MATHS

BOOKS - CENGAGE

MONOTONOCITY AND NAXINA-MINIMA OF FUNCTIONS

Single Correct Answer Type

1. If $x \in (0, \pi/2)$, then the function $f(x) = x \sin x + \cos x + \cos^2 x$ is

A. increasing

B. Decreasing

C. Neither increasing nor decreasing

D. None of these

Answer: B

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2. The function $f:(a,\infty)\to R$ where R denotes the range corresponding to the given domain, with rule $f(x)=2x-3x^2+6$, will have an inverse provided

A. $a\leq 1$

B. $a\geq 0$

C. $a\leq 0$

D. $a\geq 1$

Answer: D

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3. Let $f(x) = 1 - x - x^3$.Values of x not satisfying the inequality, $1 - f(x) - f^3(x) > f(1 - 5x)$

A. $(-2, 0)$

B. $(2, \infty)$

C. $(0, 2)$

D. None of these

Answer: C

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4. If $g(x) = 2f\left(2x^3 - 3x^2\right) + f\left(6x^2 - 4x^3 - 3\right) \forall x \in R$ and $f''(x) > 0 \forall x \in R$ then g(x) is increasing in the interval

A. $\left(-\infty, -\frac{1}{2}\right) \cup (0, 1)$

B. $\left(-\frac{1}{2}, 0\right) \cup (1, \infty)$

C. $(0, \infty)$

D. $(-\infty, 1)$

Answer: B

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5. Find the set of all values of the parameter 'a' for which the function, $f(x) = \sin 2x - 8(a+1)\sin x + \left(4a^2 + 8a - 14\right)x$ increases for all $x \in R$ and has no critical points for all $a \in R$.

A. a) $\left(-\infty, -\sqrt{5}, -2\right)$

B. b)$(1, \infty)$

C. c) $\left(\sqrt{5}, \infty\right)$

D. d) None of these

Answer: B

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6. if f(x) $= 2e^{x} - ae^{-x} + (2a+1)x - 3$ monotonically increases for $\forall x \in R$ then the minimum value of 'a' is

A. 2

B. 1

C. 0

D. -1

Answer: C

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7. If the function $f(x) = 2\cot x + (2a+1)\ln|\cos ecx| + (2-a)x$ is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$ then range of a is

A. $[0, \infty)$

B. $(-\infty, 0]$

C. $(-\infty, \infty)$

D. None of these

Answer: A

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8. If $x_1, x_2 \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\tan_{x_2}}{\tan x_1}$ is (where $x_1 < x_2$)

A. $< \frac{x_1}{x_2}$

B. $= \frac{x_1}{x_2}$

C. $< x_1 x_2$

D. $> \frac{x_2}{x_1}$

Answer: D

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9. If f(x) is a differentiable real valued function satisfying $f''(x) - 3f'(x) > 3 \forall x \geq 0$ and $f'(0) = -1$, then

$f(x) + x \, \forall x > 0$ is

A. decreasing function of x

B. increasing function of x

C. constant function

D. none of these

Answer: B

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10. The roots of $(x-41)^{49} + (x-49)^{41} + (x-2009)^{2009} = 0$ are

A. all necessarily real

B. non-real except one positive real root

C. non-real except three positive real roots

D. non-real except for three real roots of which exactly one is positive

Answer: B

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11. Let h be a twice continuously differentiable positive function on an open interval J. Let $g(x) = \ln(h(x)$ for each $x \in J$ Suppose $(h'(x))^2 > h''(x)h(x)$ for each $x \in J$. Then

A. g is increasing on H

B. g is decreasing on H

C. g is concave up on H

D. g is concave down on H

Answer: D

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12. If $\sin x + x \geq |k|x^2, \forall x \in \left[0, \frac{\pi}{2}\right]$, then the greatest value of k is

A. $\frac{-2(2+\pi)}{\pi^2}$

B. $\frac{2(2+\pi)}{\pi^2}$

C. can't be determined finitely

D. zero

Answer: B

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13. If

$4x + 8\cos x + \tan x - 2\sec x - 4\log\{\cos x(1+\sin x)\} \geq 6$

for all $x \in [0, \lambda)$ then the largest value of λ is

A. $\pi/3$

B. $\pi/6$

C. $\pi/4$

D. $3\pi/4$

Answer: B

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14. The greatest possible value of the expression $\tan x + \cot x + \cos x$ on the interval $[\pi/6, \pi/4]$ is

A. $\frac{12}{5}\sqrt{2}$

B. $\frac{11}{6}\sqrt{2}$

C. $\frac{12}{5}\sqrt{3}$

D. $\frac{11}{6}\sqrt{3}$

Answer: D

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15. Let $f(x) = \begin{cases} (x+1)^3 & -2 < x \leq -1 \\ x^{2/3} - 1 & -1 < x \leq 1 \\ -(x-1)^2 & 1 < x < 2 \end{cases}$. The total number of maxima and minima of f(x) is

A. 4

B. 3

C. 2

D. 1

Answer: B

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16. Consider the graph of the function $f(x) = x + \sqrt{|x|}$

Statement-1: The graph of $y = f(x)$ has only one critical point

Statement-2: $f'(x)$ vanishes only at one point

A. Statement 1 is true, Statement 2 is true, Statement 2 is a

correct explanation for Statement 2 is a correct explanation for Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: D

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17. The minimum value of the function $f(x) = \frac{\tan\left(x + \frac{\pi}{6}\right)}{\tan x}$ is:

A. 1

B. 0

C. $\frac{1}{2}$

D. 3

Answer: D

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18. Let $f(x) = \frac{x^2+2}{[x]}, 1 \le x \le 3$, where [.] is the greatest integer function. Then the least value of f(x) is

A. 2

B. 3

C. $3/2$

D. 1

Answer: B

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19. If $f(x) = \begin{cases} 3 - x^2, & x \leq 2 \\ \sqrt{a+14} - |x - 48|, & x > 2 \end{cases}$ and if f(x) has a local maxima at x = 2, then greatest value of a is

A. 2013

B. 2012

C. 2011

D. 2010

Answer: C

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20. The function $f(x) = x^5 - 5x^4 + 5x^3$ has

A. One minima and two maxima

B. Two minima and one maxima

C. Two minima and two maxima

D. One minima and one maxima

Answer: D

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21. If

$f(x) = |x-1| + |x-4| + |x-9| + \ldots. + |x-2500| \, \forall x \in R$

, then all the values of x where f(x) has minimum values lie in

A. (600, 700)

B. (576, 678)

C. (625, 676)

D. none of these

Answer: C

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22. Slope of tangent to the curve $y = 2e^x \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$, where $0 \leq x \leq 2\pi$ is minimum at x =

A. 0

B. π

C. 2π

D. none of these

Answer: B

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23. The value of a for which all extremum of function $f(x) = x^3 + 3ax^2 + 3(a^2 - 1)x + 1$, lie in the interval (-2, 4) is

A. $(3, 4)$

B. $(-1, 3)$

C. $(-3, -1)$

D. none of these

Answer: B

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24. If $f(x)=\begin{cases}x^3(1-x), & x\le 0\\ x\log_e x+3x, & x>0\end{cases}$ then which of the following is not true?

A. f(x) has point of maxima at x = 0

B. f(x) has point minima at $x=e^{-4}$

C. f(x) has range R

D. none of these

Answer: D

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25. The coordinates of the point on the curve $x^3=y(x-a)^2$ where the ordinate is minimum is

A. $\left(3a, \frac{27}{4}a\right)$

B. $(2a, 8a)$

C. $(a, 0)$

D. None of these

Answer: A

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26. The fraction exceeds its p^{th} power by the greatest number possible, where $p \geq 2$ is

A. $\left(\frac{1}{p}\right)^{1/(p-1)}$

B. $\left(\frac{1}{p}\right)^{p-1}$

C. $p^{1/p-1}$

D. none of these

Answer: A

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27. If $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - e^{x-1}, & 1 < x \leq 2 \\ x - e, & 2 < x \leq 3 \end{cases}$ and $g'(x) = f(x), x \in [1, 3]$, then`

A. g(x) has no local maxima

B. g(x) has no local minima

C. g(x) has local maxima at $x = 1 + \ln 2$ and local minima at x = e

D. g(x) has local minima at $x = 1 + \ln 2$ and local maxima at x = e

Answer: C

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28. If $g(x) = \max (y^2 - xy)(0 \leq y \leq 1)$, then the minimum value of g(x) (for real x) is

A. $\frac{1}{4}$

B. $3 - \sqrt{3}$

C. $3 + \sqrt{8}$

D. $\frac{1}{2}$

Answer: B

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29. If a,b $\in$ R distinct numbers satisfying |a-1| + |b-1| = |a| + |b| = |a+1| + |b+1|, Then the minimum value of |a-b| is :

A. a) 3

B. b) 0

C. c) 1

D. d) 2

Answer: D

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30. If equation $2x^3 - 6x + 2\sin a + 3 = 0$, $a \in (0, \pi)$ has only one real root, then the largest interval in which a lies is

A. $\left(0, \frac{\pi}{6}\right)$

B. $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

C. $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

D. $\left(\frac{5\pi}{6}, \pi\right)$

Answer: C

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31. Let f be a continuous and differentiable function in (x_1, x_2). If $f(x).f'(x) \geq x\sqrt{1-(f(x))^4}$ and $\lim_{x \to x_1}(f(x))^2 = 1$ and $\lim_{x \to x}\Big)(f(x))^2 = \frac{1}{2}$, then minimum value of $\left(x_1^2 - x_2^2\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{3}$

D. none of these

Answer: C

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32. If $ab = 2a + 3b, a > 0, b > 0$, then the minimum value of ab is

A. 12

B. 24

C. $\frac{1}{4}$

D. none of these

Answer: B

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33. Let a, b, c, d, e, f, g, h be distinct elements in the set $\{-7, -5, -3, -2, 2, 4, 6, 13\}$. The minimum value of $(a+b+c+d)^2+(e+f+g+h)^2$ is:(1) 30 (2) 32 (3) 34 (4) 40

A. 30

B. 32

C. 34

D. 40

Answer: B

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34. The perimeter of a sector is p. The area of the sector is maximum when its radius is

A. $\sqrt{p}$

B. $\frac{1}{\sqrt{p}}$

C. $\frac{p}{2}$

D. $\frac{p}{4}$

Answer: D

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35. Minimum integral value of k for which the equation $e^x = kx^2$ has exactly three real distinct solution,

A. 1

B. 2

C. 3

D. 4

Answer: B

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36. Let f (x)= x^3-3x+1. Find the number of different real solution of the equation f (f(x) =0

A. 2

B. 4

C. 5

D. 7

Answer: D

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Multiple Correct Answer Type

1. Which of the following statement(s) is/are true?

A. a) Differentiable function satisfying $f(-1) = f(1)$ and $f'(x) \geq 0$ for all x must be a constant function on the interval $[-1, 1]$.

B. b) There exists a function with domain R satisfying f(x) lt 0, for all x, f'(x) gt 0 for all x and f''(x0 gt 0 for all x.

C. c) If f''(x) = 0 then (c,f(c)) is an inflection point.

D. d) Suppose f(x) is a function whose derivative is the function $f(x) = 2x^2 + 2x - 12$. Then f(x) is decreasing for $-3 < x < 2$ and concave up for $x > -\frac{1}{2}$.

Answer: A::D

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2. Let $f: R \to R$, $f(x) = x + \log_e(1 + x^2)$. Then

A. f is injective

B. f is surjective

C. there is a point on the graph of y= f(x) where tangent is not parallel to any of the chords

D. inverser of f(x) exists.

Answer: A::B::C::D

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3. Let $f(x) = x - \frac{1}{x}$ then which one of the following statements is true?

A. f(x) is one-one function.

B. f(x) is increasing function.

C. f(x) = k has two distinct real roots for any real k.

D. x = 0 is point inflection.

Answer: B::C::D

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4. Let f(x) be and even function in R. If f(x) is monotonically increasing in [2, 6], then

A. $f(3) < (- 5)$

B. $f(4) < f(- 3)$

C. $f(2) > f(- 3)$

D. $f(- 3) < f(5)$

Answer: A::D

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5. If $f(x) = \begin{cases} -e^{-x} + k & , \quad x \leq 0 \\ e^{x} + 1 & , \quad 0 < x < 1 \\ ex^2 + \lambda & , \quad x \geq 1 \end{cases}$ is one-one and monotonically increasing $\forall x \in R$, then

A. maximum value of k is 1

B. maximum value of k is 3

C. minimum value of λ is 0

D. minimum value of λ is 1

Answer: B::D

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6. If the function $f(x) = axe^{-bx}$ has a local maximum at the point (2,10), then

A. a = 5e

B. a = 5

C. b = 1

D. b = 1/2

Answer: A::D

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7. Let $f(x) = \frac{e^x}{1+x^2}$ and $g(x) = f'(x)$, then

A. g(x) has two local maxima and two local minima points

B. g(x) has exactly one local maxima and one local minima point

C. x = 1 is a point of local maxima for g(x)

D. There is a point of local maxima for g(x) in the interval (-1,0)

Answer: B::D

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8. If $f'(x) = (x-a)^{2010}(x-b)^{2009}$ and $a > b$, then

A. f(x) has relative maxima at x = b

B. f(x) has relative minima at x = b

C. f(x) has relative maxima at x = a

D. f(x) has neither maxima, nor minima at x = a

Answer: B::D

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9. If $\lim_{x \to a} f(x) = \lim_{x \to a} [f(x)]$ ([.] denotes the greates integer function) and f(x) is non-constant continuous function, then

A. $\lim_{x \to a}$ f(x) is an integer

B. $\lim_{x \to a}$ f(x) is non-integer

C. f(x) has local maximum at x = a

D. f(x) has local minimum at x = a

Answer: A::D

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10. Consider the function $f(x) = In\left(\sqrt{1-x^2}-x\right)$ then which of the following is/are true?

A. f(x) increases in the on $x = \left(-1, -\frac{1}{\sqrt{2}}\right)$

B. f has local maximum at $x = -\frac{1}{\sqrt{2}}$

C. Least value of f does not exist

D. Least value of f exists

Answer: A::B::C

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Comprehension Type

1. Let

$f: R \to R, y = f(x), f(0) = 0, f'(x) > 0$ and $f''(x) > 0.$

Three point $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma)) on y = f(x)$

such that $0 < \alpha < \beta < \gamma.$

Which of the following is false ?

A. $\alpha f(\beta) > \beta(f(\alpha))$

B. $\alpha f(\beta) < \beta f(\alpha)$

C. $\gamma f(\beta) < \beta(f(\gamma))$

D. $\gamma(f(\alpha)) < \alpha f(\gamma)$

Answer: B

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2. Let

$f: R \to R, y = f(x), f(0) = 0, f'(x) > 0$ and $f''(x) > 0$.

Three point $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma)) on y = f(x)$

such that $0 < \alpha < \beta < \gamma$.

Which of the following is true?

A. $\dfrac{f(\alpha) + f(\beta)}{2} < f\left(\dfrac{\alpha + \beta}{2}\right)$

B. $\dfrac{f(\alpha) + f(\beta)}{2} > f\left(\dfrac{\alpha + \beta}{2}\right)$

C. $\dfrac{f(\alpha)+f(\beta)}{2}=f\left(\dfrac{\alpha+\beta}{2}\right)$

D. $\dfrac{2f(\alpha)+f(\beta)}{3}<f\left(\dfrac{2\alpha+\beta}{3}\right)$

Answer: B

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3. Let

$f: R \to R, y = f(x), f(0) = 0, f'(x) > 0 \text{ and } f''(x) > 0.$

Three point $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma)) on y = f(x)$

such that $0 < \alpha < \beta < \gamma$.

Which of the following is true?

A. $\gamma f(\gamma+\beta-\alpha) > (\gamma+\beta-\alpha) f(\gamma)$

B. $\gamma f(\gamma+\beta-\alpha) < (\gamma+\beta-\alpha) f(\gamma)$

C. $\alpha f(\gamma+\beta-\alpha) > (\gamma+\beta-\alpha) f(\alpha)$

D. None of these

Answer: A

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4. Let f be a twice differentiable function such that $f''(x) > 0 \forall x \in R$. Let h(x) is defined by $h(x) = f(\sin^2 x) + f(\cos^2 x)$ where $|x| < \frac{\pi}{2}$.

The number of critical points of h(x) are

A. 1

B. 2

C. 3

D. more than 3

Answer: C

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5. $f'(\sin^2 x) < f'(\cos^2 x)$ for $x \in$

A. $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

B. $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \frac{\pi}{4}, \frac{\pi}{2}\right)$

C. $\left(-\frac{\pi}{4}, 0\right) \cup \frac{\pi}{4}, \frac{\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: A

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6. h(x) is increasing for $x \in$

A. $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

B. $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \frac{\pi}{4}, \frac{\pi}{2}\right)$

C. $\left(-\frac{\pi}{4}, 0\right) \cup \frac{\pi}{4}, \frac{\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(0, \frac{\pi}{4}\right)$

Answer: B

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