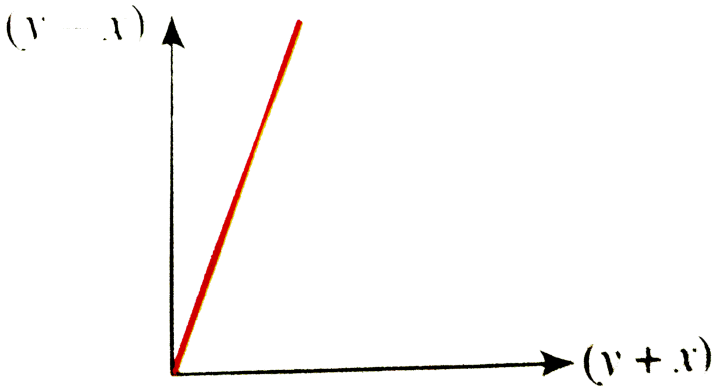
MATHS

BOOKS - CENGAGE

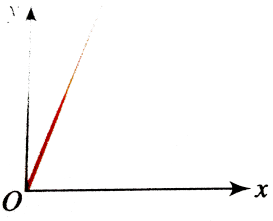
GRAPHS OF ELEMENTARY FUNCTIONS

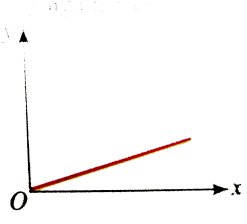
Illustrations

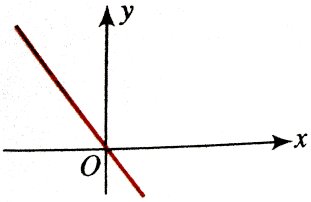
1. The graph of $(y-x)$ against $(y+x)$ is shown below.

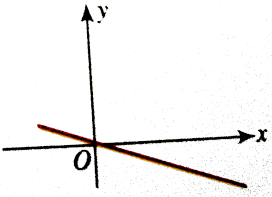


Which one of the following shows the graph of y against x ?

(a) 

(b) 

(c) 

(d) 

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2. Draw the graph of $f(x) = \dfrac{x^2 - 8x + 15}{x^2 - 2x}$.

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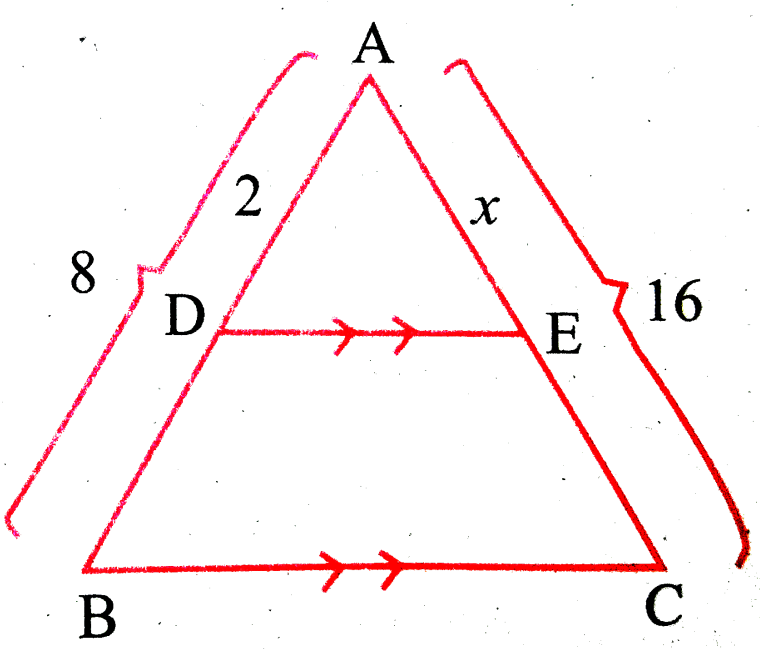
3. In the given figure, the value of x is



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4. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2} f$ or $allrealxandy$ If $f'(0)$ exists and equals $-1andf(0) = 1, thenfindf(2)$.

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5. Sketch the regions satisfying the following inequalities:

(a) $x > 2$

(b) $|y| \geq 1$

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6. Shade the regions where points satisfy $|x - y| < 1$.

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7. Plot the region satisfying $|x| + |y| \leq 2$ and $|x| + |y| > 2$.

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8. If $x < 2$, then find the values of x^2 graphically.

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9. If $x < -1$, then find the vallues of x^2 graphically.

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10. Draw the graph of $f(x) = \begin{cases} x^3, x^2 < 1 \\ x, x^2 \leq 1 \end{cases}$

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11. If $x > 2$, then find the values of $1/x$ graphically.

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12. If $x < -1$, then find the values of $1/x$ graphically.

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13. When $x > -2$, find the values of $1/x$.

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14. When $x < 3$, find the values of $1/x$.

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15. Draw the graph of $\frac{1}{x} + \frac{1}{y} = 1$.

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16. Draw the graph of $y = \frac{1}{x^2}$.

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17. Draw the graphs of (i) $y = x^2 - x - 6$ (ii) $y = 6 - x - x^2$ and find zeroes in each case.

What do you notice ?

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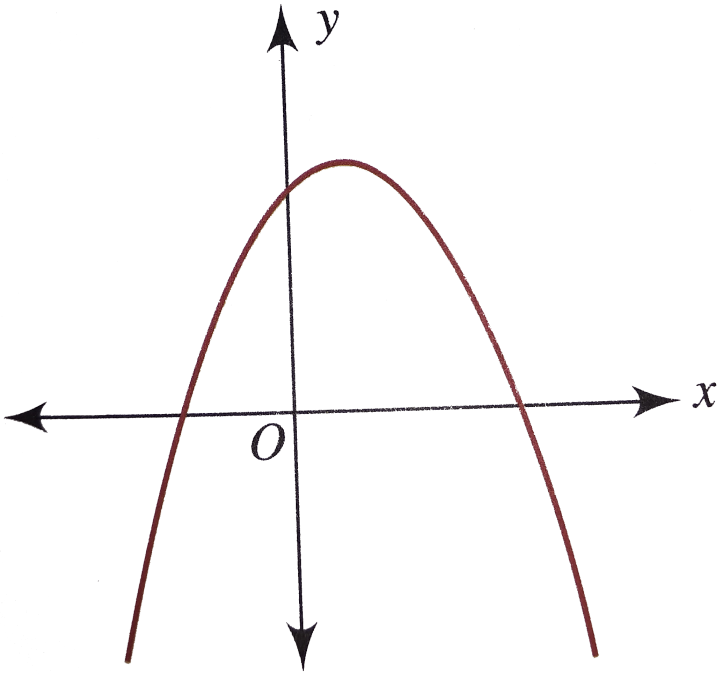
18. The following figure shows the graph of $f(x) = ax^2 + bx + c$, find the sign of a, b and c.



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19. Let $f(x) = 2x(2-x), 0 \leq x \leq 2$. Then find the number of solutions of $f(f(f(x))) = \frac{x}{2}$.

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20. If $f(x) = \begin{cases} kx & \text{for } . x \leq 2 \\ 3 & \text{for } x > 2 \end{cases}$ is continuous at x=2 then the value of k is ..... .

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21. If $f(x) = x^3 + 4x^2 + \lambda x + 1$ is a monotonically decreasing function of x in the largest possible interval $\left(-2, -\frac{2}{3}\right)$. Then $\lambda = 4$ (b) $\lambda = 2$ $\lambda = -1$ (d) λ has no real value

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22. For what real values of a do the roots of the equation $x^2 - 2x - (a^2 - 1) = 0$ lie between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$.

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23. Find the value of a for which $ax^2 + (a-3)x + 1 < 0$ for at least one positive real x .

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24. Consider the inequality, $9^x - a.3^x - a + 3 \leq 0$, where $'a'$ is a real parameter.

(a) Find the value of $'a'$ for which the inequality has at least one negative solution.

(b) Find the values of $'a'$ for which the inequality has at least one positive solution.

(c) Find the vlaues of $'a'$ for which the inequality has at least one real solution.

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25. Let a, b, c real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$

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26. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has

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27. When $x > -2$, find the values of $|x|$ graphically.

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28. When $x > -2$, find the values of $|x|$ graphically.

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29. If $|x + 2| \leq 9$, then x belongs to

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30. Draw the graph of $f(x) = x|x|$.

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31. Draw the graph of $y = |x|$.

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32. Draw the graph of $f(x) = x|x|$.

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33. Draw the graph of the function $f(x) = x - |x - x^2|, \ -1 \leq x \leq 1$ and find the points of non-differentiability.

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34. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$ is

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35. Draw the graph of $f(x) = |x| - |2x - 3|$. Find the range of the function.

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36. Draw the graph of $f(x) = x|x|$.

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37. Let $f(x) = x + 2|x + 1| + x - 1 \mid \dot{I}ff(x) = k$ has exactly one real solution, then the value of k is 3 (b) 0 (c) 1 (d) 2

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38. $Let f(x) = \left\{x^3 + x^2 + 3x + \sin x \mid \left(3 + s \in \frac{1}{x}, \right), x \neq 0.\, 0x = 0\right.$ then the number of point where $f(x)$ attains its minimum value is_____

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39. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining $\left(c - 1, e^{c-1}\right)$ and $\left(c + 1, e^{c+1}\right)$ (a) on the left of $x = c$ (b) on the right of $x = c$ (c) at no points (d) at all points

A. On the left of $x = c$

B. On the right of $x = c$

C. At no point

D. At all points

Answer:

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40. If a continous founction of defined on the real line R, assumes positive and negative values in R, then the equation f(x)=0 has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x) = 0$ has a root in R. Considetr $f(x) = ke^x - x$ for all real x where k is real constant.

The line y=x meets $y = ke^x \text{for} k \leq 0$ at

A. No point

B. One point

C. Two points

D. More than two points

Answer:

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41. If a continous founction of defined on the real line R, assumes positive and negative values in R, then the equation f(x)=0 has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x) = 0$ has a root in R. Considetr $f(x) = ke^x - x$ for all real x where k is real constant.

The line y=x meets $y = ke^x \text{for} k \leq 0$ at

A. $\frac{1}{e}$

B. 1

C. e

D. $\log_e 2$

Answer:

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42. If a continous founction of defined on the real line R, assumes positive and negative values in R, then the equation f(x)=0 has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x) = 0$ has a root in R. Considetr $f(x) = ke^x - x$ for all real x where k is real constant.

The line y=x meets $y = ke^x \text{for} k \leq 0$ at

A. $\left(0, \frac{1}{e}\right)$

B. $\left(\frac{1}{e}, 1\right)$

C. $\left(\frac{1}{e}, \infty\right)$

D. $(0, 1)$

Answer:

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43. Find the number of solutions of equation $2^x + 3^x + 4^x - 5^x = 0$

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44. Draw the graph of $y = \log_x \sqrt{x}$

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45. Find number of roots of the equation $x^3 - \log_{0.5} x = 0.$

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46. The circles $x^2 + y^2 - 12x - 12y = 0$ and $x^2 + y^2 + 6x + 6y = 0.$ touch each other externally touch each other internally intersect at two points none of these

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47. Draw the graph of $f(x) = x|x|$.

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48. Draw the graph of the function `f(x)=x^2

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49. Draw the graph of $f(x) = \left[x^2\right], x \in [0, 2)$, where $[\cdot]$ denotes the greatest integer function.

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50. Draw the graph of $f(x) = \left[\sqrt{x}\right], x \in [0, 16)$, where $[\cdot]$ denotes the greatest ineger function.

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51. Draw the graph of $y = [x] + \sqrt{x - [x]}$, where$[\cdot]$ denotes the greatest ineger function.

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52. Draw the graph of $f(x) = [\log_e x], e^{-2} < x < 10$, where $[\cdot]$ represents the greatest integer function.

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53. Solve $x^2 - 4 - [x] = 0$ (where $[]$ denotes the greatest integer function).

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54. Sketch the region of relation $[x]+[y]=5, x, y \geq 0$, where $[\cdot]$ denotes the greatest integer function.

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55. Draw the graph of $f(x)=\{2x\}$, where $\{\cdot\}$ represents the fractional part function.

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56. Find the domain of $f(x)=\sqrt{|x|-\{x\}}$ (where $\{\cdot\}$ denotes the fractional part of x).

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57. Solve : $x^2=\{x\}$, where $\{x\}$ represents the fractional part function.

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58. Draw the graph of $y = 2^{\{x\}}$, where $\{\cdot\}$ represents the fractional part function.

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59. Draw the graph of $y = \frac{1}{\{x\}}$, where $\{\cdot\}$ denotes the fractional part function.

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60. Solve : $4\{x\} = x + [x]$(where $[\cdot]$ denotes the greatest integer function and $\{\cdot\}$ denotes the fractional part function.

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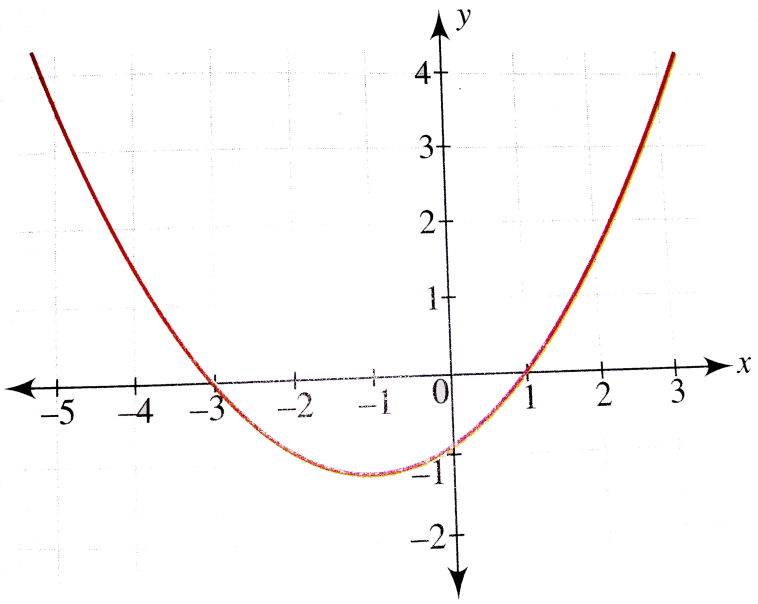
61. Given the graph of the function $y = f(x)$, draw the graph of $y = \operatorname{sgn} f(x)$.



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62. Draw the graph of $f(x) = \operatorname{sgn}\left(x^3 - x\right)$.

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63. Draw the graph of $f(x) = \text{sgn}(\log_e x)$.

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64. Let a function $f(x)$ be defined in $[-2, 2]$ as

$$f(x) = \begin{cases} \{x\}, & -2 \leq x < -1 \\ |\text{sgn } x|, & -1 \leq x \leq 1 \\ \{-x\}, & 1 < x \leq 2 \end{cases}$$

where $\{x\}$ and sgn x denote fractional part and signum functions, respectively. Then find the area bounded by the graph of $f(x)$ an the x-axis.

A. 2 sq. units

B. 3 sq. units

C. 4 sq. units

D. 5 sq. units

Answer:

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65. Let $f: R \to R$ be defined as $f(x) = e^{\text{sgn } x} + e^{x^2}$. Then find the range of the function, and also indentify the type of the function : one-one or many-one.

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66. Draw the graph of the function $f(x) = \max . \{x, x^2\}$ and write its equivalent definition.

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67. Let $f: R \to R$ be a function defined by $f(x) = \max . \{x, x^3\}$. The set of all points where $f(x)$ is NOT differenctiable is

(a) $\{-1, 1\}$

(b) $\{-1, 0\}$

(c) $\{0, 1\}$

(d) $\{-1, 0, 1\}$

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68. Find the equivalent definition of $f(x) = max x^2, (-x)^2, 2x(1-x) whre 0 \leq x \leq 1$

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69. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: R \to R$ by $h(x) = \{ \max \{f(x), g(x)\}, \text{ if } x \leq 0 \text{ and } \min \{f(x), g(x)\}, \text{ if } x >$ .The number of points at which $h(x)$ is not differentiable is

A. 1

B. 2

C. 3

D. 4

Answer:

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70. Plot the region of the points P (x,y) satisfying $|x| + |y| < 1$.

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71. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \leq 100\}$ and $B = \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then find the area of the region $A \cup B$.

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72. Draw the graphs of the following parabolas :

(i) $x = y^2 - 2y - 3$

(ii) $x = 6 + y - y^2$

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73. Find the number of roots of equation $x \sin x = 1$

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74. Let $g(x) = \sqrt{x - 2k}$, $\forall 2k \leq x < 2(k+1)$, where $k \in$ integer. Check whether $g(x)$ is periodic or not.

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75. Plot the region in the first quadrant in which points are nearer to the origin than to the line $x = 3$.

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76. Draw the graph of $y = \sqrt{x^2 - 1}$

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77. Draw the graph of $y = -\sqrt{6 - 3x^2}$

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78. The eccentricity of the ellipse $9x^2 + 25y^2 - 18x - 100y - 116 = 0$ is

$25/16$ b. $4/5$ c. $16/25$ d. $5/4$

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79. Find the area enclosed by the curves $y = \sqrt{x}$ and $x = -\sqrt{y}$ and the circle $x^2 + y^2 = 2$ above the x-axis.

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80. Plot the region in the first quadrant in which the points are nearer to the origin thatn to the line $x = 3$.

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Exercise

1. Draw the graph of $y = \dfrac{1}{(1/x)}$.

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2. (a) Draw the graph of

$$f(x) = = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, & \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2,3,\ldots \\ 0, & x = 0 \end{cases}$$

(b) Sketch the region $y \leq -1$.

(c) Sketch the region $|x| < 3$.

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3. Sketch the regions which points satisfy $|x + y| \geq 2$.

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4. Sketch the region satisfying $|x| < |y|$.

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5. Let $f: R \to R: f(x) = (x + 1)$ and $g: R \to R: g(x) = \left(x^2 - 2\right)$. Write down the formulae for (gof).

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6. Draw the graph of $y = \dfrac{x - 1}{x - 2}$.

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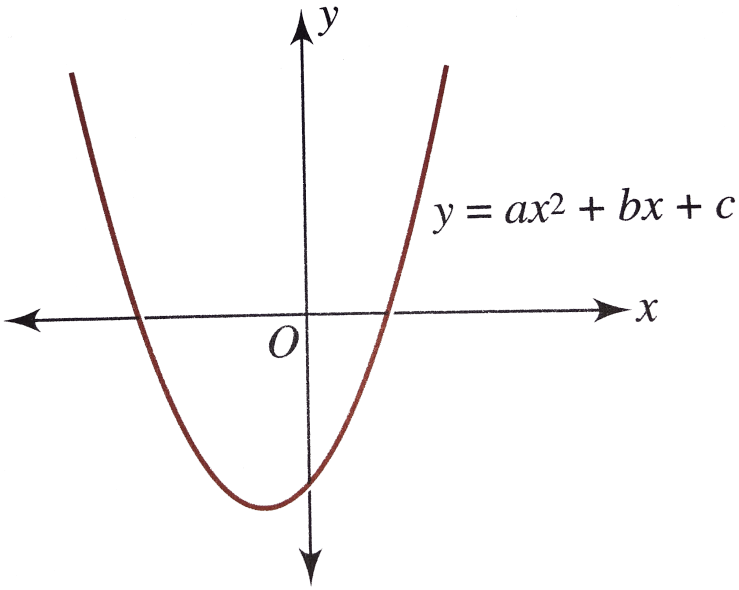
7. The following figure shows the graph of $f(x) = ax^2 + bx + c$, then find the sign of values of a, b and c.



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8. The entire graph of the equation $y = x^2 + kx - x + 9$ in strictly above the $x - a\xi s$ if and only if $k < 7$ (b) `-5-5` (d) none of these

A. $k < 7$

B. $-5 < k < 7$

C. $k > -5$

D. None of these

Answer:

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9. If $x^2 + 2ax + a < 0 \forall x \in [1, 3],$ the find the values of a.

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10. Draw the graph of $f(x) = x|x|$.

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11. Draw the graph of the function: Solve $\left|\frac{x^2}{x-1}\right| \leq 1$ using the graphical method.

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12. Draw the graph of $y = \left|x^2 - 2x\right| - x$.

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13. Draw the graph of $y = 2^x, x^2 - 2x \leq 0$

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14. Find the roots of the equation by factorization: $2x^2 - x - 1$

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15. Divide $16\left(x^2yz + xy^2z + xyz^2\right)$ by $4xyz$

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16. Find the set of real value(s) of a for which the equation $|2x+3|+2x-3\,|=ax+6$ has more than two solutions.

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17. Draw the graph of $y=|x|$.

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18. Draw the graph of $y=\dfrac{1}{\log_e x}$

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19. Find the number of real solutions to the equation $\log_{0.5} x=|x|$.

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20. Draw the graph of $f(x) = x + [x]$, where $[\cdot]$ denotes the greatest integer function.

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21. Given $f(x)$ is a periodic function with period 2 and it is defined as

$$f(x) = \begin{cases} \left[\cos \frac{\pi x}{2}\right] + 1, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \end{cases}$$

Here $[\cdot]$ represents the greatest integer $\leq x$. If $f(0) = 1$, then draw the graph of the function for $x \in [-2, 2]$.

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22. Sketch the region of relation $[x] + [y] = 5, x, y \geq 0$, where $[\cdot]$ denotes the greatest integer function.

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23. Draw the graph of $y = 2^{\{x\}}$, where $\{\cdot\}$ represents the fractional part function.

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24. Let $f(x) = \dfrac{[x]+1}{\{x\}+1}$ for $f: \left[0, \dfrac{5}{2}\right) \to \left(\dfrac{1}{2}, 3\right]$, where $[\cdot]$ represents the greatest integer function and $\{\cdot\}$ represents the fractional part of x. Draw the graph of $y = f(x)$. Prove that $y = f(x)$ is bijective. Also find the range of the function.

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25. Draw the graph of $y = 2^{\{x\}}$, where $\{\cdot\}$ represents the fractional part function.

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26. Find tha area of the region containing the points (x, y) satisfying $4 \leq x^2 + y^2 \leq 2(|x| + |y|)$.

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27. Draw the graph of $y = \sqrt{x^2 - 1}$

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28. Draw the graph of $y = |x|^{\frac{1}{2}}$ for $-1 \leq x < 1$.

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29. Draw the graph of $f(x) = \text{sgn}(\log_e x)$.

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30. Draw the graph of $y = x - \sin x$

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31. Draw the graph of the function $y = f(x) = \lim_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$. Is this function same as the function $g(x) = \text{sgn}(x^2 - 1)$.

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32. An even periodic function $f: R \to R$ with period 4 is such that $f(x) = \begin{cases} \max . (|x|, x^2), & 0 \leq x < 1 \\ x, & 1 \leq x \leq 2 \end{cases}$. Then draw the graph of $y = f(x)$ for $x \in R$

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33. IF the domain of the function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$ then range of the function is

A. A. Continuous at all points

B. B.Differentiable at all points

C. C. Differentiable at all points except at $x = 1$ and $x = -1$

D. D.Continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous

Answer:

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34. Solve $\sin^{-1} x \leq \cos^{-1} x$ graphically. Check the differentiability of f (x) =min. $\left\{\sin^{-1} x \leq \cos^{-1} x\right\}$. Also find the range of y = f(x)

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