MATHS

BOOKS - CENGAGE

VECTOR ALGEBRA

Solved Examples And Exercises

1. In a trapezium, vector $\vec{B}C = \alpha\vec{A}D^{\cdot}$ We will then find that $\vec{p} = \vec{A}C + \vec{B}D$ is collinear with$\vec{A}D^{\cdot}$ If $\vec{p} = \mu\vec{A}D,$ then which of the following is true? a) $\mu = \alpha + 2$ b) $\mu + \alpha = 2$ c) $\alpha = \mu + 1$ d) $\mu = \alpha + 1$

Watch Video Solution

2. If the vectors $\vec{a} and \vec{b}$ are linearly independent satisfying $\left(\sqrt{3}\tan\theta + 1\right)\vec{a} + \left(\sqrt{3}sec\theta - 2\right)\vec{b} = 0,$ then the most general values of θ

are a. $n\pi - \frac{\pi}{6}, n \in Z$ b. $2n\pi \pm \frac{11\pi}{6}, n \in Z$ c. $n\pi \pm \frac{\pi}{6}, n \in Z$ d. $2n\pi + \frac{11\pi}{6}, n \in Z$

Watch Video Solution

3. Given three non-zero, non-coplanar vectors $\vec{a}, \vec{b},$ and $\vec{c}. \vec{r}_1 = p\vec{a} + q\vec{b} + \vec{c} and \vec{r}_2 = \vec{a} + p\vec{b} + q\vec{c}$ If the vectors $\vec{r}_1 + 2\vec{r}_2$ and $2\vec{r}_1 + \vec{r}_2$ are collinear, then (P, q) is a. $(0, 0)$ b. $(1, -1)$ c. $(-1, 1)$ d. $(1, 1)$

Watch Video Solution

4. Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, , \vec{r}_n$ be the position vectors of points P_1, P_2, P_3, P_n relative to the origin O If the vector equation $a_1\vec{r}_1 + a_2\vec{r}_2 + \; + a_n\vec{r}_n = 0$ hold, then a similar equation will also hold w.r.t. to any other origin provided a. $a_1 + a_2 + \; + a_n = n$ b. $a_1 + a_2 + \; + a_n = 1$ c. $a_1 + a_2 + \; + a_n = 0$ d. $a_1 = a_2 = a_3 + a_n = 0$

Watch Video Solution

Watch Video Solution

5. Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}, \vec{b} = 2\hat{i} - 6\hat{j} and \vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$ Then the resolution of the vector $\vec{\alpha}$ into components with respect to $\vec{a} and \vec{b}$ is given by a. $3\vec{a} - 2\vec{b}$ b. $3\vec{b} - 2\vec{a}$ c. $2\vec{a} - 3\vec{b}$ d. $\vec{a} - 2\vec{b}$

Watch Video Solution

6. Let us define the length of a vector $a\hat{i} + b\hat{j} + c\hat{k} as |a| + |b| + |c|$ This definition coincides with the usual definition of length of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is and only if a. $a = b = c = 0$ b. any two of $a, b, and c$ are zero c. any one of $a, b, and c$ is zero d. $a + b + c = 0$

Watch Video Solution

7. Vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k},$ are so placed that the end point of one vector is the starting point of the next vector. Then the vector are (A) not coplanar (B) coplanar but cannot form a triangle (C) coplanar and form a triangle (D) coplanar and can form a right angled triangle

Watch Video Solution

8. The position vectors of the vertices $A, B, andC$ of a triangle are $\hat{i}+\hat{j}, \hat{j}+\hat{k} and \hat{i}+\hat{k}$, respectively. Find the unite vector $\hat{r}$ lying in the plane of ABC and perpendicular to $IA, whereI$ is the incentre of the triangle.

Watch Video Solution

9. A ship is sailing towards the north at a speed of 12.5 m/s. The current is taking it towards the east at the rate of 1 m/s and sailor is climbing a vertical pole on the ship at the rate of 0.5 m/s. Find the velocity of the sailor in space.

Watch Video Solution

10. ABCD is a tetrahedron and O is any point. If the lines joining O to the vertices meet the opposite faces at P, Q, R and S, prove that $\frac{OP}{AP} + \frac{OQ}{BQ} + \frac{OR}{CR} + \frac{OS}{DS} = 1.$

Watch Video Solution

11. A pyramid with vertex at point P has a regular hexagonal base $ABCDEF$, Positive vector of points A and B are $\hat{i}$ and $\hat{i} + 2\hat{j}$ The centre of base has the position vector $\hat{i} + \hat{j} + \sqrt{3}\hat{k}$ Altitude drawn from P on the base meets the diagonal AD at point G find the all possible position vectors of G It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP is 5 units.

Watch Video Solution

12. A straight line L cuts the lines $AB, AC and AD$ of a parallelogram $ABCD$ at points $B_1, C_1 and D_1,$ respectively. If

$\left(\vec{AB}\right)_1, \lambda_1\vec{AB}, \left(\vec{AD}\right)_1 = \lambda_2\vec{AD} and \left(\vec{AC}\right)_1 = \lambda_3\vec{AC},$ then prove that $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$.

Watch Video Solution

13. $A, B, C and D$ have position vectors $\vec{a}, \vec{b}, \vec{c} and \vec{d},$ respectively, such that $\vec{a} - \vec{b} = 2\left(\vec{d} - \vec{c}\right)^{\cdot}$ Then a. $AB and CD$ bisect each other b. $BD and AC$ bisect each other c. $AB and CD$ trisect each other d. $BD and AC$ trisect each other

Watch Video Solution

14. If $\vec{a} and \vec{b}$ are two unit vectors and θ is the angle between them, then the unit vector along the angular bisector of $\vec{a}$ and $\vec{b}$ will be given by a. $\frac{\vec{a} - \vec{b}}{\cos(\theta/2)}$ b. $\frac{\vec{a} + \vec{b}}{2\cos(\theta/2)}$ c. $\frac{\vec{a} - \vec{b}}{2\cos(\theta/2)}$ d. none of these

Watch Video Solution

15. $ABCD$ is a quadrilateral. E is the point of intersection of the line joining the midpoints of the opposite sides. If O is any point and $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = x\vec{O}E, thenx$ is equal to a. 3 b. 9 c. 7 d. 4

Watch Video Solution

16. If vectors $\vec{A}B = -3\hat{i} + 4\hat{k}and\vec{A}C = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a $\text{Delta}ABC$, then the length of the median through Ais a. $\sqrt{14}$ b. $\sqrt{18}$ c. $\sqrt{29}$ d. $\sqrt{5}$

Watch Video Solution

17. $ABCD$ parallelogram, and A_1andB_1 are the midpoints of sides $BCandCD$, respectivley . If $\vec{\forall}_1 + \vec{A}B_1 = \lambda\vec{A}C, then\lambda$ is equal to a. $\frac{1}{2}$ b. 1 c. $\frac{3}{2}$ d. 2 e. $\frac{2}{3}$

Watch Video Solution

18. The position vectors of the points $PandQ$ with respect to the origin O are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of $\angle POQ$, then $\vec{OM}$ is a. $2\left(\hat{i} - \hat{j} + \hat{k}\right)$ b. $2\hat{i} + \hat{j} - 2\hat{k}$ c. $2\left(-\hat{i} + \hat{j} - \hat{k}\right)$ d. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

Watch Video Solution

19. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.

Watch Video Solution

20. If $\left|\vec{a} + \vec{b}\right| < \left|\vec{a} - \vec{b}\right|$, then the angle between $\vec{a} and \vec{b}$ can lie in the interval a. $(\pi/2, \pi/2)$ b. $(0, \pi)$ c. $(\pi/2, 3\pi/2)$ d. $(0, 2\pi)$

Watch Video Solution

21. $'I'$ is the incentre of triangle ABC whose corresponding sides are $a, b, c,$ rspectively. $a\vec{IA} + b\vec{IB} + c\vec{IC}$ is always equal to a. $\vec{0}$ b. $(a+b+c)\vec{BC}$ c. $\left(\vec{a} + \vec{b} + \vec{c}\right)\vec{AC}$ d. $(a+b+c)\vec{AB}$

Watch Video Solution

22. Let $x^2 + 3y^2 = 3$ be the equation of an ellipse in the $x - y$ plane. $A and B$ are two points whose position vectors are $-\sqrt{3}\hat{i} and - \sqrt{3}\hat{i} + 2\hat{k}$ Then the position vector of a point P on the ellipse such that $\angle APB = \pi/4$ is a. $\pm\hat{j}$ b. $\pm\left(\hat{i} + \hat{j}\right)$ c. $\pm\hat{i}$ d. none of these

Watch Video Solution

23. If $\vec{x} and \vec{y}$ are two non-collinear vectors and ABC isa triangle with side lengths $a, b, and c$ satisfying $(20a - 15b)\vec{x} + (15b - 12c)\vec{y} + (12c - 20a)\left(\vec{\times} x\vec{y}\right) = 0,$ then triangle ABC is

a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. an isosceles triangle

Watch Video Solution

24. If $\hat{i} - 3\hat{j} + 5\hat{k}$ bisects the angle between $\hat{a} and - \hat{i} + 2\hat{j} + 2\hat{k}, where \hat{a}$ is a unit vector, then a. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} - 40\hat{k}\right)$ b. $\hat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} + 40\hat{k}\right)$ c. $\hat{a} = \frac{1}{105}\left(- 41\hat{i} + 88\hat{j} - 40\hat{k}\right)$ d. $\hat{a} = \frac{1}{105}\left(41\hat{i} - 88\hat{j} - 40\hat{k}\right)$

Watch Video Solution

25. If $4\hat{i} + 7\hat{j} + 8\hat{k}, 2\hat{i} + 3\hat{j} + 24 and 2\hat{i} + 5\hat{j} + 7\hat{k}$ are the position vectors of the vertices $A, B and C,$ respectively, of triangle ABC , then the position vecrtor of the point where the bisector of angle A meets BC is a. $\frac{2}{3}\left(- 6\hat{i} - 8\hat{j} - \hat{k}\right)$ b. $\frac{2}{3}\left(6\hat{i} + 8\hat{j} + 6\hat{k}\right)$ c. $\frac{1}{3}\left(6\hat{i} + 13\hat{j} + 18\hat{k}\right)$ d. $\frac{1}{3}\left(5\hat{j} + 12\hat{k}\right)$

Watch Video Solution

26. If $\vec{b}$ is a vector whose initial point divides thejoin of $5\hat{i} and 5\hat{j}$ in the ratio $k: 1$ and whose terminal point is the origin and $\left|\vec{b}\right| \leq \sqrt{37}, then k$ lies in the interval a. $[-6, -1/6]$ b. $(-\infty, -6] \cup [-1/6, \infty)$ c. $[0, 6]$ d. none of these

Watch Video Solution

27. Find the value of λ so that the points P, Q, R and S on the sides OA, OB, OC and AB, respectively, of a regular tetrahedron $OABC$ are coplanar. It is given that $\frac{OP}{OA} = \frac{1}{3}, \frac{OQ}{OB} = \frac{1}{2}, \frac{OR}{OC} = \frac{1}{3}$ and $\frac{OS}{AB} = \lambda$. (A) $\lambda = \frac{1}{2}$ (B) $\lambda = -1$ (C) $\lambda = 0$ (D) for no value of λ

Watch Video Solution

28. A uni-modular tangent vector on the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6t = 2$ is a. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ b. $\frac{1}{3}\left(\hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{6}\left(2\hat{i} + \hat{j} + \hat{k}\right)$ d. $\frac{2}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

Watch Video Solution

29. If $\vec{x} and \vec{y}$ are two non-collinear vectors and a, b, and c represent the sides of a ABC satisfying $(a - b)\vec{x} + (b - c)\vec{y} + (c - a)\left(\vec{\times} x\vec{y}\right) = 0,$ then ABC is (where $\vec{\times} x\vec{y}$ is perpendicular to the plane of $x and y$) a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. a scalene triangle

Watch Video Solution

30. The position vectors of points $A and B$ w.r.t. the origin are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}, \ \vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ respectively. Determine vector $\vec{O}P$ which bisects angle $AOB,$ where P is a point on AB.

Watch Video Solution

31. What is the unit vector parallel to $\vec{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$? What vector should be added to $\vec{a}$ so that the resultant is the unit vector $\hat{i}$?

Watch Video Solution

32. ABCD is a quadrilateral and E is the point of intersection of the lines joining the middle points of opposite side. Show that the resultant of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ = 4 $\overrightarrow{OE}$, where O is any point.

Watch Video Solution

33. $ABCD$ is a parallelogram. If $LandM$ are the mid-points of $BCandDC$ respectively, then express $\vec{A}Land\vec{A}M$ in terms of $\vec{A}Band\vec{A}D$. Also, prove that $\vec{A}L + \vec{A}M = \frac{3}{2}\vec{A}C$

Watch Video Solution

34. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are four vectors in three-dimensional space with the same initial point and such that $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, show that terminals $A, B, C and D$ of these vectors are coplanar. Find the point at which $AC and BD$ meet. Find the ratio in which P divides $AC and BD$.

Watch Video Solution

35. Find the vector of magnitude 3, bisecting the angle between the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Watch Video Solution

36. If $\vec{a} and \vec{b}$ are two vectors of magnitude 1 inclined at 120^0, then find the angle between $\vec{b} and \vec{b} - \vec{a}$.

Watch Video Solution

37. If $\vec{r}_1, \vec{r}_2, \vec{r}_3$ are the position vectors of the collinear points and scalar $pandq$ exist such that $\vec{r}_3 = p\vec{r}_1 + q\vec{r}_2,$ then show that $p + q = 1.$

Watch Video Solution

38. Show that the vectors $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c} and \vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors (where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors)

Watch Video Solution

39. Let $\vec{a}, \vec{b} and \vec{c}$ be three units vectors such that $2\vec{a} + 4\vec{b} + 5\vec{c} = 0.$ Then which of the following statement is true? a. $\vec{a}$ is parallel to $\vec{b}$ b. $\vec{a}$ is perpendicular to $\vec{b}$ c. $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$ d. none of these

Watch Video Solution

40. Four non –zero vectors will always be a. linearly dependent b. linearly independent c. either a or b d. none of these

Watch Video Solution

41. A boat moves in still water with a velocity which is k times less than the river flow velocity. Find the angle to the stream direction at which the boat should be rowed to minimize drifting.

Watch Video Solution

42. In a triangle $PQR, SandT$ are points on $QRandPR,$ respectively, such that $QS = 3SRandPT = 4TR$. Let M be the point of intersection of $PSandQT$. Determine the ratio $QM : MT$ using the vector method .

Watch Video Solution

43. In a quadrilateral $PQRS, \vec{PQ} = \vec{a}, \vec{QR}, \vec{b}, \vec{SP} = \vec{a} - \vec{b}, M$ is the midpoint of $\vec{Q}RandX$ is a point on SM such that $SX = \frac{4}{5}SM^{\cdot}$ Prove that $P, XandR$ are collinear.

Watch Video Solution

44. If $D, EandF$ are three points on the sides $BC, CAandAB,$ respectively, of a triangle ABC such that the $\frac{BD}{CD}, \frac{CE}{AE}, \frac{AF}{BF} = -1$

Watch Video Solution

45. Sow that $x_1\hat{i} + y_1\hat{j} + z_1\hat{k}, x_2\hat{i} + y_2\hat{j} + z_2\hat{k}, andx_3\hat{i} + y_3\hat{j} + z_3\hat{k},$ are non-coplanar if $|x_1| > |y_1| + |z_1|, |y_2| > |x_2| + |z_2| and |z_3| > |x_3| + |y_3|$.

Watch Video Solution

46. The position vector of the points $PandQ$ are $5\hat{i} + 7\hat{j} - 2\hat{k}$ and $-3\hat{i} + 3\hat{j} + 6\hat{k}$, respectively. Vector $\vec{A} = 3\hat{i} - \hat{j} + \hat{k}$ passes through point P and vector $\vec{B} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through point Q . A third vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ intersects vectors $AandB^{\cdot}$ Find the position vectors of points of intersection.

Watch Video Solution

47. Consider the vectors $\hat{i} + \cos(\beta - \alpha)\hat{j} + \cos(\gamma - \alpha)\hat{k}, \cos(\alpha - \beta)\hat{i} + \hat{j} + \cos(\gamma - \beta)\hat{k} and \cos(\alpha - \gamma)\hat{i} + \cos(\beta - \gamma)\hat{k}$ where α, β, and γ are different angles. If these vectors are coplanar, show that a is independent of α, β and γ

Watch Video Solution

48. If $\vec{A} And \vec{B}$ are two vectors and k any scalar quantity greater than zero, then prove that $\left|\vec{A} + \vec{B}\right|^2 \leq (1 + k)\left|\vec{A}\right|^2 + \left(1 + \frac{1}{k}\right)\left|\vec{B}\right|^{2\cdot}$

Watch Video Solution

49. The vectors $x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}, (x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k} and (x+6)\hat{i} + (x+7)\hat{j} + (x+8$

are coplanar if x is equal to a. 1 b. -3 c. 4 d. 0

Watch Video Solution

50. $\vec{A}$ isa vector with direction cosines $\cos\alpha, \cos\beta and \cos\gamma$ Assuming the $y-z$ plane as a mirror, the directin cosines of the reflected image of $\vec{A}$ in the plane are a. $\cos\alpha, \cos\beta, \cos\gamma$ b. $\cos\alpha, -\cos\beta, \cos\gamma$ c. $-\cos\alpha, \cos\beta, \cos\gamma$ d. $-\cos\alpha, -\cos\beta, -\cos\gamma$

Watch Video Solution

51. The vector $\vec{a}$ has the components $2p$ and 1 w.r.t. a rectangular Cartesian system. This system is rotated through a certain angel about the origin in the counterclockwise sense. If, with respect to a new system, $\vec{a}$ has components $(p+1)and1$, then p is equal to a. -4 b. -1/3 c. 1 d. 2

Watch Video Solution

52. The sides of a parallelogram are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. The unit vector parallel to one of the diagonals is a. $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$ b. $\frac{1}{7}\left(3\hat{i}-6\hat{j}-2\hat{k}\right)$ c. $\frac{1}{\sqrt{69}}\left(\hat{i}+6\hat{j}+8\hat{k}\right)$ d. $\frac{1}{\sqrt{69}}\left(-\hat{i}-2\hat{j}+8\hat{k}\right)$

Watch Video Solution

53. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+\mu\vec{c}and(2\lambda-1)\vec{c}$ are coplanar when a. $\mu \in R$ b. $\lambda=\frac{1}{2}$ c. $\lambda=0$ d. no value of λ

Watch Video Solution

54. If points $\hat{i} + \hat{j}, \hat{i} - \hat{j} and p\hat{i} + q\hat{j} + r\hat{k}$ are collinear, then a. $p = 1$ b. $r = 0$ c. qR d. $q \neq 1$

Watch Video Solution

55. If the vectors $\hat{i} - \hat{j}, \hat{j} + \hat{k} and \vec{a}$ form a triangle, then $\vec{a}$ may be a. $-\hat{i} - \hat{k}$ b. $\hat{i} - 2\hat{j} - \hat{k}$ c. $2\hat{i} + \hat{j} + \hat{k}$ d. $\hat{i} + \hat{k}$

Watch Video Solution

56. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \vec{F}_2 = 6\hat{i} - \hat{k} and \vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a parricle has magnitude equal to 5 units, then the value of p is a. -6 b. -4 c. 2 d. 4

Watch Video Solution

57. $\vec{a}, \vec{b}, \vec{c}$ are three coplanar unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0.$ If three vectors $\vec{p}, \vec{q}, and \vec{r}$ are parallel to $\vec{a}, \vec{b}, and \vec{c},$ respectively, and have integral but different magnitudes, then among the following options, $\left|\vec{p} + \vec{q} + \vec{r}\right|$ can take a value equal to a. 1 b. 0 c. $\sqrt{3}$ d. 2

Watch Video Solution

58. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\dot{\hat{j}} + 2\hat{k}$. Then value of x are $-\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

Watch Video Solution

59. Prove that point $\hat{i}$ +2$\hat{j}$ - 3$\hat{k}$,2$\hat{i}$ - $\hat{j}$ + $\hat{k}$ and 2$\hat{i}$ + 5$\hat{j}$ - $\hat{k}$ from a triangle in space.

Watch Video Solution

60. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

Watch Video Solution

61. If $2\vec{AC} = 3\vec{CB}$, then prove that $2\vec{OA} = 3\vec{CB}$ then prove that $2\vec{OA} + 3\vec{OB} = 5\vec{OC}$ where O is the origin.

Watch Video Solution

62. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.

Watch Video Solution

63. For given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

Watch Video Solution

64. If the projections of vector $\vec{a}$ on x -, y - and z -axes are 2, 1 and 2 units ,respectively, find the angle at which vector $\vec{a}$ is inclined to the z -axis.

Watch Video Solution

65. Find a vector in the direction of the vector $5\hat{i} + \hat{j} - 2\hat{k}$ which has magnitude 6 units.

Watch Video Solution

66. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are the position vectors representing the vertices A, B, C, D of a parallelogram then write $\vec{d}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$

Watch Video Solution

67. Show that the four points (6,-7,0),(16,-19,-4),(0,3,-6),(2,-5,10) lie on a same plane.

Watch Video Solution

68. Statement 1: The direction cosines of one of the angular bisectors of two intersecting line having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$

Statement 2: The angle between the two intersection lines having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(a) Statement 1 and Statement 2, both are correct. Statement 2 is the correct explanation for Statement 1.

(a) Statement 1 and Statement 2, both are correct. Statement 2 is not the correct explanation for Statement 1.

(c) Statement 1 is correct but Statement 2 is not correct.

(d) Statement 2 is correct but Statement 1 is not correct.

Watch Video Solution

69. Statement 1: In $\text{Delta}ABC, \vec{A}B + \vec{B}C + \vec{C}A = 0$ Statement 2: If $\vec{O}A = \vec{a}, \vec{O}B = \vec{b}, then\vec{A}B = \vec{a} + \vec{b}$

Watch Video Solution

70. Statement 1: If $\vec{u}and\vec{v}$ are unit vectors inclined at an angle $\alpha and\vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x} = (\vec{u} + \vec{v})/\left(2\sin(\alpha/2)\right.$ Statement 2: If $\text{Delta}ABC$ is an isosceles triangle with $AB = AC = 1$, then the vector representing the bisector of angel A is given by $\vec{A}D = (\vec{A}B + \vec{A}C)/2.$

Watch Video Solution

71. A vector has components p and 1 with respect to a rectangular Cartesian system. The axes are rotted through an angel αabout the origin the anticlockwise sense. Statement 1: IF the vector has component $p + 2$ and 1 with respect to the new system, then $p = -1$. Statement 2: Magnitude of the original vector and new vector remains the same.

Watch Video Solution

72. Let ABC be a triangle, the position vectors of whose vertices are $7\hat{j} + 10\hat{k}, -\hat{i} + 6\hat{j} + 6\hat{k} and -4\hat{i} + 9\hat{j} + 6\hat{k} Then \text{Delta} ABC$ is a. isosceles b. equilateral c. right angled d. none of these

Watch Video Solution

73. If non-zero vectors $\vec{a} and \vec{b}$ are equally inclined to coplanar vector $\vec{c}, then \vec{c}$ can be a. $\frac{|\vec{a}|}{|\vec{a}| + 2|\vec{b}|}a + \frac{|\vec{b}|}{|\vec{a}| + |\vec{b}|}\vec{b}$ b. $\frac{|\vec{b}|}{|\vec{a}| + |\vec{b}|}a + \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}\vec{b}$ c.

$\frac{|\vec{a}|}{|\vec{a}| + 2|\vec{b}|}a + \frac{|\vec{b}|}{|\vec{a}| + 2|\vec{b}|}\vec{b}$ d. $\frac{|\vec{b}|}{2|\vec{a}| + |\vec{b}|}a + \frac{|\vec{a}|}{2|\vec{a}| + |\vec{b}|}\vec{b}$

Watch Video Solution

74. If $A(-4, 0, 3) and B(14, 2, -5)$, then which one of the following points lie on the bisector of the angle between $\vec{O}A and \vec{O}B(O$ is the origin of reference)? a. $(2, 2, 4)$ b. $(2, 11, 5)$ c. $(-3, -3, -6)$ d. $(1, 1, 2)$

Watch Video Solution

75. Prove that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

Watch Video Solution

76. Prove that the resultant of two forces acting at point O and represented by $\vec{O}B$ and $\vec{O}C$ is given by $2\vec{O}D$,where D is the midpoint of

BC.

Watch Video Solution

77. Two forces $\vec{AB}$ and $\vec{AD}$ are acting at vertex A of a quadrilateral ABCD and two forces $\vec{CB}$ and $\vec{CD}$ at C prove that their resultant is given by $4\vec{EF}$, where E and F are the midpoints of AC and BD, respectively.

Watch Video Solution

78. ABC is a triangle and P any point on BC. if $\vec{PQ}$ is the sum of $\vec{AP} + \vec{PB} + \vec{PC}$, show that ABPQ is a parallelogram and Q , therefore , is a fixed point.

Watch Video Solution

79. If vector $\vec{a} + \vec{b}$ bisects the angle between $\vec{a}$ and $\vec{b}$, then prove that $|\vec{a}| = |\vec{b}|$.

Watch Video Solution

Watch Video Solution

80. ABCDE is a pentagon .prove that the resultant of force $\vec{A}B$, $\vec{A}E$,$\vec{B}C$, $\vec{D}C$,$\vec{E}D$ and $\vec{A}C$,is 3$\vec{A}C$.

Watch Video Solution

81. if $\vec{A}o + \vec{O}B = \vec{B}O + \vec{O}C$,than prove that B is the midpoint of AC.

Watch Video Solution

82. If the resultant of three forces $\vec{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \vec{F}_2 = 6\hat{i} - \hat{k} and \vec{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a parricle has magnitude equal to 5 units, then the value of p is a. -6 b. -4 c. 2 d. 4

Watch Video Solution

83. Statement 1: Let $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ be the position vectors of four points $A, B, C and D$ and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0.$ Then points $A, B, C, and D$ are coplanar. Statement 2: Three non-zero, linearly dependent coinitial vector $\left(\vec{P}Q, \vec{P}R and \vec{P}S\right)$ are coplanar. Then $\vec{P}Q = \lambda\vec{P}R + \mu\vec{P}S, where \lambda and \mu$ are scalars.

Watch Video Solution

84. Statement 1:Let $A(\vec{a}), B\left(\vec{b}\right) and C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 3\hat{k} and \vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$ Then $OABC$ is a tetrahedron. Statement 2: Let $A(\vec{a}), B\left(\vec{b}\right) and C(\vec{c})$ be three points such that vectors $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar. Then $OABC$ is a tetrahedron where O is the origin.

Watch Video Solution

85. Statement 1: $\vec{a} = 3\vec{i} + p\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + q\vec{k}$ are parallel vectors if $p = 9/2 and q = 2.$ Statement 2: if $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} and \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ are parallel, then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}.$

Watch Video Solution

86. The position vectors of the vertices $A, B and C$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and \vec{c}$, respectively. A vector $\vec{d}$ is such that $\dot{\vec{d}\vec{a}} = \dot{\vec{d}\vec{b}} = \dot{\vec{d}\vec{c}} and \vec{d} = \lambda\left(\vec{b} + \vec{c}\right)^{\cdot}$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

Watch Video Solution

87. a and b form the consecutive sides of a regular hexagon $\dot{ABCDEF}$ Column I, Column II If $\vec{CD} = x\vec{a} + y\vec{b}$, then, p. $x = -2$ If $\vec{CE} = x\vec{a} + y\vec{b}$, then, q.$x = -1$ If $\vec{AE} = x\vec{a} + y\vec{b}$, then, r. $y = 1$ $\vec{AD} = -x\vec{b}$, then, s.$y = 2$

Watch Video Solution

88. Statement 1: $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4 and \left|\vec{a} + \vec{b}\right| = 5, then \left|\vec{a} - \vec{b}\right| = 5.$

Statement 2: The length of the diagonals of a rectangle is the same.

Watch Video Solution

89. If $\vec{a} = 7\hat{i} - 4\hat{k} and \vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, determine vector $\vec{c}$ along the internal bisector of the angle between of the angle between vectors $\vec{a} and \vec{b} such that \left|\vec{c}\right|$ =$5\sqrt{6}$

Watch Video Solution

90. Find a unit vector $\vec{c}$ if $-\vec{i} + \vec{j} - \vec{k}$ bisects the angle between $\vec{c}$ and $3\vec{i} + 4\vec{j}$.

Watch Video Solution

Watch Video Solution

91. The vectors $2i + 3\hat{j}, 5\hat{i} + 6\hat{j}$ and $8\hat{i} + \lambda\hat{j}$ have initial points at (1, 1). Find the value of λ so that the vectors terminate on one straight line.

Watch Video Solution

92. If $\vec{a}, \vec{b} and \vec{c}$ are three non-zero vectors, no two of which ar collinear, $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then find the value of $\left|\vec{a} + 2\vec{b} + 6\vec{c}\right|^{.}$

Watch Video Solution

93. Check whether the given three vectors are coplanar or non-coplanar. $-2\hat{i} - 2\hat{j} + 4\hat{k}, -2\hat{i} + 4\hat{j}, 4\hat{i} - 2\hat{j} - 2\hat{k}$

Watch Video Solution

94. Prove that the four points $6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{j} - 6\hat{k} and 2\hat{i} + 5\hat{j} + 10\hat{5}$ form a tetrahedron in space.

Watch Video Solution

95. If $\vec{a} and \vec{b}$ are two non-collinear vectors, show that points $l_1\vec{a} + m_1\vec{b}, l_2\vec{a} + m_2\vec{b}$ and $l_3\vec{a} + m_3\vec{b}$ are collinear if $\left|l_1 l_2 l_3 m_1 m_2 m_3 111\right| = 0.$

Watch Video Solution

96. Show, by vector methods, that the angularbisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices.

Watch Video Solution

97. Find the least positive integral value of x for which the angel between vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute.

Watch Video Solution

98. If vectors $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} and \vec{c} = \text{lambdai} + \hat{\text{j}} + 2\hat{\text{k}}$ are coplanar, then find the value of $(\lambda - 4)$.

Watch Video Solution

99. Find the values of λ such that $x, y, z \neq (0, 0, 0) and \left(\hat{i} + \hat{j} + 3\hat{k}\right)x + \left(3\hat{i} - 3\hat{j} + \hat{k}\right)y + \left(-4\hat{i} + 5\hat{j}\right)z = \lambda\left(x\hat{i} + y\hat{j} + z\hat{k}\right.$, where $\hat{i}, \hat{j}, \hat{k}$ are unit vector along coordinate axes.

Watch Video Solution

100. A vector has component $A_1, A_2 and A_3$ in a right -handed rectangular Cartesian coordinate system $OXYZ$. The coordinate system is rotated about the x-axis through an angel $\pi/2$. Find the component of A in the new coordinate system in terms of $A_1, A_2, and A_3$.

Watch Video Solution

101. Let $OACB$ be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. using vector methods prove that $BD and CO$ intersect in the same ratio. Determine this ratio.

Watch Video Solution

102. In a triangle $ABC, D and E$ are points on $BC and AC$, respectivley, such that $BD = 2DC and AE = 3EC$. Let P be the point of intersection of $AD and BE$. Find BP/PE using the vector method.

Watch Video Solution

103. The axes of coordinates are rotated about the z-axis though an angle of $\pi/4$ in the anticlockwise direction and the components of a vector are $2\sqrt{2},\ \ 3\sqrt{2}, 4.$ Prove that the components of the same vector in the original system are -1,5,4.

Watch Video Solution

104. If $a^{\rightarrow}\ \ , b^{\rightarrow}$ are the vectors forming consecutive sides of a regular hexagon ABCDEF, then the vector representing side CD is

Watch Video Solution

105. If two side of a triangle are $\hat{i} + 2\hat{j} and \hat{i} + \hat{k}$, then find the length of the third side.

Watch Video Solution

106. If in parallelogram ABCD, diagonal vectors are $\vec{AC} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{BD} = -6\hat{i} + 7\hat{j} - 2\hat{k}$, then find the adjacent side vectors $\vec{AB}$ and $\vec{AD}$

Watch Video Solution

107. Check whether the three vectors $2\hat{i} + 2\hat{j} + 3\hat{k}, -3\hat{i} + 3\hat{j} + 2\hat{k} and 3\hat{i} + 4\hat{k}$ from a triangle or not

Watch Video Solution

108. The midpoint of two opposite sides of a quadrilateral and the midpoint of the diagonals are the vertices of a parallelogram. Prove that using vectors.

Watch Video Solution

109. The lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

Watch Video Solution

110. Find the angle of vector $\vec{a} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ with x -axis.

Watch Video Solution

111. If the vectors $\vec{\alpha} = a\hat{i} + a\hat{j} + c\hat{k}, \vec{\beta} = \hat{i} + \hat{k} and \vec{\gamma} = c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then prove that c is the geometric mean of $aandb$.

Watch Video Solution

112. The points with position vectors $60i + 3j, 40i - 8j, ai - 52j$ are collinear if a. $a = -40$ b. $a = 40$ c. $a = 20$ d. none of these

Watch Video Solution

113. Lett α, β and γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$; $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ and $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

Watch Video Solution

114. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1 - x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1 + x - y)\vec{k}$. Then $\left[\vec{a}\vec{b}\vec{c}\right]$ depends on (A) only x (B) only y (C) Neither x nor y (D) both x and y

Watch Video Solution

115. In a $\triangle OAB$,E is the mid point of OB and D is the point on AB such that $AD:DB = 2:1$ If OD and AE intersect at P then determine the ratio of $OP:PD$ using vector methods

Watch Video Solution

116. If $\vec{a}, \vec{b}$ are two non-collinear vectors, prove that the points with position vectors $\vec{a} + \vec{b}, \vec{a} - \vec{b}$ and $\vec{a} + \lambda\vec{b}$ are collinear for all real values of $\dot{\lambda}$

Watch Video Solution

117. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors & $\left|\vec{c}\right| = \sqrt{3}$, then ordered pair (α, β) is $(1, 1)$ (b) $(1, -1)$ $(-1, 1)$ (d) $(-1, -1)$

Watch Video Solution

118. The number of distinct real values of λ , for which the vectors $\lambda^2\hat{i} + \hat{j} + k, \hat{i} - \lambda^2\hat{j} + \hat{k}and\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar is a. zero b. one c. two d. three

Watch Video Solution

119. If $\vec{AO} + \vec{OB} = \vec{BO} + \vec{OC}$, then $A, BnadC$ are (where O is the origin) a. coplanar b. collinear c. non-collinear d. none of these

Watch Video Solution

120. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Watch Video Solution

121. Show that the points A(1, -2, -8) B (5, 0, -2) and C(11, 3, 7) are collinear and find the ratio in which B divides AC.

Watch Video Solution

122. The position vectors of $PandQ$ are $5\hat{i} + 4\hat{j} + a\hat{k}$ and $-\hat{i} + 2\hat{j} - 2\hat{k}$, respectively. If the distance between them is 7, then find the value of a

Watch Video Solution

123. Given three points are $A(-3, -2, 0), B(3, -3, 1) and C(5, 0, 2)$. Then find a vector having the same direction as that of $\vec{A}B$ and magnitude equal to $\left|\vec{A}C\right|$.

Watch Video Solution

124. Let $ABCD$ be a p[arallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P$.

Watch Video Solution

125. If $ABCD$ is quadrilateral and $E and F$ are the mid-points of $AC and BD$ respectively, prove that $\vec{A}B + \vec{A}D + \vec{C}B + \vec{C}D = 4\ \vec{E}F$.

Watch Video Solution

126. If $ABCD$ is a rhombus whose diagonals cut at the origin O, then proved that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D + \vec{O}^{\cdot}$

Watch Video Solution

127. Let $D, EandF$ be the middle points of the sides $BC, CAandAB$, respectively of a triangle $ABC^{\cdot}$ Then prove that $\vec{A}D + \vec{B}E + \vec{C}F = \vec{0}$.

Watch Video Solution

128. Consider the set of eight vector $V = \left\{a\hat{i} + b\hat{j} + c\hat{k}; a, bc \in \{-1, 1\}\right\}^{\cdot}$ Three non-coplanar vectors can be chosen from V is 2^p ways. Then p is______.

Watch Video Solution

129. Find the direction cosines of the vector joining the points A (1,2,-3) and B(-1,-2,1) directed from A to B.

Watch Video Solution

130. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Watch Video Solution

131. The median AD of the triangle ABC is bisected at E and BE meets AC at F. Find AF:FC.

Watch Video Solution

132. Vectors $\vec{a}$ and $\vec{b}$ are non-collinear. Find for what value of n vectors $\vec{c} = (n - 2)\vec{a} + \vec{b}$ and $\vec{d} = (2n + 1)\vec{a} - \vec{b}$ are collinear?

Watch Video Solution

133. Prove that the necessary and sufficient condition for any four points in three-dimensional space to be coplanar is that there exists a liner relation connecting their position vectors such that the algebraic sum of the coefficients (not all zero) in it is zero.

Watch Video Solution

134. Points $A(\vec{a}), B(\vec{b}), C(\vec{c}) and D(\vec{d})$ are relates as $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ and $x + y + z + w = 0, where x, y, z, and w$ are scalars (sum of any two of $x, y, znadw$ is not zero). Prove that if $A, B, CandD$ are concylic, then $|xy|\left|\vec{a} - \vec{b}\right|^2 = |wz|\left|\vec{c} - \vec{d}\right|^{2}$.

Watch Video Solution

135. If $\vec{a}, \vec{b} and \vec{c}$ are non-coplanar vectors, prove that the four points $2\vec{a} + 3\vec{b} - \vec{c}, \vec{a} - 2\vec{b} + 3\vec{c}, 3\vec{a} + 4\vec{b} - 2\vec{c} and \vec{a} - 6\vec{b} + 6\vec{c}$ are coplanar.

Watch Video Solution

136. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Watch Video Solution

137. Let $\vec{a}, \vec{b} and \vec{c}$ be unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}, \dot{\vec{a}}\vec{x} = 1, \dot{\vec{b}}\vec{x} = \frac{3}{2}, |\vec{x}| = 2.$ Then find the angel between and $\dot{\times}$

Watch Video Solution

138. Let $\vec{A} and \vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisects the internal angle between $\vec{A} and \vec{B}$, then find the value of $\dot{\alpha}$

Watch Video Solution

139. If the vectors $3\vec{p} + \vec{q}; 5p - 3\vec{q} and 2\vec{p} + \vec{q}; 4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, then find the angle between vectors $\vec{p} and \vec{q}$.

Watch Video Solution

140. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(, -2, -1),$ then find the projection length of $\vec{P}Q on \vec{R}S$.

Watch Video Solution

141. A, B, C, D are any four points, prove that $\vec{A}B.\vec{C}D + \vec{B}C.\vec{A}D + \vec{C}A.\vec{B}D = 0.$

Watch Video Solution

142. Let $\hat{u} = \hat{i} + \hat{j}, \hat{v} = \hat{i} - \hat{j} and \hat{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\hat{u}.\hat{n} = 0 and \hat{v}.\hat{n} = 0,$ then find the value of $\left|\hat{w}.\hat{n}\right|$.

Watch Video Solution

Watch Video Solution

143. If the angel between unit vectors $\vec{a} and \vec{b} 60^0$, then find the value of $\left|\vec{a} - \vec{b}\right|$.

Watch Video Solution

144. $\vec{a} + \vec{b} + \vec{c} = \vec{0}, \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 9$,find the angle between $\vec{a}$ and $\vec{c}$.

Watch Video Solution

145. Constant forces $P_1 = \hat{i} + \hat{j} + \hat{k}, P_2 = -\hat{i} + 2\hat{j} - \hat{k} and P_3 = -\hat{j} - \hat{k}$ act on a particle at a point A. Determine the work done when particle is displaced from position $A\left(4\hat{i} - 3\hat{j} - 2\hat{k}\right) \rightarrow B\left(6\hat{i} + \hat{j} - 3\hat{k}\right)$.

Watch Video Solution

146. If $\vec{a}, and \vec{b}$ are unit vectors , then find the greatest value of $\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|$.

Watch Video Solution

147. Let $G_1, G_2 and G_3$ be the centroids of the triangular faces $OBC, OCA and OAB,$ respectively, of a tetrahedron $OABC$. If V_1 denotes the volumes of the tetrahedron $OABC and V_2$ that of the parallelepiped with $OG_1, OG_2 and OG_3$ as three concurrent edges, then prove that $4V_1 = 9V_1$.

Watch Video Solution

148. Prove that $\hat{i} \times \left(\vec{a} \times \hat{i}\right)\hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}$.

Watch Video Solution

149. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] + \hat{j} \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \hat{k} \times \left[\left(\vec{a} - \hat{i}\right) \times \hat{k}\right] = 0,$ then find vector $\vec{a}$.

Watch Video Solution

150. Let $\vec{a}, \vec{b}, and \vec{c}$ be any three vectors, then prove that $\left[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}\right] = \left[\vec{a} \vec{b} \vec{c}\right]^2$.

Watch Video Solution

151. If $\left[\vec{a} \vec{b} \vec{c}\right] = 2,$ then find the value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right]$.

Watch Video Solution

152. If $\vec{a}, \vec{b}, and \vec{c}$ are mutually perpendicular vectors and $\vec{a} = \alpha(\vec{a} \times \vec{b}) + \beta(\vec{b} \times \vec{c}) + \gamma(\vec{c} \times \vec{a}) and [\vec{a}\vec{b}\vec{c}] = 1,$ then find the value of $\alpha + \beta + \gamma^{\cdot}$

Watch Video Solution

153. If $a, bandc$ are non-cop0lanar vector, then that prove $\left|\left(\vec{a}\dot{\vec{d}}\right)(\vec{b} \times \vec{c}) + \left(\vec{b}\dot{\vec{d}}\right)(\vec{c} \times \vec{a}) + \left(\vec{c}\dot{\vec{d}}\right)(\vec{a} \times \vec{b})\right|$ is independent of $d, wheree$ is a unit vector.

Watch Video Solution

154. Prove that vectors $\vec{u} = (al + a_1l_1)\hat{i} + (am + a_1m_1)\hat{j} + (an + a_1n_1)\hat{k}$
$\vec{v} = (bl + b_1l_1)\hat{i} + (bm + b_1m_1)\hat{j} + (bn + b_1n_1)\hat{k}$
$\vec{w} = (bl + b_1l_1)\hat{i} + (bm + b_1m_1)\hat{j} + (bn + b_1n_1)\hat{k}$ are coplanar.

Watch Video Solution

155. For any four vectors, prove that

$$\left(\vec{b} \times \vec{c}\right) \cdot \vec{a} \times \vec{d} + \left(\vec{c} \times \vec{a}\right) \cdot \vec{b} \times \vec{d} + \left(\vec{a} \times \vec{b}\right) \cdot \vec{c} \times \vec{d} = 0.$$

Watch Video Solution

156. If $\vec{b} and \vec{c}$ are two-noncollinear vectors such that $\vec{a} \mid \mid \left(\vec{b} \times \vec{c}\right)$, then prove that $\left(\vec{a} \times \vec{b}\right) . \left(\vec{a} \times \vec{c}\right)$ is equal to $\left|\vec{a}\right|^2 \left(\vec{b} \cdot \vec{c}\right).$

Watch Video Solution

157. If the vertices A,B, C of a triangle ABC are (1,2,3),(-1, 0,0), (0, 1,2), respectively, then find $\angle ABC$.

Watch Video Solution

158. Let $\vec{a}, \vec{b} and \vec{c}$ be pairwise mutually perpendicular vectors, such that $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 2.$ Then find the length of $\vec{a} + \vec{b} + \vec{c}$

Watch Video Solution

159. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$, is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Watch Video Solution

160. If $|\vec{a}| = 3, |\vec{b}| = 4 and$ the angle between $a and b$ is 120°, then find the value of $|4\vec{a} + 3\vec{b}|^{.}$

Watch Video Solution

161. If $\vec{a}, \vec{b}, and \vec{c}$ be three non-coplanar vector and $a', b' and c'$ constitute the reciprocal system of vectors, then prove that

$$\vec{r}=\left(\vec{r}\dot{\vec{a}}'\right)\vec{a}+\left(\vec{r}\dot{\vec{b}}'\right)\vec{b}+\left(\vec{r}\dot{\vec{c}}'\right)\vec{c}\ \vec{r}=\left(\vec{r}\dot{\vec{a}}'\right)\vec{a}'+\left(\vec{r}\dot{\vec{b}}'\right)\vec{b}'+\left(\vec{r}\dot{\vec{c}}'\right)\vec{c}'$$

Watch Video Solution

162. Find $|\vec{a}|and|\vec{b}|$, if $\left(\vec{a}+\vec{b}\right)\dot{\vec{a}}-\vec{b}=8,\ |\vec{a}|=8|\vec{b}|^{.}$

Watch Video Solution

163. Let $\vec{a},\vec{b},and\vec{c}and\vec{a}',\vec{b}',\vec{c}'$ are reciprocal system of vectors, then prove that $\vec{a}'\times\vec{b}'+\vec{b}'\times\vec{c}'+\vec{c}'\times\vec{a}'=\dfrac{\vec{a}+\vec{b}+\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$.

Watch Video Solution

164. If $\vec{a},\vec{b},and\vec{c}$ are three non-coplanar non-zero vecrtors, then prove that $(\vec{a}.\vec{a})\vec{b}\times\vec{c}+\left(\vec{a}.\vec{b}\right)\vec{c}\times\vec{a}+(\vec{a}.\vec{c})\vec{a}\times\vec{b}=\left[\vec{b}\vec{c}\vec{a}\right]\vec{a}$

Watch Video Solution

165. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, \hat{i}+\hat{j}+\hat{k}$.

Watch Video Solution

166. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}\neq 0, where \vec{a}, \vec{b}, and \vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a}+\vec{c}=k\vec{b}$.

Watch Video Solution

167. If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}, \vec{b}=-\vec{i}+2\vec{j}-4\vec{k} and \vec{c}=\vec{i}+\vec{j}+\vec{k}$, then find thevalue of $\left(\vec{a}\times\vec{b}\right)\cdot\vec{a}\times\vec{c}$.

Watch Video Solution

168. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k} and \vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c} and \vec{b}$ form a right-handed system, then find $\vec{\cdot}$

Watch Video Solution

169. Given that $\vec{a}\dot{\vec{b}} = \vec{a}\dot{\vec{c}}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c} and \vec{a}$ is not a zero vector. Show that $\vec{b} = \vec{\cdot}$

Watch Video Solution

170. If A, B, C, D are four distinct point in space such that AB is not perpendicular to CD and satisfies $\vec{A}B\dot{\vec{C}}D = k\left(\left|\vec{A}D\right|^2 + \left|\vec{B}C\right|^2 - \left|\vec{A}C\right|^2 - \left|\vec{B}D\right|^2\right)$, then find the value of $\dot{k}$

Watch Video Solution

171. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k} and \vec{a} \times \vec{b} = \vec{0},$ then find $(m, n)^{\cdot}$

Watch Video Solution

172. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$, find the value of $\vec{a}.\vec{b}$

Watch Video Solution

173. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)^{\cdot}$

Watch Video Solution

174. Show that for any three vectors $\vec{a}, \vec{b}$ and $\vec{c}\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right].$

Watch Video Solution

175. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors and $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$. If the angel between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$.

Watch Video Solution

176. If the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k} and 3\hat{i} - \hat{k}$ form three concurrent edges of a parallelepiped, then find the volume of the parallelepiped.

Watch Video Solution

177. If $\vec{u}, \vec{v} and \vec{w}$ are three non-cop0lanar vectors, then prove that

$$\left(\vec{u} + \vec{v} - \vec{w}\right)\dot{\vec{u}} - \vec{v} \times \left(\vec{v} - \vec{w}\right) = \dot{\vec{u}}\dot{\vec{v}} \times \vec{w}$$

Watch Video Solution

178. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k} and a\hat{i} + \hat{k}$ becomes minimum.

Watch Video Solution

Watch Video Solution

179. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k},$ then find the vaue of

$$\left| \vec{a} \cdot \vec{a} \, \vec{a} \cdot \vec{b} \, \vec{a} \cdot \vec{c} \, \vec{b} \cdot \vec{a} \, \vec{b} \cdot \vec{a} \, \vec{b} \cdot \vec{a} \, \vec{a} \cdot \vec{a} \cdot \vec{a} \cdot \vec{a} \right|.$$

Watch Video Solution

180. Prove that $\left[\vec{l}\,\vec{m}\,\vec{n}\right]\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left| \vec{l} \cdot \vec{a} \, \vec{l} \cdot \vec{b} \, \vec{l} \cdot \vec{c} \, \vec{m} \cdot \vec{a} \, \vec{m} \cdot \vec{a} \, \vec{m} \cdot \vec{a} \, \vec{n} \cdot \vec{a} \, \vec{n} \cdot \vec{a} \, \vec{n} \cdot \vec{a} \right|.$

Watch Video Solution

181. Find the altitude of a parallelepiped whose three coterminous edtges are vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} - \hat{k} and \vec{C} = \hat{i} + \hat{j} + 3\hat{k} with \vec{A} and \vec{B}$ as the sides of the base of the parallepiped.

Watch Video Solution

182. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\times\vec{b}\right| = 2,$ then find the value of $\left[\vec{a}\vec{b}\vec{a}\times\vec{b}\right]$.

Watch Video Solution

183. Prove that

$$\vec{R}+\frac{\left[\vec{R}\vec{\beta}\times\left(\vec{\beta}\times\vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha}\times\vec{\beta}\right|^2}+\frac{\left[\vec{R}\vec{\alpha}\times\left(\vec{\alpha}\times\vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha}\times\vec{\beta}\right|^2}=\frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha}\times\vec{\beta}\right)}{\left|\vec{\alpha}\times\vec{\beta}\right|^2}$$

Watch Video Solution

184. If $\vec{a}, \vec{b}, and \vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}+\vec{c}}{\sqrt{2}}, \vec{b} and \vec{c}$ are non-parallel, then prove that the angel between $\vec{a} and \vec{b} is 3\pi/4.$

Watch Video Solution

185. If $\vec{a} and \vec{b}$ are two given vectors and k is any scalar, then find the vector $\vec{r}$ satisfying $\vec{r} \times \vec{a} + k\vec{r} = \vec{b}$.

Watch Video Solution

186. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k} and 2\hat{k} - 3\hat{j}$.

Watch Video Solution

187. If $\vec{a}, \vec{b} and \vec{c}$ are three non coplanar vectors, then prove that

$$\vec{d} = \frac{\vec{a}.\vec{d}}{[\vec{a}\vec{b}\vec{c}]}\left(\vec{b} \times \vec{c}\right) + \frac{\vec{b}.\vec{d}}{[\vec{a}\vec{b}\vec{c}]}\left(\vec{c} \times \vec{a}\right) + \frac{\vec{\cdot}\vec{d}}{[\vec{a}\vec{b}\vec{c}]}\left(\vec{a} \times \vec{b}\right)$$

Watch Video Solution

188. If $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}\dot{}\vec{b}\right)^2 = 144 and \left|\vec{a}\right| = 4,$ then find the value of $\left|\vec{b}\right|$.

Watch Video Solution

189. A particle has an angualar speed of 3 rad/s and the axis of rotation passes through the point (1,1,2) and (1,1,-2) find the velocity of the particle at point p(3,6,4)

Watch Video Solution

190. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F} = 3\hat{i} + 2\hat{j}$ - $4\hat{k}$ is acting on point (1, -1, 2).

Watch Video Solution

191. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1 and \left|\vec{a} + \vec{b}\right| = \sqrt{3}$. If $\vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$, then find the value of $\vec{c} \cdot \vec{b}$

Watch Video Solution

192. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} and \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a} and \vec{b}$. If the angle between $a and b$ is $\frac{\pi}{6}$, then prove that $\left|\left(a_1a_2a_3\right)\left(b_1b_2b_3\right)\left(c_1c_2c_3\right)\right| = \frac{1}{4}(a12 + a22 + a32)(b12 + b22 + b32)$

Watch Video Solution

193. If $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ are four non-coplanar unit vector such that $\vec{d}$ make equal angles with all the three vectors $\vec{a}, \vec{b} and \vec{c}$, then prove that $\left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$.

Watch Video Solution

194. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)$

Watch Video Solution

195. Find λ if the vectors $\vec{a} = \vec{i} + 3\vec{j} + \vec{k}$,$\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ and $\vec{c} = \lambda\vec{i} + 7\vec{j} + 3\vec{k}$ are coplanar

Watch Video Solution

196. Using dot product of vectors, prove that a parallelogram , whose diagonals are equal , is a rectangle.

Watch Video Solution

197. If $a + 2b + 3c = 4$, then find the least value of $a^2 + b^2 + c^2$.

Watch Video Solution

198. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

Watch Video Solution

199. If $\vec{a}.\hat{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$, then find the unit vector $\vec{a}$

Watch Video Solution

200. Prove by vector method that $\cos(A + B)\cos A\cos B - \sin A\sin B$.

Watch Video Solution

201. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}$ on vector $2\hat{i} - \hat{j} + 5\hat{k} is \frac{1}{\sqrt{30}}$,

then find the value of x

Watch Video Solution

202. A unit vector a makes an angle $\frac{\pi}{4}$ with z-axis. If $a + i + j$ is a unit vector, then a can be equal to

Watch Video Solution

203. if $\vec{a}, \vec{b}$ and $\vec{c}$ are there mutually perpendicular unit vectors and $\vec{a}$ ia a unit vector make equal angles which $\vec{a}, \vec{b}$ and $\vec{c}$ then find the value of $\left|\vec{a} + \vec{b} + \vec{c} + \vec{d}\right|^2$

Watch Video Solution

204. If $\vec{a}, \vec{b}, and \vec{c}$ be non-zero vectors such that no tow are collinear or $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}^{\cdot}$ If θ is the acute angle between vectors $\vec{b} and \vec{c}$, then find the value of $\sin\theta^{\cdot}$

Watch Video Solution

205. If $\vec{p}, \vec{q}, \vec{r}$ denote vector $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$, respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}, \vec{b}$ is parallel $\vec{r} \times \vec{p}, \vec{c}$ is parallel to $\vec{p} \times \vec{q}^{\cdot}$

Watch Video Solution

206. If $\vec{a}, and \vec{b}$ be two non-collinear unit vector such that $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \frac{1}{2}\vec{b},$ then find the angle between $\vec{a}, and \vec{b}^{\cdot}$

Watch Video Solution

207. Prove that $\left(\vec{a}\left(\vec{b}\times\hat{i}\right)\right)\hat{i}+\left(\vec{a}\left(\vec{b}\times\hat{j}\right)\right)\hat{j}+\left(\vec{a}\left(\vec{b}\times\hat{k}\right)\right)\hat{k}=\vec{a}\times\vec{b}$

Watch Video Solution

208. If $\vec{a}, \vec{b}, and \vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}, \vec{c}\times\vec{a}=\vec{b}$, then prove that $\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|$

Watch Video Solution

209. If $\vec{a}=\vec{p}+\vec{q}, \vec{p}\times\vec{b}=0 and \vec{q}\vec{b}=0$, then prove that $\dfrac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{\vec{b}\vec{b}}=\vec{q}$

Watch Video Solution

210. If $\vec{a} = \hat{i} + \hat{j} + \hat{k} and \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, then find vector $\vec{c}$ such that $\vec{a} \cdot \vec{c} = 2 and \vec{a} \times \vec{c} = \vec{b}$.

Watch Video Solution

211. If $\vec{a}, \vec{b}, \vec{c}$ are any three mutually perpendicular vectors of equal magnitude a, then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is equal to

Watch Video Solution

212. If three unit vectors $\vec{a}, \vec{b}, and \vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = 0$, then find the angle between $\vec{a} and \vec{b}$.

Watch Video Solution

213. If $\left|\vec{a}\right| + \left|\vec{b}\right| = \left|\vec{c}\right| and \vec{a} + \vec{b} = \vec{c}$, then find the angle between $\vec{a} and \vec{b}$.

Watch Video Solution

214. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k} and 3\hat{i} - 2\hat{j} + \hat{k}$

Watch Video Solution

215. If $\vec{r}\hat{i} = \vec{r}\hat{j} = \vec{r}\hat{k} and \left|\vec{r}\right| = 3$, then find the vector $\vec{r}$

Watch Video Solution

216. If $\vec{a}, \vec{b}, and \vec{c}$ are non-zero vectors such that $\vec{a}\vec{b} = \vec{a}\vec{c}$, then find the geometrical relation between the vectors.

Watch Video Solution

217. Find the projection of vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

Watch Video Solution

Watch Video Solution

218. If θ is th angel between the unit vectors a and b , then prove that $\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} + \vec{b}\right|.,, \sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

Watch Video Solution

219. Given unit vectors $\hat{m}, \hat{n} and \hat{p}$ such that angel between $\hat{m} and \hat{n}$ is α and angle between $\hat{p} and \left(\hat{m} \times \hat{n}\right)$ is also α, if $\left[\hat{n}\hat{p}\hat{m}\right] = 1/4$, then find the value of α

Watch Video Solution

220. The base of the pyramid $AOBC$ is an equilateral triangle OBC with each side equal to $4\sqrt{2}$, O is the origin of reference, AO is perpendicualar to the plane of OBC and $\left|\vec{AO}\right| = 2.$ Then find the cosine of the angle between the skew straight lines, one passing though A and the midpoint of $OBand$ the other passing through O and the mid point of $BC^{\cdot}$

Watch Video Solution

221. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}and\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}^{\cdot}$

Watch Video Solution

222. Let the vectors $\vec{a}and\vec{b}$ be such that $|\vec{a}|=3\left|\vec{b}\right|=\frac{\sqrt{2}}{3}, then\vec{a}\times\vec{b}$ is a unit vector, if the angel between $\vec{a}and\vec{b}$ is?

Watch Video Solution

223. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)^{\cdot}$

Watch Video Solution

224. If A, B and C are the vectices of a triangle ABC, then prove sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Watch Video Solution

225. Application of cross product trigonometric proof; sin(A+B) = sinAcosB + cosAsinB

Watch Video Solution

226. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)$.

Watch Video Solution

227. If $\vec{a} and \vec{b}$ are two vectors, then prove that $\left(\vec{a} \times \vec{b}\right)^2 = \left|\dot{\vec{a}}\vec{a}\dot{\vec{a}}\vec{b}\dot{\vec{b}}\vec{a}\dot{\vec{b}}\vec{b}\right|$.

Watch Video Solution

228. In isosceles triangles ABC, $\left|\vec{A}B\right| = \left|\vec{B}C\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}Eand\vec{C}A\left(where\left|\vec{C}A\right| = 12\right)^{.}$

Watch Video Solution

229. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular , then the third pair is also perpendicular.

Watch Video Solution

230. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1$, then find the value of $\left|\vec{a} - \vec{b}\right|^{.}$

Watch Video Solution

231. If $\vec{a} = 4\hat{i} + 6\hat{j} and \vec{b} = 3\hat{j} + 4\hat{k},$ then find the component of $\vec{a} and \vec{b}^{\cdot}$

Watch Video Solution

232. If $\vec{a}, \vec{b}, and \vec{c}$ are there mutually perpendicular unit vectors and $\vec{d}$ is a unit vector which makes equal angles with $\vec{a}, \vec{b}, and \vec{c}$, the find the value off $\left|\vec{a} + \vec{b} + \vec{c} + \vec{d}\right|^{2^{\cdot}}$

Watch Video Solution

233. The position vectors of the vertices of a quadrilateral with A as origin are $B\left(\vec{b}\right), D\left(\vec{d}\right) and C\left(l\vec{b} + m\vec{d}\right)^{\cdot}$ Prove that the area of the quadrialateral is $\frac{1}{2}(l + m)\left|\vec{b} \times \vec{d}\right|^{\cdot}$

Watch Video Solution

234. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d} and \vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$, is parallel to $\vec{b} - \vec{c}$ provided $\vec{a} \ne \ \leftrightarrow \ d$ and $\vec{b} \ne \vec{c}$.

Watch Video Solution

235. Show by a numerical example and geometrically also that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does not imply $\vec{b} = \vec{c}$.

Watch Video Solution

236. In triangle $ABC, po \in tsD, EandF$ are taken on the sides $BC, CAandAB$, respectigvely, such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$. Prove that $\_\ (DEF) = \frac{n^2 - n + 1}{\left((n+1)^2\right)_{ABC}}$.

Watch Video Solution

237. Let $\vec{a} and \vec{b}$ be unit vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{3}$. Then find the value of $\left(2\vec{a}+5\vec{b}\right)3\vec{a}+\vec{b}+\vec{a}\times\vec{b}$.

Watch Video Solution

238. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

Watch Video Solution

239. $\vec{r}\times\vec{a}=\vec{b}\times\vec{a};\vec{r}\times\vec{b}=\vec{a}\times\vec{b};\vec{a}\neq\vec{0};\vec{b}\neq\vec{0};\vec{a}\neq\lambda\vec{b}, and \vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a} and \vec{b}$.

Watch Video Solution

240. If $|\vec{a}| = 2,$ then find the value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$.

Watch Video Solution

241. If $\vec{a}, \vec{b} and \vec{c}$ are the position vectors of the vertices $A, B and C$ respectively, of $ABC,$ prove that the perpendicular distance of the vertex A from the base BC of the triangle ABC is $\dfrac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{c} - \vec{b}|}$.

Watch Video Solution

242. If A, B , C ,D are any four points in space, prove that $\left|\overrightarrow{AB} \times \overrightarrow{CD} \times \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}\right|$ = 4 (area of triangle ABC).

Watch Video Solution

243. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k} and \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Watch Video Solution

244. Using vectors, find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

Watch Video Solution

245. Find the area a parallelogram whose diagonals are $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k} and \vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

Watch Video Solution

246. If $\vec{a} and \vec{b}$ are unit vectors such that $\left(\vec{a} + \vec{b}\right).\left(2\vec{a} + 3\vec{b}\right) \times \left(3\vec{a} - 2\vec{b}\right) = 0$, then angle between $\vec{a}$ and $\vec{b}$ is a. 0 b.

$\pi/2$ c. π d. indeterminate

Watch Video Solution

247. If the vectors $\vec{a}, \vec{b}, and \vec{c}$ form the sides$BC, CAandAB,$ respectively, of triangle $ABC, then$ (a)$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$ (b)$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ (c) $\dot{\vec{a}}\vec{b} = \dot{\vec{b}}\vec{c} = \dot{\vec{c}}\vec{a}$ (d) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

Watch Video Solution

248. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle $60^{0^{\cdot}}$ Suppose that $\left|\vec{u} - \hat{i}\right|$ is geometric mean of $\left|\vec{u}\right| and \left|\vec{u} - 2\hat{i}\right|, where \hat{i}$ is the unit vector along the x-axis. Then find the value of $\left(\sqrt{2} + 1\right)\left|\vec{u}\right|^{\cdot}$

Watch Video Solution

249. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{A}D = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by a. $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

Watch Video Solution

250. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, find scalars $p, q and r$ in terms of θ.

Watch Video Solution

251. Given three vectors $\vec{a}, \vec{b}, and \vec{c}$ two of which are non-collinear. Further if $\left(\vec{a} + \vec{b}\right)$ is collinear with $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is collinear with

$\vec{a}, |\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{2}$ Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ a. 3 b. -3 c. 0 d. cannot be evaluated

Watch Video Solution

252. $A_1, A_2, \ldots, A_n$ are the vertices of a regular plane polygon with n sides and O as its centre. Show that $\sum_{i=1}^{n} \overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}} = (1 - n)\left(\overrightarrow{OA_2} \times \overrightarrow{OA_1}\right)$

Watch Video Solution

253. If $\vec{c}$ is a given non-zero scalar, and $\vec{A} and \vec{B}$ are given non-zero vector such that $\vec{A} \perp B$, then find vector $\vec{X}$ which satisfies the equation $\vec{A}.\vec{X} = c$ and $\vec{A} \times \vec{X} = \vec{B}$

Watch Video Solution

254. If A, B , C ,D are any four points in space, prove that $\left|\vec{AB} \times \vec{CD} \times \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}\right|$ = 4 (area of triangle ABC).

Watch Video Solution

255. If vectors $\vec{a}, \vec{b}, and \vec{c}$ are coplanar, show that `| vec a vec b vec c vec adot vec a vec adot vec b vec adot vec c vec bdot vec a vec bdot vec b vec bdot vec c|=0

Watch Video Solution

256. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}\dot{} \vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}\dot{}\vec{A} = 0.$

Watch Video Solution

257. Determine the value of c so that for all real x , vectors $cx\hat{i} - 6\hat{j} - 3\hat{k} and x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

Watch Video Solution

258. If $\vec{r} = x_1\left(\vec{a} \times \vec{b}\right) + x_2\left(\vec{b} \times \vec{c}\right) + x_3\left(\vec{c} \times \vec{a}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1,$ then $x_1 + x_2 + x_3$ is equal to (A) $\frac{1}{2}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (B) $\frac{1}{4}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (C) $2\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$ (D) $4\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

Watch Video Solution

259. $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\left(\vec{c} \times \vec{a}\right) \times \left(\vec{a} \times \vec{b}\right)\right]$ is equal to (where $\vec{a}, \vec{b} and \vec{c}$ are nonzero non-coplanar vector) $\left[\vec{a}\vec{b}\vec{c}\right]^2$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^3$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^4$ d. $\left[\vec{a}\vec{b}\vec{c}\right]$

Watch Video Solution

260. If $\vec{a}, \vec{b} and \vec{c}$ are non coplanar vectors and $\vec{a} \times \vec{c}$ is perpendicular to $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$, then the value of $\left[a \times \left(\vec{b} \times \vec{c}\right)\right] \times \vec{c}$ is equal to a. $\left[\vec{a}\vec{b}\vec{c}\right]$ b. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{b}$ c. $\vec{0}$ d. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

Watch Video Solution

261. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q} and \vec{r}$ the vectors defined by the relation $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]} and \vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. Then the value of the expression $\left(\vec{a} + \vec{b}\right)\cdot\vec{p} + \left(\vec{b} + \vec{c}\right)\cdot\vec{q} + \left(\vec{c} + \vec{a}\right)\cdot\vec{r}$ is a.0 b. 1 c. 2 d. 3

Watch Video Solution

262. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vecrors and $\vec{r}$ be any arbitrary vector. Then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times \vec{b}\right)$ is always equal to $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ b. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ c. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ d. none of these

Watch Video Solution

263. The position vectors of point $A, B, and C$ are $\hat{i} + \hat{j} + \hat{k}, \hat{i} + 5\hat{j} - \hat{k} and 2\hat{i} + 3\hat{j} + 5\hat{k},$ respectively. Then greatest angel of triangle ABC is 120^0 b. 90^0 c. $\cos^{-1}(3/4)$ d. none of these

Watch Video Solution

264. Let $\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j} and \vec{b}(x) = (\cos 2x)\hat{i} + \left(\sin 2x\hat{j}\right)$ be two variable vectors $(x \in R)$. Then $\vec{a}(x) and \vec{b}(x)$ are a. collinear for unique value of x b. perpendicular for infinite values of x c. zero vectors for unique value of x d. none of these

Watch Video Solution

265. If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ $,\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ $,\vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ a n d $(1 + \alpha)\hat{i} + \beta(1 + \alpha)\hat{j} + \gamma(1 + \alpha)(1 + \beta)\hat{k} = \vec{a} \times \left(\vec{b} \times \vec{c}\right),$ then α, β and γ are $a.$

$-2, -4, -\frac{2}{3}$ $b.$ $2, -4, \frac{2}{3}$ $c.$ $-2, 4, \frac{2}{3}$ $d.$ $2, 4, -\frac{2}{3}$`

Watch Video Solution

266. If $\vec{a}, \vec{b} and \vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9,$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is.

Watch Video Solution

267. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then a. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$ b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$ c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar d. none of these

Watch Video Solution

268. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k} and \hat{i} + 2\hat{j} + \hat{k},$ and perpendicular to vector $\hat{i} + \hat{j} + \hat{k},$ is/are a.$\hat{j} - \hat{k}$

b. $-\hat{i}+\hat{j}$ c. $\hat{i}-\hat{j}$ d. $-\hat{j}+\hat{k}$

Watch Video Solution

269. Let $\vec{a} = -\hat{i}-\hat{k}, \vec{b} = -\hat{i}+\hat{j} and \vec{c} = \hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$, then find the value of $\vec{r}.\vec{b}$

Watch Video Solution

270. Let $\vec{a}, \vec{b}, and \vec{c}$ be vectors forming right-hand traid. Let $\vec{p} = \frac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}, and \vec{r} = \frac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. If $x \cup R^{+}$, then a. $x\left[\vec{a}\vec{b}\vec{c}\right] + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x}$ b. $x^4\left[\vec{a}\vec{b}\vec{c}\right]^2 + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x^2}$ has least value $= \left(\frac{3}{2}\right)^{2/3}$ c. $\left[\vec{p}\vec{q}\vec{r}\right] > 0$ d. none of these

Watch Video Solution

271. from a point O inside a triangle ABC, perpendiculars, OD, OE and OF are drawn to the sides, BC, CA and AB respectively , prove that the perpendiculars from A, B and C to the sides EF, FD and DE are concurrent.

Watch Video Solution

272. If $aandb$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}} and \vec{b} = \frac{\hat{2}i + \hat{j} + 3\hat{k}}{\sqrt{14}}$,

then find the value of $\left(2\vec{a} + \vec{b}\right)\dot{\left(\vec{a} \times \vec{b}\right)} \times \left(\vec{a} - 2\vec{b}\right)^{\cdot}$

Watch Video Solution

273. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acting on a particle such that the particle is displaced from point $A(- 3, \ - 4, 1) \text{ and } B(- 1, \ - 1, \ - 2)^{\cdot}$

Watch Video Solution

274. If $\vec a, \vec b, \vec c$ are three given non-coplanar vectors and any arbitrary vector $\vec r$ in space, where $\Delta 1 = \begin{vmatrix} \vec r\vec a\vec b\vec a\cdot\vec a\vec r\vec b\vec b\vec b\cdot\vec b\vec r\vec c\vec b\vec c\cdot\vec c \end{vmatrix}$,

$\Delta 2 = \begin{vmatrix} \vec a\vec a\vec r\vec a\cdot\vec a\vec a\vec b\vec r\vec b\cdot\vec b\vec a\vec c\vec r\vec c\cdot\vec c \end{vmatrix}$ $\Delta 3 = \begin{vmatrix} \vec a\vec a\vec b\vec a\vec r\vec a\vec a\vec b\vec b\vec b\vec r\vec b\vec a\vec c\vec b\vec c\vec r\vec c \end{vmatrix}$,

$\Delta = \begin{vmatrix} \vec a\vec a\vec b\vec a\cdot\vec a\vec a\vec b\vec b\vec b\cdot\vec b\vec a\vec c\vec b\vec c\cdot\vec c \end{vmatrix}$, then prove that

$\vec r = \frac{\Delta 1}{\Delta}\vec a + \frac{\Delta 2}{\Delta}\vec b + \frac{\Delta 3}{\Delta}\vec c$.

Watch Video Solution

275. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge AC. Prove that DE is equal to half the edge of tetrahedron.

Watch Video Solution

276. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i.\vec{E}_j = 1$, if $i = j and \vec{e}_i.\vec{E}_j = 0 and$ if $i \neq j$, then prove that $\left[\vec{e}_1\vec{e}_2\vec{e}_3\right]\left[\vec{E}_1\vec{E}_2\vec{E}_3\right] = 1.$

Watch Video Solution

277. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$ Determine the distance of point $A(\vec{a})$ from the line l in the form $\vec{b} - \vec{a} + \frac{\left(\vec{a} - \vec{b}\right)\vec{c}}{|\vec{c}|^2}\vec{c}$ or $\frac{\left|\left(\vec{b} - \vec{a}\right) \times \vec{c}\right|}{|\vec{c}|}$.

Watch Video Solution

278. Let a three dimensional vector $\vec{V}$ satisfy the condition, $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}$ If $3\left|\vec{V}\right| = \sqrt{m}$ Then find the value of m

Watch Video Solution

279. Given that

$\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}; \vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}; \vec{w} = \hat{i} + 3\hat{j} + 3\hat{k} and \left(\vec{u}\dot{R} - 15\right)\hat{i} + \left(\vec{v}\dot{R} - 30\right)\hat{j} + ($

Then find the greatest integer less than or equal to $\left|\vec{R}\right|$.

Watch Video Solution

280. Let $\vec{O}A - \vec{a}, \vec{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq$, then find$k$.

Watch Video Solution

281. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying

$\left[(a - 2)\alpha^2 + (b - 3)\alpha + c\right]\vec{x} + \left[(a - 2)\beta^2 + (b - 3)\beta + c\right]\vec{y} + \left[(a - 2)\gamma^2 + (b - 3)\gamma + c\right.$

are three distinct real numbers, then find the value of $\left(a^2 + b^2 + c^2 - 4\right)$.

Watch Video Solution

282. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}, and \vec{c} = 2\hat{i} + \alpha\hat{j} + \hat{k}$ Find thevalue of 6α, such that $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = 0.$

Watch Video Solution

283. Let $\vec{a}, \vec{b} and \vec{c}$ be three vectors having magnitudes 1, 5and 3, respectively, such that the angel between $\vec{a} and \vec{b} is \theta$ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = c$. Then $tan\theta$ is equal to a. 0 b. 2/3 c. 3/5 d. 3/4

Watch Video Solution

284. Two vectors in space are equal only if they have equal component in

a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

Watch Video Solution

285. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{j}-\hat{k} and \vec{c}=\hat{k}-\hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d}=0=\left[\vec{b}\vec{c}\vec{d}\right]$, then d equals a.$\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$ b. $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ c. $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ d. $\pm\hat{k}$

Watch Video Solution

286. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is $\vec{b}+\frac{\vec{b}\times\vec{a}}{|\vec{a}|^2}$ b. $\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|^2}$ c. $\vec{b}-\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}$ d. $\frac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{\left|\vec{b}\right|^2}$

Watch Video Solution

287. If $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ is perpendicular to $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$, we may have a. $\left(\vec{a}.\vec{c}\right)\left|\vec{b}\right|^2=\left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)$ b. $\vec{a}.\vec{b}=0$ c. $\vec{a}.\vec{c}=0$ d. $\vec{b}.\vec{c}=0$

Watch Video Solution

288. $\left[\vec{a}\times\vec{b}\vec{c}\times\vec{d}\vec{e}\times\vec{f}\right]$is equal to (a)$\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$ (b) $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right]-\left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$ (c)$\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$ (d) $\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

Watch Video Solution

289. $\vec{b} and \vec{c}$ are non-collinear if $\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\left(\dot{\vec{a}\vec{b}}\right)\vec{b}=(4-2x-\sin y)\vec{b}+\left(x^2-1\right)\vec{c}$ and $\left(\vec{c}.\vec{c}\right)\vec{a}=\dot{\vec{c}}$ Then

a. $x=1$ b. $x=-1$ c. $y=(4n+1)\pi/2, n\in I$ d. $y=(2n+1)\pi/2, n\in I$

Watch Video Solution

290. Let the vectors $\vec{a} and \vec{b}$ be such that $\left|\vec{a}\right|=3\left|\vec{b}\right|=\frac{\sqrt{2}}{3}, then \vec{a}\times\vec{b}$ is a unit vector, if the angel between $\vec{a} and \vec{b}$ is?

Watch Video Solution

Watch Video Solution

291. If $\vec{a} \perp \vec{b}$, then vector $\vec{v}$ in terms of $\vec{a} and \vec{b}$ satisfying the equation s $\vec{v}.\vec{a} = 0 and \vec{v}.\vec{b} = 1 and \left[\vec{v}\vec{a}\vec{b}\right] = 1$ is a. $\frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ b. $\frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ c. $\frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$ d. none of these

Watch Video Solution

292. If $\vec{a}' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}' = 2\hat{i} + \hat{j} - \hat{k}$, then the altitude of the parallelepiped formed by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is reciprocal vector $\vec{a}$)

Watch Video Solution

293. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$, then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}'$ of vector $\vec{a}$ is a. $\frac{\hat{i} + \hat{j} + \hat{k}}{2}$ b. $\frac{\hat{i} - \hat{j} + \hat{k}}{2}$ c. $\frac{-\hat{i} - \hat{j} + \hat{k}}{2}$

d. $\frac{\hat{i}+\hat{j}-\hat{k}}{2}$

Watch Video Solution

294. If unit vectors $\vec{a} and \vec{b}$ are inclined at angle 2θ such that $\left|\vec{a}-\vec{b}\right|<1 and 0\leq\theta\leq\pi, then \theta$ lies in interval a.$[0,\pi/6]$ b. $[5\pi/6,\pi]$ c. $[\pi/6,\pi/2]$ d. $[\pi/2,5\pi/6]$

Watch Video Solution

295. $A, B, C and D$ are four points such that $\vec{A}B=m\left(2\hat{i}-6\hat{j}+2\hat{k}\right), \vec{B}C=\left(\hat{i}-2\hat{j}\right) and \vec{C}D=n\left(-6\hat{i}+15\hat{j}-3\hat{k}\right)$. If CD intersects AB at some point E, then a. $m\geq1/2$ b.$n\geq1/3$ c. $m=n$ d. $m<n$

Watch Video Solution

296. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k} and \vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a} and \vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by a. $\hat{i} - 3\hat{j} + 3\hat{k}$ b. $-3\hat{i} - 3\hat{j} + 3\hat{k}$ c. $3\hat{i} - \hat{j} + 3\hat{k}$ d. $\hat{i} + 3\hat{j} - 3\hat{k}$

Watch Video Solution

297. If $\hat{a}, \hat{b}, and \hat{c}$ are unit vectors, then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\hat{c} - \hat{a}\right|^2$ does not exceed

Watch Video Solution

298. Which of the following expressions are meaningful? a.$\vec{u}\left(\vec{v} \times \vec{w}\right)$ b. $\left(\vec{u}. \vec{v}\right). \vec{w}$ c. $\left(\vec{u}. \vec{v}\right)\vec{w}$ d. $\vec{u} \times \left(\vec{v}. \vec{w}\right)$

Watch Video Solution

299. Find the value of λ if the volume of a tetrashedron whose vertices are with position vectors $\hat{i} - 6\hat{j} + 10\hat{k},\ -\hat{i} - 3\hat{j} + 3\hat{k}, 5\hat{i} - \hat{j} + \lambda\hat{k} and 7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic unit.

Watch Video Solution

300. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) \vec{a} \times \vec{d} = 0$, then which of the following may be true? a.$\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are necessarily coplanar b.$\vec{a}$ lies in the plane of $\vec{c} and \vec{d}$ c. $\vec{b}$ lies in the plane of $\vec{a} and \vec{d}$ d. $\vec{c}$ lies in the plane of $\vec{a} and \vec{d}$

Watch Video Solution

301. Vector $\frac{1}{3}(2i - 2j + k)$ is (A) a unit vector (B) makes an angle $\pi/3$ with vector $\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)$ (C) parallel to vector $\left(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$ (D) perpendicular to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Watch Video Solution

302. Let $\vec{u} and \vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$.

Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]$.

Watch Video Solution

303. The scalars$l and m$ such that $l\vec{a} + m\vec{b} = \vec{c}, where \vec{a}, \vec{b} and \vec{c}$ are given vectors, are equal to a)l=(cxb).(axb)/(axb)^2 b) b.l=(cxb).(bxa)/(bxa)^2 c) c).m=(cxb).(bxa)/(bxa)^2 d) d) m=(cxb).(axb)/(axb)^2

A. l=(cxb).(axb)/(axb)^2

B. b.l=(cxb).(bxa)/(bxa)^2

C. c).m=(cxb).(bxa)/(bxa)^2

D. d) m=(cxb).(bxa)/(bxa)^2

Answer: null

Watch Video Solution

304. If $OABC$ is a tetrahedron where O is the orogin anf $A, B, andC$ are the other three vertices with position vectors, $\vec{a}, \vec{b}, and\vec{c}$ respectively, then prove that the centre of the sphere circumscribing the tetrahedron is given by position vector $\dfrac{a^2\left(\vec{b} \times \vec{c}\right) + b^2\left(\vec{c} \times \vec{a}\right) + c^2\left(\vec{a} \times \vec{b}\right)}{2\left[\vec{a}\vec{b}\vec{c}\right]}$.

Watch Video Solution

305. Let k be the length of any edge of a regular tetrahedron (a tetrahedron whose edges are equal in length is called a regular tetrahedron). Show that the angle between any edge and a face not containing the edge is$\cos^{-1}\left(1/\sqrt{3}\right)$.

Watch Video Solution

306. In $\triangle ABC$ a point P is taken on AB such that AP/BP=1/3 and a point Q is taken on BC such that CQ/BQ=3/1 . If R is the point of intersection of the lines AQ and CP, using vector method, find the area of $\triangle ABC$ if the area of $\triangle AOC$.

Watch Video Solution

307. Let $ABCD$ be a p[arallelogram whose diagonals intersect at P and let O be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P^{\cdot}$

Watch Video Solution

308. If $\left|(a-x)^2(a-y)^2(a-z)^2(b-x)^2(b-y)^2(b-z)^2(c-x)^2(c-y)^2(c-a)^2\right| = 0$ and vectors $\vec{A}, \vec{B}, and\vec{C}, where\vec{A} = a^2\hat{i} + a\hat{j} + \hat{k}$, etc, are non-coplanar, then prove that vectors $\vec{X}, \vec{Y}and\vec{Z}, where\vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$, etc. may be coplanar.

Watch Video Solution

309. The lengths of two opposite edges of a tetrahedron are a and b; the shortest distane between these edges is d, and the angel between them is θ Prove using vectors that the volume of the tetrahedron is $\frac{abd\sin\theta}{6}$.

Watch Video Solution

310. Find the valume of a parallelepiped having three coterminus vectors of equal magnitude $|\vec{a}|$ and equal inclination θ with each other.

Watch Video Solution

311. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + \hat{j} + 5\hat{k} and \vec{C}$ form a left-handed system, then $\vec{C}$ is a.$11\hat{i} - 6\hat{j} - \hat{k}$ b.$-11\hat{i} + 6\hat{j} + \hat{k}$ c. $11\hat{i} - 6\hat{j} + \hat{k}$ d. $-11\hat{i} + 6\hat{j} - \hat{k}$

Watch Video Solution

312. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} and \vec{c} = 3\hat{j} - 2\hat{k}$ Let$\vec{x}, \vec{y}, and \vec{z}$ be thre vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$, respectively. Then $\vec{x}\dot{\vec{d}} = -1$ b. $\vec{y}\dot{\vec{d}} = 1$ c. $\vec{z}\dot{\vec{d}} = 0$ d.

$\dot{\vec{r}}\vec{d} = 0, where\vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

Watch Video Solution

313. Vectors $\vec{A} and \vec{B}$ satisfying the vector equation $\vec{A} + \vec{B} = \vec{a}, \vec{A} \times \vec{B} = \vec{b} and \vec{A} \cdot \vec{a} = 1, where \vec{a} and \vec{b}$ are given vectors, are a. $\vec{A} = \frac{\left(\vec{a} \times \vec{b}\right) - \vec{a}}{a^2}$ b. $\vec{B} = \frac{\left(\vec{b} \times \vec{a}\right) + \vec{a}\left(a^2 - 1\right)}{a^2}$ c. $\vec{A} = \frac{\left(\vec{a} \times \vec{b}\right) + \vec{a}}{a^2}$ d. $\vec{B} = \frac{\left(\vec{b} \times \vec{a}\right) - \vec{a}\left(a^2 - 1\right)}{a^2}$

Watch Video Solution

314. if $\vec{\alpha} \mid \mid \left(\vec{\beta} \times \vec{\gamma}\right)$, then $\left(\vec{\alpha} \times \beta\right)\dot{\vec{\alpha}} \times \vec{\gamma}$ equals to $\left|\vec{\alpha}\right|^2\left(\vec{\beta}\dot{\vec{\gamma}}\right)$ b. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}\dot{\vec{\alpha}}\right)$ c. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}\dot{\vec{\beta}}\right)$ d. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

Watch Video Solution

315. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k} and \vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ are three coplanar vectors with $a \neq b, and \vec{v} = \hat{i} + \hat{j} + \hat{k}$ Then v is perpendicular to $\vec{\alpha}$ b.$\vec{\beta}$ c. $\vec{\gamma}$ d. none of these

Watch Video Solution

316. $a_1, a_2, a_3, \in R - \{0\}$ and $a_1 + a_2\cos 2x + a_3\sin^2 x = 0 f$ or $all x \in R$, then (a)vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} and \vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other (b)vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} and \vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$ are parallel to each other (c)vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length$\sqrt{6}$ units, then one of the ordered triple $\left(a_1, a_2, a_3\right) = (1, -1, -2)$ (d)are perpendicular to each other if $2a_1 + 3a_2 + 6a_3 = 26, then \left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

Watch Video Solution

317. If P is any arbitrary point on the circumcirlce of the equllateral trangle of side length l units, then $\left|\vec{PA}\right|^2 + \left|\vec{PB}\right|^2 + \left|\vec{PC}\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

Watch Video Solution

318. Let $\vec{a} and \vec{b}$ be two non-zero perpendicular vectors. A vecrtor $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be $\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$ b. $2\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$ c. $\left|\vec{a}\right|\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$ d. $\left|\vec{b}\right|\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

Watch Video Solution

319. If $\vec{a} and \vec{b}$ are two vectors and angle between them is θ, then

$\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a} \dot{} \vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$ $\left|\vec{a} \times \vec{b}\right| = \left(\vec{a} \dot{} \vec{b}\right)$, if $\theta = \pi/4$

$\vec{a} \times \vec{b} = \left(\vec{a}\vec{b}\right)\hat{n}$, $\left(where\hat{n}\right.$ is unit vector,) if $\theta = \pi/4$ $\left(\vec{a} \times \vec{b}\right)\vec{a} + \vec{b} = 0$

Watch Video Solution

320. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}, where\left|\vec{a}\right| = 3and\left|\vec{b}\right| = 2.$ Then $\vec{r} = \frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$ b. $\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right.$ c. $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right.$ d. $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right.$

Watch Video Solution

321. If vector $\vec{b} = \left(tan\alpha, -1, 2\sqrt{\sin\alpha/2}\right)and\vec{c} = \left(tan\alpha, tan\alpha, \frac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is a$\alpha = \tan^{-1}2$ b. $\alpha = -\tan^{-1}2$ c. $\alpha = \tan^{-1}2$ d. $\alpha = \tan^{-1}2$

Watch Video Solution

322. Let $\vec{a}, \vec{b}, and\vec{c}$ be non-zero vectors and $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right) and \vec{V}_2 \left(\vec{a} \times \vec{b}\right) \times \dot{\vec{}}$ Vectors $\vec{V}_1 and \vec{V}_2$ are equal. Then $\vec{a} an \vec{b}$ are orthogonal b. $\vec{a} and \vec{c}$ are collinear c. $\vec{b} and \vec{c}$ are orthogonal d. $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right) when \lambda$ is a scalar

Watch Video Solution

323. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \hat{i} + \hat{j} - 2\dot{\hat{k}}$ A vector coplanar with $\vec{b} and \vec{c}$ whose projectin on $\vec{a}$ is magnitude $\sqrt{\frac{2}{3}}$ is $2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $-2\hat{i} - \hat{j} + 5\hat{k}$ c. $2\hat{i} + 3\hat{j} + 3\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$

Watch Video Solution

324. Let $\vec{PR} = 3\hat{i} + \hat{j} - 2\hat{k} and \vec{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS, and \vec{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determine by the vectors $\vec{PT}$, $\vec{PQ}$ and $\vec{PS}$ is 5 b. 20 c. 10 d. 30

Watch Video Solution

325. If in a right-angled triangle ABC, the hypotenuse $AB = p$,then $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

Watch Video Solution

326. If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \vec{a}\vec{b} = 1 and \vec{a} \times \vec{b} = \hat{j} - \hat{k}, then \hat{b}$ is $\hat{i} - \hat{j} + \hat{k}$ b. $2\hat{j} - \hat{k}$ c. $\hat{i}$ d. -i+$2\hat{i}$

Watch Video Solution

327. If a satisfies $\vec{a} \times \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - \hat{k},$ then $\vec{a}$ is equal to $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda R$ b. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda R$ c. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda R$ d. $\lambda\hat{i} - (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda R$

Watch Video Solution

. . .

328. If $\vec{r}\vec{a} = \vec{r}\vec{b} = \vec{r}\vec{c} = 0, where\vec{a}, \vec{b}, and\vec{c}$ are non-coplanar, then $\vec{r} \perp \left(\vec{c} \times \vec{a}\right)$ b. $\vec{r} \perp \left(\vec{a} \times \vec{b}\right)$ c. $\vec{r} \perp \left(\vec{b} \times \vec{c}\right)$ d. $\vec{r} = \vec{0}$

Watch Video Solution

329. The unit vector orthogonal to vector $\hat{i} + \hat{j} + 2\hat{k}$ and making equal angles with the x and y-axis a.$\pm\frac{1}{3}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$ b. $\pm\frac{1}{3}\left(\hat{i} + \hat{j} - \hat{k}\right)$ c. $\pm\frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$ d. none of these

Watch Video Solution

330. Vectors $3\vec{a} - 5\vec{b}and2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}and\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angel between $aandb$ is a.$\frac{19}{5\sqrt{43}}$ b. $\frac{19}{3\sqrt{43}}$ c. $\frac{19}{2\sqrt{45}}$ d. $\frac{19}{6\sqrt{43}}$

Watch Video Solution

331. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is $\vec{b}$ + $\frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ b. $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$ c. $\vec{b}$ - $\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}$ d. $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

Watch Video Solution

332. The value of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k} and \vec{b} = 7\hat{i} - 2\hat{j} + \hat{k}$ is obtuse and the angle between b and the z-axis acute and less that $\pi/6$ is

A. a) a

B. b. `1//2

C. c. $x > 1/2$ or $x < 0$

D. d. none of these

Answer: null

Watch Video Solution

333. Let $\vec{a}\cdot\vec{b}=0, where \vec{a} and \vec{b}$ are unit vectors and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a} and \vec{b}$. If $\vec{c}=m\vec{a}+n\vec{b}+p(\vec{a}\times\vec{b}),(m,n,p\in R)$, then a.$-\frac{\pi}{4}\leq\theta\leq\frac{\pi}{4}$ b. $\frac{\pi}{4}\leq\theta\leq\frac{3\pi}{4}$ c. $0\leq\theta\leq\frac{\pi}{4}$ d. $0\leq\theta\leq\frac{3\pi}{4}$

Watch Video Solution

334. A parallelogram is constructed on $3\vec{a}+\vec{b} and \vec{a}-4\vec{b}, where |\vec{a}|=6 and |\vec{b}|=8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is a .40 b. 64 c. 32 d. 48

Watch Video Solution

335. Let the position vectors of the points $P and Q$ be $4\hat{i}+\hat{j}+\lambda\hat{k} and 2\hat{i}-\hat{j}+\lambda\hat{k}$, respectively. Vector $\hat{i}-\hat{j}+6\hat{k}$ is perpendicular to the plane containing the origin and the points $PandQ$. Then λ equals $1/2$ b. $1/2$ c. 1 d. none of these

Watch Video Solution

336. $aandc$ are unit vectors and $|b| = 4$. The angel between $aandc$ is $\cos^{-1}(1/4)andb - 2c = \lambda a^{\cdot}$ The value of λ is $3, -4$ b. $1/4, 3/4$ c. $-3, 4$ d. $-1/4, 3/4$

Watch Video Solution

337. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|(\ddot{\vec{c}})\left(\vec{a} \times \vec{b}\right) + (\ddot{\vec{a}})\left(\vec{b} \times \vec{c}\right) + (\ddot{\vec{b}})\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$ b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$ c. $\vec{a}, \vec{b}, and\vec{c}$ are coplanar d. none of these

Watch Video Solution

338. Let $\vec{a}, \vec{b}, and\vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector, which is perpendicular to $\left(\vec{a} + \vec{b} + \vec{c}\right)^{\cdot}$ Now

$\vec{d} = \left(\vec{a} \times \vec{b}\right)\sin x + \left(\vec{b} \times \vec{c}\right)\cos y + 2\left(\vec{c} \times \vec{a}\right)^{\cdot} Then$ a. $\frac{\vec{a} + \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]} = 2$ b. $\frac{\vec{a} + \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]} = -2$ c. minimum value of $x^2 + y^2$ is $\pi^2/4$ d. minimum value of $x^2 + y^2$ is $5\pi^2/4$

Watch Video Solution

339. If $\vec{a} + 2\vec{b} + 3\vec{c} = 0, then\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$ $2\left(\vec{a} \times \vec{b}\right)$ b. $6\left(\vec{b} \times \vec{c}\right)$ c. $3\left(\vec{c} \times \vec{a}\right)$ d. $\vec{0}$

Watch Video Solution

340. $\vec{a} and \vec{b}$ are two non-collinear unit vector, and $\vec{u} = \vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b} and \vec{v} = \vec{a} \times \vec{b}$ Then $|\vec{v}|$ is $|\vec{u}|$ b. $|\vec{u}| + \left|\vec{u} \cdot \vec{b}\right|$ c. $|\vec{u}| + \left|\vec{u} \cdot \vec{a}\right|$ d. none of these

Watch Video Solution

341. prove that sec theta

Watch Video Solution

342. $\vec{a}, \vec{b}, and \vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicular to then. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angel between $\vec{a} and \vec{b}$ is $30^0, then \vec{c}$ is a. $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$ b. $\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right)/3$ c. $\left(2\hat{i} + 2\hat{j} - \hat{k}\right)/3$ d. $\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

Watch Video Solution

343. Vectors perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k} and - \hat{i} + \hat{j} + \vec{k}$ are $\hat{i} + \hat{k}$ b. $2\hat{i} + \hat{j} + \hat{k}$ c. $3\hat{i} + 2\hat{j} + \hat{k}$ d. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

Watch Video Solution

344. If side $\vec{AB}$ of an equilateral trangle ABC lying in the x-y plane $3\hat{i}$, then side $\vec{CB}$ can be $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ b. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ c. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$ d. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

Watch Video Solution

345. If $A = \{1, 2, 3, 4\}, f: R \rightarrow R = x^2 + 3x + 1$ and $g: R \rightarrow R, 8(x) = 2x - 3$ then find $fof(x)$

Watch Video Solution

346. Let two non-collinear unit vector $\hat{a}$ a n d $\hat{b}$ form an acute angle. A point P moves so that at any time t, the position vector $OP(where O$ is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t \dot{W}hen P$ is farthest from origin O, let M be the length of $OP and \hat{u}$ be the unit vector along $\dot{OP}$ Then (a) $\hat{u} = \frac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|} and M = \left(1 + \hat{a}\dot{\hat{b}}\right)^{1/2}$ (b) $\hat{u} = \frac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|} and M = \left(1 + \hat{a}\dot{\wedge}\right)^{1/2}$ (c) $\hat{u} = \frac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|} and M = \left(1 + 2\hat{a}\dot{\hat{b}}\right)^{1/2}$ (d) $\hat{u} = \frac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|} and M = \left(1 + 2\hat{a}\dot{\hat{b}}\right)^{1/2}$

Watch Video Solution

347. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k} and \vec{c} = \hat{i} + \hat{j} - \dot{\hat{k}}$ A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection of c is $1/\sqrt{3}$ is a. $4\hat{i} - \hat{j} + 4\hat{k}$ b. $3\hat{i} + \hat{j} + 3\hat{k}$ c. $2\hat{i} + \hat{j} + 2\hat{k}$ d. $4\hat{i} + \hat{j} - 4\hat{k}$

Watch Video Solution

348. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non coplanar vector $\vec{b}_1 = \vec{b} - \frac{\vec{b}\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1 = \vec{c} - \frac{\cdot \vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $, c_2 = \vec{c} - \frac{\cdot \vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}\vec{c}}{|\vec{b}_1|^2}$,

$b_1, \vec{c}_3 = \vec{c} - \frac{\cdot \vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c} - \frac{\cdot \vec{a}}{|\vec{c}|^2}\vec{a} = \frac{\vec{b}\vec{c}}{|\vec{b}|^2}\vec{b}_1$ then the set of orthogonal vectors is $\left(\vec{a}, \vec{b}_1, \vec{c}_3\right)$ b. $\left(\vec{a}, \vec{b}_1, \vec{c}_2\right)$ c. $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$ d. $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

Watch Video Solution

349. The unit vector which is orthogonal to the vector $5\hat{j} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k} and \hat{i} - \hat{j} + \hat{k}$ is $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ b. $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ c. $\frac{3\hat{i} - \hat{k}}{\sqrt{10}}$ d. $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

Watch Video Solution

350. If $\vec{a} and \vec{b}$ are unequal unit vectors such that $\left(\vec{a} - \vec{b}\right) \times \left[\left(\vec{b} + \vec{a}\right) \times \left(2\vec{a} + \vec{b}\right)\right] = \vec{a} + \vec{b}$, then angle θ between $\vec{a} and \vec{b}$ is

0 b. $\pi/2$ c. $\pi/4$ d. π

Watch Video Solution

351. If in triangle $ABC, \vec{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|} and \vec{AC} = \frac{2\vec{u}}{|\vec{u}|}, where |\vec{u}| \neq |\vec{v}|$, then

$1 + \cos 2A + \cos 2B + \cos 2C = 0$ b.$\sin A = \cos C$ c. projection of AC on BC is equal to BC d. projection of AB on BC is equal to AB

Watch Video Solution

352. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} and \vec{c} = 3\hat{j} - 2\hat{k}$ Let $\vec{x}, \vec{y}, and \vec{z}$ be thre vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$, respectively. Then $\vec{x} \cdot \vec{d} = -1$ b. $\vec{y} \cdot \vec{d} = 1$ c. $\vec{z} \cdot \vec{d} = 0$ d. $\vec{r} \cdot \vec{d} = 0, where \vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

Watch Video Solution

Watch Video Solution

353. If $a \times (b \times c) = (a \times b) \times c$, then $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$ b.$\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$ c. $\vec{b} \times \left(\vec{c} \times \vec{a}\right)\vec{0}$ d. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

Watch Video Solution

354. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ. The maximum value of θ is

Watch Video Solution

355. Let the pairs $a, b and c, d$ each determine a plane. Then the planes are parallel if $\left(\vec{a} \times \vec{c}\right) \times \left(\vec{b} \times \vec{d}\right) = \vec{0}$ b. $\left(\vec{a} \times \vec{c}\right) \cdot \vec{b} \times \vec{d} = \vec{0}$ c. $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$ d. $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c} \times \vec{d} = \vec{0}$

Watch Video Solution

356. $P\left(\vec{p}\right)$ and $Q\left(\vec{q}\right)$ are the position vectors of two fixed points and $R\left(\vec{r}\right)$ is the position vectorvariable point. If R moves such that $\left(\vec{r}-\vec{p}\right)\times\left(\vec{r}-\vec{q}\right)=0$ then the locus of R is

Watch Video Solution

357. Two adjacent sides of a parallelogram $ABCD$ are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. Then the value of $|AC\times BD|$ is $20\sqrt{5}$ b. $22\sqrt{5}$ c. $24\sqrt{5}$ d. $26\sqrt{5}$

Watch Video Solution

358. If $\hat{a},\hat{b},and\hat{c}$ are three unit vectors, such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector and $\theta_1,\theta_2andth\eta_3$ are angles between the vectors $\hat{a},\hat{b};\hat{b},\hat{c}and\hat{c},\hat{a}$ respectively, then among $\theta_1,\theta_2,andth\eta_3$ a. all are acute angles b. all are right angles c. at least one is obtuse angle d. none of these

Watch Video Solution

359. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}\dot{\vec{b}} = 0 = \vec{a}\dot{\vec{c}}$ and the angle between $\vec{b} and \vec{c}$ is $\pi/3$, then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is $1/2$ b. 1 c. 2 d. none of these

Watch Video Solution

360. Let $\vec{a} = \hat{i} + \hat{j}; \vec{b} = 2\hat{i} - \dot{\hat{k}}$ Then vector $\vec{r}$ satisfying $\vec{r} \times \vec{a} = \vec{b} \times \vec{a} and \vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is $\hat{i} - \hat{j} + \hat{k}$ b. $3\hat{i} - \hat{j} + \hat{k}$ c. $3\hat{i} + \hat{j} - \hat{k}$ d. $\hat{i} - \hat{j} - \hat{k}$

Watch Video Solution

361. If $\vec{a} and \vec{b}$ are two vectors, such that $\vec{a}\dot{\vec{b}} < 0 and \left|\vec{a}\dot{\vec{b}}\right| = \left|\vec{a} \times \vec{b}\right|$, then the angle between vectors $\vec{a} and \vec{b}$ is π b. $7\pi/4$ c. $\pi/4$ d. $3\pi/4$

Watch Video Solution

362. $\vec{a}, \vec{b}, and \vec{c}$ are three vectors of equal magnitude. The angel between each pair of vectors is $\pi/3$ such that $\left|\vec{a} + \vec{b} + \vec{c}\right| = 6.$ Then $\left|\vec{a}\right|$ is equal to 2 b. -1 c. 1 d. $\sqrt{6}/3$

Watch Video Solution

363. If $\vec{a}, \vec{b} and \vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is a.$\vec{a} + \vec{b} + \vec{c}$ b. $\frac{\vec{a}}{\left|\vec{a}\right|} + \frac{\vec{b}}{\left|\vec{b}\right|} + \frac{\vec{c}}{\left|\vec{c}\right|}$ c. $\frac{\vec{a}}{\left|\vec{a}\right|^2} + \frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{c}}{\left|\vec{c}\right|^2}$ d. $\left|\vec{a}\right|\vec{a} - \left|\vec{b}\right|\vec{b} + \left|\vec{c}\right|\vec{c}$

Watch Video Solution

364. $\vec{a} and \vec{b}$ are two non-collinear unit vector, and $\vec{u} = \vec{a} - \left(\vec{a}\dot{\vec{b}}\right)\vec{b} and \vec{v} = \vec{a} \times \dot{\vec{b}}$ Then $\left|\vec{v}\right|$ is $\left|\vec{u}\right|$ b.$\left|\vec{u}\right| + \left|\vec{u}\dot{\vec{b}}\right|$ c. $\left|\vec{u}\right| + \left|\vec{u}\dot{\vec{a}}\right|$ d. none of these

Watch Video Solution

Watch Video Solution

365. The vertex A triangle ABC is on the line $\vec{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i}and\hat{j}$ Let Delta be the area of the triangle and Delta$\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to A is a.$[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

Watch Video Solution

366. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4}\tan A + a\tan B + \sqrt{a^2 + 4}\tan c = 6a,$ then the least vale of $\tan^2 A + \tan^2 b + \tan^2 Cis\ 6$ b. 10 c. 12 d. 3

Watch Video Solution

367. The position vectors of the vertices $A, BandC$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and\vec{c}$, respectively. A vector $\vec{d}$ is such that

$\vec{d}\vec{a} = \vec{d}\vec{b} = \vec{d}\vec{c} and \vec{d} = \lambda\left(\vec{b} + \vec{c}\right)$ Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

Watch Video Solution

368. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b}\cdot\vec{q} = 0 and \left|\vec{b}\right|^2 = 1, where \mu$ is a scalar. Then $\left|\left(\vec{a}\vec{q}\right)\vec{p} - \left(\vec{p}\vec{q}\right)\vec{a}\right|$ is equal to (a) $2\left|\vec{p}.\vec{q}\right|$ (b) $(1/2)\left|\vec{p}.\vec{q}\right|$ (c) $\left|\vec{p}\times\vec{q}\right|$ (d) $\left|\vec{p}.\vec{q}\right|$

Watch Video Solution

369. In fig. $AB, DEandGF$ are parallel to each other and $AD, BGandEF$ are parallel to each other. If $CD : CE = CG : CB = 2 : 1$, then the value of area (AEG) : area (ABD) is equal to $7/2$ b. 3 c. 4 d. $9/2$

Watch Video Solution

370. In a quadrilateral $ABCD, \vec{AC}$ is the bisector of $\vec{A}Band\vec{A}D$, angle between $\vec{A}Band\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|$. Then the angle between $\vec{B}Aand\vec{C}D$ is $\frac{\cos^{-1}\left(\sqrt{14}\right)}{7\sqrt{2}}$ b. $\frac{\cos^{-1}\left(\sqrt{21}\right)}{7\sqrt{3}}$ c. $\frac{\cos^{-1}2}{\sqrt{7}}$ d. $\frac{\cos^{-1}\left(2\sqrt{7}\right)}{14}$

Watch Video Solution

371. Position vector $\hat{k}$ is rotated about the origin by angle 135^0 in such a way that the plane made by it bisects the angle between $\hat{i}and\hat{j}$. Then its new position is a.$\pm\frac{\hat{i}}{\sqrt{2}} \pm \frac{\hat{j}}{\sqrt{2}}$ b. $\pm\frac{\hat{i}}{2} \pm \frac{\hat{j}}{2} - \frac{\hat{k}}{\sqrt{2}}$ c. $\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{k}}{\sqrt{2}}$ d. none of these

Watch Video Solution

372. A non-zero vector $\vec{a}$ is such that its projections along vectors $\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal, then unit vector along $\vec{a}$ is $\frac{\sqrt{2}\hat{j} - \hat{k}}{\sqrt{3}}$ b. $\frac{\hat{j} - \sqrt{2}\hat{k}}{\sqrt{3}}$

c. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

Watch Video Solution

373. Let $\vec{a}=2i+j+k, \vec{b}=i+2j-k$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is $\frac{1}{\sqrt{2}}(-j+k)$ b. $\frac{1}{\sqrt{3}}(-i-j-k)$ c. $\frac{1}{\sqrt{5}}(-k-2j)$ d. $\frac{1}{\sqrt{3}}(i-j-k)$

Watch Video Solution

374. Let $\vec{a}=2i+j-2k and \vec{b}=i+\dot{j}$ If $\vec{c}$ is a vector such that $\dot{\vec{a}}\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ between $\vec{a}\times\vec{b}$ and $\vec{c} is 30^0, then\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ I equal to a.2/3 b. 3/2 c. 2 d. 3

Watch Video Solution

375. Vector $\vec{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j} and \vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b} and \vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$ The value of $\vec{a}$ is a. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$ b. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ c. $\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$ d. $\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$

Watch Video Solution

376. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{b + c}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

Watch Video Solution

377. Let $\vec{u}, \vec{v} and \vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $\left|\vec{u}\right| = 3, \left|\vec{v}\right| = 4 and \left|\vec{w}\right| = 5$, then $\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}$ is a. 47 b. -25 c. 0 d. 25

Watch Video Solution

378. If $\vec{a}, \vec{b} and \vec{c}$ are three non-coplanar vectors, then $\left(\vec{a} + \vec{b} + \vec{c}\right)\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)$ equals a. 0 b. $\left[\vec{a}\vec{b}\vec{c}\right]$ c. $2\left[\vec{a}\vec{b}\vec{c}\right]$ d. $-\left[\vec{a}\vec{b}\vec{c}\right]$

Watch Video Solution

379. $\vec{p}, \vec{q}, and \vec{r}$ are three mutually perpendicular vectors of the same magnitude. If vector $\vec{x}$ satisfies the equation $\vec{p} \times \left(\left(\vec{x} - \vec{q}\right) \times \vec{p}\right) + \vec{q} \times \left(\left(\vec{x} - \vec{r}\right) \times \vec{q}\right) + \vec{r} \times \left(\left(\vec{x} - \vec{p}\right) \times \vec{r}\right) = 0, then \vec{x}$ is given by a. $\frac{1}{2}\left(\vec{p} + \vec{q} - 2\vec{r}\right)$ b. $\frac{1}{2}\left(\vec{p} + \vec{q} + \vec{r}\right)$ c. $\frac{1}{3}\left(\vec{p} + \vec{q} + \vec{r}\right)$ d. $\frac{1}{3}\left(2\vec{p} + \vec{q} - \vec{r}\right)$

Watch Video Solution

380. If vectors $b, c and d$ are not coplanar, then prove that vector $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) + \left(\vec{a} \times \vec{c}\right) \times \left(\vec{d} \times \vec{b}\right) + \left(\vec{a} \times \vec{d}\right) \times \left(\vec{b} \times \vec{c}\right)$ is parallel to $\vec{a}$.

Watch Video Solution

Watch Video Solution

381. The position vectors of the vertices, A, B and C of a tetrahedron are $\hat{i}+\hat{j}+\hat{k}, \hat{i}$ and $3\hat{i}$ respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of triangle ABC at a point E. if the length of the side AD is 4 and the volume of the tetrahedron is $2\sqrt{2}/3$ find teh position vectors of the point E for all its possible positions .

Watch Video Solution

382. If $\vec{A}, \vec{B}$ and $\vec{C}$ are vectors such that $\left|\vec{B}\right| = \left|\vec{C}\right|$. Prove that $\left[\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)\right]\times\left(\vec{B}+\vec{C}\right).\left(\vec{B}+\vec{C}\right) = 0$

Watch Video Solution

383. A parallelogram is constructed on $3\vec{a}+\vec{b} and \vec{a}-4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is a .40 b. 64 c. 32 d. 48

Watch Video Solution

384. Statement 1: Vector $\vec{c} = 5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angel between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k} and \vec{b} = -8\hat{i} + \hat{j} - 4\hat{k}^{\cdot}$ Statement 2: $\vec{c}$ is equally inclined to $\vec{a} and \vec{b}^{\cdot}$

Watch Video Solution

385. Statement 1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}^{\cdot}$ Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is $2\hat{i} + 2\hat{j} + 2\hat{k}^{\cdot}$

Watch Video Solution

386. Statement 1 : Points $A(1, 0), B(2, 3), C(5, 3), and D(6, 0)$ are concyclic. Statement 2 : Points $A, B, C, and D$ form an isosceles trapezium or $AB and CD$ meet at $E^{\cdot}$ Then $EA. EB = EC. ED^{\cdot}$

Watch Video Solution

387. Let $\vec{r}$ be a non-zero vector satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ for given non-zero vectors $\vec{a}, \vec{b} and \vec{\cdot}$ Statement 1: $\left[\vec{a} - \vec{b}\vec{b} - \vec{c}\vec{c} - \vec{a}\right] = 0$ Statement 2: $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

Watch Video Solution

388. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}; \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}; \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}\&\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$

Watch Video Solution

389. Statement 1: If $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k} and \vec{C} = \hat{i} + 2\hat{j} + \hat{k}$, then

$\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = 243$

Statement 2: $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = \left|\vec{A}\right|^2 \left|\left[\vec{A}\vec{B}\vec{C}\right]\right|$

a. Statement 1 and Statement 2 , both are true and Statement 2 is the correct explanation for Statement 1.

b. Statement 1 and Statement 2 , both are true and Statement 2 is not the correct explanation for Statement 1.

c. Statement 1 is true but Statement 2 is false.

c. Statement 2 is true but Statement 1 is false.

Watch Video Solution

390. Statement 1: $\vec{a}, \vec{b}, and \vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are non-coplanar. If $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1, then \vec{d} = \vec{a} + \vec{b} + \vec{c}$ Statement 2: $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

Watch Video Solution

391. Let vectors $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ be such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = 0.$ Let $P_1 and P_2$ be planes determined by the pair of vectors $\vec{a}, \vec{b}, and \vec{c}, \vec{d},$ respectively. Then the angle between $P_1 and P_2$ is a.0 b. $\pi/4$ c. $\pi/3$ d. $\pi/2$

Watch Video Solution

392. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

Watch Video Solution

393. For any two $\vec{a} and \vec{b}$, $\left(\vec{a} \times \hat{i}\right)\vec{b} \times \hat{i} + \left(\vec{a} \times \hat{j}\right)\vec{b} \times \hat{j} + \left(\vec{a} \times \hat{k}\right)\vec{b} \times \hat{k}$ is always equal to a.$\vec{a}.\ \vec{b}$ b. $2\vec{a}.\ \vec{b}$ c. zero d. none of these

Watch Video Solution

394. Let $\overrightarrow{f(t)} = [t]\hat{i} + (t - [t])\hat{j} + [t + 1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors $\overrightarrow{f\left(\frac{5}{4}\right)} and f(t), 0 < t < i$ are(a) parallel to each other(b) perpendicular(c) inclined at $\cos^{-1}2\left(\sqrt{7\left(1 - t^2\right)}\right)$ (d)inclined at $\cos^{-1}\left(\frac{8 + t}{\sqrt{1 + t^2}}\right)$;

Watch Video Solution

395. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, then $\left(\vec{a} \times \vec{b}\right)\dot{\vec{a}} \times \vec{c}$ is equal to a. $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$ b. $|\vec{b}|^2\left(\vec{a}.\vec{c}\right)$ c. $|\vec{c}|^2\left(\vec{a}.\vec{b}\right)$ d. none of these

Watch Video Solution

396. The three vectors $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume:

Watch Video Solution

397. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|(\ddot{\vec{c}})\left(\vec{a} \times \vec{b}\right) + (\ddot{\vec{a}})\left(\vec{b} \times \vec{c}\right) + (\ddot{\vec{b}})\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$ b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$ c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar d. none of these

Watch Video Solution

398. If $|a| = 2 and |b| = 3$ and $a\dot{b} = 0, then(a \times (a \times (a \times (a \times b))))$ is equal to a.$48\hat{b}$ b. $-48\hat{b}$ c. $48\hat{a}$ d. $-48\hat{a}$

Watch Video Solution

399. If the two diagonals of one its faces are $6\hat{i} + 6\hat{k} and 4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $c = 4\hat{j} - 8\hat{k},$ then the volume of a parallelepiped is a.60 b. 80 c. 100 d. 120

Watch Video Solution

400. The volume of a tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, and \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a} + \vec{b}, \vec{b} + \vec{c} and \vec{c} + \vec{a}$ is a.6 b. 18 c. 36 d. 9

Watch Video Solution

401. If $\vec{a}, \vec{b}$, and $\vec{c}$ are three mutually orthogonal unit vectors, then the triple product $\left[\vec{a} + \vec{b} + \vec{c}\vec{a} + \vec{b}\vec{b} + \vec{c}\right]$ equals: (a.) 0 (b.) 1 or -1 (c.) 1 (d.) 3

Watch Video Solution

402. Vector $\vec{c}$ is perpendicular to vectors $\vec{a} = (2, -3, 1) and \vec{b} = (1, -2, 3)$ and satisfies the condition $\vec{c}.\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 10.$ Then vector $\vec{c}$ is equal to a.$(7, 5, 1)$ b. $-7, -5, -1$ c. $1, 1, -1$ d. none of these

Watch Video Solution

403. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j}; \vec{a} \perp \vec{b}, \vec{a}.\vec{c} = 4.$ Then

a. $\left[\vec{a}\vec{b}\vec{c}\right]^2 = |\vec{a}|$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^= |\vec{a}|$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^= 0$ d. $\left[\vec{a}\vec{b}\vec{c}\right]^= |\vec{a}|^2$

Watch Video Solution

404. $\vec{a} and \vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that is equally inclined to $\vec{a}, \vec{b} and \vec{a} \times \vec{b}$ is a. $\frac{1}{\sqrt{2}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$ b. $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{a} + \vec{b}\right)$ c. $\frac{1}{\sqrt{3}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$ d. $\frac{1}{3}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

Watch Video Solution

405. If $\vec{r} and \vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r} + b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2 + \left|\vec{r} + b\vec{s}\right|^2$ is equal to a. $2\left|\vec{r}\right|^2$ b. $\left|\vec{r}\right|^2/2$ c. $3\left|\vec{r}\right|^2$ d. $|r|^2$

Watch Video Solution

406. The scalar $\vec{A}\left(\vec{B}+\vec{C}\right)\times\left(\vec{A}+\vec{B}+\vec{C}\right)$ equals a.0 b. $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$ c. $\left[\vec{A}\vec{B}\vec{C}\right]$ d. none of these

Watch Video Solution

407. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2, j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is a. $\frac{4}{13}$ b. 4 c. $\frac{2}{7}$ d. 2

Watch Video Solution

408. For non-zero vectors $\vec{a}, \vec{b}, and\vec{c}$, $\left|\left(\vec{a}\times\vec{b}\right)\dot{\vec{c}}\right| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if a. $\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0$ b. $\vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$ c. $\vec{c}\cdot\vec{a}=0, \vec{a}\cdot\vec{b}=0$ d. $\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$

Watch Video Solution

409. For three vectors $\vec{u}, \vec{v} and \vec{w}$ which of the following expressions is not equal to any of the remaining three ? a.$\vec{u}.\left(\vec{v} \times \vec{w}\right)$ b. $\left(\vec{v} \times \vec{w}\right).\vec{u}$ c. $\vec{v}.\left(\vec{u} \times \vec{w}\right)$ d. $\left(\vec{u} \times \vec{v}\right).\vec{w}$

Watch Video Solution

410. Let $\vec{A}$ be a vector parallel to the line of intersection of planes $P_1 and P_2^{\cdot}$ Plane P_1 is parallel to vectors $2\hat{j} + 3\hat{k} and 4\hat{j} - 3k and P_2$ is parallel to $\hat{j} - \hat{k} and 3\hat{i} + 3\hat{j}^{\cdot}$ Then the angle betweenvector $\vec{A}$ and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is a.$\pi/2$ b. $\pi/4$ c. $\pi/6$ d. $3\pi/4$

Watch Video Solution

411. If $\vec{a}.\vec{b} = \beta and \vec{a} \times \vec{b} = \vec{c}, then \vec{b}$ is a.$\frac{\left(\beta\vec{a} - \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$ b. $\frac{\left(\beta\vec{a} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$ c. $\frac{\left(\beta\vec{c} - \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$ d. $\frac{\left(\beta\vec{a} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

Watch Video Solution

412. $\vec{b} and \vec{c}$ are unit vectors. Then for any arbitrary vector $\vec{a}, \left(\left(\left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{c}\right)\right) \times \left(\vec{b} \times \vec{c}\right)\right). \left(\vec{b} - \vec{c}\right)$ is always equal to a.$|\vec{a}|$ b. $\frac{1}{2}|\vec{a}|$ c. $\frac{1}{3}|\vec{a}|$ d. none of these

Watch Video Solution

413. Let $\vec{a} and \vec{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$, a. $\vec{r} = \left(\vec{r}.\hat{a}\right)\hat{a} + \left(\vec{r}.\hat{b}\right)\hat{b} + \left(\vec{r}.\left(\hat{a} \times \hat{b}\right)\right)\left(\hat{a} \times \hat{b}\right)$ b. $\vec{r} = \left(\vec{r}.\hat{a}\right) - \left(\vec{r}.\hat{b}\right)\hat{b} - \left(\vec{r}.\left(\hat{a} \times \hat{b}\right)\right)\left(\hat{a} \times \hat{b}\right)$ c. $\vec{r} = \left(\vec{r}.\hat{a}\right)\hat{a} - \left(\vec{r}.\hat{b}\right)\hat{b} + \left(\vec{r}.\left(\hat{a} \times \hat{b}\right)\right)\left(\hat{a} \times \hat{b}\right)$ d.none of these

Watch Video Solution

414. Value of $\left[\vec{a} \times \vec{b} \vec{a} \times \vec{c} \vec{d}\right]$ is always equal to $\left(\vec{a}.\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$ b. $\left(\vec{a}.\vec{c}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ c. $\left(\vec{a}.\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ d. none of these

Watch Video Solution

415. Let $\vec{a} and \vec{b}$ be unit vectors that are perpendicular to each other. Then $\left[\vec{a} + \left(\vec{a} \times \vec{b}\right)\vec{b} + \left(\vec{a} \times \vec{b}\right)\vec{a} \times \vec{b}\right]$ will always be equal to a.1 b. 0 c. -1 d. none of these

Watch Video Solution

416. Let $\vec{r}, \vec{a}, \vec{b} and \vec{c}$ be four nonzero vectors such that $\vec{r}\dot{\vec{a}} = 0, \left|\vec{r} \times \vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right| and \left|\vec{r} \times \vec{c}\right| = \left|\vec{r}\right|\left|\vec{c}\right|^{\cdot}$ Then $[abc]$ is equal to a. $|a||b||c|$ b. -$|a||b||c|$ c. 0 d. none of these

Watch Video Solution

417. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} and \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three nonzero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a} and \vec{b}^{\cdot}$ If the angle between $\vec{a} and \vec{b}$ is $\pi/6$, then the value of

$\left|a_1b_1c_1a_2b_2c_2a_3b_3c_3\right|$ is a.0 b. 1 c. $\frac{1}{4}\left(a_12 + a22 + a32\right)(b12 + b22 + b32)$ d. $\frac{3}{4}(a12 + a22 + a32)(b12 + b22 + b32)$

Watch Video Solution

418. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0,$ then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to

a. vector perpendicular to the plane ofa, b, c b. a scalar quantity c. $\vec{0}$ d. none of these

Watch Video Solution

419. If $\vec{a}, \vec{b}, and\vec{c}$ are such that $\left[\vec{a}\vec{b}\vec{c}\right] = 1, \vec{c} = \lambda\vec{a} \times \vec{b},$ angle, between $\vec{a}and\vec{b}$ is $\frac{2\pi}{3}, \left|\vec{a}\right| = \sqrt{2}, \left|\vec{b}\right| = \sqrt{3}and\left|\vec{c}\right| = \frac{1}{\sqrt{3}}$, then the angel between $\vec{a}and\vec{b}$ is a.$\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$

Watch Video Solution

420. A vector of magnitude $\sqrt{2}$ coplanar with the vecrtor $\vec{a} = \hat{i} + \hat{j} + 2\hat{k} and \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$, is a.$-\hat{j} + \hat{k}$ b. $\hat{i} - \hat{k}$ c. $\hat{i} - \hat{j}$ d. $\hat{i} - \hat{j}$

Watch Video Solution

421. Let P be a point interior to the acute triangle ABC If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

Watch Video Solution

422. G is the centroid of triangle $ABCandA_1andB_1$ are rthe midpoints of sides $ABandAC$, respectively. If Delta_1 is the area of quadrilateral $GA_1AB_1and\text{Delta}$ is the area of triangle ABC, then $\text{Delta}/\text{Delta}_1$ is equal to a.$\frac{3}{2}$ b. 3 c. $\frac{1}{3}$ d. none of these

Watch Video Solution

423. Points $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = 0.$ Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma is$ a. $\frac{1}{14}$ b. 14 c. 6 d. $1/\sqrt{6}$

Watch Video Solution

424. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 1 and 2, respectively, and $\left(1 - 3\vec{a}\dot{\vec{b}}\right)^2 + \left|2\vec{a} + \vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2 = 47,$ then the angel between $\vec{a} and \vec{b}$ is a.$\pi/3$ b. $\pi - \cos^{-1}(1/4)$ c. $\frac{2\pi}{3}$ d. $\cos^{-1}(1/4)$

Watch Video Solution

425. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 2 and 3, respectively, such that $\left|2\left(\vec{a} \times \vec{b}\right)\right| + \left|3\left(\vec{a}\dot{\vec{b}}\right)\right| = k,$ then the maximum value of k is a.

$\sqrt{13}$ b. $2\sqrt{13}$ c. $6\sqrt{13}$ d. $10\sqrt{13}$

Watch Video Solution

426. $\vec{a}, \vec{b} and \vec{c}$ are unit vectors such that $\left|\vec{a} + \vec{b} + 3\vec{c}\right| = 4$. Angle between $\vec{a} and \vec{b} is \theta_1$, between $\vec{b} and \vec{c}$ is θ_2 and between $\vec{a} and \vec{c}$ varies $[\pi/6, 2\pi/3]$. Then the maximum of $\cos\theta_1 + 3\cos\theta_2 is$ a. 3 b. 4 c. $2\sqrt{2}$ d. 6

Watch Video Solution

427. If the vector product of a constant vector $\vec{OA}$ with a variable vector $\vec{OB}$ in a fixed plane OAB be a constant vector, then the locus of B is a. a straight line perpendicular to $\vec{OA}$ b. a circle with centre O and radius equal to $\left|\vec{OA}\right|$ c. a straight line parallel to $\vec{OA}$ d. none of these

Watch Video Solution

428. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $\left|\vec{u}\right| = 1, \left|\vec{v}\right| = 2 and \left|\vec{w}\right| = 3.$ If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $\left|\vec{u} - \vec{v} + \vec{w}\right|$ equals a. 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

Watch Video Solution

429. If the two adjacent sides of two rectangles are represented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} + \vec{b},$ respectively, then the angel between the vector $\vec{x} = \frac{1}{3}\left(\vec{p} + \vec{r} + \vec{s}\right)$ and $\vec{y} = \frac{1}{5}\left(\vec{r} + \vec{s}\right)$ is a.$\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ b. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ c. $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ d. cannot be evaluate

Watch Video Solution

430. Let P, Q, R and S be the points on the plane with position vectors $-2i - j, 4i, 3i + 3j and - 3j + 2j,$ respectively. The quadrilateral $PQRS$ must be a Parallelogram, which is neither a rhombus nor a rectangle Square Rectangle, but not a square Rhombus, but not a square

Watch Video Solution

431. $\vec{u}, \vec{v} and \vec{w}$ are three non-coplanar unit vecrtors anf $\alpha, \beta and \gamma$ are the angles between $\vec{u} and \vec{v}, \vec{v} and \vec{w}, and \vec{w} and \vec{u},$ respectively, and $\vec{x}, \vec{y} and \vec{z}$ are unit vectors along the bisectors of the angles $\alpha, \beta and \gamma$, respectively. Prove that $\left[\vec{\times} x\vec{y}\vec{y} \times \vec{z}\vec{z} \times \vec{x}\right] = \frac{1}{16}\left[\vec{u}\vec{v}\vec{w}\right]^2 \frac{\sec^2\alpha}{2}\frac{\sec^2\beta}{2}\frac{\sec^2\gamma}{2}\cdot$

Watch Video Solution

432. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(t, 1, 1)$ is minimum.

Watch Video Solution

433. The condition for equations $\vec{r} \times \vec{a} = \vec{b} and \vec{r} \times \vec{c} = \vec{d}$ to be consistent

.

is a.$\vec{b}.\ \vec{c} = \vec{a}.\ \vec{d}$ b. $\vec{a}.\ \vec{b} = \vec{c}.\ \vec{d}$ c. $\vec{b}.\ \vec{c} + \vec{a}.\ \vec{d} = 0$ d. $\vec{a}\vec{b} + \vec{c}.\ \vec{d} = 0$

Watch Video Solution

434. If $aandb$ are nonzero non-collinear vectors, then $\left[\vec{a}\vec{b}\hat{i}\right]\hat{i} + \left[\vec{a}\vec{b}\hat{j}\right]\hat{j} + \left[\vec{a}\vec{b}\hat{k}\right]\hat{k}$ is equal to a.$\vec{a} + \vec{b}$ b. $\vec{a} \times \vec{b}$ c. $\vec{a}$ - $\vec{b}$ d. $\vec{b} \times \vec{a}$

Watch Video Solution

435. If $\vec{r}.\ \vec{a} = \vec{r}.\ \vec{b} = \dot{\vec{r}}\vec{c} = \frac{1}{2}$ or some nonzero vector $\vec{r}$, then the area of the triangle whose vertices are $A\left(\vec{a}\right), B\left(\vec{b}\right) andC\left(\vec{c}\right) is \left(\vec{a}, \vec{b}, \vec{c}\right.$ are non-coplanar) a.$\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$ b. $\left|\vec{r}\right|$ c. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$ d. none of these

Watch Video Solution

436. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be (A) $6\hat{i} + 8\hat{j}$ (B) $-8\hat{i} + 3\hat{j}$ (C) $6\hat{i} - 8\hat{j}$ (D) $8\hat{i} + 6\hat{j}$

Watch Video Solution

437. If $a\left(\vec{\alpha} \times \vec{\beta}\right) + b\left(\vec{\beta} \times \vec{\gamma}\right) + c\left(\vec{\gamma} \times \vec{\alpha}\right) = 0$ and at least one of $a, bandc$ is nonzero, then vectors $\vec{\alpha}, \vec{\beta}and\vec{\gamma}$ are a. parallel b. coplanar c. mutually perpendicular d. none of these

Watch Video Solution

438. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right) = \vec{b}, where\vec{a}, \vec{b}, and\vec{c}$ are nonzero vectors, then (a)$\vec{a}, \vec{b}, and\vec{c}$ can be coplanar (b)$\vec{a}, \vec{b}, and\vec{c}$ must be coplanar (c)$\vec{a}, \vec{b}, and\vec{c}$ cannot be coplanar (d)none of these

Watch Video Solution

439. If $\vec{a}, \vec{b}, \vec{c}$ are any three noncoplanar vector, then the equaltion

$$\left[\vec{b} \times \vec{c}\vec{c} \times \vec{a}\vec{a} \times \vec{b}\right]x^2 + \left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right]x + 1 + \left[\vec{b} - \vec{c}\vec{c} - \vec{a}\vec{a} - \vec{b}\right] = 0$$

has roots a. real and distinct b. real c. equal d. imaginary

Watch Video Solution

440. If $\vec{x} + \vec{c} \times \vec{y} = \vec{a}$ and $\vec{y} + \vec{c} \times \vec{x} = \vec{b}$,where $\vec{c}$ is a nonzero vector, then which of the following is not correct?

a. $\vec{x} = \dfrac{\vec{b} \times \vec{c} + \vec{a} + \left(\vec{c}.\vec{a}\right)\vec{c}}{1 + \vec{c}.\vec{c}}$

b. $\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + \left(\vec{c}.\vec{a}\right)\vec{c}}{1 + \vec{c}.\vec{c}}$

c. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + \left(\vec{c}.\vec{b}\right)\vec{c}}{1 + \vec{c}.\vec{c}}$

d. none of these

Watch Video Solution

441. If $\vec{a} and \vec{b}$ are two unit vectors incline at angle $\pi/3$, then

$\left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\}\dot{\vec{b}}$ is equal to a. $\frac{-3}{4}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$ d. $\frac{1}{2}$

Watch Video Solution

442. If $\vec{a}$ and $\vec{b}$ are orthogonal unit vectors, then for a vector $\vec{r}$ noncoplanar with $\vec{a}$ and $\vec{b}$, vector $r \times a$ is equal to a. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{b} - \left(\vec{r}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)$ b. $\left[\vec{r}\vec{a}\vec{b}\right]\left(\vec{a} + \vec{b}\right)$ c. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{a} - \left(\vec{r}.\vec{a}\right)\vec{a} \times \vec{b}$ d. none of these

Watch Video Solution

443. Find the volume of a parallelopiped whose edges are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

Watch Video Solution

444. find three- dimensional vectors, $\vec{v}1, \vec{v}2$ and $\vec{v}3$ satisfying $\vec{v}_1.\vec{v}_2 = -2, \vec{v}_1.Vecv_3 = 6, \vec{v}_2, \vec{v}_2 = 2\vec{v}_2.Vecv_3 = -5$

Watch Video Solution

445. For any two vectors $\vec{u} and \vec{v}$ prove that $(\vec{u}.\vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2$

Watch Video Solution

446. If the incident ray on a surface is along the unit vector $\vec{v}$, the reflected ray is along the unit vector $\vec{w}$ and the normal is along the unit vector $\vec{a}$ outwards, express $\vec{w}$ in terms of $\vec{a}$ and $\vec{v}$

Watch Video Solution

447. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} and \vec{a} \times \vec{b} = \vec{c} \times \vec{d}$, prove that $(\vec{a} - \vec{d})\vec{b} - \vec{c} \neq 0$,

Watch Video Solution

448. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is a.-1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

Watch Video Solution

449. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and l,m,n are distinct real numbers, then $[(l\vec{a} + m\vec{b} + n\vec{c})(l\vec{b} + m\vec{c} + n\vec{a})(l\vec{c} + m\vec{a} + n\vec{b})]$ = 0, implies (A) lm+mn+nl = 0 (B) l+m+n = 0 (C) $l^2 + m^2 + n^2 = 0$

Watch Video Solution

450. If $\vec{a}, \vec{b} and \vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b} 2\vec{b} - \vec{c} 2\vec{c} - \vec{a}\right]$ is a.0 b. 1 c. $-\sqrt{3}$ d. $\sqrt{3}$

Watch Video Solution