MATHS

BOOKS - KC SINHA ENGLISH

ALGEBRA - JEE MAINS AND ADVANCED QUESTIONS - FOR COMPETITION

Exercise

1. If ω is an imaginary cube root of unity, then $\left(1+\omega-\omega^2\right)^7$ is equal to

128ω (b) -128ω $128\omega^2$ (d) $-128\omega^2$

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2. Let z_1 and z_2 be theroots of the equation $z^2+az+b=0$ z being compex. Further, assume that the origin z_1 and z_2 form an equilatrasl triangle then

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3. Let Z and w be two complex number such that $|zw| = 1$ and $arg(z) - arg(w) = \pi/2$ then

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4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

n $\in$ N.

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5. about to only mathematics

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6. If $z=x-iy$ and $z'^{\frac{1}{3}}=p+iq$, then $\frac{1}{p^2+q^2}\left(\frac{x}{p}+\frac{y}{q}\right)$ is equal to

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7. If $|z^2-1|=|z|^2+1$, then z lies on (a) The Real axis (b)The imaginary axis (c)A circle (d)An ellipse

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8. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3+8=0$ are a. $-1, 1+2\omega, 1+2\omega^2$ b. $-1, 1-2\omega, 1-2\omega^2$ c. $-1, -1, -1$ d. none of these

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9. If $z_1 and z_2$ are two nonzero complex numbers such that = $|z_1+z_2|=|z_1|+|z_2|$, then $arg z_1 - arg z_2$ is equal to $-\pi$ b. $\frac{\pi}{2}$ c. 0 d.

$\frac{\pi}{2}$ e. π

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10. If $w=z/[z-(1/3)i]and|w|=1,$ then find the locus of z.

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11. If $S=\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11}-i\cos\frac{2\pi k}{11}\right)$ then

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12. If $z^2+z+1=0,$ where z is a complex number, the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots+\left(z^6+\frac{1}{z^6}\right)^2$ is

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13. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is (1) 4 (B) 10 (3) 6 (4) 0

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14. If the conjugate of a complex numbers is $\frac{1}{i-1}$, where $i=\sqrt{-1}$. Then, the complex number is

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15. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is (1) greater than 4ab (2) less than 4ab (3) greater than $4ab$ (4) less than $4ab$

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16. If $\left|z-\frac{4}{z}\right|=2$, then the maximum value of $|Z|$ is equal to (1) $\sqrt{3}+1$ (2) $\sqrt{5}+1$ (3) 2 (4) $2+\sqrt{2}$

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17. If α and β are the roots of the equation x^2-$x+1=0$, then $\alpha^{2009}+\beta^{2009}=$ (1) 4 (2) 3 (3) 2 (4) 1

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18. The number of complex numbersd z, such that $|z-1|=|z+1|=|z-i|$, where $i=\sqrt{-1}$ equals to

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19. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line Re $z=1$, then it is necessary that : (1)

$b \in (0, 1)$ (2) $b \in (-1, 0)$ (3) $|b| = 1$ (4) $b \in (1, \infty)$

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20. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

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21. If $z \neq 1$ and $\dfrac{z^2}{z-1}$ is real, then the point represented by the complex number z lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis

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22. If z is complex number of unit modulus and argument θ then arg $\left(\dfrac{1+z}{1+\bar{z}}\right)$ equals

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23. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ (1) is equal to $\frac{5}{2}$ (2) lies in the interval (1, 2) (3) is strictly greater than $\frac{5}{2}$ (4) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

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24. A complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z_2}}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a

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25. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ purely imaginary, is

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26. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$

If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$ then k is equal to

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27. If ω is a non-real complex cube root of unity and $(5 + 3\omega^2 - 5\omega)^{4n+3} + (5\omega + 3 - 5\omega^2)^{4n+3} + (5\omega^2 + 3\omega - 5)^{4n+3} = 0,$ then possible value of n is

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28. Let $z = \dfrac{-1 + \sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is _____.

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29. Let a,b,x and y be real numbers such that a-b =1 and $y \neq 0$. If the complex number $z = x + iy$ satisfies $Im\left(\frac{az+b}{z+1}\right) = y$ then which of the following is (are) possible value (s) of x?

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30. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is :

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31. The number of real roots of $3^2 x^2 - 7x + 7 = 9$ is (A) 0 (B) 2 (C) 1 (D) 4

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32. If the sum of the roots of the quadratic equaion $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then prove that

$\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in HP

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33. If the difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then

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34. If $2a + 3b + 6c = 0$, then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

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35. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ a. is always +ve b. is always-ve c. does not exist d. none of these

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36. Number of real solutions of the equation $x^2 + 3|x| + 2 = 0$ is:

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37. Find the value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other.

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38. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then find its roots.

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39. The one root of the equation $x^2 + px + 12 = 0 is 4,$ while the equation $x^2 + px + q = 0$ has equal roots, the value of q is $49/4$ (b) $4/49$ (c) 4 (d) none of these

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40. Let two humbers have arithmatic mean 9 and geometric mean 4. Then these numbers are roots of the equation :

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41. If $2a+3b+6c=0$, then prove that at least one root of the equation $ax^2+bx+c=0$ lies in the interval (0,1).

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42. In a triangle PQR, $\angle R=\frac{\pi}{2}$.If $\tan\left(\frac{P}{2}\right)$&$\tan\left(\frac{Q}{2}\right)$, are the roots of the equation $ax^2+bx+c=(a\neq 0)$ then

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43. If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is (a)0 (b) 1 (c) 2 (d) none of these

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44. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval.

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45. Find the value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2) - x - a - 1 = 0$ assumes the least value.

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46. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less then 4 lie in the

interval

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47. If the roots of the equation $x^2 + px - q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ then the value of 2-q-p is

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48. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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49. The quadratic equations $x^2 6x + a = 0 and x^2 cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is (1) 1 (2) 4 (3) 3 (4) 2

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50. Let for $a \neq a_1 \neq 0 f(x) = ax^2 + bx + c, g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = (-1)$ and $p(-2) = 2$, the value of $p(2)$ is

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51. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are:

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52. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

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53. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0, a, b, c \in R$, have a common root, then $a : b : c$ is (1) 3 : 2 : 1 (2) 1 : 3 : 2 (3) 3 : 1 : 2 (4) 1 : 2 : 3

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54. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0. If$ p , q, r are in A.P . And $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is

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55. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $\alpha_n = a^n - \beta^n$ for $a \neq 1$, then the values of $\frac{a_{10} - 2a_8}{2a_9}$ is

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56. The sum of all real values of x satisfying the equation

$\left(x^2-5x+5\right)^{x^2+4x-60}=1$ is

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57. Let $a \in R$ and let $f: R^{\rightarrow}$ be given by $f(x)=x^5-5x+a,$ then (a) $f(x)$ has three real roots if $a>4$ (b) $f(x)$ has only one real roots if $a>4$ (c) $f(x)$ has three real roots if $a<-4$ (d) $f(x)$ has three real roots if `-4

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58. The quadratic equation $p(x)=0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x))=0$ has only purely imaginary roots at real roots two real and purely imaginary roots neither real nor purely imaginary roots

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59. Let S be the set of all non-zero real numbers such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots $x_1 and x_2$ satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is (are) a subset (s) of S? $\left(\frac{1}{2}, \frac{1}{\sqrt{5}}\right)$ b. $\left(\frac{1}{\sqrt{5}}, 0\right)$ c. $\left(0, \frac{1}{\sqrt{5}}\right)$ d. $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

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60. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1, are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals:

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61. The least value of $\alpha \in R$ for which $4ax^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

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62. Let p ,q be integers and let α, β be the roots of the equation,

$x^2 - x - 1 = 0$ where $\alpha \neq \beta$ For n = 0,1,2,..., let $a_n = p\alpha^n + q\beta^n$.

Fact : If a and b are rational number and $a + b\sqrt{5} = 0$, then

a = 0 = b .

If $a_4 = 28, thenP + 2p =$

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63. Let $S(k) = 1 + 3 + 5 + \ldots.. + (2k - 1) = 3 + k^2$. Which of the following is true ?

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64. Statement-1: For every natural number $n \geq 2, \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots.. \frac{1}{\sqrt{n}} > \sqrt{n}$

Statement-2: For every natural number $n \geq 2, \sqrt{n(n+1)} < n + 1$

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65. Statement -1 For each natural number $n, (n+1)^7 - n^7 - 1$ is divisible by 7.

Statement -2 For each natural number $n, n^7 - n$ is divisible by 7.

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66. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then

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67. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ Then only correct statement about the matrix A is (A) A is a zero matrix (B) $A^2 = 1$ (C) A^{-1} does not exist (D) $A = (-1)$ I where I is a unit matrix

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68. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$.

If B is the inverse of A, then find the value α.

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69. If $A^2 - A + I = 0$, then the inverse of A is

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70. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction)

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71. If A and B f are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$ which of the following will be always true?

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72. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$ Then,

(a) there exist exactly one B such that $AB = BA$

(b) there exist exactly infinitely many B's such that AB=BA

(c) there cannot exist any B such that $AB = BA$

(d) there exist more than one but finite number of B's such that $AB = BA$

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73. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ $.If|A^2| = 25,$ then α equals to:

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74. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? (1) If $detA = \pm 1, thenA^1$ exists but all its entries are not necessarily integers (2) If $detA \neq \pm 1, thenA^1$ exists and all its entries are non-integers (3) If $detA = \pm 1, thenA^1$ exists and all its entries are integers (4) If $detA = \pm 1, thenA^1$ need not exist

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75. Assertion: If $A \neq I$ and $A \neq -I, then \det A = -1$, Reason: If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) both A and R is false.

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76. Assertion: $adj(adjA) = (\det A)^{n-2}A$ Reason: $|adjA| = |A|^{n-1}$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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77. consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3,$

$3x_1 + 5x_2 + 2x_3 = 1$

the system has

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78. Assertion: $Tr(A) = 0$ Reason: $|A| = 1$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) both A and R is false.

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79. The number of 3 x 3 non-singular matrices, with four entries as 1 and all other entries as 0, is:- (1) 5 (2) 6 (3) at least 7 (4) less than 4

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80. Let A and B two symmetric matrices of order 3.

Statement 1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement 2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

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81. Assertion: Determinant of a skew symmetric matrix of order 3 is zero.

Reason: For any matix A, $\det(A^T) = \det(A)$ and $\det(-S) = -\det(S)$ (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true and R is not the correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

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82. If $\omega = 1$ is the complex cube root of unity and matrix $H = \begin{vmatrix} \omega & 0 \\ 0 & \omega \end{vmatrix}$, then H^{70} is equal to:

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83. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

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84. IF $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then α is equal to :

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85. If A is a 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$ then BB' equals to

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86. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to

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87. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = AA^T$, then $5a + b$ is equal to

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88. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if The first column of M is the transpose of the second row of M The second row of M is the transpose of the first column of M M is a diagonal matrix with non-zero entries in the main diagonal The product of entries in the main diagonal of M is not the square of an integer

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89. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

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90. Let $X \, and \, Y$ be two arbitrary, 3×3 , non-zero, skew-symmetric matrices and Z be an arbitrary 3×3 , non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

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91. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and Q = $\left[q_{ij}\right]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\dfrac{q_{21} + q_{31}}{q_{32}}$ is equal to

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92. Let $p = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = \left[q_{ij}\right]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}, k \neq 0$ and $I$is the identity matrix of order 3. If $q_{23} = -\dfrac{k}{8}$ and $\det(Q) = \dfrac{k^2}{2}$, then

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93. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5 ?

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94. For a real number α , if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations , the infinitely many solutions , then $1 + \alpha + \alpha^2$=

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96. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy,$ then a.$x = 3, y = 1$ b. $x = 1, y = 3$ c. $x = 0, y = 3$ d. $x = 0, y = 0$

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97. If $\omega(\neq 1)$ is a cube root of unity, then value of the determinant

$\left|11+i+\omega^2\omega^21-i-1\omega^2-1-i-i+\omega-1-1\right|$ is 0 b. 1 c. i d. ω

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98. If the system of linear equations

$x+2ay+az=0$
$x+3by+bz=0$
$x+4cy+cz=0$

has a non zero solutions, then a, b, c are in

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99. If ω is a non-real cube root of unity and n is not a multiple of 3, then

$=\left|1\omega^n\omega^{2n}\omega^{2n}1\omega^n\omega^n\omega^{2n}1\right|$ is equal to (a) 0 (b) ω (c) ω^2 (d) 1

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100. if $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ form a G.P. and $a_1 > 0$, for all $I \geq 1$

$$\begin{vmatrix} \log a_n, & \log a_n + \log a_{n+2}, & \log a_{n+2} \\ \log a_{n+3}, & \log a_{n+3} + \log a_{n+5}, & \log a_{n+5} \\ \log a_{n+6}, & \log_{n+6} + \log a_{n+8}, & \log a_{n+8} \end{vmatrix}$$

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101. If $a^2 + b^2 + c^2 = -2 and f(x) =$

$|1 + a^2 x(1 + b^2)x(1 + c^2)x(1 + a^2)x1 + b^2 x(1 + c^2)x(1 + a^2)x(1 + b^2)x$

, then $f(x)$ is a polynomial of degree 0 b. 1 c. 2 d. 3

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102. The system of equations $\alpha x + y + z = \alpha - 1, x + \alpha y + z = \alpha - 1$ and $x + y + \alpha z = \alpha - 1$ has no solution, if α is :

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103. if $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ form a G.P. and $a_1 > 0$, for all $I \geq 1$

$$\begin{vmatrix} \log a_n, & \log a_n + \log a_{n+2}, & \log a_{n+2} \\ \log a_{n+3}, & \log a_{n+3} + \log a_{n+5}, & \log a_{n+5} \\ \log a_{n+6}, & \log_{n+6} + \log a_{n+8}, & \log a_{n+8} \end{vmatrix}$$

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104. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is (1) divisible by neither x nor y (2) divisible by both x and y (3) divisible by x but not y (4) divisible by y but not x

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105. If $x = cy + bz, y = az + cx, z = x + ay, where. x, y, z$ are not all zeros, then find the value of $a^2 + b^2 + c^2 + 2abc\cdot$

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106. Let a,b,c be such that b(a+c) $\neq$ 0. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$$ then the

value of n is

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107. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is

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108. If the trivial solution is the only solution of the system of equations x - ky + z =0 , kx + 3y-kz=0 and 3x+y-z=0 . Then , set of all values of k is :

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109. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

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110. The number of values of k for which the system of equations (k+1)x+8y=4k,kx+(k+3)y=3k-1 has no solution is

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111. if $\alpha, \beta, \neq 0 \quad \text{and} \quad f(n) = \alpha^n + \beta^n$

$$\text{and} \begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$ then k is equal to

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112. The set of all values of λ for which the system of linear equations

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

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113. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for

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114. If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution then S is

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115. The total number of distinct $x \in R$ for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10 \quad \text{is} \quad \_\_\_\_$$

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116. Let T_n denote the number of triangles, which can be formed using the vertices of a regular polygon of n sides. It $T_{n+1} - T - n = 21, the \cap$ equals a.5 b. 7 c. 6 d. 4

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117. A students is to answer 10 out of 13 questions in an examminations such that he must choose at least 4 from the first five questions. Find the numbers of choices available to him.

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118. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by.

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119. For ${}^{n}C_{r}$ + $2{}^{n}C_{r-1}$ + ${}^{n}C_{r-2}$ =

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120. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

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121. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

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122. If the letters of the word SACHIN arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number:

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123. Find the value of : $^{50}C_4 + \sum_{r=1}^{6} . ^{56-r}C_3$.

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124. At an election, a voter may vote for any number of candidates, not greater than number to be elected. There are 10 condidates and 4 are to be selected. If a voter votes for atleast one candidate, then number of ways in which he can vote, is

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125. The set $S = \{1, 2, 3, , 12)$ is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \varphi$. The number of ways to partition S is (1) $\frac{12!}{3!(4!)^3}$ (2) $\frac{12!}{3!(3!)^4}$ (3) $\frac{12!}{(4!)^3}$ (4) $\frac{12!}{(4!)^4}$

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126. In a shop there are five types of ice-creams available. A child buys six ice-creams. Statement -1: The number of different ways the child can buy the six ice-creams is $\hat{} 10C_5$. Statement -2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 As and 4 Bs in a row.

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127. How many different words can be formed by Jumbling the letter in the word MISSISSIPPI iin which no two S's are adjancent?

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128. From 6 different novels and 3 different dictionaries, 4 novels annd 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then, the number of such arrangements is

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129. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn, two balls are taken out at random and then transferred to the other. The number of ways in which this can be done. Is

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130. Statement-1 : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $\hat{\ } 9C_3$. Statement-2 : The number of ways of choosing any 3 places from 9 different places is $\hat{\ } 9C_3$. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. Statement-1 is true, Statement-2 is false. Statement-1 is false, Statement-2 is true.

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131. There are 10 points in a plane , out of these 6 are collinear .if N is number of triangles formed by joining these points , then

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132. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls, is

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133. let T_n be the number of all possible triangels formed by joining vertices of an n-sided regular polygon. Iff $T_{n+1} - T_n = 10$, the value of n is

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134. The number of integers greater than 6000 that can be formed using the digits 3,5,6,7 and 8 without repetition, is

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135. Let A and B be two sets containing four and two elements, respectively. Then, the number of subjects of the set $A \times B$, each having atleast three elements is

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136. If all the words (with or without meanining having five letters, formed usingg the letters of the word SMAL and arranged as in a dictionary, then the position of the word SMALL is

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137. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y has no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies annd 3 men, so that 3 friends of each of X and Y are in this party, is

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138. Let `n_1

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139. Six cards and six envelopes are numbered 1,2,3,4,5,6 and cards are to be placed in envelopes, so that each envelope containns exactly one card and nno card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered. then the number of ways it cann be done, is

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140. r

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141. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the tem. If the team has to include at most one boy, then the number of ways of selecting the team is

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142. Word of length 10 are formed using the letters A,B,C,D,E,F,G,H,I,J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. The, $\frac{y}{9x} =$

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143. The coefficient of $x^5 \in \left(1 + 2x + 3x^2 + \right)^{-3/2} is(|x| < 1)$ 21 b. 25 c. 26 d. none of these

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144. If $|x| < 1,$ then the coefficient of x^n in expansion of $\left(1 + x + x^2 + x^3 + \right)^2$ is a. n b. $n - 1$ c. $n + 2$ d. $n + 1$

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145. The numberof integral terms in the expansion of $\left(\sqrt{3} + \sqrt[8]{5}\right)^{256}$ is (A) 32 (B) 33 (C) 34 (D) 35

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146. If x is positive, the first negative term in the expansion of $(1+x)^{27/5} is (|x|<1)$ $5th term$ b. $8th term$ c. $6th term$ d. $7th term$

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147. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals :

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148. The coefficient of x^n in the expansion of $(1-x)(1-x)^n$ is $n-1$ b. $(-1)^n(1-n)$ c. $(-1)^{n-1}(n-1)^2$ d. $(-1)^{n-1}n$

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149. If the coefficient of rth, $(r+1)^{th}$, and $(r+2)th$ terms in the binamial expansion of $(1+y)^m$ are in A.P. then prove that

$m^2 - m(4r+1) + 4r^2 - 2 = 0.$

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150. If the coefficient of $x^7 \in \left[ax^2 - \left(\frac{1}{bx^2}\right)\right]^{11}$ equal the coefficient of x^{-7} in satisfy the $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, $thenaandb$ satisfy the relation $a + b = 1$ b. $a - b = 1$ c. $b = 1$ d. $\frac{a}{b} = 1$

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151. If x is so small that x^3 and higher powers of x may be neglectd, then $\frac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as a. $3x + \frac{3}{8}x^2$ b. $1 - \frac{3}{8}x^2$ c. $\frac{x}{2} - \frac{3}{\times^2}$ d. $-\frac{3}{8}x^2$

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152. If the expansion in powers of x of the function $1/[(1-ax)(1-bx)]$ is a$a_0 + a_1x + a_2x^2 + a_3x^3 +, then a_n is$ a.$\dfrac{b^n - a^n}{b-a}$ b. $\dfrac{a^n - b^n}{b-a}$ c. $\dfrac{b^{n+1} - a^{n+1}}{b-a}$ d. $\dfrac{a^{n+1} - b^{n+1}}{b-a}$

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153. For natural numbers $m, n,$ if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots,$ and $a_1 = a_2 = 10, t$

a. $m < n$ b. $m > n$ c. $m + n = 80$ d. $m - n = 20$

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154. In the binomial expansion of $(a-b)^{\cap} \geq 5,$ the sum of the 5th and 6th term is zero. Then a/b equals $(n-5)/6$ b. $(n-4)/5$ c. $n/(n-4)$ d. $6/(n-5)$

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155. Find the sum

$\hat{} \ 20C_{10}.^{15}C_0 +^{20}C_9.^{15}C_1 +^{20}C_8.^{15}C_2 + ... +^{20}C_0.^{15}C_{10}$

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156. Statement 1: $\sum_{r=0}^{n}(r+1)^n c_r = (n+2)2^{n-1}$.

Statement 2: $\sum_{r=0}^{n}(r+1)^n c_r = (1+x)^n + nx(1+x)^{n-1}$.

(1) Statement 1 is false, Statement 2 is true.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

(3) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(4) Statement 1 is true, Statement 2 is false.

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157. The reamainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

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158. Find the coefficient of x^7 in the expansion of $\left(1-x-x^2+x^3\right)^6$.

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159. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers

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160. The term independent of x in expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$ is (1) 120 (2) 210 (3) 310 (4) 4

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161. If the coefficient of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in power of x are both zero, then (a, b) is equal to

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162. The sum of coefficient of integral powers of x in the binomial expansion of $(1 - 2\sqrt{x})^{50}$ is

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163. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)x \neq 0$, is 28, then the sum of coefficient of all the terms in this expansion, is

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164. Coefficient of x^{11} in the expansion of $(1+x^2)(1+x^3)^7(1+x^4)^{12}$ is 1051 b. 1106 c. 1113 d. 1120

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165. The coefficient of x^9 in the expansion of $(1+x)(16x^2)(1+x^3)(1+x^{100})$ is

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166. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+\ldots\ldots+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1).{}^{51}C_3$ for some positive integer n, then the value of n is ______.

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167. If 1,$\log_9\left(3^{1-x}+2\right)$ and $\log_3(4.3^x-1)$ are A.P. then x is

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168. The product $2^{\frac{1}{2}}.\,4^{\frac{1}{8}}.\,8^{\frac{1}{16}}\ldots\text{to}\infty$ equal to

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169. The sum of series

$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+\ldots\ldots$ is

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170. Let T_r be the rth term of an A.P., for $r=1,2,3,$ If for some positive integers $m,n,$ we have $T_m=\frac{1}{n}andT_n=\frac{1}{m}, thenT_{mn}$ equals $\frac{1}{mn}$ b. $\frac{1}{m}+\frac{1}{n}$ c. 1 d. 0

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171. The sum of series $\frac{1}{2}! + \frac{1}{4}! + 16! + \dots\dots\dots$ is (A) $\frac{e^2-1}{2}$ (B) $\frac{e^2-2}{e}$ (C) $\frac{e^2-1}{2e}$ (D) $\Big)(e-1)^2\frac{)}{2e}$

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172. The sum of the first n terms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 \dots is \frac{n(n+1)^2}{2}$ when n is even .Then find the sum when n is odd.

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173. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n, wherera, b, andc$ are in A.P. and $|a| < , |b| < 1, and|c| < 1,$ then prove that $x, yandz$ are in H.P.

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174. Let $x_1, x_2, \ldots.., x_n$ be n observation such that $\sum (x_i)^2 = 400$ and $\sum x_i = 40$, then a possible value of n among the following is

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175. The sum of the series $1 + \frac{1}{4.2}!\frac{1}{16.4}! + \frac{1}{64.6}! + \ldots\ldots\ldots \to \infty$ is (A) $\frac{e+1}{2\sqrt{e}}$ (B) $\frac{e-1}{\sqrt{e}}$ (C) $\frac{e-1}{2\sqrt{e}}$ (D) $\frac{e+1}{2}\sqrt{e}$

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176. If $a_1, a_2, a_3,$ be terms of an A.P. and $\frac{a_1 + a_2 + \ldots. + a_p}{a_1 + a_2 + \ldots. + a_q} = \frac{p^2}{q^2}, p \neq q, then \frac{a_6}{a_{21}}$ equals to (a).41/11 (b). 7/2 (c). 2/7 (d). 11/41

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177. If $a_1, a_2, a_3, ...a_n$ are in HP, then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to

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178. If in a geometric progression consisting of positive terms, each term equals the sum of the next two terms, then the common ratio of this progression equals

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179. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is (1) 2 (2) 1/2 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

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180. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$ upto infinity is (1) e^{-2} (2) e^{-1} (3) $e^{-1/2}$ (4) $e^{1/2}$

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181. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is (1) 4 (2) 12 (3) 12 (4) 4

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182. The sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{14}{3^4} + \dots is$

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183. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts is the n^{th} minute .If $a_1 = a_2$ =...... $= a_{10}$= 150 and $a_{10}, a_{11}. . ,$ are in A.P with common difference -2, then the time to count all notes

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184. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after how many months

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185. Let a_n be the nth term of an AP, if $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the AP is

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186. Statement 1: The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \ldots\ldots$ $+ (361 + 380 + 400) is 8000$. Statement 2: $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$ for any natural number n. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

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187. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is (1) 150 (2) 150 times its 50^{th} term (3) 150 (4) zero

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188. The sum of first 20 terms of the sequence 0.7 ,0.77 , 0.777 ...., is

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189. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is $2-\sqrt{3}$ b. $2+\sqrt{3}$ c. $\sqrt{3}-2$ d. $3+\sqrt{2}$

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190. If $(10)^{9}+2(11)^{2}(10)^{7}+\ldots.+10(11)^{9}=k(10)^{9}$

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191. The sum of first 9 terms of the series $\frac{1^{3}}{1}+\frac{1^{3}+2^{3}}{1+3}+\frac{1^{3}+2^{3}+3^{3}}{1+3+5}+\ldots\ldots..$ is

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192. If m is the A.M of two distict real numbers l and n $(l, n > 1)$and G_1, G_2 and G_3 are three geomatric means between l and n, then $(G_1)^4 + 2(G_2)^4 + (G_3)^4$ equals

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193. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots\dots\dots$ is $\frac{16}{5}m$, then m equal to

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194. For any three positive real numbers a, b ans c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then

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195. Let $a, b, c \in R. If f(x) = ax^2 + bx + c$ is such that a +b+c =3 and $f(x+y) = f(x) + f(y) + xy, \forall x, y \in R, then \sum_{n=1}^{10} f(n)$ is equal to

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196. If, for a positive integer n, the quadratic equation, $x(x+1) + (x-1)(x+2) + \ + (x+n-1)(x+n) = 10n$ has two consecutive integral solutions, then n is equal to : $(1)10$ (2) 11 (3) 12 (4) 9

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197. about to only mathematics

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198. about to only mathematics

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199. Let $b_i > 1$ for $i = 1, 2. \dots, 101$ .Suppose loge b_1 loge b_2 ....., loge b_{101} are in arihtmetic progression (A.P) with the common difference $\log_e$ 2. Suppose $a_1, a_2, \dots, a_{101}$ are in A.P such that $a_1 = b_1$ and $a_{51}. If t = b_1 + b_2 + \dots. + b_{51}$ and $s = a_1 + a_2 + \dots + a_{51}$ then

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200. The sides of a right angled triangle are in arithmetic progression .If the triangle has aera 24, then what is the length of its smallest side ?

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