

#### **MATHS**

#### **BOOKS - CENGAGE**

### **GRAPHS OF POLYNOMIAL AND RATIONAL FUNCTIONS**

Illustrations

**1.** Draw the rough sketch of the curve  $y=\left(x-1
ight)^2(x-3)^3$ 



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**2.** Draw the rough sketch of the curve  $y=(x-1)^2(x-2)(x-3)^3$ .



**3.** Draw the rough sketch of the curve  $y = x^4 - x^2$ .



**4.** Draw the graph of the function  $y=3x^4-4x^3$ . Discuss the points of local extremum, inflection and intervals of monotonicity.



**5.** Draw the graph of |y| = (x-1)(x-2)(x-3).



**6.** If  $f(x) = 3\sqrt{1-x^2} + 3\sqrt{1-x^2}$ , then f(x) is



7. Whether the given equation  $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} = \cot\theta-\csc\theta$ ?

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**8.** Draw the graph of  $y = x^4 + 2x^2 - 8x + 3$ 

Find the number of real roots of the equation  $x^4 + 2x^2 - 8x + 3 = 0$ .

Also find the sum of the integral parts of all real roots.



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- **9.** Let  $\mathsf{a} \in \mathsf{R}$  and  $\mathsf{f} \colon R \to R$  be given by  $f(x) = x^5 5x + a$ , then
- (a) f(x) = 0 has three real roots if a > 4
- (b) f(x) = 0 has only one real root if a > 4
- (c) f(x) = 0 has three real roots if a < -4
- (d) f(x) = 0 has three real roots if -4 < a < 4



**10.** If  $\cos\theta = 3/5$  then the value of  $(\sin\theta - \tan\theta + 1)/2\tan^2\theta$ 



**11.** Find the area bounded by the curves  $y=\sqrt{1-x^2}$  and  $y=x^3-x$  without using integration.



**12.** Draw the graph of  $f(x)=4x^3-3x$  and hence draw the graph of  $g(x)=\cos^{-1}\bigl(4x^3-3x\bigr).$ 



Let

13.

 $g(x)=\{max\!:\!f(t;x+1\leq t\leq x+2,\;-3\leq x\leq 01-x,x\geq\;\;$  .Find continuity and differentiability of g(x) for x in [-3,1]

 $f(x) = x^3 - 3x^2 + 6 \forall x \in R$ 

and

**14.** Find the value of k if  $x^3 = 3x + a = 0$  has three real distinct roots.



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**15.** If t is a real number satisfying the equation  $2t^3-9t^2+30-a=0,$  then find the values of the parameter a for which the equation  $x+\frac{1}{x}=t$  gives six real and distinct values of x.



- **16.** Let  $f(x)=x^3-9x^2+24x+c=0$  have three real and distinct roots  $lpha,\,eta$  and  $\lambda.$
- (i) Find the possible values of c.
- (ii) If  $[\alpha] + [\beta] + [\lambda] = 8$ , then find the values of c, where  $[\cdot]$  represents the greatest integer function.

(ii) If  $[lpha]+[eta]+[\lambda]=7$ , then find the values of c, where  $[\ \cdot\ ]$  represents

the greatest integer function.s



**17.** Draw the graph of  $f(x)=rac{x^2-5x+6}{x^2-x}$ 



**18.** Draw the graph of  $y=rac{1}{x^2}$ .



**19.** Write a possible rational function h with a hole at x=5, a vertical asymptote at x=-1, a horizontal asymptote at y=2 and x-intercept at x=2.



**20.** Write a rational function g with vertical asymptotes at x=3 and

$$x=-3$$
, a horizontal asymptote at  $y=-4$  and with no  $x$ -intercept.



**21.** Draw the graph of  $y=f(x)=rac{x+1}{x^2+1}$ 



**22.** Draw the graph of the function  $f\colon R-\{-1,1\} o R.\ f(x)=rac{x}{1-|x|}.$ 



**23.** Draw the graph of  $f(x)=rac{1}{x^2-2x+2}$ .



**24.** From the graph of  $y=x^2-4$ , draw the graph of  $y=\frac{1}{x^2-4}$ .



**25.** Draw the graph of  $y=x^2+rac{1}{x^2}, x
eq 0.$ 



**26.** Draw the graph of  $f(x) = \left| \frac{x^2 - 2}{x^2 - 1} \right|$  .



**27.** Draw the graph of  $y=\frac{1-x^2}{1+x^2}$  and hence draw the graph of  $y=\cos^{-1}.$   $\frac{1-x^2}{1+x^2}.$ 



**28.** Write a possible rational function h with a hole at x = 5, a vertical asymptote at x = -1, a horizontal asymptote at y = 2 and x-intercept at x = -1



2.

# **29.** Draw the graph of $y = \frac{x-1}{x-2}$ .



**30.** Draw the graph of 
$$y=\frac{3x-x^3}{1-3x^2}$$
 and hence the graph of  $y=\tan^{-1}.\,\frac{3x-x^3}{1-3x^2}.$ 



**31.** Draw the graph of 
$$y=\dfrac{x^3}{3(x+1)}.$$



- **32.** Draw the graph of  $y=\dfrac{1}{x+1}+\dfrac{1}{x}+\dfrac{1}{x-2}.$ 
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- **33.** Find the greatest value of  $f(x) \frac{1}{2ax x^2 5a^2} \in [-3, 5]$ depending upon the parameter a
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**Exercise** 

- **1.** Draw the graph of  $y = (x 1)(x^2 x + 1)$ .
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**2.** Draw the graph of  $y=\left(x^2-x^5
ight)\left(x-2
ight)^3$ .

## 3. Draw the graphs of

- (i)  $y = x^2(x-1)|x-2|$
- (ii)  $y = x^3(x-1)|x-2|$ 
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**4.** Write a possible rational function f that has a vertical asymptote at x = 2, a horizontal asymptote y = 3 and a zero at x = -5. Also draw the graph of the function.



- **5.** Draw the graph of  $y=f(x)=rac{x^2}{x^2+1}$ .
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**6.** Draw graph of  $y = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$ .



**7.** Draw the graph of  $f(x)=rac{x^2-8x+15}{x^2-2x}$ .

**8.** Draw the graph of  $y=f(x)=rac{x^2}{x^2-1}$ .

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**9.** Draw the graph of  $y = x + \frac{1}{x}$ 

**11.** Draw graph of 
$$y=rac{x^3-2x^2}{3(x+1)^2}.$$



**12.** Draw graph of 
$$y=rac{x^3-2x^2}{3{(x+1)}^2}.$$



**13.** If  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta t hen \frac{dy}{dx} =$ 



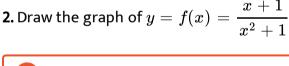
**Exercises** 

**1.** Sketch the graph of an example of a rational function f that satisfies all the given conditions.

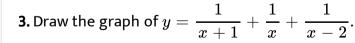
(i) 
$$f(0) = 0, \, f(1), \, \lim_{x \, o \, \infty} \, f(x) = 0, f \, \, \, ext{is odd}$$

(ii) 
$$\lim_{x o 0^+}f(x)=\infty,\ \lim_{x o 0^-}f(x)=-\infty,\ \lim_{x o \infty}f(x)=1,\ \lim_{x o -\infty}f(x)=$$
 (iii)  $\lim_{x o -2}f(x)=\infty,\ \lim_{x o -\infty}f(x)=3,\ \lim_{x o \infty}f(x)=3, f(0)=0$ 











**4.** Draw the graph of  $y=\left|x\right|+2$ 

