



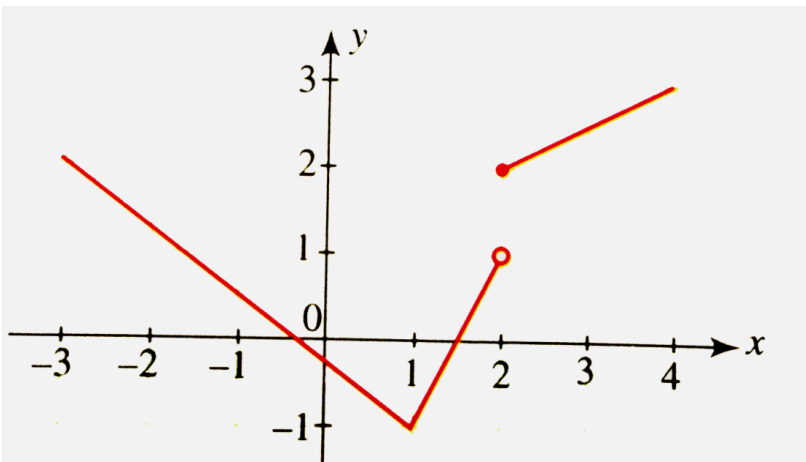
## MATHS

### BOOKS - CENGAGE

## GETTING STARTED WITH GRAPHS

### Illustrations

1. Does the following graph represent a function or a relation?

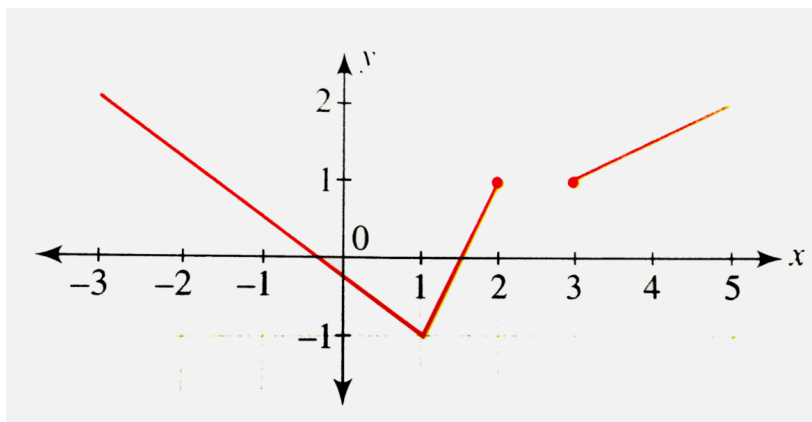


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2. Find the angle between  $\vec{a}$  and  $\vec{b}$ , when  $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$

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3. Does the following graph pass the vertical or horizontal line test?



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4. In how many points graph of  $y = x^3 - 3x^2 + 5x - 3$  intersect the x-axis?

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5. Find the sum of the vectors  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{c} = 2\hat{i} - 6\hat{j} - 7\hat{k}$



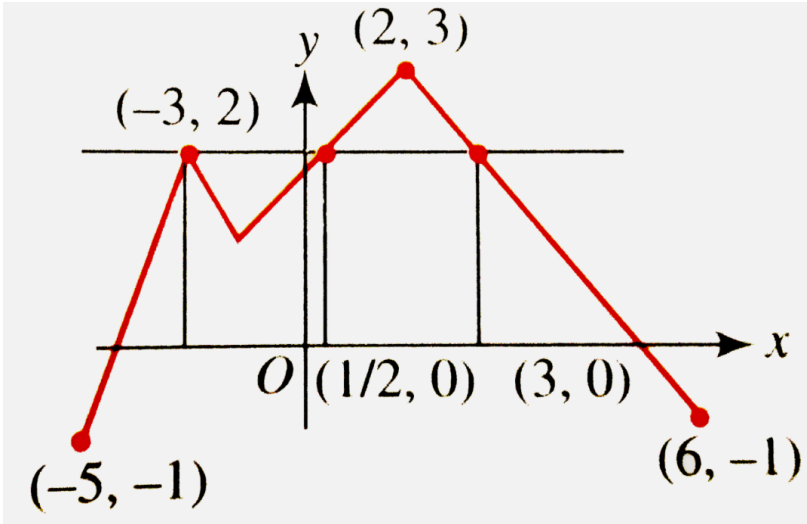
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6. Which of the following pair of graphs intersect?  $y = x^2 - x$  and  $y = 1$   
 $y = x^2 - 2x + 3$  and  $y = \sin x$   $y = x^2 - x + 1$  and  $y = x - 4$



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7. The graph of  $y = f(x)$  is shown, find the number of solution of  $f(f(x)) = 2$ .



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8. Does the graph of the function  $f(x) = x^2 - 3$  have  $y$ -axis symmetry?

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9. Does the graph of the function  $f(x) = 1/x^3$  have origin symmetry?

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10. Which of the following functions has (have) y-symmetry or origin symmetry?

(i)  $f(x) = x^2 \sin x$

(ii)  $f(x) = \log(x + \sqrt{1 + x^2})$

(iii)

$$f(x) = \frac{e^x + e^{-x}}{2}$$

$$(iv) f(x) = \begin{cases} 0, & \text{If } x \text{ is rational} \\ 1, & \text{If } x \text{ is irrational} \end{cases}$$



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11. Let  $f: R \rightarrow R$  be a continuous onto function satisfying  $f(x) + f(-x) = 0 \forall x \in R$ . If  $f(-3) = 2$  and  $f(5) = 4 \in [-5, 5]$ , then the minimum number of roots of the equation  $f(x) = 0$  is



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13. If the lines  $x - 2y - 6 = 0$ ,  $3x + y - 4 = 0$  and  $\lambda x + 4y + \lambda^2 = 0$  can be concurrent, then



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14. Draw the graph of

$$f(x) = \begin{cases} x - 2n, & 2n \leq x < 2n + 1, \end{cases} ($$

periodic? If yes, what is its period?



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15. If  $f: R^+ \rightarrow R^+$  and  $g: R^+ \rightarrow R^+$ , defined as

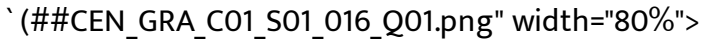
$$f(x) = x^2, g(x) = \sqrt{x},$$

then find  $gof$  and  $fog$  whether are they equivalent?



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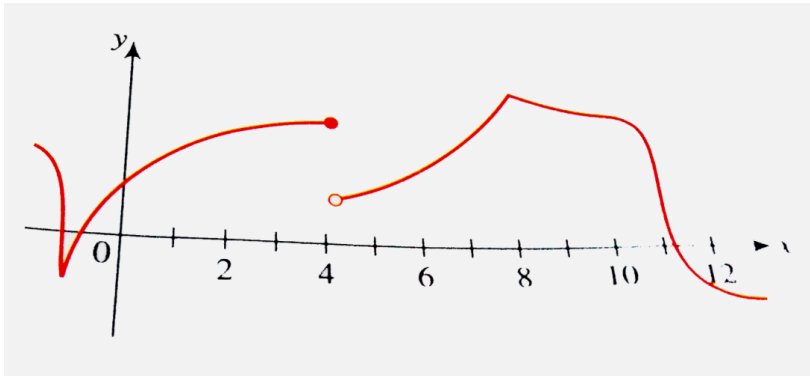
16. The graph of  $y = f(x)$  is as shown in the following figure.



Identify the points of discontinuity and give the reason for the same.

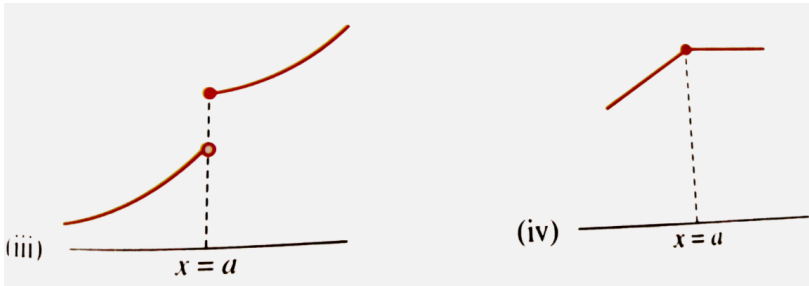
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17. The graph of  $f(x)$  is given. State with reasons the number at which the function is non-differentiable.



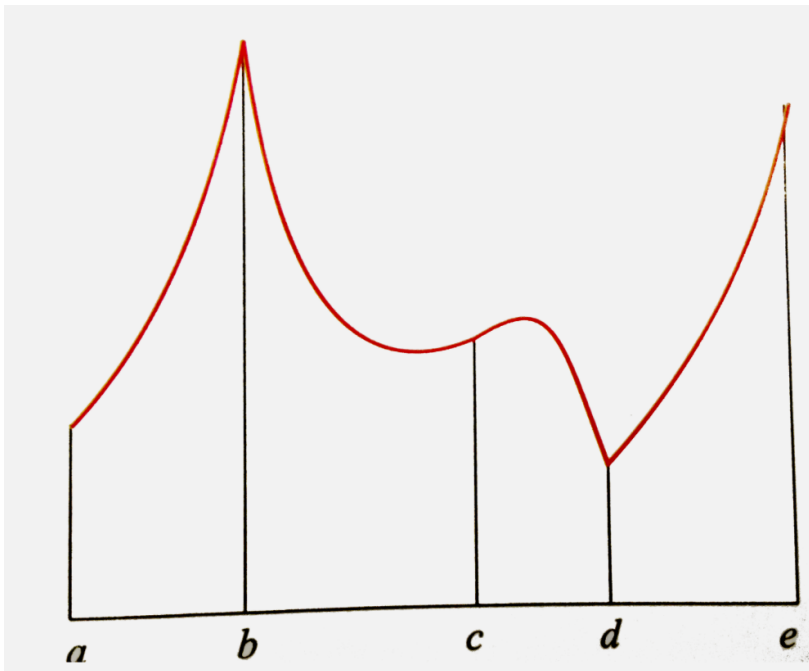
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18. For each of the following graphs, comment whether  $f(x)$  is increasing or decreasing or neither increasing nor decreasing at  $x = a$ .



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19. The graph of a function is shown in the following figure.



Determine the intervals on which the function is concave up and the

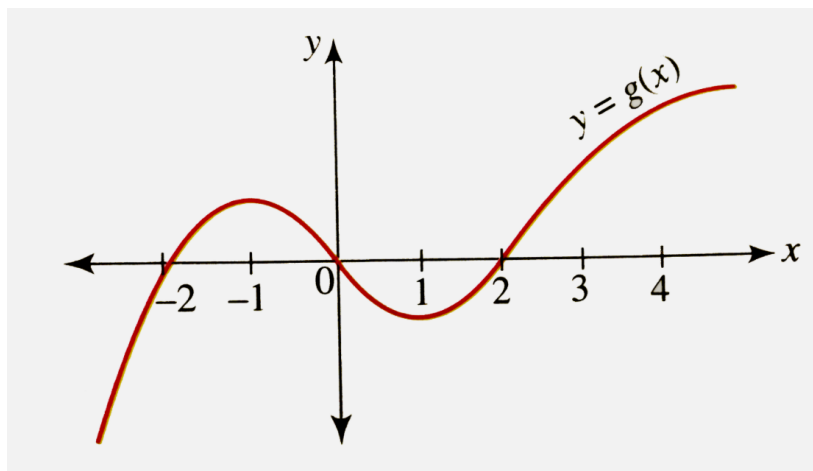


intervals on which it is concave down. Find the  $x$  - coordinates of any inflection points.

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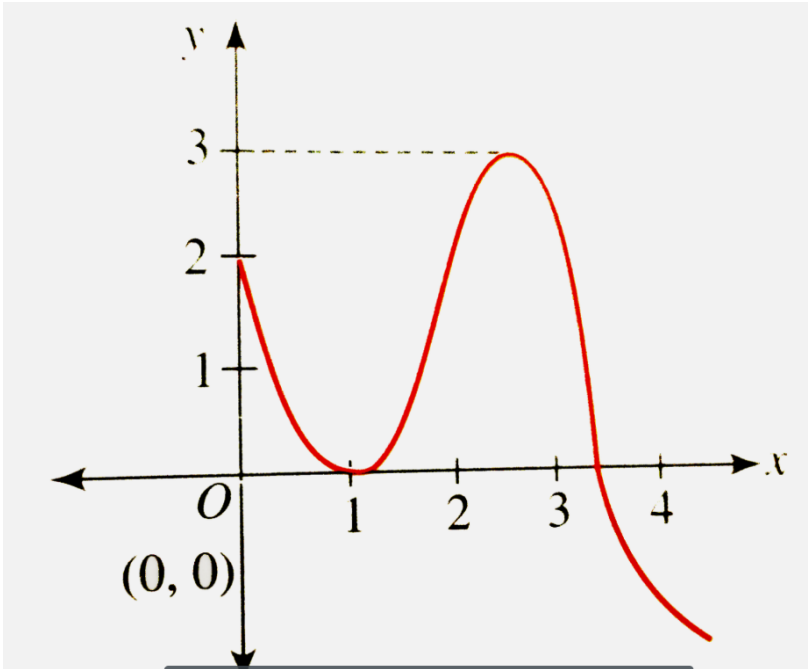
20. For the function  $g$  whose graph is given. Arrange the following numbers in increasing order and explain your reasoning.

$g(0)$ ,  $g'(-2)$ ,  $g'(0)$ ,  $g'(2)$ ,  $g'(4)$



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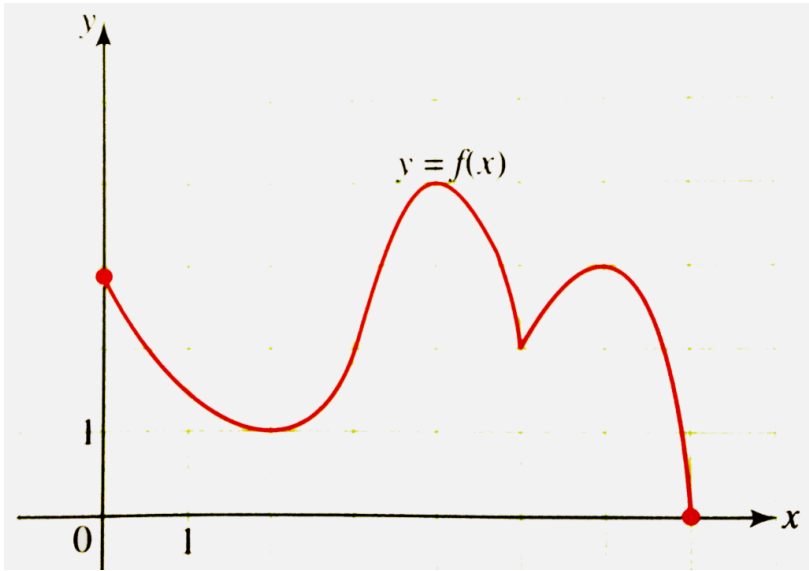
21. The diagram shows the graph of the derivative of a function  $f(x)$  for  $0 \leq x \leq 4$  with  $f(0) = 0$ . Which of the following could be correct statements for  $y = f(x)$ ?



- (a) Tangent line to  $y = f(x)$  at  $x = 0$  makes an angle of  $\sec^{-1} \sqrt{5}$  with the  $x$ -axis.
- (b)  $f$  is increasing in  $(0, 3)$ .
- (c)  $x = 1$  is both an inflection point and the point of local extremum.
- (d) Number of critical point on  $y = f(x)$  is two.

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22. In the following graph, state the absolute and local maximum and minimum values of the function.

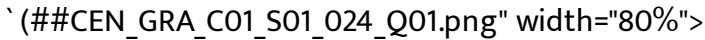


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23. Evaluate  $\int (2x - 5)^2 dx$

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24. The graph of the second derivation  $f''(x)$  is given in the following figure. State the  $x$  - coordinate of the point of inflection of  $f$ . Given reasons for your answer.



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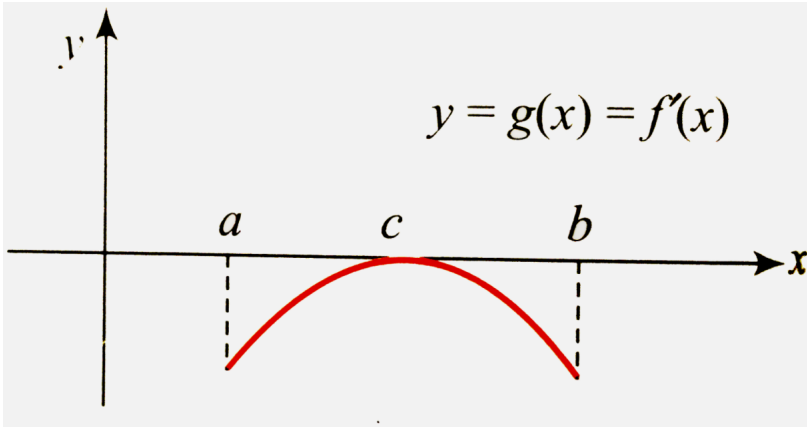
25. If  $P(E) = 0.10$ , what is the probability of not  $E$ ?

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26. Following is the graph of  $y = f'(x)$ , given that  $f(c) = 0$ . Analyse the graph and answer the following questions.

- How many times the graph of  $y = f(x)$  will intersect the  $x$  - axis?
- Discuss the type of roots of the equation  $f(x) = 0$ ,  $a \leq x \leq b$ .
- How many points of inflection the graph of  $y = f(x)$ ,  $a \leq x \leq b$ , has?
- Find the points of local maxima/minima of  $y = f(x)$ ,  $a < x < b$ .

(e) How many roots equation  $f''(x) = 0$  has?



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27. Find the asymptote of the function  $y = \frac{2x^2 + 3x + 1}{x}$  if any.

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28. Find the horizontal, vertical and oblique asymptotes of each of the curves.

$$(a) \quad y = \frac{x}{x+4}$$

$$(b) \quad y = \frac{x^2+4}{x^2-1}$$

$$(c) \quad y = \frac{x^3}{x^2+3x-10}$$

$$(d) \quad y = \frac{x^3+1}{x^3+x}$$

$$(e) \quad y = \frac{x}{\sqrt[4]{x^4+1}}$$

$$(f) \quad y = \frac{x-9}{\sqrt{4x^2+3x+2}}$$

$$(g) \quad y = \frac{1}{2^x-1}$$

$$(h) \quad y = \frac{1}{\log_e x}$$

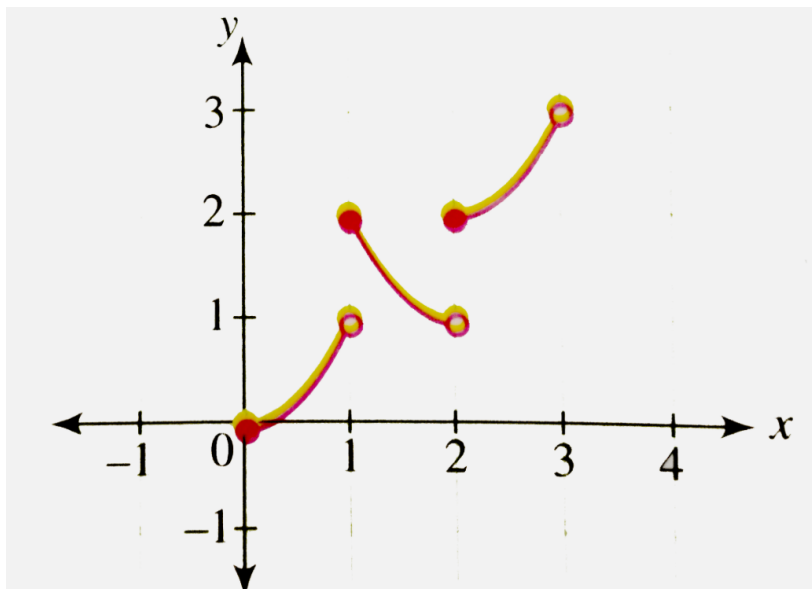
$$(i) \quad y = \frac{1}{2^x-1}$$



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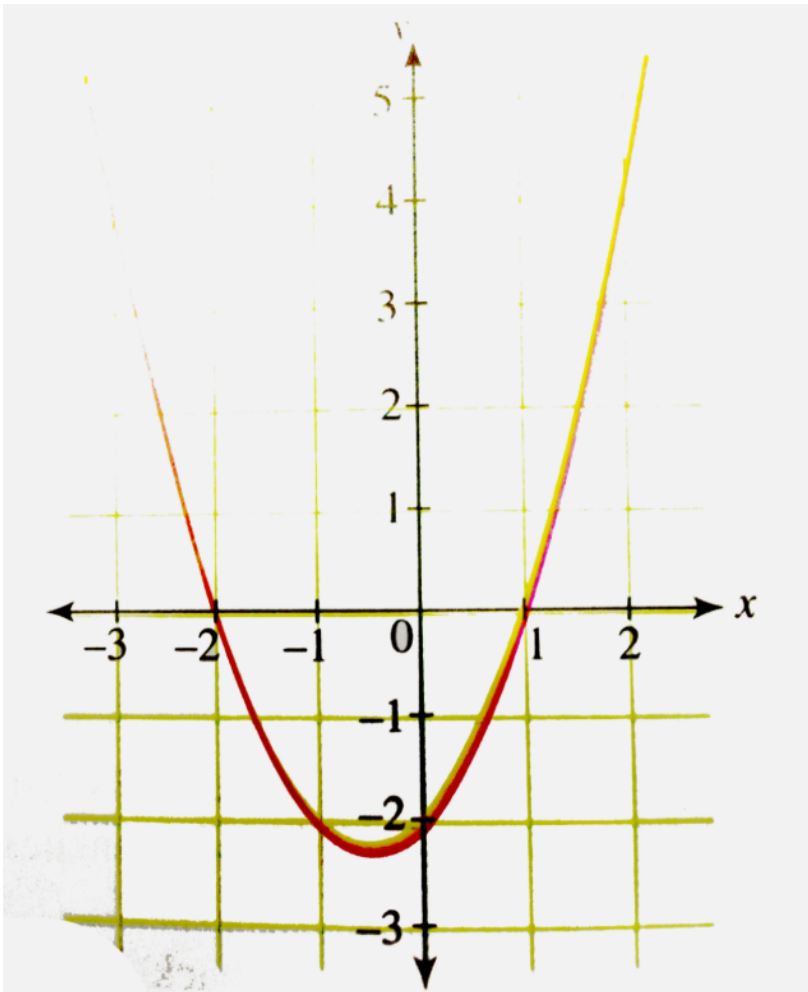
## Exercise

1. Does the following graph pass the vertical or horizontal line test ?



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2. Consider the graph of  $y = f(x)$  as shown in the following figure.



- (i) Find the sum of the roots of the equation  $f(x) = 0$ .
- (ii) Find the product of the roots of the equation  $f(x) = 4$ .

(iii) Find the absolute value of the difference of the roots of the equation

$$f(x) = x+2 .$$



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3. Which of the following functions has (have) y-symmetry or origin symmetry?

(i)  $f(x) = x^2 \sin x$

(ii)  $f(x) = \log\left(\frac{1-x}{1+x}\right)$

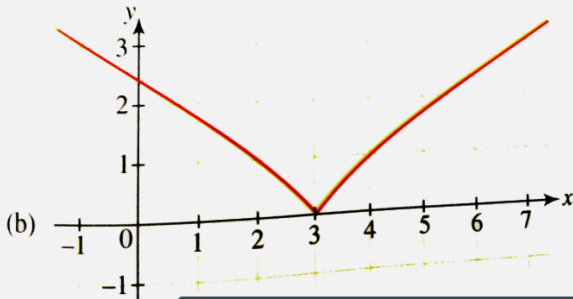
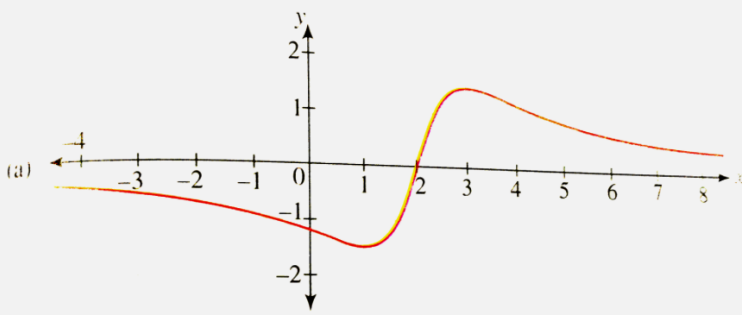
(iii)  $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$



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4. The graph of functions are given in the following figure. Discuss the symmetry.

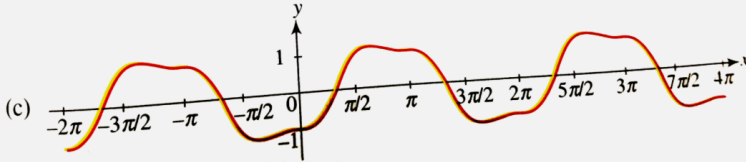
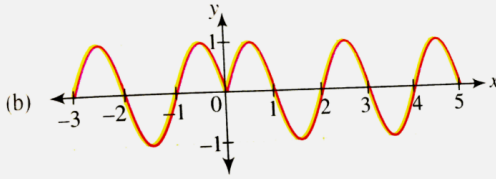
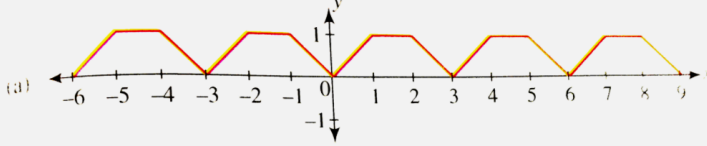




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5. Check whether the following function/functions is/are periodic or not?

Find the period in case the function is periodic.

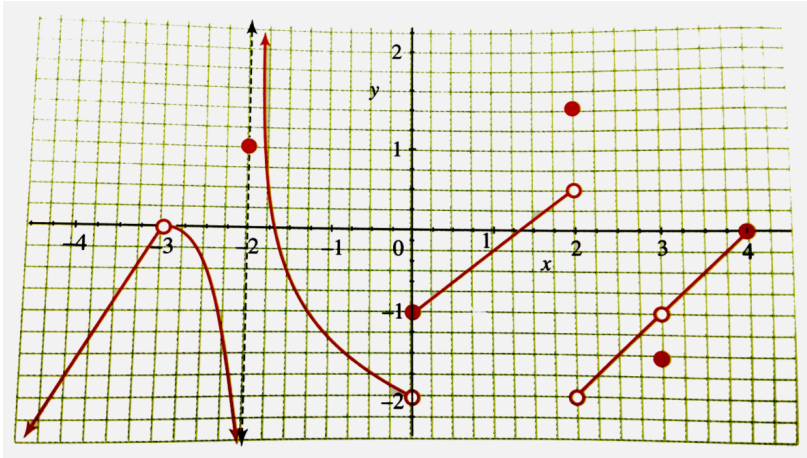


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6. Suppose that  $f$  is even, periodic function with period 2, and that  $f(x) = x$  for all  $x$  in the interval  $[0, 1]$ . Then draw the graph of  $y = f(x)$ .

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7. The graph of  $y = f(x)$  is as shown in the following figure.



Find the following values

(i)  $f(-3)$

(ii)  $f(-2)$

(iii)  $f(0)$

(iv)  $f(2)$

(v)  $f(3)$

(vi)  $\lim_{x \rightarrow -3} f(x)$

(vii)  $\lim_{x \rightarrow 0} f(x)$

(viii)  $\lim_{x \rightarrow 2} f(x)$

(ix)  $\lim_{x \rightarrow 3} f(x)$

(x)  $\lim_{x \rightarrow 2^-} f(x)$

(xi)  $\lim_{x \rightarrow -2^+} f(x)$

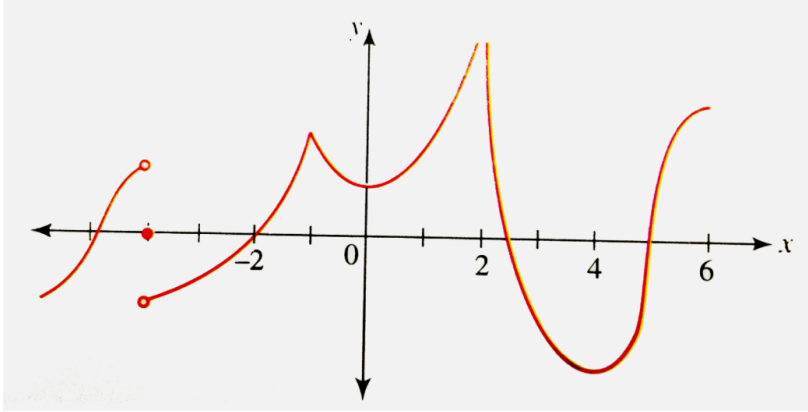
(xii)  $\lim_{x \rightarrow 0^-} f(x)$

(xiii)  $\lim_{x \rightarrow 0^+} f(x)$



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8. The graph of  $f$  is shown. State, with reason, the numbers at which  $f$  is not differentiable.



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9. If  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$  are given by  $f(x) = (x + 1)^2$  and  $g(x) = x^2 + 1$ , then write the value of  $f \circ g(-3)$ .

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10. (a) Can the graph of the function intersect the horizontal asymptote?  
 (b) Can the graph of the function intersect the vertical asymptote?

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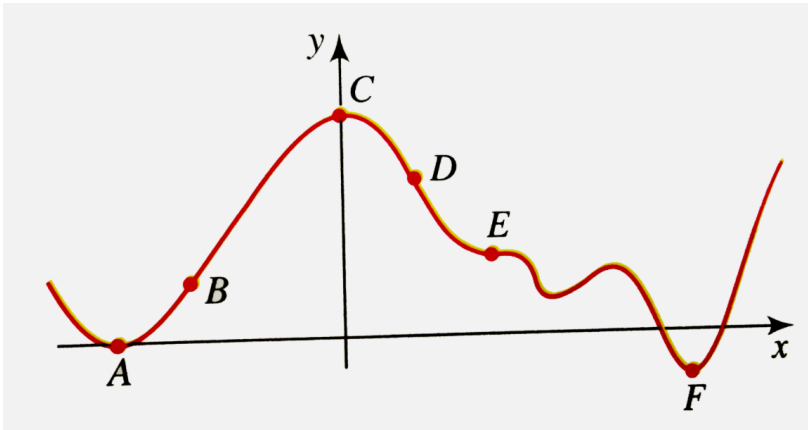
11. The graph of  $y = f(x)$  is given with six labelled points. Answer the following questions.

(a) At which point  $f'(x)$  has the greatest value?

(b) At which point  $f(x)$  and  $f'(x)$  both are zero?

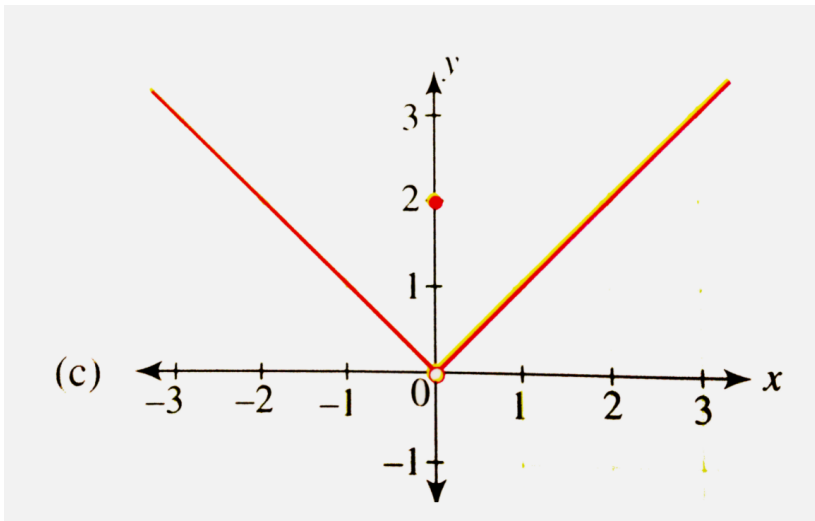
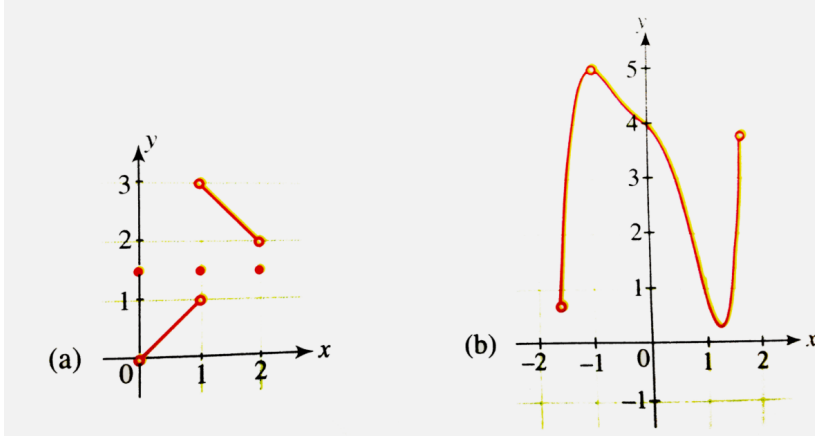
(c) At how many point  $f'(x)$  is negative ?

(d) Which is the point of infection?



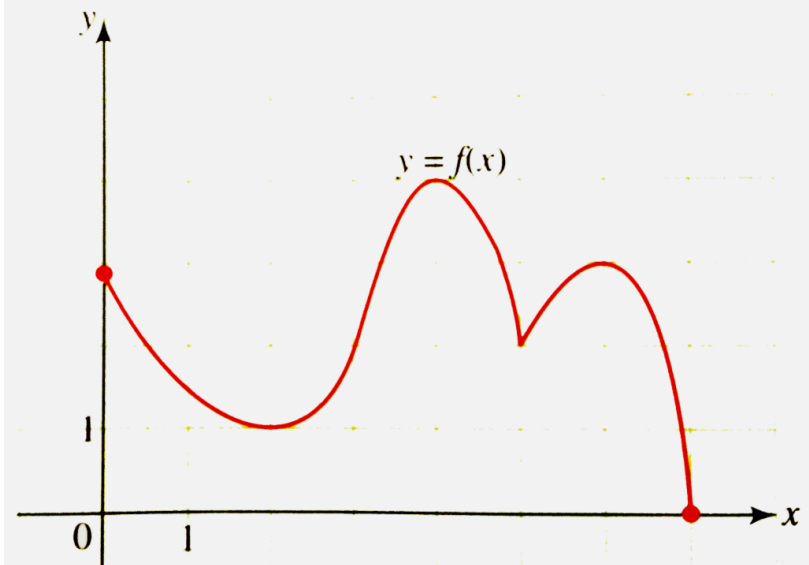
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12. Graph of functions are given in the following figure. Check the functions for absolute extremum.



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13. In the following graph, state the absolute and local maximum and minimum values of the function.



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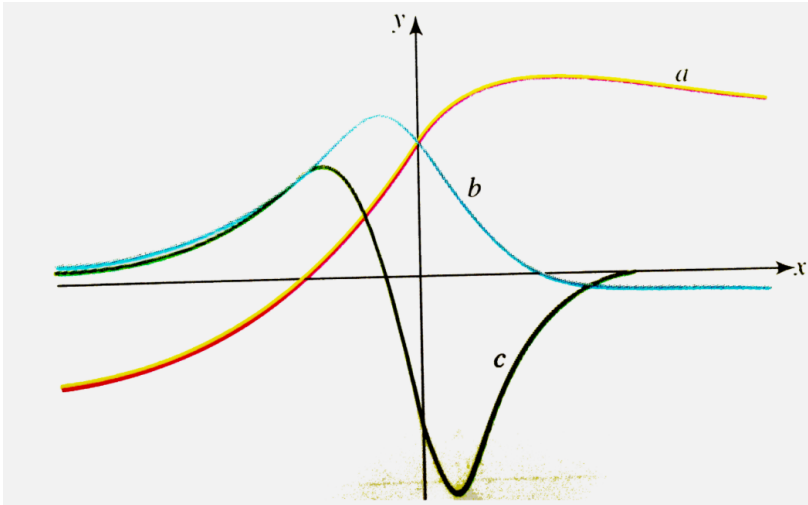
14. Prove :  $\left( \frac{1 + \cot^2 A}{1 + \tan^2 A} \right) = \left( \frac{1 - \cot A}{1 - \tan A} \right)^2 = \tan^2 A$

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15. Evaluate  $\int_0^{\frac{\pi}{2}} \left( \frac{\sin x}{\cos x + \sin x} \right) dx$

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16. The figure shows the graphs of  $f$ ,  $f'$  and  $f''$ . Identify each curve and explain your choices.



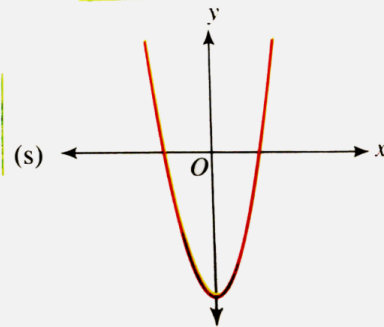
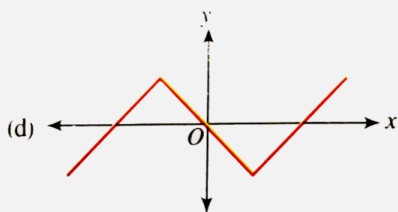
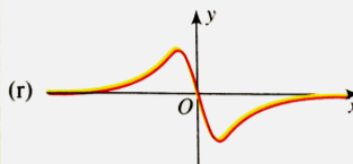
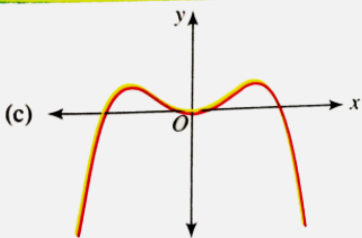
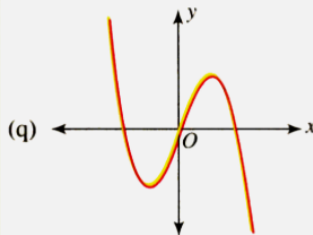
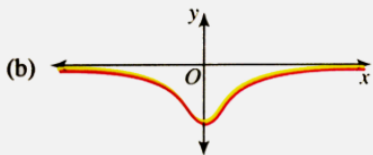
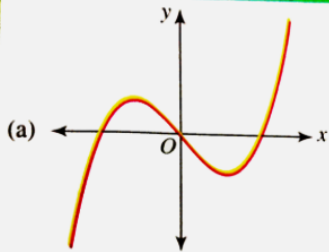
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17. Match the graph of  $y = f(x)$  in Column I with the corresponding graph of  $y = f'(x)$  in Column II.



**Column I**

**Column II**



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18. If  $f(x)$  is an even function, then  $f'(x)$ . If in a  $\triangle ABC$ ,  $a \tan A + b \tan B = (a + b) \tan\left(\frac{A + B}{2}\right)$  then



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