MATHS

BOOKS - CENGAGE

DIFFERENT PRODUCTS OF VECTORS AND THEIR GEOMETRICAL APPLICATIONS

Examples

1. Find the angel between the following pairs of vectors $3\hat{i} + 2\hat{j} - 6\hat{k}, 4\hat{i} - 3\hat{j} + \hat{k}$

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2. If $\vec{a}, \vec{b}, and \vec{c}$ are non-zero vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the geometrical relation between the vectors.

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3. if $\vec{r}.\hat{i} = \vec{r}.\hat{j} = \vec{r}.\hat{k}$ and $\left|\vec{r}\right| = 3$, then find vector $\vec{r}$.

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4. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

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5. if $\vec{a}, \vec{b}$ and $\vec{c}$ are mutally perpendicular vectors of equal magnitudes, then find the angle between vectors and $\vec{a} + \vec{b} + \vec{c}$.

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6. If $\left|\vec{a}\right| + \left|\vec{b}\right| = \left|\vec{c}\right|$ and $\vec{a} + \vec{b} = \vec{c}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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7. If $\left|\vec{a}\right| + \left|\vec{b}\right| = \left|\vec{c}\right|$ and $\vec{a} + \vec{b} = \vec{c}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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8. If θ is the angle between the unit vectors $\vec{a}$ and $\vec{b}$, then prove that $\frac{\sin(\theta)}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

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9. find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

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10. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}$ on vector $2\hat{i} - \hat{j} + 5\hat{k} is \frac{1}{\sqrt{30}}$,

then find the value of x

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11. If $\vec{a} = x\hat{i} + (x - 1)\hat{j} + \hat{k}$ and $\vec{b} = (x + 1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a.

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12. If $\vec{a}.\vec{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$. Then find the unit vector $\vec{a}$.

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13. Prove by vector method that $\cos(A + B)\cos A\cos B - \sin A\sin B$

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14. Projection formula:

Prove that $a = b\cos C + c\cos B$.

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15. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

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16. Using dot product of vectors, prove that a parallelogram, whose diagonals are equal, is a rectangle

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17. If $a + 2b + 3c = 4,$ then find the least value of $a^2 + b^2 + c^2$.

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18. about to only mathematics

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19. vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are of the same length and when taken pair-wise they form equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ then find vector $\vec{c}$.

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20. Vectors $a, bandc$ are of the same length and when taken pair-wise they form equal angles. If $\vec{a} = \hat{i} + \hat{j} and \vec{b} = \hat{j} + \hat{k},$ then find vector $\vec{\cdot}$

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21. A particle acted on by constant forces $4\vec{i} + \vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ is displaced from the point $\vec{i} + 2\vec{j} + 3\vec{k}$ to the point $5\vec{i} + 4\vec{j} + \vec{k}$. Find the total work done by the forces

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22. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes , show that the vector $\vec{c} \cdot \vec{d} = 15$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

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23. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$ find the vector component of $\vec{a}alond\vec{b}$.

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24. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1$ then prove that $\left|\vec{a} - \vec{b}\right| = \sqrt{3}$.

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25. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 0\hat{j} + \hat{k}$ then find vector $\vec{c}$ satisfying the following conditions, (i) that it is coplaner with $\vec{a}$ and $\vec{b}$, (ii) that it is $\perp$ to$\vec{b}$ and (iii) that $\vec{a}.\vec{c} = 7$.

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26. If $\vec{a}, \vec{b}$ and $\vec{c}$ are vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ and $(\vec{a} + \vec{b})$ is perpendicular to vecc,$(\vec{b} + \vec{c})$ is perpendicular to veca and $(\vec{c} + \vec{a})$ is perpendicular to $\vec{b}$ then $|\vec{a} + \vec{b} + \vec{c}| =$ (A) $4\sqrt{3}$ (B) $5\sqrt{2}$ (C) 2 (D) 12

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27. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular , then the third pair is also perpendicular.

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28. In isosceles triangles ABC, $\left|\vec{A}B\right| = \left|\vec{B}C\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}Eand\vec{C}A\left(where\left|\vec{C}A\right| = 12\right)^{\cdot}$

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29. An arc AC of a circle subtends a right angle at then the center O. the point B divides the are in the ratio $1:2$, If $\vec{O}A = a\&\vec{O}B = b$. then the vector $\vec{O}C$ in terms of $\vec{a}\&\vec{b}$, is

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30. Vector $\vec{O}A = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is $\dfrac{4\hat{i} - \hat{j} - \hat{k}\cdot}{\sqrt{2}}$

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31. The base of the pyramid $AOBC$ is an equilateral triangle OBC with each side equal to $4\sqrt{2}$, O is the origin of reference, AO is perpendicular to the plane of OBC and $\left|\vec{AO}\right| = 2$. Then find the cosine of the angle between the skew straight lines, one passing though A and the midpoint of $OBand$ the other passing through O and the mid point of BC.

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32. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

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33. Let the vectors$\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

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34. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$

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35. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

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36. If $A, BandC$ are the vetices of a triangle ABC, then prove sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

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37. Prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

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38. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)^{.}$

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39. If $\vec{a}, \vec{b}$ are any two vectors, then prove that $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$

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40. If $\left|\vec{a}\right| = 2,$ then find the value of $\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^{2^{.}}$

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41. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a} \neq \lambda\vec{b}$ and $\vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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42. If A, B , C ,D are any four points in space, prove that $\left|\overrightarrow{AB}\times\overrightarrow{CD}\times\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right|$ = 4 (area of triangle ABC).

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43. If $\vec{a}, \vec{b} and \vec{c}$ are the position vectors of the vertices $A, B and C$ respectively, of ABC, prove that the perpendicular distance of the vertex A from the base BC of the triangle ABC is $\dfrac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\left|\vec{c}-\vec{b}\right|}$.

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44. Find the area of the triangle with vertices A(1,1,2)B(2,3,5) and C(1,5,5).

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45. Find the area of the parallelogram whsoe adjacent sides are given by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$`

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46. If the area of the parallelogram having diagonals $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is :

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47. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \neq 0, |\vec{a}| = |\vec{c}| = 1, |\vec{b}| = 4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then find the value of λ .

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48. Find the moment about (1,-1,-1) of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ acting at (1,0,-2)

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49. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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50. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are parallel.

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51. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$.

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52. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the vertices of a cycle quadrilateral ABCD, then

$$\frac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{d} + \vec{d} \times \vec{a}\right|}{\left(\vec{b} - \vec{a}\right).\left(\vec{d} - \vec{a}\right)} + \frac{\left|\vec{b} \times \vec{c} + \vec{c} \times \vec{d} + \vec{d} + \vec{d} \times \vec{b}\right|}{\left(\vec{b} - \vec{c}\right).\left(\vec{d} - \vec{c}\right)}$$ = 0 is,

A. True

B. False

C.

D.

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53. The postion vectors of the vertrices fo aquadrilateral with A as origian are $B\left(\vec{b}\right), D\left(\vec{d}\right)$ and $C\left(l\vec{b} + m\vec{d}\right)$. Prove that the area of the quadrilateral is $\frac{1}{2}(l + m)\left|\vec{b} \times \vec{d}\right|$.

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54. Let $\vec{a}$ and $\vec{b}$ be unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. Then find the value of $\left(2\vec{a} + 5\vec{b}\right).\left(3\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

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55. Find the moment about (1,-1,-1) of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ acting at (1,0,-2)

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56. In triangle $ABC, po \in tsD, EandF$ are taken on the sides $BC, CAandAB,$ respectigvely, such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$ Prove that

$$\_\,(DEF) = \frac{n^2 - n + 1}{\left((n+1)^2\right)_{ABC}}$$

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57. Determine whether the three vectors $2\hat{i} + 3\hat{j} + \hat{k}, \hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 3\hat{k}$ are coplanar.

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58. Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$. If the ordered set $\left[\vec{b}\vec{c}\vec{a}\right]$ is left handed, then find the value of x.

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59. If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 1$ then the value of $\frac{\vec{a}.\left(\vec{b} \times \vec{c}\right)}{\left(\vec{c} \times \vec{a}\right).\vec{b}} + \frac{\vec{b}.\left(\vec{c} \times \vec{a}\right)}{\left(\vec{a} \times \vec{b}\right).\vec{c}} + \frac{\vec{c}.\left(\vec{a} \times \vec{b}\right)}{\left(\vec{c} \times \vec{b}\right).\vec{a}}$ is_______

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60. if the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} - \hat{k}$ from three concurrent edges of a parallelpiped, then find the volume of the parallelepied.

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61. The postion vectors of the four angular points of a tetrahedron are $A\left(\hat{j} + 2\hat{k}\right), B\left(3\hat{i} + \hat{k}\right), C\left(4\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $D\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)$ find the volume of the tetrahedron ABCD.

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62. If $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$. Prove that $\vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$

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63. Prove that $\left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$

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64. Show that : $\left[\vec{l}\,\vec{m}\,\vec{n}\right]\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$

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65. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \hat{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then find the value of

$$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

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66. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k} and a\hat{i} + \hat{k}$ becomes minimum.

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67. If $\vec{u}, \vec{v} and \vec{w}$ are three non-cop0lanar vectors, then prove that

$\left(\vec{u} + \vec{v} - \vec{w}\right)\dot{\vec{u}} - \vec{v} \times \left(\vec{v} - \vec{w}\right) = \dot{\vec{u}}\dot{\vec{v}} \times \vec{w}$

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68. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\left|veaa \times \vec{b}\right| = 2$, then find the value of $\left[\vec{a}\vec{b}\vec{a} \times \vec{b}\right]$

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69. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - -7\hat{k}.$

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70. If $\left[\vec{a}\vec{b}\vec{c}\right] = 2$, then find the value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right]$

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71. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vector and $\vec{a} = \alpha\left(\vec{a} \times \vec{b}\right) + \beta\left(\vec{b} \times \vec{c}\right) + \gamma\left(\vec{c} \times \vec{a}\right)$ and $\left[\vec{a}\vec{b}\vec{c}\right] = 1 then \vec{\alpha} + \vec{\beta} + \vec{\gamma} =$ (A) $\left|\vec{a}\right|^2$ (B) - $\left|\vec{a}\right|^2$ (C) 0 (D) none of these

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72. If $\vec{a}, \vec{b} a$ and $\vec{c}$ are non- coplanar vecotrs, then prove that $|\left(\vec{a}.\vec{d}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{b}.\vec{d}\right)\left(\vec{c} \times \vec{a}\right) + \left(\vec{c}.\vec{d}\right)\left(\vec{a} \times \vec{b}\right)$ is independent of $\vec{d}$ where $\vec{d}$ is a unit vector.

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73. Prove that vectors $\vec{u} = \left(al + a_1 l_1\right)\hat{i} + \left(am + a_1 m_1\right)\hat{j} + \left(an + a_1 n_1\right)\hat{k}$ $\vec{v} = \left(bl + b_1 l_1\right)\hat{i} + \left(bm + b_1 m_1\right)\hat{j} + \left(bn + b_1 n_1\right)\hat{k}$ $\vec{w} = \left(bl + b_1 l_1\right)\hat{i} + \left(bm + b_1 m_1\right)\hat{j} + \left(bn + b_1 n_1\right)\hat{k}$

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74. Let $G_1, G_2 and G_3$ be the centroids of the triangular faces $OBC, OCA and OAB$, respectively, of a tetrahedron $OABC$. If V_1 denotes the volumes of the tetrahedron $OABC and V_2$ that of the parallelepiped with $OG_1, OG_2 and OG_3$ as three concurrent edges, then prove that $4V_1 = 9V_1$.

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75. For any vector $\vec{a}$, prove that $\hat{i} \times \left(\vec{a} \times \vec{i}\right) + \hat{j} \times \left(\vec{a} \times \vec{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}.$

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76. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \vec{k} \times \left[\left(\vec{a} - \vec{i}\right) \times \hat{k}\right] = 0$, then find vector $\vec{a}$.

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77. Prove that $\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^{2}$

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78. Show that

$\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)+\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{b}\times\vec{d}\right)=0.$

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79. If $\vec{b}$ and $\vec{c}$ are two non-collinear such that $\vec{a}$ | | $\left(\vec{b}\times\vec{c}\right)$. Then prove that $\left(\vec{a}\times\vec{b}\right)$. $\left(\vec{a}\times\vec{c}\right)$ is equal to $\left|\vec{a}\right|^{2}\left(\vec{b}.\vec{c}\right)$`

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80. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$

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81. Let $\hat{a}, \vec{b}$ and $\vec{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c} is \alpha$ between $\hat{c}$ and $\hat{a} is \beta$ and between $\hat{a}$ and $\hat{b} is \gamma$. If $A(\hat{a}\cos\alpha), B(\hat{b}\cos\beta)$ and $C(\hat{c}\cos\gamma)$, then show that in triangle ABC,

$$\frac{\left|\hat{a} \times \left(\hat{b} \times \hat{c}a\right)\right|}{\sin A} = \frac{\left|\hat{b} \times \left(\hat{c} \times \hat{a}\right)\right|}{\sin B} = \frac{\left|\hat{c} \times \left(\hat{a} \times \hat{b}\right)\right|}{\sin C} = \frac{\prod | \hat{a} \times \left(\hat{\times} \hat{c} |\right)}{\sum \sin\alpha - \cos\beta . \cos\gamma\hat{n}_1}$$

where $\hat{n}_1 = \dfrac{\hat{b} \times \hat{c}}{\left|\hat{b} \times \hat{c}\right|}, \hat{n}_2 = \dfrac{\hat{c} \times \hat{a}}{\left|\hat{c} \times \hat{a}\right|}$ and $\hat{n}_3 = \dfrac{\hat{a} \times \hat{b}}{\left|\hat{a} \times \hat{b}\right|}$

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82. Let $\hat{a}, \vec{b}$ and $\vec{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c} is \alpha$ between $\hat{c}$ and $\hat{a} is \beta$ and between $\hat{a}$ and $\hat{b} is \gamma$. If $A(\hat{a}\cos\alpha), B(\hat{b}\cos\beta)$ and $C(\hat{c}\cos\gamma)$, then show that in triangle ABC,

$$\frac{\left|\hat{a}\times\left(\hat{b}\times\hat{c}a\right)\right|}{\sin A}=\frac{\left|\hat{b}\times\left(\hat{c}\times\hat{a}\right)\right|}{\sin B}=\frac{\left|\hat{c}\times\left(\hat{a}\times\hat{b}\right)\right|}{\sin C}=\frac{\prod \mid \hat{a}\times\left(\hat{\times}\hat{c}\mid\right)}{\sum \sin\alpha - \cos\beta.\cos\gamma\hat{n}_1}$$

where $\hat{n}_1=\dfrac{\hat{b}\times\hat{c}}{\left|\hat{b}\times\hat{c}\right|}, \hat{n}_2=\dfrac{\hat{c}\times\hat{a}}{\left|\hat{c}\times\hat{a}\right|}$ and $\hat{n}_3=\dfrac{\hat{a}\times\hat{b}}{\left|\hat{a}\times\hat{b}\right|}$

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83. If $\vec{b}$ is not perpendicular to $\vec{c}$. Then find the vector $\vec{r}$ satisfying the equation $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ and $\vec{r}.Ve=0$

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84. If $\vec{a}and\vec{b}$ are two given vectors and k is any scalar, then find the vector $\vec{r}$ satisfying $\vec{r}\times\vec{a}+k\vec{r}=\vec{b}$

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85. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a} \neq \lambda \vec{b}$ and $\vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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86. if vector $\vec{x}$ satisfying $\vec{x} \times \vec{a} + \left(\vec{x}. \vec{b}\right)\vec{c} = \vec{d}$ is given by

$$\vec{x} = \lambda\vec{a} + \vec{a} \times \frac{\vec{a} \times \left(\vec{d}x\vec{c}\right)}{\left(\vec{a}. \vec{c}\right)\left|\vec{a}\right|^2}$$

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87. a , b , c are three non coplanar non zero vectors and r is any vector in space, then (a × b)×(r × c)+(b × c)×(r × a)+(c × a)×(r × b) is equal to)`[⃗ [2a b⃗ c]⃗ r b. 2 [⃗ a⃗ b⃗ c]⃗ r c. 3 [⃗ a⃗ b⃗ c]⃗ r d. none of these

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88. If $\vec{a}, \vec{b}, \vec{c}$ are three non - coplanar vector such that

$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is ....

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89. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta} \times \left(\vec{\beta} \times \vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha} \times \left(\vec{\alpha} \times \vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha} \times \vec{\beta}\right)}{\left|\vec{\alpha} \times \vec{\beta}\right|^2}$$

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90. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar non-zero vectors, then prove that

$$\left(\vec{a}.\vec{a}\right)\vec{b} \times \vec{c} + \left(\vec{a}.\vec{b}\right)\vec{c} \times \vec{a} + \left(\vec{a}.\vec{c}\right)\vec{a} \times \vec{b} = \left[\vec{b}\vec{c}\vec{a}\right]\vec{a}$$

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91. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, \hat{i}+\hat{j}+\hat{k}$

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92. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vector and $\vec{p}$, $\vec{q}$ $\vec{r}$ are defind by the relations $\vec{p}=\dfrac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{q}=\dfrac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{r}=\dfrac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, then $\vec{p}.\left(\vec{a}+\vec{b}\right)+\vec{q}.\left(\vec{b}+\vec{c}\right)+\vec{r}.\left(\vec{c}+\vec{a}\right)$ =...........

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93. Prove that $\vec{a}'\times\vec{b}'+\vec{b}'\times\vec{c}'+\vec{c}'\times\vec{a}'=\dfrac{\vec{a}+\vec{b}+\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$

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94. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors and a',b' and c' constitute the reciprocal system of vectors, then prove that

i. $\vec{r} = \left(\vec{r}.\vec{a}'\right)\vec{a} + \left(\vec{r}.\vec{b}'\right)\vec{b} + \left(\vec{r}.\vec{c}'\right)\vec{c}$

ii. $\vec{r} = \left(\vec{r}.\vec{a}\right)\vec{a}' + \left(\vec{r}.\vec{b}\right)\vec{b}' + \left(\vec{r}.\vec{c}\right)\vec{c}'$

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95. Find the angel between the following pairs of vectors $3\hat{i} + 2\hat{j} - 6\hat{k}, 4\hat{i} - 3\hat{j} + \hat{k}\hat{i} - 2\hat{j} + 3\hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}$

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96. If $\vec{a}, \vec{b}, and\vec{c}$ are non-zero vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$, then find the geometrical relation between the vectors.

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97. if $\vec{r}.\hat{i} = \vec{r}.\hat{j} = \vec{r}.\hat{k}$ and $\left|\vec{r}\right| = 3$, then find vector $\vec{r}$.

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98. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

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99. if $\vec{a}, \vec{b}$ and $\vec{c}$ are mutally perpendicular vectors of equal magnitudes, then find the angle between vectors and $\vec{a} + \vec{b} = \vec{c}$.

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100. If $\left|\vec{a}\right| + \left|\vec{b}\right| = \left|\vec{c}\right|$ and $\vec{a} + \vec{b} = \vec{c}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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101. If three unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

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102. If θ is the angle between the unit vectors $\vec{a}$ and $\vec{b}$, then prove that $\frac{\sin(\theta)}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

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103. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

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104. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}$ on vector $2\hat{i} - \hat{j} + 5\hat{k} is \frac{1}{\sqrt{30}}$,

then find the value of x

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105. If $\vec{a} = x\hat{i} + (x - 1)\hat{j} + \hat{k}$ and $\vec{b} = (x + 1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a.

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106. If $\vec{a}.\vec{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$. Then find the unit vector $\vec{a}$.

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107. Prove by vector method that $\cos(A + B)\cos A\cos B - \sin A\sin B$

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108. Projection formula:

Prove that $a = b\cos C + c\cos B$.

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109. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

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110. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle.

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111. If $a + 2b + 3c = 4$, then find the least value of $a^2 + b^2 + c^2$.

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112. about to only mathematics

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113. Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are of the same length and when taken pair-wise they form equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ then find vector $\vec{c}$.

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114. Vectors $a, bandc$ are of the same length and when taken pair-wise they form equal angles. If $\vec{a} = \hat{i} + \hat{j} and \vec{b} = \hat{j} + \hat{k},$ then find vector $\vec{\cdot}$

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115. A particale acted upon by constant forces $3\hat{i} + 2\hat{j} + 2\hat{k}$ and $2\hat{k} - \hat{j} - \hat{k}$ is displaced from the piont (1,3,-1) to the point (4,-1, λ). If the wrok done by the forces is 16 units , find the value of λ

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116. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$

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117. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$ find the vector component of $\vec{a} alond \vec{b}$.

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118. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1$ then prove that $\left|\vec{a} - \vec{b}\right| = \sqrt{3}$.

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119. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 0\hat{j} + \hat{k}$ then find vector $\vec{c}$ satisfying the following conditions, (i) that it is coplaner with $\vec{a}$ and $\vec{b}$, (ii) that it is $\perp$ to $\vec{b}$ and (iii) that $\vec{a}.\vec{c} = 7$.

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120. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 12$ and each one of them being perpendicular to the sum of the other two. Find $|\vec{a} + \vec{b} + \vec{c}|$.

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121. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular , then the third pair is also perpendicular.

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122. In isosceles triangles ABC, $\left|\vec{A}B\right| = \left|\vec{B}C\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}Eand\vec{C}A\left(where\left|\vec{C}A\right| = 12\right)^{.}$

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123. An arc AC of a circle subtends a right angle at then the center O. the point B divides the are in the ratio $1:2$, If $\vec{O}A = a\&\vec{O}B = b$. then the vector $\vec{O}C$ in terms of $\vec{a}\&\vec{b}$, is

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124. vecctor $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new postion is

$$\frac{4\hat{i} - \hat{j} - \hat{k}}{\sqrt{2}}$$

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125. The base of the pyramid $AOBC$ is an equilateral triangle OBC with each side equal to $4\sqrt{2}$, O is the origin of reference, AO is perpendicualar to the plane of OBC and $\left|\vec{A}O\right| = 2$. Then find the cosine of the angle between the skew straight lines, one passing though A and the midpoint of $OBand$ the other passing through O and the mid point of BC.

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126. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

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127. Let the vectors$\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

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128. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$

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129. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

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130. Sine formula:

With usual notation in a ΔABC

Prove that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

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131. Prove that $\sin(A + B) = \sin A\cos B + \cos A\sin B$.

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132. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)$.

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133. If $\vec{a}$ and $\vec{b}$ are two vectors, then prove that

$$\left(\vec{a} \times \vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$$

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134. If $|\vec{a}| = 2$, then find the value of $\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^2$.

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135. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a} \neq \lambda\vec{b}$ and $\vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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136. If A, B , C ,D are any four points in space, prove that $\left|\overrightarrow{AB} \times \overrightarrow{CD} \times \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}\right|$ = 4 (area of triangle ABC).

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137. If $\vec{a}, \vec{b} and \vec{c}$ are the position vectors of the vertices $A, B and C$ respectively, of ABC, prove that the perpendicular distance of the vertex A from the base BC of the triangle ABC is $\dfrac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|.}{\left|\vec{c} - \vec{b}\right|}$

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138. Find the area of the triangle with vertices A(1,1,2)B(2,3,5) and C(1,5,5).

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139. Find the area of the parallelogram whsoe adjacent sides are given by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$`

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140. If the area of the parallelogram having diagonals $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is :

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141. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \neq 0, |\vec{a}| = |\vec{c}| = 1, |\vec{b}| = 4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then find the value of λ .

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142. Find the moment about (1,-1,-1) of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ acting at (1,0,-2)

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143. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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144. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are parallel.

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145. Show by a numerical example and geometrically also that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does not imply $\vec{b} = \vec{c}$.

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146. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the vertices of a cycle quadrilateral ABCD, then $\frac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{d} + \vec{d} \times \vec{a}\right|}{\left(\vec{b} - \vec{a}\right).\left(\vec{d} - \vec{a}\right)} + \frac{\left|\vec{b} \times \vec{c} + \vec{c} \times \vec{d} + \vec{d} + \vec{d} \times \vec{b}\right|}{\left(\vec{b} - \vec{c}\right).\left(\vec{d} - \vec{c}\right)}$ = 0 is,

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147. The postion vectors of the vertices of a quadrilateral with A as origin are $B\left(\vec{b}\right), D\left(\vec{d}\right)$ and $C\left(l\vec{b} + m\vec{d}\right)$. Prove that the area of the quadrilateral is $\frac{1}{2}(l + m)\left|\vec{b} \times \vec{d}\right|$.

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148. Let $\vec{a}$ and $\vec{b}$ be unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$, then the value of $\left(2\vec{a} + 5\vec{b}\right)$.

$\left(3\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right) =$

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149. $\hat{u}$ and $\hat{v}$ are two non-collinear unit vectors such that $\left|\frac{\hat{u} + \hat{v}}{2} + \hat{u} \times \vec{v}\right| = 1$. Prove that $\left|\hat{u} \times \hat{v}\right| = \left|\frac{\hat{u} - \hat{v}}{2}\right|$

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150. In triangle $ABC, po \in tsD, EandF$ are taken on the sides $BC, CAandAB,$ respectigvely, such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$. Prove that $\_ (DEF) = \frac{n^2 - n + 1}{\left((n + 1)^2\right)_{ABC}}$.

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151. Let A,B,C be points with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j} + 2\hat{k}$ respectively. Find the shortest distance between point B and plane OAC.

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152. Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$. If the ordered set $\left[\vec{b}\vec{c}\vec{a}\right]$ is left handed, then find the value of x.

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153. If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 1$ then the value of $\frac{\vec{a}.\left(\vec{b} \times \vec{c}\right)}{\left(\vec{c} \times \vec{a}\right).\vec{b}} + \frac{\vec{b}.\left(\vec{c} \times \vec{a}\right)}{\left(\vec{a} \times \vec{b}\right).\vec{c}} + \frac{\vec{c}.\left(\vec{a} \times \vec{b}\right)}{\left(\vec{c} \times \vec{b}\right).\vec{a}}$ is_______

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154. if the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} - \hat{k}$ from three concurrent edges of a parallelpiped, then find the volume of the parallelepied.

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155. The postion vectors of the four angular points of a tetrahedron are $A\left(\hat{j} + 2\hat{k}\right), B\left(3\hat{i} + \hat{k}\right), C\left(4\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $D\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)$ find the volume of the tetrahedron ABCD.

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156. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors and $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then find the value of $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

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157. Prove that $\left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$

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158. Prove that $\left[\vec{l}\vec{m}\vec{n}\right]\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$

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159. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \hat{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then find the value of

$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

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160. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k}anda\hat{i} + \hat{k}$ becomes minimum.

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161. If $\vec{u}, \vec{v} and \vec{w}$ are three non-cop0lanar vectors, then prove that

$\left(\vec{u}+\vec{v}-\vec{w}\right)\dot{\vec{u}}-\vec{v}\times\left(\vec{v}-\vec{w}\right)=\dot{\vec{u}}\dot{\vec{v}}\times\vec{w}$

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162. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\left|\vec{a}\times\vec{b}\right|=2$, then find the value of $\left[\vec{a}\vec{b}\vec{a}\times\vec{b}\right]$

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163. Find the altitude of a parallelepiped whose three coterminous edges are vectors $\vec{A}=\hat{i}+\hat{j}+\hat{k}, \vec{B}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{C}=\hat{i}+\hat{j}+3\hat{k} \text{with} \vec{A}$ and $\vec{B}$ as the sides of the base of the parallelepiped .

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164. If $\left[\vec{a}\vec{b}\vec{c}\right]=2$, then find the value of $\left[\left(\vec{a}+2\vec{b}-\vec{c}\right)\left(\vec{a}-\vec{b}\right)\left(\vec{a}-\vec{b}-\vec{c}\right)\right]$

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165. If $\vec{a}, \vec{b}$ and $\vec{c}$ are , mutually perpendicular vcetors and $\vec{a}=\alpha\left(\vec{a}\times\vec{b}\right)+\beta\left(\vec{b}\times\vec{c}\right)+\gamma\left(\vec{c}\times\vec{a}\right)$ and $\left[\vec{a}\vec{b}\vec{c}\right]=1$, then find the value of $\alpha+\beta+\gamma$

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166. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar vecotrs, then prove that $|\left(\vec{a}.\vec{d}\right)\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}.\vec{d}\right)\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}.\vec{d}\right)\left(\vec{a}\times\vec{b}\right)$ is independent of $\vec{d}$ where $\vec{d}$ is a unit vector.

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167. Prove that vectors

$$\vec{u} = (al + a_1 l_1)\hat{i} + (am + a_1 m_1)\hat{j} + (an + a_1 n_1)\hat{k}$$
$$\vec{v} = (bl + b_1 l_1)\hat{i} + (bm + b_1 m_1)\hat{j} + (bn + b_1 n_1)\hat{k}$$
$$\vec{w} = (cl + c_1 l_1)\hat{i} + (cm + c_1 m_1)\hat{j} + (cn + c_1 n_1)\hat{k}$$

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168. Let $G_1, G_2 and G_3$ be the centroids of the triangular faces $OBC, OCA and OAB$, respectively, of a tetrahedron $OABC$. If V_1 denotes the volumes of the tetrahedron $OABC and V_2$ that of the parallelepiped with $OG_1, OG_2 and OG_3$ as three concurrent edges, then prove that $4V_1 = 9V_1$.

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169. For any vector $\vec{a}$ prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$

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170. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] - \vec{j} \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \vec{k} \times \left[\left(\vec{a} - \vec{i}\right) \times \hat{k}\right] = 0$, then find vector $\vec{a}$.

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171. Prove that $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = \left[\vec{a}, \vec{b}, \vec{c}\right]^2$

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172. For any four vectors prove that

$$\left(\vec{b} \times \vec{c}\right). \left(\vec{a} \times \vec{d}\right) + \left(\vec{c} \times \vec{a}\right). \left(\vec{b} \times \vec{d}\right) + \left(\vec{a} \times \vec{b}\right). \left(\vec{c} \times \vec{d}\right) = 0$$

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173. If $\vec{b}$ and $\vec{c}$ are two non-collinear such that $\vec{a} \mid \mid \left(\vec{b} \times \vec{c}\right)$. Then prove that $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$`

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174. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$

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175. Let $\hat{a}, \vec{b}$ and $\vec{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c} is \alpha between \hat{c}$ and $\hat{a} is \beta$ and $between \hat{a}$ and $\hat{b} is \gamma$. If $A(\hat{a}\cos\alpha, 0), B\left(\hat{b}\cos\beta, 0\right)$ and $C(\hat{c}\cos\gamma, 0)$, then show that in triangle ABC, $\dfrac{\left|\hat{a} \times \left(\hat{b} \times \hat{c}a\right)\right|}{\sin A} = \dfrac{\left|\hat{b} \times (\hat{c} \times \hat{a})\right|}{\sin B} = \dfrac{\left|\hat{c} \times \left(\hat{a} \times \hat{b}\right)\right|}{\sin C}$

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176. Let $\hat{a}, \vec{b}$ and $\vec{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c} is \alpha between \hat{c}$ and $\hat{a} is \beta$ and between $\hat{a}$ and $\hat{b} is \gamma$. If $A(\hat{a}\cos\alpha), B(\hat{b}\cos\beta)$ and $C(\hat{c}\cos\gamma)$, then show that in triangle ABC,

$$\frac{\left|\hat{a}\times\left(\hat{b}\times\hat{c}a\right)\right|}{\sin A}=\frac{\left|\hat{b}\times\left(\hat{c}\times\hat{a}\right)\right|}{\sin B}=\frac{\left|\hat{c}\times\left(\hat{a}\times\hat{b}\right)\right|}{\sin C}=\frac{\Pi\ |\ \hat{a}\times\left(\hat{\times}\ \hat{c}\ |\ \right)}{\sum\sin\alpha-\cos\beta.\cos\gamma\hat{n}_1}$$

where $\hat{n}_1=\dfrac{\hat{b}\times\hat{c}}{\left|\hat{b}\times\hat{c}\right|}, \hat{n}_2=\dfrac{\hat{c}\times\hat{a}}{\left|\hat{c}\times\hat{a}\right|}$ and $\hat{n}_3=\dfrac{\hat{a}\times\hat{b}}{\left|\hat{a}\times\hat{b}\right|}$

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177. If $\vec{b}$ is not perpendicular to $\vec{c}$. Then find the vector $\vec{r}$ satisfying the equation $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ and $\vec{r}.\vec{c}=0$

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178. If $\vec{a} and \vec{b}$ are two given vectors and k is any scalar, then find the vector $\vec{r}$ satisfying $\vec{r}\times\vec{a}+k\vec{r}=\vec{b}^{\cdot}$

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179. If $\vec{a}, \vec{b}$ are any two vectors, then prove that $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$

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180. if vector $\vec{x}$ satisfying $\vec{x} \times \vec{a} + \left(\vec{x}.\vec{b}\right)\vec{c} = \vec{d}$ is given by

$$\vec{x} = \lambda\vec{a} + \vec{a} \times \frac{\vec{a} \times \left(\vec{d} x \vec{c}\right)}{\left(\vec{a}.\vec{c}\right)\left|\vec{a}\right|^2}$$

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181. $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors and $\vec{r}$. Is any arbitrary vector. Prove that $\left[\vec{b}\vec{c}\vec{r}\right]\vec{a} + \left[\vec{c}\vec{a}\vec{r}\right]\vec{b} + \left[\vec{a}\vec{b}\vec{r}\right]\vec{c} = \left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$.

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182. If $\vec{a}, \vec{b}, \vec{c}$ are three non - coplanar vector such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is ....

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183. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta} \times \left(\vec{\beta} \times \vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha} \times \left(\vec{\alpha} \times \vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha} \times \vec{\beta}\right)}{\left|\vec{\alpha} \times \vec{\beta}\right|^2}$$

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184. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar non-zero vectors, then prove that $(\vec{a}.\vec{a})\vec{b} \times \vec{c} + \left(\vec{a}.\vec{b}\right)\vec{c} \times \vec{a} + (\vec{a}.\vec{c})\vec{a} \times \vec{b} = \left[\vec{b}\vec{c}\vec{a}\right]\vec{a}$

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185. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, \hat{i}+\hat{j}+\hat{k}$

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186. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vector and $\vec{p}$, $\vec{q}$ $\vec{r}$ are defind by the relations $\vec{p}=\dfrac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{q}=\dfrac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{r}=\dfrac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, then $\vec{p}.\left(\vec{a}+\vec{b}\right)+\vec{q}.\left(\vec{b}+\vec{c}\right)+\vec{r}.\left(\vec{c}+\vec{a}\right)$ =............

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187. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, then prove that $\vec{a}'\times\vec{b}'+\vec{b}'\times\vec{c}'+\vec{c}'\times\vec{a}'=\dfrac{\vec{a}+\vec{b}+\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$

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188. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors and a',b' and c' constitute the reciprocal system of vectors, then prove that

i. $\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')\vec{b} + (\vec{r}.\vec{c}')\vec{c}$

ii. $\vec{r} = (\vec{r}.\vec{a})\vec{a}' + (\vec{r}.\vec{b})\vec{b}' + (\vec{r}.\vec{c})\vec{c}'$

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Exercise

1. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

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2. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.

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3. If the vertices A,B, C of a triangle ABC are (1,2,3),(-1, 0,0), (0, 1,2), respectively, then find $\angle ABC$.

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4. If $|a| = 3, |b| = 4 and$ the angle between $a and b$ is 120^0 , then find the value of $|4a + 3b|$

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5. If vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then find the locus of th point (x,y).

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6. Let $\vec{a}\vec{b}$ and $\vec{c}$ be pairwise mutually perpendicular vectors, such that $|\vec{a}| = 1, \left|\vec{b}\right| = 2, |\vec{c}| = 2$, the find the length of $\vec{a} + \vec{b} + \vec{c}$.

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7. If $\vec{a}+\vec{b}+\vec{c}=0, \left|\vec{a}\right|=3, \left|\vec{b}\right|=5, \left|\vec{c}\right|=7$ then the angle between $\vec{a}$ and $\vec{b}$ is :

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8. If the angle between unit vectors $\vec{a}$ and $\vec{b} is 60^{\circ}$. Then find the value of $\left|\vec{a}-\vec{b}\right|$.

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9. Let $\vec{u}=hai+\hat{j}, \vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ isa unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0, \left|\vec{w}.\hat{n}\right|$ is equal to (A) 0 (B) 1 (C) 2 (D) 3

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10. A, B, C, D are any four points, prove that $\vec{A}B\vec{C}D + \vec{B}C\vec{A}D + \vec{C}A\vec{B}D = 0.$

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11. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(, -2, -1),$ then find the projection length of $\vec{P}Qon\vec{R}S$

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12. If the vectors $3\vec{p} + \vec{q}; 5p - 3\vec{q} and 2\vec{p} + \vec{q}; 3\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, then find the angle between vectors $\vec{p} and \vec{q}$

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13. Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisets the internal angle between $\vec{A}$ and $\vec{B}$ then find the value of α.

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14. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + 2\vec{b} + \vec{c} = \vec{0}$, and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

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15. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the greatest value of $|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$.

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16. Constant forces $P_1 = \hat{i} + \hat{j} + \hat{k}, P_2 = \hat{i} + 2\hat{j} - \hat{k} and P_3 = \hat{j} - \hat{k}$ act on a particle at a point $\dot{A}$ Determine the work done when particle is displaced from position $A(4\hat{i} - 3\hat{j} - 2\hat{k}) \rightarrow B(6\hat{i} + \hat{j} - 3\hat{k})^{\cdot}$

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17. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$.

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18. If A, B, C, D are four distinct point in space such that AB is not perpendicular to CD and satisfies $\vec{A}B\vec{C}D=k\left(\left|\vec{A}D\right|^2+\left|\vec{B}C\right|^2-\left|\vec{A}C\right|^2=\left|\vec{B}D\right|^2\right)$, then find the value of k

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19. If $\vec{a}=2\hat{i}+3\hat{j}-5\hat{k}$, $\vec{b}=m\hat{i}+n\hat{j}+12\hat{k}$ and $\vec{a}\times\vec{b}=\vec{0}$ then find (m,n)

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20. If $\left|\vec{a}\right|=2$, $\left|\vec{b}\right|=5$ and $\left|\vec{a}\times\vec{b}\right|=8$ then find the value of $\vec{a}.\vec{b}$

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21. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}\neq 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a}+\vec{c}=kb\vec{b}$.

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22. If $\vec{a}=2\vec{j}+3\vec{j}-\vec{k}, \vec{b}=-\vec{i}+2\vec{j}-4\vec{k}$ and $\vec{c}=\vec{i}+\vec{j}+\vec{k}$, then find the value of $\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{c}\right)$

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23. if the vectors $\vec{c}, \vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{b}=\hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ from a right -handed system, then find $\vec{c}$.

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24. given that $\vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a}$ is not a zero vector. Show that $\vec{b} = \vec{c}$.

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25. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$

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26. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\left|\vec{z}\right| = \frac{2}{\sqrt{7}}$ such that $\vec{z} + \vec{z} \times \vec{x} = \vec{y}$ then find the angle θ between $\vec{x}$ and $\vec{z}$

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27. prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a} \times \hat{i}\right) + \left(\vec{a}.\hat{j}\right)\left(\vec{a} \times \hat{j}\right) + \left(\vec{a}.\hat{k}\right)\left(\vec{a} \times \hat{k}\right) = \vec{0}$

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28. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

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29. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)^{\cdot}$ Find the velocity of the particle at point $P(3, 6, 4)^{\cdot}$

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30. If $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$. Prove that $\vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$

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31. if $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right)^2 = 144$ and $\left|\vec{a}\right| = 4$ the find the value of $\left|\vec{b}\right|$

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32. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3} \text{if} \vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$ then find the value of $\vec{c}. Vecb$.

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33. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is acting on point (1, -1, 2).

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34. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-coplanar unit vectors such that $\vec{d}$ makes equal angles with all the three vectors $\vec{a}, \vec{b}, \vec{c}$ then prove that $\left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$

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35. If vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, show that $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$

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36. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)$.

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37. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$

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38. If $\vec{x}.\vec{a} = 0$, $\vec{x}.\vec{b} = 0$, $\vec{x}.\vec{c} = 0$ and $\vec{x} \neq \vec{0}$ then show yhat $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar .

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39. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$

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40. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$ then prove that $|\vec{a}| = |\vec{b}| = |\vec{c}|$

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41. If $\vec{a} = \vec{P} + \vec{q}, \vec{P} \times \vec{b} = \vec{0}$ and $\vec{q}.\vec{b} = 0$ then prove that $\dfrac{\vec{b} \times (\vec{a} \times \vec{b})}{\vec{b}.\vec{b}} = \vec{q}$

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42. prove that $\left(\vec{a}.\left(\vec{b}\times\hat{i}\right)\right)\hat{i}+\left(\vec{a}.\left(\vec{b}\times\hat{j}\right)\right)\hat{j}+\left(\vec{a}.\left(\vec{b}\times\hat{k}\right)\right)\hat{k}=\vec{a}\times\vec{b}$

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43. for any four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that $\vec{d}.\left(\vec{a}\times\left(\vec{b}\times\left(\vec{c}\times\vec{d}\right)\right)\right)=\left(\vec{b}.\vec{d}\right)\left[\vec{a}\,\vec{c}\,\vec{d}\right]$

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44. If $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors such that $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\frac{1}{2}\vec{b}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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45. If $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ for non coplanar $\vec{a}, \vec{b}, \vec{c}$ then......

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46. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the non zero vectors such that $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. if theta is the acute angle between the vectors $\vec{b}$ and $\vec{a}$ then theta equals (A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $2\frac{\sqrt{2}}{3}$

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47. If $\vec{p}, \vec{q}, \vec{r}$ denote vector $\vec{b}\times\vec{c}, \vec{c}\times\vec{a}, \vec{a}\times\vec{b}$, respectively, show that $\vec{a}$ is parallel to $\vec{q}\times\vec{r}, \vec{b}$ is parallel $\vec{r}\times\vec{p}, \vec{c}$ is parallel to $\vec{p}\times\vec{q}$.

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48. Let $\vec{a}, \vec{b}, \vec{c}$ be non -coplanar vectors and let equations $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c}$ then prove that $\vec{a}\times\vec{a}'+\vec{b}\times\vec{b}'+\vec{c}\times\vec{c}'$ is a null vector.

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49. Given unit vectors $\hat{m}\hat{n}$ and $\hat{p}$ such that angle between $\hat{m}$ and $\hat{n} is \alpha$ and angle between $\hat{p}$ and

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50. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angleθ . If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, find scalars $p, qandr$ in terms of θ.

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51. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both vectors, $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is \pi/6 then \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

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52. If $\left|(a-x)^2(a-y)^2(a-z)^2(b-x)^2(b-y)^2(b-z)^2(c-x)^2(c-y)^2(c-a)^2\right| = 0$ and vectors $\vec{A}, \vec{B}, and \vec{C}, where \vec{A} = a^2\hat{i} + a\hat{j} + \hat{k}$, etc, are non-coplanar, then prove that vectors $\vec{X}, \vec{Y} and \vec{Z}, where \vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$, etc. may be coplanar.

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53. OABC is a tetrahedron where O is the origin and A,B,C have position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively prove that circumcentre of tetrahedron OABC is $\dfrac{a^2\left(\vec{b}\times\vec{c}\right)+b^2\left(\vec{c}\times\vec{a}\right)+c^2\left(\vec{a}\times\vec{b}\right)}{2\left[\vec{a}\vec{b}\vec{c}\right]}$

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54. Prove that the smaller angle between any two diagonals of a cube is $\cos^{-1}\frac{1}{3}$.

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55. In ABC, a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP$, ising vedctor method, find the are of ABC if the area of BRC is 1 unit

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56. Let O be an interior point of ΔABC such that $\overline{OA} + 2\overline{OB} + 3\overline{OC} = 0$. Then the ratio of a ΔABC to area of ΔAOC is

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57. The lengths of two opposite edges of a tetrahedron of $a and b;$ the shortest distane between these edgesis $d,$ and the angel between them if $\dot{\theta}$ Prove using vector4s that the volume of the tetrahedron is $\frac{abdisn\theta}{6}$.

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58. Find the volume of a parallelopiped having three coterminus vectors of equal magnitude $|a|$ and equal inclination θ with each other.

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59. Find the derivative of $y = 4\tan^{-1}3x^4$.

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60. Given that $\vec{A}, \vec{B}, \vec{C}$ form triangle such that $\vec{A} = \vec{B} + \vec{C}$. Find a,b,c,d such that area of the triangle is $5\sqrt{6}$ where

$\vec{A} = a\vec{i} + b\vec{i} + c\vec{k}. \vec{B} = d\vec{i} + 3\vec{j} + 3\vec{k}$ and $\vec{C} = 3\vec{i} + \vec{j} - 2\vec{k}$.

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61. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$. Determine the distance of point A$(\vec{a})$ from the line l in from $\left|\vec{b} - \vec{a} + \frac{(\vec{a} - \vec{b})\vec{c}}{|\vec{c}|^2}\vec{c}\right|$ or $\frac{\left|(\vec{b} - \vec{a}) \times \vec{c}\right|}{|\vec{c}|}$

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62. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i \cdot \vec{E}_j = 1$, if $i = j and \vec{e}_i \cdot \vec{E}_j = 0 and$ if $i \neq j$, then prove that $[\vec{e}_1 \vec{e}_2 \vec{e}_3][\vec{E}_1 \vec{E}_2 \vec{E}_3] = 1$.

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63. In a quadrilateral ABCD, it is given that $AB \mid \mid CD$ and the diagonals AC and BD are perpendicular to each other. Show that $AD. BC \geq AB. CD$.

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64. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge $\dot{AC}$ Prove that DE is equal to half the edge of tetrahedron.

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65. If A$(\vec{a})$. $B(\vec{b})$ and $C(\vec{c})$ are three non-collinear point and origin does not lie in the plane of the points A, B and C, then for any point $P(\vec{P})$ in the plane of the $\triangle ABC$ such that vector $\overrightarrow{OP}$ is $\perp$ to plane of $trianglABC$, show that $\overrightarrow{OP} = \dfrac{\left[\vec{a}\vec{b}\vec{c}\right]\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)}{4\Delta^2}$

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66. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors and any arbitrary vector $\vec{r}$ in space, where

$$\Delta_1 = \begin{vmatrix} \vec{r}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{r}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \Delta_2 = \left|\left(\vec{a}.\vec{a}, \vec{r}.\vec{a}, \vec{c}.\vec{a}\right), \left(\vec{a}.\vec{b}, \vec{r}.\vec{b}, \vec{c}.\vec{b}\right), \left(\vec{a}.\vec{c}, \vec{r}., \vec{c}\right.\right.$$

$$\Delta_3 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{r}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{r}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{r}.\vec{c} \end{vmatrix}' \Delta = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \text{then prove that } \vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}$$

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67. Two vectors in space are equal only if they have equal component in a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

A. a given direction

B. two given directions

C. three given direction

D. in any arbitrary direaction

Answer: c

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68. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{3}{4}$

Answer: d

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69. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of equal magnitude such that the angle between each pair is $\frac{\pi}{3}$. If $\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{6}$, then $\left|\vec{a}\right| =$

A. 2

B. -1

C. 1

D. $\sqrt{6}/3$

Answer: c

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70. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is (A) $\vec{a}+\vec{b}+\vec{c}$ (B) $\frac{\vec{a}}{\left|\vec{a}\right|}+\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{c}}{\left|\vec{c}\right|}$ (C) $\frac{\vec{a}}{\left|\vec{a}\right|^2}+\frac{\vec{b}}{\left|\vec{b}\right|^2}+\frac{\vec{c}}{\left|\vec{c}\right|^2}$ (D) $\left|\vec{a}\right|\vec{a}-\left|\vec{b}\right|\vec{b}+\left|\vec{c}\right|\vec{c}$

A. $\vec{a}+\vec{b}+\vec{c}$

B. $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$

C. $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$

D. $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

Answer: b

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71. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. Then the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is

A. $\hat{i} - \hat{j} + \hat{k}$

B. $3\hat{i} - \hat{j} + \hat{k}$

C. $3\hat{i} + \hat{j} - \hat{k}$

D. $\hat{i} - \hat{j} - \hat{k}$

Answer: c

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72. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\vec{a}.\vec{b} < 0$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between angles between the vectors $\vec{a}$ and $\vec{b}$ is

A. π

B. $7\pi/4$

C. $\pi/4$

D. $3\pi/4$

Answer: d

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73. If $\hat{a}, \hat{b}, and\hat{c}$ are three unit vectors, such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and $\theta_1, \theta_2 andth\eta_3$ are angles between the vectors $\hat{a}, \hat{b}; \hat{b}, \hat{c}and\hat{c}, \hat{a}$

respectively, then among $\theta_1, \theta_2, andth\eta_3$ a. all are acute angles b. all are right angles c. at least one is obtuse angle d. none of these

A. all are acute angles

B. all are right angles

C. at least one is obtuse angle

D. none of these

Answer: c

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74. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c} is \pi/3$ then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is

A. $1/2$

B. 1

C. 2

D. none of these

Answer: b

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75. about to only mathematics

A. a plane containing the origian O and parallel to two non-collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

B. the surface of a sphere described on PQ as its diameter

C. a line passing through points P and Q

D. a set of lines parallel to line PQ

Answer: c

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76. Two adjacent sides of a parallelogram ABCD are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Then the value of $\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$ is

A. $20\sqrt{5}$

B. $22\sqrt{5}$

C. $24\sqrt{5}$

D. $26\sqrt{5}$

Answer: b

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77. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ. The maximum value of θ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{5}$

Answer: c

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78. Let the pair of vector $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ each determine a plane. Then the planes are parallel if

A. $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$

B. $(\vec{a} \times \vec{c}).(\vec{b} \times \vec{d}) = \vec{0}$

C. $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

D. $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = \vec{0}$

Answer: c

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79. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar, then

A. $\vec{r} \perp (\vec{c} \times \vec{a})$

B. $\vec{r} \perp (\vec{a} \times \vec{b})$

C. $\vec{r} \perp (\vec{b} \times \vec{c})$

D. $\vec{r} = \vec{0}$

Answer: d

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80. If $\vec{a}$ satisfies $\vec{a} \times (\hat{i} + 2\hat{j} + \hat{k}) = \hat{i} - \hat{k}$ then $\vec{a}$ is equal to

A. $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

B. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

C. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

D. $\lambda\hat{i} + (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

Answer: c

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81. Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a} nad \vec{b}$ is

A. $\dfrac{19}{5\sqrt{43}}$

B. $\dfrac{19}{3\sqrt{43}}$

C. $\dfrac{19}{\sqrt{45}}$

D. $\dfrac{19}{6\sqrt{43}}$

Answer: a

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82. The units vectors orthogonal to the vector $-\hat{i} + 2\hat{j} + 2\hat{k}$ and making equal angles with the X and Y axes islare) :

A. $\pm \frac{1}{3}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

B. $\frac{19}{5\sqrt{43}}$

C. $\pm \frac{1}{3}\left(\hat{i} + \hat{j} - \hat{k}\right)$

D. none of these

Answer: a

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83. The value of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + \hat{k}$ is obtuse and the angle between $\vec{b}$ and the z-axis is acute and less then $\pi/6$

A. $a < x < 1/2$

B. $1/2 < x < 15$

C. $x < 1/2$ or $x < 0$

D. none of these

Answer: b

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84. If vectors $\vec{a}$ and $\vec{b}$ are two adjacent sides of parallelograsm then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is (A) $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ (B) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$ (C) $\vec{b} - \frac{\vec{b}.\vec{a}}{(|\vec{a}|)^2}\vec{a}$ (D) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

A. $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$

B. $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$

C. $\vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$

D. $\frac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

Answer: a

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85. A parallelogram is constructed on $3\vec{a} + \vec{b} and \vec{a} - 4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is a .40 b. 64 c. 32 d. 48

A. 40

B. 64

C. 32

D. 48

Answer: c

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86. Let $\vec{a}.\vec{b}=0$ where $\vec{a}$ and $\vec{b}$ are unit vectors and the vector $\vec{c}$ is inclined an anlge θ to both $\vec{a}$ and $\vec{b}$. $If\vec{c}=m\vec{a}+n\vec{b}+p\left(\vec{a}\times\vec{b}\right),(m,n,p\in R)$ then

A. $\frac{\pi}{4}\leq\theta\leq\frac{\pi}{4}$

B. $\frac{\pi}{4}\leq\theta\leq\frac{3\pi}{4}$

C. $0\leq\theta\leq\frac{\pi}{4}$

D. $0\leq\theta\leq\frac{3\pi}{4}$

Answer: a

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87. $\vec{a}$ and $\vec{c}$ are unit vectors and $\left|\vec{b}\right|=4$ the angle between $\vec{a}$ and $\vec{b}is\cos^{-1}(1/4)$ and $\vec{b}-2\vec{c}=\lambda\vec{a}$ the value of λ is

A. $3,\ -4$

B. $\frac{1}{4},\frac{3}{4}$

C. $-3, 4$

D. $-1/4, \frac{3}{4}$

Answer: a

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88. Let the position vectors of the points $P and Q$ be $4\hat{i} + \hat{j} + \lambda\hat{k} and 2\hat{i} - \hat{j} + \lambda\hat{k}$, respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $P and Q$. Then λ equals $1/2$ b. $1/2$ c. 1 d. none of these

A. $-1/2$

B. $1/2$

C. 1

D. none of these

Answer: a

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89. A vector of magnitude $\sqrt{2}$ coplanar with the vectors $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ is

A. $-\hat{j} + \hat{k}$

B. $\hat{i}$ and $\hat{k}$

C. $\hat{i} - \hat{k}$

D. $\hat{i} - \hat{j}$

Answer: a

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90. Let P be a point interior to the acute triangle ABC. If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

A. centroid

B. orthocentre

C. incentre

D. circumcentre

Answer: a

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91. G is the centroid of triangle $ABCandA_1andB_1$ are rthe midpoints of sides $ABandAC,$ respectively. If Delta_1 is the area of quadrilateral $GA_1AB_1and\text{Delta}$ is the area of triangle $ABC,$ then $\text{Delta}/\text{Delta}_1$ is equal to $\frac{3}{2}$ b. 3 c. $\frac{1}{3}$ d. none of these

A. $\frac{3}{2}$

B. 3

C. $\frac{1}{3}$

D. none of these

Answer: b

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92. Points $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ are coplanar and $(s \in \alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = 0.$ Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma is \frac{1}{14}$ b. 14 c. 6 d. $1/\sqrt{6}$

A. $1/14$

B. 14

C. 6

D. $1/\sqrt{6}$

Answer: a

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93. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1and 2. respectively, and $\left(1 - 3\vec{a}.\vec{b}\right)^2 + \left|2\vec{a} + \vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2 = 47$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\pi/3$

B. $\pi - \cos^{-1}(1/4)$

C. $\frac{2\pi}{3}$

D. $\cos^{-1}(1/4)$

Answer: c

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94. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 2 and 3, respectively, such that $\left|2\left(\vec{a} \times \vec{b}\right)\right| + \left|3\left(\vec{a} \cdot \vec{b}\right)\right| = k,$ then the maximum value of k is a.$\sqrt{13}$ b. $2\sqrt{13}$ c. $6\sqrt{13}$ d. $10\sqrt{13}$

A. $\sqrt{13}$

B. $2\sqrt{13}$

C. $6\sqrt{13}$

D. $10\sqrt{13}$

Answer: c

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95. $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vecrtors such that $\left|\vec{a}+\vec{b}+3\vec{c}\right| = 4$ Angle between $\vec{a}$ and $\vec{b} is \theta_1$, between $\vec{b}$ and $\vec{c} is \theta_2$ and between $\vec{a}$ and $\vec{b}$ varies $[\pi/6, 2\pi/3]$. Then the maximum value of $\cos\theta_1 + 3\cos\theta_2$ is

A. 3

B. 4

C. $2\sqrt{2}$

D. 6

Answer: b

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96. If the vector product of a constant vector $\vec{O}A$ with a variable vector $\vec{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is a straight line perpendicular to $\vec{O}A$ b. a circle with centre O and radius equal to $\left|\vec{O}A\right|$ c. a straight line parallel to $\vec{O}A$ d. none of these

A. a straight line perpendicular to $\overrightarrow{OA}$

B. a circle with centre O and radius equal to $\left|\overrightarrow{OA}\right|$

C. a striaght line parallel to $\overrightarrow{OA}$

D. none of these

Answer: c

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97. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2 and |\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals a. 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: c

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98. If the two adjacent sides of two rectangles are reprresented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}, \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}, \vec{s} = -\vec{a} + \vec{b}$, respectively, then the angle between the vectors $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$ is

A. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $\pi\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. cannot of these

Answer: b

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99. if $\vec{\alpha} \mid \mid \left(\vec{\beta} \times \vec{\gamma}\right)$, then $\left(\vec{\alpha} \times \beta\right)\dot{\vec{\alpha}} \times \vec{\gamma}$ equals to $\left|\vec{\alpha}\right|^2\left(\vec{\beta}\dot{\vec{\gamma}}\right)$ b. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}\dot{\vec{\alpha}}\right)$

c. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}\dot{\vec{\beta}}\right)$ d. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

A. $\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$

B. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$

C. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$

D. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

Answer: a

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100. The position vectors of the points P,Q,R,S are $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{k}+2\hat{j}-3\hat{k}$, and $\hat{i}-6\hat{j}-\hat{k}$ respectively. Prove that the line PQ and RS are parallel.

A. 120°

B. 90°

C. $\cos^{-1}(3/4)$

D. none of these

Answer: b

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101. Given three vectors e$\vec{a}, \vec{b}$ and $\vec{c}$ two of which are non-collinear. Futrther if $(\vec{a}+\vec{b})$ is collinear with $\vec{c}, (\vec{b}+\vec{c})$ is collinear with $\vec{a}, |\vec{a}|=|\vec{b}|=|\vec{c}|=\sqrt{2}$ find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

A. 3

B. -3

C. 0

D. cannot of these

Answer: b

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102. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $(\vec{a}+\vec{b}).(2\vec{a}+3\vec{b})\times(3\vec{a}-2\vec{b})=\vec{0}$ then angle between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. π

D. indeterminate

Answer: d

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103. If in a right-angled triangle ABC, the hypotenuse $AB = p, then\vec{A}\dot{B}\vec{A}C + \vec{B}\dot{C}\vec{B}A + \vec{C}\dot{A}\vec{C}B$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

A. $2p^2$

B. $\frac{p^2}{2}$

C. p^2

D. none of these

Answer: c

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104. Resolved part of vector $\vec{a}$ and along vector $\vec{b}$ is $\vec{a}_1$ and that prependicular to $\vec{b}$ is $\vec{a}_2$ then $\vec{a}_1 \times \vec{a}_2$ is equl to

A. $\dfrac{\left(\vec{a} \times \vec{b}\right). \vec{b}}{\left|\vec{b}\right|^2}$

B. $\dfrac{\left(\vec{a}. \vec{b}\right)\vec{a}}{\left|\vec{a}\right|^2}$

C. $\dfrac{\left(\vec{a}. \vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

D. $\dfrac{\left(\vec{a}. \vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b} \times \vec{a}\right|}$

Answer: c

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105. Let $\vec{a} = 2\hat{i} = \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors . A vector in the pland of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\left(\frac{2}{3}\right)}$ is (A) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (B) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (C) $-2\hat{i} - \hat{j} + 5\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $-2\hat{i} - \hat{j} + 5\hat{k}$

C. $2\hat{i} + 3\hat{j} + 3\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: b

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106. If P is any arbitrary point on the circumcirlce of the equllateral trangle of side length l units, then $\left|\vec{PA}\right|^2 + \left|\vec{PB}\right|^2 + \left|\vec{PC}\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

A. $2l^2$

B. $2\sqrt{3}l^2$

C. l^2

D. $3l^2$

Answer: a

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107. If $\vec{r} and \vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r}+b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2+\left|\vec{r}+b\vec{s}\right|^2$ is equal to $2\left|\vec{r}\right|^2$ b. $\left|\vec{r}\right|^2/2$ c. $3\left|\vec{r}\right|^2$ d. $|r|^2$

A. $2\left|\vec{r}\right|^2$

B. $\left|\vec{r}\right|^2/2$

C. $3\left|\vec{r}\right|^2$

D. $\left|\vec{r}\right|^2$

Answer: b

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108. $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that if equally inclined to $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ is equal to

A. $\frac{1}{\sqrt{2}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

B. $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{a} + \vec{b}\right)$

C. $\frac{1}{\sqrt{3}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

D. $\frac{1}{3}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

Answer: a

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109. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b}.\vec{q} = 0$ and $\left(\vec{b}\right)^2 = 1$ where μ is a sclar. Then $\left|\left(\vec{a}.\vec{q}\right)\vec{p} - \left(\vec{p}.\vec{q}\right)\vec{a}\right|$ is equal to

A. $2\left|\vec{p}\vec{q}\right|$

B. $(1/2)\left|\vec{p}.\,\vec{q}\right|$

C. $\left|\vec{p}\times\vec{q}\right|$

D. $\left|\vec{p}.\,\vec{q}\right|$

Answer: d

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110. The position vectors of the vertices $A, BandC$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and\vec{c}$, respectively. A vector $\vec{d}$ is such that $\ddot{\vec{a}} = \ddot{\vec{b}}and\vec{d} = \lambda\left(\vec{b}+\vec{c}\right)^{.}$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

A. acute angled

B. obtuse angled

C. right angled

D. none of these

Answer: a

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111. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4\tan A} + a\tan B\sqrt{a^2 + 4\tan c} = 6a,$ then the least vale of $\tan^2 A + \tan^2 b + \tan^2 Cis\ 6$ b. 10 c. 12 d. 3

A. 6

B. 10

C. 12

D. 3

Answer: d

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112. The vertex A triangle ABC is on the line $\vec{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i} and \hat{j}$. Let Delta be the area of the triangle and Delta$\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to A is $[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

A. [-8, -4]cup[4,8]`

B. $[-4, 4]$

C. [-2,2]

D. $[-4, -2] \cup [2, 4]$

Answer: c

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113. A non-zero vector $\vec{a}$ is such that its projections along vectors $\frac{\hat{i} + \hat{j}}{\sqrt{2}}, \frac{-\hat{i} + \hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal, then unit vector along $\vec{a}$ is $\frac{\sqrt{2}\hat{j} - \hat{k}}{\sqrt{3}}$ b. $\frac{\hat{j} - \sqrt{2}\hat{k}}{\sqrt{3}}$

c. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

A. $\frac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$

B. $\frac{\hat{j}-\sqrt{2}\hat{k}}{\sqrt{3}}$

C. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$

D. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

Answer: a

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114. Position vector $\hat{k}$ is rotated about the origin by angle 135^{0} in such a way that the plane made by it bisects the angel between $\hat{i} and \hat{j}$. Then its new position is $\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}$ b. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$ c. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$ d. none of these

A. $\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}$

B. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$

C. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$

D. none of these

Answer: d

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115. In a quadrilateral $ABCD, \vec{A}C$ is the bisector of $\vec{A}Band\vec{A}D$, angle between $\vec{A}Band\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|^{.}$ Then the angle between $\vec{B}Aand\vec{C}D$ is $\frac{\cos^{-1}\left(\sqrt{14}\right)}{7\sqrt{2}}$ b. $\frac{\cos^{-1}\left(\sqrt{21}\right)}{7\sqrt{3}}$ c. $\frac{\cos^{-1}2}{\sqrt{7}}$ d. $\frac{\cos^{-1}\left(2\sqrt{7}\right)}{14}$

A. $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$

B. $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\cos^{-1}\frac{2}{\sqrt{7}}$

D. $\cos^{-1}\frac{2\sqrt{7}}{14}$

Answer: c

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116. In fig. 2.33 AB, DE and GF are parallel to each other and AD, BG and EF ar parallel to each other . If CD: CE = CG:CB = 2:1 then the value of area $(\triangle AEG) : area(\triangle ABD)$ is equal to

A. $7/2$

B. 3

C. 4

D. $9/2$

Answer: b

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117. A unit vector $\vec{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that angle between $\vec{a}$ and $\vec{d}$ where $\vec{d} = \vec{j} + 2\vec{k}$ is

A. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

B. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

C. $\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$

D. $\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$

Answer: b

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118. Let $ABCD$ be a tetrahedron such that the edges $AB, ACandAD$ are mutually perpendicular. Let the area of triangles $ABC, ACDandADB$ be 3, 4 and 5sq. units, respectively. Then the area of triangle BCD is $5\sqrt{2}$ b. 5 c. $\dfrac{\sqrt{5}}{2}$ d. $\dfrac{5}{2}$

A. $5\sqrt{2}$

B. 5

C. $\frac{\sqrt{5}}{2}$

D. $\frac{5}{2}$

Answer: a

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119. Let $\overrightarrow{f}(t) = [t]\hat{i} + (t - [t])\hat{j} + [t + 1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors ` vecf(5/4)a n df(t),0

A. parallel to each other

B. perpendicular to each other

C. inclined at $\frac{\cos^{-1}2}{\sqrt{7}\left(1 - t^2\right)}$

D. inclined at $\frac{\cos^{-1}(8 + t)}{9\sqrt{1 + t^2}}$

Answer: d

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120. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, $then\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to

A. $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$

B. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$

C. $|\vec{c}|^2\left(\vec{a}.\vec{b}\right)$

D. none of these

Answer: a

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121. about to only mathematics

A. $1/3$

B. 4

C. $\left(3\sqrt{3}\right)/4$

D. $4\sqrt{3}$

Answer: d

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122. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a on zero vector and $\left|\left(\vec{d}.\vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d}.\vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d}.\vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0$ then (A) $|\vec{a}| + \left|\vec{b}\right| + |\vec{c}| = \left|\vec{d}\right|$ (B) $|\vec{a}| = \left|\vec{b}\right| = |\vec{c}|$ (C) $\vec{a}, \vec{b}, \vec{c}$ are coplanar (D) $\vec{a} + \vec{c} = 2\vec{b}$

A. $|\vec{a}| = \left|\vec{b}\right| = |\vec{c}|$

B. $|\vec{a}| + \left|\vec{b}\right| + |\vec{c}| = \left|\vec{d}\right|$

C. $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

D. none of these

Answer: c

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123. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 0$, then $\left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)\right)$ is equal

A. $48\hat{b}$

B. $-48\hat{b}$

C. $48\hat{a}$

D. $-48\hat{a}$

Answer: a

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124. If the two diagonals of one its faces are $6\hat{i} + 6\hat{k} and 4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $c = 4\hat{j} - 8\hat{k}$, then the volume of a parallelepiped is a.60 b. 80 c. 100 d. 120

A. 60

B. 80

C. 100

D. 120

Answer: d

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125. The volume of a tetrahedron fomed by the coterminus edges $\vec{a}, \vec{b}$ and $\vec{c} is 3$. Then the volume of the parallelepiped formed by the coterminus edges $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ is

A. 6

B. 18

C. 36

D. 9

Answer: c

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126. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors , then the triple product $\left[\vec{a} + \vec{b} + \vec{c}\,\vec{a} + \vec{b}\,\vec{b} + \vec{c}\right]$ equals

A. 0

B. 1 or -1

C. 1

D. 3

Answer: b

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127. Vector $\vec{c}$ is perpendicular to vectors $\vec{a} = (2, -3, 1) and \vec{b} = (1, -2, 3)$ and satisfies the condition $\vec{c}.\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 10.$ Then vector $\vec{c}$ is equal to a.$(7, 5, 1)$ b. $-7, -5, -1$ c. $1, 1, -1$ d. none of these

A. 7,5,1

B. (-7, -5, -1)

C. 1,1,-1

D. none of these

Answer: a

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128. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j}, \vec{a} \perp \vec{b}, \vec{a}.\vec{c} = 4$ then find the value of $\left[\vec{a}\vec{b}\vec{c}\right]$

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129. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ gve three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is \frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} p = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

D. $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

Answer: c

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130. Let $\vec{r}, \vec{a}, \vec{b}$ and $\vec{c}$ be four non-zero vectors such that $\vec{r}.\vec{a} = 0, \left|\vec{r} \times \vec{b}\right| = |\vec{r}|\left|\vec{b}\right|, |\vec{r} \times \vec{c}| = |\vec{r}||\vec{c}|$ then

$\left[\vec{a}\quad\vec{b}\quad\vec{c}\right] =$

A. |a||b||c|

B. - $|a||b||c|$

C. 0

D. none of these

Answer: c

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131. If $\vec{a}, \vec{b}$ and $\vec{c}$are such that $\left[\vec{a}\vec{b}\vec{c}\right] = 1, \vec{c} = \lambda\vec{a} \times \vec{b}$, angle between $\vec{a}$ and $\vec{b} is 2\pi/3, \left|\vec{a}\right| = \sqrt{2}\left|\vec{b}\right| = \sqrt{3}$ and $\left|\vec{c}\right| = \dfrac{1}{\sqrt{3}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: b

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132. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0$, then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to

a. vector perpendicular to the plane ofa, b, c b. a scalar quantity c. $\vec{0}$ d. none of these

A. a vector perpendicular to the plane of $\vec{a}, \vec{b}$ and $\vec{c}$

B. a scalar quantity

C. $\vec{0}$

D. none of these

Answer: c

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133. Value of $\left[\vec{a}\times\vec{b}\vec{a}\times\vec{c}\vec{d}\right]$ is always equal to $\left(\vec{a}.\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$ b. $\left(\vec{a}.\vec{c}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ c. $\left(\vec{a}.\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ d. none of these

A. $\left(\vec{a}.\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$

B. `(veca.vecc)[veca vecb vecd]

C. $\left(\vec{a}.\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$

D. none of these

Answer: a

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134. Let $\vec{a} and \vec{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$, a. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}+\left(\vec{r}.\hat{b}\right)\hat{b}+\left(\vec{r}.\left(\hat{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$ b. $\vec{r}=\left(\vec{r}.\hat{a}\right)-\left(\vec{r}.\hat{b}\right)\hat{b}-\left(\vec{r}.\left(\hat{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$ c. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}-\left(\vec{r}.\hat{b}\right)\hat{b}+\left(\vec{r}.\left(\hat{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$ d.none of these

A. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}+\left(\vec{r}.\hat{b}\right)\hat{b}+\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

B. $\vec{r} = \left(\vec{r}.\hat{a}\right) - \left(\vec{r}.\hat{b}\right)\hat{b} - \left(\vec{r}.\left(\vec{a} \times \hat{b}\right)\right)\left(\hat{a} \times \hat{b}\right)$

C. $\vec{r} = \left(\vec{r}.\hat{a}\right)\hat{a} - \left(\vec{r}.\hat{b}\right)\hat{b} - \left(\vec{r}.\left(\vec{a} \times \hat{b}\right)\right)\left(\hat{a} \times \hat{b}\right)$

D. none of these

Answer: a

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135. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other I. then $\left[\vec{a} + \left(\vec{a} \times \vec{b}\right)\vec{b} + \left(\vec{a} \times \vec{b}\right)\vec{a} \times \vec{b}\right]$ will always be equal to

A. 1

B. 0

C. -1

D. none of these

Answer: a

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136. $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a}. Vecb = 2. If \vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ then find angle between $\vec{b}$ and $\vec{c}$.

A. $\dfrac{\pi}{3}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{3\pi}{4}$

D. $\dfrac{5\pi}{6}$

Answer: d

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137. Then for any arbitary vector $\vec{a}, (((\vec{a} \times \vec{b}) + (\vec{a} \times \vec{b})) \times (\vec{b} \times \vec{c}))(\vec{b} - \vec{c})$ is always equal to

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138. If $\vec{a}.\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is

A. $\dfrac{\left(\beta\vec{a}-\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

B. $\dfrac{\left(\beta\vec{a}+\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

C. $\dfrac{\left(\beta\vec{c}+\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

D. $\dfrac{\left(\beta\vec{c}+\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

Answer: a

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139. If $a\left(\vec{\alpha}\times\vec{\beta}\right)+b\left(\vec{\beta}\times\vec{\gamma}\right)+c\left(\vec{\gamma}\times\vec{\alpha}\right)=0$ and at least one of $a, bandc$ is nonzero, then vectors $\vec{\alpha},\vec{\beta}and\vec{\gamma}$ are a. parallel b. coplanar c. mutually perpendicular d. none of these

A. parallel

B. coplanar

C. mutually perpendicular

D. none of these

Answer: b

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140. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a} + \vec{c} = kb\vec{b}$.

A. $\vec{a}, \vec{b}$ and $\vec{v}$ can be coplanar

B. $\vec{a}, \vec{b}$ and $\vec{c}$ must be coplanar

C. $\vec{a}, \vec{b}$ and $\vec{c}$ cannot be coplanar

D. none of these

Answer: c

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141. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = \frac{1}{2}$ for some non zero vector $\vec{r}$ and $\vec{a}, \vec{b}, \vec{c}$ are non coplanar, then the area of the triangle whose vertices are $A(\vec{a}), B(\vec{b})$ and $C(\vec{c}0$ is (A) $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$ (B) $|\vec{r}|$ (C) $\left|\left[\vec{a}\vec{b}\vec{r}\right]\vec{r}\right|$ (D) none of these

A. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

B. $|\vec{r}|$

C. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$

D. none of these

Answer: c

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142. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be $6\hat{i} + 8\hat{j}$ b. $-8\hat{i} + 3\hat{j}$ c. $6\hat{i} - 8\hat{j}$ d. $8\hat{i} + 6\hat{j}$

A. $6\hat{i} + 8\hat{j}$

B. $-8\hat{i} + 3\hat{j}$

C. $6\hat{i} - 8\hat{j}$

D. $8\hat{i} + 6\hat{j}$

Answer: a

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143. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3 then \left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\}.\vec{b}$ is equal to

A. $\frac{-3}{4}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: a

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144. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r}\times\vec{a}$ is equal to

A. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{b}-\left(\vec{r}.\vec{b}\right)\left(\vec{b}\times\vec{a}\right)$

B. $\left[\vec{r}\vec{a}\vec{b}\right]\left(\vec{a}+\vec{b}\right)$

C. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{a}+\left(\vec{r}.\vec{a}\right)\vec{a}\times\vec{b}$

D. none of these

Answer: a

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145. If $\vec{a}+\vec{b},\vec{c}$ are any three non- coplanar vectors then the equation $\left[\vec{b}\times\vec{c}\vec{c}\times\vec{a}\vec{a}\times\vec{b}\right]x^2+\left[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}\right]x+1+\left[\vec{b}-\vec{c}\vec{c}-\vec{c}-\vec{a}\vec{a}-\vec{b}\right]=0$ has roots

A. real and distinct

B. real

C. equal

D. imaginary

Answer: c

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146. Sholve the simultasneous vector equations for `vecx aedn vecy: vecx+veccxxvecy=veca and vecy+veccxxvecx=vecb, vec!=0

A. $\vec{x} = \dfrac{\vec{b} \times \vec{c} + \vec{a} + (\vec{c}.\vec{a})\vec{c}}{1 + \vec{c}.\vec{c}}$

B. $\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + (\vec{c}.\vec{a})\vec{c}}{1 + \vec{c}.\vec{c}}$

C. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + (\vec{c}.\vec{b})\vec{c}}{1 + \vec{c}.\vec{c}}$

D. none of these

Answer: b

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147. The condition for equations $\vec{r} \times \vec{a} = \vec{b} and \vec{r} \times \vec{c} = \vec{d}$ to be consistent

is a.$\vec{b}.\vec{c} = \vec{a}.\vec{d}$ b. $\vec{a}.\vec{b} = \vec{c}.\vec{d}$ c. $\vec{b}.\vec{c} + \vec{a}.\vec{d} = 0$ d. $\vec{a}\vec{b} + \vec{\cdot}\vec{d} = 0$

A. $\vec{b}.\vec{c} = \vec{a}.\vec{d}$

B. $\vec{a}.\vec{b} = \vec{c}.\vec{d}$

C. $\vec{b}.\vec{c} + \vec{a}.\vec{d} = 0$

D. $\vec{a}.\vec{b} + \vec{c}.\vec{d} = 0$

Answer: c

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148. If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, then $\left[\vec{a}\vec{b}\vec{c}\right] =$

A. 30

B. -30

C. 15

D. -15

Answer: b

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149. If

$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $(1 + \alpha)\hat{i} + \beta(1 + \alpha)\hat{j} + \gamma(1 + \alpha)($

A. $-2, -4, -\frac{2}{3}$

B. $2, -4, \frac{2}{3}$

C. $-2, 4, \frac{2}{3}$

D. $2, 4, -\frac{2}{3}$

Answer: a

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150. Let $\left(\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j}\right.$ and $\vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)$. Then $\vec{a}(x)$ and $\vec{b}(x)$ are

A. collinear for unique value of x

B. perpendicular for infinte values of x.

C. zero vectors for unique value of x

D. none of these

Answer: b

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151. For any vectors $\vec{a}$ and $\vec{b}$, $\left(\vec{a} \times \hat{i}\right) + \left(\vec{b} \times \hat{i}\right) + \left(\vec{a} \times \hat{j}\right).\left(\vec{b} \times \hat{j}\right) + \left(\vec{a} \times \hat{k}\right).\left(\vec{b} \times \hat{k}\right)$ is always equal to

A. $\vec{a}.\vec{b}$

B. $2\vec{a}.Vecb$

C. zero

D. none of these

Answer: b

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152. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors and $\vec{r}$ is any vector in space, then $\left(\times \vec{b}\right), \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times \vec{b}\right) =$

(A) $\left[\vec{a}\vec{b}\vec{c}\right]$ (B) $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ (C) $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ (D) $4\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

B. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

C. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

D. none of these

Answer: b

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153. If $\vec{P} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$ and $\vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are three non- coplanar vectors then the value of the expression $\left(\vec{a} + \vec{b} + \vec{c}\right). \left(\vec{p} + \vec{q} + \vec{r}\right)$ is

A. 3

B. 2

C. 1

D. 0

Answer: a

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154. $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are the vertices of triangle ABC and $R(\vec{r})$ is any point in the plane of triangle ABC, then $\vec{r}, (\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$ is always equal to

A. zero

B. $[\vec{a}\vec{b}\vec{c}]$

C. - $[\vec{a}\vec{b}\vec{c}]$

D. none of these

Answer: b

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155. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar vectors and $\vec{a}\times\vec{c}$ is perpendicular to $\vec{a}\times(\vec{b}\times\vec{c})$, then the value of $[\vec{a}\times(\vec{b}\times\vec{c})]\times\vec{c}$ is equal to

A. $[\vec{a}\vec{b}\vec{c}]\vec{c}$

B. $[\vec{a}\vec{b}\vec{c}]\vec{b}$

C. $\vec{0}$

D. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

Answer: c

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156. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV'$, $thenK$ is equal to a.9 b. 12 c. 27 d. 81

A. 9

B. 12

C. 27

D. 81

Answer: c

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157. $\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right)\right]$ is equal to

(where $\vec{a}, \vec{b}$ and $\vec{c}$ are non - zero non- colanar vectors).

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2$

B. $\left[\vec{a}\vec{b}\vec{c}\right]^3$

C. $\left[\vec{a}\vec{b}\vec{c}\right]^4$

D. $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: c

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158. If

$\vec{r} = x_1\left(\vec{a}\times\vec{b}\right) + x_2\left(\vec{b}\times\vec{c}\right) + x_3\left(\vec{c}\times\vec{a}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1$ then $x_1 + x_2 + x_3$

is equal to

A. $\frac{1}{2}\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

B. $\frac{1}{4}\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

C. $2\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

D. $4\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

Answer: d

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159. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other then a vector $\vec{v}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v}.\vec{a}=0, \vec{v}.\vec{b}=1$ and $\left[\left(\vec{v}.\vec{a}\times\vec{b}\right)\right]=1$ is

A. $\frac{\vec{b}}{\left|\vec{b}\right|^2}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^2}$

B. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^2}$

C. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

D. none of these

Answer: a

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160. If $\vec{a}' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k} nad \vec{c}' = 2\hat{i} - \hat{j} + \hat{k}$ then the altitude of the parallelepiped formed by the vectors, $\vec{a}, \vec{b}$ and $\vec{c}$ having baswe formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is recipocal vector $\vec{a}$)

A. 1

B. $3\sqrt{2}/2$

C. $1/\sqrt{6}$

D. $1/\sqrt{2}$

Answer: d

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161. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$ then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is

A. $\frac{\hat{i} + \hat{j} + \hat{k}}{2}$

B. $\frac{\hat{i} - \hat{j} + \hat{k}}{2}$

C. $\frac{-\hat{i} - \hat{j} + \hat{k}}{2}$

D. $\frac{\hat{i} + \hat{j} - \hat{k}}{2}$

Answer: d

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162. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1$ and $0 \leq \theta \leq \pi$, then θ lies in the interval

A. $[0, \pi/6)$

B. $(5\pi/6, \pi]$

C. $[\pi/6, \pi/2]$

D. $(\pi/2, 5\pi/6]$

Answer: a,b

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163. Differentiate $y = \cos^4 4x$

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164. Unit vectors $\vec{a}$ and $\vec{b}$ ar perpendicular , and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}. If \alpha\vec{a} + \beta\vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$ then.

A. $\alpha = \beta$

B. $\gamma^2 = 1 - 2\alpha^2$

C. $\gamma^2 = -\cos 2\theta$

D. $\beta^2 = \dfrac{1 + \cos 2\theta}{2}$

Answer: a,b,c,d

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165. If vectors $\vec{a}$ and $\vec{b}$ are two adjacent sides of parallelograsm then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is (A) $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ (B) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$ (C) $\vec{b} - \frac{\vec{b}.\vec{a}}{(|\vec{a}|)^2}\vec{a}$ (D) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

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166. If $\vec{a} \times (\vec{b} \times \vec{c})$ is perpendicular to $(\vec{a} \times \vec{b}) \times \vec{c}$, we may have a. $(\vec{a}.\vec{c})|\vec{b}|^2 = (\vec{a}.\vec{b})(\vec{b}.\vec{c})$ b. $\vec{a}\vec{b} = 0$ c. $\vec{a}\vec{c} = 0$ d. $\vec{b}\vec{c} = 0$

A. $(\vec{a}.\vec{b})|\vec{b}|^2 = (\vec{a}.\vec{b})(\vec{b}.\vec{c})$

B. $\vec{a}.\vec{b} = 0$

C. $\vec{a}.\vec{c} = 0$

D. $\vec{b}.\vec{c} = 0$

Answer: a,c

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167. If $\vec{P} = \dfrac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$ and $\vec{r} = \dfrac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are three non- coplanar vectors then the value of the expression $(\vec{a} + \vec{b} + \vec{c}).(\vec{p} + \vec{q} + \vec{r})$ is

A. $x[\vec{a}\vec{b}\vec{c}] + \dfrac{[\vec{p}\vec{q}\vec{r}]}{x}$ has least value 2

B. $x^2[\vec{a}\vec{b}\vec{c}]^2 + \dfrac{[\vec{p}\vec{q}\vec{r}]}{x^2}$ has least value $(3/2^{2/3})$

C. $[\vec{p}\vec{q}\vec{r}] > 0$

D. none of these

Answer: a,c

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168. $a_1, a_2, a_3 \in R - \{0\}$ and $a_1 + a_2\cos2x + a_3\sin^2 x = 0$ " for all $x \in R$ then

A. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other

C. if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered trippplet $\left(a_1, a_2, a_3\right) = (1, -1, -2)$

D. if $2a_1 + 3a_2 + 6a_3 = 26,$ then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

Answer: a,b,c,d

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169. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ, then

A. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$

B. $\left|\vec{a} \times \vec{b}\right| = \left(\vec{a}.\vec{b}\right)$, if $\theta = \pi/4$

C. $\vec{a} \times \vec{b} = \left(\vec{a}.\vec{b}\right)\hat{n}$ (where $\hat{n}$ is a normal unit vector) $\text{ if } \theta f = \pi/4$

D. $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} + \vec{b}\right) = 0$

Answer: a,b,c,d

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170. Let $\vec{a}$ and $\vec{b}$ are two given perpendicular vectors, which are non-zero.

A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$, can be

A. $\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

B. $2\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

C. $\left|\vec{a}\right|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

D. $\left|\vec{b}\right|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

Answer: a,b,cd,

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171. If vectors $\vec{b} = \left(\tan\alpha, -1, 2\sqrt{\sin\alpha/2}\right)$ and $\vec{c} = \left(\tan\alpha, \tan\alpha, \dfrac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis , then the value of α is

A. $\alpha = (4n+1)\pi + \tan^{-1}2$

B. $\alpha = (4n+1)\pi - \tan^{-1}2$

C. $\alpha = (4n+2)\pi + \tan^{-1}2$

D. $\alpha = (4n+2)\pi - \tan^{-1}2$

Answer: b,d

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172. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $\left|\vec{a}\right| = \sqrt{3}$ and $\left|\vec{b}\right| = \sqrt{2}$

A. $\vec{r} = \frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

B. $\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

C. $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$

D. $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

Answer: b,d

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173. If $\vec{a}$ and $\vec{b}$ are unequal unit vectors such that $\left(\vec{a} - \vec{b}\right) \times \left[\left(\vec{b} + \vec{a}\right) \times \left(2\vec{a} + \vec{b}\right)\right] = \vec{a} + \vec{b}$ then angle θ between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. $\pi/4$

D. π

Answer: b,d

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174. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpenicualar to each other and $\vec{c}=\lambda_1\vec{a}+\lambda_2\vec{b}+\lambda_3\left(\vec{a}\times\vec{b}\right)$, then which of the following is (are) true ?

A. $\lambda_1=\vec{a}.\vec{c}$

B. $\lambda_2=\left|\vec{b}\times\vec{c}\right|$

C. $\lambda_3= | \left(\vec{a}\times\vec{b}\right| \times\vec{c}\right|$

D. $\lambda_1\vec{a}+\lambda_2\vec{b}+\lambda_3\left(\vec{a}\times\vec{b}\right)$

Answer: a,d

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175. If vectors $\vec{a}$ and $\vec{b}$ are non collinear then $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|}$ is

A. a unit vector

B. in the plane of $\vec{a}$ and $\vec{b}$

C. equally inclined to $\vec{a}$ and $\vec{b}$

D. perpendicular to $\vec{a} \times \vec{b}$

Answer: b,c,d

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176. If $\vec{a}$ and $\vec{b}$ are non - zero vectors such that $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - 2\vec{b}\right|$ then

A. $2\vec{a}.\vec{b} = \left|\vec{b}\right|^2$

B. $\vec{a}.\vec{b} = \left|\vec{b}\right|^2$

C. least value of $\vec{a}.\vec{b} + \frac{1}{\left|\vec{b}\right|^2 + 2}$ is $\sqrt{2}$

D. least value of $\vec{a}.\vec{b} + \frac{1}{\left|\vec{b}\right|^2 + 2}$ is $\sqrt{2} - 1$

Answer: a,d

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177. Let $\vec{a}\vec{b}$ and $\vec{c}$ be non- zero vectors aned $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ and $\vec{V}_2 = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$.vectors $\vec{V}_1$ and $\vec{V}_2$ are equal . Then

A. $\vec{a}$ and $\vec{b}$ ar orthogonal

B. $\vec{a}$ and $\vec{c}$ are collinear

C. $\vec{b}$ and $\vec{c}$ ar orthogonal

D. $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right)$ when λ is a scalar

Answer: b,d

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178. Vectors $\vec{A}$ and $\vec{B}$ satisfying the vector equation $\vec{A}+\vec{B}=\vec{a}, \vec{A}\times\vec{B}=\vec{b}$ and $\vec{A}.\vec{a}=1$. Vectors and $\vec{b}$ are given vectosrs, are

A. $\vec{A}=\dfrac{\left(\vec{a}\times\vec{b}\right)-\vec{a}}{a^2}$

B. $\vec{B}=\dfrac{\left(\vec{b}\times\vec{a}\right)+\vec{a}\left(a^2-1\right)}{a^2}$

C. $\vec{A}=\dfrac{\left(\vec{a}\times\vec{b}\right)+\vec{a}}{a^2}$

D. $\vec{B}=\dfrac{\left(\vec{b}\times\vec{a}\right)-\vec{a}\left(a^2-1\right)}{a^2}$

Answer: b,c,

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179. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}, \vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{j}-2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{;} \vec{c}, \vec{a},$ respectively. Then

A. $\vec{x}.\vec{d} = -1$

B. $\vec{y}.\vec{d} = 1$

C. vecz.vecd=0`

D. vecr.vecd=0, " where " vecr=lambda vecx + mu vecy +deltavecz`

Answer: c.d

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180. Vectors perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ are (A) $\hat{i} + \hat{k}$ (B) $2\hat{i} + \hat{j} + \hat{k}$ (C) $3\hat{i} + 2\hat{j} + \hat{k}$ (D) $-4\hat{i} - 2\hat{j} - 2\hat{k}$

A. $\hat{i} + \hat{k}$

B. $2\hat{i} + \hat{j} + \hat{k}$

C. $3\hat{i} + 2\hat{j} + \hat{k}$

D. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

Answer: b,d

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181. If side $\vec{AB}$ of an equilateral trangle ABC lying in the x-y plane $3\hat{i}$, then side $\vec{CB}$ can be $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ b. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ c. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$ d. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

A. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$

B. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$

C. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

D. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

Answer: b,d

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182. Let $\hat{a}$ be a unit vector and $\hat{b}$ a non zero vector non parallel to $\vec{a}$. Find the angles of the triangle tow sides of which are represented by the vectors. $\sqrt{3}\left(\hat{}\times\vec{b}\right)$ and $\vec{b}-\left(\hat{a}.\vec{b}\right)\hat{a}$

A. $\tan^{-1}\left(\sqrt{3}\right)$

B. $\tan^{-1}\left(1/\sqrt{3}\right)$

C. $\cot^{-1}(0)$

D. tant^(-1)(1)`

Answer: a,b,c

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183. $\vec{a},\vec{b}$ and $\vec{c}$ are unimdular and coplanar. A unit vector $\vec{d}$ is perpendicualt to them , $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\frac{1}{6}\hat{i}-\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b}is30^{\circ}$ then $\vec{c}$ is

A. $\left(\hat{i}-2\hat{j}+2\hat{k}\right)/3$

B. $\left(-\hat{i}+2\hat{j}-2\hat{k}\right)/3$

C. $\left(-\hat{i}+2\hat{j}-\hat{k}\right)/3$

D. $\left(-2\hat{i}-2\hat{j}+\hat{k}\right)/3$

Answer: a,b

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184. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=$

A. $2\left(\vec{a}\times\vec{b}\right)$

B. $6\left(\vec{b}\times\vec{c}\right)$

C. $3\left(\vec{c}\times\vec{a}\right)$

D. $\vec{0}$

Answer: c,d

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185. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right| + \left|\vec{u}.\vec{b}\right|$

C. $\left|\vec{u}\right| + \left|\vec{u}.\vec{a}\right|$

D. none of these

Answer: d

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186. if $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$, where $\vec{c} \neq \vec{0}$ then

A. $\left|\vec{a}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| = \left|\vec{b}\right|$

C. $\left|\vec{b}\right| = 1$

D. $\left|\vec{a}\right| = \vec{b}| = |\vec{c}\ | \ = 1$

Answer: a,c

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187. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non- coplanar vectors and $\vec{d}$ be a non -zero , which is perpendicular to $\left(\vec{a} + \vec{b} + \vec{c}\right)$. $Now \vec{d} = \left(\vec{a} \times \vec{b}\right)\sin x + \left(\vec{b} \times \vec{c}\right)\cos y + 2\left(\vec{c} \times \vec{a}\right)$. Then

A. $\dfrac{\vec{d}.\left(\vec{a} + \vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = 2$

B. $\dfrac{\vec{d}.\left(\vec{a} + \vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = -2$

C. minimum value of $x^2 + y^2 is \pi^2/4$

D. minimum value of $x^2 + y^2 is 5\pi^2/4$

Answer: b,d

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188. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{b}$, then $\left(\vec{b}\right.$ and $\vec{c}$ being non parallel)

A. angle between $\vec{a}$ and $\vec{b} is \pi/3$

B. angle between $\vec{a}$ and $\vec{c} is \pi/3$

C. angle between $\vec{a}$ and $\vec{b} is \pi/2$

D. angle between $\vec{a}$ and $\vec{c} is \pi/2$

Answer: b,c

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189. If in triangle ABC, $\overrightarrow{AB} = \frac{\vec{u}}{\left|\vec{u}\right|} - \frac{\vec{v}}{\left|\vec{v}\right|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{\left|\vec{u}\right|}$, where $\left|\vec{u}\right| \neq \left|\vec{v}\right|$, then

A. $1 + \cos 2A + \cos 2B + \cos 2C = 0$

B. $\sin A = \cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: a,b,c

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190. $\left[\vec{a}\times\vec{b} \quad \vec{c}\times\vec{d} \quad \vec{e}\times\vec{f}\right]$ is equal to

A. $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

B. $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right] - \left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

C. $\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$

D. $\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

Answer: a,b,c

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191. The scalars l and m such that $l\vec{a} + m\vec{b} = \vec{c}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are given vectors, are equal to

A. $l = \dfrac{(\vec{c} \times \vec{b}).(\vec{a} \times \vec{b})}{(\vec{a} \times \vec{b})^2}$

B. $l = \dfrac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})}$

C. $m = \dfrac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})^2}$

D. $m = \dfrac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})}$

Answer: a,c

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192. If $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}).(\vec{a} \times \vec{d}) = 0$ then which of the following may be true ?

A. $\vec{a}, \vec{b}$ and $\vec{d}$ are nenessarily coplanar

B. $\vec{a}$ lies iin the plane of $\vec{c}$ and $\vec{d}$

C. $\vec{b}$ lies in the plane of $\vec{a}$ and $\vec{d}$

D. $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{d}$

Answer: b,c,d

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193. A,B C and dD are four points such that $\overrightarrow{AB} = m\left(2\hat{i} - 6\hat{j} + 2\hat{k}\right)\overrightarrow{BC} = \left(ahti - 2\hat{j}\right)$ and $\overrightarrow{CD} = n\left(-6\hat{i} + 15\hat{j} - 3\hat{k}\right)$. If CD intersects AB at some points E, then

A. $m \geq 1/2$

B. $n \geq 1/3$

C. m= n

D. $m < n$

Answer: a,b

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194. about to only mathematics

A. $l + m + n = 0$

B. roots of the equation $lx^2 + mx + n = 0$ are equal

C. $l^2 + m^2 + n^2 = 0$

D. $l^3 + m^2 + n^3 = 3lmn$

Answer: a,b,d

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195. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\vec{v}$ is perpendicular to

A. $\vec{\alpha}$

B. $\vec{\beta}$

C. $\vec{\gamma}$

D. none of these

Answer: a,b,c

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196. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + \hat{j} + 5\hat{k}$ and $\vec{C}$ form a left handed system then $\vec{C}$ is (A) $11\hat{i} - 6\hat{j} - \hat{k}$ (B) $-11\hat{i} + 6\hat{j} + \hat{k}$ (C) $-11\hat{i} + 6\hat{j} - \hat{k}$ (D) $-11\hat{i} + 6\hat{j} - \hat{k}$

A. $11\hat{i} - 6\hat{j} - \hat{k}$

B. $-11\hat{i} - 6\hat{j} - \hat{k}$

C. $-11\hat{i} - 6\hat{j} + \hat{k}$

D. $-11\hat{i} + 6\hat{j} - \hat{k}$

Answer: b,d

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197. If

$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k},$, then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

Answer: a,b,c,d

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198. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$ for non coplanar $\vec{a}, \vec{b}, \vec{c}$ then......

A. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$

B. $\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$

C. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

D. $\vec{c} \times \vec{a} \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

Answer: a,c,d

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199. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{;} \vec{c}, \vec{a},$ respectively. Then

A. $\vec{z} . \vec{d} = 0$

B. $\vec{x} . \vec{d} = 1$

C. $\vec{y} . \vec{d} = 32$

D. $\vec{r} . \vec{d} = 0, \text{ where } \vec{r} = \lambda\vec{x} + \mu\vec{y} + \gamma\vec{z}$

Answer: a,d

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200. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}. If\left|\vec{\alpha}\right| = \left|\vec{\beta}\right| = 2$ and angle between $\vec{\alpha}$ and $\vec{\beta} is \frac{\pi}{3}$ then the length of a diagonal of the parallelogram is (A) $4\sqrt{5}$ (B) $4\sqrt{3}$ (C) $4\sqrt{7}$ (D) none of these

A. $4\sqrt{5}$

B. $4\sqrt{3}$

C. $4\sqrt{7}$

D. none of these

Answer: b,c

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201. Statement 1: Vector $\vec{c} = -5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angle between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 8\hat{i} + \hat{j} - 4\hat{k}$.

Statement 2 : $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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202. Statement1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$

Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k} is 2\hat{i} + 2\hat{j} + 2\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: c

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203. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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204. Let $\vec{r}$ be a non - zero vector satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ for given non- zero vectors $\vec{a}\vec{b}$ and $\vec{c}$

Statement 1: $\left[\vec{a} - \vec{b}\vec{b} - \vec{c}\vec{c} - \vec{a}\right] = 0$

Statement 2: $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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205. Statement 1: If $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ are three mutually perpendicular unit vectors then $a_1\hat{i}+b_1\hat{j}+c_1\hat{k}, a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ and $a_3\hat{i}+b_3\hat{j}+c_3\hat{k}$ may be mutually perpendicular unit vectors.

Statement 2 : value of determinant and its transpose are the same.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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206. Statement 1: $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$ then

$\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = 243$

Statement 2: $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = \left|\vec{A}\right|^2 \left|\left[\vec{A}\vec{B}\vec{C}\right]\right|$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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207. Statement 1: $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are non- coplanar. If $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1$, then $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

Statement 2: $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Option A: Both the statements are true and statement 2 is the correct explanation for statement 1. Option B: Both statements are true but statement 2 is not the correct explanation for statement 1. Option C: Statement 1 is true and Statement 2 is false Option D: Statement 1 is false and Statement 2 is true.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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208. Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement 1: $\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right).\vec{b}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right).\vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a}\right).\vec{b}\right)\hat{k}$

Statement 2: $\vec{c} = \left(\hat{i}.\vec{c}\right)\hat{i} + \left(\hat{j}.\vec{c}\right)\hat{j} + \left(\hat{k}.\vec{c}\right)\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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209. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and |

Vector $\vec{u}$ is

A. $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: b

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210. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{a}, \vec{u}\times(\vec{v}\times\vec{w})=\vec{b}, (\vec{u}\times\vec{v})\times\vec{w}=\vec{c}, \vec{a}.\vec{u}=3/2, \vec{a}.\vec{v}=7/4$ and

Vector $\vec{u}$ is

A. $2\vec{a} - 3\vec{c}$

B. $3\vec{b} - 4c$

C. $-4\vec{c}$

D. $\vec{a}+\vec{b}+2\vec{c}$

Answer: c

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211. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and Vector $\vec{u}$ is

A. $\frac{2}{3}\left(2\vec{c} - \vec{b}\right)$

B. $\frac{1}{3}\left(\vec{a} - \vec{b} - \vec{c}\right)$

C. $\frac{1}{3}\vec{a} - \frac{2}{3}\vec{b} - 2\vec{c}$

D. $\frac{4}{3}\left(\vec{c} - \vec{b}\right)$

Answer: d

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212. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{\times} x\left(\vec{y} \times \left(\vec{z} \times \vec{x}\right) = \vec{b}nd \vec{\times} x\vec{y} = \vec{c}, f \in d\vec{x}, \vec{y}, \vec{z}\right.$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$.

A. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \left(\vec{a} + \vec{b}\right)\right]$

B. $\frac{1}{2}\left[\left(\vec{a}+\vec{b}\right)\times\vec{c}+\left(\vec{a}-\vec{b}\right)\right]$

C. $\frac{1}{2}\left[-\left(\vec{a}+\vec{b}\right)\times\vec{c}+\left(\vec{a}+\vec{b}\right)\right]$

D. $\frac{1}{2}\left[\left(\vec{a}+\vec{b}\right)\times\vec{c}-\left(\vec{a}+\vec{b}\right)\right]$

Answer: d

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213. vertors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make an angle of 60° with each other . $\vec{x}\times\left(\vec{y}\times\vec{z}\right)=\vec{a}, \vec{y}\times\left(\vec{z}\times\vec{x}\right)=\vec{b}$ and $\vec{x}\times\vec{y}=\vec{c}$

Vector $\vec{x}$ is

A. $\frac{1}{2}\left[\left(\vec{a}+\vec{c}\right)\times\vec{b}-\vec{b}-\vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a}-\vec{c}\right)\times\vec{b}+\vec{b}+\vec{a}\right]$

C. $\frac{1}{2}\left[\left(\vec{a}-\vec{b}\right)\times\vec{c}+\vec{b}+\vec{a}\right]$

D. $\frac{1}{2}\left[\left(\vec{a}-\vec{c}\right)\times\vec{a}+\vec{b}-\vec{a}\right]$

Answer: c

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214. vertors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make an angle of 60° with each other . $\vec{x} \times \left(\vec{y} \times \vec{z}\right) = \vec{a}, \vec{y} \times \left(\vec{z} \times \vec{x}\right) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$

Vector $\vec{x}$ is

A. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{c} - \vec{b} + \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \vec{b} - \vec{a}\right]$

C. $\frac{1}{2}\left[\vec{c} \times \left(\vec{a} - \vec{b}\right) + \vec{b} + \vec{a}\right]$

D. none of these

Answer: b

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215. If $\vec{x} \times \vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}. \vec{b} = \gamma, \vec{x}. \vec{y} = 1$ and $\vec{y}. \vec{z} = 1$ then find x,y,z in terms of $\vec{a}, \vec{b}$ and γ.

A. $\dfrac{1}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: b

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216. Find the derivative of $y = \cos^{-1}(1 - x)$.

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217. Find the derivative of $y = \sin^{-1}\left(1 - x^2\right)$.

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218. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then $\left(\vec{P} \times \vec{B}\right) \times \vec{B}$ is equal to

A. $\vec{P}$

B. $-\vec{P}$

C. $2\vec{B}$

D. $\vec{A}$

Answer: b

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219. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then $\vec{P}$ is equal to

A. $\frac{\vec{A}}{2} + \frac{\vec{A} \times \vec{B}}{2}$

B. $\frac{\vec{A}}{2} + \frac{\vec{B} \times \vec{A}}{2}$

C. $\frac{\vec{A} \times \vec{B}}{2}$ - $\frac{\vec{A}}{2}$

D. $\vec{A} \times \vec{B}$

Answer: b

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220. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A}$ - $\vec{P}$. then which of the following statements is false ?

A. vectors $\vec{P}, \vec{A}$ and $\vec{P} \times \vec{B}$ ar linearly dependent.

B. vectors $\vec{P}, \vec{B}$ and $\vec{P} \times \vec{B}$ ar linearly independent

C. $\vec{P}$ is orthogonal to $\vec{B}$ and has length $\frac{1}{\sqrt{2}}$.

D. none of these

Answer: d

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221. Find the derivative of $y = \cos 2x^6$.

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222. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a} on \vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1 on \vec{c}$. Then $\vec{a}_1 . \vec{b}$ is equal to

A. -41

B. -41/7

C. 41

D. 287

Answer: a

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223. Find the derivative of $y = 2\sin 3x + 5\cos 3x^4$.

A.

B.

C.

D.

Answer: c

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224. Consider a triangular pyramid ABCD the position vectors of whose anglar points are A(3,0,1) , B(-1,4,1) C(5,2,3) and D(0,-5,4) . Let G be the point of intersection of the medians of tiangle BCD

The length of the perpendicular from vertex D on the opposite face is

A. $\sqrt{17}$

B. $\sqrt{51}/3$

C. $3/\sqrt{6}$

D. $\sqrt{59}/4$

Answer: b

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225. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the vector $\overline{AG}$ is

A. 24

B. $8\sqrt{6}$

C. $4\sqrt{6}$

D. none of these

Answer: c

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226. Consider a triangular pyramid ABCD the position vectors of whose anglar points are A(3,0,1) , B(-1,4,1) C(5,2,3) and D(0,-5,4) . Let G be the point of intersection of the medians of tiangle BCD

The length of the perpendicular from vertex D on the opposite face is

A. $14/\sqrt{6}$

B. $2/\sqrt{6}$

C. $3/\sqrt{6}$

D. none of these

Answer: a

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227. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D.

The distance between the parallel lines AB and CD is

A. $\sqrt{6}$

B. $3\sqrt{6/5}$

C. $2\sqrt{2}$

D. 3

Answer: c

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228. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D.

the orthogonal projections of the parallelgram on the three coordinate planes xy, yz nad zx. Respectively, are

A. $\frac{4\sqrt{6}}{9}$

B. $\frac{32\sqrt{6}}{9}$

C. $\frac{16\sqrt{6}}{9}$

D. none

Answer: b

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229. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D.

The distance between the parallel lines AB and CD is

A. 14, 4,2

B. 2,4,14

C. 4,2,14

D. 2,14,4

Answer: d

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230. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j}-8\hat{i}-\vec{r}\right)=40$ and $p_1 = \max\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. p1+p2 equals

A. 2

B. 10

C. 18

D. 5

Answer: d

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231. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $p_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. p1+p2 is equal to

A. 2

B. 10

C. 18

D. 5

Answer: c

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232. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $p_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. p1+p2 is equal to

A. 2

B. 10

C. 18

D. 5

Answer: c

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233. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: a

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234. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{|\vec{a}|^2} + \frac{3\left(\vec{b} \times \vec{a}\right)}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{|\vec{a}|^2} - \frac{3\left(\vec{b} \times \vec{a}\right)}{|\vec{a}|^2}$

D. none of these

Answer: b

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235. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{\left|\vec{a}\right|^2} + \frac{3\left(\vec{b} \times \vec{a}\right)}{\left|\vec{a}\right|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{\left|\vec{a}\right|^2} - \frac{3\left(\vec{b} \times \vec{a}\right)}{\left|\vec{a}\right|^2}$

D. none of these

Answer: c

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236. Find the derivative of $y = 2\sin 3x$.

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237. Find the derivative of $y = \ln 2x$

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238. Differentiate $y = \cos\left(3x^2 + 2\right)$.

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239. Let $\vec{p}$ and $\vec{q}$ any two othogonal vectors of equal magnitude 4 each. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be any three vectors of lengths $7\sqrt{15}$ and $2\sqrt{33}$, mutually perpendicular to each other. Then find the distance of the vector $(\vec{a}.\vec{p})\vec{p} + (\vec{a}.\vec{q})\vec{q} + (\vec{a}.(\vec{p} \times \vec{q}))(\vec{p} \times \vec{q}) + (\vec{b}.\vec{p})\vec{p} + (\vec{b}.\vec{p})\vec{q} + (\vec{b}.(\vec{b}.\vec{q}))($ from the origin.

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240. Four lines $x + 3y - 10 = 0, x + 3y - 20 = 0, 3x - y + 5 = 0$ and $3x - y - 5 = 0$ form a figure which is.

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241. Draw the graph of $y = x - \sin x$

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242. Valume of parallelpiped formed by vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ is 36 sq. units.

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243. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\frac{3\left|\vec{a} + \vec{b}\right|}{2} + 2\left|\vec{a} - \vec{b}\right|$

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244. Let $\vec{u}$ be a vector on rectangular coodinate system with sloping angle 60° suppose that $|\vec{u} - \hat{i}|$ is geomtric mean of $|\vec{u}|$ and $|\vec{u} - 2\hat{i}|$, where $\hat{i}$ is the unit vector along the x-axis . Then find the value of $(\sqrt{2} - 1)|\vec{u}|$

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245. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2)$; $B(1, 2, 3) and C(5, 1, 1)$ is minimum.

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246. Ifveca=a_(1)hati+a_(2)hatj+a_(3)hatk, vecb= b_(1)hati+b_(2)hatj + b_(3)hatk, vecc=c_(1)hati+c_(2)hatj+c_(3)hatk and [3veca+vecb=vecc 3vecc + veca] =lambda|{:(veca.hati,veca.hatj,veca.hatk),(vecb.hati,veca.hatj,hatb.hatk),(vecc.hati,vecc.hatj,vecc.hatk):}| " then find the value of " lambda/4`

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247. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value of 6α. Such that $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = 0$

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248. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a-2)\alpha^2 + (b-3)\alpha + c\right]\vec{x} + \left[(a-2)\beta^2 + (b-3)\beta + c\right]\vec{y} + \left[(a-2)\gamma^2 + (b-3)\gamma + c\right.$

are three distinct real numbers, then find the value of $\left(a^2 + b^2 + c^2 - 4\right)$.

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249. Let $\vec{u} and \vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$.

Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]$.

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250. The volume of the tetrahedronwhose vertices are the points with position vectors $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 7\hat{k}$, $5\hat{i} - \hat{j} + \lambda\hat{k}$ and $7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic units then the value of λ is (A) 7 (B) 1 (C) -7 (D) -1

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251. Given that vec u= hat i-2 hat j+3 hat k ; vec v=2 hat i+ hat j+4 hat k ; vec w= hat i+3 hat j+3 hat ka n d(vec udot vec R-15) hat i+(vec vdot vec R-30) hat j+(vec wdot vec R-20) hat k=0. Then find the greatest integer less than or equal to | vec R|dot

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252. Let a three dimensional vector $\vec{V}$ satisfy the condition, $2\vec{V} + \vec{V} \times (\hat{i} + 2\hat{j}) = 2\hat{i} + \hat{k}$. If $3|\vec{V}| = \sqrt{m}$. Then find the value of m.

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253. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c} is \pi/3$ then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is

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254. Let $\vec{O}A - \vec{a}, \vec{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB,$ and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq,$ then find$k$.

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255. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acting on a particle such that the particle is displaced from point $A(-3, -4, 1)$ and $B(-1, -1, -2)$.

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256. from a point O inside a triangle ABC, perpendiculars, OD, OE and OF are drawn to the sides, BC, CA and AB respectively , prove that the perpendiculars from A, B and C to the sides EF, FD and DE are concurrent.

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257. $A_1, A_2, \ldots. A_n$ are the vertices of a regular plane polygon with n sides and O ars its centre. Show that

$$\sum^{n-1} i = 1\left(\overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}}\right) = (1-n)\left(\overrightarrow{OA_2} \times \overrightarrow{OA_1}\right)$$

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258. If c is a given non - zero scalar, and $\vec{A}$ and $\vec{B}$ are given non- zero , vectors such that $\vec{A} \perp \vec{B}$. Then find vector, $\vec{X}$ which satisfies the equations $\vec{A}.\vec{X} = c$ and $\vec{A} \times \vec{X} = \vec{B}$.

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259. If A, B , C ,D are any four points in space, prove that

$\left|\overrightarrow{AB} \times \overrightarrow{CD} \times \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}\right|$ = 4 (area of triangle ABC).

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260. If vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, show that $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$

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261. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$ Determine a vector $\vec{R}$ satisfying

$\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}\vec{A} = 0$.

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262. Determine the value of c so that for all real x , vectors $cx\hat{i} - 6\hat{j} - 3\hat{k} and x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

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263. Prove that:

$$\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) + \left(\vec{a} \times \vec{c}\right) \times \left(\vec{d} \times \vec{b}\right) + \left(\vec{a} \times \vec{d}\right) \times \left(\vec{b} \times \vec{c}\right) = -2\left[\vec{b}\vec{c}\vec{d}\right]\vec{a}$$

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264. about to only mathematics

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265. $Let \vec{a}, \vec{b},$ and $\vec{c}$ be three non-coplanar ubit vectors such the angle between every pair of them is $\frac{\pi}{3}$. if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p,q and r are scalars , then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

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266. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\left|\vec{b}\right| = \left|\vec{c}\right|$ then $\left\{\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)\right\} \times \left(\vec{b} \times \vec{c}\right).\left(\vec{b} + \vec{c}\right) =$

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267. For any two vectors $\vec{u}$ and $\vec{v}$ prove that $\left(1 + \left|\vec{u}\right|^2\left(1 + \left|\vec{v}\right|^{20} = \left(1 - \vec{u}.\vec{c}\right)^2 + \ \left| \ \vec{u} + \vec{v} + \vec{u} \times \vec{|}^2\right.\right.\right.$

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268. Find the derivative of $y = 3\cos^{-1}\left(x^2 + 0.5\right)$.

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269. find three- dimensional vectors, $\vec{v}1, \vec{v}2$ and $\vec{v}3$satisfying $\vec{v}_1 . \vec{v}_2 = -2, \vec{v}_1 . Vecv_3 = 6, \vec{v}_2, \vec{v}_2 = 2\vec{v}_2 . Vecv_3 = -5,$

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270. Let V be the volume of the parallelepied formed by the vectors, $\vec{a} = a_1\hat{i} = a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. if $a_r b_r nadc_r$ are non- negative real numbers and $\sum^{3} r=1\left(a_r + b_r + c_r\right) = 3L$ show that $V \leq L^3$

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271. $\vec{u}, \vec{v}$ and $\vec{w}$ are three nono-coplanar unit vectors and α, β and γ are the angles between $\vec{u}$ and $\vec{u}, \vec{v}$ and $\vec{w}$ and $\vec{w}$ and $\vec{u}$, respectively and $\vec{x}, \vec{y}$ and $\vec{z}$ are unit vectors along the bisectors of the angles α, β and γ. respectively, prove that $\left[\vec{x} \times \vec{y}\vec{y} \times \vec{z}\vec{z} \times \vec{x}\right) = \frac{1}{16}\left[\vec{u}\vec{v}\vec{w}\right]^2 \frac{\sec^2\alpha}{2} \frac{\sec^2\beta}{2} \frac{\sec^2\gamma}{2}$.

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272. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$. Prove that $\left(\vec{a} - \vec{d}\right).\left(\vec{c} - \vec{b}\right) \neq 0, i.e., \vec{a}.\vec{b} + \vec{d}.\vec{c} \neq \vec{d}.\vec{b} + \vec{a}.\vec{c}$.

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273. $P_1 ndP_2$ are planes passing through origin $L_1 andL_2$ are two lines on $P_1 andP_2$, respectively, such that their intersection is the origin. Show that there exist points $A, BandC$, whose permutation $A', B' andC'$, respectively, can be chosen such that A is on $L_1, BonP_1$ but not on $L_1 andC$ not on P_1; A' is on $L_2, B' onP_2$ but not on $L_2 andC'$ not on P_2

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274. about to only mathematics

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275. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be vectors of legth , 3,4and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to$\vec{A}+\vec{B}$ then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is ________.

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276. The unit vector perendicular to the plane determined by P (1,-1,2) ,C(3,-1,2) is _______.

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277. the area of the triangle whose vertices are A (1,-1,2) , B (1,2, -1) ,C (3, -1, 2) is _______.

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278. If $\left[\vec{a}, \vec{b}, \vec{c}\right] = 1$ then the value of $\frac{\vec{a}.\left(\vec{b} \times \vec{c}\right)}{\left(\vec{c} \times \vec{a}\right).\vec{b}} + \frac{\vec{b}.\left(\vec{c} \times \vec{a}\right)}{\left(\vec{a} \times \vec{b}\right).\vec{c}} + \frac{\vec{c}.\left(\vec{a} \times \vec{b}\right)}{\left(\vec{c} \times \vec{b}\right).\vec{a}}$ is_______

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279. If $\vec{A} = (1, 1, 1)$ and $\vec{C} = (0, 1, -1)$ are given vectors the vector $\vec{B}$ satisfying the equations $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{A}.\vec{B} = 3$ is _______.

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280. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy- plane. All vectors in the sme plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$., respectively, are given by _______

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281. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are _______ and _________, respectively.

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282. A unit vector coplanar with $\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ and perpendicular to $\vec{i} + \vec{j} + \vec{k}$ is ______

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283. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by vectors $\hat{i}$ and $\hat{i} + \hat{j}$ and the plane determined by verctors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{k}$. The angle between $\vec{a}$ and vectors $\hat{i} - 2\hat{j} + 2\hat{k}$ is ______

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284. if $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors and $\vec{a}$ is any vector, then $(\vec{a}.\vec{b})\vec{b} + (\vec{a}.\vec{c})\vec{c} + \frac{\vec{a}.(\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|}(\vec{b} \times \vec{c}) =$ ________

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285. let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2, respectively, if $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is ______

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286. A, B C and D are four points in a plane with position vectors, $\vec{a}, \vec{b}\vec{c}$ and $\vec{d}$ respectively, such that $(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$ then point D is the _____ of triangle ABC.

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287. If $\vec{A} = \lambda(\vec{u} \times \vec{v}) + \mu(\vec{v} \times \vec{w}) + v(\vec{w} \times \vec{u})$ and $[\vec{u}\vec{v}\vec{w}] = \frac{1}{5} then \lambda + \mu + v =$ (A) 5 (B) 10 (C) 15 (D) none of these

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288. If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$ $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle , then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is ______

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289. If $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$

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290. If $\vec{x}.\vec{a}=0$, $\vec{x}.\vec{b}=0$, $\vec{x}.\vec{c}=0$ and $\vec{x}\neq\vec{0}$ then show yhat $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar .

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291. for any three vectors, $\vec{a},\vec{b}$ and $\vec{c}$, $\left(\vec{a}-\vec{b}\right).\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)=2\vec{a}.\vec{b}\times\vec{c}$.

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292. The scalar $\vec{A}.\left(\vec{B}+\vec{C}\right)\times\left(\vec{A}+\vec{B}+\vec{C}\right)$ equals (A) 0 (B) $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$ (C) $\left[\vec{A}\vec{B}\vec{C}\right]$ (D) none of these

A. 0

B. $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$

C. $\left[\vec{A}\vec{B}\vec{C}\right]$

D. none of these

Answer: a

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293. For non zero vectors $\vec{a}, \vec{b}, \vec{c}$

$\left|\left(\vec{a} \times \vec{b}\right). \vec{c}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$ holds iff

A. $\vec{a}. \vec{b} = 0, \vec{b}. \vec{c} = 0$

B. $\vec{b}. \vec{c} = 0, \vec{c}, \vec{a} = 0$

C. $\vec{c}. \vec{a} = 0, \vec{a}, \vec{b} = 0$

D. $\vec{a}. \vec{b} = \vec{b}. \vec{c} = \vec{c}. \vec{a} = 0$

Answer: d

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294. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2, j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is $4/13$ b. 4 c. $2/7$ d. 2

A. $4/13$

B. 4

C. $2/7$

D. 2

Answer: d

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295. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vector and $\vec{p}$, $\vec{q}$ $\vec{r}$ are defind by the relations $\vec{p}=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{q}=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{r}=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, then $\vec{p}.\left(\vec{a}+\vec{b}\right)+\vec{q}.\left(\vec{b}+\vec{c}\right)+\vec{r}.\left(\vec{c}+\vec{a}\right)$ =............

A. 0

B. 1

C. 2

D. 3

Answer: d

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296. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\vec{a}.\hat{d} = 0 = \left[\vec{b}, \vec{c}, \vec{d}\right]$ then hat$d equals (A)$+-(hati+hatj-2hatk)/sqrt(6)(B)+-(hati+hatj-hatk)/sqrt(3)(C)+-(hati+hatj+hatk)/sqrt(3)(D)+-hatk`

A. $\pm\dfrac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

B. $\pm\dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

C. $\pm\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

D. $\pm\hat{k}$

Answer: a

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297. If $\vec{a}, \vec{b}, \vec{c}$ are three non - coplanar vector such that $\vec{a} \times (\vec{b} \times \vec{c}) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is ....

A. $3\pi/4$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: a

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298. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vector such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $|\vec{u}| = 3, |\vec{v}| = 4$ and $|\vec{w}| = 5$ then $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is ....

A. 47

B. -25

C. 0

D. 25

Answer: b

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299. If $\vec{a}, \vec{b}$ and $\vec{c}$1 are three non-coplanar vectors, then $\left(\vec{a}+\vec{b}+\vec{c}\right).\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)\right]$ equals

A. 0

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. $2\left[\vec{a}\vec{b}\vec{c}\right]$

D. - $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: d

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300. $\vec{p}, \vec{q}, and \vec{r}$ are three mutually perpendicular vectors of the same magnitude. If vector $\vec{x}$ satisfies the equation $\vec{p} \times ((\vec{x} - \vec{q}) \times \vec{p}) + \vec{q} \times ((\vec{x} - \vec{r}) \times \vec{q}) + \vec{r} \times ((\vec{x} - \vec{p}) \times \vec{r}) = 0, then \vec{x}$ is given by $\frac{1}{2}(\vec{p} + \vec{q} - 2\vec{r})$ b. $\frac{1}{2}(\vec{p} + \vec{q} + \vec{r})$ c. $\frac{1}{3}(\vec{p} + \vec{q} + \vec{r})$ d. $\frac{1}{3}(2\vec{p} + \vec{q} - \vec{r})$

A. $\frac{1}{2}(\vec{p} + \vec{q} - 2\vec{r})$

B. $\frac{1}{2}(\vec{p} + \vec{q} + \vec{r})$

C. $\frac{1}{3}(\vec{p} + \vec{q} + \vec{r})$

D. $\frac{1}{3}(2\vec{p} + \vec{q} - \vec{r})$

Answer: b

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301. Let $\vec{a} = 2\vec{j} + \vec{j} - 2\vec{k}, \vec{b} = \vec{i} + \vec{j}.$ If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $30°$. Find the value of $|(\vec{a} \times \vec{b}) \times \vec{c}|$

A. 2/3

B. 3/2

C. 2

D. 3

Answer: b

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302. Let $\vec{a} = 2i + j + k, \vec{b} = i + 2j - k$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is $\frac{1}{\sqrt{2}}(-j + k)$ b. $\frac{1}{\sqrt{3}}(-i - j - k)$ c. $\frac{1}{\sqrt{5}}(-k - 2j)$ d. $\frac{1}{\sqrt{3}}(i - j - k)$

A. $\frac{1}{\sqrt{2}}(-j + k)$

B. $\frac{1}{\sqrt{3}}(i - j - k)$

C. $\frac{1}{\sqrt{5}}(i - 2j)$

D. $\frac{1}{\sqrt{3}}(i - j - k)$

Answer: a

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303. If the vectors $\vec{a}, \vec{b}, \vec{c}$ form the sides BC,CA and AB respectively of a triangle ABC then (A) $\vec{a}.\left(\vec{b}\times\vec{c}\right)=\vec{0}$ (B) $\vec{a}\times\left(\vec{b}x\vec{c}\right)=\vec{0}$ (C) $\vec{a}.\vec{b}=\vec{c}=\vec{c}=\vec{a}.a\neq 0$ (D) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\vec{0}$

A. $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$

B. $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

C. $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$

D. $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

Answer: b

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304. Consider the vectors, $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$ Let P_1 and P_2 be the planes determined by the pairs of vectors, $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

A. 0

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: a

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305. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors , then the scalar triple porduct $\left[2\vec{a} - \vec{b} \quad 2\vec{b} - \vec{c} \quad 2\vec{c} - \vec{a}\right]$is

A. 0

B. 1

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: a

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306. If $\hat{a}, \hat{b}, and \hat{c}$ are unit vectors, then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\hat{c} - \hat{a}\right|^2$ does not exceed 45^0 b. 60^0 c. $\cos^{-1}(1/3)$ d. $\cos^{-1}(2/7)$

A. 4

B. 9

C. 8

D. 6

Answer: b

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307. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is (A) 45^0 (B) 60^0 (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

A. $45°$

B. $60°$

C. $\cos^{-1}(1/3)$

D. $\cos^{-1}(2/7)$

Answer: b

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308. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is a.-1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

A. -1

B. $\sqrt{10}+\sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: c

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309. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i}+a\hat{j}+k, \hat{j}+a\hat{k}anda\hat{i}+\hat{k}$ becomes minimum.

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: c

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310. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{k}$, then $\vec{b}.\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right) =$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{i} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: c

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311. The unit vector which is orthogonal to the vector $5\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$is

A. $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

C. $\frac{3\hat{i} - \hat{k}}{\sqrt{10}}$

D. $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

Answer: c

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312. if $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non- coplanar vectors and

$$\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1, \vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}}{|\vec{b}}$$

, then the set of orthogonal vectors is

A. $(\vec{a}, \vec{b}_1, \vec{c}_3)$

B. $(\vec{c}a, \vec{b}_1, \vec{c}_2)$

C. $(\vec{a}, \vec{b}_1, \vec{c}_1)$

D. $(\vec{a}, \vec{b}_2, \vec{c}_2)$

Answer: c

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313. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vectors $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c} is \frac{1}{\sqrt{3}}$ is given by

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $3\hat{i} + \hat{j} - 3\hat{k}$

C. $2\hat{i} + \hat{j} - 2\hat{k}$

D. $4\hat{i} + \hat{j} - 4\hat{k}$

Answer: a

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314. Let two non-collinear unit vector $\hat{a} and \hat{b}$ form an acute angle. A point P moves so that at any tgiem t, the position vector $OP(where O$ is the origin) is given by $\hat{a}\cot t + \hat{b}\sin t When P$ is farthest from origin O, let M be the length of $OP and \hat{u}$ be the unit vector along OP Then

$\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} and M = \left(1 + \hat{a}\dot{\hat{b}}\right)^{1/2}$ $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|} and M = \left(1 + \hat{a}\dot{\hat{b}}\right)^{1/2}$

$\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} and M = \left(1 + 2\hat{a}\dot{\hat{b}}\right)^{1/2}$ $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|} and M = \left(1 + 2\hat{a}\dot{\hat{b}}\right)^{1/2}$

A. , $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{1/2}$

B. , $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{1/2}$

C. $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{1/2}$

D. , $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{1/2}$

Answer: a

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315. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right).\left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$ then (A) $\vec{a}, \vec{b}, \vec{c}$ are non coplanar (B)

$\vec{b}, \vec{c}, \vec{d}$ are non coplanar (C) $\vec{b}, \vec{d}$ are non paralel (D) $\vec{a}, \vec{d}$ are paralel and $\vec{b}, \vec{c}$ are parallel

A. $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar

B. $\vec{b}, \vec{c}$ and $\vec{d}$ are non-coplanar

C. $\vec{b}$ and $\vec{d}$ are non- parallel

D. $\vec{a}$ and $\vec{d}$ are parallel and $\vec{b}$ and $\vec{c}$ are parallel

Answer: c

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316. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by a. $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: b

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317. Let P, Q, R and S be the points on the plane with position vectors $-2i - j, 4i, 3i + 3jand - 3j + 2j,$ respectively. The quadrilateral $PQRS$ must be a Parallelogram, which is neither a rhombus nor a rectangle Square Rectangle, but not a square Rhombus, but not a square

A. Parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square.

Answer: a

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318. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vectors $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c} is \frac{1}{\sqrt{3}}$ is given by

A. $\hat{i} - 3\hat{j} + 3\hat{k}$

B. $-3\hat{i} - 3\hat{j} + \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

D. $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: c

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319. Let $\overline{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overline{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overline{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overline{PT}, \overline{PQ}$ and $\overline{PS}$ is

A. 5

B. 20

C. 10

D. 30

Answer: c

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320. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ gve three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both

$\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is \frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} p = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)$

D. $\frac{3}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)\left(c_1^2 + c_2^2 + c_2^2\right)$

Answer: c

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321. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

A. one

B. two

C. three

D. infinite

Answer: b

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322. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{j} + 2\hat{j} - \hat{k}, \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$. A vector coplanar with $\vec{b}$ and $\vec{c}$. Whose projection on $\vec{a}$ is magnitude $\sqrt{\frac{2}{3}}$ is

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $2\hat{i} + 3\hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: a,c

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323. For three vectors $\vec{u}, \vec{v} and \vec{w}$ which of the following expressions is not equal to any of the remaining three ? a.$\vec{u}.\left(\vec{v}\times\vec{w}\right)$ b. $\left(\vec{v}\times\vec{w}\right).\vec{u}$ c. $\vec{v}.\left(\vec{u}\times\vec{w}\right)$ d. $\left(\vec{u}\times\vec{v}\right).\vec{w}$

A. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$

B. $\left(\vec{v}\times\vec{w}\right).\vec{u}$

C. $\vec{v}.\left(\vec{u}\times\vec{w}\right)$

D. $\left(\vec{u}\times\vec{v}\right).\vec{w}$

Answer: c

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324. Which of the following expressions are meaningful? a.$\vec{u}\left(\vec{v}\times\vec{w}\right)$ b. $\left(\vec{u}.\vec{v}\right).\vec{w}$ c. $\left(\vec{u}.\vec{v}\right)\vec{w}$ d. $\vec{u}\times\left(\vec{v}.\vec{w}\right)$

A. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$

B. $\left(\vec{u}.\vec{v}\right).\vec{w}$

C. $(\vec{u}.\vec{v})\vec{w}$

D. $\vec{u} \times (\vec{v}.Vecw)$

Answer: a,c

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325. Let $A = \{2, 3, 4\}, B = \{7, 8, 9, 10\}$ and $f = \{(2, 7), (3, 8), (4, 9)\}$ be a function from A to B. Show that f is one to one but not onto function.

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326. Vector $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$ is

A. a unit vector

B. makes an angle $\pi/3$ with vector $(2\hat{i} - 4\hat{j} + 3\hat{k})$

C. parallel to vector $\left(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$

D. perpendicular to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Answer: a,c,d

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327. Let $\vec{A}$ be a vector parallel to the line of intersection of planes $P_1andP_2\dot{}$ Plane P_1 is parallel to vectors $2\hat{j}+3\hat{k}and4\hat{j}-3kandP_2$ is parallel to $\hat{j}-\hat{k}and3\hat{i}+3\hat{j}\dot{}$ Then the angle betweenvector $\vec{A}$ and a given vector $2\hat{i}+\hat{j}-2\hat{k}$ is $\pi/2$ b. $\pi/4$ c. $\pi/6$ d. $3\pi/4$

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/4$

Answer: b,d

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328. The unit vector parallel to the resultant of the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ is:

A. $\hat{j} - \hat{k}$

B. $-\hat{i} + \hat{j}$

C. $\hat{i} - \hat{j}$

D. $-\hat{j} + \hat{k}$

Answer: a,d

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329. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vector each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. if $vcea$ is a non - zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

A. $\vec{b} = \left(\vec{b}. \vec{z}\right)\left(\vec{z} - \vec{x}\right)$

B. $\vec{a} = (\vec{a}.\vec{y})(\vec{y} - \vec{z})$

C. $\vec{a}.\vec{b} = -(\vec{a}.\vec{y})(\vec{b}.\vec{z})$

D. $\vec{a} = (\vec{a}.\vec{y})(\vec{z} - \vec{y})$

Answer: a,b,c

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330. Let ΔPQR be a triangle Let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$ if $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true ?

A. $\dfrac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

B. $\dfrac{|\vec{c}|^2}{2} - |\vec{a}| = 30$

C. $\left|\vec{a} \times \vec{v}b + \vec{c} \times \vec{a}\right| = 48\sqrt{3}$

D. $\vec{a}.\vec{b} = -72$

Answer: a,c,d

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331. Draw the graph of $y = \log_e(-x)$, flip the graph of $y = \log_e x$ over the y-axis

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332. Find the derivative of $y = \ln x^2$

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333. Find the derivative of $y = 2\ln\left(3x^2 - 1\right)$

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334. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$

$\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$, is

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335. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$ then find the value of $\vec{r}.\vec{b}$.

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336. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is

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337. $Let \vec{a}, \vec{b},$ and $\vec{c}$ be three non-coplanar ubit vectors such the angle between every pair of them is $\frac{\pi}{3}$. if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p,q and r are scalars , then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

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