



## MATHS

### BOOKS - CENGAGE

## GRAPHS OF POLYNOMIAL AND RATIONAL FUNCTIONS

### Illustrations

1. Draw the rough sketch of the curve  $y = (x - 1)^2(x - 3)^3$

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2. Draw the rough sketch of the curve  $y = (x - 1)^2(x - 2)(x - 3)^3$ .

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3. Draw the rough sketch of the curve  $y = x^4 - x^2$ .



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4. Draw the graph of the function  $y = 3x^4 - 4x^3$ . Discuss the points of local extremum, inflection and intervals of monotonicity.



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5. Draw the graph of  $|y| = (x - 1)(x - 2)(x - 3)$ .



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6. If  $f(x) = 3\sqrt{1 - x^2} + 3\sqrt{1 - x^2}$ , then  $f(x)$  is



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7. Whether the given equation  $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \cot\theta - \csc\theta$  ?

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8. Draw the graph of  $y = x^4 + 2x^2 - 8x + 3$

Find the number of real roots of the equation  $x^4 + 2x^2 - 8x + 3 = 0$ .

Also find the sum of the integral parts of all real roots.

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9. Let  $a \in \mathbb{R}$  and  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^5 - 5x + a$ , then

(a)  $f(x) = 0$  has three real roots if  $a > 4$

(b)  $f(x) = 0$  has only one real root if  $a > 4$

(c)  $f(x) = 0$  has three real roots if  $a < -4$

(d)  $f(x) = 0$  has three real roots if  $-4 < a < 4$

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10. If  $\cos\theta = 3/5$  then the value of  $(\sin\theta - \tan\theta + 1)/2\tan^2\theta$

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11. Find the area bounded by the curves  $y = \sqrt{1 - x^2}$  and  $y = x^3 - x$  without using integration.

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12. Draw the graph of  $f(x) = 4x^3 - 3x$  and hence draw the graph of  $g(x) = \cos^{-1}(4x^3 - 3x)$ .

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13. Let  $f(x) = x^3 - 3x^2 + 6 \forall x \in R$  and  $g(x) = \{ \max : f(t); x + 1 \leq t \leq x + 2, -3 \leq x \leq 0 \}$  and  $g(x) = 1 - x, x \geq 0$ . Find continuity and differentiability of  $g(x)$  for  $x$  in  $[-3, 1]$

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14. Find the value of  $k$  if  $x^3 = 3x + a = 0$  has three real distinct roots.



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15. If  $t$  is a real number satisfying the equation  $2t^3 - 9t^2 + 30 - a = 0$ , then find the values of the parameter  $a$  for which the equation  $x + \frac{1}{x} = t$  gives six real and distinct values of  $x$ .



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16. Let  $f(x) = x^3 - 9x^2 + 24x + c = 0$  have three real and distinct roots  $\alpha, \beta$  and  $\lambda$ .

(i) Find the possible values of  $c$ .

(ii) If  $[\alpha] + [\beta] + [\lambda] = 8$ , then find the values of  $c$ , where  $[\cdot]$  represents the greatest integer function.

(ii) If  $[\alpha] + [\beta] + [\lambda] = 7$ , then find the values of  $c$ , where  $[\cdot]$  represents the greatest integer function.

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17. Draw the graph of  $f(x) = \frac{x^2 - 5x + 6}{x^2 - x}$

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18. Draw the graph of  $y = \frac{1}{x^2}$ .

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19. Write a possible rational function  $h$  with a hole at  $x = 5$ , a vertical asymptote at  $x = -1$ , a horizontal asymptote at  $y = 2$  and x-intercept at  $x = 2$ .

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20. Write a rational function  $g$  with vertical asymptotes at  $x = 3$  and  $x = -3$ , a horizontal asymptote at  $y = -4$  and with no  $x$ -intercept.

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21. Draw the graph of  $y = f(x) = \frac{x + 1}{x^2 + 1}$

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22. Draw the graph of the function

$$f: \mathbb{R} - \{-1, 1\} \rightarrow \mathbb{R}. f(x) = \frac{x}{1 - |x|}.$$

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23. Draw the graph of  $f(x) = \frac{1}{x^2 - 2x + 2}$ .

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24. From the graph of  $y = x^2 - 4$ , draw the graph of  $y = \frac{1}{x^2 - 4}$ .

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25. Draw the graph of  $y = x^2 + \frac{1}{x^2}$ ,  $x \neq 0$ .

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26. Draw the graph of  $f(x) = \left| \frac{x^2 - 2}{x^2 - 1} \right|$ .

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27. Draw the graph of  $y = \frac{1 - x^2}{1 + x^2}$  and hence draw the graph of  $y = \cos^{-1} \cdot \frac{1 - x^2}{1 + x^2}$ .

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28. Write a possible rational function  $h$  with a hole at  $x = 5$ , a vertical asymptote at  $x = -1$ , a horizontal asymptote at  $y = 2$  and  $x$ -intercept at  $x = 2$ .



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29. Draw the graph of  $y = \frac{x - 1}{x - 2}$ .



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30. Draw the graph of  $y = \frac{3x - x^3}{1 - 3x^2}$  and hence the graph of  $y = \tan^{-1} \cdot \frac{3x - x^3}{1 - 3x^2}$ .



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31. Draw the graph of  $y = \frac{x^3}{3(x + 1)}$ .



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32. Draw the graph of  $y = \frac{1}{x+1} + \frac{1}{x} + \frac{1}{x-2}$ .

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33. Find the greatest value of  $f(x) \frac{1}{2ax - x^2 - 5a^2} \in [-3, 5]$  depending upon the parameter  $a$ .

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## Exercise

1. Draw the graph of  $y = (x-1)(x^2 - x + 1)$ .

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2. Draw the graph of  $y = (x^2 - x^5)(x-2)^3$ .



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3. Draw the graphs of

(i)  $y = x^2(x - 1)|x - 2|$

(ii)  $y = x^3(x - 1)|x - 2|$



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4. Write a possible rational function  $f$  that has a vertical asymptote at  $x = 2$ , a horizontal asymptote  $y = 3$  and a zero at  $x = -5$ . Also draw the graph of the function.



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5. Draw the graph of  $y = f(x) = \frac{x^2}{x^2 + 1}$ .



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6. Draw graph of  $y = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$ .

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7. Draw the graph of  $f(x) = \frac{x^2 - 8x + 15}{x^2 - 2x}$ .

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8. Draw the graph of  $y = f(x) = \frac{x^2}{x^2 + 1}$ .

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9. Draw the graph of  $y = x + \frac{1}{x}$ .

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10. Draw graph of  $y = \frac{1}{x^2} - x$ .

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11. Draw graph of  $y = \frac{x^3 - 2x^2}{3(x + 1)^2}$ .

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12. Draw graph of  $y = \frac{x^3 - 2x^2}{3(x + 1)^2}$ .

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13. If  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$  then  $\frac{dy}{dx} =$

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1. Sketch the graph of an example of a rational function  $f$  that satisfies all the given conditions.

(i)  $f(0) = 0$ ,  $f(1)$ ,  $\lim_{x \rightarrow \infty} f(x) = 0$ ,  $f$  is odd

(ii)

$$\lim_{x \rightarrow 0^+} f(x) = \infty, \lim_{x \rightarrow 0^-} f(x) = -\infty, \lim_{x \rightarrow \infty} f(x) = 1, \lim_{x \rightarrow -\infty} f(x) =$$

(iii)  $\lim_{x \rightarrow -2} f(x) = \infty$ ,  $\lim_{x \rightarrow -\infty} f(x) = 3$ ,  $\lim_{x \rightarrow \infty} f(x) = 3$ ,  $f(0) = 0$



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2. Draw the graph of  $y = f(x) = \frac{x + 1}{x^2 + 1}$



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3. Draw the graph of  $y = \frac{1}{x + 1} + \frac{1}{x} + \frac{1}{x - 2}$ .



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4. Draw the graph of  $y = |x| + 2$



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