx$

A. $3+2\pi$

B. $4-\pi$

C. $2+\pi$

D. none of these

Answer: B

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14. The value of the integral $\int_0^1 \frac{dx}{x^2 + 2x \cos\alpha + 1}$ is equal to $\sin\alpha$ (b) $\alpha \sin\alpha$ $\frac{\alpha}{2\sin\alpha}$ (d) $\frac{\alpha}{2}\sin\alpha$

A. $\sin\alpha$

B. $\alpha\sin\alpha$

C. $\frac{\alpha}{\sin\alpha}$

D. $\frac{\alpha}{2}\sin\alpha$

Answer: C

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15. $\int_0^\infty \frac{dx}{\left[x + \sqrt{x^2+1}\right]^3} isequa < o \frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) none of these

A. $\frac{3}{8}$

B. $\frac{1}{8}$

C. $-\frac{3}{8}$

D. none of these

Answer: A

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16. $If f(y) = e^y, g(y) = y; y > 0, and F(t) = \int_0^t f(t-y)g(y)dy, then$

a) $F(t) = e^t - (1+t)$ b) $F(t) = te^t$ c) $F(t) = te^{-t}$ (d) $F(t) = 1 - e^t(1+t)$

A. $F(t) = e^t - (1+t)$

B. $F(t) = te^t$

C. $F(t) = te^{-t}$

D. $F(t) = 1 - e^t(1+t)$

Answer: A

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17. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6 and p(3) = 2$, then $p'(0)$ is_____

A. $\frac{17}{4}$

B. $\frac{13}{4}$

C. $\frac{19}{4}$

D. $\frac{5}{4}$

Answer: C

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18. The numbers of possible continuous $f(x)$ defined in $[0, 1]$ for which

$$I_1 = \int_0^1 f(x)dx = 1, I_2 = \int_0^1 x f(x)dx - a, I_3 = \int_0^1 x^2 f(x)dx = a^2 is / a$$

1 (b) ∞ (c) 2 (d) 0

A. 1

B. ∞

C. 2

D. 0

Answer: D

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19. Suppose that $F(x)$ is an anti-derivative of $f(x)=\frac{\sin x}{x}, wherex > 0$. Then $\int_1^3 \tan^{-1} dx$ can be expressed as $F(6)-F(2)$ (b) $\frac{1}{2}(F(6)-f(2))$ $\frac{1}{2}(F(3)-f(1))$ (d) $2(F(6))-F(2))$

A. $F(6)-F(2)$

B. $\frac{1}{2}(F(6)-F(2))$

C. $\frac{1}{2}(F(3)-F(1))$

D. $2(F(6)-F(2))$

Answer: A

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20. $\int_{-\frac{\pi}{3}}^{0}\left[\cot^{-1}\left(\frac{2}{2\cos x-1}\right)+\cot^{-1}\left(\cos x-\frac{1}{2}\right)\right]dx$ i s equal to (a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{3\pi^2}{8}$

A. $\frac{\pi^2}{6}$

B. $\frac{\pi^2}{3}$

C. $\frac{\pi^2}{8}$

D. $\frac{3\pi^2}{8}$

Answer: A

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21. Evaluate the definite integrals

$$\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$$

A. $\frac{1}{20}\log 3$

B. $\frac{1}{40}\log 3$

C. $\frac{1}{20}\log 6$

D. $10\log 3$

Answer: A

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22. $\int_1^2 x^2 dx$ is equal to

A. $\frac{\pi}{2} = 2\tan^{-1} e$

B. $\frac{\pi}{2} - 2\cot^{-1} e$

C. $2\tan^{-1} e$

D. $\pi - 2\tan^{-1} e$

Answer: D

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23. $\int_0^{\frac{\pi}{2}} \sin^7 x dx$ is :

A. $\pi / 2$

B. $\pi / 4$

C. $\pi / 6$

D. $3\pi / 2$

Answer: B

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24. The range of the function $f(x) = \int_{-1}^{1} \frac{\sin x dt}{(1 - 2t \cos x + t^2) is}$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $[0, \pi]$ $\{0, \pi\}$ (d) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $[0, \pi]$

C. $\{0, \pi\}$

D. $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

Answer: D

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25. If the function $f:[0,8]\overrightarrow{R}$ is differentiable, then for `0

A. $3\left[\alpha^3 f\left(\alpha^2\right)+\beta^2 f\left(\beta^2\right)\right]$

B. $3\left[\alpha^3 f(\alpha)+\beta^3 f(\beta)\right]$

C. $3\left[\alpha^2 f\left(\alpha^3\right)+\beta^2 f\left(\beta^3\right)\right]$

D. $3\left[\alpha^2 f\left(\alpha^2\right)+\beta^2 f\left(\beta^2\right)\right]$

Answer: C

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26. If $f(x)=x\tan^{-1}x$, then $f'(1)$ is

A. 0

B. $\log_e 3$

C. $\log_e 4$

D. $\log_e 7$

Answer: D

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27. Let $f(0) = 0 and \int_0^2 f'(2t)e^{f(2t)}\,dt = 5. thenvalueoff f(4) is$ log 2 (b) log 7 (c) log 11 (d) log 13

A. $\log 2$

B. $\log 7$

C. $\log 11$

D. $\log 13$

Answer: C

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28. If $f(x) = \cos\left(\tan^{-1} x\right)$, then the value of the integral $\int_0^1 x f^x dx \text{i s}$ $\frac{3-\sqrt{2}}{2}$ (b) $\frac{3+\sqrt{2}}{2}$ 1 (d) $1 - \frac{3}{2\sqrt{2}}$

A. $\frac{3-\sqrt{2}}{2}$

B. $\frac{3+\sqrt{2}}{2}$

C. 1

D. $1 - \frac{3}{2\sqrt{2}}$

Answer: D

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29. The equation of the curve is $y = f(x)$. The tangents at $[1, f(1), [2, f(2)], and[3, f(3)]$ make angles $\frac{\pi}{6}, \frac{\pi}{3}, and\frac{\pi}{4}$, respectively, with the positive direction of x-axis. Then the value of

$\int_2^3 f'(x) f^x dx + \int_1^3 f^x dx$ is equal to $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (e) 0 (d) none of these

A. $-1/\sqrt{3}$

B. $1/\sqrt{3}$

C. 0

D. none of these

Answer: A

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30. The value of $\int_1^e \left(\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right) dx is$ $\tan e$ (b) $\tan^{-1} e$ $\tan^{-1}\left(\frac{1}{e}\right)$ (d) none of these

A. $\tan e$

B. $\tan^{-1} e$

C. $\tan^{-1}(1/e)$

D. none of these

Answer: B

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31. $If f(\pi) = 2\int_0^{\pi} (f(x) + f^x)\sin x dx = 5, then f(0)$ is equal to (it is given that $f(x)$ is continuous in $[0, \pi]$)$\cdot$ 7 (b) 3 (c) 5 (d) 1

A. 7

B. 3

C. 5

D. 1

Answer: B

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32. If $\int_1^2 e^x \hat{} 2dx = a, then \int_e^{e^4} \sqrt{1nx} dxisequa < o\ 2e^4 - 2e - a$ (b) $2e^4 - e - a\ 2e^4 - e - 2a$ (d) $e^4 - e - a$

A. $2e^4 - 2e - a$

B. $2e^4 - e - a$

C. $2e^4 - e - 2a$

D. $e^4 - e - a$

Answer: B

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33. If $f(x)$ is continuous for all real values of x, then $\sum_{r=1}^{n} f(r - 1 + x) dxisequa < o \int_0^n f(x) dx$ (b) $\int_0^1 f(x) dx\ n \int_0^1 f(x) dx$ (d) $(n - 1) \int_0^1 f(x) dx$

A. $\int_0^n f(x) dx$

B. $\int_0^1 f(x)dx$

C. $n\int_0^1 f(x)dx$

D. $(n-1)\int_0^1 f(x)dx$

Answer: A

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34. $The value of \int_0^{\frac{\pi}{2}} \sin|2x-\alpha|dx, where \alpha \in [0, \pi], is \quad 1-\cos\alpha$ (b) $1+\cos\alpha$ 1 (d) $\cos\alpha$

A. $1-\cos\alpha$

B. $1+\cos\alpha$

C. 1

D. $\cos\alpha$

Answer: C

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35. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \frac{n^2}{2} \forall n \in I$. Then $\int_{-3}^{5} f(|x|)dx$ is equal to $\frac{19}{2}$ (b) $\frac{35}{2}$ (c) $\frac{17}{2}$ (d) none of these

A. $19/2$

B. $35/2$

C. $17/2$

D. none of these

Answer: B

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36. If $f(x) = \int_0^x t \cos t dt$, then $\frac{df}{dx}$

A. $\frac{1}{2}(1 - x^2)$

B. $\frac{1}{2}x^2$

C. $\frac{1}{2}(1+x^2)$

D. none of these

Answer: C

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37. If $a > 0$ and $A = \int_0^a \cos^{-1} x dx,$ and $\int_{-a}^{a} \left(\cos^{-1} x - \sin^{-1} \sqrt{1-x^2}\right) dx = \pi a - \lambda A$. Then λ is

A. 0

B. 2

C. 3

D. none of these

Answer: B

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38. The value of $\int_1^a [x]f'(x)dxf'(x)dx, wherea > 1, and[x]$ denotes the greatest integer not exceeding x, is $af(a)-\{f(1)f(2)+\ +f([a])\}$ $[a]f(a)-\{f(1)+f(2)+\ +f([a])\}$ $[a]f(a)-\{f(1)+f(2)+\ +fA\}$ $af([a])-\{f(1)+f(2)+\ +fA\}$

A. $af(a)-(f(1)+f(2)+\ldots\ldots\ldots\ +f([a]))$

B. $[a]f(a)-(f(1)+f(2))+\ldots\ldots\ldots\ +f([a]))$

C. $[a]f([a])-(f(1)+f(2)+\ldots\ldots\ldots\ +f(a))$

D. $af([a])-(f(1)+f(2)+\ldots\ldots\ldots\ldots+f(a))$

Answer: B

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39. $\int_3^{10} [\log[x]]dxisequa < o(where[.]represetn\hbox{h}egreatest$ integer function) 9 (b) $16-e$ (c) 10 (d) $10+e$

A. 9

B. $16 - e$

C. 10

D. $10 + e$

Answer: A

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40. $\int_{-1}^{2}\left[\frac{[x]}{1+x^2}\right]dx, where[.]$ denotes the greater integer function, is equal to -2 (b) -1 $zero$ (d) none of these

A. -2

B. -1

C. zero

D. none of these

Answer: B

Watch Video Solution

41. $Thevalueof \int_{-2}^{1} \left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right]+1\right]dx,$ where [.] denotes the greatest integer function, is 1 (b) $1/2$ (c) 2 (d) none of these

A. 1

B. $1/2$

C. 2

D. none of these

Answer: C

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42. Evaluate: $\int_{0}^{2\pi}[\sin x]dx, where[x]$ denotes the greatest integer function.

A. $\frac{-5\pi}{3}$

B. $-\pi$

C. $\frac{5\pi}{3}$

D. -2π

Answer: B

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43.

$I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx, I_2 = \int_0^{2\pi} \cos^6 x dx, I_3 = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x dx, I_4 =$

$I_2 = I_3 = I_4 = 0, I_1 \neq 0$ $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

$I_1 = I_2 = I_3 = 0, I_4 \neq 0$ $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

A. $I_2 = I_3 = I_4 = 0, I_1 \neq 0$

B. $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

C. $I_1 = I_3 = I_4 = 0, I_2 \neq 0$

D. $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

Answer: C

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44. $Given \int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x+\cos x}=\log 2$. Then the value of integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\sin x+\cos x} dx$ is equal to (a) $\frac{1}{2}\log 2$ (b) $\frac{\pi}{2}-\log 2$ (c) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (d) $\frac{\pi}{2}+\log 2$

A. $\frac{1}{2}A$

B. $\frac{\pi}{2}-A$

C. $\frac{\pi}{4}-\frac{1}{2}A$

D. $\frac{\pi}{2}+A$

Answer: C

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45. $\mathrm{I\ f} I_1=\int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ and$I_2=\int_{-100}^{101} \frac{dx}{5+2x-2x^2}$, $then \frac{I_1}{I_2}$ i s (a)2 (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{1}{2}$

A. 2

B. $\frac{1}{2}$

C. 1

D. $-\frac{1}{2}$

Answer: B

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46. The value of $\int_0^\infty \frac{x dx}{(1+x)(1+x^2)}$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none of these

Answer: A

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47. about to only mathematics

A. π

B. 1

C. 0

D. none of these

Answer: C

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48. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf([x(1-x)]dx$, $I_2 = \int_{1-k}^{k} f[x(1-x)]dx, where 2k - 1 > 0. Then \frac{I_1}{I_2} is$ 2 (b) k (c) $\frac{1}{2}$ (d) 1

A. 2

B. k

C. $\frac{1}{2}$

D. 1

Answer: C

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49. If

$$f(x) = \frac{e^x}{1+e^x}, I_1 = \int (f(-a))^{f(a)} xg\left(x(1-x)dx, \text{ and } I_2 = \int_{f(-a)}^{f(a)} g\right.$$

then the value of $\frac{I_2}{I_1}$ is (a) -1 (b) -2 (c) 2 (d) 1

A. -1

B. -2

C. 2

D. 1

Answer: C

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50. The value of $\int_1^3 \left(\sqrt{1+(x-1)^3}+(x^2-1)^{\frac{1}{3}}+1\right)dx$ is ________.

A. $\frac{\pi}{8}\log_e 2$

B. $\frac{\pi}{2}\log_e 2$

C. $-\frac{\pi}{2}\log_e 2$

D. none of these

Answer: A

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51. The value of the definite integral $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}dx$ is $\sqrt{2}\pi$ (b) $\frac{\pi}{\sqrt{2}}$ $2\sqrt{2}\pi$ (d) $\frac{\pi}{2\sqrt{2}}$

A. $\sqrt{2}\pi$

B. $\frac{\pi}{\sqrt{2}}$

C. $2\sqrt{2}\pi$

D. $\dfrac{\pi}{2\sqrt{2}}$

Answer: B

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52. $f(x) > 0 \forall x \in RandisboundeÏf$

$$(\lim)_{n \to \infty}\left[\int_0^a \frac{f(x)dx}{f(x)+f(a-x)} + a^2 + a\int_a^{2a} \frac{f(x)dx}{f(x)+f(3a-x)} + \int_{2a}^{3a}$$

(where $a < 1$), then a is equal to $\dfrac{2}{7}$ (b) $\dfrac{1}{7}$ (c) $\dfrac{14}{19}$ (d) $\dfrac{9}{14}$

A. $\dfrac{2}{7}$

B. $\dfrac{1}{7}$

C. $\dfrac{14}{19}$

D. $\dfrac{9}{14}$

Answer: C

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53. $If \int_0^1 \cot^{-1}(1 - x + x^2)dx = \lambda \int_0^1 \tan^{-1} x dx, then \lambda is equa < o$ 1

(b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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54. The value of the definite integral $\int_0^1 (1 + e^{-x} \hat{} 2)dx$ (b) 2 $1 + e^{-1}$

(d) none of these

A. π

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: D

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55. $Thevalueofthe \in tegral \int_{-\frac{3\pi}{4}}^{\frac{5\pi}{4}} \left(\frac{\sin x + \cos x}{e^{x - \frac{\pi}{4}} + 1} dx\right.$ 0 (b) 1 (c) 2 (d) none of these

A. 0

B. 1

C. 2

D. none of these

Answer: A

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56. $I_1=\int_0^{\frac{\pi}{2}}\ln(\sin x)dx,\ I_2=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\ln(\sin x+\cos x)dx\cdot$ Then (a) $I_1=2I_2$ (b) $I_2=2I_1$ (c)$I_1=4I_2$ (d) $I_2=4I_1$

A. $I_1=2I_2$

B. $I_2=2I_1$

C. $I_1=4I_2$

D. $I_2=4I_1$

Answer: A

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57. $IfI_1=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{1+\cos^2 x}dx,\ I_2=\int_0^{\frac{\pi}{2}}\frac{\sin^2 x}{1+\sin^2 x}dx$ $I_3=\int_0^{\frac{\pi}{2}}\frac{1+2\cos^2 x\sin^2 x}{4+2\cos^2 x\sin^2 x}dx,\ then$ $I_1=I_2>I_3$ (b) $I_3>I_1=I_2$ $I_1=I_2=I_3$ (d) none of these

A. $I_1=I_2>I_3$

B. $I_3 > I_1 = I_2$

C. $I_1 = I_2 = I_3$

D. none of these

Answer: C

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58. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

A. $\frac{\pi^2}{2}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{8}$

D. $\frac{\pi^2}{16}$

Answer: D

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59. For $x \in R$ and a continuous function f, let $I_1 = \int_{s \in^2 t}^{1+\cos^2 t} x f\{x(2-x)\} dx and I_2 = \int_{\sin^2 t}^{1+\cos^2} x f\{x(2-x)\} dx \dot{T}hen \frac{I_1}{I_2}$ is -1 (b) 1 (c) 2 (d) 3

A. -1

B. 1

C. 2

D. 3

Answer: B

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60.

$If \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{e^{\frac{\pi}{4}} dx}{\left(e^x + e^{\frac{\pi}{4}}\right)(\sin x + \cos x)} = k \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec x dx, thenthevalueofkis\right.$

$\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$

D. $-\frac{1}{\sqrt{2}}$

Answer: C

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61. The value of the definite integral $\int_2^4 x(3-x)(4+x)(6-x)(10-x)+\sin x\Big)dxequals \quad \cos 2+\cos 4$

(b) $\cos 2-\cos 4 \sin 2+\sin 4$ (d) $\sin 2-\sin 4$

A. $\cos 2+\cos 4$

B. $\cos 2-\cos 4$

C. $\sin 2+\sin 4$

D. $\sin 2-\sin 4$

Answer: B

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62. $IfI = \int_{-20\pi}^{20\pi} |\sin x| [\sin x] dx (where [.]$ denotes the greatest integer function), then the value of I is -40 (b) 40 (c) 20 (d) -20

A. -40

B. 40

C. 20

D. -20

Answer: A

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63. Which of the following statement is always true? (a)If $f(x)$ is increasing, then $f^{-1}(x)$ is decreasing. (b)If $f(x)$ is increasing, then $\frac{1}{f(x)}$

is also increasing. (c)If $f and g$ are positive functions and f is increasing and g is decreasing, then $\frac{f}{g}$ is a decreasing function. (d)If $f and g$ are positive functions and f is decreasing and g is increasing, the $\frac{f}{g}$ is a decreasing function.

A. is always non-positive

B. is always non-negative

C. can take positive and negative values

D. none of these

Answer: A

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64. $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx$ is

A. $\frac{\pi^2}{4}$

B. $\frac{\pi^2}{2}$

C. $\frac{3\pi^2}{2}$

D. $\frac{\pi^2}{3}$

Answer: A

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65. If $f(x) = \int_0^{\pi} \frac{t \sin t dt}{\sqrt{1 + \tan^2 x \sin^2 t}}$ for $0 < x < \frac{\pi}{2}$ then

A. $f(0^+) = -\pi$

B. $f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8}$

C. f is continuous and differntiable in $\left(0, \frac{\pi}{2}\right)$

D. f is continuous but not differentiable in $\left(0, \frac{\pi}{2}\right)$

Answer: C

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66. $\int_{-3}^{3} x^{8}\{x^{11}\}dx$=

A. 3^{8}

B. 3^{7}

C. 3^{9}

D. none of these

Answer: B

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67. The value of $\int_{0}^{4\pi} \log_{e}\left|3\sin x + 3\sqrt{3}\cos x\right|dx$ then the value of I is equal to

A. $\pi \log_{e} 3$

B. $2\pi \log_{e} 3$

C. $4\pi \log_{e} 3$

D. $8\pi \log_e 3$

Answer: C

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68. Evaluate $\int_0^1 \left(\frac{x^2}{1+x^2}\right) dx$

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69. The value f the integral $\int_{-\pi}^{\pi} \sin mx \sin nx dx,$ for $m \neq n(m, n \in I), is$ 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π

A. 0

B. π

C. $\pi / 2$

D. 2π

Answer: A

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70. If $f(x)$ and $g(x)$ are continuous functions, then $\int_{In\lambda}^{In(1/\lambda)} \frac{f(x^2/4)[f(x)-f(-x)]}{g(x^2/4)[g(x)+g(-x)]}dx$ is

A. depenent on λ

B. a non zero constant

C. zero

D. none of these

Answer: C

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71. The value of $\int_0^1 \frac{\tan^{-1}\left(\frac{x}{x+1}\right)}{0\tan^{-1}\left(\frac{1+2x-2x^2}{2}\right)}dx$ is

A. $1/4$

B. $1/2$

C. 1

D. 2

Answer: B

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72. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{|\sin x|}\cos x}{(1+e^{\tan x})dxisequa < o}$ $e+1$ (b) $1-e$ $e-1$ (d) none of these

A. $e+1$

B. $2e$

C. $e-1$

D. $e-2$

Answer: C

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73. The value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

A. π

B. π^2

C. $2\pi^2$

D. $\pi^2 / 2$

Answer: B

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74. The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is.......

A. 2π

B. 999π

C. 0

D. π

Answer: A

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75. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)\cdot$ If $I = \int_0^T f(x)dx,$ then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}I$

B. $2I$

C. $3I$

D. $6I$

Answer: C

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76. $(\lim)_{x \overrightarrow{1}} \frac{1}{\sqrt{|x| - \{-x\}}}(where\{x\}$ denotes the fractional part of (x) is equal to does not exist (b) 1 ∞ (d) $\frac{1}{2}$

A. 13

B. 6.3

C. 1.5

D. 7.5

Answer: C

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77. The value of $\int_0^x [\cos t]dt,\ x \in \left[(4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2}\right] andn \in N,$ is equal to where [.] represents greatest integer function. $\frac{\pi}{2}(2n-1) - 2x$ $\frac{\pi}{2}(2n-1) + x$ $\frac{\pi}{2}(2n+1) - x$ (d) $\frac{\pi}{2}(2n+1) + x$

A. $\frac{\pi}{2}(2n-1) - 2x$

B. $\frac{\pi}{2}(2n-1)+x$

C. $\frac{\pi}{2}(2n+1)-x$

D. $\frac{\pi}{2}(2n+1)+x$

Answer: C

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78. Evaluate $\int_0^2 x(2-x)^n dx$

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79. $\int_0^x \frac{2^t}{2^{[t]}} dt, where[.]$ denotes the greatest integer function and $x \in R^+$, is equal to

A. $\frac{1}{1n2}\left([x]+2^{\{x\}}-1\right)$

B. $\frac{1}{1n2}\left([x]+2^{\{x\}}\right)$

C. $\frac{1}{1n2}\left([x]-2^{\{x\}}\right)$

D. $\frac{1}{1n2}\left([x]+2^{\{x\}}+1\right)$

Answer: A

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80. If $\int \sin x d(\sec x) = f(x) - g(x) + c, then \quad f(x) = \sec x$ (b) $f(x) = \tan x$ $g(x) = 2x$ (d) $g(x) = x$

A. $g(x)$ is odd

B. $2(n) = 0, n\varepsilon N$

C. $g(2n) = 0, n\varepsilon N$

D. $g(x)$ is non-periodic

Answer: C

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81. $Ifg(x) = \int_0^x (|\sin t| + |\cos t|)dt, theng\left(x + \frac{\pi n}{2}\right)$ is equal to, where $n \in N$, $g(x) + g(\pi)$ (b) $g(x) + g\left(\frac{n\pi}{n2}\right)$ $g(x) + g\left(\frac{\pi}{2}\right)$ (d) none of these

A. $g(x) + g(\pi)$

B. $g(x) + ng\left(\frac{\pi}{2}\right)$

C. $g(x) + g\left(\frac{\pi}{2}\right)$

D. none of these

Answer: B

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82. $Ifx = \int_c^{\sin t} \sin^{-1} zdz, y = \int_k^{\sqrt{t}} \frac{\sin z^2}{z} dz, then \frac{dy}{dx} isequa < o$ $\frac{\tan t}{2t}$ (b) $\frac{\tan t}{t^2}$ $\frac{t}{2t^2}$ (d) $\frac{ta \hat{} 2}{2t^2}$

A. $\frac{\tan t}{2t}$

B. $\frac{\tan t}{t^2}$

C. $\dfrac{\tan t}{2t^2}$

D. $\dfrac{\tan t^2}{2t^2}$

Answer: C

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83. about to only mathematics

A. 1

B. 17

C. $\sqrt{17}$

D. none of these

Answer: C

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84. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals $\frac{2}{5}$ (b) $-\frac{5}{2}$ 1 (d) $\frac{5}{2}$

A. $2/5$

B. $-5/2$

C. 1

D. $5/2$

Answer: A

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85. If $f(x) = \cos x - \int_0^x (x-t) f(t) dt, then f'(x) + f(x)$ is equal to $-\cos x$ (b) $-\sin x$ $\int_0^x (x-t) f(t) dt$ (d) 0

A. $-\cos x$

B. $-\sin x$

C. $\int_0^x (x-t)f(t)dt$

D. 0

Answer: A

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86. A function f is continuous for all x (and not everywhere zero) such that $f^2(x) = \int_0^x f(t)\frac{\cos t}{2+\sin t}dt\cdot$ Then $f(x)$ is $\frac{1}{2}\ln\left(\frac{x+\cos x}{2}\right); x \neq 0$ $\frac{1}{2}\ln\left(\frac{3}{x+\cos x}\right); x \neq 0$ $\frac{1}{2}\ln\left(\frac{2+\sin x}{2}\right); x \neq n\pi, n \in I$ $\frac{\cos x + \sin x}{2+\sin x}; x \neq n\pi + \frac{3\pi}{4}, n \in I$

A. $\frac{1}{2}In\left(\frac{x+\cos x}{2}\right)$

B. $\frac{1}{2}In\left(\frac{3}{2+\cos x}\right)$

C. $\frac{1}{2}In\left(\frac{2+\sin x}{2}\right)$

D. $\frac{\cos x+\sin x}{2+\sin x}$

Answer: C

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87. $(\lim)_{n \overrightarrow{\infty}} \frac{1}{x}\left[\int_y^a e^{\sin} \hat{} (2t)dt - \int_{x+y}^a e^{\sin} \hat{} (2t)dt - \right] isequa < o$

$(0, 1]$ (b) $[1, \infty)$ $(0, \infty)$ (d) none of these

A. $e^{\sin^2 y}$

B. $\sin 2ye^{\sin^2 y}$

C. 0

D. none of these

Answer: A

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88. Let $f(x) = \sqrt{\log_{10} x^2}$.Find the set of all values of x for which f (x) is real.

A. $(0, 1]$

B. $[1, \infty)$

C. $(0, \infty)$

D. none of these

Answer: A

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89. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of f(1) is

A. $1/2$

B. 0

C. 1

D. $-1/2$

Answer: A

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90. $Iff(x) = 1 + \frac{1}{\xi} nt_1^x f(t)dt, thenthevalueof\left(e^{-1}\right)is$ 1 (b) 0 (c) -1

(d) none of these

A. 1

B. 0

C. -1

D. none of these

Answer: B

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91. If $\left[f\left(\frac{\sqrt{3}}{2}\right)\right]$ is [.] denotes the greatest integer function) 4 (b) 5 (c) 6

(d) -7

A. 4

B. 5

C. 6

D. -7

Answer: B

Watch Video Solution

92. $f(x)$ is continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$. Then the value of a is equal to: $-\frac{1}{24}$ (b) $\frac{17}{168}$ (c) $\frac{1}{7}$ (d) $-\frac{167}{840}$

A. $-\frac{1}{24}$

B. $\frac{17}{168}$

C. $\frac{1}{7}$

D. $-\frac{167}{840}$

Answer: D

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93. The value of $\int_{\frac{1}{e}}^{\tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e}}^{\cot x} \frac{dt}{t(1+t^2)}$, where $x \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$, is equal to: 0 (b) 2 (c) 1 (d) none of these

A. 0

B. 2

C. 1

D. none of these

Answer: C

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94. $\lim_{x \to \infty} \frac{x^2 \tan \frac{1}{x}}{\sqrt{8x^2+7x+1}}$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 1

D. π

Answer: A

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95. A function f is defined by $f(x)=\int_0^{\pi}\cos t\cos(x-t)dt, 0\leq x\leq 2\pi$ then which of the following.hold(s) good?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{-\pi}{2}$

D. $\frac{-\pi}{4}$

Answer: C

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96. If f' is a differentiable function satisfying $f(x) = \int_0^x \sqrt{1 - f^2(t)}dt + \frac{1}{2}$ then the value of $f(\pi)$ is equal to

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{2}$

Answer: B

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97. $If \int_0^1 e^x \hat{} \, 2(x - \alpha)dx = 0, then$ `a

A. $1 < \alpha < 2$

B. $\alpha < 0$

C. $0 < \alpha < 1$

D. $\alpha = 0$

Answer: C

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98. The function x^x decreases in the interval $(0, e)$ (b) $(0, 1)$ $\left(0, \frac{1}{e}\right)$ (d) none of these

A. $(0, 1)$

B. $(-1, 0)$

C. $(1, e)$

D. none of these

Answer: C

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99. Given that f satisfies $|f(u) - f(v)| \leq |u - v| f$ or $u and v$ in $[a, b]$.

Then $\left|\int_a^b f(x)dx - b(b - a)f(a)\right| \leq \dfrac{(b - a)}{2}$ (b) $\dfrac{(b - a)^2}{2}$ $(b - a)^2$ (d) none of these

A. $\dfrac{(b - a)}{2}$

B. $\dfrac{(b - a)^2}{2}$

C. $(b - a)^2$

D. none of these

Answer: B

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100. $(\lim)_{x \to \infty} \dfrac{x(\log x)^3}{1 + x + x^2} equal$ 0 (b) -1 (c) 1 (d) none of these

A. 0

B. $\log 7$

C. $5 \log 13$

D. none of these

Answer: A

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101. $\int_0^\infty \left(\frac{\pi}{1+\pi^2x^2} - \frac{1}{1+x^2}\right) \log x dx isequa < o \quad -\frac{\pi}{21} n\pi$ (b) 0 $\frac{\pi}{21} n2$ (d) none of these

A. $-\frac{\pi}{2} In\pi$

B. 0

C. $\frac{\pi}{2} In2$

D. none of these

Answer: A

Watch Video Solution

102. $IfA = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx, then \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{x+1} dxisequa < o$

$\frac{1}{2} + \frac{1}{\pi + 2} - A$ (b) $\frac{1}{\pi + 2} - A$ $1 + \frac{1}{\pi + 2} - A$ (d) $A - \frac{1}{2} - \frac{1}{\pi + 2}$

A. $\frac{1}{2} + \frac{1}{\pi + 2} - A$

B. $\frac{1}{\pi + 2} - A$

C. $1 + \frac{1}{\pi + 2} - A$

D. $A - \frac{1}{2} - \frac{1}{\pi + 2}$

Answer: A

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103. $\int_0^4 \frac{(y^2 - 4y + 5)\sin(y-2)dy}{[2y^2 - 9y + 11]} isequa < o$ 0 (b) 2 (c) -2 (d) none of these

A. 0

B. 2

C. -2

D. none of these

Answer: A

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104. IF $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$ then $\cos\left(\theta - \frac{\pi}{4}\right)$ is equal to

A. $-\cos\theta \int_1^{\tan\theta} f(x \sin\theta) dx$

B. $-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x) dx$

C. $\sin\theta \int_1^{\tan\theta} f(x \cos\theta) dx$

D. $\frac{1}{\tan\theta} \int_{\sin\theta}^{\sin\theta \tan\theta} f(x) dx$

Answer: A

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105. $Let I_1 = \theta\int_0^1 \frac{e^x dx}{1+x} and I_2 \theta\int_0^1 \frac{x^2 dx}{e^x \hat{} 3(2-x^3)} \dot{t}ehn \frac{I_1}{I_2} isequa < o$

$\frac{3}{e}$ (b) $\frac{e}{3}$ (c) $3e$ (d) $\frac{1}{3e}$

A. $3/e$

B. $e/3$

C. $3e$

D. $1/3e$

Answer: C

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106. $Let I_1 = \int_{-2}^2 \frac{x^6+3x^5+7x^4}{x^4+2} dx and$

$I_2 = \int_{-3}^1 \frac{2(x+1)^2 + 11(x+1) + 14}{(x+1)^4 + 2} dt$. Then the value of $I_1 + I_2 is$ 8

(b) $\frac{200}{3}$ (c) $\frac{100}{3}$ (d) $none of these$

A. 8

B. $200/3$

C. $100/3$

D. noe

Answer: C

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107. Let f beintegrable over $[0, a]$ for any real value of a.

If $I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta)\,d\theta$ and $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta)\,d\theta$, then

A. $I_1 = -2I_2$

B. $I_1 = I_2$

C. $2I_1 = I_2$

D. $I_1 = -I_2$

Answer: B

Watch Video Solution

108. The value of $\int_a^b (x-a)^3(b-x)^4 dx is$ $\frac{(b-a)^4}{6^4}$ (b) $\frac{(b-a)^8}{280}$ $\frac{(b-a)^7}{7^3}$ (d) none of these

A. $\frac{(b-a)^4}{6^4}$

B. $\frac{(b-a)^8}{280}$

C. $\frac{(b-a)^7}{7^3}$

D. none of these

Answer: B

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109. $If I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx, (m, n \in I, m, n \geq 0), then$ $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}}dx$ $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ $I(m, n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$ $I(m, n) = \int_0^\infty \frac{x^n}{(1+x)^{m+n}}dx$

A. $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}} dx$

B. $I(m, n) = \int_0^\infty \frac{x^m}{(1+x)^{m+n}} dx$

C. $I(m, n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$

D. $I(m, n) = \int_0^\infty \frac{x^n}{(1+x)^{m+n}} dx$

Answer: C

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110. The value of the definite integral $\int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx is$ 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

A. 0

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: A

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111. $If I_n = \int_0^{\pi} e^x (\sin x)^n dx, then \frac{I_3}{I_1} isequa < o \frac{3}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

A. $3/5$

B. $1/5$

C. 1

D. $2/5$

Answer: A

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112. If $f'(x) = f(x) + \int_0^1 f(x)dx$,given $f(0) = 1$, then the value of $f(\log_e 2)$ is

A. $\frac{1}{3+e}$

B. $\frac{5-e}{3-e}$

C. $\frac{2+e}{e-2}$

D. none of these

Answer: B

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113. Let $f(x)$ be positive, continuous, and differentiable on the interval $(a, b) and (\lim)_{x \rightarrow a^+} f(x) = 1, (\lim)_{x \rightarrow b^-} f(x) = 3^{\frac{1}{4}} \dot{I} f f'(x) \geq f^3(x) + \frac{1}{f(x)}$ then the greatest value of $b-a$ is $\frac{\pi}{48}$ (b) $\frac{\pi}{36}$ $\frac{\pi}{24}$ (d) $\frac{\pi}{12}$

A. $\frac{\pi}{48}$

B. $\frac{\pi}{36}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{12}$

Answer: C

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Exercise (Multiple)

1. $\int_1^2 x^2 dx$ is equal to

A. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

B. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

C. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$

D. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

Answer: B::C

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2. If $L = \lim_{n \to \infty} \frac{n^3\left(e^{1/n} + e^{2/n} + \dots\dots\dots + e\right)}{(n+1)^m\left(1^m + 4^m + \dots. + n^{2m}\right)}$ is non zer finite real,

then

A. $L = 3(e-1)$

B. $L = 2(e - 1)$

C. $m = 1/3$

D. $m = 1/3$

Answer: A::C

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3. Evaluate $\int_0^1 x(1-x)^6 dx$

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4. Let $S_n = \sum_{k=0}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$,for $n = 1, 2, 3, \ldots\ldots.,$ then

A. $S_n < \frac{\pi}{3\sqrt{3}}$

B. $S_n > \frac{\pi}{3\sqrt{3}}$

C. $T_n < \frac{\pi}{3\sqrt{3}}$

D. $T_n > \frac{\pi}{3\sqrt{3}}$

Answer: A::D

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5. $Thevalueof \int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)} dxis \quad \frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$ $\frac{\pi}{4} + 2\log 2 - \frac{\tan^{-1} 1}{3}$ $2\log 2 - \cot^{-1} 3$ (d) $-\frac{\pi}{4} + \log 4 + \cot^{-1} 2$

A. $\frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$

B. $\frac{\pi}{4} + 2\log 2 - \tan^{-1} \frac{1}{3}$

C. $2\log 2 - \cot^{-1} 3$

D. $-\frac{\pi}{4} + \log 4 + \cot^{-1} 2$

Answer: A::C::D

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6. Let $f(x)=\int_1^x \frac{3^t}{1+t^2}dt$, where $x>0$, Then

A. for $0<\alpha<\beta, f(\alpha)<f(\beta)$

B. for $0<\alpha<\beta, f(\alpha)>f(\beta)$

C. $f(x)+\pi/4<\tan^{-1}x\,\forall x\geq 1$

D. $f(x)+\pi/4>\tan^{-1}x\,\forall x\geq 1$

Answer: A::D

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7. about to only mathematics

A. $a+b=\frac{9\pi}{2}$

B. $|a=b|=4\pi$

C. $\frac{a}{b}=15$

D. $\int_a^b \sec^2 x dx=0$

Answer: A::B

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8. $If f(x) = \int_0^x 2|t|dt, then\ \ g(x) = x|x|\ \ g(x)$ is monotonic $g(x)$ is differentiable at $x = 0$ $g'(x)$ is differentiable at $x = 0$

A. $g(x) = x|x|$

B. g(x) is monotonic

C. g(x) is differentiable at x = 0

D. g'(x) is differentiable at x = 0

Answer: A::B::C

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9. $If A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx,\ b_n = \int_0^{\frac{\pi}{2}} \left(\frac{\sin nx}{\sin x}\right)^2 dx f\ \text{or}\ n \in N,$

Then $A_{n+1} = A_n$ (b) $B_{n+1} = B_n$ $A_{n+1} - A_n = B_{n+1}$ (d)

$B_{n+1} - B_n = A_{n+1}$

A. $A_{n+1} = A_n$

B. $B_{n+1} = B_n$

C. $A_{n+1} - A_n = B_{n+1}$

D. $B_{n+1} - B_n = A_{n+1}$

Answer: A::D

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10. $Thevalueof \int_0^{\infty} \frac{dx}{1+x^4} is \quad sameastha \rightarrow f \int_0^{\infty} \frac{x^2+1dx}{1+x} \quad \frac{\pi}{2\sqrt{2}}$ $sameastha \rightarrow f \int_0^{\infty} \frac{x^2+1dx}{1+x^4}$ (d) $\frac{\pi}{\sqrt{2}}$

A. same as that of $\int_0^{\infty} \frac{x^2+1dx}{1+x^4}$

B. $\frac{\pi}{2\sqrt{2}}$

C. same as that of $\int_0^{\infty} \frac{x^2dx}{1+x^4}$

D. $\frac{\pi}{\sqrt{2}}$

Answer: B::C

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11. The value of $\int_0^1 e^{x^2 - x} dx$ is (a) < 1 (b) > 1 (c) $> e^{-\frac{1}{4}}$ (d) `none of these

A. < 1

B. > 1

C. $> e^{-\frac{1}{4}}$

D. $< e^{-\frac{1}{4}}$

Answer: A::C

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12. $If \int_a^b \frac{f(x)}{f(a) + f(a + b - x)} dx = 10, then \quad b = 22, a = 2$ (b) $b = 15, 1 = -5\, b = 10, a = -10$ (d) $b = 10, a = -2$

A. $b = 22, a = 2$

B. $b = 15, a = -5$

C. $b = 10, a = -10$

D. $b = 10, a = -2$

Answer: A::B::C

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13. The values of a for which the integral $\int_0^2 |x-a| dx \geq 1$ is satisfied are $(2, \infty)$ (b) $(-\infty, 0)$ $(0, 2)$ (d) none of these

A. $[2, \infty)$

B. $(-\infty, 0]$

C. $(0, 2)$

D. none of these

Answer: A::B::C

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14. If $f(x)=\int_0^x |t-1|dt$, where $0 \le x \le 2$, then

A. range of $f(x)$ is $[0, 1]$

B. $f(x)$ is differentiable at $x=1$

C. $f(x)=\cos^{-1}x$ has two real roots

D. $f'(1/2)=1/2$

Answer: B

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15. Evaluate $\int_0^1 x(1-x)^3 dx$

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16. $If f(x) = \int_0^x (\cos(\sin t) + \cos(\cos t) dt, then f(x+\pi) is$

$f(x) + f(\pi)$ (b) $f(x) + 2(\pi)$ $f(x) + f\left(\frac{\pi}{2}\right)$ (d) $f(x) + 2f\left(\frac{\pi}{2}\right)$

A. $f(x) + f(\pi)$

B. $f(x) + 2f(\pi)$

C. $f(x) + f\left(\frac{\pi}{2}\right)$

D. $f(x) + 2f\left(\frac{\pi}{2}\right)$

Answer: A::D

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17. If $I_n = \int_0^{\pi/4} \tan^n x dx$, $(n > 1$ is an integer), then

A. $I_7 + I_5 = \frac{1}{6}$

B. $I_{10} + I_8 = \frac{1}{9}$

C. $I_8 - I_{12} = \frac{2}{99}$

D. $I_{12} + 2I_{10} + I_8 = \frac{20}{99}$

Answer: B::C::D

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18. $If I_n = \int_0^1 \frac{dx}{(1+x^2)^n}, where n \in N,$ which of the following statements hold good? $2nI_{n+1} = 2^{-n} + (2n-1)I_n$ $I_2 = \frac{\pi}{8} + \frac{1}{4}$ (c) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

A. $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

B. $I_2 = \frac{\pi}{8} + \frac{1}{4}$

C. $I_2 = \frac{\pi}{8} - \frac{1}{4}$

D. $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

Answer: A::B::D

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19. $Let f:[1,\infty)\overrightarrow{R} and f(x)=\int_1^x \frac{e^t}{t}dt - e^x \dot{T}hen\ f(x)$ is an increasing function $(\lim)_{x\overrightarrow{\infty}} f(x)\overrightarrow{\infty}$ $f'(x)$ has a maxima at $x=e$ $f(x)$ is a decreasing function

A. $f(x)$ is an increasing function

B. $\lim_{x\to\infty} f(x)\to\infty$

C. $f'(x)$ has a maxima at $x=e$

D. $f(x)$ is a decreasing function

Answer: A::B

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20. $If f(x)=\int_a^x [f(x)]^{-1} dx and \int_a^1 [f(x)]^{-1} dx=\sqrt{2}, then\ f(2)=2$
(b) $f'(2)=\frac{1}{2}$ $f'(2)=2$ (d) $\int_0^1 f(x)dx=\sqrt{2}$

A. $f(2)=2$

B. $f'(2)=1/2$

C. $f^{-1}(2) = 2$

D. $\int_0^1 f(x)dx = \sqrt{2}$

Answer: A::B::C

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21. A continuous function $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 e^x f(t)dt$ then $f(1) =$

A. $\frac{1}{2-e}$

B. $-\frac{1}{2-e}$

C. $-\frac{e}{2-e}$

D. $\frac{e}{2-e}$

Answer: A::B

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22. $\int_0^x \left\{\int_0^u f(t)dx\right\} duisequa < o$ $\int_0^x (x-u)f(u)du$ $\int_0^x uf(x-u)du\ \xi nt_0^x f(u)du$ (d) $\xi nt_0^x uf(u-x)du$

A. $\int_0^x (x-u)f(u)du$

B. $\int_0^x uf(x-u)du$

C. $x\int_0^x f(u)du$

D. $x\int_0^x uf(u-x)du$

Answer: A::B

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23. Evaluate $\int_0^1 6^x dx$

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24. Evaluate $\int_0^2 7^x dx$

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25. Evaluate $\int_0^1 x(1-x)dx$

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26. Check whether the given equation is a quadratic equation.

i) $(x+1)^2 = 3(x-1)$

ii) $\left(x^2 - 4x\right) = (-2)(4-x)$.

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27. Let f(x) be a non-constant twice differentiable function defined on (∞, ∞) such that $f(x) = f(1-x)$ and f"(1/4) = 0`. Then

A. $f'(x)$ vanishes at least twice on $[0, 1]$

B. $f'\left(\frac{1}{2}\right) = 0$

C. $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x dx = 0$

D. $\int_0^{1/2} f(t)e^{\sin xt}dt = \int_{t/2}^{1} f(1-t)e^{\sin \pi t}dt$

Answer: A::B::C::D

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Exercise (Comprehension)

1. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The range of $y = f(x)$ is

A. $[0, \infty)$

B. R

C. $(-\infty, 0]$

D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer: D

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2. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The range of $y = f(x)$ is

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3. $y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The value of x for which $f(x)$ is increasing is

A. $(-\infty, 1]$

B. $[-1, \infty)$

C. $[-1, 1]$

D. none of these

Answer: C

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4. Let $f: R \rightarrow R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$

$f(x)$ increases for

A. $x > 1$

B. $x < -2$

C. $x > 2$

D. none of these

Answer: B

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5. Let $f: R \rightarrow R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$

$y = f(x)$ is

A. injective but not surjective

B. surjective but not injective

C. bijective

D. neither injective nor surjective

Answer: B

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6. Let $f(x)$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t)dt$ then $\int_0^1 f(x)dx =$

A. $\frac{1}{4}$

B. $-\frac{1}{12}$

C. $\frac{5}{12}$

D. $\frac{12}{7}$

Answer: C

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7. $f(x)$ satisfies the relation $f(x)-\lambda\int_0^{\pi/2}\sin x\cdot\cos t f(t)dt=\sin x$ If $\lambda>2$ then $f(x)$ decreases in

A. $(0,\pi)$

B. $\left(\frac{\pi}{2},3\pi/2\right)$

C. $(-\pi/2,\pi/2)$

D. none of these

Answer: C

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8. $f(x)$ satisfies the relation $f(x)-\lambda\int_0^{\pi/2}\sin x\cos t f(t)dt=\sin x$

If $f(x)=2$ has the least one real root, then

A. $\lambda\varepsilon[1,4]$

B. $\lambda\varepsilon[-1,2]$

C. $\lambda \varepsilon [0, 1]$

D. $\lambda \varepsilon [1, 3]$

Answer: D

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9. $f(x)$ satisfies the relation $f(x) - \lambda \int_0^{\pi/2} \sin x \cos t f(t) dt = \sin x$

If $\lambda > 2$, then $f(x)$ decreases in which of the following interval?

A. 1

B. $3/2$

C.

D.

Answer: C

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10. Check whether the equation is quadratic equation: $(x-1)(x-5)=(x-1)(x-3)$

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11. Let $f(x)$ and $\phi(x)$ are two continuous function on R satisfying $\phi(x)=\int_a^x f(t)dt, a\neq 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha)+g(x)=0 \forall x \varepsilon R, \alpha>0,$ and $\int_b^{2k} g(t)dt$ is independent of b

If $f(x)$ is an even function, then

A. $\phi(x)$ is also an even function

B. $\phi(x)$ is an odd function

C. if $f(a-x)=-f(x)$, then $\phi(x)$ is an even function

D. if $f(a-x)=-f(x)$ then $\phi(x)$ is an odd function

Answer: D

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12. If $f(x)$ is a function satisfying $f(x+a)+f(x)=0$ for all $x \in R$ and positive constant a such that $\int_{b}^{c+b} f(x)dx$ is independent of b, then find the least positive value of $\cdot$

A. 0

B. 1

C. α

D. 2α

Answer: D

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13. Evaluate $\int_{0}^{2} \left(x^2+x+2\right)dx$

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14. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter with in the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

The value of $\int_0^1 \frac{x^a - 1}{\log x} dx$ is

A. $\log(a-1)$

B. $\log(a+1)$

C. $a\log(a+1)$

D. none of these

Answer: B

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15. Evaluate $\int_2^3 (x^2+1)\,dx$

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16. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

The value of $\frac{dI}{da}$ when $I = \int_0^{\pi/2} \log\left(\frac{1 + a\sin x}{1 - a\sin x}\right)\frac{dx}{\sin x}$ (where $|a| < 1$) is

A. $\frac{\pi}{\sqrt{1 - a^2}}$

B. $-\pi\sqrt{1 - a^2}$

C. $\sqrt{1 - a^2}$

D. $\frac{\sqrt{1 - a^2}}{\pi}$

Answer: A

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17. Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

If $\int_0^{\pi} \frac{dx}{(a - \cos x)} = \frac{\pi}{\sqrt{a^2 - 1}}$, then the value of $\frac{dx}{\left(\sqrt{10} - \cos x\right)}$ is

A. $\frac{\pi}{81}$

B. $\frac{7\pi}{162}$

C. $\frac{7\pi}{81}$

D. none of these

Answer: C

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18. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t) dt$

The range of $f(x)$ is

A. $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

B. $\left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$

C. $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

D. none of these

Answer: B

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19. $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x) f(t) dt$

$f(x)$ is not invertible for

A. $x\varepsilon\left[-\frac{\pi}{2} - \tan^{-1} 2, \frac{\pi}{2} - \tan^{-1} 2\right]$

B. $x\varepsilon\left[\frac{\tan^{-1} 1}{2}, \pi + \tan^{-1}\frac{1}{2}\right]$

C. $x\varepsilon\left[\pi+\cot^{-1}2, 2\pi+\cot^{-1}2\right]$

D. none of these

Answer: D

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20. $f(x)=\sin x+\int_{-\pi/2}^{\pi/2}(\sin x+t\cos x)f(t)dt$

The value of $\int_0^{\pi/2} f(x)dx$ is

A. 1

B. -2

C. -1

D. 2

Answer: C

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21. Let $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$ and $v = \int_0^{\infty} \frac{x^2 dx}{x^4 + 7x^2 + 1}$ then

A. $v > u$

B. $6v = \pi$

C. $3u + 2v = \frac{5\pi}{6}$

D. $u + v = \frac{\pi}{3}$

Answer: B

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22. Let $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$ and $v = \int_0^{\infty} \frac{x^2 dx}{x^4 + 7x^2 + 1}$ then

A. $v > u$

B. $6v = \pi$

C. $3u + 2v = \frac{5\pi}{6}$

D. $u + v = \frac{\pi}{3}$

Answer: B

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23. If $f(x) = \int_0^1 \frac{dt}{1+|x-t|}, then f'\left(\frac{1}{2}\right) is equa < o$ 0 (b) $\frac{1}{2}$ (c) 1 (d) none of these

A. $1/2$

B. 0

C. 1

D. 2

Answer: B

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24. If $f(x) = \int_0^1 \frac{dt}{1+|x-t|}, x \varepsilon R$

Which of the following is not true about $f(x)$?

A. $f(x)$ is decreasing for $x > 1$

B. $f(x)$ is increasing for $x < 1$

C. $f(1) = \log_e 2$

D. $f(1/2) = \log_e(3/2)$

Answer: D

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25. Let f be a differentiable function satisfying
$\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t)dt = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$

The value of $\int_0^{\pi/2} f(x)dx$ lies in the interval

A. $\left(\frac{2}{\pi}, 1\right)$

B. $\left(1, \frac{\pi}{2}\right)$

C. $\left(\frac{3}{2}, \frac{\pi}{2}\right)$

D. $\left(0, \frac{2}{\pi}\right)$

Answer: B

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26. Let f be a differentiable function satisfying $\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t)dt = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$

The value of $\lim_{x \to 0} \frac{\cos x}{f(x)}$ is equal to where [.] denotes greatest integer function

A. 0

B. 1

C. $1/2$

D. 2

Answer: B

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27. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx$ where n is positive integer of zero, then The value of U_n is

A. $\pi/2$

B. π

C. $n\pi/2$

D. $n\pi$

Answer: D

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28. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx,$ where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$. Hence, deduce that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} = \frac{1}{2} n\pi$.

A. $\pi/2$

B. π

C. $n\pi / 2$

D. $n\pi$

Answer: C

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29. Evaluate $\int_0^2 (2x^2 + x + 1)dx$

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30. Evaluate $\int_0^4 (x^2 + 2x + 8)dx$

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31. Let the definite integral be defined by the formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a) + f(b))$. For more accurate result, for

$c\varepsilon(a, b)$, we can use $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$ so that for $c = \frac{a+b}{2}$ we get $\int_a^b f(x)dx = \frac{b-a}{4}(f(a) + f(b) + 2f(c))$.

If $f''(x) < 0 \forall x \varepsilon(a, b)$ and c is a point such that $a < c < b$, and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum then $f'(c)$ is equal to

A. $\frac{f(b) - f(a)}{b - a}$

B. $\frac{2(f(b) - f(a))}{b - a}$

C. $\frac{2f(b) - f(a)}{2b - a}$

D. 0

Answer: B

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Exercise (Matrix)

1. Evaluate $\int_{-1}^{1}[x+[x+[x]]]dx$, where [.] denotes the greatest integer function

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2. Evaluate $\int_{0}^{1.5}x\left[x^2\right]dx$, where [.] denotes the greatest integer function

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3. If the point $(\lambda+1, 1)$, $(2\lambda+1, 3$ and $(2\lambda+2, 2\lambda)$ are collinear then the possible value of λ is

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4. If line $y-x-1+\lambda=0$ is equally inclined to axes and equidistant from the point $(1, \ -2)$ and $(3, 4)$,the λ is

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5. Evaluate $\int_{-1}^{3}(x-[x])dx$,where [.] denotes the greatest integer function.

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6. Find the roots of the equation $100x^2 - 20x + 1$

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Exercise (Numerical)

1. $Ifthevalueof(\lim)_{n\vec{\infty}}\left(n^{-\frac{3}{2}}\right)\sum_{j=1}^{.6n}\sqrt{j}isequa < o\sqrt{N}, then$ the value of $\frac{N}{12}$ is ________

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2. $\left(\lim\right)_{n\overrightarrow{\infty}}\frac{n}{2^n}\int_0^2 x^n dx equals\_\_$

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3. A continuous real function f satisfies $f(2x)=3\left(f(x)\forall x\in R\dot{I}f\int_0^1 f(x)dx=1,\right.$ then find the value of $\int_1^2 f(x)dx$

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4. Consider the polynomial $f(x)=ax^2+bx+\cdot$ If $f(0), f(2)=2,$ then the minimum value of $\int_0^2 |f'(x)dx is\_\_$

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5. $IfI = \int_0^{\frac{3\pi}{5}} (1+x)\sin x + (1-x)\cos x \Big) dx,$ then value of $(\sqrt{2}-1)I$ is____

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6. If $\int_0^{100} f(x)dx = 7,$ then $\sum_{r=1}^{100} \int_0^1 (r-1+x)dx =$ ________.

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7. The value of $\int_0^{\frac{3\pi}{2}} \frac{|\tan^{-1}\tan x| - |\sin^{-1}\sin x|}{|\tan^{-1}\tan x| + |\sin^{-1}\sin x|} dx$ is equal to

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8. $Let f(x) = x^3 = \frac{3x^2}{2} + x + \frac{1}{4}$ Then the value of $\left(\int_{\frac{1}{4}}^{\frac{3}{4}} f(f(x))dx\right)^{-1}$ os___

A. 1

B. 2

C. 3

D. 4

Answer: 4

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9. The value of $\int_0^1 \frac{\tan^{-1} x}{\cot^{-1}(1 - x + x^2)dx}$ is____.

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10. Let $f(x)$ be differentiate function symmetric about $x = 2$, then the value of $\int_0^4 \cos(\pi x) f'(x) dx$ is equal to_______.

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11. Let $f:[0, \infty)\vec{R}$ be a continuous strictly increasing function, such that $f^3(x)=\int_0^x tf^{\cdot 2}(t)dt$ for every $x \geq 0$. Then value of $f(6)$ is______

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12. If f is continuous function and $F(x)=\int_0^x\left((2t+3)\int_t^{\cdot 2} f(u)du\right)dt$, then $\left|\frac{F^2}{f(2)}\right|$ is equal to____

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13. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$ is $\frac{\pi^2}{\sqrt{n}}$ (where $n \in N$), then the value of $\frac{n}{27}$ is

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14. about to only mathematics

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15. Let $g(x)$ be differentiable on R and $\int_{\sin t}^{1} x^2 g(x)dx = (1 - \sin t)$, where $t \in \left(0, \frac{\pi}{2}\right)$. Then the value of $g\left(\frac{1}{\sqrt{2}}\right)$ is____

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16. If $\int_0^\infty x^{2n+1}\dot{e}^{-xdx} = 360$, then the value of n is___

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17. Let $f(x)$ be a derivable function satisfying $f(x) = \int_0^x e^t \sin(x - t)dtandg(x) = f^x - f(x)$ Then the possible integers in the range of $g(x)$ is______

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18. Let $f(x) = \frac{1}{x^2}\int_0^x \left(4t^2 - 2f'(t)\right)dt$ then find $f'(4)$

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19. If the value of the definite integral $\int_0^1 \hat{} \, (2007)C_7 x^{2000} 1 \dot{-} x^{7} dx$ is equal to $\frac{1}{k}, wherek \in N,$ then the value of $\frac{k}{26}$ is____

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20. $IfI_n = \int_0^1 \left(1 - x^5\right)^n dx, then \frac{55}{7}\frac{I_{10}}{I_{11}} isequa < o\_\_$

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21. Evaluate: $5050\frac{\int 01\left(1 - x^{50}\right)^{100} dx}{\int 01\left(1 - x^{50}\right)^{101} dx}$

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22. $Let J = \int_{-5}^{-4} (3 - x^2)\tan(3 - x^2)\, dx and K = \int_{-2}^{-1} (6 - 6x + x^2)$ $\tan(6x - x^2 - 6)\, dx \dot{T}hen (J + K)$ equals ____

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23. The value of the definite integral $\int_{2-1}^{\sqrt{2}+1} \frac{x^4 + x^2 + 2}{(x^2 + 1)^2} dx equals_{-}-$

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24. Consider a real valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t(f(t)) dt$. If M and m are maximum and minimum values of function f, then the value of M/m is__________.

A. 1

B. 2

C. 3

D. 4

Answer: 3

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25. $Iff(x) = x + \int_0^1 t(x+t)f(t)dt, thenthevalueo\frac{f(23)}{2}f(0)$ is equal to ________

A. 4

B. 7

C. 9

D. 12

Answer: 9

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26. Let $y = f(x) = 4x^3 + 2x - 6,$ then the value of $\int_0^2 f(x)dx + \int_0^{30} f^{-1}(y)dy$ is equal to _______.

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27. The value of $\int_1^3 \left(\sqrt{1+(x-1)^3}+(x^2-1)^{\frac{1}{3}}+1\right)dx$ is ________.

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28. The value of $\int_0^1 x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ (x>0) is equal to ________.

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JEE Main Previous Year

1. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to

A. $\frac{\pi}{2}$

B. 1

C. -1

D. $-\frac{\pi}{2}$

Answer: D

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2. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$ for all $x\varepsilon[0, 1], p(0) = 1,$ and $p(1) = 41.$

Then $\int_0^1 p(x)dx$ is equal to

A. 42

B. $\sqrt{41}$

C. 21

D. 41

Answer: C

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3. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

A. $\log 2$

B. $\pi \log 2$

C. $\frac{\pi}{8}\log 2$

D. $\frac{\pi}{2}\log 2$

Answer: B

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4. For $x\varepsilon\left(0, \frac{5\pi}{2}\right)$, definite $f(x) = \int_0^x \sqrt{t}\sin t dt$. Then f has

A. local maximum at π and local minima at 2π

B. local maximum at π and 2π

C. local minimum at π and 2π

D. local minimum at π and local maximum at `2pi

Answer: A

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5. If $g(x) = \int_0^x \cos^4 t dt$, then prove that $g(x + \pi) = g(x) + g(\pi)$.

A. $\frac{g(x)}{g(\pi)}$

B. $g(x) + g(\pi)$

C. $g(x) - g(\pi)$

D. $g(x).\ g(\pi)$

Answer: B

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6. Statement I: The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement II: $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

A. Statement I is true, statement II is true, statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is a not a correct explanation for statement I

C. Statement I is true, statement II is false

D. Statement I is false, statement II is true

Answer: D

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7. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t| dt, x \in R,$ which are parallel to the line $y = 2x$, are equal to (1) ± 2 (2) ± 3 (3) ± 4 (4) ± 1

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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8. The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx$ equals

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - \sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \frac{\pi}{3}$

Answer: D

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9. The integral $\int_{2}(4)\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ is equal to

A. 2

B. 4

C. 1

D. 6

Answer: C

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10. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)(n+3).......3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

A. $\frac{27}{e^20}$

B. $\frac{9}{e^2}$

C. $3\log 3-2$

D. $\frac{18}{e^4}$

Answer: A

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11. Find the roots of the equation $x^2 + \sqrt{3}x + \frac{3}{4}$

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12. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$ is

A. $\pi/4$

B. $\pi/8$

C. $\pi/2$

D. 4π

Answer: A

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1. Solve: $(x+2)^3 = 2x(x^2-1)$

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2. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$ is

A. $\frac{22}{7} - \pi$

B. $\frac{2}{105}$

C. 0

D. $\frac{71}{15} - \frac{3\pi}{2}$

Answer: A

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3. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt$, for all, $x \in (-1, 1) and let f^{-1}$ be the inverse function of f. Then $(f^{-1})'(2)$ is equal to 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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4. Find the roots of the following quadratic equations $x^2 - 2x - 3$

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5. Let $f:[-1,2]\overrightarrow{0,\infty}$ be a continuous function such that $f(x)=f(1-x)f$ or $allx\in[-1,2]$. Let $R_1=\int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by $y=f(x), x=-1, x=2,$ and the $x-a\xi s$. Then $R_1=2R_2$ (b) $R_1=3R_2$ $2R_1$ (d) $3R_1=R_2$

A. $R_1=2R_2$

B. $R_1=3R_2$

C. $2R_1=R_2$

D. $3R_1=R_2$

Answer: C

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6. Let $f:\left[\frac{1}{2},1\right]\overrightarrow{R}$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x)<2f9x\Big)andf\left(\frac{1}{2}\right)=1$. Then the value of $\int_{\frac{1}{2}}^{1} f(x)dx$ lies in the interval $(2e-1, 2e)$ (b) $(3-1, 2e-1)$ $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

A. $(2e-1, 2e)$

B. $(e-1, 2e-1)$

C. $\left(\frac{e-1}{2}, e-1\right)$

D. $\left(0, \frac{e-1}{2}\right)$

Answer: D

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7. Let $f:[0,2]\overrightarrow{R}$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0)=1$ $Let: F(x)=\int_0^{x^2} f(\sqrt{t})\,dt f$ or $x\in[0,2]\dot{I}fF'(x)=f'(x)$. for all $x\in(0,2)$, then $F(2)$ equals e^2-1 (b) e^4-1 $e-1$ (d) e^4

A. e^2-1

B. $e^{4}-1$

C. $e-1$

D. e^{4}

Answer: B

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8. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\cos ecx)^{17}$ dx

A. $\int_{0}^{\log\left(1+\sqrt{2}\right)} 2\left(e^{u}+e^{-u}\right)^{16}du$

B. $\int_{0}^{\log\left(1+\sqrt{2}\right)} 2\left(e^{u}+e^{-u}\right)^{17}du$

C. $\int_{0}^{\log\left(1+\sqrt{2}\right)} 2\left(e^{u}-e^{-u}\right)^{17}du$

D. $\int_{0}^{\log\left(1+\sqrt{2}\right)} 2\left(e^{u}-e^{-u}\right)^{16}du$

Answer: A

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9. Find $A \times B$, $A \times A$ and $B \times A$: $A = \{1, 2, 3\}$ and $B = \{1, -4\}$.

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10. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

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11. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots$ then which one of the following is not true ?

A. $I_n = I_{n+2}$

B. $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{10} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: A::B::C

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12. about to only mathematics

A. $f''(x)$ exists for al $x\varepsilon(0, \infty)$

B. $f'(x)$ exists for all $x\varepsilon(0, \infty)$ and f' is continuous on $(0, \infty)$ but not differentiable on $(0, \infty)$.

C. There exists $\alpha > 1$ suchthat $|f'(x)| < \ |f(x) +$ for all $x\varepsilon(\alpha, \infty)$

D. There exists $\beta > 0$ such that $|f(x)| + \ |f'(x) + \ \leq \beta$ for all $x\varepsilon(0, \infty)$

Answer: B::C

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13. Let S be the area of the region enclosed by $y = e^{-x} \ \hat{} \ 2, y = 0, x = 0, andx = 1.$ Then $S \geq \frac{1}{e}$ (b) $S \geq 1 = \frac{1}{e}$

$S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (d) $S \le \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

A. $S \ge \frac{1}{e}$

B. $S \ge 1-\frac{1}{e}$

C. $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$

D. $S \le \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Answer: A::B::D

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14. For $a \in R$ (the set of all real numbers), $a \ne -1$),

$$(\lim)_{n\overrightarrow{\infty}}\left(\frac{1^a+2^a+\ +n^a}{(n+1)^{a-a}[(na+1)+(na+2)+\&(na+n)]}=\frac{1}{60.}\right.$$

Then $a =$ 5 (b) 7 (c) $\frac{-15}{2}$ (d) $\frac{-17}{2}$

A. 5

B. 7

C. $\frac{-15}{2}$

D. $\frac{-17}{2}$

Answer: B::D

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15. Let f be a continuous function on $[a, b]$. Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dt = \int_x^b f(t)dt$.

A. $g(x)$ is continuous but not differentiable at a

B. $g(x)$ is differentiable on R

C. $g(x)$ is continuous but not differentiable at b

D. $g(x)$ is continuous and differentiable at either a or b but not both

Answer: A::C

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16. Let $f:(0,\infty)\in R$ be given

$$f(x)=\int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{1}{t}dt,\text{ then}$$

A. $f(x)$ is monotonically increasing on $[1,\infty)$

B. $f(x)$ is monotonocally decreasing on $(0,1)$

C. $f(x)+f\left(\frac{1}{x}\right)=0$, for all $x\varepsilon(0,\infty)$

D. $f(2^x)$ is an odd function of x on R

Answer: A::C::D

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17. The option(s) with the values of $aandL$ that satisfy the following equation is (are) $\dfrac{\int 04\pi e^t\left(s\in^6 at+\cos^4 at\right)dt}{\int 0\pi e^t(s\in^6 at+\cos^4 at)dt}=L$

$a=2,\ L=\dfrac{e^{4\pi}-1}{e^{\pi}-1}$ (b) $a=2,\ L=\dfrac{e^{4\pi}+1}{e^{\pi}+1}$ $a=4,\ L=\dfrac{e^{4\pi}-1}{e^{\pi}-1}$ (d) $a=4,\ L=\dfrac{e^{4\pi}+1}{e^{\pi}+1}$

A. $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

B. $a = 2, L = \frac{e^{4\pi + 1}}{e^{\pi} + 1}$

C. $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

D. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

Answer: A::C

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18. Given $A = \{2, 4, 5\}, B = \{2, 5\}, C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

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19. $(\lim)_{x \overrightarrow{\infty}} \left(\frac{n^2}{n^2}\right)^{n(n-1)isequa<o}$ e (b) e^2 (c) e^{-1} (d) 1

A. $f\left(\frac{1}{2}\right) \geq f(1)$

B. $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$

C. $f'(2) \leq 0$

D. $\dfrac{f'(3)}{f(3)} \geq \dfrac{f'(2)}{f'(2)}$

Answer: B::C

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20. Let $f: R\overrightarrow{R}$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(1n5)$ is_____

A. $e^x - \int_0^x f(t)\sin t dt$

B. $x^9 - f(x)$

C. $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t dt$

D. $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t dt$

Answer: B::D

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21. $\int \frac{1}{x(\log x)\log(\log x)}dx =$

A. $I > \log_e 99$

B. $I < \log_e 99$

C. $I < \frac{49}{50}$

D. $I > \frac{49}{50}$

Answer: B::D

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22. Evaluate the following integrals using properties of integration :

$$\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}dt + \int_0^{\cos^2 x} \sqrt{t}dt$$

A. $g'\left(\frac{\pi}{2}\right) = -2\pi$

B. $g'\left(-\frac{\pi}{2}\right) = 2\pi$

C. $g'\left(\frac{\pi}{2}\right) = 2\pi$

D. $g'\left(-\frac{\pi}{2}\right) = -2\pi$

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23. The total number for distinct $x\varepsilon[0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x - 1$ is ________.

A. π

B. 2π

C. $\frac{\pi}{2}$

D. 0

Answer: A

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24. Given that for each $a\varepsilon(0, 1)$, $\lim_{h \to 0^+} \int_h^{1-h} f^{-a}(1-t)^{a-1}dt$ exists. Let this limit be $g(a)$. In addition it is given the function $g(a)$ is

differentiable on $(0, 1)$.

The value of $g'\left(\frac{1}{2}\right)$ is

A. $\frac{\pi}{2}$

B. π

C. $-\frac{\pi}{2}$

D. 0

Answer: D

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25. Let $f: R \to R$ be a twice differentiable function such that $f(x+\pi) = f(x)$ and $f''(x) + f(x) \geq 0$ for all $x \in R$. Show that $f(x) \geq 0$ for all $x \in R$.

A. $f'(1) < 0$

B. $f(2) < 0$

C. $f'(x) \neq 0$ for an $x\varepsilon(1, 3)$

D. $f'(x) = 0$ for some $x\varepsilon(1, 3)$

Answer: A::B::C

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26. Let $F: R \to R$ be a thrice differntiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F'(x) < 0$ for all $x\varepsilon(1/2, 3)$. Let $f(x) = xF(x)$ for all $x\varepsilon R$.

The correct statement(s) is (are)

A. $9f'(3) + f'(1) - 32 = 0$

B. $\int_1^3 f(x)dx = 12$

C. $9f'(3) - f'(1) + 32 = 0$

D. $\int_1^3 f(x)dx = -12$

Answer: C::D

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27. If f:R → R and g:R → R be two functions defined as below, then find (fog)(x) and (gof)(x); f(x)=2x+3,g(x)=x ^2+5

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28. about to only mathematics

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29. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0), x \in R,$ where $f'(x)$ denotes $\frac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0.$ Then the value of $y(2)$ is_____

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30. The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} \left(1 - x^2\right)^5 \right\} dxis$

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31. Let $f: R\overrightarrow{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}.$ Suppose that $F(x) = \int_{-1}^{x} f(t) dtf \text{ or } allx \in [-1, 2] andG(x) = \int_{-1}^{x} t|f(f(t))| dtf \text{ or } a$

Then the value of $f\left(\frac{1}{2}\right)$ is

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32. If $\alpha = \int_0^1 \left(e^9 x + 3\tan^{(-1)x}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right) dxwherm^{-1}$ takes only principal values, then the value of $\left((\log)_e |1 + \alpha| - \frac{3\pi}{4}\right) is$

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34. Let $f: R\overrightarrow{R}$ be a function defined by $f(x) = \{[x], x \leq 20, x > 2$ where $[x]$ is the greatest integer less than or equal to $x\cdot$ If $I = \int_{-1}^{2} \frac{x f(x^2)}{2 + f(x+1)} dx, thenthevalueof f(4I - 1) is$

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35. The total number for distinct $x\varepsilon[0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is ________.

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36. Let $f: R \rightarrow R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$

$y = f(x)$ is

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37. Evaluate: $(\lim)_{n \overrightarrow{\infty}} \left(\frac{(n+1)(n+2)(n+n)^{\frac{1}{n}}}{n} \right.$

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38. Choose the correct answer

The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

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Single Correct Answer Type

1. $\lim_{n \to \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n} \right]$ is equal to

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2. The value of $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{\sin\left\{\frac{(n+r)\pi}{4n}\right\}} \cdot \frac{\pi}{n}$ is equal to

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3. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k^{1/a}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$ is equal to

A. 1

B. 2

C. 43467

D. 4

Answer: A

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4. If $\int_0^3 (3ax^2 + 2bx + c)dx = \int_1^3 (3ax^2 + 2bx + c)dx$ where a, b, c are constants then $a + b + c =$

A. $a + b + c = 3$

B. $a + b + c = 1$

C. $a + b + c = 0$

D. $a + b + c = 2$

Answer: C

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5. The number of solution of the equation $\int_{-2}^1 |\cos x| dx = 0, 0 < x < \frac{\pi}{2}$, is

A. 0

B. 1

C. 2

D. 4

Answer: A

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6. $\int_0^1 e^{2x}e^{e^x}$ dx =)

A. $e^e(2e-1)$

B. $e^e(e-1)$

C. $e^{2e}(e-1)$

D. none of these

Answer: B

Watch Video Solution

7. Let $f(x) = \lim_{n \to \infty} \frac{\cos x}{1 + (\tan^{-1} x)^n}$. Then the value of $\int_o^\infty f(x)dx$ is equal to

A. $\cos(\tan 1)$

B. $\sin(\tan 1)$

C. $\tan(\tan 1)$

D. none of these

Answer: B

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8. The value of difinite integral $\int_0^1 \frac{dx}{\sqrt{(x+1)^3(3x+1)}}$ equals

A. $\sqrt{2} - 1$

B. $\tan. \frac{\pi}{12}$

C. $\tan. \frac{5\pi}{12}$

D. none of these

Answer: A

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9. If f(x) is continuous and $\int_0^9 f(x)dx = 4$, then the value of the integral $\int_0^3 x.\ f(x^2)dx$ is

A. 2

B. 18

C. 16

D. 4

Answer: A

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10. $\lim_{t \to 0} \int_0^{2\pi} \frac{|\sin(x+t) - \sin x|}{|t|} dx$ equals

A. a) 2

B. b) 4

C. c) 1/4

D. d) 1

Answer: B

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11. The value of the integral $\int_0^1 \frac{1}{(1+x^2)^{\frac{3}{2}}} dx$ is

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12. The value of $I = \int_0^{\pi} x\left(\sin^2(\sin x) + \cos^2(\cos x)\right) dx$ is

A. π^2

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{4}$

D. none of these

Answer: B

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13. $\int_0^a \log(\cot a + \tan x)dx$ where $a \in \left(0, \frac{\pi}{2}\right)$ is (A) $a \ln \sin a$ (B) $-a \ln \sin a$ (C) $-a \ln \cos a$ (D) none of these

A. $a \ln(\sin a)$

B. $-a \ln(\sin a)$

C. $-a \ln(\cos a)$

D. none of these

Answer: B

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14. IF $f(x+f(y)) = f(x) + y \forall x, y \in R$ and $f(0) = 1$, then $\int_0^{10} f(10-x)dx$ is equal to

A. 1

B. 10

C. $\int_0^1 f(x)dx$

D. $10\int_0^1 f(x)dx$

Answer: D

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15. $u = \int_0^{\frac{\pi}{2}} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx$ and $v = \int_0^{\frac{\pi}{2}} \cos\left(\frac{\pi}{3}\sin x\right)dx$

A. 2u = v

B. 2u = 3v

C. u = y

D. u = 2v

Answer: A

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16. $\int_0^{100\pi}\left(\sum_{r=1}^{10}\tan rx\right)dx$ is equal to

A. 0

B. 100π

C. -50π

D. 50π

Answer: A

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17. $\int_0^{\pi/2} \sin x \sin 2x \sin 3x \sin 4x dx =$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{16}$

D. $\frac{\pi}{32}$

Answer: C

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18. $\int_1^{2013} [(x-1)(x-2)...(x-2013)]dx$

A. $(2013)^2$

B. $(2012)(2013)(2014)$

C. $2013!$

D. 0

Answer: D

Watch Video Solution

19. $f : [0, 5] \rightarrow R, y = f(x)$ such that $f''(x) = f''(5 - x) \, \forall x \in [0, 5] \, f'(0) = 1$ and $f'(5) = 7$, then the value of $\int_1^4 f'(x) dx$ is

A. 4

B. 6

C. 8

D. 10

Answer: C

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20. $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 dx}{e^{2x} - 1}$ is equal to (i)0 (ii)2 (iii)e (iv)none of these

A. 0

B. 2

C. e

D. 2e

Answer: A

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21. If f and g are two continuous functions being even and and odd, respectively, then $\int_{-a}^{a} \frac{f(x)}{b^{g(x)}+1} dx$ is equal to (a being any non-zero number and b is positive real number, $b \neq 1$)

A. independent of f

B. independent of g

C. independent of both f and g

D. none of these

Answer: B

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22. If $\int_0^{4\pi} \ln\left|13\sin x + 3\sqrt{3}\cos x\right| dx = k\pi \ln 7$, then the value of k is

A. 2

B. 4

C. 8

D. 16

Answer: B

Watch Video Solution

23. f is a real valued function from R to R such that $f(x) + f(-x) = 2$, then $\int_{1-x}^{1+X} f^{-1}(t)dt =$

A. -1

B. 0

C. 1

D. none of these

Answer: B

Watch Video Solution

24. if $\int_{\log 2}^{x} \frac{du}{(e^u - 1)^{\frac{1}{2}}} = \frac{\pi}{6}$ then $e^x =$

A. 1

B. 2

C. $\sqrt{3} + 1$

D. -1

Answer: C

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25. The value of $\int_{e}^{\pi^2} [\log_{\pi} x] d(\log_e x)$ (where [.] denotes greatest integer function) is

A. $2\log_e \pi$

B. $\log_e \pi$

C. 1

D. 0

Answer: B

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26. The value of $\frac{\int_0^1 \frac{dt}{\sqrt{1-t^4}}}{\int_0^1 \frac{1}{\sqrt{1+t^4}} dt}$ is

A. 1

B. 2

C. $2\sqrt{3}$

D. $\sqrt{2}$

Answer: D

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27. Let a and b be two positive real numbers. Then the value of $\int_a^b \frac{e^{x/a} - e^{b/x}}{x} dx$ is

A. 0

B. ab

C. 1/ab

D. e^{ab}

Answer: A

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28. The value of $\int_0^{\infty} \frac{\log x}{a^2 + x^2} dx$ is

A. $\frac{2\pi \log a}{a}$

B. $\frac{\pi \log a}{2a}$

C. $\pi \log a$

D. 0

Answer: B

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29. $\int_{1/3}^{3} \frac{1}{x} \log_e \left(\left| \frac{x + x^2 - 1}{x - x^2 + 1} \right| \right) dx$ is equal to

A. $\frac{8}{3}$

B. $-\frac{8}{3}$

C. 0

D. 3

Answer: C

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30. If I_(n)=int_(0)^(1)(1+x+x^(2)+....+x^(n-1))(1+3x+5x^(2)+....+(2n-3)x^(n-2)+(2n-1)x^(n-1))dx,n in N, then the value of sqrt(I_(9)) is

A. 3

B. 6

C. 9

D. 12

Answer: C

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31. A function f(x) satisfie $f(x) = f\left(\frac{c}{x}\right)$ for some real number c(gt 1) and all positive number 'x'. If $\int_1^{\sqrt{c}} \frac{f(x)}{x} dx = 3$, then $\int_1^{c} \frac{f(x)}{x} dx$ is

A. 4

B. 6

C. 8

D. 9

Answer: B

Watch Video Solution

32. Let $I_1 = \int_0^{\infty} \frac{x^2\sqrt{x}}{(1+x)^6} dx, I_2 = \int_0^{\infty} \frac{x\sqrt{x}}{(1+x)^6} dx$, then

A. $I_1 = 2I_2$

B. $I_2 = 2I_1$

C. $I_1 = I_2$

D. $I_1 = -I_2$

Answer: D

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33. If $\int_0^{x^2(1+x)} f(t)dt = x$, then the value of f(2) is.

A. $1/2$

B. $1/3$

C. $1/4$

D. $1/5$

Answer: D

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34. If $f(x) = \int_0^x \log_{0.5}\left(\frac{2t-8}{t-2}\right)dt$, then the interval in which f(x) is increasing is

A. $(-\infty, 2) \cup (6, \infty)$

B. $(4, 6)$

C. $(-\infty, 2) \cup (4, \infty)$

D. (2,6)

Answer: B

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35. If a, b and c are real numbers, then the value of $\lim_{t \to 0} \ln\left(\frac{1}{t}\int_0^t (1 + a\sin bx)^{c/x} dx\right)$ equals

A. abc

B. $\frac{ab}{c}$

C. $\frac{bc}{a}$

D. $\frac{ca}{b}$

Answer: A

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36. If $f(x) = \int_2^x \frac{dt}{1+t^4}$, then

A. $f(3) < \frac{1}{17}$

B. $f(3) > \frac{1}{17}$

C. $f(3) = \frac{1}{17}$

D. $f(3) > 1$

Answer: A

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37. If $\int_0^x f(x) \sin t dt = \text{ constant}, \ 0 < x < 2\pi \text{ and } f(\pi) = 2,$ then the value of $f(\pi/2)$ is

A. 3

B. 2

C. 4

D. 8

Answer: C

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38. A function f, continuous on the positive real axis, has the property that for all choices of x gt 0 and y gt 0, the integral $\int_{x}^{xy} f(t)dt$ is independent of x (and therefore depends only on y). If f(2) = 2, then $\int_{1}^{e} f(t)dt$ is equal to

A. e

B. 4e

C. 4

D. none of these

Answer: C

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39. The maximum value of the integral $\int_{a-1}^{a+1} \frac{1}{1+x^4} dx$ is attained

A. exactly at two values of a

B. only at one value of a which is positive

C. only a one value of a which is negative

D. only at a = 0

Answer: D

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40. $\lim_{x \to 0} \frac{\int_0^x \frac{t^2}{\sqrt{a+t}} dt}{x - \sin x} = 1(a > 0)$. Then the value of a is

A. $1/2$

B. $1/4$

C. 2

D. 4

Answer: D

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41. Let f(x) be a differentiable non-decreasing function such that $\int_0^x (f(t))^3 dt = \frac{1}{x^2}\left(\int_0^x f(x)dt\right)^3 \forall x \in R - \{0\}$ and $f(1) = 1$. If $\int_0^x f($

is

A. always equal to 1

B. always equal to -2

C. may be 1 or -2

D. not independent of x

Answer: A

Watch Video Solution

42. Let f be continuous and the function g is defined as $g(x) = \int_0^x \left(t^2 \int_0^t f(u)du\right)dt$ where $f(1) = 3$. then the value of $g'(1) + g''(1)$ is

A. 1

B. 2

C. 3

D. 4

Answer: C

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43. Let $I_n = \int_0^{\pi/2} (\sin x + \cos x)^n dx (n \geq 2)$. Then the value of n. $I_n - 2(n-1)I_{n-1}$ is

A. 5

B. 9

C. 2

D. 7

Answer: C

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44. Let $I_n = \int_0^1 x^n \sqrt{1-x^2} dx$. Then $\lim_{n \to \infty} \frac{I_n}{I_{n-2}} =$

A. 2

B. 1

C. -1

D. -2

Answer: B

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45. If $\int_0^{\infty} e^{-ax} dx = \frac{1}{a}$, then $\int_0^{\infty} (x^n) e^{-ax} dx$ is

A. $\frac{(-1)^n n!}{a^{n+1}}$

B. $\frac{(-1)^n (n-1)!}{a^n}$

C. $\frac{n!}{a^{n+1}}$

D. none of these

Answer: C

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46. Let $n \geq 1, n \in Z$. The real number $a \in 0, 1$ that minimizes the integral $\int_0^1 |x^n - a^n| dx$ is

A. $\frac{1}{2}$

B. 2

C. 1

D. $\frac{1}{3}$

Answer: A

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47. Let f be a continuous function satisfying $f'(Inx) = [1$ for $0 < x \leq 1, x$ for $x > 1$ and $f(0) = 0$ then $f(x)$ can be defined as

A. $f(x) = \begin{cases} 1 & \text{if} \quad x \leq 1 \\ 1 - e^x & \text{if} \quad x > 1 \end{cases}$

B. $f(x) = \begin{cases} 1 & \text{if} \quad x \leq 1 \\ e^x - 1 & \text{if} \quad x > 1 \end{cases}$

C. $f(x) = \begin{cases} 1 & \text{if} \quad x < 1 \\ e^x & \text{if} \quad x > 1 \end{cases}$

D. $f(x) = \begin{cases} 1 & \text{if} \quad x \leq 1 \\ e^x - 1 & \text{if} \quad x > 1 \end{cases}$

Answer: D

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48. $\dfrac{626\int_0^{\infty} e^{-x} \sin^{25} x dx}{\int_0^{\infty} e^{-x} \sin^{23} x dx}$ is equal to

A. 300

B. 625

C. 600

D. 1200

Answer: C

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49. If $g(x)$ is the inverse of $f(x)$ and $f(x)$ has domain $x \in [1, 5]$, where $f(1) = 2$ and $f(5) = 10$ then the values of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$ equals

A. 72

B. 56

C. 36

D. 48

Answer: D

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50. If $fx = x + \sin x$, then $\int_{\pi}^{2\pi} f^{-1}(x)dx$ is equal to

A. $\frac{3\pi^2}{2} - 2$

B. $\frac{3\pi^2}{2} + 2$

C. $3\pi^2$

D. none of these

Answer: B

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51. Given a real-valued function f which is monotonic and differentiable. Then $\int_{f(a)}^{f(b)} 2x\left(b - f^{-1}(x)\right)dx =$

A. $\int_a^b \left(f^2(x) - 2f^2(a)\right)dx$

B. $\int_a^b \left(2f^2(x) - f^2(a)\right)dx$

C. $\int_a^b \left(f^2(x) - f^2(a)\right)dx$

D. none of these

Answer: C

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52. Let $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$ then

A. $\frac{1}{3} < I < \frac{1}{\sqrt{8}}$

B. $\frac{1}{4} < I < \frac{1}{3}$

C. $\frac{1}{4} < I < 0$

D. none of these

Answer: A

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53. Consider the function $h(x) = \frac{g^2(x)}{2} + 3x^3 - 5$, where g(x) is a continuous and differentiable function. It is given that h(x) is a monotonically increasing function and g(0) = 4. Then which of the following is not true ?

A. $g^2(1) > 10$

B. $h(5) > 3$

C. $h\left(\frac{5}{2}\right) < 2$

D. $g^{-1} < 22$

Answer: C

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Multiple Correct Answer Type

1. If $A(x+y)=A(x)A(y)$ and $A(0)\neq 0$ and $B(x)=\dfrac{A(x)}{1+(A(x))^2}$,

then

A. $\int_{-2010}^{2010} B(x)dx = \int_0^{2011} B(x)dx$

B. $\int_{-2010}^{2011} B(x)dx = \int_0^{2010} B(x)dx + \int_0^{2011} B(x)dx$

C. $\int_{-2010}^{2011} B(x)dx = 0$

D. $\int_{-2010}^{2010} (2B(-x)-B(x))dx = 2\int_0^{2010} B(x)dx$

Answer: B:D

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2. A function f is defined by $f(x)=\int_0^{\pi} \cos t\cos(x-t)dt, 0\leq x\leq 2\pi$. Which of the following hold(s) good? (A) $f(x)$ is continuous but not differentiable in $(0, 2\pi)$ (B) There exists at least one $c\in(0, 2\pi)$ such that $f'(c)=0$ (C) Maximum value of f is $\frac{\pi}{2}$ (D) Minimum value of f is $-\frac{\pi}{2}$

A. f(x) is continuous but not differentiable in $(0, 2\pi)$.

B. Maximum value of f is $\pi / 2$

C. There exists atleast one $c \in (0, 2\pi)$ such that $f'(c) = 0$

D. Minimum value of f is $-\frac{\pi}{2}$.

Answer: B::C::D

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3. $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ then

A. a) $\int_{+0}^\infty e^{-2x^2} dx = \frac{\sqrt{\pi}}{2\sqrt{2}}$

B. b) $\int_0^\infty x e^{-x^2} dx = \frac{1}{2}$

C. c) $\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$

D. d) $\int_0^\infty x^2 e^{-x^2} dx = \frac{\pi}{4}$

Answer: A::B::C

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4. Let $f(x) = \int_0^x \frac{e^t}{t} dt (x > 0)$,

then $e^{-a}[f(x+1) - f(1+a)] =$

A. a) $\int_0^x = \frac{e^t}{(t+a)} dt$

B. b) $\int_1^x \frac{e^t}{t+a} dt$

C. c) $e^{-a} \int_{1+a}^{x+a} \frac{e^t}{t} dt$

D. d) $\int_0^x \frac{e^{t-a}}{(t+a)} dt$

Answer: B::C

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5. Let $f(x) = \int_2^x f(t^2 - 3t + 4) dt$. Then

A. f(2) = 0

B. $f(-2) = 0$

C. $f'(2) = 0$

D. $f'(2) = 2$

Answer: A::C

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6. If $\int_0^x f(t)dt = e^x - ae^{2x}\int_0^1 f(t)e^{-t}dt$, then

A. $a = \frac{1}{3 - 2e}$

B. $f(x) = e^x - 2e^{2x}$

C. $a = \frac{1}{e}$

D. $f(x) = e^x - e^{-x}$

Answer: A::B

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7. A function $f(x)$ satisfies $f(x) = \sin x + \int_0^x f'(t)\left(2\sin t - \sin^2 t\right)dt$ is

A. $f\left(\frac{\pi}{6}\right) = 1$

B. $g(x) = \int_0^x f(t)$ dt is increasing on $(0, \pi)$

C. $f(0) = 0$

D. f(x) is increasing on $(0, \pi)$

Answer: A::B::C

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Comprehension Type

1. Consider the unction

$$f(x) = \int_0^x \left(5\ln\left(1+t^2\right) - 10t\tan^{-1}t + 16\sin t\right)dt$$

f(x) is

A. negative for all $x \in (0, 1)$

B. increasing for all $x \in (0, 1)$

C. decreasing for all $x \in (0, 1)$

D. non-monotonic function for $x \in (0, 1)$

Answer: B

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2. Consider the unction

$$f(x) = \int_0^x \left(5\ln\left(1+t^2\right) - 10t\tan^{-1}t + 16\sin t\right)dt$$

Which is not true for $\int_0^x f(t)dt$ gt?

A. positive for all $x \in (0, 1)$

B. increasing for all $x \in (0, 1)$

C. non-monotonic for all $x \in (0, 1)$

D. none of these

Answer: C

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3. Let m,n be two positive real numbers and define $f(n) = \int_0^\infty x^{n-1} e^{-x} dx$ and $g(m, n) = \int_0^1 x^{m-1}(1-m)^{n-1} dx.$

It is known that f(n) for n $>$ 0 is finite and g(m, n) = g(n, m) for m, n $>$ 0.

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx =$$

A. g(m,n)

B. $g(m-1, n)$

C. $g(m-1, n-1)$

D. $g(m, n-1)$

Answer: A

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4. Let m,n be two positive real numbers and define

$f(n)=\int_0^{\infty} x^{n-1}e^{-x}dx$ and $g(m,n)=\int_0^1 x^{m-1}(1-m)^{n-1}dx.$

It is known that f(n) for n gt 0 is finite and g(m, n) = g(n, m) for m, n gt 0.

$\int_0^1 x^m\left(\log_e \cdot \frac{1}{x}\right)dx =$

A. $\frac{f(n+1)}{(m+1)^n}$

B. $\frac{f(n)}{(m+1)^{n+1}}$

C. $\frac{f(n+1)}{(m+1)^{n+1}}$

D. $g(m+1),n+1)$

Answer: C

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5. Let m,n be two positive real numbers and define

$f(n)=\int_0^{\infty} x^{n-1}e^{-x}dx$ and $g(m,n)=\int_0^1 x^{m-1}(1-m)^{n-1}dx.$ It is known that $f(n)$ for $n > 0$ is finite and $g(m,n) = g(n,m)$ for $m, n > 0$.

$$\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx =$$

A. g(n, m)

B. $g(m-1, n+1)$

C. $g(m-1, n-1)$

D. $g(m+1, n-1)$

Answer: A

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Subjective Type

1. Prove that $\int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n} \text{for n} \in N$

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