MATHS

BOOKS - CENGAGE

DIFFERENTIATION

Examples

1. $\int \cot^{-1} \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}\, dx$ is:

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2. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x using the first principle.

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3. If $f(x) = x\tan^{-1}x,$ find $f'(\sqrt{3})$ using the first principle.

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4. $f(x) = [2x]\sin 3\pi x and f'(k') = \lambda k\pi(-1)^k$ (where [.] denotes the greatest integer function and $k \in N),$ then find the value of λ .

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5. Let $f: R^{\rightarrow}$ satisfying $|f(x)| \leq x^2 \forall x \in R$ be differentiable at $x = 0.$ The find $f'(0).$

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6. A function $f: R \to R$ is such that $f(x+y) = f(x).f(y)$ for all x.y $\in R$ and $f(x) \neq 0$ for all $x \in R.$ If $f'(0) = 2$ then $f'(x)$ is equal to

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7. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$, then find $\frac{dy}{dx}$.

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8. If $f(x) = x|x|$, then prove that $f'(x) = 2|x|$

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9. If $y = 1 = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + + \frac{x^n}{n!}$, show that $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0.$

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10. Find $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x)$, where $x \in (0, 2\pi)$.

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11. Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to x is

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12. simplify : $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)\frac{-\pi}{2} < x < \frac{\pi}{2}, \frac{a}{b}\tan x > -1$

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13. $y = \sin^{-1}\left(\frac{x}{1+x^2}\right) + \cos^{-1}\left(\frac{x}{1+x^2}\right)$, where `0

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14. Find $\frac{dy}{dx}$ for $y = \sin\left(x^2+1\right)$.

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15. If $y = \sqrt{\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}$, $then f \in d\frac{dy}{dx}$.

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16. Differentiate the function $f(x) = \sec\left(\tan\left(\sqrt{x}\right)\right.$ with respect to x.

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17. Find $\frac{dy}{dx} f$ or $y = \log\left(x + \sqrt{a^2 + x^2}\right)$.

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18. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$, $where$ `-1

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19. Find $\frac{dy}{dx}$ for `y=tan^(-1)sqrt((a-x)/(a+x),)-a

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20. The value of $\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$ is equal to $\sin^{-1}x+\sin^{-1}\sqrt{x}\ \sin^{-1}x-\sin^{-1}\sqrt{x}\ \sin^{-1}\sqrt{x}-\sin^{-1}x$ none of these

Watch Video Solution

21. If $y=\frac{\tan^{-1}1}{1+x+x^2}+\frac{\tan^{-1}1}{x^2+3x+3}+\frac{\tan^{-1}1}{x^2+5x+7}++$ upto n terms, then find the value of $y'(0)$.

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22. Let $f:R\overrightarrow{R}$ be a one-one onto differentiable function, such that $f(2)=1 and f'(2)=3$. The find the value of $\left(\left(\frac{d}{dx}\left(f^{-1}(x)\right)\right)\right)_{x=1}$

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23. If $f(x) = \cos x \cos 2x \cos 4x \cos(8x).\cos 16x$ then find $f'\left(\frac{\pi}{4}\right)$

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24. If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

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25. Find $\frac{dy}{dx} f$ or $y = x \sin x \log x$.

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26. Evaluate $\lim_{h \to 0} \frac{(a+h)^2.\sin^{-1}(a+h) - a^2 \sin^{-1} a}{h}$.

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27. Differentiate $y = \dfrac{e^x}{1 + \sin x}$

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31. Find the sum of the series $1 + 2x + 3x^2 + (n-1)x^{n-2})$ using differentiation.

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32. If $\sqrt{x} + \sqrt{y} = 4$, then find $\frac{dy}{dx}$.

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33. If $xy + y^2 = \tan x + y, then f \in d\frac{dy}{dx}$.

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34. If $y = x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \cdots}}}$, prove that $\frac{dy}{dx} = \frac{y}{2y - x}$.

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35. $\sec(x+y) = xy$

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36. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dot{\longrightarrow} \infty}}}$, prove that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

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37. If $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

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38. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to_____

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39. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$.

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40. If $x = a\ sec^3\theta$ and $y = a\ tan^3\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

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41. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$

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42. Let $y = x^3 - 8x + 7 and x = f(t)$ and $\frac{dy}{dx} = 2 and x = 3att = 0$, then find the value of $\frac{dx}{dt} att = 0$

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43. Find the derivative of $\frac{\sqrt{x}(x+4)^{\frac{3}{2}}}{(4x-3)^{\frac{4}{3}}}$

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44. If $x^m y^n = (x+y)^{m+n}$, $prove \frac{dy}{dx} = \frac{y}{x}$.

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45. Differentiate $(\log x)^{\cos x}$ with respect to x.

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46. If $f(x) = |x|^{|\sin x|}$, then find $f'\left(-\frac{\pi}{4}\right)$

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47. If $y = x^{x\ \hat{}\ x \wedge ((((\infty))))}$, find $\frac{dy}{dx}$.

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48. Let $f(x) = (\lim)_{h \to 0} \frac{(\sin(x+h))^{1n(x+h)} - (\sin x)^{1nx}}{h}$. Then $f\left(\frac{\pi}{2}\right)$ equal to 0 (b) equal to 1 In $\frac{\pi}{2}$ (d) non-existent

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49. If $x < 1, provethat \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \infty = \frac{1}{1-x}$

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50. Differentiate log sin x w.r.t. $\sqrt{x}$.

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51. If $\frac{\tan^{-1}\left(\sqrt{1+x^2}-1\right)}{x}=4^0$ then

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52. Find the derivative of $f(\tan x)wrtg(\sec x)atx=\frac{\pi}{4}$, where $f'(1)=2andg'\left(\sqrt{2}\right)=4.$

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53. Let $=\begin{vmatrix}\cos x & \sin x & \cos x\\ \cos 2x & \sin 2x & 2\cos 2x\\ \cos 3x & \sin 3x & 3\cos 3x\end{vmatrix}$ then find the values of f(0) and f'$(\pi/2)$.

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54. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ then find the value of $\lim_{x \to 0} \frac{f(x)}{x}$

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55. If $y = \cos^{-1} x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.

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57. If $y = x \log\left(\frac{x}{a+bx}\right), then x^3 \frac{d^2y}{dx^2} =$ (a) $\left(x\frac{dy}{dx} - y\right)$^2 (b) $x\left(\frac{dy}{dx}\right) - y(c)$ y(dy/dx)-x(d)(y(dy/dx)-x)^2`

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58. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

is a constant independent of a and b.

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59. If $y=e^{a\cos^{(-1)}x}, -1\le x<1$, *showthat*

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$$

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60. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{d^2y}{dx^2}$.

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61. If g is inverse of f then prove that $f''(g(x)) = -g''(x)(f'(g(x)))^3$.

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62. Let f(x) and g(x) be real valued functions such that f(x)g(x)=1, $\forall x \in R$. If $f''(x)$ and $g''(x)$ exists $\forall x \in R$ and $f'(x)$ and $g'(x)$ are never zero, then prove that $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)} = \frac{2f'(x)}{f(x)}$

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63. Prove that $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

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64. Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}f\ \text{or}\ allrealxandy$ If $f'(0)$ exists and equals $-1andf(0)=1,thenfindf(2).$

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65. $f(x)+f\left(y_{=}f\left(\frac{x+y}{1-xy}\right)\right.$ for all$x,y\in R.$ $(xy\neq 1),and(\lim)_{x\ 0}\rightarrow\frac{f(x)}{x}=2.F\in df\left(\frac{1}{\sqrt{3}}\right)andf'(1).$

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66. Let $f:R-\{0\}\to R$ be a function which is differentiable in its domain and satisfying the equation $f(x+y)=f(x)+f(y)+\frac{x+y}{xy}-\frac{1}{x+y}$, also f'(1)=2. Then find the function.

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67. Find function f(x) which is differentiable and satisfy the relation $f(x+y)=f(x)+f(y)+(e^x-1)(e^y-1)\,\forall x,y\in R,$ and $f'(0)=2.$

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68. If $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$,$\forall y,\ f(y)\neq 0$ and $f'(1)=2$, find f(x) .

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69. Let $f:R\overrightarrow{R}$ be a function satisfying condition $f\left(x+y^3\right)=f(x)+[f(y)]^3 f$ or $allx,y\in R\cdot$ If $f'(0)\geq 0,$ find $f(10)\cdot$

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70. Let $f(x+y)=f(x)+f(y)+2xy-1$ for all real $xandy$ and $f(x)$ be a differentiable function. If $f'(0)=\cos\alpha,$ then prove that $f(x)>0\forall x\in R\cdot$

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71. If $f(x) = (\log)_x(\ln x), then f'(x)$ at $x = e$ is equal to $\frac{1}{e}$ (b) e (c) 1 (d) zero

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72. If $\frac{\cos x}{2}\frac{\dot{\cos} x}{2^2}\frac{\dot{\cos} x}{2^3}\infty = \frac{\sin x}{x}$, then find the value of $\frac{1}{2^2}\frac{\sec^2 x}{2} + \frac{1}{2^4}\frac{\sec^2 x}{2^2} + \frac{1}{2^6}\frac{\sec^2 x}{2^3} + \infty$

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73. If $y = f(a^x) and f'(\sin x) = (\log)_e x, then f \in d\frac{dy}{dx}$, if it exists, where `pi/2

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74. If P_n is the sum of a $G\dot{P}$. upto n terms $(n \geq 3)$, then prove that $(1-r)\frac{dP_n}{dr} = (1-n)P_n + nP_{n-1}$, where r is the common ratio of $G\dot{P}$.

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75. If $g(x) - \frac{f(x)}{(x-a)(x-b)(x-c)}, where f(x)$ is a polynomial of degree < 3, then prove that $\frac{dg(x)}{dx} = \left|1af(a)(x-a)^{-2}1bf(b)(x-b)^{-2}1cf(c)(x-c)^{-2}\right| + \left|a^2a1b^2b1c\right.$

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76. If $x = \cos ec\theta - \sin\theta and y = \cos ec^n\theta - \sin^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$.

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77. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then prove that $\frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$

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78. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitary constants.

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79. If $y = \frac{1}{(2)^{n-1}}\cos\left(n\cos^{-1}x\right)$, then prove that y satisfies the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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80. Let $f(x) and g(x)$ be two function having finite nonzero third-order derivatives $f^x and g^x$ for all $x \in R$. If $f(x)g(x) = 1$ for all $x \in R$, then

prove that $f'f' - g'g' = 3\left(f'f - g'g\right)$.

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81. If a curve is represented parametrically by the equation $x = f(t)$ and $y = g(t)$ then prove that $\frac{d^2y}{dx^2} = -\left[\frac{g'(t)}{f'(t)}\right]^3\left(\frac{d^2x}{dy^2}\right)$

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82. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real $xandy$ and $f'(2) = 2$, then determine $y = f(x)$.

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83. If $f(x) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than $0andf(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}, thenf \in df(x)$.

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84. If $|a_1 \sin x + a_2 \sin 2x + \ + a_n \sin nx| \leq \ | \ |\sin x|$ for $x \in R$, then prove that $|a_1 + 2a_1 + 3a + 3 + na_n| \leq |$

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85. Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \ + a_n x^n$. If $|p(x)| \leq e^{x-1} - 1|$ for all $x \geq 0$, prove that $|a_1 + 2a_2 + \ + na_n| \leq 1$.

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86. If $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}, x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then find $\frac{dy}{dx}$.

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87. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x using the first principle.

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91. A function $f: R\overrightarrow{R}$ satisfies the equation $f(x+y) = f(x)f(y)$ for all $x, y \in Randf(x) \neq 0f$ or $allx \in R.$ If $f(x)$ is differentiable at

$x = 0 and f'(0) = 2$, then prove that $f'(x) = 2f(x)$.

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121. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots \to \infty}}}$, $prove that \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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136. Find the derivative of $f(\tan x) wr\dot{t} g(\sec x) at x = \frac{\pi}{4}$, where $f'(1) = 2 and g'\left(\sqrt{2}\right) = 4.$

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137. Let $= \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the values of f(0) and f'$(\pi/2)$.

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is a constant independent of a and b.

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143. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1,$ show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

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144. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

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145. If g is inverse of f then prove that $f''(g(x)) = -g''(x)(f'(g(x)))^3$.

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146. Let f(x) and g(x) be real valued functions such that f(x)g(x)=1, $\forall x \in R$. If $f''(x)$ and $g''(x)$ exists $\forall x \in R$ and $f'(x)$ and $g'(x)$ are never zero, then prove that $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)} = \frac{2f'(x)}{f(x)}$

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147. If $f(x), g(x) and h(x)$ are three polynomials of degree 2, then prove that

$\varphi(x) = |f(x)g(x)h(x)f'(x)g'(x)h'(x)f^x g^x h^x| is a constant polynomial$

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148. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2} f$ or $all real x and y$ If $f'(0)$ exists and equals $-1 and f(0) = 1, then find f(2).$

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149. $f(x) + f\left(y_{=} f\left(\frac{x+y}{1-xy}\right)\right.$ for all $x, y \in R.$ $(xy \neq 1), and (\lim)_{x \to 0} \frac{f(x)}{x} = 2. F \in df\left(\frac{1}{\sqrt{3}}\right) and f'(1).$

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150. Find function f(x) which is differentiable and satisfy the relation $f(x+y)=f(x)+f(y)+(e^x-1)(e^y-1)\,\forall x,y\in R,\ \text{and}\ f'(0)=2.$

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151. Find function f(x) which satisfy the relation $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}\,\forall x,y\in R, y\neq 0, f(y)\neq 0\ \text{and}\ f'(1)=2$

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152. Let $f:R\overrightarrow{R}$ be a function satisfying condition $f\left(x+y^3\right)=f(x)+[f(y)]^3 f\ \text{or}\ allx,y\in R\cdot$ If $f'(0)\geq 0,$ find $f(10)\cdot$

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153. Let $f(x+y)=f(x)+f(y)+2xy-1$ for all real $xandy$ and $f(x)$ be a differentiable function. If $f'(0)=\cos\alpha,$ then prove that

$f(x) > 0 \forall x \in R$.

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154. If $f(x) = (\log)_x(\ln x), then f'(x)$ at $x = e$ is equal to $\frac{1}{e}$ (b) e (c) 1 (d) zero

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155. If $\frac{\cos x}{2}\frac{\cos x}{2^2}\frac{\cos x}{2^3}\infty = \frac{\sin x}{x}$, then find the value of $\frac{1}{2^2}\frac{\sec^2 x}{2} + \frac{1}{2^4}\frac{\sec^2 x}{2^2} + \frac{1}{2^6}\frac{\sec^2 x}{2^3} + \infty$

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156. If $y = f(a^x) and f'(\sin x) = (\log)_e x, then f \in d\frac{dy}{dx}$, if it exists, where `pi/2

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157. If P_n is the sum of a $G\dot{P}$. upto n terms $(n \geq 3)$, then prove that $(1-r)\dfrac{dP_n}{dr} = (1-n)P_n + nP_{n-1}$, where r is the common ratio of $G\dot{P}$.

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158. If $x = \cos ec\theta - \sin\theta and y = \cos ec^n\theta - \sin^n\theta$, then show that $(x^2+4)\left(\dfrac{dy}{dx}\right)^2 = n^2(y^2+4)$.

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159.

If $= \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{x-c} + 1$, prove tha

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160. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitary constants.

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161. If $y = \frac{1}{(2)^{n-1}}\cos\left(n\cos^{-1}x\right)$, then prove that y satisfies the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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162. If a curve is represented parametrically by the equation $x = f(t)$ and $y = g(t)$ then prove that $\frac{d^2y}{dx^2} = -\left[\frac{g'(t)}{f'(t)}\right]^3\left(\frac{d^2x}{dy^2}\right)$

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163. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real $xandy$ and $f'(2) = 2$, then determine $y = f(x)$.

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164. If $f(x) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than $0 and f(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}, then f \in df(x)\cdot$

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165. If $|a_1 \sin x + a_2 \sin 2x + \ + a_n \sin nx| \leq \ | \ |\sin x|$ for $x \in R,$ then prove that $|a_1 + 2a_1 + 3a + 3 + na_n| \leq |$

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166. Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \ + a_n x^n\cdot$ If $|p(x)| \leq e^{x-1} - 1|$ for all $x \geq 0,$ prove that $|a_1 + 2a_2 + \ + na_n| \leq 1.$

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Exercise 3.1

1. Using the definition of derivative find the derivative of $\sqrt{\sin x}$

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2. Find the derivative of $\sqrt{4-x}$ w.r.t. x using the first principle.

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3. Let $f(x)=\frac{\left(2^{x}+2^{-x}\right)\sin x\sqrt{\tan^{-1}\left(x^{2}-x+1\right)}}{\left(7x^{2}+3x+1\right)^{3}}$. Then find the value of f'(0).

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4. Statement 1: Let $f: R\overrightarrow{R}$ be a real-valued function $\forall x, y \in R$ such that $|f(x)-f(y)| \leq |x-y|^{3}$. Then $f(x)$ is a constant function. Statement

2: If the derivative of the function w.r.t. x is zero, then function is constant.

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5. Find the derivative of $\sqrt{4-x}$ w.r.t. x using the first principle.

Watch Video Solution

6. Let $f(x)=\frac{\left(2^{x}+2^{-x}\right)\sin x\sqrt{\tan^{-1}\left(x^{2}-x+1\right)}}{\left(7x^{2}+3x+1\right)^{3}}$. Then find the value of f'(0).

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7. Statement 1: Let $f: R\overrightarrow{R}$ be a real-valued function $\forall x, y \in R$ such that $|f(x)-f(y)| \leq |x-y|^{3}$. Then $f(x)$ is a constant function. Statement 2: If the derivative of the function w.r.t. x is zero, then function is constant.

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Exercise 3.2

1. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then find $\frac{dy}{dx}$

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2. Find $\frac{dy}{dx}$ in the following :

$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \ -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}.$

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3. Find $\frac{dy}{dx}$ in the following :

$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \ \frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}.$

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4. Find $\frac{dy}{dx}$ if $y = \frac{\tan^{-1}(4x)}{1+5x^2} + \frac{\tan^{-1}(2+3x)}{3-2x}$

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5. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$

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6. $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

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7. Find $\frac{dy}{dx}$ for the function: $y = \sin^{-1}\sqrt{(1-x)} + \cos^{-1}\sqrt{x}$

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8. Find $\frac{dy}{dx}$ for the function: $y = \sqrt{\sin\sqrt{x}}$

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9. Differentiate : $y = e^{\sin 2x}$

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10. Find $\frac{dy}{dx}$ for the function: $y = \log\sqrt{\sin\sqrt{e^x}}$

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11. Find $\frac{dy}{dx}$ for the function: $y = a^{(\sin^{-1}x)} \hat{} 2$

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12. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

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13. $y = \sin^{-1}[\sqrt{x-ax} - \sqrt{a-ax}] then prove t \frac{\hat{1}}{2\sqrt{x}\sqrt{1-x}}$

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14. Find $\frac{dy}{dx}$ for the functions: $y = x^3 e^x \sin x$

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15. Find $\frac{dy}{dx}$ for the function: $y = (\log)_e \sqrt{\frac{1+\sin x}{1-\sin x}}, where x = \frac{\pi}{3}$

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16. Find $\frac{dy}{dx}$ for the functions: $y = \frac{\sin x}{x + \cos x}$

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17. If $y = (1+x)(1+x^2)(1+x^4).....(1+x^{2n})$, then find $\frac{dy}{dx} atx = 0.$

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18. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

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19. If g is the inverse function of $f and f'(x) = \sin x, then g'(x)$ is $\cos ec\{g(x)\}$ (b) $\sin\{g(x)\} - \frac{1}{\sin\{g(x)\}}$ (d) none of these

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20. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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21. Find $\frac{dy}{dx}$ in the following :

$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}.$

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22. Find $\frac{dy}{dx}$ in the following :

$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), 0 < x < \frac{1}{\sqrt{2}}.$

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23. Find $\frac{dy}{dx}$ if $y = \frac{\tan^{-1}(4x)}{1 + 5x^2} + \frac{\tan^{-1}(2 + 3x)}{3 - 2x}$

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24. $\mathrm{Find}\ \frac{dy}{dx}\ \text{if}\ y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right),$ where $x \neq 0$

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25. $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

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26. Find $\frac{dy}{dx}$ for the function: $y = \sin^{-1}\sqrt{(1-x)} + \cos^{-1}\sqrt{x}$

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27. Find $\frac{dy}{dx}$ for the function: $y = \sqrt{\sin\sqrt{x}}$

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28. Differentiate : $y = e^{\sin 2x}$

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29. Find $\frac{dy}{dx}$ for the function: $y = \log \sqrt{\sin \sqrt{e^x}}$

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30. Find $\frac{dy}{dx}$ for the function: $y = a^{(\sin^{-1} x)} \hat{} 2$

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31. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x \left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

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32. $y = \sin^{-1}[\sqrt{x - ax} - \sqrt{a - ax}] then prove t \frac{\hat{1}}{2\sqrt{x}\sqrt{1 - x}}$

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33. Find $\frac{dy}{dx}$ for the functions: $y = x^3 e^x \sin x$

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34. Find $\frac{dy}{dx}$ for the function: $y = (\log)_e \sqrt{\frac{1 + \sin x}{1 - \sin x}}, where x = \frac{\pi}{3}$

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35. $y = \frac{x + \sin x}{x + \cos x}$

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36. If $y=(1+x)\left(1+x^2\right)\left(1+x^4\right).....\left(1+x^{2n}\right)$, then find $\frac{dy}{dx}atx=0.$

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37. If $x\sqrt{1+y}+y\sqrt{1+x}=0,$ prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

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38. If g is the inverse function of $fandf'(x)=\sin x, theng'(x)$ is $\cos ec\{g(x)\}$ (b) $\sin\{g(x)\}-\frac{1}{\sin\{g(x)\}}$ (d) none of these

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Exercise 3.3

1. If $x^3 + y^3 + 3axy = 0$, $find$

$\frac{dy}{dx}$.

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2. If $y = b\tan^{-1}\left(\frac{x}{a} + \frac{\tan^{-1} y}{x}\right)$, $f \in d\frac{dy}{dx}$.

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3. Find the derivative of $y = x\tan x$

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4. If

$y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \infty}}}}, then prove that \frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

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5. If $x^3 + y^3 + 3axy = 0$, *find*

$\frac{dy}{dx}$.

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6. If $y = b\tan^{-1}\left(\frac{x}{a} + \frac{\tan^{-1} y}{x}\right), f \in d\frac{dy}{dx}$.

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7.

If $\log_e(\log_e x - \log_e y) = e^{x^2 y}(1 - \log_e x)$, then find the value of $y'(e)$.

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8. If

$y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \infty}}}}, then prove that \frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

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Exercise 3.4

1. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$ then $\frac{dy}{dx}$ at $t = 2$.....

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2. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

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3. Find $\frac{dy}{dx}$ if x= 3 cos theta - cos 2theta and y= sin theta - sin 2theta.`

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4. the solution of $\sin^3\theta\cos\theta - \sin\theta\cos^3\theta = \frac{1}{4}$ is

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5. about to only mathematics

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6. $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$ then $\frac{dy}{dx}$ is

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7. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

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8. Find $\frac{dy}{dx}$ if $x = \cos\theta - \cos 2\theta$

and $y = \sin\theta - \sin 2\theta$

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9. Find $\frac{dy}{dx}$ if $x = 3\cos\theta - 2\cos^3\theta$, $y = 3\sin\theta - 2\sin^3\theta$.

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Exercise 3.5

1. Differentiate the functions given in w.r.t. x.

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}.$$

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2. If $x^y = e^{x-y}$, Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

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3. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

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4. If $x^y = y^x$, then find $\frac{dy}{dx}$.

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5. If $x = e^y + e^{(y+\to\infty)}$, where $x > 0, thenf \in d\frac{dy}{dx}$

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6. Find $\frac{dy}{dx} f$ or $y = x^x$.

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7. Differentiate $(\cos x)^x$ with respect to x.

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8. If $y = (\tan x)^{(\tan x)^{\tan x}}$, then find $\frac{dy}{dx}$.

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9. Differentiate the functions given in w.r.t. x.

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}.$$

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10. If $x^y = e^{x-y}$, Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

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11. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

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12. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

Watch Video Solution

13. If $x = e^y + e^{(y+\to\infty)}$, where $x > 0, then f \in d\frac{dy}{dx}$

Watch Video Solution

14. Find $\frac{dy}{dx} f$ or $y = x^x$.

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15. Differentiate $(\cos x)^x$ with respect to x.

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16. If $y = (\tan x)^{(\tan x)^{\tan x}}$, then find $\frac{dy}{dx}$.

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Exercise 3.6

1. Find the derivative of $\frac{\tan^{-1}(2x)}{1-x^2} w\dot{r}t \frac{\sin^{-1}(2x)}{1+x^2}$

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2. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right) w\dot{r}\dot{t} \sqrt{1-x^2}$ at $x = \frac{1}{2}$

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3. Differentiate $\frac{x}{\sin x}$ w.r.t . $\sin x$.

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4. If $y = f(x^3), z = g(x^5), f'(x) = \tan x, and g'(x) = \sec x,$ then find the value of $\left(\lim\right)_{x \to 0} \frac{\left(\frac{dy}{dz}\right)}{x}$

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5. Find the derivative of $\frac{\tan^{-1}(2x)}{1-x^2} w.r.t \frac{\sin^{-1}(2x)}{1+x^2}$

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6. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right) w.r.t \sqrt{1-x^2}$ at $x = \frac{1}{2}$

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7. Differentiate $\frac{x}{\sin x}$ w.r.t . $\sin x$.

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8. If $y = f(x^3), z = g(x^5), f'(x) = \tan x, and g'(x) = \sec x,$ then find the value of $\left(\lim\right)_{x \to 0} \frac{\left(\frac{dy}{dz}\right)}{x}$

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Exercise 3.7

1. If $f(x) = \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ ac & bc & x + c^2 \end{vmatrix}$, then prove that $f'(x) = 3x^2 + 2x(a^2 + b^2 + c^2)$.

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2. Let $f(x) = \begin{vmatrix} \cos(x + x^2) & \sin(x + x^2) & -\cos(x + x^2) \\ \sin(x - x^2) & \cos(x - x^2) & \sin(x - x^2) \\ \sin 2x & 0 & \sin(2x^2) \end{vmatrix}$.

Find the value of f'(0).

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3. Let

$g(x)\left|f(x+c)f(x+2c)f(x+3c)f(c)f(2c)f(3c)f'(c)f'(2c)f'(3c)\right|,$

where c is constant, then find $\left(\lim\right)_{x\to 0}\frac{g(x)}{x}$

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4. If $f(x)=\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, then prove that $f'(x)=3x^2+2x\left(a^2+b^2+c^2\right).$

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5. Let $f(x)=\begin{vmatrix} \cos\left(x+x^2\right) & \sin\left(x+x^2\right) & -\cos\left(x+x^2\right) \\ \sin\left(x-x^2\right) & \cos\left(x-x^2\right) & \sin\left(x-x^2\right) \\ \sin 2x & 0 & \sin\left(2x^2\right) \end{vmatrix}.$

Find the value of f'(0).

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6. Let

$g(x)|f(x+c)f(x+2c)f(x+3c)f(c)f(2c)f(3c)f'(c)f'(2c)f'(3c)|$,

where c is constant, then find $\left(\lim\right)_{x\to 0}\frac{g(x)}{x}$

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Exercise 3.8

1. If $f(x)=(1+x)^2$, then the value of $f(x0)+f'(0)+\frac{f^0}{2!}+\frac{f^0}{3!}+\frac{f^n(0)}{n!}$.

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2. If $e^y(x+1)=1, showthat\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

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3. Prove that $\frac{d^n}{dx^n}\left(e^{2x}+e^{-2x}\right)=2^n\left[e^{2x}+(-1)^n e^{-2x}\right]$

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4. If y =sin (sin x) and $\frac{d^2y}{dx^2}+\frac{dy}{dx}$ tan x + f(x) = 0, then find f(x).

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5. If y=log (1+ sin x), prove that $y_4+y_3y_1+y_2^2=0$.

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6. If $f(x)=\left|x^{\cap}!2\cos x\frac{\cos(n\pi)}{2}4\sin x\frac{\sin(n\pi)}{2}8\right|$ then find the value of $\frac{d^n}{dx^n}([f(x)])_{x=0}n\dot{\in}z\cdot$

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7. If $x = a\cos\theta, y = b\sin\theta,$ then prove that $\frac{d^3y}{dx^3} = \frac{3b}{a^3}\cos ec^4\theta\cot\theta$.

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8. If $x = a\cos^3\theta, y = b\sin^3\theta, find\frac{d^3y}{dx^3}$ at $\theta = 0$.

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9. If $f(x) = (1+x)^2,$ then the value of $f(x0) + f'(0) + \frac{f^0}{2!} + \frac{f^0}{3!} + \frac{f^n(0)}{n!}$.

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10. If $e^y(x+1) = 1, showthat\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

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11. Prove that $\frac{d^n}{dx^n}\left(e^{2x}+e^{-2x}\right)=2^n\left[e^{2x}+(-1)^n e^{-2x}\right]$

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12. If y= sin (sin x) and $\frac{d^2y}{dx^2}+\frac{dy}{dx}$ tan x + f(x) = 0, then find (x).

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13. If y=log (1+ sin x), prove that $y_4+y_3y_1+y_2^2=0$.

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14. If $x=a\cos\theta, y=b\sin\theta,$ then prove that $\frac{d^3y}{dx^3}=\frac{3b}{a^3}\cos ec^4\theta\cot\theta$.

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15. If $x=a\cos^3\theta, y=b\sin^3\theta, find\frac{d^3y}{dx^3}$ at $\theta=0$.

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Exercise 3.9

1. Let $f(x+y)=f(x)f(y)$ for all $x and y$. Suppose $f(5)=2 and f'(0)=3$. Find $f'(5)$.

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2. Let $f(xy)=f(x)f(y)\,\forall x, y \in R and f$ is differentiable at $x=1$ such that $f'(1)=1$. Also, $f(1)\neq 0$, $f(2)=3$. Then find $f'(2)$.

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3. Let f be a function such that $f(x+y)=f(x)+f(y)$ for all $x and y and f(x)=\left(2x^2+3x\right)g(x)$ for all x, where $g(x)$ is continuous and $g(0)=3$. Then find $f'(x)$.

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4. Let $g: R \overrightarrow{R}$ be a differentiable function satisfying $g(x) = g(y)g(x-y)\,\forall x, y \in R$ and $g'(0) = a and g'(3) = b$. Then find the value of $g'(-3)$.

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5. Let $f(x^m y^n) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$. If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $(\lim)_{x \to 0} \frac{f(1+x)}{x}$

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6. If $f\left(\frac{x+2y}{3}\right) = \frac{f(x)+2f(y)}{3}\,\forall x, y \in R and f'(0) = 1, f(0) = 2,$ then find $f(x)$.

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7. Prove that $\lim_{x \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$ (without using L' Hospital srule).

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8. Let f(x+y) = f(x). F(y) for all x and y. Suppose f(5) = 2 and f'(0) = 3. Find f'(5).

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9. Let $f(xy) = f(x)f(y) \forall x, y \in R and f$ is differentiable at $x = 1$ such that $f'(1) = 1$. Also, $f(1) \neq 0$, $f(2) = 3$. Then find $f'(2)$.

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10. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all $x and y and f(x) = \left(2x^2 + 3x\right) g(x)$ for all x, where $g(x)$ is continuous

and $g(0) = 3$. Then find $f'(x)$.

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11. Let $g: R \overrightarrow{R}$ be a differentiable function satisfying $g(x) = g(y)g(x-y)\,\forall x, y \in R$ and $g'(0) = a and g'(3) = b$. Then find the value of $g'(-3)$.

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12. Let $f(x^m y^n) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$. If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $(\lim)_{x \overrightarrow{0}} \frac{f(1+x)}{x}$

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13. If $f\left(\frac{x+2y}{3}\right) = \frac{f(x)+2f(y)}{3}\,\forall x, y \in R and f'(0) = 1, f(0) = 2,$ then find $f(x)$.

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14. Prove that $\lim_{x \to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2} = f''(x)$ (without using L' Hospital srule).

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Exercise (Single)

1. If $y = a\sin x + b\cos x, then y^2 + \left(\frac{dy}{dx}\right)^2$ is a function of x (b) function of$y$ function of $xandy$ (d) constant

A. function of x

B. function of y

C. function of x and y

D. constant

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2. `d/(dx)sqrt((1-sin2x)/(1+sin2x))i se q u a lto,(0

A. $\sec^2 x$

B. $-\sec^2\left(\frac{\pi}{4}-x\right)$

C. $\sec^2\left(\frac{\pi}{4}+x\right)$

D. $\sec^2\left(\frac{\pi}{4}-x\right)$

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3. If $f(x)=|\cos x|+|\sin x|$, then $\frac{dy}{dx}$ at $x=\frac{2\pi}{3}$ is equal to

A. $\frac{1-\sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}(\sqrt{3}-1)$

D. none of these

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4. If f(x) $= |\log_e |x||,$ then f'(x) equals

A. $\frac{1}{|x|}$, where $x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

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5. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ is equal to

$(a) -(\cos x + \sin x), f \text{ or } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$(b) \cos x + \sin x, f \text{ or } x \in \left(0, \frac{\pi}{4}\right)$

$(c) -(\cos x + \sin x), f \text{ or } x \in \left(0, \frac{\pi}{4}\right)$

$(d) \cos x - \sin x, f \text{ or } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

A. $-(\cos x + \sin x)$, for x $\in (\pi/4, \pi/2)$

B. cosx + sin x for x $\in (0, \pi/4)$

C. $-(\cos x + \sin x)$, for $x \in (0, \pi/4)$

D. $\cos x - \sin x$, for $x \in (\pi/4, \pi/2)$

Answer: C

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6. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D^{\cdot}F(x)$ by the formula $D \cdot (x) = (\lim)_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}, where f^2(x)$ mean $[f(x)]^2 and \Leftrightarrow (x) = x \log x, then D^{\cdot} f(x)(\ |\ )_{x=e}$ has the value e (B) 2e (c) 4e (d) none of these

A. e

B. 2e

C. 4e

D. none of these

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7. $y = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), f \in d\frac{dy}{dx}.$

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. 3

D. 1

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8. If $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{a}, then \frac{dy}{dx}$ wherever it is defined is $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$ (b) $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}} - \frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$ (d) $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

A. $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$

B. $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$

C. $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

D. $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

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9. The derivative of $y = (1-x)(2-x)(n-x) at x = 1$ is (b) $(-1)(n-1)!\, n! - 1$ (d) $(-1)^{n-1}(n-1)!$

A. 0

B. $(-1)(n-1)!$

C. $n! - 1$

D. $(-1)^{n-1}(n-1)!$

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10. If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $\frac{(1-x^2)dy}{dx} + y = 0$

A. y^2

B. $1/y$

C. $-y$

D. $-y/x$

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11. If $y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}, then \frac{dy}{dx} isequa < o \quad \frac{ay}{x\sqrt{a^2 - x^2}}$ (b) $\frac{ay}{\sqrt{a^2 - x^2}} \quad \frac{ay}{x\sqrt{a^2 - x^2}}$ (d) none of these

A. $\frac{ay}{x\sqrt{a^2 - x^2}}$

B. $\frac{ay}{\sqrt{a^2 - x^2}}$

C. $\frac{ay}{x\sqrt{x^2 - a^2}}$

D. none of these

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12. Let $u(x)$ and $v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7$. If (u'(x)/(v'(x))= p and ((u'(x))/(v'(x)))' = q,$then$(p+q)/(p-q)$hasthevalueof \rightarrow$

(a) 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

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13. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a, then \frac{dy}{dx}$ is equal to $\frac{x}{y}$ (b) $\frac{y}{x^2}$ $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2-y^2}{x^2+y^2}$

D. $\frac{y}{x}$

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14. Let h(x) be differentiable for all x and let $f(x) = (kx = e^x)h(x) where k is some cons\tan t. If h(0) = 5$, h^(')(0) = -2 and f^(')(0) =18` then the value of k is equal to

A. 5

B. 4

C. 3

D. 2.2

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15. If $(\lim)_{t \to x} \frac{e^t f(x) - e^x f(t)}{(t-x)(f(x))^2} = 2 and f(0) = \frac{1}{2}$, then find the value of $f'(0)$. a) 4 (b) 2 (c) 0 (d) 1

A. 4

B. 2

C. 0

D. 1

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16. If $f(0) = 0, f'(0) = 2,$ then the derivative of $y = f(f(f(x)))$ at $x = 0$ is 2 (b) 8 (c) 16 (d) 4

A. 2

B. 8

C. 16

D. 4

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17. If $f(x) = \sqrt{1 + \cos^2(x^2)}, then f'\left(\frac{\sqrt{\pi}}{2}\right)$ is $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\pi/6}\ 1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

A. $\sqrt{\pi/6}$

B. $-\sqrt{\pi/6}$

C. $1/\sqrt{6}$

D. $\pi/\sqrt{6}$

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18. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x} isequa < o \quad \frac{1}{2}\sqrt{1 + secx}$ (b) $\sqrt{1 + \sec x}$ $-\frac{1}{2}\sqrt{1 + \sec x}$ (d) $-\sqrt{1 + \sec x}$

A. $\frac{1}{2}\sqrt{1 + \sec x}$

B. $\sqrt{1 + \sec x}$

C. $-\frac{1}{2}\sqrt{1 + \sec x}$

D. $-\sqrt{1+\sec x}$

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19. If $y=\tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right), then \frac{dy}{dx} atx=0$ is 1 (b) 2 (c) 1n 2 (d) none of these

A. 1

B. 2

C. In 2

D. $-\frac{1}{10}In2$

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20. If $y = \left(x + \sqrt{x^2 + a^2}\right)^n$, $then \frac{dy}{dx}$ is $\frac{ny}{\sqrt{x^2 + a^2}}$ (b) $-\frac{ny}{\sqrt{x^2 + a^2}}$ $\frac{nx}{\sqrt{x^2 + a^2}}$ (d) $-\frac{nx}{\sqrt{x^2 + a^2}}$

A. $\frac{ny}{\sqrt{x^2 + a^2}}$

B. $-\frac{ny}{\sqrt{x^2 + a^2}}$

C. $\frac{nx}{\sqrt{x^2 + a^2}}$

D. $-\frac{nx}{\sqrt{x^2 + a^2}}$

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21. If $y = (\log)_{\sin x}(\tan x), then\left(\left(\frac{dy}{dx}\right)\right)_{\frac{\pi}{4}} isequa < o$ $\frac{4}{\log 2}$ (b) $-4\log 2$ $\frac{-4}{\log 2}$ (d) none of these

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. none of these

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22. $\int \sqrt{\frac{1-x}{1+x}} dx$ is

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

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23. The differential coefficient of $f\left((\log)_e x\right)$ with respect to x, where $f(x) = (\log)_e x$, is $\frac{x}{(\log)_e x}$ (b) $\frac{1}{x}(\log)_e x$ $\frac{1}{x(\log)_e x}$ (d) none of these

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x \log_e x}$

D. none of these

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24. If $f'(x) = \sqrt{2x^2 - 1} and y = f(x^2), then \frac{dy}{dx} at x = 1$ is 2 (b) 1 (c) -2 (d) none of these

A. 2

B. 1

C. -2

D. none of these

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25. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2}x\cos x^3 \cos ecx^2 \frac{2}{3}\sin x^3 \sec x^2 \tan x$ (d) none of these

A. $\frac{3}{2}x \cos x^2 \operatorname{cosec} x^2$

B. $\frac{3}{2} \sin x^3 \sec x^2$

C. $\tan x$

D. none of these

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26. A function f, defined for all positive real numbers, satisfies the equation $f(x^2) = x^3$ for every $x > 0$. Then the value of $f'(4)$ is 12 (b) 3 (c) $3/2$ (d) cannot be determined

A. 12

B. 3

C. $3/2$

D. cannot be determined

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27. Let $f:(-5,5) \to R$ be a differentiable function with f(4) = 1, $f'(4) = 1, f(0) = -1$ and f^(')(0) = 1, Ifg(x) = f(2f^(2)(x)+2))^(2), $then$ - g^(')(0)` equals

A. 4

B. -4

C. 8

D. -8

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28. The function $f(x) = e^x + x$, being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{In2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. none of these

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29. If $f(x) = x + \tan x and f$ is the inverse of g, then $g'(x)$ equals $\frac{1}{1+[g(x)-x]^2}$ (b) $\frac{1}{2-[g(x)-x]^2}$ $\frac{1}{2+[g(x)-x]^2}$ (d) none of these

A. $\frac{1}{1+[g(x)-x]^2}$

B. $\frac{1}{2-[g(x)-x]^2}$

C. $\frac{1}{2+[g(x)-x]^2}$

D. none of these

Answer: C

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30. If $f(x) = x^3 + 3x + 4$ and g is the inverse function of f(x), then the value of $\frac{d}{dx}\left(\frac{g(x)}{g(g(x))}\right)$ at x = 4 equals

A. $\frac{-1}{6}$

B. 6

C. $\frac{-1}{3}$

D. non-existent

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31. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}, then \frac{(1-x^2)dy}{dx}$ is equal to (a)$x + y$ (b) $1 + xy$ (c) $1 - xy$ (d) $xy - 2$

A. x+y

B. 1+xy

C. 1-xy

D. xy-2

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32. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \infty}}}, then \frac{dy}{dx} is$ $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

A. $\frac{x}{2y-1}$

B. $\frac{x}{2y+1}$

C. $\frac{1}{x(2y-1)}$

D. $\frac{1}{x(1-2y)}$

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33. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right] = \frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$ $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

A. $\frac{1}{2(1+x)\sqrt{x}}$

B. $\frac{3}{(1+x)\sqrt{x}}$

C. $\frac{2}{(1+x)\sqrt{x}}$

D. $\frac{3}{2(1+x)\sqrt{x}}$

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34. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative $7atx = 2.$ The derivative of the function $f(x) - f(4x)atx = 1$ has the value equal to 19 (b) 9 (c) 17 (d) 14

A. 19

B. 9

C. 17

D. 14

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35. If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}1 - x\right), showthat\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

A. 0

B. 1

C. 2

D. none of these

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36.

Let $e^y = \frac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}$ and $\tan\frac{x}{2} = \sqrt{\frac{1-\alpha}{1+\alpha}}, \alpha \in [-1, 0] \cup (0, 1$

A. $1/2$

B. 1

C. 2

D. $1/3$

Answer: C

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37. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ is $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

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38. If $\ln\left((e-1)e^{xy}+x^2\right)=x^2+y^2$ then $\left(\frac{dy}{dx}\right)_{1,0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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39. If $y = (x^x)$ then $\frac{dy}{dx}$ is

A. $y[x^x(\log ex)\log x + x^x]$

B. $y[x^x(\log ex)\log x + x]$

C. $y\left[x^x(\log ex)\log x + x^{-1}\right]$

D. $y\left[x^x(\log_e x)\log x + x^{-1}\right]$

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40. The first derivative of the function $\left[\cos^{-1}\left(s \in \sqrt{\frac{1+x}{2}}\right) + x^x\right]$ with respect to x at $x = 1$ is $3/4$ (b) 0 (c) $1/2$ (d) $-1/2$

A. $3/4$

B. 0

C. $1/2$

D. $-1/2$

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41. $f(x) = x^x, x \in (0, \infty)$ and let $g(x)$ be inverse of f(x) , then $g(x)'$ must be

A. $x(1 + \log x)$

B. $x(1 + \log(x))$

C. $\frac{1}{x(1 + \log g(x))}$

D. non-existent

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42. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

A. n(n-1) y

B. n(n+1)y

C. ny

D. n^2y

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43. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

A. $m^2\left(ae^{mx} - be^{-mx}\right)$

B. 1

C. 0

D. none of these

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44. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of $\frac{|ab|}{3}$ is ___

A. 25

B. 9

C. -15

D. -9

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45. $\frac{d^{20}y}{dx^{20}}(2\cos x \cos 3x) isequa < o$ $2^{20}\left(\cos 2x - 2^{20} os3x\right)$

$2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$ $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$

$2^{20}(\sin 2x - 2^{20}\sin 4x)$

A. $2^{20}(\cos 2x - 2^{20}\cos 3x)$

B. $2^{20}(\cos 2x + 2^{20}\cos 4x)$

C. $2^{20}(\sin 2x + 2^{20}\sin 4x)$

D. $2^{20}(\sin 2x - 2^{20}\sin 4x)$

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46. $\frac{d^n}{dx^n}(\log x) = \frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

A. $\frac{(n-1)!}{x^n}$

B. $\frac{n!}{x^n}$

C. $\frac{(n-2)!}{x^n}$

D. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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47. The nth derivative of the function $f(x) = \frac{1}{1-x^2}$ [where $x \in (-1, 1)$ at the point $x = 0$ where n is even is (a) 0 (b) $n!$ (c) ${}^nC_2$ (d) ${}^{2^n}C_2$

A. 0

B. $n!$

C. ${}^nC_2$

D. ${}^{2^n}C_2$

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48. If $y = x\log\left(\frac{x}{a+bx}\right), then x^3\frac{d^2y}{dx^2} =$ (a) $\left(x\frac{dy}{dx} - y\right)$^2 (b) $x\left(\frac{dy}{dx}\right) - y(c)$ y(dy/dx)-x(d)(y(dy/dx)-x)^2`

A. $x\frac{dy}{dx} - y$

B. $\left(x\frac{dy}{dx}-y\right)^2$

C. $y\frac{dy}{dx}-x$

D. $\left(y\frac{dy}{dx}-x\right)^2$

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49. If $ax^2+2hxy+by^2=1, then\frac{d^{2y}}{dx^2}$ is $\frac{h^2-ab}{(hx+by)^2}$ (b) $\frac{ab-h^2}{(hx+by)^2}$ $\frac{h^2+ab}{(hx+by)^2}$ (d) none of these

A. $\frac{h^2-ab}{(hx+by)^3}$

B. $\frac{ab-h^2}{(hx+by)^2}$

C. $\frac{h^2+ab}{(hx+by)^2}$

D. none of these

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50. If $y^{\frac{1}{m}} = \left(x + \sqrt{1+x^2}\right), then\left(1+x^2\right)y_2 + xy_1$ is (where y_r represents the rth derivative of y w.r.t. $x \cdot m^2y$ (b) my^2 (c) m^2y^2 (d) none of these

A. m^2y

B. my^2

C. m^2y^2

D. none of these

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51. If $(\sin x)(\cos y) = \frac{1}{2}, then\frac{d^2y}{dx^2}at\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -4 (b) -2 (c) -6 (d) 0

A. -4

B. -2

C. -6

D. 0

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52. A function f satisfies the condition $f(x) = f'(x) + f^x + f^x, where f(x)$ is a differentiable function indefinitely and dash denotes the order of derivative. If $f(x) = 1, then f(x)$ is $e^{\frac{x}{2}}$ (b) e^x (c) e^{2x} (d) e^{4x}

A. $e^{x/2}$

B. e^x

C. e^{2x}

D. e^{4x}

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53. Let $f(x)$ be a polynomial of degree 3 such that $f(3)=1, f'(3)=-1, f^3=0, and f^3=12.$ Then the value of $f'(1)$ is 12 (b) 23 (c) -13 (d) none of these

A. 12

B. 23

C. -13

D. none of these

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54. If $y^2=ax^2+bx+c, then y^3\frac{d^{2y}}{dx^2}$ is a constant a function of x only a function of y only a function of $x and y$

A. a constant

B. a function of x only

C. a function of y only

D. a function of x and y

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55. If $y = \sin x + e^x, then \frac{d^{2x}}{dy^2} = (-\sin x + e^x)^{-1} \frac{\sin x - e^x}{(\cos x + e^x)^2}$ (c) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$ (d) $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

A. $(-\sin x + e^x)^{-1}$

B. $\frac{\sin x - e^x}{(\cos x + e^x)^2}$

C. $\frac{\sin x - e^x}{(\cos x + e^x)^3}$

D. $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

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56. if y= sin mx, them the value of the determinant

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} \text{ Where } y_n = \frac{d^n y}{dx^n} \text{ is}$$

A. 1

B. 0

C. -1

D. none of these

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57. If $f^x = -f(x) and g(x) = f'(x) and F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5,$ then $F(10)$ is 5 (b) 10 (c) 0 (d) 15

A. 5

B. 10

C. 0

D. 15

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58. Let $y = 1n(1+\cos x)^2$. Then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$ equal (b) $\frac{2}{1+\cos x}$ $\frac{4}{1+\cos x}$ (d) $\frac{-4}{(1+\cos x)^2}$

A. 0

B. $\frac{2}{1+\cos x}$

C. $\frac{4}{1+\cos x}$

D. $\frac{-4}{(1+\cos x)^2}$

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59. $x = t\cos t, y = t + \sin t\cdot$ Then $\frac{d^{2x}}{dy^2}att = \frac{\pi}{2}$ is $(a)\frac{\pi+4}{2}$ (b) $-\frac{\pi+4}{2}$ (c) -2 (d) none of these

A. $\frac{\pi+4}{2}$

B. $-\frac{\pi+4}{2}$

C. -2

D. none of these

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60. If $f(x) = (x-1)^4(x-2)^3(x-3)^2(x-4),$ then the value of $f'''(1) + f''(2) + f'(3) + f'(4)$ equals

A. 0

B. 50

C. 324

D. 648

Answer: 648

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61. If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

A. $\frac{5}{2}t$

B. $20t^8$

C. $\frac{5}{16t^6}$

D. none of these

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62. If x and y are positive real numbers such that $2\log(2y-3x)=\log x+\log y$, then find the value of $\frac{x}{y}$.

A. $\frac{d^2y}{dx^2}-2p=0$

B. $\frac{d^2y}{dx^2}+y=0$

C. $\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

D. $\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

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63. If $x=\varphi(t), y=\psi(t), then\frac{d^{2y}}{dx^2}$ is $\frac{\varphi'\psi^{-\psi}{}'\varphi''}{(\varphi')^2}$ (b) $\frac{\varphi'\psi^{-\psi}{}'\varphi''}{(\varphi')^3}$

φ/ψ (d) ψ/φ

A. $\frac{\phi'\psi''-\psi'\phi''}{(\phi')^2}$

B. $\frac{\phi'\psi''-\psi'\phi''}{(\phi')^3}$

C. $\frac{\phi''}{\psi''}$

D. $\frac{\psi''}{\phi''}$

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64. If $f(x) = x^4 \tan(x^3) - x1n(1 + x^2)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is 0 (b) 6 (c) 12 (d) 24

A. 0

B. 6

C. 12

D. 24

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65. If graph of $y = f(x)$ is symmetrical about the y-axis and that of $y = g(x)$ is symmetrical about the origin and if

$h(x) = f(x)g(\dot{x}), then \frac{d^3h(x)}{dx^3} atx = 0$ is cannot be determined (b) $f(0)g(\dot{0})$ 0 (d) none of these

A. cannot be determined

B. $f(0) \cdot g(0)$

C. 0

D. none of these

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66. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g^{f(x)}$ equals. $-\frac{f^x}{(f'(x))^3}$ (b) $\frac{f'(x)f^x - (f'(x))^3}{f'(x)}$ $\frac{f'(x)f^x - (f'(x))^2}{(f'(x))^2}$ (d) none of these

A. $-\frac{f''(x)}{(f'(x))^3}$

B. $\frac{f'(x)f''(x) - (f(x))^3}{f'(x)}$

C. $\frac{f'(x)f''(x) - (f'(x))^2}{(f'(x))^2}$

D. none of these

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67. $f(x) = e^x - e^{-x} - 2\sin x - \frac{2}{3}x^3$. Then the least value of n for which $\frac{d^n}{dx^n} f(x)(\ |\ )_{x=0} is non zero is$ (a) 5 (b) 6 (c) 7 (d) 8

A. 5

B. 6

C. 7

D. 8

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68. Let $y = f(x)$ and $x = \frac{1}{z}$. If $\frac{d^2y}{dx^2} = \lambda(z^3)\frac{dy}{dz} + z^4\frac{d^2y}{dz^2}$, then the value of λ is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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69. Let x = f(t) and y = g(t), where x and y are twice differentiable function If $f'(0) = g'(0) = f''(0) = 2, g(0) = 6,$ then the value of $\left(\frac{d^2y}{\left(dx^2\right)}\right)_{t=0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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70. If $f(x)$ satisfies the relation $f\left(\frac{5x-3y}{2}\right)=\frac{5f(x)-3f(y)}{2}\forall x, y \in R,$ and $f(0)=3 and f'(0)=2,$ then the period of $\sin(f(x))$ is 2π (b) π (c) 3π (d) 4π

A. 2π

B. π

C. 3π

D. 4π

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71. A function $f: R \to R$ satisfies $\sin x \cos y(f(2x+2y) - f(2x-2y) = \cos x \sin y(f(2x+2y) + f(2x-2y)$ If $f'(0) = \frac{1}{2}, then$ (a)$f''(x) = f(x) = 0$ (b)$4f''(x) + f(x) = 0$ (c) $f''(x) + f(x) = 0$ (d)$4f''(x) - f(x) = 0$

A. $f''(x) = f(x) = 0$

B. $4f''(x) + f(x) = 0$

C. $f''(x) + f(x) = 0$

D. $4f''(x) - f(x) = 0$

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72. If $y = a\sin x + b\cos x, theny^2 + \left(\frac{dy}{dx}\right)^2$ is a function of x (b) function of$y$ function of $xandy$ (d) constant

A. function of x

B. function of y

C. function of x and y

D. constant

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73. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ is equal to, $(0 < x < \pi/4)$,

A. $\sec^2 x$

B. $-\sec^2\left(\frac{\pi}{4}-x\right)$

C. $\sec^2\left(\frac{\pi}{4}+x\right)$

D. $\sec^2\left(\frac{\pi}{4}-x\right)$

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74. If y = |cos x| + |sin x|,then $\frac{dy}{dx}$ at $x\frac{2\pi}{3}$ is

A. $\frac{1-\sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}(\sqrt{3}-1)$

D. none of these

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75. If f(x) $= |\log_e|x||,$ then f'(x) equals

A. $\frac{1}{|x|}$, where $x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

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76. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ is equal to

$(a) -(\cos x + \sin x), f \text{ or } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$(b) \cos x + \sin x, f \text{ or } x \in \left(0, \frac{\pi}{4}\right)$

$(c) -(\cos x + \sin x), f \text{ or } x \in \left(0, \frac{\pi}{4}\right)$

$(d) \cos x - \sin x, f \text{ or } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

A. $-(\cos x + \sin x)$, for x $\in (\pi/4, \pi/2)$

B. cosx + sin x for x $\in (0, \pi/4)$

C. $-(\cos x + \sin x)$, for $x \in (0, \pi/4)$

D. $\cos x - \sin x$, for $x \in (\pi/4, \pi/2)$

Answer: C

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77. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D \cdot F(x)$ by the formula $D \cdot (x) = (\lim)_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}, where f^2(x)$ mean

$[f(x)]^2 and \Leftrightarrow (x) = x \log x, then D \cdot f(x)(\ |\ )_{x=e}$ has the value e (B) 2e

(c) 4e (d) none of these

A. e

B. 2e

C. 4e

D. none of these

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78. If $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{a}, then \frac{dy}{dx}$ wherever it is defined is $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$ (b) $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}} - \frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$ (d) $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

A. $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$

B. $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$

C. $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

D. $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

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79. The derivative of $y=(1-x)(2-x)(n-x)atx=1$ is (b) $(-1)(n-1)!\,n!-1$ (d) $(-1)^{n-1}(n-1)!$

A. 0

B. $(-1)(n-1)!$

C. $n!-1$

D. $(-1)^{n-1}(n-1)!$

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80. If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $\frac{\left(1-x^2\right)dy}{dx}+y=0$

A. y^2

B. $1/y$

C. $-y$

D. $-y/x$

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81. If $y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}, then \frac{dy}{dx} isequa < o \quad \frac{ay}{x\sqrt{a^2 - x^2}}$ (b) $\frac{ay}{\sqrt{a^2 - x^2}} \quad \frac{ay}{x\sqrt{a^2 - x^2}}$ (d) none of these

A. $\frac{ay}{x\sqrt{a^2 - x^2}}$

B. $\frac{ay}{\sqrt{a^2 - x^2}}$

C. $\frac{ay}{x\sqrt{x^2 - a^2}}$

D. none of these

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82. Let $u(x)$ and $v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7$. If (u'(x)/(v'(x))= p and ((u'(x))/(v'(x)))' = q,$then$(p+q)/(p-q) $hasthevalueof \rightarrow$ (a) 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

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83. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a, then\frac{dy}{dx}$ is equal to $\frac{x}{y}$ (b) $\frac{y}{x^2}$ $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2 - y^2}{x^2 + y^2}$

D. $\frac{y}{x}$

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84. Let h(x) be differentiable for all x and let $f(x) = (kx = e^x)h(x) where k is some cons\tan t. If h(0) = 5$, h^(')(0) = -2 and f^(')(0) =18` then the value of k is equal to

A. 5

B. 4

C. 3

D. 2.2

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85. If $(\lim)_{t\vec{x}} \frac{e^t f(x) - e^x f(t)}{(t-x)(f(x))^2} = 2 and f(0) = \frac{1}{2}$, then find the value of $f'(0)$. a) 4 (b) 2 (c) 0 (d) 1

A. 4

B. 2

C. 0

D. 1

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86. If $f(0) = 0, f'(0) = 2,$ then the derivative of $y = f(f(f(x)))$ at $x = 0$ is 2 (b) 8 (c) 16 (d) 4

A. 2

B. 8

C. 16

D. 4

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87. If $f(x) = \sqrt{1 + \cos^2(x^2)}, then f'\left(\frac{\sqrt{\pi}}{2}\right)$ is $\frac{\sqrt{\pi}}{6}$ (b) $-\sqrt{\pi/6}\ 1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

A. $\sqrt{\pi/6}$

B. $-\sqrt{\pi/6}$

C. $1/\sqrt{6}$

D. $\pi/\sqrt{6}$

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88. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x} is equa < o \quad \frac{1}{2}\sqrt{1+secx}$ (b) $\sqrt{1+\sec x}$ $-\frac{1}{2}\sqrt{1+\sec x}$ (d) $-\sqrt{1+\sec x}$

A. $\frac{1}{2}\sqrt{1+\sec x}$

B. $\sqrt{1+\sec x}$

C. $-\frac{1}{2}\sqrt{1+\sec x}$

D. $-\sqrt{1+\sec x}$

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89. If $y=\tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right), then \frac{dy}{dx} atx = 0$ is 1 (b) 2 (c) 1n 2 (d) none of these

A. 1

B. 2

C. In 2

D. $-\frac{1}{10}In2$

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90. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$, $then\frac{dy}{dx}$ is $\frac{ny}{\sqrt{x^2+a^2}}$ (b) $-\frac{ny}{\sqrt{x^2+a^2}}$ $\frac{nx}{\sqrt{x^2+a^2}}$ (d) $-\frac{nx}{\sqrt{x^2+a^2}}$

A. $\frac{ny}{\sqrt{x^2+a^2}}$

B. $-\frac{ny}{\sqrt{x^2+a^2}}$

C. $\frac{nx}{\sqrt{x^2+a^2}}$

D. $-\frac{nx}{\sqrt{x^2+a^2}}$

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91. If $y = (\log)_{\sin x}(\tan x), then\left(\left(\frac{dy}{dx}\right)\right)_{\frac{\pi}{4}} isequa < o \quad \frac{4}{\log 2}$ (b) $-4\log 2 \ \frac{-4}{\log 2}$ (d) none of these

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. none of these

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92. Prove that $\sin\left[2\tan^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right] = \sqrt{1-x^2}$

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

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93. The differential coefficient of $f\left((\log)_e x\right)$ with respect to x, where $f(x) = (\log)_e x$, is $\frac{x}{(\log)_e x}$ (b) $\frac{1}{x}(\log)_e x$ $\frac{1}{x(\log)_e x}$ (d) none of these

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x \log_e x}$

D. none of these

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94. If $f'(x) = \sqrt{2x^2 - 1} and y = f\left(x^2\right), then \frac{dy}{dx} at x = 1$ is 2 (b) 1 (c) -2 (d) none of these

A. 2

B. 1

C. -2

D. none of these

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95. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2}x\cos x^3 \cos ecx^2 \frac{2}{3}\sin x^3 \sec x^2 \tan x$ (d) none of these

A. $\frac{3}{2}x \cos x^2 \operatorname{cosec} x^2$

B. $\frac{3}{2} \sin x^3 \sec x^2$

C. $\tan x$

D. none of these

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96. A function $f,$ defined for all positive real numbers, satisfies the equation $f\left(x^2\right)=x^3$ for every $x>0$. Then the value of $f'(4)$ is 12 (b) 3 (c) $3/2$ (d) cannot be determined

A. 12

B. 3

C. $3/2$

D. cannot be determined

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97. Let $f:(-5,5)\to R$ be a differentiable function with f(4) = 1, $f'(4)=1, f(0)=-1$ and f^(')(0) = 1, Ifg(x) = f(2f^(2)(x)+2))^(2), $then$ - g^(')(0)` equals

A. 4

B. -4

C. 8

D. -8

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98. The function $f(x) = e^x + x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{In2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. none of these

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99. If $f(x) = x + \tan x and f$ is the inverse of g, then $g'(x)$ equals

$\frac{1}{1+[g(x)-x]^2}$ (b) $\frac{1}{2-[g(x)-x]^2}$ $\frac{1}{2+[g(x)-x]^2}$ (d) none of these

A. $\frac{1}{1+[g(x)-x]^2}$

B. $\frac{1}{2-[g(x)-x]^2}$

C. $\frac{1}{2+[g(x)-x]^2}$

D. none of these

Answer: C

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100. If $f(x) = x^3 + 3x + 4$ and g is the inverse function of f(x), then the value of $\frac{d}{dx}\left(\frac{g(x)}{g(g(x))}\right)$ at x = 4 equals

A. $\frac{-1}{6}$

B. 6

C. $\frac{-1}{3}$

D. non-existent

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101. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}, then \frac{(1-x^2)dy}{dx}$ is equal to (a)$x + y$ (b) $1 + xy$ (c) $1 - xy$ (d) $xy - 2$

A. x+y

B. 1+xy

C. 1-xy

D. xy-2

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102. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \infty}}}$, then $\frac{dy}{dx}$ is $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

A. $\frac{x}{2y-1}$

B. $\frac{x}{2y+1}$

C. $\frac{1}{x(2y-1)}$

D. $\frac{1}{x(1-2y)}$

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103. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right] = \frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$ $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

A. $\frac{1}{2(1+x)\sqrt{x}}$

B. $\frac{3}{(1+x)\sqrt{x}}$

C. $\frac{2}{(1+x)\sqrt{x}}$

D. $\frac{3}{2(1+x)\sqrt{x}}$

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104. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative $7atx = 2$. The derivative of the function $f(x) - f(4x)atx = 1$ has the value equal to 19 (b) 9 (c) 17 (d) 14

A. 19

B. 9

C. 17

D. 14

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105. If

$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}-x\right), showthat\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

A. 0

B. 1

C. 2

D. none of these

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106. $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right), f \in d\frac{dy}{dx}.$

A. $1/2$

B. 1

C. 2

D. $1/3$

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107. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ is $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

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108. If $\log_e\left((e-1)e^{xy}+x^2\right)=x^2+y^2, then\frac{dy}{dx}$ at (1, 0) is equal to

A. 0

B. 1

C. 2

D. 3

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109. If $y = (x^x)$ then $\frac{dy}{dx}$ is

A. $y[x^x(\log ex)\log x + x^x]$

B. $y[x^x(\log ex)\log x + x]$

C. $y\left[x^x(\log ex)\log x + x^{-1}\right]$

D. $y\left[x^x(\log_e x)\log x + x^{-1}\right]$

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110. The first derivative of the function $\left[\cos^{-1}\left(s \in \sqrt{\frac{1+x}{2}}\right)+x^{x}\right]$ with respect to x at $x=1$ is $3/4$ (b) 0 (c) $1/2$ (d) $-1/2$

A. $3/4$

B. 0

C. $1/2$

D. $-1/2$

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111. Let $f(x)=x^{3}, x \in (0, \infty)$ and let g(x) be inverse of f(x), then g'(x) must be

A. $x(1+\log x)$

B. $x(1+\log(x))$

C. $\frac{1}{x(1+\log g(x))}$

D. non-existent

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112. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

A. n(n-1) y

B. n(n+1)y

C. ny

D. n^2y

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113. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

A. $m^2\left(ae^{mx} - be^{-mx}\right)$

B. 1

C. 0

D. none of these

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114. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of $\frac{|ab|}{3}$ is ___

A. 25

B. 9

C. -15

D. -9

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115. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x)isequa < o$ $2^{20}\left(\cos 2x - 2^{20}os3x\right)$

$2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$ $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$

$2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

A. $2^{20}\left(\cos 2x - 2^{20}\cos 3x\right)$

B. $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$

C. $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$

D. $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

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116. $\frac{d^n}{dx^n}(\log x) = \frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

A. $\frac{(n-1)!}{x^n}$

B. $\frac{n!}{x^n}$

C. $\frac{(n-2)!}{x^n}$

D. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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117. If $y = x\log\left(\frac{x}{a+bx}\right), then x^3\frac{d^2y}{dx^2} =$ (a) $\left(x\frac{dy}{dx} - y\right)$^2 (b) $x\left(\frac{dy}{dx}\right) - y$(c) y(dy/dx)-x(d)(y(dy/dx)-x)^2`

A. $x\frac{dy}{dx} - y$

B. $\left(x\frac{dy}{dx} - y\right)^2$

C. $y\frac{dy}{dx} - x$

D. $\left(y\frac{dy}{dx} - x\right)^2$

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118. If $ax^2 + 2hxy + by^2 = 1, then \frac{d^2y}{dx^2}$ is $\frac{h^2 - ab}{(hx + by)^2}$ (b) $\frac{ab - h^2}{(hx + by)^2}$ $\frac{h^2 + ab}{(hx + by)^2}$ (d) none of these

A. $\frac{h^2 - ab}{(hx + by)^3}$

B. $\frac{ab - h^2}{(hx + by)^2}$

C. $\frac{h^2 + ab}{(hx + by)^2}$

D. none of these

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119. If $y^{\frac{1}{m}} = \left(x + \sqrt{1 + x^2}\right), then\left(1 + x^2\right)y_2 + xy_1$ is (where y_r represents the rth derivative of y w.r.t. $x \cdot m^2y$ (b) my^2 (c) m^2y^2 (d) none of these

A. m^2y

B. my^2

C. m^2y^2

D. none of these

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120. If $(\sin x)(\cos y) = \frac{1}{2}, then \frac{d^2y}{dx^2} at \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -4 (b) -2 (c) -6 (d) 0

A. -4

B. -2

C. -6

D. 0

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121. A function f satisfies the condition $f(x) = f'(x) + f^x + f^x, where f(x)$ is a differentiable function indefinitely and dash denotes the order of derivative. If $f(x) = 1, then f(x)$ is $e^{\frac{x}{2}}$ (b) e^x (c) e^{2x} (d) e^{4x}

A. $e^{x/2}$

B. e^x

C. e^{2x}

D. e^{4x}

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122. Let $f(x)$ be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f^3 = 0, and f^3 = 12.$ Then the value of $f'(1)$ is 12 (b) 23 (c) -13 (d) none of these

A. 12

B. 23

C. -13

D. none of these

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123. If $y^2 = ax^2 + bx + c, then y^3 \frac{d^{2y}}{dx^2}$ is a constant a function of x only a function of y only a function of $x and y$

A. a constant

B. a function of x only

C. a function of y only

D. a function of x and y

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124. If $y = \sin x + e^x, then \frac{d^{2x}}{dy^2} = (-\sin x + e^x)^{-1} \frac{\sin x - e^x}{(\cos x + e^x)^2}$ (c) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$ (d) $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

A. $(-\sin x + e^x)^{-1}$

B. $\frac{\sin x - e^x}{(\cos x + e^x)^2}$

C. $\frac{\sin x - e^x}{(\cos x + e^x)^3}$

D. $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

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125. If $f^x = -f(x) and g(x) = f'(x) and F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5$, then $F(10)$ is 5 (b) 10 (c) 0 (d) 15

A. 5

B. 10

C. 0

D. 15

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126. Let $y = 1n(1+\cos x)^2$. Then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$ equal (b) $\frac{2}{1+\cos x}$ $\frac{4}{1+\cos x}$ (d) $\frac{-4}{(1+\cos x)^2}$

A. 0

B. $\frac{2}{1+\cos x}$

C. $\frac{4}{1+\cos x}$

D. $\frac{-4}{(1+\cos x)^2}$

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127. $x = t\cos t, y = t + \sin t\cdot$ Then $\frac{d^{2x}}{dy^2}att = \frac{\pi}{2}$ is $(a)\frac{\pi+4}{2}$ (b) $-\frac{\pi+4}{2}$ (c) -2 (d) none of these

A. $\frac{\pi+4}{2}$

B. $-\frac{\pi+4}{2}$

C. -2

D. none of these

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128. Let $y = t^{10} + 1$ and $x = t^8 + 1.$ Then $\frac{d^2y}{dx^2}$ is

A. $\frac{5}{2}t$

B. $20t^8$

C. $\frac{5}{16t^6}$

D. none of these

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129. If x = log p and $y = \frac{1}{p}$, then

A. $\frac{d^2y}{dx^2} - 2p = 0$

B. $\frac{d^2y}{dx^2} + y = 0$

C. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

D. $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

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130. If $x = \varphi(t), y = \psi(t), then \frac{d^{2y}}{dx^2}$ is $\frac{\varphi'\psi^{-\psi}{}'\varphi''}{(\varphi')^2}$ (b) $\frac{\varphi'\psi^{-\psi}{}'\varphi''}{(\varphi')^3}$

φ/ψ (d) ψ/φ

A. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$

B. $\frac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$

C. $\frac{\phi''}{\psi''}$

D. $\frac{\psi''}{\phi''}$

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131. If $f(x) = x^4 \tan(x^3) - x1n(1 + x^2)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is 0 (b) 6 (c) 12 (d) 24

A. 0

B. 6

C. 12

D. 24

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132. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g^{f(x)}$ equals. $-\frac{f^x}{(f'(x))^3}$ (b) $\frac{f'(x)f^x-(f'(x))^3}{f'(x)}$ $\frac{f'(x)f^x-(f'(x))^2}{(f'(x))^2}$ (d) none of these

A. $-\frac{f''(x)}{(f'(x))^3}$

B. $\frac{f'(x)f''(x)-(f(x))^3}{f'(x)}$

C. $\frac{f'(x)f''(x)-(f'(x))^2}{(f'(x))^2}$

D. none of these

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133. $f(x)=e^x-e^{-x}-2\sin x-\frac{2}{3}x^3$. Then the least value of n for which $\frac{d^n}{dx^n}f(x)(\ |\ )_{x=0} isnonzero is$ (a) 5 (b) 6 (c) 7 (d) 8

A. 5

B. 6

C. 7

D. 8

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134. Let $y = f(x)$ and $x = \frac{1}{z}$. If $\frac{d^2y}{dx^2} = \lambda_{z^3}\frac{dy}{dz} + z^4\frac{d^2y}{dz^2}$, then the value of λ is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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135. Let x = f(t) and y = g(t), where x and y are twice differentiable function If $f'(0) = g'(0) = f''(0) = 2, g(0) = 6,$ then the value of $\left(\frac{d^2y}{\left(dx^2\right)}\right)_{t=0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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136. If $f(x)$ satisfies the relation $f\left(\frac{5x-3y}{2}\right) = \frac{5f(x)-3f(y)}{2} \forall x, y \in R,$ and $f(0) = 3 and f'(0) = 2,$ then the period of $\sin(f(x))$ is 2π (b) π (c) 3π (d) 4π

A. 2π

B. π

C. 3π

D. 4π

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137. A function $f: R \to R$ satisfies $\sin x \cos y(f(2x+2y) - f(2x-2y) = \cos x \sin y(f(2x+2y) + f(2x-2y)$ If $f'(0) = \frac{1}{2}, then$ (a)$f''(x) = f(x) = 0$ (b)$4f''(x) + f(x) = 0$ (c) $f''(x) + f(x) = 0$ (d)$4f''(x) - f(x) = 0$

A. $f''(x) = f(x) = 0$

B. $4f''(x) + f(x) = 0$

C. $f''(x) + f(x) = 0$

D. $4f''(x) - f(x) = 0$

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Exercise (Multiple)

1. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, then $\frac{dy}{dx} isequa < o \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$ $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$ (d) $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

A. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$

B. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$

C. $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$

D. $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

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2. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \infty}}}$, $\frac{dy}{dx}$ is equal to $\frac{1}{2y - 1}$ (b) $\frac{x}{x + 2y}$ $\frac{1}{\sqrt{1 + 4x}}$ (d) $\frac{y}{2x + y}$

A. $\frac{1}{2y - 1}$

B. $\frac{x}{x + 2y}$

C. $\frac{1}{\sqrt{1 + 4x}}$

D. $\frac{y}{2x + y}$

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3. If $\cos 2\theta = (\sqrt{2} + 1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right)$, then the general value of $\theta (n \in Z)$

A. $f\left(\frac{\pi}{4}\right) = 1$

B. $f\left(\frac{\pi}{4}\right) = \sqrt{2}$

C. $\frac{d(f(\theta))}{d(\cos\theta)}$ at $\theta = \frac{\pi}{4} is - 2$

D. $f'\left(\frac{\pi}{4}\right) = \sqrt{2}$

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4. $f(x) = \left|x^2 - 3|x| + 2\right|$. Then which of the following is/are true

A. $f'(x) = 2x - 3 \quad \text{for} \quad x \in (0, 1) \cup (2, \infty)$

B. $f'(x) = 2x + 3f \text{ or } x \in (-\infty, -2) \cup (-1, 0)$

C. $f'(x) = -2x - 3 \quad \text{for} \quad x \in (-2, -1)$

D. None of these

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5. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1}x$. Then $f'(10 = 1 \quad f'\left(\frac{3}{2}\right) = -1$

$doma \in off(x)isx \geq 1$ range of $f(x)is(-2 - 1) \cup (2, \infty)$

A. $f(10) = 1$

B. $f'(3/2) = -1$

C. domain of $f(x) is x \geq 1$

D. range of $f(x) is (-2, -1)] \cup (2, 00)$

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6. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0 and y(1) = 1, then \ y'(1) = \frac{4}{3}$ (b) $y^1 = -\frac{4}{3} \ y^1 = -8\frac{22}{27}$ (d) $y'(1) = \frac{2}{3}$

A. $y'(1) = 4/3$

B. $y''(1) = -4/3$

C. $y''(1) = -8\frac{22}{27}$

D. $y'(1) = 2/3$

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7. If $y = x^{\log x \hat{}}((\log(\log \dot{x})))$, $then \frac{dy}{dx} is$

$\frac{y}{x}\left(1nx^{\infty x-1}\right) + 21nx1n(1nx)\Big)$ $\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x)+1)$

$\frac{y}{x1nx}\left[(1nx)^2 + 21n(1nx)\right]$ $\frac{y}{x}\frac{\log y}{\log x}[2\log(\log x)+1]$

A. $\frac{y}{x}\left((Inx^{x-1}\right) + 2 \text{ In x In(In x)})$

B. $\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x)+1)$

C. $\frac{y}{\text{x In x}}\left[(Inx)^2 + 2In(Inx)\right]$

D. $\frac{y}{x}\frac{\log y}{\log x}[2\log(\log x)+1]$

Answer: B

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8. If $f(x-y), f(x)f(y), and f(x+y)$ are in A.P. for all $x, y, and f(0) \neq 0,$ then (a)$f(4) = f(-4)$ (b)$f(2)+f(-2)=0$ (c) $f'(4)+f'(-4)=0$ (d)$f'(2) = f'(-2)$

A. $f(4) = f(-4)$

B. $f(2) + f(-2) = 0$

C. $f'(4) + f'(-4) = 0$

D. $f'(2) = f'(-2)$

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9. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), then \frac{dy}{dx} is$ $\frac{-2}{1+x^2}$ for all x (b) $\frac{-2}{1+x^2}$ for all $|x| < 1$ $\frac{2}{1+x^2}$ for $|x| > 1$ (d) none of these

A. $\frac{-2}{1+x^2}$ for all x

B. $\frac{-2}{1+x^2}$ for all $|x| < 1$

C. $\frac{2}{1+x^2}$ for $|x| > 1$

D. none of these

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10. $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N and f_0(x) = x, then \frac{d}{dx}\{f_n(x)\}$ is $\frac{f_n(x)d}{dx}\{f_{n-1}(x)\}$ (b) $f_n(x)f_{n-1}(x)$ $f_n(x)f_{n-1}(x)f_2(x)\dot{f}_1(x)$ $noneofthese$

A. $f_n(x)\frac{d}{dx}\{f_{n-1}(x)\}$

B. $f_n(x)f_{n-1}(x)$

C. $f_n(x)f_{n-1}(x)...f_2(x).\ f_1(x)$

D. None of these

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11. Suppose $f and g$ are functions having second derivative f and g'' everywhere. If $f(x)g(\dot{x}) = 1$ for all $x and f' and g'$ are never zero, then $\frac{f^x}{f'(x)} - \frac{g^x}{g'(x)} equal \frac{-2f'(x)}{f}$ (b) $\frac{2g'(x)}{g(x)}$ $\frac{-f'(x)}{f(x)}$ (d) $\frac{2f'(x)}{f(x)}$

A. $\frac{-2f'(x)}{f(x)}$

B. $\frac{-2g'(x)}{g(x)}$

C. $\frac{-f'(x)}{f(x)}$

D. $\frac{2f'(x)}{f(x)}$

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12. If $y = e^{-x}\cos x and y_n + k_n y = 0, where y_n = \frac{d^{ny}}{dx^n} and k_n$ are constants $\forall n \in N,$ then $k_4 = 4$ (b) $k_8 = -16$ $k_{12} = 20$ (d) $k_{16} = -24$

A. $k_4 = 4$

B. $k_8 = -16$

C. $k_{12} = 20$

D. $k_{16} = -24$

Answer: B

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13. If a function is represented parametrically be the equations $x = \frac{1 + (\log)_e t}{t^2}; y = \frac{3 + 2(\log)_e t}{t}$, then which of the following statements are true? $y^{x - 2xy'} = y \quad yy' = 2x\left(y'\right)^2 + 1$ $xy' = 2y\left(y'\right)^2 + 2\, y^{y - 4xy'} = \left(y'\right)^2$

A. $y''(x - 2xy') = y$

B. $yy' = 2x(y')^2 + 1$

C. $xy' = 2y(y')^2 + 2$

D. $y''(y - 4xy') = (y')^2$

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14. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2 + 1} + (\log)_e\sqrt{x + \sqrt{x^2 + 1}}$, prove that $2y = xy' + (\log)_e y'$, $where y'$ denotes the derivative w.r.t x.

A. $y' = x + \sqrt{x^2 + 1}$

B. $y' = \frac{1}{x + \sqrt{x^2 + 1}}$

C. $2y = xy' + \log_e y'$

D. $2y = xy' - \log_e y'$

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15. A curve parametrically given by $x = t + t^3$ and $y = t^2$, where $t \in R$. For what vlaue(s) of t is $\frac{dy}{dx} =$?

A. $1/3$

B. 2

C. 3

D. 1

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16. If $e^{\sin(x^2+y^3)} = \tan\frac{y^2}{4} + \sin^{-1}x$, then y'(0) can be

A. $\frac{1}{3\sqrt{\pi}}$

B. $-\frac{1}{3\sqrt{\pi}}$

C. $-\frac{1}{5\sqrt{\pi}}$

D. $-\frac{1}{3\sqrt{5\pi}}$

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17. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, then $\frac{dy}{dx} isequa < o \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$ $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$ (d) $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

A. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$

B. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$

C. $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$

D. $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

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18. Let $y=\sqrt{x+\sqrt{x+\sqrt{x+\infty}}}$, $\frac{dy}{dx}$ is equal to $\frac{1}{2y-1}$ (b) $\frac{x}{x+2y}$ $\frac{1}{\sqrt{1+4x}}$ (d) $\frac{y}{2x+y}$

A. $\frac{1}{2y-1}$

B. $\frac{x}{x+2y}$

C. $\frac{1}{\sqrt{1+4x}}$

D. $\frac{y}{2x+y}$

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19. If $f(x-y), f(x)f(y), and f(x+y)$ are in A.P. for all $x, y, and f(0) \neq 0$, then (a)$f(4) = f(-4)$ (b)$f(2) + f(-2) = 0$ (c) $f'(4) + f'(-4) = 0$ (d)$f'(2) = f'(-2)$

A. $f(4) = f(-4)$

B. $f(2) + f(-2) = 0$

C. $f'(4) + f'(-4) = 0$

D. $f'(2) = f'(-2)$

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20. If a function is represented parametrically be the equations $x = \frac{1 + (\log)_e t}{t^2}; y = \frac{3 + 2(\log)_e t}{t}$, then which of the following statements are true? $y^{x-2xy'} = y$ $yy' = 2x(y')^2 + 1$ $xy' = 2y(y')^2 + 2$ $y^{y-4xy'} = (y')^2$

A. $y''(x - 2xy') = y$

B. $yy' = 2x(y')^2 + 1$

C. $xy' = 2y(y')^2 + 2$

D. $y''(y - 4xy') = (y')^2$

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21. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2+1} + (\log)_e\sqrt{x + \sqrt{x^2+1}}$, prove that $2y = xy' + (\log)_e y'$, $where y'$ denotes the derivative w.r.t x.

A. $y' = x + \sqrt{x^2+1}$

B. $y' = \frac{1}{x + \sqrt{x^2+1}}$

C. $2y = xy' + \log_e y'$

D. $2y = xy' - \log_e y'$

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22. A curve parametrically given by $x = t + t^3$ and $y = t^2$, where $t \in R$. For what vlaue(s) of t is $\frac{dy}{dx} =$?

A. $1/3$

B. 2

C. 3

D. 1

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Exercise (Comprehension)

1. f(x) is a polynomial function, $f: R \to R$, such that $f(2x) = f'(x)f''(x)$. The value of f(3) is

A. 4

B. 12

C. 15

D. none of these

Answer: B

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2. f(x) is a polynomial function, $f: R \to R$, such that $f(2x) = f'(x)f''(x)$. f(x) is (A) one-one and onto (B) one-one and into (C) many-one and onto (D) many-one and into

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

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3. f(x) is a polynomial function, $f: R \to R$, such that $f(2x) = f'(x)f''(x)$. Equation f(x) = x has (A) three real and positive roots (B) three real and negative roots (C) one real root (D) three real roots such that sum of roots is zero

A. three real and positive roots

B. three real and negative roots

C. one real root

D. three real roots such that sum of roots is zero

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4.

$f: R \to R$, $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$.

The value of $f'(1) + f''(2) + f'''(3)$ is

A. 2

B. 3

C. -1

D. 4

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5.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \quad \text{for all} \quad x \in R.$

The value of f(1) is

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

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6.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R.$

The value of $f'(1) + f''(2) + f'''(3)$ is

A. 0

B. -1

C. 2

D. 3

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7. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x - \alpha)^2 (x - \beta) \ldots = 0,$ from which we can conclude that

$f(x) = 0$ or $2(x-\alpha)[(x-\beta)...] + (x-\alpha)^2[(x-\beta)...]' = 0$ or $(x-\alpha)$ has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f'''(x)=0. If x-c is a factor of order m of the polynomial f(x) of degree n $(1 < m < n)$, then x=c is a root of the polynomial [where $f^r(x)$ represent rth derivative of f(x) w.r.t. x]

A. $f^m(x)$

B. $f^{m-1}(x)$

C. $f^n(x)$

D. none of these

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8. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0

is equivalent to $(x-\alpha)^2(x-\beta)\ldots=0,$ from which we can conclude that

$f(x)=0$ or $2(x-\alpha)[(x-\beta)\ldots]+(x-\alpha)^2[(x-\beta)\ldots]'=0$ or $(x-\alpha)$ has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f'''(x)=0. If $a_1x^3+b_1x^2+c_1x+d_1=0$ and $a_2x^3+b_2x^2+c_2x+d_2=0$ have a pair of repeated roots common, then

$$\begin{vmatrix} 3a_1 & 2b_1 & c_1 \\ 3a_2 & 2b_2 & c_2 \\ a_2b_1-a_1b_2 & c_1a_2-c_2a_1 & d_1a_2-d_2a_1 \end{vmatrix}=$$

A. 0

B. 1

C. -1

D. 2

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9. If roots of an equation $x^n - 1 = 0$ are $1, a_1, a_2, . . , a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3). . (1 - a_{n-1})$ will be

A. $n^2/2$

B. n

C. $(-1)^n n$

D. none of these

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10. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_{n-1}$.

The value of $\sum_{r=1}^{n-1} \frac{1}{2 - a_r}$ is

A. $\frac{2^{n-1}(n-2)+1}{2^n - 1}$

B. $\frac{2^n(n-2)+1}{2^n - 1}$

C. $\frac{2^{n-1}(n-1)-1}{2^n-1}$

D. none of these

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11. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_{n-1}.$

The value of $\sum_{n-1}^{r=1} \frac{1}{1-a_r}$ is

A. $\frac{n}{4}$

B. $\frac{n(n-1)}{2}$

C. $\frac{n-1}{2}$

D. none of these

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12.

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f'(x).$

The value of f(3) is

A. 1

B. 0

C. -1

D. -2

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13.

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f'(x).$

The value of g(0) is

A. 0

B. -3

C. 2

D. none of these

Answer: C

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14.

$f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f'(x).$

The domain of the function $\sqrt{\frac{f(x)}{g(x)}}$ is

A. $(-\infty, 1] \cup (2, 3]$

B. $(-2, 0] \cup (1, \infty)$

C. $(-\infty, 0] \cup (2/3, 3]$

D. none of these

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15.

$g(x+y)=g(x)+g(y)+3xy(x+y)\,\forall x,y\in R$ and $g'(0)=-4.$

Number of real roots of the equation g(x) = 0 is

A. 2

B. 0

C. 1

D. 3

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16.

$g(x+y)=g(x)+g(y)+3xy(x+y)\,\forall x,y\in R$ and $g'(0)=-4.$

For which of the following values of x is $\sqrt{g(x)}$ not defined ?

A. $[-2,0]$

B. $[2,\infty)$

C. $[-1, 1]$

D. none of these

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17.

$g(x+y) = g(x) + g(y) + 3xy(x+y) \forall x, y \in R$ and $g'(0) = -4.$

The value of g'(1) is

A. 0

B. 1

C. -1

D. none of these

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18. A curve is represented parametrically by the equations $x = f(t) = a^{In(b')}$ and $y = g(t) = b^{-In(a^t)}\, a, b > 0$ and $a \neq 1, b \neq 1$ W

Which of the following is not a correct expression for $\frac{dy}{dx}$?

A. $\frac{1}{f(t)^2}$

B. $-(g(t))^2$

C. $\frac{-g(t)}{f(t)}$

D. $\frac{-f(t)}{g(t)}$

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19. A curve is represented parametrically by the equations $x = f(t) = a^{In(b^t)}$ and $y = g(t) = b^{-In(a^t)}\, a, b > 0$ and $a \neq 1, b \neq 1$ W

The value of $\frac{d^2y}{dx^2}$ at the point where f(t)=g(t) is

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

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20. A curve is represented parametrically by the equations $x = f(t) = a^{In(b')}$ and $y = g(t) = b^{-In(a^t)}\, a, b > 0$ and $a \neq 1, b \neq 1$ V

The value of $\frac{f(t)}{f'(t)} \cdot \frac{f''(-t)}{f'(-t)} + \frac{f(-t)}{f'(-t)} \cdot \frac{f''(t)}{f'(t)} \forall t \in R$ is

A. -2

B. 2

C. -4

D. 4

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21. Let $f: R \to R$ be a differentiable function satisfying $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, then

The value of f'(3) is

A. 8

B. 10

C. 12

D. 18

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22. Let $f: R \to R$ be a differentiable function satisfying $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, then

The value of f(9) is

A. 240

B. 356

C. 252

D. 730

Answer: C

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23. If roots of an equation $x^n - 1 = 0$ are $1, a_1, a_2, . . , a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3). . (1 - a_{n-1})$ will be

A. $n^2/2$

B. n

C. $(-1)^n n$

D. none of these

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Linked Comprehension Type

1. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x-\alpha)^2(x-\beta)\ldots=0,$ from which we can conclude that

$f(x)=0$ or $2(x-\alpha)[(x-\beta)\ldots]+(x-\alpha)^2[(x-\beta)\ldots]'=0$ or $(x-\alpha)$ has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f'''(x)=0. If α root occurs p times and β root occurs q times in polynomial equation f(x)=0 of degree $n(1<p,q<n)$, then which of the following is not ture [where $f^r(x)$ represents rth derivative of f(x) w.r.t x] ?

A. If $p<q<n$, then α and β are two of the roots of the equation $f^{p-1}(x)=0.$

B. If $q < p < n$, then α and β are two of the roots of the equation $f^{q-1}(x) = 0$.

C. If $p < q < n$, then equations f(x) = 0 and $f^p(x) = 0$ have exactly one root common

D. If $q < p < n$, then equations $f^q(x) = 0$ and $f^p(x) = 0$ have exactly two roots common.

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Exercise (Matrix)

1. Find the derivative of $x = \theta^3 \csc 2\theta$.

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2. Find the derivative of $y = \sec^4 3x$.

Watch Video Solution

3. Find the derivative of $y = \csc^2(2x^2)$.

Watch Video Solution

4. Find the derivative of $y = \sec^2 2x$

Watch Video Solution

5. Match the relation for derivatives given in List II with the relation given in List I and then choose the correct code.

A. $\begin{matrix} a & b & c & d \\ q & p & s & r \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ r & q & s & p \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ q & p & r & s \end{matrix}$

Watch Video Solution

Exercise (Numerical)

1. $f'(x) = \varphi'(x) = f(x)$ for all x. Also, $f(3) = 5 and f'(3) = 4$. Then the value of $[f(10)]^2$ is _____

Watch Video Solution

2. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then |f'(-3)|$ equals_____.

Watch Video Solution

3. If $x^3 + 3x^2 - 9x = c$ is of the form $(x - \alpha)^2(x - \beta)$, then c is equal to 27 b. -27 c. 5 d. -5

Watch Video Solution

4. If graph of $y = f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7) = 3,$ then the value of $f'(3)$ is_____

Watch Video Solution

5. Let $g(x) = f(x)\sin x, where f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f(-\pi) = 1.$ The value of $|g^n(-\pi)|$ equals _____

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6. Let $f(x) = (x-1)(x-2)(x-3)(x-n), n \in N, and f(n) = 5040.$

Then the value of n is_____

Watch Video Solution

7. $y = f(x)$, where f satisfies the relation $f(x+y) = 2f(x) + xy(y) + y\sqrt{f(x)}\,\forall x, y \in R and f'(0) = 0.$ Then $f(6)$ is equal of $f(-3)$ is _____

Watch Video Solution

8. If function f satisfies the relation $f(x)xf'(-x) = f(-x)xf'(x)f$ or $all x, and f(0) = 3, and \Leftrightarrow (3) = 3,$

then the value of $f(-3)$ is _____

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9. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}} and y' = 0 at x = 5,$ then the value of $\frac{a^2}{b^2}$ is_____

Watch Video Solution

10. Prove that $\frac{2^{(\log)_2 \frac{1}{4} x} - 3^{\log}\_(27)\left(x^2+1\right)^3 - 2x >}{7^{4(\log)_{49} x} - x - 1} 0$

Watch Video Solution

11. $(\lim)_{h \to 0} \frac{(e+h)^{1n(e+h)} - e}{h} is\_\_$

Watch Video Solution

12. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3} and g(x) = f^{-1}(x),$ then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is_____

Watch Video Solution

13. Suppose that $f(0) = 0 and f'(0) = 2,$ and let $g(x) = f(-x + f(f(x)))$. The value of g' (0) is equal to _____

Watch Video Solution

14. f(x) is polynomial of degree 4 with real coefficients such that f(x)=0 satisfied by x=1, 2, 3 only then $f'(1)f'(2)f'(3)$ is equal to -

Watch Video Solution

15. A nonzero polynomial with real coefficient has the property that $f(x) = f'(x)\dot{f}''(x)$. If a is the leading coefficient of $f(x)$, then the value of $\frac{1}{2a}$ is____

Watch Video Solution

16. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}\dot{T}henthevalueof\left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is_____

Watch Video Solution

17. Let $z = (\cos x)\hat{5}andy = \sin x\cdot$ Then the value of $2\frac{d^2z}{dy^2}atx = \frac{2\pi}{9}$ is_____.

Watch Video Solution

18. Let $g(x) = \left\{\frac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x}, x \neq 00, x = 0\right.$ If $g'(0)$ exists and is equal to nonzero value b, then $52\frac{b}{a}$ is equal to ____

Watch Video Solution

19. Let $f(x) = x + \frac{1}{2x + \frac{1}{2x + \frac{1}{2x+\infty}}}$ Compute the value of $f(50)\dot{f}'(50)$

Watch Video Solution

20. Let $F(x) = f(x)g(x)h(x)$ for all real $x, where f(x), g(x), and h(x)$ are differentiable functions. At some point $x_0, F'(x_0) = 21F(x_0), f'(x_0)4f(x_0), g'(x_0) = -7g(x_0)$, then the value of $g'(1)$ is _____

Watch Video Solution

21. If $y = \frac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$, then y'(0) is equal to -

Watch Video Solution

22. $f'(x) = \varphi'(x) = f(x)$ for all x. Also, $f(3) = 5 and f'(3) = 4$. Then the value of $[f(10)]^2$ is _____

Watch Video Solution

23. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then|f'(-3)|$ equals_____.

Watch Video Solution

24. If $x^3 + 3x^2 - 9x = c$ is of the form $(x-\alpha)^2(x-\beta)$, then c is equal to 27 b. -27 c. 5 d. -5

Watch Video Solution

25. If graph of $y = f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7) = 3,$ then the value of $f'(3)$ is_____

Watch Video Solution

26. Let $f(x) = (x-1)(x-2)(x-3)(x-n), n \in N, andf(n) = 5040.$ Then the value of n is_____

Watch Video Solution

27. $y = f(x)$, where f satisfies the relation $f(x+y) = 2f(x) + xy(y) + y\sqrt{f(x)}\,\forall x, y \in R and f'(0) = 0.$ Then $f(6)$ is equal of $f(-3)$ is _____

Watch Video Solution

28. If a function 'f' satisfies the relation $f(x)f''(x) - f(x)f'(x) - f'(x)^2 = 0 \forall x \in R$ and $f(0) = 1 = f'(0).$ Then find $f(x).$

Watch Video Solution

29. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}} and y' = 0 at x = 5,$ then the value of $\frac{a^2}{b^2}$ is_____

Watch Video Solution

30. $\left(\lim\right)_{h \to 0} \frac{(e+h)^{1n(e+h)} - e}{h} is\_\_$

Watch Video Solution

31. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3} and g(x) = f^{-1}(x),$ then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is_____

Watch Video Solution

32. Suppose that $f(0) = 0 and f'(0) = 2,$ and let $g(x) = f(-x + f(f(x)))$. The value of g' (0) is equal to _____

Watch Video Solution

33. A nonzero polynomial with real coefficient has the property that $f(x) = f'(x)\dot{f}''(x)$. If a is the leading coefficient of $f(x)$, then the

value of $\frac{1}{2a}$ is____

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34. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}\dot{T}henthevalueof\left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is_____

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35. Let $z = (\cos x)\hat{5}andy = \sin x \cdot$ Then the value of $2\frac{d^2z}{dy^2}atx = \frac{2\pi}{9}$ is_____.

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36. Let $g(x) = \begin{cases} \frac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x}, x \neq 00, x = 0 \end{cases}$ If $g'(0)$ exists and is equal to nonzero value b, then $52\frac{b}{a}$ is equal to ____

Watch Video Solution

37. Let $f(x) = x + \dfrac{1}{2x + \dfrac{1}{2x + \dfrac{1}{2x + \infty}}}$ Compute the value of $f(50)\dot{f}'(50)$

Watch Video Solution

38. Let $F(x) = f(x)g(x)h(x)$ for all real $x, where f(x), g(x), and h(x)$ are differentiable functions. At some point $x_0, F'(x_0) = 21F(x_0), f'(x_0)4f(x_0), g'(x_0) = -7g(x_0),$ then the value of $g'(1)$ is _____

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39. If $y = \dfrac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$, then y'(0) is equal to -

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40. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0.$

Then y' (1) equals

A. -1

B. 1

C. log 2

D. `-log 2

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41. Let $f: (1, 1)\overrightarrow{R}$ be a differentiable function with $f(0) = 1 and f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ (1) 4 (2) 0 (3) 2 (4) -4

A. -2

B. 4

C. -4

D. 0

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JEE Previous Year

1. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0.$

Then $y'(1)$ equals: 1 b. $\log 2$ c. $-\log 2$ d. -1

A. -1

B. 1

C. log 2

D. `-log 2

Answer: A

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2. Let $f:(1,1)\overrightarrow{R}$ be a differentiable function with $f(0)=1 and f'(0)=1$. Let $g(x)=[f(2f(x)+2)]^2$. Then $g'(0)=$ (1) 4 (2) 0 (3) 2 (4) -4

A. -2

B. 4

C. -4

D. 0

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3. $\frac{d^2x}{dy^2}$ equals: (1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dx}{dy}\right)^3$ (3) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

A. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

B. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

C. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

D. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

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4. If $y = \sec\left(\tan^{-1} x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

A. $1/2$

B. 1

C. $\sqrt{2}$

D. $1\sqrt{2}$

Watch Video Solution

5. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$ then g(x) is equal to (1) $1+x^5$ (2) $5x^4$ (3) $\frac{1}{1+\{g(x)\}^5}$ (4) $1+\{g(x)\}^5$

A. $1+x^5$

B. $5x^4$

C. $\frac{1}{1+\{g(x)\}^5}$

D. $1+\{g(x)\}^5$

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6. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g(x)$, then $g(x)$ equals: $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

A. $\frac{3}{1+9x^3}$

B. $\frac{9}{1+9x^3}$

C. $\frac{3x\sqrt{x}}{1-9x^3}$

D. $\frac{3x}{1-9x^3}$

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7. $\frac{d^2x}{dy^2}$ equals: (1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dx}{dy}\right)^3$ (3) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

A. -1

B. 1

C. log 2

D. `-log 2

Answer: A

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8. If $y = \sec\left(\tan^{-1} x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

A. -2

B. 4

C. -4

D. 0

Watch Video Solution

9. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then g'(x) is equal to

A. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

B. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

C. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

D. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

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10.

If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g(x) equals

A. $1/2$

B. 1

C. $\sqrt{2}$

D. $1\sqrt{2}$

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11. Let $f(x) = x \sin \pi x, x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at a unique point in the interval $\left(n, n + \frac{1}{2}\right)$ a unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$ a unique point in the interval $(n, n + 1)$ two points in the interval $(n, n + 1)$

A. $1 + x^5$

B. $5x^4$

C. $\frac{1}{1 + \{g(x)\}^5}$

D. $1 + \{g(x)\}^5$

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12.

Let $f: (0. \infty) \to R$ be a differentiable function such that $f'(x) = 2 -$

Then

A. $\frac{3}{1 + 9x^3}$

B. $\frac{9}{1+9x^3}$

C. $\frac{3x\sqrt{x}}{1-9x^3}$

D. $\frac{3x}{1-9x^3}$

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13. Let `f(theta)=sin(tan^(-1)((sintheta)/(sqrt(cos2theta)))),w h e r e-pi/4

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14. The slope of the tangent to the curve $\left(y-x^5\right)^2=x\left(1+x^2\right)^2$ at the point $(1,3)$ is.

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15. Let $f: R \to R$ be a differentiable function with f(0) = 1 and satisfying the equation $f(x+y) = f(x)f'(y) + f'(x)f(y)$ for all x, $y \in R$. Then, the value of $\log_e(f(4))$ is .....

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Multiple Correct Answers Type

1. Let $f(x) = x \sin \pi x, x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at a unique point in the interval $\left(n, n+\frac{1}{2}\right)$ a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$ a unique point in the interval $(n, n+1)$ two points in the interval $(n, n+1)$

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

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2. Let $f:(0,\infty)\to R$ be a differentiable function such that $f'(x)=2-\dfrac{f(x)}{x}$ for all $x\in(0,\infty)$ and $f(1)=1$, then

A. $\lim\limits_{x\to 0^+} f'\left(\dfrac{1}{x}\right)=1$

B. $\lim\limits_{x\to 0^+} xf\left(\dfrac{1}{x}\right)=2$

C. $\lim\limits_{x\to 0^+} x^2f'(x)=0$

D. $|f(x)|\le 2$ for all $x\in(0,2)$

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3. Let $f:R\to R$ and $h:R\to R$ be differentiable functions such that $f(x)=x^3+3x+2, g(f(x))=x$ and $h(g(x))=x$ for all $x\in R$.

Then,

A. $g'(2) = \frac{1}{15}$

B. $h'(1) = 666$

C. $h(0) = 16$

D. $h(g(3)) = 36$

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Matrix Match Type

1. Find the derivative of $y = \cos^{-1} 5x$.

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Numerical Value Type

1. Let `f(theta)=sin(tan^(-1)((sintheta)/(sqrt(cos2theta)))),w h e r e-pi/4

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2. The slope of the tangent to the curve $\left(y-x^5\right)^2=x\left(1+x^2\right)^2$ at the point $(1,3)$ is.

Watch Video Solution

3. Let $f: R \to R$ be a differentiable function with f(0) = 1 and satisfying the equation $f(x+y)=f(x)f'(y)+f'(x)f(y)$ for all $x, y \in R$. Then, the value of $\log_e(f(4))$ is .....

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