MATHS

BOOKS - CENGAGE

MONOTONICITY AND MAXIMA MINIMA OF FUNCTIONS

Examples

1. Check the nature of the following differentiable functions (i) f(x) =e^x +sin x ,x $\in R^+$ (ii)$f(x)=\sin x+\tan x-2x, x \in (0, \pi/2)$

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2. Prove that the function $f(x)=(\log)_e(x^2+1)-e^{-x}+1$ is strictly increasing $\forall x \in R$.

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3. Find the least value of k for which the function $x^2 + kx + 1$ is an increasing function in the interval `1

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4. Find the values of a if $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ is increasing for all values of $x \cdot$

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5. If f(x) and g(x) are differentiable and increasing functions then which of the following functions alwasys increases?

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6. Let

$g(x) = (f(x))^3 - 3(f(x))^2 + 4f(x) + 5x + 3\sin x + 4\cos x \ \forall x \in R$.

Then prove that g is increasing whenever is increasing.

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7. Find the complete set of values of $\alpha$for which the function

f(x) =$\{(x+1, x<1$

α, x =1 , is strictly increasing x=1

$x^2 - x + 3, x > 1$

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8. Prove that function f(x) =$\{-2x^3 + 3x^2 - 6x + 5, x < 0$

$-x^2 - x + 1, x \geq 0$ is decreasing for all x.

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9. If $fogoh(x)$ is an increasing function, then which of the following is not possible? $f(x), g(x), andh(x)$ are increasing $f(x)andg(x)$ are decreasing and $h(x)$ is increasing $f(x), g(x), andh(x)$ are decreasing

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10. Let $f(x)andg(x)$ be two continuous functions defined from $R\vec{R}$, such that `f(x_1)>f(x_2)a n dg(x_1)f(g(3alpha-4))`

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11. Let $f : [0, \infty)\overrightarrow{0, \infty}andg : [0, \infty)vec[0, \infty)$be non-increasing and non-decreasing functions, respectively, and h(x)=g(f(x))dot If fa n dg are differentiable functions, h(x)=g(f(x))dot If fa n dg are differentiable for all points in their respective domains and h(0)=0, then show h(x) is always, identically zero. check-circle

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12. Find the values of p if $f(x) = \cos x - 2px$ is invertible.

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13. Find the critical points(s) and stationary points (s) of the function $f(x) = (x - 2)^{2/3}(2x + 1)$

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14. In X-Y plane, the path defined by the equation $\frac{1}{x^m} + \frac{1}{y^m} + \frac{k}{(x + y)^n} = 0$, is a hyperbola if $m = 1, k = -1, n = 0$

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15. Find the interval of monotonocity of the function $f(x) = |x - 1|x^2$.

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16. Find the intervals of decrease and increase for the function $f(x) = \cos\left(\frac{\pi}{x}\right)$

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17. Let $g(x) = f(x) + f(1-x) and f^{x} > 0 \forall x \in (0, 1)$. Find the intervals of increase and decrease of $g(x)$.

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18. Find the range of the function $f(x) = x \sin x - \frac{1}{2} \sin^2$ x for $x \in \left(0, \frac{\pi}{2}\right)$

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19. Find the range of f(x)=$\frac{1}{\pi} \sin^{-1} x + \tan^{-1} + \frac{x+1}{x^2+2x+5}$

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20. Find the number of roots of the equation $\log_e(1+x) - \frac{\tan^{-1} x}{1+x} = 0$

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21. Find the number of roots of the function $f(x) = \frac{1}{(x+1)^3} - 3x + \sin x \cdot$

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22. Prove that $\log_e(1+x) < xf \text{ or } x > 0$

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23. Let $fandg$ be differentiable on R and suppose $f(0) = g(0) and f'(x) \leq g'(x)$ for all $x \geq 0.$ Then show that $f(x) \leq g(x)$ for all $x \geq 0.$

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24. Show that $1 + x \in \left(x + \sqrt{x^2 + 1}\right) \geq \sqrt{1 + x^2}$ for all $x \geq 0$.

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25. In X-Y plane, the path defined by the equation $\frac{1}{x^m} + \frac{1}{y^m} + \frac{k}{(x+y)^n} = 0$, is a pair of lines if $m = k = n = 1$

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26. Prove that $|\cos\alpha - \cos\beta| \leq |\alpha - \beta|$

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27. If $P(1) = 0 and \frac{dP(x)}{dx}, \sin x + 2x \geq \frac{3x(x+1)}{\pi}$. Explain the identity, if any, used in the proof.

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28. For $0 < x < \frac{\pi}{2}$,prove that $x > \sin x$and hence $\cos(\sin x) > \sin(\cos x)$

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29. Prove that $e^x \geq 1 + x$ and hence $e^x + \sqrt{1 + e^{2x}} \geq (1 + x) + \sqrt{2 + 2x + x^2} \forall$ x $\in$ R

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30. Separate the intervals of concavity of the following functions

(i) f(x)=$\sin^{-1} x$,(ii) $f(x) = x + \sin x$

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31. If graph of the function f(x) $=3x^4 + 2x^3 + ax^2 - x + 2$ is concave upward for all real x, then find values of a,

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32. Prove that for any two numbers x_1 and x_2

$$\frac{2e^{x_1} + e^{x_2}}{3} > e\frac{2x_1 + x_2}{3}$$

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33. Prove that in triangle ABC, sin $A + \sin B + \sin C \leq \frac{3\sqrt{3}}{2}$

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34. Prove that $\frac{a_1^m + a_2^m + \dots + a_n^m}{n} < \frac{a_1 + a_2 + .. + a_n}{(n)^m}$

If $0 < m < 1$ and $a_i > 0$ for all I.

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35. Find the points of inflection for (i) $f(x) = \sin x$ (ii) $f(x) = 3x^4 - 4x^3$

(iii) $f(x) = x^{\frac{1}{3}}$

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36. Find the coordinates of the point of inflection of the curve f(x)

$= e^{-x^2}$

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37. The function y=$\dfrac{ax+b}{x-1}(x-4)$ has turning point at P(2,-1) Then find the values of a and b.

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38. How many ciritcal points functionf(x)=(x-1)|x-3|-4x+12 has? How many of these are points of local extrema?

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39. Discuss the extremum of the function f(x)=(x-1)(x-2)(x-3).How many criticla points f(x) has?

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40. Find the points of extrema of the function f(x)=2 sec x+3 cosec x

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41. Prove that for all a,b in the function f(X) $= 3x^4 - 4x^3 + 6x^2 + ax + b$ has exactly one extremium.

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42. The function $f(x) = (x^2 - 4)^n (x^2 - x + 1), n \in N,$ assumes a local minimum value at $x = 2.$ Then find the possible values of n

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43. Test f(x)={x} for the existence of alocal maximum and minimum at x=1, where{.} represents the fractional part function.

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44. Let f(x)=$\{x^3 + x^2 + 10x, x < 0$.Investigate x=0 for local

$-3\sin x, x \geq 0$

maxima/minima.

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45. Let $f(x) = \frac{a}{x} + x^2$. If it has a maximum at $x = -3$, then find the value of a.

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46. Discuss the extremum of $f(x) = \sin x(1 + \cos x)$, $x \in \left(0, \frac{\pi}{2}\right)$

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47. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima and local minima and points of inflexion.

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48. Discuss the extremum of $f(x) = 40\left(3x^4 8x 3 - 18x^2 + 60\right)$.

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49. Discuss extrema of the function

$$f(x) = \int_l^x 2(x-1)(x-2)^3 + 3(x-1)^2(x-2)^2 dx$$

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50. Let `f(x)=sin^3x+lambdasin^2x ,-pi/2

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51. Let `f(x)=sin^3x+lambdasin^2x ,-pi/2

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52. Discuss the extrema of $f(x) = \frac{x}{1 + x \tan x}, x \in \left(0, \frac{\pi}{2}\right)$

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53. In X-Y plane, the path defined by the equation $\frac{1}{x^m} + \frac{1}{y^m} + \frac{k}{(x+y)^n} = 0$, is a pair of lines if $m = k = -1, n = 1$

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54. Discuss the extreama of the following functions

$(i) f(x) = |x|, (ii) f(x) = e^{-|x|}, (iii) f(x) = x^{2/3}$

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55. If $1 f(x) = \{x^2, x \leq 0$.Investigate the functions at x for maxima/manima

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56. Discuss the extremum of $f(x) = 2x + 3x^{\frac{2}{3}}$

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57. $f(x) = \begin{cases} \dfrac{\cos(\pi x)}{2}, x > 0 \; x + a, x \leq 0 \end{cases}$ Find the values of a if $x = 0$ is a point of maxima.

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58. let $f(x) = -x^3 + \dfrac{b^3 - b^2 + b - 1}{b^2 + 3b + 2}$ if x is 0 to 1 and $f(x) = 2x - 3$ if x if 1 to 3.All possible real values of b such that $f(x)$ has the smallest value at $x = 1$ are

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59. $f(x) = |ax - b| + c|x| \, \forall x \in (-\infty, \infty)$, where $a > 0, b > 0, c > 0$. Find the condition if $f(x)$ attains the minimum value only at one point.

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60. Let $f(x) = 2x^3 = 9x^2 + 12x + 6$. Discuss the global maxima and minima of $f(x) \in [0, 2] and (1, 3)$ and, hence, find the range of $f(x)$ for corresponding intervals.

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61. Find the absolute maximum and absolute minimum values of $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ in $[0,\ 3]$

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62. Find the range of the function $f(x) = 4\cos^3 x - 8\cos^2 x + 1$.

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63. Find the range of the function $f(x) = 2\sqrt{x - 2} + \sqrt{4 - x}$

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64. A functiony =f(x) is represented parametrically as following

$x = \phi(t) = t^5 - 20t + 7$

$y = \psi(t) = 4t^3 - 3t^2 - 18t + 3$

where t in [-2,2]

Find the intervals of monotonicity and also find the points of extreama.Also find the range of function.

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65. Find how many roots of the equations $x^4 + 2x^2 - 8x + 3 = 0.$

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66. Draw the graph of y=$\dfrac{x^2}{\sqrt{x+1}}$

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67. Draw the graph of $y = xe^x$.Also

(i) Find the range of the functionf

(ii) Find the point of inflection.

(iii)Find the values of k if $1xe^x$=k has two distinct real roots

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68. Minimum integral value of k for which the equation $e^x = kx^2$ has exactly three real distinct solution,

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69. Draw the graph of $f(x) = \log_e\left(\sqrt{1 - x^2}\right)$.Find the range of the function .Also find the values of k if (k) has two distinct real roots.

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70. Draw the graph of $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - x}$

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71. Find the value of a if $x^3 - 3x + a = 0$ has three distinct real roots.

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72. Prove that there exist exactly two non-similar isosceles triangles ABC such that $\tan A + \tan B + \tan C = 100.$

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73. If t is a real number satisfying the equation $2t^3 - 9t^2 + 30 - a = 0,$ then find the values of the parameter a for which the equation $x + \dfrac{1}{x} = t$ gives six real and distinct values of x .

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74. The tangent to the parabola $y = x^2$ has been drawn so that the abscissa x_0 of the point of tangency belongs to the interval [1,2]. Find x_0 for which the triangle bounded by the tangent, the axis of ordinates, and the straight line $y = x02$ has the greatest area.

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75. Find the point $(\alpha,)\beta$ on the ellipse $4x^2 + 3y^2 = 12,$ in the first quadrant, so that the area enclosed by the lines $y = x, y = \beta, x = \alpha$, and the x-axis is maximum.

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76. LL' is the latus sectum of the parabola $y^2 = 4axandPP'$ is a double ordinate drawn between the vertex and the latus rectum. Show that the area of the trapezium $PP'LL'$ is maximum when the distance PP' from the vertex is $a/9$.

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77. Find the points on the curve $5x^2 - 8xy + 5y^2 = 4$ whose distance from the origin is maximum or minimum.

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78. Rectangles are inscribed inside a semi-circle of radius r. Find the rectangle with maximum area.

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79. A running track of 440 ft is to be laid out enclosing a football field, the shape of which is a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, then find the length of its sides.

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80. If the sum of the lengths of the hypotenuse and another side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between these sides is $\frac{\pi}{3}$.

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81. The tangent to the parabola $y = x^2$ has been drawn so that the abscissa x_0 of the point of tangency belongs to the interval [1,2]. Find x_0 for which the triangle bounded by the tangent, the axis of ordinates, and the straight line $y = x02$ has the greatest area.

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84. Find the points on the curve $5x^2 - 8xy + 5y^2 = 4$ whose distance from the origin is maximum or minimum.

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85. A sheet of area $40m^2$ is used to make an open tank with square base. Find the dimensions of the base such that the volume of this tank is maximum.

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86. A sheet of area $40m^2$ is used to make an open tank with square base. Find the dimensions of the base such that the volume of this tank is maximum.

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87. The latest edge of a regular hexagonal pyramid is $1cm$. If the volume is maximum, then find its height.

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88. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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89. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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90. Find the possible values of a such that $f(x) = e^{2x} - (a+1)e^x + 2x$ is monotonically increasing for $x \in R$.

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91. Of

$$f(x) = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) - \log(x^2 + x + 1)\left(\lambda^2 - 5\lambda + 3\right)x + 10$$

is a decreasing function for all $x \in R$, find the permissible values of λ.

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92. Leg $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function on the set R. Then find the condition on $aandb$.

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93. Prove that $\left[\lim_{x \to 0} \frac{\sin x . \tan x}{x^2}\right] = 1$,where [.] represents greatest integer function.

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94. Using the relation `2(1-cosx)

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95. If $f'(x) > 0, f(x) > 0 \forall x \in (0, 1)$ and $f(0) = 0, f(1) = 1$,then prove that `f(x)f^-1(x)

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96. Discuss the monotonocity of $Q(x)$, where $Q(x) = 2f\left(\frac{x^2}{2}\right) + f(6 - x^2)\,\forall x \in R$ It is given that $f^x > 0\,\forall x \in R$.

Find also the point of maxima and minima of $Q(x)$.

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97. Prove that $\left(\frac{\tan^{-1}(1)}{e}\right)^2 + \frac{2e}{\sqrt{e^2+1}}$

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98. Prove that `sin^2theta

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99. $F(x) = x^3 - 3x^2 + 6$ and $g(x) = \max\{F(t) : x+1 \leq t \leq x+2\} - 3 \leq x < 0$ and $1 - x$ for $x \geq 0$ Find continuity and differentiability of g(x) for $x \in [-3, 1]$

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100. Show that $5x \le 8\sin x - \sin 2x \le 6xf$ or $x \le x \le \frac{\pi}{3}$

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101. Let $f(x), x \ge 0,$ be a non-negative continuous function. If $f'(x)\cos x \le f(x)\sin x \,\forall x \ge 0,$ then find $f\left(\frac{5\pi}{3}\right)$

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102. If $ax^2 + \frac{b}{x} \ge c$ for all positive x where $a > 0$ and $b > 0,$ show that $27ab^2 \ge 4c^3.$

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103. Prove that for $x \in \left[0, \frac{\pi}{2}\right]$, $\sin x + 2x \ge \frac{3x(x+1)}{\pi}.$

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104. Find the set of values of p for which line $2px - 4y + 2\pi - 9 = 0$ intersect the curve $y = \cos^{-1}$at three distinct points.

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105. From a fixed point A on the circumference of a circle of radius $r,$ the perpendicular AY falls on the tangent at $P\cdot$ Find the maximum area of triangle $APY\cdot$

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106. $PandQ$ are two points on a circle of centre C and radius α . The angle PCQ being 2θ , find the value of $\sin\theta$ when the radius of the circle inscribed in the triangle CPQ is maximum.

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107. The lower corner of a leaf in a book is folded over so as to reach the inner edge of the page. Show that the fraction of the width folded over when the area of the folded part is minimum is $2/3$.

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108. Let $A(p^2, -p), B(q^2, q), C(r^2, -r)$ be the vertices of triangle ABC. A parallelogram AFDE is drawn with D,E, and F on the line segments BC, CA and AB, respectively. Using calculus, show that the maximum area of such a parallelogram is $\frac{1}{2}(p+q)(q+r)(p-r)$.

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109. A window of perimeter P (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

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Illustration

1. Let $A = \{2, 4, 6\}, B = \{8, 10, 12, 14\}$ and $f = \{(2, 8), (4, 10), (6, 12)\}$ be a function from A to B. Show that f is one to one but not onto function.

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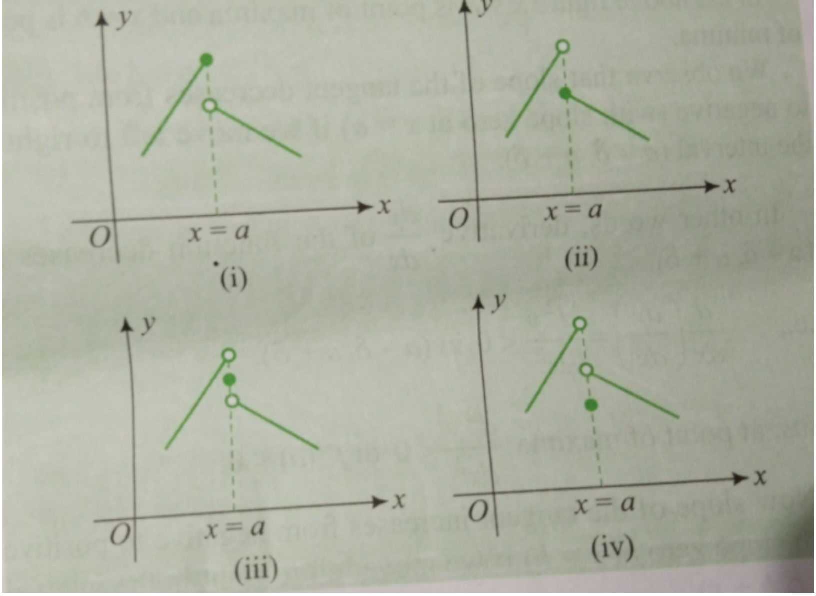
2. Consider the following graphs of the functions. Check each for the extrema at x=a



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Exercise 6.1

1. Prove that the following functions are strictly increasing:
$$f(x) = \log(1+x) - \frac{2x}{2+x}$$

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2. Separate the intervals of monotonocity for the function $f(x) = -2x^3 - 9x^2 - 12x + 1$

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3. Discuss monotonocity of $f(x) = \frac{x}{\sin x} and$ `g(x)=x/(tanx),w h e r e

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4. A function $y = f(x)$ is given by $x = \frac{1}{1+t^2}$ and $y = \frac{1}{t(1+t^2)}$ for all $t > 0$ then f is

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5. Find the value of a for which the function $(a+2)x^3 - 3ax^2 + 9ax - 1$ decreases montonically for all real $x\cdot$

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6. Find the value of a in order that $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real values of $x\cdot$

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7. Discuss the monotonocity of function $f(x) = 2\log|x - 1| - x^2 + 2x + 3.$

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8. If $f(x) = \sin x + \log_e|\sec x + \tan x| - 2xf$ or $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then check the monotonicity of f(x)

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9. Find the interval of the monotonicity of the function f(x)= $\log_e\left(\frac{\log_e x}{x}\right)$

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10. Let $g(x)=f(\log x)+f(2-\log x) and f^x<0 \forall x \in (0,3)\cdot$ Then find the interval in which $g(x)$ increases.

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Exercise 6.2

1. Find the range of f(X) =$\tan^{-1}x-\frac{1}{2}\log_e x \in \frac{1}{\sqrt{3}},(\sqrt{3})$

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2. Find the range of f(x) =$\frac{\sin x}{x}+\frac{x}{\tan x} \in \left(0,\frac{\pi}{2}\right)$

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3. Range of $f(x) = \sin^{-1} x + \cos^{-1} x + \cot^{-1} x$

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4. Find the number of solution of the equation

$x^3 + 2x + \cos x + \tan x = 0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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5. Show that $\frac{x}{(1+x)} \langle 1n(1+x) f \text{ or } x \rangle 0$

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6. For `0

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7. Show that $\tan^{-1} x > \dfrac{x}{1+\frac{x^2}{3}}$ if $x \in (0, \infty)$.

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8. Show that $2\sin x + \tan x \geq 3x$, where $0 \leq x < \dfrac{\pi}{2}$

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9. Prove that $f(x) = \dfrac{\sin x}{x}$ is monotonically decreasing in $\left[0, \dfrac{\pi}{2}\right]$. Hence, prove that `(2x)/pi

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10. For `0

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Exercise 6.3

1. Show that graph of the function (x) $=\log_e(x-2)-\frac{1}{x}$ always concave downwards.

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2. Separate the interval of convaity of y $=x \log_e x-\frac{x^2}{2}+\frac{1}{2}$

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3. consider f(X)= $\cos 2x+2x\lambda^2+(2\lambda+1)(\lambda-1)x^2, \lambda \in R$

If $\alpha \neq \beta$ and $\frac{f(\alpha+\beta)}{2}<\frac{f(\alpha)+f(\beta)}{2}$ for α and β then find the values of λ

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4. Find the values of x where function $f(X) = (\sin x + \cos x)(e^x)$ in $(0, 2\pi)$ has point of inflection

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5. Prove that $\frac{a_1^m + a_2^m + \ldots . + a_n^m}{n} < \frac{a_1 + a_2 + . . + a_n}{(n)^m}$

If $0 < m < 1$ and $a_i > 0$ for all I.

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6. Find the least value of $\sec A + \sec B + \sec C$ in an acute angled triangle.

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Exercise 6.4

1. Find the critical (stationary) points of the function f(X)=$\frac{x^5}{20}-\frac{x^4}{12}$ +5Name these points .Also find the point of inflection

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2. The curve $f(x)=\frac{x^2+ax+6}{x-10}$ has a stationary point at $(4,1)$. Find the values of $aandb$. Also, show that $f(x)$ has point of maxima at this point.

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3. If the function f(x) =axe^{bx^2} has maximum value at x=2 such that f(2) =1 , then find the values of a and b

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4. Discuss the extremum of $f(x)=\frac{1}{3}\left(x+\frac{1}{x}\right)$

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5. Discuss the extremum of $f(x) = 1 + 2\sin x + 3\cos^2 x, x \leq x \leq \frac{2\pi}{3}$

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6. Discuss the extremum of $f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x, 0 \leq x \leq \pi\cdot$

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7. Let $f(x) = -\sin^3 x + 3\sin^2 x + 5 on \left[0, \frac{\pi}{2}\right]$. Find the local maximum and local minimum of $f(x)\cdot$

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8. discuss the extremum of $f(\theta) = \sin^p \theta \cos^q \theta, p, q > 0, 0 < \theta < \frac{\pi}{2}$

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9. Find the maximum and minimum values of the function $y = (\log)_e\left(3x^4 - 2x^3 - 6x^2 + 6x + 1\right) \forall x \in (0, 2)$ Given that $\left(3x^4 - 2x^3 - 6x^2 + 6x^2 + 6x + 1\right) > 0 A x \in (0, 2)$

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10. Discuss the extremum of $f(x) = x\left(x^2 - 4\right)^{-\frac{1}{3}}$

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11. Discuss the maxima and minima of the function $f(x) = x^{\frac{2}{3}} - x^{\frac{4}{3}}$. Draw the graph of $y = f(x)$ and find the range of $f(x)$.

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12. Discuss the extremum of

`f(x)={|x^2-2|,-1lt=x

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13. Discuss the extremum of

$f(x) = \{1 + \sin x, x < 0x^2 - x + 1, x \geq 0atx = 0$

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14. Find the minimum value of $|x| + \left|x + \frac{1}{2}\right| + |x - 3| + \left|x - \frac{5}{2}\right|.$

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15. Let $f(x)$ be defined as `f(x)={tan^(-1)alpha-5x^2,0

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16. Let

$f(x) = \{x^3 - x^2 + 10x - 5, x \leq 1 - 2x + (\log)_2(b^2 - 2), x > 1$ Find

the values of b for which $f(x)$ has the greatest value at $x = 1$.

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Exercise 6.5

1. Draw the graph of $f(x) = (x - 1)|(x - 2)(x - 3)|$. .

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2. Discuss the number of roots of the equation $e(k - x \log x) = 1$ for different value of $k\cdot$

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3. Draw the graph of $y = (x+1)^{2/3} + (x-1)^{2/3}$

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4. Draw the graph of $f(X) = \log_e(1 - \log_e x)$. Find the point of inflection

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5. Draw and graph of $f(x) = \frac{4\log_e x}{x^2}$. Also find the range.

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Exercise 6.6

1. A private telephone company serving a small community makes a profit of Rs. 12.00 per subscriber, if it has 725 subscribers. It decides to reduce the rate by a fixed sum for each subscriber over 725, thereby reducing the profit by 1 paise per subscriber. Thus, there will be profit of Rs. 11.99 on each of the 726 subscribers, Rs. 11.98 on each of the 727 subscribers, etc. What is the number of subscribers which will give the company the maximum profit?

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2. Find the derivative of $y = \sin e^{3x}$

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3. A figure is bounded by the curves $y = x^2 + 1, y = 0, x = 0, and x = 1.$ At what point (a, b) should a tangent be drawn to curve $y = x^2 + 1$ for it to cut off a trapezium of greatest area from the figure?

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4. Find the minimum length of radius vector of the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

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5. Find the point at which the slope of the tangent of the function $f(x) = e^x \cos x$ attains minima, when $x \in [0, 2\pi]$.

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6. An electric light is placed directly over the centre of a circular plot of lawn 100 m in diameter. Assuming that the intensity of light varies directly as the sine of the angle at which it strikes an illuminated surface and inversely as the square of its distance from its surface, how should the light be hung in order that the intensity may be as great as possible at the circumference of the plot?

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7. A swimmer S is in the sea at a distance dkm from the closest point A on a straight shore. The house of the swimmer is on the shore at distance Lkm from A. He can swim at speed of ukm/h. walk at speed at vkm/h. At what point on shore should he land so that hhe reaches his house in the shortest time.

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8. Let (h, k) be a fixed point, where $h > 0, k > 0$. A straight line passing through this point cuts the positive direction of the coordinate axes at the point $PandQ$. Find the minimum area of triangle OPQ, O being the origin.

Watch Video Solution

9. A point P is given on the circumference of a circle of radius r. Chords QR are parallel to the tangent at P. Determine the maximum possible area of triangle PQR.

Watch Video Solution

Exercise 6.7

1. A metal box with a square base and vertical sides is to contain 1024 cm3 of water, the material for the top and bottom costs Rs 5 per cm2 and the material for the sides costs Rs 2.50 per cm2. Find the least cost of the box.

Watch Video Solution

2. Find the derivative of $y = e^{\sin x}$

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3. The lateral edge of a regular rectangular pyramid is $acmlong$. The lateral edge makes an angle α with the plane of the base. Find the value of α for which the volume of the pyramid is greatest.

Watch Video Solution

4. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

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5. A regular square based pyramid is inscribed in a sphere of given radius R so that all vertices of the pyramid belong to the sphere. Find the greatest value of the volume of the pyramid.

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Exercise (Single)

1. A function is matched below against an interval where it is supposed to be increasing. Which of the following parts is incorrectly matched? Interval, Function (a)$[2, \infty)$, $2x^3 - 3x^2 - 12x + 6$ (b)$(-\infty, \infty)$, $x^3 = 3x^2 + 3x + 3$ (c)$(-\infty - 4)$, $x^3 + 6x^2 + 6$ (d)$\left(-\infty, \frac{1}{3}\right)$, $3x^2 - 2x + 1$

A. $[2, \infty), 2x^3 - 3x^2 - 12x + 6$

B. $[-\infty, \infty), x^3 - 3x^2 - 3x + 3$

C. $[-\infty, -4), x^3 - 6x^2 + 6$

D. $\left[-\infty, \frac{1}{3}\right], 3x^2 - 2x + 1$

Answer: 4

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2. On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing? $\left(, \frac{\pi}{2}\right)$ (b) $(0, 1)$ $\left(\frac{\pi}{2}, \pi\right)$ (d) none of these

A. $(0, \pi / 2)$

B. (0,1)

C. $(\pi / 2, \pi)$

D. None of these

Answer: 4

Watch Video Solution

3. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

A. $(0, \pi / 2)$

B. $\left(0, \frac{\pi}{2}\right)$

C. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Answer: 1

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4. Consider the following statements in S and R S: Both $\sin x and \cos x$ are decrerasing function in the interval $\left(\frac{\pi}{2}, \pi\right)$ R: If a differentiable function decreases in an interval (a, b), then its derivative also decrease in $a, b)$. Which of the following it true? Both S and R are wrong. Both S and R are correct, but R is not the correct explanation of S. S is correct and R is the correct explanation for S. S is correct and R is wrong.

A. Both S and R are wrong

B. Both S and R are correct , but R is not the correct explanation ofS.

C. S is correct and R is the correct explanation for S.

D. S is correct and R is wrong.

Answer: 4

Watch Video Solution

5. The length of the longest interval in which the function $3\sin x-4\sin^3 x$ is increasing is $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. (π)

Answer: 1

Watch Video Solution

6. The function x^x decreases in the interval $(0, e)$ (b) $(0, 1)$ $\left(0, \frac{1}{e}\right)$ (d) none of these

A. (0,e)

B. (0,1)

C. $\left(0, \frac{1}{e}\right)$

D. none of these

Answer: 3

Watch Video Solution

7. Let f(x) =$x\sqrt{4ax-x^2}$, $(a>0)$ Then f(x) is

A. increasing in (0,3a) decreasing in (3a,4a)

B. increasing in (a,4a),decreasing in $(5a, \infty)$

C. increasing in $(0, 4a)$

D. none of these

Answer: 1

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8. Function $f(x)=|x|-|x-1|$ is monotonically increasing when $x<0$ (b) $x>1$ $x<1$ (d) `0

A. $x<0$

B. $x>0$

C. $x<0$

D. $0<x<1$

Answer: 4

Watch Video Solution

9. If $f'(x)=|x|-\{x\}$, where {x} denotes the fractional part of x, then $f(x)$ is decreasing in $\left(-\frac{1}{2},0\right)$ (b) $\left(-\frac{1}{2},2\right)$ $\left(-\frac{1}{2},2\right)$ (d) $\left(\frac{1}{2},\infty\right)$

A. $\left(\frac{-1}{2},0\right)$

B. $\left(\frac{-1}{2},2\right)$

C. $\left(\frac{-1}{2},2\right)$

D. $\left(\frac{1}{2}, \infty\right)$

Answer: 1

Watch Video Solution

10. The length of the largest continuous interval in which the function $f(x) = 4x - \tan 2x$ is monotonic is $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{16}$

A. $\pi/2$

B. $\pi/4$

C. $\pi/8$

D. $\pi/16$

Answer: 2

Watch Video Solution

11. $f(x)=(x-2)|x-3|$ is monotonically increasing in $\left(-\infty, \frac{5}{2}\right) \cup (3, \infty)$ (b) $\left(\frac{5}{2}, \infty\right)$ $(2, \infty)$ (d) $(-\infty, 3)$

A. $(-\infty, 5/2) \cup (3, \infty)$

B. $5/2, \infty)$

C. $(2, \infty)$

D. $(-\infty, 3)$

Answer: 1

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12. $f(x)=(x-8)^4(x-9)^5, 0 \leq x \leq 10,$ monotonically decreases in $\left(\frac{76}{9, 10}\right]$ (b) $\left(\frac{8, 76}{9}\right)$ $(0, 8)$ (d) $\left(\frac{76}{9}, 10\right)$

A. $\left(\frac{76}{9}, 10\right)$

B. $\left((8), \frac{76}{9}\right)$

C. [0,8)

D. $\left(\frac{76}{9}, 10\right)$

Answer: 2

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13. For all $x \in (0, 1)$

(a)$e^{x} < 1 + x$

(b) $(\log)_e(x + 1) < x$

(c)$\sin x > x$

(d) $(\log)_e x > x$

A. $e^{x} < 1 + x$

B. $\log e^{1+x} < x$

C. $\sin x > x$

D. $\log_e x > x$

Answer: 2

Watch Video Solution

14. If $f(x) = xe^{x(x-1)}$, then $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$ decreasing on R increasing on R (d) decreasing on $\left[-\frac{1}{2}, 1\right]$

A. increasing on$[-1/2, 1]$

B. decreasing on $[-1/2, \infty]$

C. incresing on R

D. decreasing on $[-1/2, \infty]$

Answer: 1

Watch Video Solution

15. If $f(x) = kx^3 - 9x^2 + 9x + 3$ monotonically increasing in R, then $k < 3$ (b) $k \leq 2$ $k \geq 3$ (d) none of these

A. $k < 3$

B. $k \leq 2$

C. $k \geq 3$

D. none of these

Answer: 3

Watch Video Solution

16. If the function $f(x) = \frac{K \sin x + 2 \cos x}{\sin x + \cos x}$ is strictly increasing for all values of x, then $K < 1$ (b) $K > 1$ $K < 2$ (d) $K > 2$

A. $k < 1$

B. $k \leq 2$

C. $k \geq 3$

D. none of these

Answer: 4

Watch Video Solution

17. Let $f: R\overrightarrow{R}$ be a function such that $f(x) = ax + 3\sin x + 4\cos x$. Then $f(x)$ is invertible if $a \in (-5, 5)$ (b) $a \in (-\infty, 5)$ $a \in (-5, +\infty)$ (d) none of these

A. $k < 1$

B. $k > 1$

C. $k < 2$

D. $k > 2$

Answer: 4

Watch Video Solution

18. Which of the following statement is always true? (a)If $f(x)$ is increasing, then $f^{-1}(x)$ is decreasing. (b)If $f(x)$ is increasing, then $\frac{1}{f(x)}$ is also increasing. (c)If $fandg$ are positive functions and f is increasing and g is decreasing, then $\frac{f}{g}$ is a decreasing function. (d)If $fandg$ are positive functions and f is decreasing and g is increasing, the $\frac{f}{g}$ is a decreasing function.

A. If f(x) is increasing then f^{-1}(x) is also decreasing

B. If f(x) is increasing then $1/f(x)$ is also increasing

C. If f and g are positive functions and f is increasing and g is decreasing then f/g is decreasing function

D. If f and g are positive functions and f is decreasing and g is increasing then f/g is decreasing function

Answer: 4

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19. Let $f:\overrightarrow{RR}$ be a differentiable function for all values of x and has the property that $f(x) and f'(x)$ has opposite signs for all value of x. Then, $f(x)$ is an increasing function $f(x)$ is an decreasing function $f^2(x)$ is an decreasing function $|f(x)|$ is an increasing function

A. f(x) is an iincreasing function

B. f(x) is a decreasing function

C. $f^2(x)$ is a decreasing function

D. |f(x)| is an increasing function

Answer: 3

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20. Let $f: R^{\rightarrow}$ be a differentiable function $\forall x \in R$. If the tangent drawn to the curve at any point $x \in (a, b)$ always lies below the curve, then $f'(x) < 0, f^x < 0 \forall x \in (a, b)$ $f'(x) > 0, f^x > 0 \forall x \in (a, b)$ $f'(x) > 0, f^x > 0 \forall x \in (a, b)$ $noneofthese$

A. $f'(x) > 0, f''(x) < 0 \forall x \in)(a, b)$

B. $f'(x) < 0, f''(x) < 0 \forall x \in (a, b)$

C. $f'(x) > 0, f''(x) > 0 \forall x \in (a, b)$

D.

Answer: 3

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21. Let $f(x)$ be a function such that $f(x) = (\log)_{\frac{1}{2}}\left[(\log)_3(\sin x + a\right]\cdot$ If $f(x)$ is decreasing for all real values of $x,$ then $a \in (1, 4)$ (b) $a \in (4, \infty)$ $a \in (2, 3)$ (d) $a \in (2, \infty)$

A. $a \in (1, 4)$

B. $a \in (4, \infty)$

C. $a \in (2, 3)$

D. $a \in (2, \infty)$

Answer: 2

Watch Video Solution

22. If $f(x) = x^3 + 4x^2 + \lambda x + 1$ is a monotonically decreasing function of x in the largest possible interval $\left(-2, -\frac{2}{3}\right)$. Then $\lambda = 4$ (b) $\lambda = 2$ $\lambda = -1$ (d) λ has no real value

A. $\lambda = 4$

B. $\lambda = 2$

C. $\lambda = -1$

D. λ has no real value

Answer: 1

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23. $f(x) = |x \log_e x|$ monotonically decreases in $\left(0, \frac{1}{e}\right)$ (b) $\left(\frac{1}{e}, 1\right)$ $(1, \infty)$ (d) $\left(\frac{1}{e}, \infty\right)$

A. $(0, 1/e)$

B. $(1/e, 1)$

C. $(1. \infty)$

D. $(1/e, \infty)$

Answer: 2

Watch Video Solution

24. The set of value(s) of a for which the function $f(x) = \frac{ax^3}{3} + (a+2)x^2 + (a-1)x + 2$ possesses a negative point of inflection is $(-\infty, -2) \cup (0, \infty)$ (b) $\left\{-\frac{4}{5}\right\}$ $(-2, 0)$ (d) empty set

A. $(-\infty, -2) \cup (0, \infty)$

B. $\{-4/5\}$

C. $(-2, 0)$

D. empty set

Answer: 1

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25. The maximum value of the function $f(x) = \frac{(1+x)^{0.6}}{1+x^{0.6}}$ in the interval $[0, 1]$ is $2^{0.4}$ (b) $2^{-0.4}$ 1 (d) $2^{0.6}$

A. $2^{0.4}$

B. $2^{-0.4}$

C. 1

D. $2^{0.6}$

Answer: 3

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26. Suppose that f is a polynomial of degree 3 and that $f^{x} \neq 0$ at any of the stationary point. Then f has exactly one stationary point f must have no stationary point f must have exactly two stationary points f has either zero or two stationary points.

A. f has exactly one stationary point

B. f must have no stationary pint

C. f must have exactly two stationary points

D. f has either zero or two stationary points

Answer: 4

Watch Video Solution

27. A function $g(x)$ is defined as $g(x)=\frac{1}{4}f(2x^2-1)+\frac{1}{2}f(1-x^2) and f(x)$ is an increasing function. Then $g(x)$ is increasing in the interval. $(-1,1)$ $\left(-\sqrt{\frac{2}{3}},0\right)\cup\left(\sqrt{\frac{2}{3}},\infty\right)$ $\left(-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}\right)$ (d) none of these

A. $(-1,1)$

B. $-\left(\frac{\sqrt{2}}{3},0\right)\cup\left(\frac{\sqrt{2}}{3},\infty\right)$

C. $-\frac{\sqrt{2}}{3}.\frac{\sqrt{2}}{3}$

D. none of these

Answer: 2

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28. If $\varphi(x)$ is a polynomial function and $\varphi'(x) > \varphi(x)\,\forall x \geq 1 and \varphi(1) = 0,$ then $\varphi(x) \geq 0\,\forall x \geq 1$ `varphi(x)

A. $\phi(x) \geq 0\,\forall x \geq 1$

B. $\phi(x) < 0\,\forall x \geq 1$

C. $\phi(x) = 0\,\forall x \geq 1$

D. none of these

Answer: 1

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29. If

$f''(x) > \forall \in R,\ f(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4y)0 < x <$

,then g(x) is increasing in

A. $\left(0, \frac{\pi}{4}\right)$

B. $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

C. $\left(0, \frac{\pi}{3}\right)$

D. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Answer: 4

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30. If $f(x) = x + \sin x, g(x) = e^{-x}, u = \sqrt{c+1} - \sqrt{c}$ $v = \sqrt{c}$ $-\sqrt{c-1}, (c > 1),$ then `fog(u)gof(v)(d)fog(u)

A. $fog(u) < fog(v)$

B. $gof(u) < gof(v)$

C. $gof(u) > gof(v)$

D. $fog(u) < fog(v)$

Answer: 4

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31. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $(0, 2\pi]$ is

A. one

B. two

C. three

D. zero

Answer: 4

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32. Let $f(x) = \cos \pi x + 10x + 3x^2 + x^3, \ -2 \leq x \leq 3.$ The absolute minimum value of $f(x)$ is 0 (b) -15 (c) $3 - 2\pi$ none of these

A. 0

B. -15

C. $3 - 2\pi$

D. none of these

Answer: 2

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33. The global maximum value of $f(x0 = (\log)_{10}(4x^3 - 12x^2 + 11x - 3), x \in [2, 3],$ is $-\frac{3}{2}(\log)_{10}3$ (b) $1 + (\log)_{10}3$ $(\log)_{10}3$ (d) $\frac{3}{2}(\log)_{10}3$

A. $-\frac{3}{2}\log_{10}^3$

B. $1 + \log 10^3$

C. $\log 10^3$

D. $\frac{3}{2}\log 10^3$

Answer: 2

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34. $f: R\overrightarrow{R}, f(x)$ is differentiable such that $f(f(x)) = k(x^5 + x), k \neq 0)$. Then $f(x)$ is always increasing (b) decreasing either increasing or decreasing non-monotonic

A. increasing

B. decreasing on $[-1/2, \infty]$

C. either increasing or decreasing

D. non monotonic

Answer: 3

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35. The value of a for which the function $f(x) = a\sin x + \left(\frac{1}{3}\right)\sin 3x$ has an extremum at $x = \frac{\pi}{3}$ is 1 (b) -1 (c) 0 (d) 2

A. 1

B. -1

C. 0

D. 2

Answer: 5

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36. If $f(x) = a\log|x| + bx^2 + x$ has its extremum values at $x = -1 and x = 2,$ then $a = 2, b = -1$ $a = 2, b = -1/2$ $a = -2, b = 1/2$ (d) none of these

A. a=2,b=-1

B. a=2,b=$-1/2$

C. a=-2,b=$1/2$

D. none of these

Answer: 2

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37. If a function $f(x)$ has $f'(a)=0 and f^a=0,$ then $x=a$ is a maximum for $f(x)$ $x=a$ is a minimum for $f(x)$ it is difficult to say $(a)and(b)$ $f(x)$ is necessarily a constant function.

A. x=a is a maximum for f(x)

B. x=a is a minimum for(x)

C. it is difficult to say (a) and (b)

D. f(x) is necessarily a constant function

Answer: 3

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38. The function $f(x) = \sin^4 x + \cos^4 x$ increases, if

A. It is monotonic increasing $\forall$ x in R.

B. f(x) fails to exist for three disticnt real values of x

C. f(x) changes its sign twice as x varifes from $-\infty \rightarrow \infty$

D. The function attains its extreme values at x_1 and x_2 such that $x_1 x_2 > 0$

Answer: 3

Watch Video Solution

39. The function $f(x) = \sin^4 x + \cos^4 x$ increases, if

A. $0 < x < \pi / 8$

B. $\pi / 40 < x < 3\pi / 8$

C. $3\pi / 8 < x < 5\pi / 8$

D. $5\pi/8 < x < 3\pi/4$

Answer: 2

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40. If $f(x) = x^5 - 5x^4 + 5x^3 - 10$ has local maximum and minimum at $x = p and x = q$, respectively, then $(p, q) \equiv$ $(0, 1)$ (b) (1,3) (c) (1,0) (d) none of these

A. (0,1)

B. (1,3)

C. (1,0)

D. none of these

Answer: 2

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41. Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \; + a_nx^{2n}$ be a polynomial in a real variable x with `0

A. neither a maximum nor a minimum

B. only one maximum

C. only one minimum

D. only one maximum and only one minimum

Answer: 3

Watch Video Solution

42. $Let f(x) = \{|x|, f \text{ or } 0 < |x| \leq 21, f \text{ or } x = 0$ Then at $x = 0, f$ has a local maximum (b) no local maximum a local minimum (d) no extremum

A. a local maximum

B. no local maximum

C. a local minimum

D. no extremum

Answer: 1

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43. If $f(x) = x^3 + bx^2 + cx + d$ and $0 \leq b^2 \leq c$, then a) $f(x)$ is a strictly increasing function b)f(x) has local maxima c) $f(x)$ is a strictly decreasing function d) $f(x)$ is bounded

A. f(x) is strictly increasing function

B. f(x) has local maxima

C. f(x) is a strictly decreasing function

D. f(x) is bounded

Answer: 1

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44. If $f(x) = \{(\sin(x^2 - 3x), x \leq 0$

$.^{x} + 5x^2, x > 0$

A. f(x) has a local minima

B. f(x) has a local maxima

C. f(x) has point of inflection

D. none of these

Answer: 1

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45. The greatest value of $f(x) = \cos\left(xe^{[x]} + 7x^2 - 3x\right), x \in [-1, \infty],$ is (where [.] represents the greatest integer function). -1 (b) 1 (c) 0 (d) none of these

A. -1

B. 1

C. 0

D. none of these

Answer: 2

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46. The function $f(x) = \left(4\sin^2 x - 1\right)^n \left(x^2 - x + 1\right), n \in N,$ has a local minimum at $x = \dfrac{\pi}{6}$. Then n is any even number n is an odd number n is odd prime number n is any natural number

A. n is any even integer

B. n is an odd integer

C. n is odd prime number

D. n is any natural number

Answer: 1

Watch Video Solution

47. The true set of real values of x for which the function $f(x) = x \ln x - x + 1$ is positive is

A. $(1, \infty)$

B. $(1/e, \infty)$

C. $[e, \infty)$

D. $(0, 1) \cup (1, \infty)$

Answer: 4

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48. All possible value of $f(x) = (x+1)^{\frac{1}{3}} - (x-1)^{\frac{1}{3}}$ on [0,1] is 1 (b) 2 (c) 3 (d) $\frac{1}{3}$

A. 1

B. 2

C. 3

D. $\frac{1}{3}$

Answer: 2

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49. The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is increasing in $(0, \infty)$ decreasing in $(0, \infty)$ increasing in $\left(0, \frac{\pi}{e}\right)$, decreasing in $\left(\frac{\pi}{e}, \infty\right)$ decreasing in $\left(0, \frac{\pi}{e}\right)$, increasing in $\left(\frac{\pi}{e}, \infty\right)$

A. increasing in $(0, \infty)$

B. decreasing oin $(0, \infty)$

C. increasing in $(0, \pi / e), decrea\sin g \in (\pi / e, \infty)$

D. decreasing in $(0, \pi / e)$ increasing in $(\pi / e, \infty)$

Answer: 2

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50. If the function $f(x) = 2x^3 - 9ax^2 + 12x^2x + 1, wherea > 0,$ attains its maximum and minimum at $pandq$, respectively, such that $p^2 = q,$ then a equal to 1 (b) 2 (c) $\frac{1}{2}$ (d) 3

A. 1

B. 2

C. $\frac{1}{2}$

D. 3

Answer: 2

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51. Let $f(x) = \{x + 2, \ -1 \leq \ < 0$

$1, x = 0\frac{x}{2}, 0 < x \leq 1$

A. a point of minima

B. a point of maxima

C. both points of minima and maxima

D. neither a pointof minimanor that of maxima

Answer: 4

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52. If $f(x) = \{\sin^{-1}(\sin x), x > 0$

$\frac{\pi}{2}, x = 0, then \cos^{-1}(\cos x), x < 0$

A. x=0is apoint of maxima

B. x=0 is a point of minima

C. x=0 is a point of intersectionn

D. none of these

Answer: 1

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53. A function f is defined by $f(x) = |x|^m |x-1|^n \forall x \in R$. The local maximum value of the function is $(m, n \in N)$, 1 (b) $m^{\cap} \hat{} m$ $\frac{m^m n^n}{(m+n)^{m+n}}$ (d) $\frac{(mn)^{mn}}{(m+n)^{m+n}}$

A. 1

B. $m^n n^m$

C. $\frac{m^m n^n}{(m+n)^{m+n}}$

D. $\frac{mn^{mn}}{m+n^{m+n}}$

Answer: 3

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54. Let the function f(x) be defined as follows:

$f(x) = \{x^3 + x^2 - 10x, \; -1 \le x \le 0$

$\cos x, 0 \leq x \leq \pi/2$

$1 + \sin x, \pi/2 \leq x \leq \pi$

A. a local minimum at x=$\pi/2$

B. a global maximum at x=$\pi/2$

C. a absolute maximum at x=-1

D. a absolute maximum at x=π

Answer: 3

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55. Consider the function $f: (-\infty, \infty) \overrightarrow{-\infty, \infty}$ defined by $f(x) = \dfrac{x^2 + a}{x^2 + a}, a > 0,$ which of the following is not true? maximum value of f is not attained even though f is bounded. $f(x)$ is increasing on $(0, \infty)$ and has minimum at , $= 0$ $f(x)$ is decreasing on $(-\infty, 0)$ and has minimum at $x = 0$. $f(x)$ is increasing on $(-\infty, \infty)$ and has neither a local maximum nor a local minimum at $x = 0$.

A. Maximum value of f is not attained even though f is bounded

B. f(x) is increasing on $(0, \infty)$ and has minimum at x=0

C. f(x) is decreasing on $(-\infty, 0)$ and has minimum at x=0

D. f(x) is increasing on $(-\infty, \infty)$ and has neither a local maximum nor a local minimum at x=0

Answer: 4

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56. $F(x) = 4\tan x - \tan^2 x + \tan^3 x, x \neq n\pi + \frac{\pi}{2}$

A. is monotonically increasing

B. is monotonically decreasing

C. has a point of maxima

D. has a point of minima

Answer: 1

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57. Let $h(x) = x^{\frac{m}{n}}$ for $x \in R,$ where +m and n are odd numbers and 0 less than m less than n Then $y = h(x)$ has

a)no local extremums

b)one local maximum

c)one local minimum

d)none of these

A. no local extremums

B. one local maximum

C. one local minimum

D. none of these

Answer: 1

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58. The greatest value of the function $f(x) = \frac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)}$ on the interval $\left(0, \frac{\pi}{2}\right) is \frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) $-\sqrt{2}$

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. 1

D. $-\sqrt{2}$

Answer: 3

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59. The minimum value of $e^{2x^2 - 2x + 1} \sin^2 x$ is e (b) $\frac{1}{e}$ (c) 1 (d) 0

A. e

B. $1/e$

C. 1

D. 0

Answer: 3

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60. The maximum value of $x^4 e \widehat{-} x^2$ is e^2 (b) e^{-2} (c) $12e^{-2}$ (d) $4e^{-2}$

A. e^2

B. e^{-2}

C. $12e^{-2}$

D. $4e^{-2}$

Answer: 4

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61. If $a^2x^4 + b^2y^4 = c^6$, then the maximum value of xy is (a)$\dfrac{c^2}{\sqrt{ab}}$ (b) $\dfrac{c^3}{ab}$

(c) $\dfrac{c^3}{\sqrt{2ab}}$ (d) $\dfrac{c^3}{2ab}$

A. $\dfrac{c^2}{\sqrt{ab}}$

B. $\dfrac{c^3}{\sqrt{ab}}$

C. $\dfrac{c^3}{\sqrt{2ab}}$

D. $\dfrac{c^3}{\sqrt{2ab}}$

Answer: 3

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62. The least natural number a for which $x + ax^{-2} > 2 \forall x \in (0, \infty)$ is 1

(b) 2 (c) 5 (d) none of these

A. 1

B. 2

C. 5

D. none of these

Answer: 2

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63. $f(x) = \{4x - x^3 + \ln(a^2 - 3a + 3), 0 \leq x < 3$

$x - 18, x \geq 3$ complete set of values of a such that $f(x)$ as a local minima at $x = 3$ is $[-1, 2]$ $(-\infty, 1) \cup (2, \infty)$ $[1, 2]$ (d) $(-\infty, -1) \cup (2, \infty)$

A. $[-1, 2]$

B. $(-\infty, 1) \cup (2, \infty)$

C. $[1, 2]$

D. $(-\infty, -1) \cup (2, \infty)$

Answer: 2

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64.

$f(x) = \{2 - |x^2 + 5x + 6|, x \neq 2a^2 + 1, x = -2 Thentheran \geq ofa,$

so that $f(x)$ has maxima at $x = -2,$ is $|a| \geq 1$ (b) $|a| < 1$ $a > 1$ (d)

$a < 1$

A. $|a| \geq 1$

B. $|a| \leq 1$

C. $a > 1$

D. $a < 1$

Answer: 4

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65. A differentiable function $f(x)$ has a relative minimum at $x = 0$. Then the function $f = f(x) + ax + b$ has a relative minimum at $x = 0$ for all

a and all$b$ (b) all b if $a = 0$ all $b > 0$ (d) all $a > 0$

A. all a and all b

B. all b if a =0

C. all b $>$ 0

D. all a $>$ 0

Answer: 2

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66. if $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \{ \min \{f(t) : 0 \leq t \leq x; 0 \leq x \leq 1, 3 - x : 1\}$ then g(1/4)+g(3/4)+g(5/4) is equal to

A. $7/4$

B. $9/4$

C. $13/4$

D. $5/2$

Answer: 4

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67. If $f: R\vec{R} and g: R\vec{R}$ are two functions such that $f(x)+f^{x}=-xg(x)f'(x)andg(x)>0 \forall x \in R$. Then the function $f^2(x)+f('(x))^2$ has a maxima at $x=0$ a minima at $x=0$ a point of inflexion at $x=0$ none of these

A. a maxima at x =0

B. a minima at x =0

C. a point of inflexion at x =0

D. a point of inflexion at x =0

Answer: 1

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68. If A > 0, B > 0, and A + B = $\frac{\pi}{3}$ then the maximum value of tan A tan B is

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{3}$

C. 3

D. $\sqrt{3}$

Answer: 2

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69. If $f(x) = \frac{t + 3x - x^2}{x - 4}$, where t is a parameter that has minimum and maximum, then the range of values of t is $(0, 4)$ (b) $(0, \infty)$ $(-\infty, 4)$ (d) $(4, \infty)$

A. $(0, 4)$

B. $(0, \infty)$

C. $-(\infty, 4)$

D. $(4, \infty)$

Answer: 3

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70. The least value of a for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$ has at least one solution in the interval $\left(0, \frac{\pi}{2}\right)$ 9 (b) 4 (c) 8 (d) 1

A. 9

B. 4

C. 8

D. 1

Answer: 3

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71. If f(x)=$-x^3-3x^2-2x+a,\ a\in R$ then the real values of x satisfying $f(x^2+1)>f(2x^2+2x+3)$ will be

A. $(-\infty,\infty)$

B. $(0,\infty)$

C. $-(\infty,0)$

D. ϕ

Answer: 1

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72. Which one of the following denotes the greatest positive proper fraction?

A. $\left(\frac{1}{4}\right)^{(\log)_2 6}$

B. $l\left(\frac{1}{3}\right)^{(\log)_3 5}$

C. $3^{(\log)_3 2}$

D. $8^{\left(\frac{1}{(\log)_3 2}\right)}$

Answer: 4

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73. If the equation $4x^3 + 5x + k = 0 (k \in R)$ has a negative real root then

A. k=0

B. $-\infty < k < 0$

C. $0 < k < \infty$

D. $-\infty < k < \infty$

Answer: 3

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74. A tangent is drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta\left(0, \frac{\pi}{2}\right)\right.$.

Then find the value of θ such that the sum of intercepts on the axes made by this tangent is minimum.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{4}$

Answer: 2

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75. The largest term in the sequence $a_n = \frac{n^2}{n^3 + 200}$ is given by $\frac{529}{49}$ (b) $\frac{8}{89}$ $\frac{49}{543}$ (d) none of these

A. $\frac{529}{49}$

B. $\frac{8}{89}$

C. $\frac{49}{543}$

D. none of these

Answer: 3

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76. A factory D is to be connected by a road with a straight railway line on which a town A is situated. The distance DB of the factory to the railway line is $5\sqrt{3}km$. Length AB of the railway line is 20km. Freight charges on the road are twice the charges on the railway. The point `P(A P

A. BP=5 km

B. AP=5 km

C. BP=7.5 km

D. none of these

Answer: 1

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77. The volume of the greatest cylinder which can be inscribed in a cone of height 30 cm and semi-vertical angle 30^0 is $4000\frac{\pi}{\sqrt{3}}$ (b) $400\frac{\pi}{3}cm^3$ $4000\frac{\pi}{\sqrt{3}cm^3}$ (d) none of these

A. $4000\pi / 3cm^3$

B. $400\pi / 3cm^3$

C. $4000\pi / \sqrt{3}cm^3$

D. $4000\pi / 3cm^3$ none of these

Answer: 1

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78. A rectangle of the greatest area is inscribed in a trapezium $ABCD$, one of whose non-parallel sides AB is perpendicular to the base, so that one of the rectangles die lies on the larger base of the trapezium. The base of trapezium are 6cm and 10 cm and AB is 8 cm long. Then the maximum area of the rectangle is $24cm^2$ (b) $48cm^2$ $36cm^2$ (d) none of these

A. $24cm^2$

B. $48cm^2$

C. $36cm^2$

D. none of these

Answer: 2

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79. A bell tent consists of a conical portion above a cylindrical portion near the ground. For a given volume and a circular base of a given radius, the amount of the canvas used is a minimum when the semi-vertical angle of the cone is $\frac{\cos^{-1}2}{3}$ (b) $\frac{\sin^{-1}2}{3}$ $\frac{\cos^{-1}1}{3}$ (d) none of these

A. $\cos^{-1}2/3$

B. $\sin^{-1} 2/3$

C. $\cos^{-1} 1/3$

D. none of these

Answer: 1

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80. A rectangle is inscribed in an equilateral triangle of side length $2a$ units. The maximum area of this rectangle can be $\sqrt{3}a^2$ (b) $\frac{\sqrt{3}a^2}{4}$ a^2 (d) $\frac{\sqrt{3}a^2}{2}$

A. $\sqrt{3a^2}$

B. $\frac{\sqrt{3a^2}}{4}$

C. a^2

D. $\frac{\sqrt{3a^2}}{2}$

Answer: 4

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81. Tangents are drawn to $x^2 + y^2 = 16$ from the point $P(0, h)$. These tangents meet the $x - a\xi s$ at $AandB$. If the area of triangle PAB is minimum, then $h = 12\sqrt{2}$ (b) $h = 6\sqrt{2}$ $h = 8\sqrt{2}$ (d) $h = 4\sqrt{2}$

A. h=$12\sqrt{2}$

B. h=$6\sqrt{2}$

C. h=$8\sqrt{2}$

D. h=$4\sqrt{2}$

Answer: 4

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82. The largest area of the trapezium inscribed in a semi-circle or radius R, if the lower base is on the diameter, is $\frac{3\sqrt{3}}{4}R^2$ (b) $\frac{\sqrt{3}}{2}R^2$ $\frac{3\sqrt{3}}{8}R^2$ (d) R^2

A. $\left(\frac{3\sqrt{3}}{4}R^2\right)$

B. $\left(\frac{\sqrt{3}}{2}R^2\right)$

C. $\left(\frac{3\sqrt{3}}{8}R^2\right)$

D. R^2

Answer: 1

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83. In the formula $\angle A + \angle B + \angle C = 180^\circ$, if $\angle A = 90^\circ$ and $\angle B = 55^\circ$, then $\angle C =$ ______

A. 45°

B. 25°

C. 35°

D. none of these

Answer: 3

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84. Two runner A and B start at the origin and run along positive x axis ,with B running three times as fast as A. An obsever, standeing one unit above the origin , keeps A and B in view.Then the maximum angle theta of sight between the observer's view of A and B is

A. $\pi/8$

B. $\pi/6$

C. $\pi/3$

D. $\pi/4$

Answer: 2

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85. The fuel charges for running a train are proportional to the square of the speed generated in km/h, and the cost is Rs. 48 at 16 km/h. If the fixed charges amount to Rs. 300/h, the most economical speed is 60 km/h (b) 40 km/h 48 km/h (d) 36 km/h

A. 60 km / h

B. 40 km / h

C. 48 km / h

D. 36 km / h

Answer: 2

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86. A cylindrical gas container is closed at the top and open at the bottom. If the iron plate of the top is 5/4 times as thick as the plate forming the cylindrical sides, the ratio of the radius to the height of the cylinder using minimum material for the same capacity is 3:4 (b) 5:6 (c) 4:5 (d) none of these

A. $3 : 4$

B. $5:6$

C. $4:5$

D. none of these

Answer: 3

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87. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

A. $4\sqrt{3r}$

B. $2\sqrt{3r}$

C. $6\sqrt{3r}$

D. $8\sqrt{3r}$

Answer: 3

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88. A given right cone has volume p, and the largest right circular cylinder that can be inscribed in the cone has volume q. Then $p:q$ is 9:4 (b) 8:3 (c) 7:2 (d) none of these

A. $9:4$

B. $8:3$

C. $7:2$

D. none of these

Answer: 1

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89. Find the cosine of the angle at the vertex of an isoceles triangle having the greatest area for the given constant length e of the median drawn to its lateral side.

A. 0.4

B. 0.5

C. 0.6

D. 0.8

Answer: 4

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90. A box, constructed from a rectangular metal sheet, is 21 cm by 16cm by cutting equal squares of sides x from the corners of the sheet and then turning up the projected portions. The value of x os that volume of the box is maximum is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C.

D. 3

Answer: 3

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91. The vertices of a triangle are (0,0), $(x, \cos x)$, and `(sin^3x ,0),w h e r

e0

A. $3\frac{\sqrt{3}}{32}$

B. $\frac{\sqrt{3}}{32}$

C. $\frac{4}{32}$

D. $6\frac{\sqrt{3}}{32}$

Answer: 1

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92. The maximum area of the rectangle whose sides pass through the vertices of a given rectangle of sides $aandb$ is $2(ab)$ (b) $\frac{1}{2}(a+b)^2$

$\frac{1}{2}(a^2 + b^2)$ (d) $noneofthese$

A. 2(ab)

B. $\frac{1}{2}(a+b)^2$

C. $\frac{1}{2}(a+b)^2$

D. none of these

Answer: 2

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93. The base of prism is equilateral triangle. The distance from the centre of one base to one of the vertices of the other base is l. Then altitude of the prism for which the volume is greatest is $\frac{l}{2}$ (b) $\frac{l}{\sqrt{3}}$ (c) $\frac{l}{3}$ (d) $\frac{l}{4}$

A. $\frac{l}{2}$

B. $\frac{l}{\sqrt{3}}$

C. $\frac{l}{3}$

D. $\frac{l}{4}$

Answer: 2

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Exercise (Multiple)

1. Let $f(x) = \begin{cases} x^2 + 3x, & -1 \le x < 0 \\ -\sin x, & 0 \le x < \pi/2 \\ -1 - \cos x, & \frac{\pi}{2} \le x \le \pi \end{cases}$. Draw the graph of the function and find the following

(a) Range of the function

(b) Point of inflection

(c) Point of local minima

A. f(x) has global minimum value -2

B. global maximum value occurs at x=0

C. global maximum value occurs at x=π

D. $x = \pi / 2$ is point of local minima

Answer: 1,2,3,4

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2. Leg $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$. Then, f increase on $[1, \infty]$ f decreases on $[1, \infty]$ f has a minimum at $x = 1$ f has neither maximum nor minimum

A. f increases on $[1, \infty]$

B. f decreases on $[1, \infty]$

C. f has a minimum at x=1

D. f has neither maximum nor minimum

Answer: 1,3

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3. Let $f(x) = 2x - \sin x$ and $g(x) = 3\sqrt{x}$. Then

A. range of gof is R

B. gof is one-one

C. both f and g are one-one

D. both f and g are onto

Answer: 1,2,3,4

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4. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1 + x^2} - x\right)$ is increasing in R

A. increases in $[0, \infty)$

B. idecreases in $[0, \infty)$

C. neither increases nor decreases in $[0, \infty)$

D. increases in $(-\infty, \infty)$

Answer: 1,4

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5. Let $f(x) = |x^2 - 3x - 4|, \ -1 \le x \le 4.$ Then

A. f(x) is monotonically increasing in $[-1, 3/2]$

B. f(x) is monotonically decreasing in $(3/2, 4]$

C. the maximum value of f(x) is $\frac{25}{4}$

D. the minimum value of f(x) is 0

Answer: 1,2,3,4

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6. If $f(x) = \int_0^x \frac{\sin t}{t} dt, x > 0,$ then (a)$f(x)$ has a local maxima at $x = n\pi(n = 2k, k \in I^+)$ (b)$f(x)$ has a local minima at $x = n\pi(n = 2k, k \in I^+)$ (c)$f(x)$ has neither maxima nor minima at

$x = n\pi(n \in I^+)$ (d)$(f)x$ has local maxima at $x = n\pi(n = 2k + 1, k \in I^+)$

A. f(x) has a local maxima at x =$n\pi(n = 2k, k \in I^+)$

B. f(x) has a local minimum at x =$n\pi(n = 2k, k \in I^+)$

C. f(x) has neither maxima nor minima at x =$n\pi(n \in I^+)$

D. f(x) has local maxima at x =$n\pi(n = 2k - 1, k \in I^+)$

Answer: 2,4

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7. The values of parameter a for which the point of minimum of the function $f(x) = 1 + a^2x - x^3$ satisfies the inequality $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0 are$ $(2\sqrt{3}, 3\sqrt{3})$ (b) $-3\sqrt{3}, -2\sqrt{3})$ $(-2\sqrt{3}, 3\sqrt{3})$ (d) $(-2\sqrt{2}, 2\sqrt{3})$

A. $2\sqrt{3}, 3\sqrt{3}$

B. $-3\sqrt{3}, -2\sqrt{3}$

C. $-2\sqrt{3}, 3\sqrt{3}$

D. $-3\sqrt{2}, 2\sqrt{3}$

Answer: 1,2

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8. Let $f(x) = ax^2 - b|x|$, where $a and b$ are constants. Then at $x = 0$, $f(x)$ has a maxima whenever $a > 0, b > 0$ a maxima whenever $a > 0, b < 0$ minima whenever $a > 0, b < 0$ neither a maxima nor a minima whenever $a > 0, b < 0$

A. a maxima whenever $a > 0, b > 0$

B. a maxima whenever $a > 0, b < 0$

C. minima whenever $a > 0, b < 0$

D. neither a maxima nor a minima whenver $a > 0, b < 0$

Answer: 1,3

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9. The function $y = \frac{2x-1}{x-2}, (x \neq 2)$

A. is its own inverse

B. decreases at all values of x in the domain

C. has a graph intirely above the x axis

D. is unbounded

Answer: 1,2,4

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10. Let $f(x) = ax^2 + bx + c$ and $f(-1) < 1, f(1) > -1, f(3) < -4$ and $a \neq 0$, then

A. $a > 0$

B. $a < 0$

C. sign of a cannot be defined

D. none of these

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11. If $f(x) = x^3 - x^2 + 100x + 2002, then$ $f(1000) > f(1001)$ $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$ $f(x-1) > f(x-2)$ $f(2x-3) > f(2x)$

A. $f(1000) < f(1001)$

B. $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

C. $f(x-1) > f(x-2)$

D. $f(2x-3) > f(2x)$

Answer: 2,3

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12. If $f'(x) = g(x)(x-a)^2$, where g(a) $\neq 0$ and g is continuous at x = a, then :

A. f is increasing in the neighborhood of a if $g(x) > 0$

B. f is increasing in the neighborhood of a if $g(x) < 0$

C. f is decreasing in the neighborhood of a if $g(x) > 0$

D. f is decreasing in the neighborhood of a if $g(x) < 0$

Answer: 1,4

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13. The value of a for which the function $f(x) = (4a-3)(x+\log 5) + 2(a-7)\frac{\cot x}{2}\frac{\sin^2 x}{2}$ does not possess critical points is $\left(-\infty, -\frac{4}{3}\right)$ (b) $(-\infty, -1)$ $[1, \infty)$ (d) $(2, \infty)$

A. $(-, \infty, -4/3)$

B. $(-\infty, -1)$

C. $[1, \infty)$

D. $(2, \infty)$

Answer: 1,4

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14. Let $f(x) = (x^2 - 1)^{n+1} + (x^2 + x + 1)$. Then f(x) has local extremum at $x = 1$, when n is (A) $n = 2$ (C) $n = 4$ (B) $n = 3$ (D)$n = 5$

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15. Let $f(x) = \sin x + ax + b$.Then which of the following is/are true?

A. f(x) =0 has only one real root which is positive if $> 1, b < 0$

B. f(x) =0 has only one real root which is negative if $a > 1, b > 0$

C. f(x) =0 has only one real root which is negative if $a < -1, b < 0$

D. None of these

Answer: 1,2,3

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16. The function $\frac{\sin(x+a)}{\sin(x+b)}$ has no maxima or minima if $b-a=n\pi, n \in I$ $b-a=(2n+1)\pi, n \in I$ $b-a=2n\pi, n \in I$ (d) none of these

A. $b-a=n\pi, n \in I$

B. $b-a=(2n+1)\pi, n \in I$

C. $b-a=2n\pi, n \in I$

D. none of these

Answer: 1,2,3

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17. Consider $f(x) = ax^4 + cx^2 + dx + e$ has no point o inflection Then which of the following is/are possible?

A. $a > 0, c < 0$

B. $a < 0, c > 0$

C. $a, c < 0$

D. $a, c > 0$

Answer: 3,4

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18. $Let f(x) = \begin{cases} \frac{(x-1)(6x-1)}{2x-1}, & \text{if } x \neq \frac{1}{2} 0, & \text{if } x = \frac{1}{7} \end{cases}$ Then at $x = \frac{1}{2}$, which of the following is/are not true? f has a local maxima f has a local minima f has an inflection point. f has a removable discontinuity.

A. f has a local maxima

B. f has a local minima

C. f has an inflection point

D. f has a removable dicontinuity

Answer: 1,2,4

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19. If $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $f: A \to B$ is an onto function defined by $f(x) = 2x^2 + x - 2$ and find B.

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20. Let $f(x)$ be an increasing function defined on $(0, \infty)$. If $f(2a^2 + a + 1) > f(3a^2 - 4a + 1)$, then the possible integers in the range of a is/are 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: 2,3,4

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21. If $f(x) = \left(\sin^2 x - 1\right)^{\mathrm{n}}$, then $x = \frac{\pi}{2}$ is a point of local maximum, if n is odd local minimum, if n is odd local maximum, if n is even local minimum, if n is even

A. local maximum , if n is odd

B. local minimum, if n is odd

C. local maximum if n is even

D. local minimum if n is even

Answer: 1,4

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22. For the cubic function $f(x) = 2x^3 + 9x^2 + 12x + 1,$ which one of the following statement/statements hold good? $f(x)$ is non-monotonic. $f(x)$ increases in $(-\infty, -2) \cup (-1, \infty)$ and decreases in $(-2, -1)$ $f: R\overrightarrow{R}$ is bijective. Inflection point occurs at $x = -\frac{3}{2}$.

A. f(x) is non monotonic

B. f(x) increses in $(-\infty, -2) \cup (-1, \infty)$ and decreases in (-2,-1)

C. f:R $\rightarrow$ R is objective

D. O inflection point occurs at x =-$3/2$

Answer: 1,2,4

View Text Solution

23. Let $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x,$ where $a_i's$ are real and $f(x) = 0$ has a positive root α_0. Then $f'(x) = 0$ has a positive root

α_1 such that `0

A. f(x)=0 has a root α_1 such that $0 < \alpha_1 < \alpha_0$

B. f(x) =0 has at least two real roots

C. f(x) =00 has at least one real root

D. none of these

Answer: 1,2,3

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24. If $f(x) and g(x)$ are two positive and increasing functions, then which of the following is not always true? $[f(x)]^{g(x)}$ is always increasing $[f(x)]^{g(x)}$ is decreasing, when $f(x) < 1$ $[f(x)]^{g(x)}$ is increasing, then $f(x) > 1$. If $f(x) > 1, then [f(x)]^{g(x)}$ is increasing.

A. $[f(x)]^{g(x)}$ is always increasing

B. $[f(x)]^{g(x)}$ is decreasing when f(x) $<$ 1

C. $If [f(x)]^{g(x)}$ is increasing then f(x) $>$ 1

D. If f(x) $>$ 1, then $[f(x)]^{g(x)}$ is increasing

Answer: 1,2,3

View Text Solution

25. An extremum of the function $f(x) = \frac{2-x}{\pi}\cos\pi(x+3) + \frac{1}{\pi^2}\sin\pi(x+3), 0x < 4,$ occurs at $x = 1$ (b) $x = 2$ $x = 3$ (d) $x = \pi$

A. x=1

B. x=2

C. x=3

D. x=π

Answer: 1,3

View Text Solution

26. For the function $f(x) = x^4\left(12(\log)_e x - 7\right)$

A. the point (1,-7) is the point of minima

B. x=$e^{1/3}$ is the point of minima

C. the graph is concave downwards in (0,1)

D. the graph is concave upwards in $(1, \infty)$

Answer: 1,2,3,4

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27. Let $f(x) = \log\left(2x - x^2\right) + \frac{\sin(\pi x)}{2}$. Then which of the following is/are true? Graph of f is symmetrical about the line $x = 1$ Maximum value of $f is 1$. Absolute minimum value of f does not exist. none of these

A. Graph of f is symmeterical about the line x=1

B. maximum value of f is 1

C. absolute mimumum value of f does not exist

D. none of these

Answer: 1,2,3

View Text Solution

28. Which of the following hold(s) good for the function $f(x) = 2x - 3x^{\frac{2}{3}}$? $f(x)$ has two points of extremum. $f(x)$ is convave upward $\forall x \in R$. $f(x)$ is non-differentiable function. $f(x)$ is continuous function.

A. f(x) has two points of extremum

B. f(x) is convace upward $\forall x \in R$

C. f(x) is non differentiable function

D. f(x) is continuous function

Answer: 1,2,3,4

View Text Solution

29. For the function $f(x) = \dfrac{e^x}{1+e^x}$, which of the following hold good? f is monotonic in its entire domain. Maximum of f is not attained even though f is bounded f has a point of inflection. f has one asymptote.

A. f is monotonic in its entire domain

B. maximum of f is not attained even thought

C. f is bounded

D. f ahs a point of inflection

Answer: 1,2,3

View Text Solution

30. Which of the following is true about point of extremum $x = a$ of function $y = f(x)$?

A. At x =a, function y = f(x) may be discontinous

B. At x =a function y = f(x) may be continous but non differentiable

C. At x =a function y= f(x) may have point of inflection

D. none of these

Answer: 1,2,3

View Text Solution

31. which of the folloiwng function has point of extremum at x =0?

A. f(x) =$e^{-|x|}$

B. f(x) = sin |x|

C. f(x)=$\begin{cases} x^2+4x+3 & x<0 \\ -x & x \geq 0 \end{cases}$

D.

Answer: 1,2,3

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32. Which of the following function/functions has/have point of inflection? $f(x) = x^{\frac{6}{7}}$ (b) $f(x) = x^6$ $f(x) = \cos x + 2x$ (d) $f(x) = x|x|$

A. f(x)=$x^{6/7}$

B. $f(x) = x^6$

C. f(x)=cosx+2x

D. f(X)=x|x|

Answer: 3,4

View Text Solution

33. The funciton f(x)=$x^2 + \dfrac{\lambda}{x}$has a

A. minimum at x =2 if λ =16

B. maximum at x =2 if λ =16

C. maximum for no real value of λ

D. point of inflectin at x=1 if λ=-1

Answer: 1,3,4

View Text Solution

34. The function $f(x) = x^{\frac{1}{3}}(x-1)$ has two inflection points has one point of extremum is non-differentiable has range $\left[-3x2^{-\frac{8}{3}}, \infty\right)$

A. has two inflection points

B. has one point of extremum

C. is non differentiable

D. has range $\left[-3 \times 2^{-8/3}, \infty\right)$

Answer: 1,2,3,4

View Text Solution

35. Let f be function $f(x) = \cos x - \left(1 - \frac{X^2}{2}\right)$.Then

A. f(x) is an increasing function in $(0, \infty)$

B. f(X) is a decresing function in $(-\infty, \infty)$

C. f(X) is an increasing function in $(-\infty, \infty)$

D. f(x) is a decreasing function in $(-\infty, 0)$

Answer: 1,4

View Text Solution

36. Consider the function f(x)= x cos x - sin x , then identify the statement which is correct

A. f is odd

B. f is monotonic decreasing at x=0

C. f has point of inflection at x=0

D. f has a maxima at x= π

Answer: 1,2,3

View Text Solution

37. If $f(x) = \dfrac{x^2}{2 - 2\cos x}$; $g(x) = \dfrac{x^2}{6x - 6\sin x}$ where $0 < x < 1$, then

(A) both 'f' and 'g' are increasing functions

A. f is increasing function

B. g is increasing function

C. f is decreasing function

D. g is decreasing function

Answer: 1,4

Watch Video Solution

38. Find the greatest value of $f(x)\dfrac{1}{2ax - x^2 - 5a^2} \in [-3, 5]$ depending upon the parameter a.

A. f(5) if a =1

B. f(-3) if a =1

C. f(5) if a $<$ 1

D. f(-3) if a $>$ 1

Answer: 1,2,3,4

View Text Solution

39. For any acute angled $\triangle ABC, \frac{\sin A}{A}+\frac{\sin B}{B}+\frac{\sin C}{C}$ can

A. 1

B. 2

C. 3

D. 4

Answer: 1,2

Watch Video Solution

40. Let f(x) be a non negative continuous and bounded function for all $x \geq 0$ .If $(\cos x) f(x) < (\sin x - \cos x) f(x) \, \forall x \geq 0$, then which of the following is/are correct?

A. $f(6) + f(5) > 0$

B. $x^2 - 3x + 2 + f(7) = 0$ has 2 distinct solution

C. f(5)f(7)-f(5)=0

D. $\lim_{x \to 6} \dfrac{f(x) - \sin(\pi x)}{x - 6} = 1$

Answer: 2,3

View Text Solution

Exercise (Comprehension)

1. $f(x) = \sin^{-1} x + x^2 - 3x + \dfrac{x^3}{3}, x \in [0, 1]$

A. f(x) has a point of maxima

B. f(x) has a point of minima

C. f(x) is increasing

D. f(X) is decreasing

Answer: 2

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2. which of the following is true for $x \in [0, 1]$?

A. $\sin^{-1} x + x^2 - x\frac{9 - x^2}{3} \leq 0$

B. $\sin^{-1} x + x^2 - x\frac{9 - x^2}{3} \geq 0$

C. $\sin^{-1} x + x^2 - x\frac{9 - x^2}{3} \leq 0$

D. $\sin^{-1} x + x^2 - x\frac{9 - x^2}{3} \geq 0$

Answer: 1

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3. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$ and $g(x) = f(\sin x) + f(\cos x)$, then

A. g' is increasing

B. g' is decreasing

C. g' has a point of minima

D. g' has a point of maxima

Answer: 1

Watch Video Solution

4. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$ and g(x) =f(sinx)+f(cosx)

which of the following is true?

A. g(x) is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

B. g(x) increasing in $\left(0, \frac{\pi}{4}\right)$

C. g(x) is nonotonically increasing in $\left(0, \frac{\pi}{2}\right)$

D. none of these

Answer: 4

View Text Solution

5. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$ and g(x) =f(sinx)+f(cosx)

consider function f(x)= $\begin{cases} -x^2 + 4x + a & x \leq 3 \\ ax + b & 3 < x < 4 \\ -\frac{b}{4}x + 6 & x \geq 4 \end{cases}$

which of the following is true?

A. f(x) is discounting function for all ordered pairs (a,b)

B. f(x) is contionuous for finite number of ordered pairs (,ab)

C. f(x) can be differentiable

D. f(x) is continous for infinite ordered pairs (a,b)

Answer: 4

View Text Solution

6. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$ and g(x) =f(sinx)+f(cosx)

If x = 3 is the only point of minima in its neighborhood and x=4 is neither a point of maxima nor a point minima, then which of the following can be true?

A. $a > 0, b < 0$

B. $a < , b < 0$

C. $a > 0, b \in R$

D. none of these

Answer: 1

View Text Solution

7. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$ and g(x) =f(sinx)+f(cosx)

If x=4 is the only point of maxima in its neighborhood but x=3 is neither a point of maxima nor a point of minima then which of the following can be true?

A. $a < 0, b > 0$

B. $ahy0, b < 0$

C. $aht0, b < 0$

D. not possible

Answer: 4

View Text Solution

8. Let $f'(\sin x) < 0$ and $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$ and g(x) =f(sinx)+f(cosx)

If x=3 is a point of minima and x=4 ios a popint of maximan then which of the following is true?

A. $a < 0, b > 0$

B. $a > 0, b < 0$

C. $a > 0, b > 0$

D. not possible

Answer: 3

View Text Solution

9. If ϕ (x) is a differentiable real valued function satisfying $\phi(x) + 2\phi \leq 1$, then it can be adjucted as $e^{2x}\phi(x) + 2e^{2x}\phi(x) \leq e^{2x}$ or $\frac{d}{dx}\left(e^{2}\phi(x) - \frac{e^{2x}}{2}\right) \leq$ or $\frac{d}{dx}e^{2x}\left(\phi(x) -\right.$ Here e^{2x} is called integrating factor which helps in creating single differential coefficeint as shown above. Answer the following question:

If p(1)=0 and $dP\frac{x}{dx} < P(x)$ for all $x \geq 1$ then

A. $P(x) > 0 \forall x > 1$

B. P(x) is a constant function

C. $P(x) < 0 \forall x > 1$

D. none of these

Answer: 1

View Text Solution

10. If $H(x_0)$=0 for some x=$x_0$and $\frac{d}{dx}H(x) > 2cxH(x)$ for all $x \geq x_0$ where $c > 0$ then

A. H(x) = 0 has root for $x > x_0$

B. H(x) = 0 has no root for $x > x_0$

C. H(x) is a constant functio

D. none of these

Answer: 2

Watch Video Solution

11. Let $h(x) = f(x) - a(f(x))^3$ for every real number x

h(x) increase as f(x) increses for all real values of x if

A. $a \in (0, 3)$

B. $a \in (-2, 2)$

C. $[3, \infty)$

D. none of these

Answer: 1

View Text Solution

12. H(x) increase as f(x) decreases for all real values of x if

A. $a \in (0, 3)$

B. $a \in (-2, 2)$

C. $[3, \infty)$

D. none of these

Answer: 4

Watch Video Solution

13. If f(x) is strictly increasing function then h(x) is non monotonic function given

A. $a \in (0, 3)$

B. $a \in (-2, 2)$

C. $(3, \infty)$

D. $a \in (-\infty, 0) \cup (3, \infty)$

Answer: 4

View Text Solution

14. If the function $f(x) = x^3 - 9x^2 + 24x + c$ has three real and distinct roots α, β and γ, then the value of `[alpha]+[beta]+[gamma] is

A. (-20 , -16)

B. (-20,-18)

C. (-18,-16)

D. none of these

Answer: 1

View Text Solution

15. $f(x) = x^3 - 9x^2 + 24x + c$ has real roots α, β, γ then find the value of $[\alpha] + [\beta] + [\gamma]$. Where[] represents greatest integer function

A. (-20, -16)

B. (-20,-18)

C. (-18,-16)

D. none of these

Answer: 3

View Text Solution

16. If $[\alpha]+[\beta]+[\gamma]$=7 then the values of c, where [.] represent the greates integer function are,

A. (-20,-16)

B. (-20,-18)

C. (-18,-16)

D. none of these

Answer: 2

View Text Solution

17. Let f(x) = $4x^2 - 4ax + a^2 - 2a + 2$ and the golbal minimum value of f(x) for x in [0,2] is equal to 3

The number of values of a for which the global minimum value equal to 3 for x in [0,2] occurs at the endpoint of interval[0,2] is

A. 1

B. 2

C. 3

D. 0

Answer: 2

View Text Solution

18. Let f(x) = $4x^2 - 4ax + a^2 - 2a + 2$ and the golbal minimum value of f(x) for x in [0,2] is equal to 3

The number of values opf a for which the global minimum value equal to 3 for x in [0,2] occurs for the vlue of x lying in (0,2) is

A. 1

B. 2

C. 3

D. 0

Answer: 4

View Text Solution

19. Let f(x) = $4x^2 - 4ax + a^2 - 2a + 2$ and the global minimum value of f(x) for x in [0,2] is equal to 3

The values of a for which f(x) is monotonic for x $\in$ [0,2]

A. $a \leq$ 0 or a ≥ 4

B. $0 \leq a \leq 4$

C. $a > 0$

D. none of these

Answer: 1

View Text Solution

20. Let f(x) =$x^3 - 3(7-a)X^2 - 3(9-a^2)x + 2$

The values of parameter a if f(x) has a negative point of local minimum are

A. π

B. (-3,3)

C. $\left(-\infty, \frac{58}{14}\right)$

D. none of these

Answer: 1

View Text Solution

21. Let f(x) =$x^3 - 3(7-a)X^2 - 3\left(9-a^2\right)x + 2$

The values of parameter a if f(x) has a positive point of local maxima are

A. π

B. $-\infty, \ -3 \cup \left(3, \frac{29}{7}\right)$

C. $-\infty, \ \frac{58}{14}$

D. none of these

Answer: 2

View Text Solution

22. Let f(x) =$x^3 - 3(7-a)X^2 - 3\left(9-a^2\right)x + 2$

The values of parameter a if f(x) has points of extrema which are opposite in sign are

A. π

B. (-3,3)

C. $\left(-\infty, \frac{58}{14}\right)$

D. none of these

Answer: 2

View Text Solution

23. consider the function f(x) =1$\left(1+\frac{1}{x}\right)^{x}$

The domain of f(x) is

A. $(-1, 0) \cup (0, \infty)$

B. R-{0}

C. $(-\infty, -1) \cup (0, \infty) \cup (0, \infty)$

D. (0,oo)`

Answer: 3

Watch Video Solution

24. consider the function f(x) =$1\left(1+\frac{1}{x}\right)^{x}$

The function f(x)

A. has a maximum but no minima

B. has a minima but no maxima

C. has exactly one maxima and one minima

D. is monotnic

Answer: 4

View Text Solution

25. consider the function f(x) =$1+\frac{1}{(x)^{x}}$

The range of the function f(x) is

A. $(0, p\infty)$

B. $(-\infty, e)$

C. $1, \infty)$

D. $(1, e) \cup (e, \infty)$

Answer: 4

View Text Solution

26. consider the function f(X) =x+cosx -a

which of the following is not true about y =f(x)?

A. It is an increasing function

B. It is a monotonic function

C. It has infinite points of inflection s

D. None of these

Answer: 4

View Text Solution

27. consider the function f(X) =x+cosx -a

values of a which f(X) =0 has exactly one positive root are

A. (0,1)

B. $(-\infty, 1)$

C. (-1,1)

D. $(1, \infty)$

Answer: 4

Watch Video Solution

28. consider the function f(X) =x+cosx -a

values of a for which f(X) =0 has exactly one negative root are

A. (0,1)

B. $(-\infty, 1)$

C. (-1,1)

D. $(1, \infty)$

Answer: 2

Watch Video Solution

29. consider the function f(X) =$3x^4 + 4x^3 - 12x^2$

Y= f(X) increase in the inerval

A. $(-1, 0) \cup (2, \infty)$

B. $(-\infty, 0) \cup (1, 2)$

C. $(-2, 0) \cup (1, \infty)$

D. none of these

Answer: 3

Watch Video Solution

30. consider the function f(X) =$3x^4 + 4x^3 - 12x^2$

The range of the function y=f(x) is

A. $(-\infty, \infty)$

B. $[-32, \infty)$

C. $[0, \infty)$

D. none of these

Answer: 2

Watch Video Solution

31. consider the function f(X) =$3x^4 + 4x^3 - 12x^2$

The range of values of a for which f(x) = a has no real

A. $(4, \infty)$

B. $(10, \infty)$

C. $(20, \infty)$

D. none of these

Answer: 4

Watch Video Solution

32. consider the function $f: R \to R, f(x) = \dfrac{x^2 - 6x + 4}{x^2 + 2x + 4}$

f(x) is

A. unbounded function

B. one one function

C. onto function

D. none of these

Answer: 4

View Text Solution

33. consider the function $f: R \to R$, $f(x) = \dfrac{x^2 - 6x + 4}{x^2 + 2x + 4}$

which of the following is not true about f(x)?

A. f(x) has two points of extremum

B. f(x) has only one asymptote

C. f(x) is differentiable for all x in R

D. none of these

Answer: 4

View Text Solution

34. Draw graph of $y = \dfrac{x^2 - 6x + 4}{x^2 + 2x + 4}$.

A. $\left(-\infty \quad -\frac{2}{3}\right] \cup [2, 0)$

B. $\left[\dfrac{-1}{3}, 5\right]$

C. $(-\infty, 2) \cup \left[\dfrac{7}{3}, \infty\right)$

D. $(20, \infty)$

Answer: 2

Watch Video Solution

35. Consider a polynomial y = P(x) of the least degree passing through A(-1,1) and whose graph has two points of inflection B(1,2) and C with abscissa 0 at which the curve is inclined to the positive axis of abscissa at an angle of $\sec^{-1}\sqrt{2}$

The value of P(2) ius

A. -1

B. $\frac{-3}{2}$

C. $\frac{5}{2}$

D. $\frac{7}{2}$

Answer: 3

View Text Solution

36. Consider a polynomial y = P(x) of the least degree passing through A(-1,1) and whose graph has two points of inflection B(1,2) and C with abscissa 0 at which the curve is inclined to the positive axis of abscissa at an angle of $\sec^{-1}\sqrt{2}$

The value of P(0) is

A. 1

B. 0

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: 4

View Text Solution

37. Consider a polynomial y = P(x) of the least degree passing through A(-1,1) and whose graph has two points of inflection B(1,2) and C with abscissa 0 at which the curve is inclined to the positive axis of abscissa at an angle of $\sec^{-1}\sqrt{2}$

The equation P(x) =0 has

A. three distinct real roots

B. one real root

C. three real roots such that one root is repeated

D. none of these

Answer: 3

View Text Solution

38. Let f(X) be real valued continous funcion on R defined as f(X) =$x^2 e^{-|x|}$

The values of k for which the equation $x^2 e^{-|x|}$=k has four real roots are

A. $0 < k < e$

B. $0 < k < \frac{8}{e^2}$

C. $0 < k < \frac{4}{e^2}$

D. none of these

Answer: 3

View Text Solution

39. Let f(X) be real valued continous funcion on R defined as f(X) =$x^2e^{-|x|}$

Number of points of inflection for y = f(X) is

A. y = f(x) has two points of maxima

B. y=f(X) has only one asymptote

C. f(X) =0 has three real roots

D. none of these

Answer: 4

Watch Video Solution

40. Let f(X) be real valued continous funcion on R defined as f(X) = $x^2 e^{-|x|}$

Number of points of inflection for y = f(X) is

A. 1

B. 2

C. 3

D. 4

Answer: 4

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41. P(x) be a polynomial of degree 3 satisfying P(-1) =10 , P(1) =-6 and p(x) has maxima at x = -1 and p(x) has minima at x=1 then The value of P(2) is

A. -15

B. -16

C. -17

D. -22

Answer: 3

View Text Solution

42. P(x) be a polynomial of degree 3 satisfying P(-1) =10 , P(1) =-6 and p(x) has maxima at x = -1 and p(x) has minima at x=1 then The value of P(1) is

A. -12

B. -10

C. 15

D. 21

Answer: 1

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Linked comprehension type

1. consider the graph of y=g(x)=f'(x) given that f(c) =0, where y=f(x) is a polynomial funtion

The graph of y=f(x) will intersect the x axis

A. twice

B. once

C. never

D. none of these

Answer: 2

View Text Solution

2. consider the graph of y=g(x)=f'(x) given that f(c) =0, where y=f(x) is a polynomial funtion

The equation $f(x) = 0, a \leq x \leq b$,has

A. four real roots

B. no real roots

C. two distinct real roots

D. at least three repeated roots

Answer: 4

View Text Solution

3. consider the graph of y=g(x)=f'(x) given that f(c) =0, where y=f(x) is a polynomial funtion

The graph of y =$f(x), a \leq x \leq b$ has

A. two points of inflection

B. one point of inflection

C. no point of inflection

D. none of these

Answer: 2

View Text Solution

4. consider the graph of y=g(x)=f'(x) given that f(c) =0, where y=f(x) is a polynomial funtion

The function y = $f(x), a \leq x \leq b$,has

A. exactly one local maxima

B. one local minima and one maxima

C. exactly one local minima

D. none of these

Answer: 4

View Text Solution

5. consider the graph of y=g(x)=f'(x) given that f(c) =0, where y=f(x) is a polynomial funtion

The equation f''(x)=0

A. has no real root

B. has at least one real root

C. has at least two distinct roots

D. none of these

Answer: 2

View Text Solution

6. The graph of y =g(x) =f(X) is as shown in the following figure analyse this graph and answer the following question

The graph of y=f(x) for $a < x < b$has

A. no point of extream

B. one point of extrema

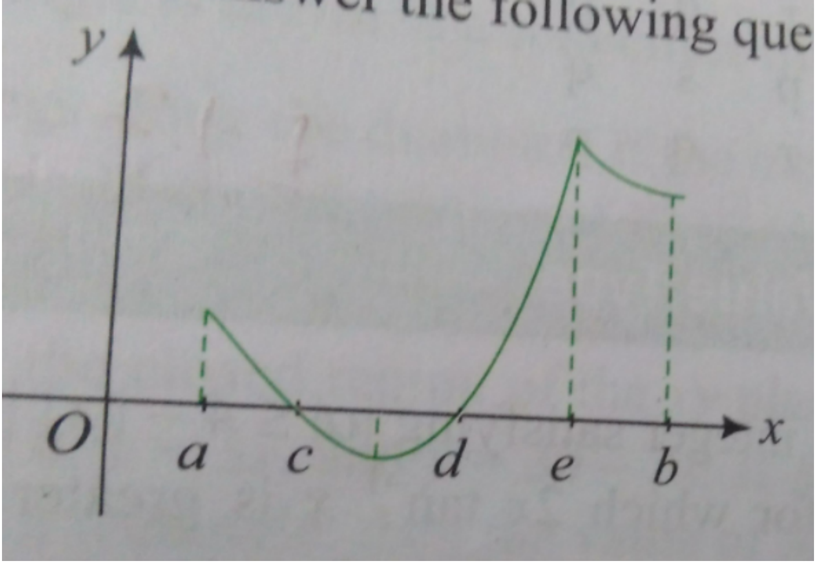
C. two points extrema

D. can't say anything

Answer: 3

View Text Solution

7. The graph of y =g(x) =f(X) is as shown in the following figure analyse this graph and answer the following question



Number of points of inflectionn the graph of y = f(x) for $a < x < b$ has

A. 0

B. 1

C. 2

D. can't say anything

Answer: 3

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8. The graph of y =g(x) =f(X) is as shown in the following figure analyse this graph and answer the following question

Which of the following is not true about the graph of y = f(x) ,$a < x < b$

A. always increasing

B. discontinous at one point

C. first increases then decreases

D. none of these

Answer: 3

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Matrix Match Type

1. Let f be a function $f: N \to N$ be defined by $f(x) = 5x - 3$, Find the image of $3, 4, 5$.

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2. If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ and find B.

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3. Let $f(x) = (x-1)^m (2-x)^n, mn \in N$ and $m, n < 2$. Then match the following lists:

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4. The function f(x) $= \sqrt{(ax^3 + bx^3 + cx + d)}$ has its nonzero local minimum and maximum values at x=-2 and x=2 respectively.If a is a root of $x^2 - x - 6 = 0$ then match the following lists:

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5. Let f be a function $N \rightarrow N$ be defined by $f(x) = x + 3$, Find the pre-Image of $20, 25, 50$.

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6. Consider $f: \{4, 5, 6\} \rightarrow \{p, q, r\}$ given by $f(4) = p$, $f(5) = q$ and $f(6) = r$. Find f^{-1} and show that $\left(f^{-1}\right)^{-1} = f$.

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7. If $f: [-1, 2] \rightarrow B$ is given by $f(x) = 3x^2$ then find B so that f is onto.

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8. Find k if $fof(k) = 4$ where $f(k) = 3k - 2$.

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9. Find k if $fog = gof$ where $f(x) = 3x - 1, g(x) = 4x + k.$

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10. f(x) is polynomial function of degree 6 which $\lim_{x \to 0} 1 + \dfrac{f(x)}{\left(x^3\right)^{1/x}} = e^2$ and has local maximum at x 1 and local minimum at x=0 and x=2 Now match the following lists and then choose the correct code.

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11. Find k if $fog = gof$ where $f(x) = 2x + 1, g(x) = x + k.$

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12. Let $f(x) = x + \log_e x - x\log_e x, x \in (0, \infty)$

List I contains information about zero of f(X) ,f'(x) and f''(x)

List II contains information about the limiting behaviour of f(x), f(x) and f''(x) at infinty

List III contains information about increasing /decreasing nature of f(x) and f'(x)

which o fhte following options is the only CORRECT comibination?

A. (iv)(i)(S)

B. (I)(ii)(R)

C. (III)(iv)(P)

D. (II)(ii)(S)

Answer: 4

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Exercise (Numerical)

1. If α is an integer satisfying $|\alpha| \leq 4 - |[x]|$, where x is a real number for which $2x \tan^{-1} x$ is greater than or equal to $\ln(1 + x^2)$, then the number of maximum possible values of a (where [.] represents the greatest integer function) is____

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2. From a given solid cone of height H, another inverted cone is carved whose height is h such that its volume is maximum. Then the ratio $\frac{H}{h}$ is

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3. $Let f(x) = \left\{x^3 + x^2 + 3x + \sin x \mid \left(3 + s \in \frac{1}{x}, \right), x \neq 0. 0x = 0\right.$ then the number of point where $f(x)$ attains its minimum value is____

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4. Leg $f(x)$ be a cubic polynomial which has local maximum at $x = -1 and f(x)$ has a local minimum at $x = 1.$ If $f(-1) = 10 and f(3) = -22,$ then one fourth of the distance between its two horizontal tangents is __________

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5. consider P(x) to be a polynomial of degree 5 having extremum at x=-1,1 and $\lim_{x \to 0} p\frac{x}{x^3} - 2 = 4$ Then the value of P(x)_____.

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6. If m is the minimum value of $f(x, y) = x^2 - 4x + y^2 + 6y$ when $x and y$ are subjected to the restrictions $0 \leq x \leq 1 and 0 \leq y \leq 1,$ then the value of $|m|$ is________

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7. For a cubic function $y = f(x)$, $f^{x} = 4x$ at each point (x, y) on it and it crosses the $x - a\xi s$ at $(-2, 0)$ at an angle of 45^{0} with positive direction of the x-axis. Then the value of $\left|\frac{f(1)}{5}\right|$ is______

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8. Number of integral values of b for which the equation $\frac{x^3}{3} - x = b$ has three distinct solutions is___

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9. $Let f(x) = \{x + 2, x < -1x^2, -1 \leq x < 1(x - 2)^2, x \geq 1$ Then number of times $f'(x)$ changes its sign in $(-\infty, \infty)$ is__

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10. The number of nonzero integral values of a for which the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ is concave upward along the entire real line is________

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11. $Legf(x) = \{x^{\frac{3}{5}}, \text{ if } x \leq 1 - (x-2)^3, \text{ if } x > 1$ Then the number of critical points on the graph of the function is___

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12. A right triangle is drawn in a semicircle of radius $\frac{1}{2}$ with one of its legs along the diameter. If the maximum area of the triangle is M , then the value of $32\sqrt{3}Mc$ is_____

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13. A rectangle with one side lying along the x-axis is to be inscribed in the closed region of the xy plane bounded by the lines

$y = 0, y = 3x, andy = 30 - 2x$. If M is the largest area of such a rectangle, then the value of $\frac{2M}{27}$ is______

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14. The least integral value of x where $f(x) = (\log)_{\frac{1}{2}}\left(x^2 - 2x - 3\right)$ is monotonically decreasing is_______

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15. The least area of a circle circumscribing any right triangle of area $\frac{9}{\pi}$ is __________

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16. $Let f(x) = \left\{|x^2 - 3x| + a, 0 \leq x < \frac{3}{2} - 2x + 3, x \geq \frac{3}{2}\right.$ If $f(x)$ has a local maxima at $x = \frac{3}{2}$, then greatest value of $|4a|$ is ________

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17. Let $f(x) = 30 - 2x - x^3$, the number ofthe positive integral values of x which does satisfy $f(f(f(x))) > f(f(-x))$ is

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18. Let f(x) =$\{(x(x-1)(x-2), (0 \le x < n), \sin(\pi x), (n \le x \le 2n)$ least value of n for which f(x) has more points of minima than maxima in [0,2n] is _____.

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19. Number of critical point of the function f(X) =x+$\sqrt{|x|}$ is ______.

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20. consider $f(X) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ Let x_1 and x_2 be point wher f(x) attains local minmum and global maximum respectively .If $k = f(x_1) + f(x_2)$ then 6k-9=_______.

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JEE Main Previous Year

1. Given P(x) $= x^4 + ax^3 + bx^2 + cx + d$ such that x=0 is the only real root of P'(x) =0 . If P(-1) lt P(1),$then \in the \int erval$[-1,1]`

A. P(-1) is the minimum and P(1) is the maximum of P

B. P(-1) is not minimum but P(1) is the maximum of P

C. P(-1) is not minimum and P(1) is not the maximum of P

D. neither P(-1) is the minimum nor P(1) is the maximum of P

Answer: 2

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2. Let $f: R \to R$ be defined by $f(x)=\begin{cases} k-2x & \text{if } x \leq -1 \\ 2x+3 & \text{if } x > -1 \end{cases}$. If f has a local minimum at $x=1$, then a possible value of k is (1) 0 (2) $-\frac{1}{2}$ (3) -1 (4) 1

A. -1

B. 1

C. 0

D. $\frac{1}{2}$

Answer: 1

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3. Let f: R R be a continuous function defined by $f(x)=\frac{1}{e^x+2e^{-x}}$. Statement-1: $f(c)=\frac{1}{3}$, for some $c \in R$. Statement-2: $0 < f(x) \leq \frac{1}{2\sqrt{2}}$, for all $x \in R$. (1) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1 (2) Statement-1 is true, Statement-2 is false (3) Statement-1 is false, Statement-2 is true (4) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1

A. statement 1 is false statement 2 is true

B. satement 1 si true , statement 2 is true statement 2 is a correct explanation for statement 1

C. statement 1 is true statement 2 is true statement 2 is not a correct explanation for statement 2

D. statement 1 is true , statement 2 is false

Answer: 2

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4. Let a, b R be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = 1$ and

$x = 2$. Statement 1: f has local maximum at $x = 1$ and at $x = 2$. Statement 2: $a = \frac{1}{2} \text{and} b = \frac{-1}{4}$ (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

A. statement 1 is false statement 2 is true

B. statement 1 is true statement 2 is true , statement 2 is a correct explanation for statement 1

C. statement 1 is true statement 2 is true , statement 2 is not a correct explanation for statement 1

D. statement 1 is true statement 2 is false

Answer: 2

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5. find the values k for which the quadratic equation $2x^2 + Kx + 3 = 0$ has two real equal roots

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6. If x = -1 and x = 2 are extreme points of f(x) = $\alpha \log|x| + \beta x^2 + x$, then

A. $\alpha = -6, \beta = \frac{1}{3}$

B. $\alpha = -6, \beta = -\frac{1}{2}$

C. $\alpha = 2, \beta = -\frac{1}{2}$

D. $\alpha = 2, \beta = \frac{1}{2}$

Answer: C

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7. Let f(X) be a polynomila of degree four having extreme values at x =1 and x=2.If $\lim_{x \to 0} \left[1 + \frac{f(x)}{x^2}\right] = 3$ then f(2) is equal to

A. -8

B. -4

C. 0

D. 4

Answer: 3

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8. A wire of length 2 units is cut into two parts which are bent respectively to form a square of $side = x$ units and a circle of $radius = r$ units. If the sum of the areas of the square and the circle so formed is minimum, then : (1) $2x = (\pi + 4)r$ (2) $(\pi + 4)x = \pi r$ (3) $x = 2r$ (4) $2x = r$

A. $(4-\pi)x = \pi r$

B. x=2r

C. 2x=r

D. $2x = (\pi + 4)r$

Answer: 2

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9. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in $sq.\ m$) of the flower-bed is: (a) 25 (2) 30 (3) 12.5 (4) 10

A. 30

B. 12.5

C. 10

D. 25

Answer: 4

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10. Let $f(x) = x^2 + \left(\frac{1}{x^2}\right)$ and $g(x) = x - \frac{1}{x}$ $x \in R - \{-1, 0, 1\}$. If $h(x) = \left(\frac{f(x)}{g(x)}\right)$ then the local minimum value of $h(x)$ is: (1) 3 (2) -3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

A. $2\sqrt{2}$

B. 3

C. -3

D. $-2\sqrt{2}$

Answer: 1

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1. The function $f:[0,3]\overrightarrow{1,29}$, defined by $f(x)=2x^3-15x^2+36x+1$, is one-one and onto onto but not one-one one-one but not onto neither one-one nor onto

A. one-one and onto

B. onto but not one -one

C. one-one but not onto

D. neither one-one nor on to

Answer: 2

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2. The number of points in $(-\infty,\infty)$, for which $x^2-x\sin x-\cos x=0$, is 6 (b) 4 (c) 2 (d) 0

A. 6

B. 4

C. 2

D. 0

Answer: 3

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3. If $f: R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R, \text{ and } f\left(\frac{1}{2}\right) = \frac{1}{2}. f(1) = 1,$ then

A. $0 < f'(1) \le \frac{1}{2}$

B. $f'(1) \le 0$

C. $f'(1) > 1$

D. $\frac{1}{2} < f'(1) \le 1$

Answer: 3

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4. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio $8 : 15$ is converted into anopen rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the length of the sides of the rectangular sheet are 24 (b) 32 (c) 45 (d) 60

A. 24

B. 32

C. 45

D. 60

Answer: 1,3

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5. The function $f(x) = 2|x| + |x + 2| = ||x|2| - 2|x||$ has a local minimum or a local maximum at $x = -2$ (b) $-\frac{2}{3}$ (c) 2 (d) $\frac{2}{3}$

A. -2

B. $-2/3$

C. 2

D. $2/3$

Answer: 1,2

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6. For every pair of continuous functions $f, g: [0, 1] \to R$ such that $\max\{f(x): x \in [0, 1]\} = \max\{g(x): x \in [0, 1]\}$ then which are the correct statements

A. $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some c $\in$ [0,1]

B. $(f(c))^2 + f(c) = (g(c))^2$+3g(c) for some c $\in$ [0,1]

C. $(f(c))^2 + 3f(c) = (g(c))^2$+g(c) for some c $\in$ [0,1]

D. $(f(c))^2 = (g(c))^2$ for some c $\in$ [0,1]

Answer: 1,4

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7. Let $a \in R$ and $f, R \to R$ be given by $f(x) = x^5 - 5x + a$ Then

A. f(x) has three real roots if $a > 4$

B. f(X) has only one real root if $a > 4$

C. f(x) has three real roots if $a < -4$

D. f(X) has threee real roots if $-4 < a < 4$

Answer: 2,4

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8. Let $f: R \to (0, \infty)$ and $g: R \to$ R be twice differntiable function such that f'' and g'' ar continous fucntion on R. Suppose

$f(2) = g(2) = 0, f''(2) \neq 0$ and $g''(2) \neq 0. If \lim_{x \to 2} \frac{f(X)g(x)}{f'(x)g'(x)} = 1$

then

A. f has a local minimum at x=2

B. f has a local maximum at x=2

C. $f''(2) > f(x)$

D. `f(X)-f''(x)=0 for at least one x in R

Answer: 1,4

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9. If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(x) > 2f(x)$ for all $x \in \mathbb{R}$ and $f(0) = 1,$ then

A. $f(x) > e^{2x} \in (0, \infty)$

B. f(X) is decreasing in $(0, \infty)$

C. f(X) is increasing in $(0, \infty)$

D. $f'(x) < e^{2x}$ in $(0, \infty)$

Answer: 1,3

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10. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

A. f'(x) =0 at exactly three points in $(-\pi, \pi)$

B. f(x) attains its maximum at x=0

C. f(x) attains its mionimum at x=0

D. f'(x) =0 at more than three points in $(-\pi, \pi)$

Answer: 2,4

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11. Let f:(0,pi)rarrR be a twice differentiable fucntion such that $\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$ for all x $\in (0, \pi)$

If $f\left(\frac{\pi}{6}\right) = \left(-\frac{\pi}{12}\right)$ then which of the following statement (s) is (are) TRUE?

A. $f\left(x\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

B. $f(x) < \frac{x^4}{6} - x^2 f \text{ or } allx \in (0, \pi)$

C. There exist $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

D. $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right)$=0

Answer: 2,3,4

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12. Let $f[0, 1] \to R$ (the set of all real numbers be a function.Suppose the function f is twice differentiable, $f(0) = f(1) = 0$,and satisfies $f'(x) - 2f'(x) + f(x) \leq e^x, x \in [0, 1]$.Which of the following is true for $0 < x < 1$?

A. $0 < f(x) < \infty$

B. $-\frac{1}{2} < f(x) < \frac{1}{2}$

C. $-\frac{1}{4} < f(x) < 1$

D. $-\infty < f(x) < 0$

Answer: 4

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13. If the function e^{-x} f(X) assumes its minimum in the interval [0,1] at $x = 1/4$ which of the following is true ?

A. $f'(x) < f(X), 1/4 < x < 3/4$

B. $f'(x) > f(X), 0 < x < 1/4$

C. $f'(x) < f(X), 0 < x < 1/4$

D. $f'(x) < f(X), 3/4 < x < 1$

Answer: 3

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14. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x))$, for all $x \in R$, then the number of point is R at which g has a local maximum is ___

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15. Let $IRI\overrightarrow{}R$ be defined as $f(x) = |x| + +x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is______

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16. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If

$p(1) = 6 and p(3) = 2,$ then $p'(0)$ is____

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17. A cylindrica container is to be made from certain solid material with the following constraints: It has a fixed inner volume of $V mm^3$, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2mm and is of radius equal to the outer radius of the container. If the volume the material used to make the container is minimum when the inner radius of the container is $10 mm$. then the value of $\frac{V}{250\pi}$ is

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