MATHS

BOOKS - CENGAGE

INEQUALITIES INVOLVING MEANS

Single Correct Answer

1. If a, b, c be three positive numbers in $A.P.$ and $E = \frac{a+8b}{2b-a} + \frac{8b+c}{2b-c}$, then a value of E can be

A. 16

B. 15

C. 17

D. 18

Answer: D

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2. Let $ab = 1$, then the minimum value of $\frac{1}{a^4} + \frac{1}{4b^4}$ is

A. 1

B. 2

C. $1/4$

D. $1/2$

Answer: A

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3. If $x > 0, y > 0, z > 0$, the least value of

$x^{\log_e y - \log_e z} + y^{\log_e z - \log_e x} + Z^{\log_e x - \log_e y}$ is

A. 3

B. 1

C. 5

D. 6

Answer: A

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4. Let $p, q, r \in R^+$ and $27pqr \geq (p+q+r)^3$ and $3p+4q+5r=12.$

Then the value of $8p+4q-7r=$

A. 2

B. 3

C. 4

D. 5

Answer: D

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5. Minimum value of $f(x) = \cos^2 x + \frac{\sec x}{4}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

A. $3/2$

B. $3/4$

C. $3/8$

D. none of these

Answer: B

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6. Let $x, y, z \in R^+$ and $2xy + 3yz + 4xz = 18$. If α, β and γ be the values of x, y and z respectively, for which xyz attains its maximum value, then the value of $2\alpha + \beta + \gamma =$

A. 4

B. 6

C. 8

D. 12

Answer: B

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7. The minimum value of $\dfrac{x^4+y^4+z^2}{xyz}$ for positive real numbers x, y, z is $\sqrt{2}\ 2\sqrt{2}\ 4\sqrt{2}\ 8\sqrt{2}$

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. $8\sqrt{2}$

Answer: B

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8. If n_1, n_2, n_3,.....,n_{100} are positive real numbers such that $n_1 + n_2 + n_3 + \ldots + n_{100} = 20$ and $k = n_1(n_2 + n_3 + n_4)(n_5 + n_6 + \ldots + n_9)(n_{10} + \ldots . + n_{16})\ldots(\ldots + n_{10}$, then k belongs to

A. $(o, 100]$

B. $(o, 128]$

C. $(o, 144]$

D. $(o, 1024]$

Answer: D

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9. If a, b, c are the sides of triangle , then the least value of $\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a}$ is

A. $1/3$

B. 1

C. 3

D. 6

Answer: B

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10. If $x > 0, \frac{x^n}{1 + x + x^2 + \dots + x^{2n}}$ is

A. $\leq \frac{1}{2n+1}$

B. $< \frac{2}{2n+1}$

C. $\geq \frac{1}{2n+1}$

D. $> \frac{2}{2n+1}$

Answer: A

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11. If positive quantities a, b, c are in $H.P.$, then which of the following is not true ?

A. $b > \frac{a+c}{2}$

B. $\frac{1}{a-b} - \frac{1}{b-c} > 0$

C. $ac > b^2$

D. none of these

Answer: A:B

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12. Given that x, y, z are positive real numbers such that $xyz = 32$, the minimum value of $\sqrt{(x+2y)^2 + 2z^2 - 15}$ is equal to

A. 6

B. 8

C. 9

D. 12

Answer: C

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13. If x, y, z are positive real numbers such that $x^2 + y^2 + Z^2 = 7$ and $xy + yz + xz = 4$ then the minimum value of xy is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: C

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14. If x lies between -5 and 11, then the greatest value of $(11-x)^3(x+5)^5$ is

A. $6^5 \cdot 10^3$

B. $6^3 \cdot 10^3$

C. $6^3 \cdot 10^4$

D. $6^3 \cdot 10^5$

Answer: D

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15. The least integral value of

$$f(x) = \frac{(x-1)^7 + 3(x-1)^6 + (x-1)^5 + 1}{(x-1)^5}, \forall x > 1 \text{ is}$$

A. 8

B. 6

C. 12

D. 18

Answer: B

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16. If a, b, c are positive real numbers and $2a + b + 3c = 1$, then the maximum value of $a^4b^2c^2$ is equal to

A. $\frac{1}{3 \cdot 4^8}$

B. $\frac{1}{9 \cdot 4^7}$

C. $\frac{1}{9 \cdot 4^8}$

D. $\frac{1}{27 \cdot 4^8}$

Answer: C

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17. If x, y, z be three positive numbers such that xyz^2 has the greatest value $\frac{1}{64}$, then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is

A. 6

B. 8

C. 10

D. 12

Answer: C

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18. If x_1, x_2 and x_3 are the positive roots of the equation $x^3-6x^2+3px-2p=0$, πnR, then the value of $\sin^{-1}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)+\cos^{-1}\left(\frac{1}{x_2}+\frac{1}{x_3}\right)-\tan^{-1}\left(\frac{1}{x_3}+\frac{1}{x_1}\right)$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. π

Answer: A

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Comprehension

1. If a, b, x, y are real number and $x, y > 0$, then $\frac{a^2}{x} + \frac{b^2}{y} \geq \frac{(a+b)^2}{x+y}$ so on solving it we have $(ay - bx)^2 \geq 0$.

Similarly, we can extend the inequality to three pairs of numbers, i.e,

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq \frac{(a+b+c)^2}{x+y+z}$$

Now use this result to solve the following questions.

The value of $\frac{a^2+b^2}{a+b} + \frac{b^2+c^2}{b+c} + \frac{a^2+c^2}{a+c}$ is

A. $\geq (a+b+c)$

B. $\geq \frac{1}{2}(a+b+c)$

C. $\frac{3}{2} \leq (a+b+c)$

D. None of these

Answer: A

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2. If a, b, x, y are real number and $x, y > 0$, then $\frac{a^2}{x}+\frac{b^2}{y} \geq \frac{(a+b)^2}{x+y}$ so on solving it we have $(ay-bx)^2 \geq 0$.

Similarly, we can extend the inequality to three pairs of numbers, i.e,

$$\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z} \geq \frac{(a+b+c)^2}{x+y+z}$$

Now use this result to solve the following questions.

If $abc=1$, then the minimum value of

$\frac{1}{a^3(b+c)}+\frac{1}{b^3(a+c)}+\frac{1}{c^3(a+b)}$ is

A. 3

B. $3/2$

C. 6

D. 9

Answer: B

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Examples

1. Prove that $(ab+xy)(ax+by)>4abxy(a, b, x, y>0)$.

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2. Prove that $b^2c^2+c^2a^2+a^2b^2>abc(a+b+c)$, where $a, b, c>0$.

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3. Prove that $\frac{2}{b+c}+\frac{2}{c+a}+\frac{2}{a+b}<\frac{1}{a}+\frac{1}{b}+\frac{1}{c}, wherea, b, c>0.$

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4. If $a, b, andc$ are distinct positive real numbers such that $a+b+c=1,$ then prove tha $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}>8.$

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5. Find the minimum value of $\left(4^{\sin^2 x}+4^{\cos^2 x}\right)$.

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6. If $(\log)_2(a+b)+(\log)_2(c+d)\geq 4.$ Then find the minimum value of the expression $a+b+c+d$

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7. Find all real solutions to $2^x + x^2 = 2 - \frac{1}{2^x}$.

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8. Find all positive real solutions to $4x + \frac{18}{y} = 14,\ 2y + \frac{9}{z} = 15,\ 9z + \frac{16}{x} = 17.$

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9. Let A;G;H be the arithmetic; geometric and harmonic means between three given no. a;b;c then the equation having a;b;c as its root is $x^3 - 3Ax^2 + 3\frac{G^3}{H}x - G^3 = 0$

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10. .Let $a_1, a_2, \ldots\ldots\ldots$ be positive real numbers in geometric progression. For each n, let $A_n G_n, H_n$, be respectively the arithmetic mean, geometric mean & harmonic mean of $a_1, a_2 \ldots\ldots\ldots a_n$. Find an expression ,for the geometric mean of $G_1, G_2, \ldots\ldots. G_n$ in terms of $A_1, A_2, \ldots\ldots., A_n, H_1, H_2, \ldots\ldots., H_n$.

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11. If $a+b+c=1$, then prove that $\frac{8}{27abc} > \left\{\frac{1}{a}-1\right\}\left\{\frac{1}{b}-1\right\}\left\{\frac{1}{c}-1\right\} > 8$.

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12. If $yz+zx+xy=12, wherex, y, z$ are positive values, find the greatest value of xyz.

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13. If a, b, c are positive, then prove that $a/(b+c)+b/(c+a)+c/(a+b) \geq 3/2$.

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14. Prove that $2^n > 1+n\sqrt{2^{n-1}}, \forall n > 2$ where n is a positive integer.

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15. If $S+a_1+a_2+a_3+\ +a_n, a_1 \in R^+$ for $i=1 \rightarrow n$, then prove that $\frac{S}{S-a_1}+\frac{S}{S-a_2}+\ +\frac{S}{S-a_n} \geq \frac{n^2}{n-1}, \forall n \geq 2$

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16. If $a_1+a_2+a_3+\ldots+a_n=1 \forall a_i > 0, i=1, 2, 3, , n$, then find the maximum value of $a_1a_2a_3a_4a_5\ldots. a_n$.

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17. If $a, b, c,$ are positive real numbers, then prove that (2004, 4M)

$\{(1+a)(1+b)(1+c)\}^7 > 7^7 a^4 b^4 c^4$

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18. Prove that $\frac{\sec^4 \alpha}{\tan^2 \beta} + \frac{\sec^4 \beta}{\tan^2 \alpha} \geq 8$. If each term in the expression is well defined.

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19. Prove that

$$\left[\frac{x^2+y^2+z^2}{x+y+z}\right]^{x+y+z} > x^x y^y z^z > \left[\frac{x+y+z}{3}\right]^{x+y+z} \quad (x, y, z > 0)$$

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20. Prove that $1^2 + 2^2 + 3^2 + \ldots.. + n^2 = \frac{n(n+1)(2n+1)}{6}$

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21. Find the greatest value of $x^2y^3, wherexandy$ lie in the first quadrant on the line $3x+4y=5$.

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22. Find the maximum value of $(7-x)^4(2+x)^5 whenx$ lies between $-2and7$.

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23. Find the maximum value of xyz when $\frac{x}{1}+\frac{y^2}{4}+\frac{z^3}{27}=1$, where $x, y, z>0$.

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24. If a,$b>0$ such that $a^3+b^3=2$, then show that $a+b \leq 2$.

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25. If $m > 1, n \in N$ show that $1m + 2m + 2^{2m} + 2^{3m} + \ + 2^{nm-m} > n^{i-m}(2^n - 1)^m$.

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26. Prove that n in acute angled triangle $ABC, \sec A + \sec B + \sec C \geq 6.$

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27. Prove that $\frac{b^2 + c^2}{b + c} + \frac{c^2 + a^2}{c + a} + \frac{a^2 + b^2}{a + b} > a + b + c$, where $a, b, c > 0.$

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28. Prove that $\frac{a^8+b^8+c^8}{a^3b^3c^3} > \frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

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29. If $a, b, and c$ are positive and $a+b+c=6,$ show that $(a+1/b)2+(b+1/c)2+(c+1/a)2 \geq 75/4.$

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30. Let $x_1, x_2, , x_n$ be positive real numbers and we define $S = x_1 + x_2 + \ + x_n.$ Prove that $(1+x_1)(1+x_2)(1+x_n) \leq 1+S+\frac{S^2}{2!}+\frac{S^3}{3!}+\ +\frac{S^n}{n!}$

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31. If $2x^3+ax^2+bx+4=0$ (a and b are positive real numbers) has 3 real roots, then prove that $a+b \geq 6\left(2^{\frac{1}{3}}+4^{\frac{1}{3}}\right)$

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32. Calculate the greatest and least values of the function
$$f(x) = \frac{x^4}{x^8 + 2x^6 - 4x^4 + 8x^2 + 16}$$

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33. If a,b,c are three distinct positive real numbers in G.P., than prove that $c^2 + 2ab > 3ac$.

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34. If x and y are real numbers such that $16^{x^2+y} + 16^{x+y^2} = 1$, then find the values of x and y.

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35. In how many parts an integer $N \geq 5$ should be dissected so that the product of the parts is maximized.

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36. If $x + y + z = 1 and x, y, z$ are positive, then show that $\left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2 + \left(z + \frac{1}{z}\right)^2 > \frac{100}{3}$

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37. If a, b,c are three positive real numbers , then find minimum value of $\frac{a^2 + 1}{b + c} + \frac{b^2 + 1}{c + a} + \frac{c^2 + 1}{a + b}$

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Exercise 6 1

1. Prove that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{1}{\sqrt{(bc)}}+\frac{1}{\sqrt{(ca)}}+\frac{1}{\sqrt{(ab)}}$, where a,b,c $>$ 0

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2. If a,b,c are real numbers such that $0<a<1, 0<b<1, 0<c<1, a+b+c=2,$ then prove that $\frac{a}{1-a}\frac{b}{1-b}\frac{c}{1-c}\geq 8$

.

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3. If x,y are positive real numbers and m, n are positive integers, then prove that $\frac{x^n y^m}{(1+x^{2n})(1+y^{2m})}\leq\frac{1}{4}$

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4. If $a_i > 0$ (I - 1,2,3,....n) prove that

$$\sum_{I \geq I \geq j \geq n} \sqrt{a_i a_j} \leq \frac{n-1}{2}(a_1 + a_2 + \ldots. + a_n)$$

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5. Find the minimum value of $2^{\sin x} + 2^{\cos x}$

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6. If $(\log)_{10}(x^3 + y^3) - (\log)_{10}(x^2 + y^2 - xy) \leq 2, where x, y$ are positive real number, then find the maximum value of xy.

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7. Prove that the greatest value of xy is $c^3 / \sqrt{2ab}$. if $a^2 x^4 + b^4 y^4 = c^6$.

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8. If $x, y \in R^{+}$ such that x + y = 8, then find the minimum value of $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)$

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Exercise 6 2

1. Prove that $\left(\frac{n+1}{2}\right)^{n} > n!$

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2. If $a_1, a_2, , a_n > 0,$ then prove that $\frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_4}++\frac{a_{n-1}}{a_n}+\frac{a_n}{a_1} > n$

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3. In ABC , prove that $tanA + tanB + tanC \geq 3\sqrt{3}, whereA, B, C$ are acute angles.

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4. If n $\geq$ 1 is a positive integer, then prove that $3^n \geq 2^n + n.\, 6^{\frac{n-1}{2}}$

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5. If $abcd = 1$ where a,b,c,d are positive reals then the minimum value of $a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + cd$ is

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6. If x,y,z $>$ 0 and x + y + z = 1, the prove that

$$\frac{2x}{1-x} + \frac{2y}{1-y} + \frac{2z}{1-z} \geq 3.$$

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Exercise 6 3

1. Prove that

$$\left[\frac{a^2+b^2}{a+b}\right]^{a+b} > a^a b^b > \left\{\frac{a+b}{2}\right\}^{a+b}$$, where a,b $>$ 0

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2. Prove that $a^p b\&q > \left(\frac{ap+bp}{p+q}\right)^{p+q}$.

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3. Prove that $px^{q-r}+qx^{r-p}+rx^{p-q}>p+q+r, where p, q, r$ are distinct and$x \neq 1$.

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4. Given are positive rational numbers a, b, c such that $a + b + c = 1$, then prove that $a^a b^b c^c + a\hat{}\quad\hat{}\ a + a^c b^a c^b \leq 1$.

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5. If a and b are positive numbers such that $a^2 + b^2 = 4$, then find the maximum value of $a^2 b$.

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6. Find the greatest value of $x^2 y^3 z^4$ if $x^2 + y^2 + z^2 = 1, wherex, y, z$ are positive.

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Exercise 6 4

1. Prove that $a^4 + b^4 + c^4 > abc(a + b + c), wherea, b, c > 0.$

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2. If $C_r = \frac{n!}{r!(n-r)!}$, then prove that

$\sqrt{C_1} + \sqrt{C_2} + \dots . + \sqrt{C_n}\sqrt{n(2^n - 1)}$

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3. If a + b =1, a $>$ 0,b $>$ 0, prove that $\left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2 \geq \frac{25}{2}$

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4. In ΔABC, prove that sin A + sin B + sin C $\geq \frac{3\sqrt{3}}{2}$

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Exercise Single

1. The minimum value of $\dfrac{x^4+y^4+z^2}{xyz}$ for positive real numbers x, y, z is

$\sqrt{2}$ $2\sqrt{2}$ $4\sqrt{2}$ $8\sqrt{2}$

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. $8\sqrt{2}$

Answer: B

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2. A rod of fixed length k slides along the coordinates axes, If it meets the axes at $A(a, 0) and B(0, b)$, then the minimum value of $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2$ 0 8 $k^2=4+\frac{4}{k^2}$ $k^2+4+\frac{4}{k^2}$

A. 0

B. 8

C. $k^2 - 4 + \frac{4}{k^2}$

D. $k^2 + 4 + \frac{4}{k^2}$

Answer: D

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3. The least value of $6\tan^2\varphi + 54\cot^2\varphi + 18$ is 54 when A.M. $\geq$ GM. Is applicable for $6\tan^2\varphi, 54\cot^2\varphi, 18$ 54 when A.M. $\geq$ GM. Is applicable for $6\tan^2\varphi, 54\cot^2\varphi, 18$ is added further 78 when $\tan^2\varphi = \cot^2\varphi$ (I) is correct, (II) is false (I) and (II) are correct (III) is correct None of the above are correct

A. (I) is correct, (II) is false

B. (I) and (II) are correct

C. (III) is correct

D. (III) is correct

Answer: B

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4. If $ab^2c^3, a^2b^3c^4, a^3b^4c^5$ are in A.P. $(a, b, c > 0)$, then the minimum value of $a+b+c$ is $1\ 3\ 5\ 9$

A. 1

B. 3

C. 5

D. 9

Answer: B

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5. If $y = 3^{x-1} + 3^{-x-1}$, then the least value of y is

A. 2

B. 6

C. $2/3$

D. $3/2$

Answer: C

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6. Minimum value of $(b+c)/a + (c+a)/b + (a+b)/c$ (for real positive numbers a, b, c) is

A. 1

B. 2

C. 3

D. 6

Answer: D

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7. If the product of n positive numbers is n^n , then their sum is a positive integer b. divisible by n equal to $n + 1/n$ never less than n^2

A. a positive integer

B. divisible by n

C. equal to $n + l/n$

D. never less than n^2

Answer: D

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8. The minimum value of $P = bcx + cay + abz$, when $xyz = abc$, is

A. 3abc

B. 6abc

C. abc

D. 4abc

Answer: A

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9. If l, m, n are the three positive roots of the equation $x^3 - ax^2 + bx - 48 = 0,$ then the minimum value of $(1/l) + (2/m) + (3/n)$ equals $1\ 2\ 3/2\ 5/2$

A. 1

B. 2

C. $3/2$

D. $5/2$

Answer: C

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10. If positive numbers a, b, c are in H.P., then equation $x^2 - kx + 2b^{101} - a^{101} - c^{101} = 0 (k \in R)$ has both roots positive both roots negative one positive and one negative root both roots imaginary

A. both roots positive

B. both roots

C. one positive and one negative root

D. both roots imaginary

Answer: C

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11. For $x^2-(a+3)|x|=4=0$ to have real solutions, the range of a is $(-\infty,-7]\cup[1,\infty)$ $(-3,\infty)$ $(-\infty,-7]$ $[1,\infty)$

A. $(-\infty,-7]\cup[1,\infty)$

B. $(-3,\infty)$

C. $(-\infty,-7]$

D. $[1,\infty)$

Answer: D

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12. If a, b, c are the sides of a triangle, then the minimum value of $\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}$ is equal to 3 6 9 12

A. 3

B. 6

C. 9

D. 12

Answer: A

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13. If $a, b, c, d \in R^{\pm}\{1\}$, then the minimum value of $(\log)_d a + (\log)_b d + (\log)_a c + (\log)_c b$ is 4 2 1 none of these

A. 4

B. 2

C. 1

D. none of these

Answer: A

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14. If $a, b, c \in R^{+}, then \frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b}$ is always $\leq \frac{1}{2}(a+b+c)$ $\geq \frac{1}{3}\sqrt{abc}$ $\leq \frac{1}{3}(a+b+c)$ $\geq \frac{1}{2}\sqrt{abc}$

A. $\leq \frac{1}{2}(a+b+c)$

B. $\geq \frac{1}{2}\sqrt{abc}$

C. $\leq \frac{1}{3}(a+b+c)$

D. $\geq \frac{1}{2}\sqrt{abc}$

Answer: A

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15. If $a, b, c \in R^{+} then (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is always ≥ 12 ≥ 9 ≤ 12 none of these

A. ≥ 12

B. ≥ 9

C. ≤ 12

D. none of these

Answer: B

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16. If $a, b, c \in R^{+}$, then the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is equal to abc $2abc$ $3abc$ $6abc$

A. abc

B. 2abc

C. 3abc

D. 6abc

Answer: D

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17. If $a, b, c, \in R^{+}$, and a,b,c, d are in H.P. then

A. $a + d > b + c$

B. $a + b > c + d$

C. $a + c > b + d$

D. none of these

Answer: A

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18. If $a, b, c, \in R^{+}$, such that $a + b + c = 18$, then the maximum value of a^2, b^3, c^4 is equal to

A. $2^{18} \times 3^2$

B. $2^{18} \times 3^3$

C. $2^{19} \times 3^2$

D. $2^{19} \times 3^3$

Answer: D

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19. $f(x) = \frac{(x-2)(x-1)}{x-3}, \forall x > 3$. The minimum value of $f(x)$ is equal to

A. $3 + 2\sqrt{2}$

B. $3 + 2\sqrt{3}$

C. $3\sqrt{2} + 2$

D. $3\sqrt{2} - 2$

Answer: A

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20. If $a > 0$, the least value of $\left(a^3 + a^2 + a + 1\right)^2$ is

A. $64a^2$

B. $16a^4$

C. $16a^3$

D. none of these

Answer: C

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Exercise Multiple

1. If A is the area and 2s the sum of the sides of a triangle,then

A. $A \leq \frac{s^2}{4}$

B. $A \leq \frac{s^2}{3\sqrt{3}}$

C. $A < \frac{s^2}{\sqrt{3}}$

D. none of these

Answer: A::B

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2. If x,y,z are positive numbers in A.P., then

A. $y^2 \geq xz$

B. $xy + yz \geq 2xz$

C. $\dfrac{x+y}{2y-x} + \dfrac{y+z}{2y-z} \geq 4$

D. none of these

Answer: A::C

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3. For positive real numbers a, bc such that $a + b + c = p,$ which one holds? $(p-a)(p-b)(p-c) \leq \dfrac{8}{27}p^3$ $(p-a)(p-b)(p-c) \geq 8abc$ $\dfrac{bc}{a} + \dfrac{ca}{b} + \dfrac{ab}{c} \leq p\, noneofthese$

A. $(p-a)(p-b)(p-c) \leq \frac{8}{27}p^3$

B. $(p-a)(p-b)(p-c) > 8abc$

C. $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \leq p$

D. none of these

Answer: A::B

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4. If first and $(2n-1)^{th}$ terms of A.P., G.P. and H.P. are equal and their nth terms are a,b,c respectively, then

A. $a = b = c$

B. $a + c = b$

C. $a > b > c$

D. $ac - b^2 = 0$

Answer: C::D

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5. If a, b, c are positive real numbers and $2a+b+3c=1$, then the maximum value of $a^4b^2c^2$ is equal to

A. $a^4b^2c^2$ is greatest then $a=\frac{1}{4}$

B. $a^4b^2c^2$ is greatest then $b=\frac{1}{4}$

C. $a^4b^2c^2$ is greatest then $c=\frac{1}{12}$

D. greatest value of $a^4b^2c^2 is \frac{1}{9.4^8}$

Answer: A::B::C::D

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Exercise Comprehension

1. If roots of the equation $f(x)=x^6-12x^5+bx^4+cx^3+dx^2+ex+64=0$are positive, then

Which has the greatest absolute value ?

A. b

B. c

C. d

D. e

Answer: C

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2. If roots of the equation $f(x) = x^6 - 12x^5 + bx^4 + cx^3 + dx^2 + ex + 64 = 0$ are positive, then

Which has the greatest absolute value ?

A. b

B. c

C. d

D. e

Answer: A

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3. If roots of the equation $f(x) = x^6 - 12x^5 + bx^4 + cx^3 + dx^2 + ex + 64 = 0$ are positive, then remainder when $f(x)$ is divided by $x - 1$ is

A. 2

B. 1

C. 3

D. 10

Answer: B

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4. Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a,b,c are non-negative).

Minimum non-negative real value of a is

A. 10

B. 9

C. 6

D. 4

Answer: D

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5. Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a,b,c are non-negative).

Minimum non-negative real value of b is

A. 12

B. 15

C. 6

D. 10

Answer: C

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6. Equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ has real roots (a,b,c are non-negative). Minimum non-negative real value of c is

A. 10

B. 9

C. 6

D. 4

Answer: D

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Exercise Numerical Jee Previous Year

1. For $x \geq 0$, the smallest value of the function $f(x) = \frac{4x^2 + 8x + 13}{6(1+x)}$, is _______.

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2. Let $x^2 - 3x + p = 0$ has two positive roots $a and b$, then minimum value if $\left(\frac{4}{a} + \frac{1}{b}\right)$ is,

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3. If x,y,and z are positive real numbers and $x = \frac{12 - yz}{y + z}$. The maximum value of (xyz) equals_________.

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4. If a,b, and c are positive and $9a + 3b + c = 90$, then the maximum value of $(\log a + \log b + \log c)$ is (base of the logarithm is 10)_______.

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5. Given that x, y, z are positive reals such that $xyz = 32$. The minimum value of $x^2 + 4xy + 4y^2 + 2z^2$ is _________.

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6. If $x, y, \in R^+$ satisfying $x + y = 3$, then the maximum value of $x^2 y$ is ___________.

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7. For any $x, y, \in R, xy > 0$. Then the minimum value of $\frac{2x}{y^3} + \frac{x^3 y}{3} + \frac{4y^2}{9x^4}$ is _________.

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8. Let a,b,c,d and e be positive real numbers such that $a+b+c+d+e=15$ and $ab^2c^3d^4e^5=(120)^3\times 50$. Then the value of $a^2+b^2+c^2+d^2+e^2$ is ________.

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9. Consider the system of equations $x_1+x_2^2+x_3^3+x_4^4+x_5^5=5$ and $x_1+2x_2+3x_3+4x_4+5x_5=15$ where x_1, x_2, x_3, x_4, x_5 are positive real numbers. Then numbers of $(x_1, x_2, x_3, x_4, x_5)$ is ________.

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10. The least value of $a\in\mathbb{R}$ for which $4ax^2+\frac{1}{x}\geq 1$, for all $x>0$, is

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{27}$

D. $\frac{1}{25}$

Answer: C

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11. The minimum value of the sum of real number $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8, and a^{10} with a > 0$ is

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